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Explain 'conjectural variation' in Cournot duopoly, evaluate its impacts and discuss the policy implication

To fully understand the impacts and policy implications of conjectural variation, one must first understand the part it played in the great indeterminacy debate within marginal economics during the first half of the 20th century. Therefore, this paper shall begin with a short description and history of conjectural variation, followed by how the debates surrounding it led to the final resolution of the indeterminacy issue. The policy implications of that resolution for imperfect markets are then briefly discussed.

Definition & History

At the beginning of the 20th century, political economics, as it was still known then, was wrestling with the indeterminacy issue i.e.; whether an economy tended towards an equilibrium or not[1]. The prevailing mood of the time felt a strong need that an economy *should* be determinate as it was not understood then how otherwise an economy should not fall apart[2].

The problem was that empirical evidence strongly suggested indeterminacy, as did some of the most notable Economists of the time. F. Y. Edgeworth (1897) was considered to have given the authoritative judgement on the indeterminacy of Cournot duopoly by showing that the imposition of a quantity constraint caused the market price to oscillate inside an interval[3].

A. L. Bowley, in his 1924 *Mathematical Groundwork*, explicitly introduced the firms' conjectures so one could experiment mathematically with how firms respond. A conjectural variation (CV) is a

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[1] This realisation is relatively recent – the Economists at the time probably wouldn't have expressed their debate this way mainly as it took the Great Depression to sharpen up the direction of research. Giocoli (2003, 2005) gives an excellent history from this perspective.

[2] The reasoning was simple: if the equations governing an economy were convergent, stability would arise. If divergent, according to 19th century mathematics, the economy should tear itself apart with runaway feedback loops. As the market economy had not collapsed in two centuries or so, the reasoning followed that it **must** be determinate. See the Hicks quote overleaf for an example.

[3] Edgeworth's 1897 paper originated "the hog's back", better known today as a saddle point, whereby a system if disturbed from equilibrium will tend to diverge from that equilibrium rather than converge toward it.

conjecture by one firm in a duopoly about how the other firm will adjust its action with respect to potential adjustments in the first firm's action. Expressed mathematically, the parameterised first order conditions (FOCs) for this are[4]:

$$\text{For i, j} = 1, 2: \partial\pi_i(y_i, y_j)/\partial y_i = p(Y) + p'(Y)[1 + v_{ij}]y_i - c'_i(y_i) = 0$$

Thus depending on the values of v_{12} and v_{21}, one can yield the FOCs for various competitive models: when $v_{12} = v_{21} = 0$, one yields the Cournot quantity model whereby each firm believes the other firm's choice is independent from its own; when $v_{12} = v_{21} = -1$, one yields the perfectly competitive model since the FOCs reduce to price being equal to marginal cost; when $v_{12} = y_2/y_1$ and $v_{21} = y_1/y_2$ one yields the FOCs of joint profit maximisation; and lastly when $v_{ij} = f'_j(y_i)$, the slope of the reaction curve of the other firm, one yields the Stackelberg model where the first firm chooses its output on the basis of how it conjectures the other firm will respond[5].

Bowley did note at the time some of the special cases which made the outcome determinate, though he himself felt that the only realistic determinate solution was collusion. This, along with A. C. Pigou's endorsement, interested a great number of Economists as perhaps in CV theory was a solution to the indeterminacy Edgeworth had found.

The *consistency* restriction, proposed by Harrod (1934)[6], adds the requirement that firm *i*'s conjecture as to how firm *j* will react to a variation of *i*'s output is correct, such that:

$$\text{For i, j} = 1,2: y_i = f_i(y_j) \text{ and } v_{ij}(y_i) = \partial f_j(y_i)/\partial y_i$$

This leads to the determinate outcome of a *consistent conjectural equilibrium* (CCE), but this did not solve other problems, namely that of dynamic inconsistency.

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[4] Derived from Varian's *Microeconomic Analysis*, ch. 16.

[5] Note that the Stackelberg model was only published in 1933 – hence Bowley did not include it in his conjectural proposal. This was done by Leontief in his 1936 review of Stackelberg's *Marktform und Gleichewicht*.

[6] Harrod was actually arguing *against* determinacy and proposed the consistency condition to show how unrealistic it is.

The 1940's saw considerable criticism, beginning with Stigler (1940), who anticipated the Nash equilibrium of collusive monopoly when the game was infinitely long. More importantly though, Kahn (1937) and Fellner (1949) finally realised that the dynamic inconsistency of the CV model was its undoing, and thus paved the way for a modern game-theoretic approach. The problem, as Fellner clearly showed, was that the response functions, being static, were destroyed as soon as one deviated from the static equilibrium of the intersection of the response functions. As Fellner put it, they were the right response functions "for the wrong reason".

Harrod was a believer in indeterminacy, and this quote by John Hicks neatly summarises the stakes:

> [Harrod] welcomes the instability of his system, because he believes it to be an explanation of the tendency to fluctuation which exists in the real world … But mathematical instability does not in itself elucidate fluctuation. A mathematically unstable system does not fluctuate; it just breaks down. The unstable position is one in which it will not tend to remain. (Hicks, 1949)

While CV theory was obsolete, the contributions it made to the indeterminacy debate were far reaching.

The Impact on Neo-Classical Economics

While the one-shot simultaneous game nature of the CV model had been abandoned in favour of a game-theoretic approach, the consistency condition was still required to maintain determinacy. As Giocoli (2005) details, much of the debate about CV in the 1920-1940's concerned what conditions are required, whether generality (or determinacy) should be chosen over realism (or indeterminacy), how much weight should be given to which factors etc. This debate can be seen as part of the larger long-term debate over marginal economics which was becoming dominant since the 19th century. Repeatedly, debate returned to how to incorporate the future, most specifically *uncertainty* when dealt with by rationally modelled humans and firms[7].

This led to the obvious reverse question: how *few* conditions are minimally required to make Neo-Classical Economics, specifically Walrasian general equilibrium, viable? Arrow (1951) and Debreu

[7] Sadly ignoring Veblen's 1904 critique of how marginal economics assumed the nature of rationality and uncertainty. Galbraith later quoted Veblen's work as the most influential on his own.

(1959) took on the challenge of finding out how few conditions are required on uncertainty for Walrasian general equilibrium to be possible[8]. Surprisingly, the restrictions were onerous:

1. Each person must have perfect information about their environment, including all its future possible states though not necessarily which of those states might emerge.
2. Everyone must have identical and correct beliefs regarding the prices in each potential state at every point into the future.

Radner (1968) reduced condition two to a more realistic:

2. Each person may have differing beliefs about the future, but they also must have infinite computational power.

These conditions looked more like a description of God than a real man[9]. Things got worse in the 1970's with the Sonnenshein-Mantel-Debreu (SMD) conditions of which the most important is that there can only be one representative agent and one representative commodity aggregated to create market demand i.e.; every person must be identical, and they all must consume the same thing. This representative agent must still have perfect knowledge of their environment, both past, presently and all possible futures. Failing to observe the SMD conditions causes multiple equilibria to appear[10], each satisfying Pareto optimality, but at the same time breaking social utility (welfare) optimisation.

The final nail was hammered into the coffin by Newbury & Stiglitz (1982) which proved that in an uncertain world, a competitive equilibrium can **not** be a Pareto optimum due to the effects of the risk of price volatility even when handled by rational agents. This work led to Stiglitz's work on asymmetric information which ultimately earned him the Nobel Prize in Economics in 2001.

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[8] Arrow received the Nobel prize in Economics in 1972 for this work. Debreu received it in 1983 for similar work. Both were strong believers in determinacy, though Debreu wound down his academic work in Economics after his discoveries.

[9] Oddly enough, these conditions for handling uncertainty might have seemed obvious to a more theistic man like Adam Smith than is common today.

[10] The proof of this and implications thereof is lengthy, and occupies a reasonable slice of Mas-Colell's (1995) *Microeconomic Theory*. Start at section 17.

The Impact on Economics

As Mas-Colell's (1995) *Microeconomic Theory* says in Section 20F (Equilibrium and Time), indeterminacy from relaxing the SMD conditions really means *chaotic* behaviour i.e.; that the results are "structured randomness". Mas-Colell like most Neo-Classical Economists calls such results "pathological" and certainly within the axioms of Neo-Classical Economics (where enlightened rational self-interest is supposed to always increase social welfare) they are. However, the Economic value of working with chaotic behaviour was reported as early as in Hotelling (1929).

What the discovery of fundamental indeterminacy due to inclusion of the future definitely did was to cause a schism in Economics. A number of Nobel laureates including Friedman, Stiglitz, Coase, North, Leontief and Solow muttered misgivings quietly[11], but it took until the year 2000 for a unified revolutionary group to emerge in Economics (The Post-Autistic ECOnomics Network – PAECON) who start with the assumption of fundamentally uncertain, non-linear behaviour[12]. Time will tell if this challenge to Neo-Classical orthodoxy will do better than Sraffan or Marxist critiques.

[11] According to the PAECON website at http://www.paecon.net/. 'Quietly' varied between Friedman's view that the monetarist ideology was more important than economics anyway to Stiglitz's public denunciation of IMF polices which eventually meant he had to move from the US to the UK. None stuck to the Neo-Classical Economics when proscribing advice.

[12] Ormerod (1998) is an accessible account of early post-autistic economics.

Conclusion

Having seen the impacts that the debate over conjectural variation played in the great indeterminacy debate, we can state the policy implications for duopolies. From just the theory alone:

1. If the firms are not identical in every way apart from size, or that conjectures are not consistent, multiple equilibria exist which may not maximise social welfare. These multiple equilibria mean that conjectures could be absolutely correct, but which equilibrium is chosen is chaotic and even worse, such an equilibrium is not stable (though that does not imply collapse[13]). This does tally with empirical evidence.

2. If any realistic level of uncertainty is permitted, deadweight loss will not be eliminated by even perfect competition. This has particularly disturbing implications as it gives no conclusive reason why monopolies should be prevented, leading to the horrible idea that the past break-up of monopolies was a bad idea[14]. The Sraffan idea of the ideal market structure being that of joint profit maximisation yielding at least some consumer surplus may be the best thing we can hope for.

[13] For much of the 20th century (since Poincaré) it has been known in maths that chaotic equations can be quasi-stable – in fact in Poincaré's case, he discovered that gravity is fundamentally chaotic and yet our planets aren't collapsing around us! Ormerod (1998) and Keen (2000) illustrate many examples and Ormerod in particular builds economic models from them.

[14] Indeed the damming empirical evidence against breaking up certain types of monopolies was given in my EC2001 essay last year which is available at http://www.nedprod.com/NeoCapitalism/.

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