MPRA
Munich Personal RePEc Archive

The impossibility of effective enforcement mechanisms in collateralized credit markets

Ferreira, Thiago Revil T. and Torres-Martínez, Juan Pablo

March 2009

Online at https://mpra.ub.uni-muenchen.de/13781/
MPRA Paper No. 13781, posted 05 Mar 2009 10:09 UTC

THE IMPOSSIBILITY OF EFFECTIVE ENFORCEMENT MECHANISMS IN COLLATERALIZED CREDIT MARKETS

THIAGO REVIL T. FERREIRA AND JUAN PABLO TORRES-MARTÍNEZ

Abstract. We analyze the possibility of the simultaneous presence of three key features in price-taking credit markets: infinity horizon, collateralized credit operations and effective additional enforcement mechanisms, i.e. those implying payments besides the value of the collateral guarantees.

We show that these additional mechanisms, instead of strengthening, actually weaken the restrictions that collateral places on borrowing. In fact, when collateral requirements are not large enough in relation to the effectiveness of the additional mechanisms, lenders anticipate total payments exceeding the value of the collateral requirements. Thus, by non-arbitrage, they lend more than the value of these guarantees. In turn, in the absence of other market frictions such as borrowing constraints, agents may indefinitely postpone their debts, implying the collapse of the agent's maximization problem and of such credit markets.

Keywords. Effective default enforcements, Collateral guarantees, Individual's optimality.
JEL classification: D50; D52.

1. Introduction

In modern financial markets, collateral guarantees play an important role in enforcing borrowers not to entirely default on their financial promises. These guarantees are used in several credit operations, from corporate bonds to Collateralized Mortgages Obligations,[1] allowing markets to reduce credit risk and increase portfolio diversification. However, to protect investors from the excess of losses induced by large negative shocks in the value of collateral guarantees, financial markets may create and implement additional enforcement mechanisms against default. In this paper, we focus on the theoretical effects of such a policy on the agent's maximization problem and, consequently, on the price-taking credit market.

Date: February 21, 2009.

We are indebted with Filipe Martins-da-Rocha for suggestions on previous versions of this paper. We thank Marcelo Abreu, Helen Harris, Carlos Hervés-Beloso, Paulo Klinger Monteiro, and Emma Moreno-García for useful comments. T. Ferreira acknowledges support from CNPq-Brasil. J.P.Torres-Martínez receives a CNPq research grant through project 307554/2004-0.

[1]That is, derivative assets secured by pools of individual mortgages, each of which is backed mostly by real estates.

In the infinite horizon context, the incentives provided by the collateralization of financial contracts are mostly addressed when the only enforcement mechanism against default is the seizure of the associated physical collateral guarantees. In fact, for incomplete market economies, Araujo, Páscoa and Torres-Martínez (2002) prove the existence of equilibrium independent of the choice of collateral guarantees. One important consequence is that such simple financial structure keeps the credit market from collapsing. Essentially, since agents default only when the value of the collateral requirements is smaller than that of the respective financial promise, the net payoff of lending is always less than or equal to the one associated to holding the amount of the required collateral. Therefore, by the absence of arbitrage, the value of any loan has to be less than the value of the respective collateral, precluding agents to become leveraged and eliminating Ponzi schemes.

However, Páscoa and Seghir (2007) have shown that the results above may not hold when linear utility penalties for default act as an additional enforcement mechanism besides the seizure of collateral guarantees. They provide examples of deterministic economies where sufficiently harsh penalties induce agents to pay more than the depreciated collateral, which, by non-arbitrage, may lead the value of collateral requirements to be persistently lower than that of the loan. In such a context, given any budget feasible plan, agents may improve their utilities by taking new loans, constituting the associated collateral guarantees and increasing their consumption. Therefore, there is no individual's optimal plan and we cannot define a credit market.

In this paper, we analyze how the interaction between the infinite horizon, collateral guarantees and generic additional enforcement mechanisms can extend the result from Páscoa and Seghir's particular examples to more general economies. Doing so, we identify the effectiveness of these additional mechanisms as an important economic concept responsible for the result. Additionally, we argue that two new features of our approach enable us to reach further conclusions than the ones already presented in the literature. First, instead of using an general equilibrium framework, we only analyze the decision problem of one agent. Second, we work with a reduced form approach to model the inclusion of additional enforcement mechanisms against default.

Focusing on the maximization problem of a price taker agent in an economy analogous to that studied by Araujo, Páscoa and Torres-Martínez (2002), we introduce *effective* additional enforcement mechanisms, i.e. mechanisms enforcing payments besides the value of the collateral guarantees. We represent these additional mechanisms by their effectiveness on enforcing payments besides the value of the collateral requirements. Thus, we do not intend to explicitly model how the market imposes additional payments on borrowers besides the value of collateral guarantees. However, with this reduced form approach, we can concentrate on the pricing and market effects of these additional mechanisms. In fact, we derive an explicit relationship between the primitives of the economy, such as the effectiveness and collateral requirements, implying the collapse of the agent's maximization problem.

Essentially, we only need one agent to reach conclusions about asset pricing in competitive credit markets. Additionally, with the reduced form approach for the additional mechanisms against default such pricing becomes well tractable. Then, as we include enforcement mechanisms in addition to the seizure of collateral requirements, lenders may expect sufficiently large payments for their loans besides the value of the these requirements. In such situation, these additional mechanisms, instead of strengthening, actually weaken the restrictions that collateral places on borrowing. In fact, lenders anticipate that, even in case of default, they still receive more than just the value of the collateral guarantees. Thus, by non-arbitrage, they lend more than the value of these guarantees. On the other end, borrowers have the incentive and the possibility to take new credits in order to pay their older ones, since there is no debt constraints or monitoring precluding agents to incur in a Ponzi scheme. Their behavior, then, leads to the non-existence of a physical feasible solution for the agent's problem and to the collapse of the credit market.

Regarding the relationship found between the primitives of the economy, we may view it from two different perspectives. From the first one, given a level of effectiveness of the additional mechanisms, we show that there are strictly positive upper bounds for collateral requirements under which agents have incentives to indefinitely postpone their debts through new credits, leading to the non-existence of an optimal utility maximizing plan. Therefore, the market choice of collateral guarantees becomes relevant. From the second one, we provide theoretical foundations to the examples given by Páscoa and Seghir (2007). That is, given collateral requirements, we show that any sufficiently effective additional enforcement mechanism implies the non-existence of physical feasible individuals' optimal plans. Hence, it is the effectiveness of these mechanisms that brings the main result, not any mechanism *per se*.

The remainder of the paper is organized as follows: Section 2 presents an infinite horizon economy with assets subject to default and with effective enforcement mechanisms in addition to collateral repossession. In Section 3 we show our main result. Some extensions are discussed in Sections 4.

2. Model

Consider a discrete-time infinite-horizon economy with uncertainty and symmetric information. Let S be the set of states of nature and $\mathbb{F}_t$ the information available at period $t \in T := \mathbb{N} \cup \{0\}$. $\mathbb{F}_t$ is a partition of S, and if $t' > t$, make $\mathbb{F}_{t'}$ finer than $\mathbb{F}_t$. Summarizing the uncertainty structure, define an event-tree as $D = \{(t,\sigma) \in T \times 2^S : t \in T, \sigma \in \mathbb{F}_t\}$, where a pair $\xi := (t,\sigma) \in D$ is called a node and $t(\xi) := t$ is the associated period of time. For simplicity, at $t = 0$ there is no information, that is $\mathbb{F}_0 := \{S\}$ and, therefore, there is only one node, which is denoted by ξ_0.

A node $\xi' = (t', \psi')$ is a successor of $\xi = (t, \psi)$, denoted by $\xi' \geq \xi$, if $t' \geq t$ and $\psi' \subseteq \psi$. Given $\xi \in D$, the set of its successors is given by the subtree $D(\xi) := \{\mu \in D : \mu \geq \xi\}$. Also, for each $\xi \neq \xi_0$, since $\mathbb{F}_{t(\xi)}$ is finer than $\mathbb{F}_{t(\xi)-1}$, there is only one predecessor $\xi^- \in D$. We define ξ' as an immediate successor of ξ when it is in the set $\xi^+ := \{\xi' \in D : \xi' \geq \xi, t(\xi') = t(\xi) + 1\}$.

At each node ξ in the event-tree D there is a non-empty and finite set of commodities, L. These commodities may be traded in a competitive market at unitary prices $p_\xi = (p_{(\xi,l)})_{l \in L} \in \mathbb{R}^L_+$ by a non-empty set of consumers. Also, at any node $\xi > \xi_0$, there is a technology represented by a matrix with non-negative entries, $Y_\xi := ((Y_\xi)_{l,l'}; (l, l') \in L \times L)$, which transform commodity bundles consumed at ξ^-, and allows for durable commodities. Thus, for each $(l, l') \in L \times L$, $(Y_\xi)_{l,l'}$ is the amount of commodity l obtained at ξ if one unit of commodity l' is consumed at ξ^-. Also, let $W_\xi \in \mathbb{R}^L_+$ be the aggregate physical resources up to node ξ, while $W = (W_\xi)_{\xi \in D}$ is the plan of such resources.

There is a finite set of real assets $J(\xi)$ at each node $\xi \in D$. Each j in $J(\xi)$ is short-lived, has promises $A_{(\mu,j)} \in \mathbb{R}^L_{++} \cup \{0\}$ at $\mu \in \xi^+$, and is traded in competitive markets by a unitary price $q_{(\xi,j)} \in \mathbb{R}_+$. Note that, when financial promises are non-trivial, its market value take into account all the commodities prices. This assumption may be intuitively understood as an indexation for asset payments using as a price index a referential bundle that may vary with the uncertainty of the economy. Thus, independently of prices, when at least a percentage of original promises is honored by borrowers, lenders maintain a minimal purchase power for every commodity.

Since assets are subject to credit risk, borrowers are burdened to constitute physical collateral guarantees in order to limit lenders' losses. Particularly, for every unit of an asset $j \in J(\xi)$ sold, borrowers must establish—and may consume—a bundle $C_{(\xi,j)} \in \mathbb{R}^L_+ \setminus \{0\}$ that is seized by the market in case of default. For the sake of notation, let $J(D) := \{(\xi, j) \in D \times \cup_{\mu \in D} J(\mu) : j \in J(\xi)\}$ and $J^+(D) := \{(\mu, j) \in D \times \cup_{\eta \in D} J(\eta) : (\mu^-, j) \in J(D)\}$.

Furthermore, additional default enforcement mechanisms may exist. For each unit of asset $j \in J(\xi)$ sold, we let financial markets recover amounts of payments $(F_{(\mu,j)}(p_\mu))_{\mu \in \xi^+}$ that may be higher than the value of depreciated collateral guarantees in case of default. We allow generality in the type of additional enforcement mechanisms assuming that borrowers pay, and lenders expect to receive, a fixed percentage of the remaining debt, $\lambda_{(\mu,j)} \in [0, 1]$. More formally, for every unit of asset $j \in J(\xi)$, each borrower pays at each $\mu \in \xi^+$ an amount

$$F_{(\mu,j)}(p_\mu) := \min\{p_\mu A_{(\mu,j)}, p_\mu Y_\mu C_{(\xi,j)}\} + \lambda_{(\mu,j)} [p_\mu A_{(\mu,j)} - p_\mu Y_\mu C_{(\xi,j)}]^+,$$

where $\lambda_{(\mu,j)} \in [0, 1]$ is the *effectiveness* of additional enforcement mechanisms on asset j at node μ, and, for any $z \in \mathbb{R}$, $[z]^+ := \max\{z, 0\}$.

Our approach allows us to include in our analysis economic (i.e. those induced by legal contracts) and non-economic (e.g. moral sanctions, loss of reputations) default enforcement mechanisms, provided that these mechanisms may be summarized by a family of parameters of effectiveness, $(\lambda_{(\mu,j)})_{(\mu,j)\in J^+(D)}$. However, this last requirement do not induce loss of generality, since traders perfect foresee asset payments. In fact, we can always normalize financial payments as done above. Also, with this approach, it is possible to focus on the consequences of the effectiveness of such mechanisms on the individual's decision.

Definition. *Given $(\mu,j)\in J^+(D)$, additional enforcement mechanisms are effective on asset j at a node μ when $\lambda_{(\mu,j)}A_{(\mu,j)}$ is a non-zero vector. Additional enforcement mechanisms are persistently effective in a subtree $D(\xi)$, if for any $\mu>\xi$, there is $j\in J(\mu^-)$ on which additional enforcement mechanisms are effective at μ.*

Definitions above not only depend on parameters $(\lambda_{(\mu,j)})_{(\mu,j)\in J^+(D)}$, but also on the non-triviality of the original promises. Thus, *effective additional enforcement mechanisms* means that, in the case of default, a strictly positive amount of resources is seized besides the depreciated collateral value.

In contrast to any equilibrium model, we focus in the non-existence of a physically feasible solution for the individual's problem. For these reason, it is sufficient to study a decision model where there is an infinitely lived agent, namely i, who perfectly foresees both market prices and the effectiveness of additional enforcement mechanisms.

Agent i has physical endowments $(w^i_\xi)_{\xi\in D}\in\mathbb{R}^{D\times L}_+$ and preferences represented by a utility function $U^i:\mathbb{R}^{D\times L}_+\to\mathbb{R}_+\cup\{+\infty\}$. As commodities may be durable, we denote by W^i_ξ the cumulated endowments of agent i up to node ξ, which are recursively defined by: $W^i_\xi=w^i_\xi+Y_\xi W^i_{\xi^-}$, when $\xi>\xi_0$, and $W^i_{\xi_0}=w^i_{\xi_0}$, otherwise. Also, we assume that, for any $\xi\in D$, $W^i_\xi\le W_\xi$.

Let $x_\xi\in\mathbb{R}^L_+$ be a bundle of autonomous consumption at node ξ (i.e. non-collateralized commodities). Also, define $\theta_{(\xi,j)}$ and $\varphi_{(\xi,j)}$ as the quantities of asset $j\in J(\xi)$ purchased and sold at the same node. Given $(p,q)\in\Pi:=\mathbb{R}^{D\times L}_+\times\mathbb{R}^{J(D)}_+$, a plan

$$(x,\theta,\varphi):=\big((x_\xi,\theta_{(\xi,j)},\varphi_{(\xi,j)});\xi\in D,\,j\in J(\xi)\big)\in\mathbb{E}:=\mathbb{R}^{D\times L}_+\times\mathbb{R}^{J(D)}_+\times\mathbb{R}^{J(D)}_+$$

is *budget feasible* for agent i at prices (p,q) when

$$p_{\xi_0}(x_{\xi_0}-w^i_{\xi_0})+p_{\xi_0}\sum_{j\in J(\xi_0)}C_{(\xi_0,j)}\varphi_{(\xi_0,j)}+\sum_{j\in J(\xi_0)}q_{(\xi_0,j)}(\theta_{(\xi_0,j)}-\varphi_{(\xi_0,j)})\quad\le\quad 0, \tag{1}$$

$$p_\xi(x_\xi-w^i_\xi)+p_\xi\sum_{j\in J(\xi)}C_{(\xi,j)}\varphi_{(\xi,j)}+\sum_{j\in J(\xi)}q_{(\xi,j)}(\theta_{(\xi,j)}-\varphi_{(\xi,j)}) \tag{2}$$

$$\le p_\xi Y_\xi x_{\xi^-}+\sum_{j\in J(\xi^-)}\big(p_\xi Y_\xi C_{(\xi^-,j)}\varphi_{(\xi^-,j)}+F_{(\xi,j)}(p_\xi)(\theta_{(\xi^-,j)}-\varphi_{(\xi^-,j)})\big)\,,\ \ \forall\xi>\xi_0.$$

Also, $(x,\theta,\varphi)\in\mathbb{E}$ is *physically feasible* if $x_\xi+\sum_{j\in J(\xi)}C_{(\xi,j)}\varphi_{(\xi,j)}\leq W_\xi$, for any $\xi\in D$. Finally, given $(p,q)\in\Pi$, the objective of agent i is to maximize the utility of his consumption, $U^i((x^i_\xi+\sum_{j\in J(\xi)}C_{(\xi,j)}\varphi^i_{(\xi,j)})_{\xi\in D})$, choosing a budget feasible plan $(x^i,\theta^i,\varphi^i)\in\mathbb{E}$.

3. Enforcement mechanisms and the size of collateral bundles

In this section, we prove our main result: in contrast to the polar case studied by Araujo, Páscoa and Torres-Martínez (2002), the market choice of collateral bundles becomes relevant when there are *persistently effective* additional enforcement mechanisms besides collateral repossession. To achieve our objective, we impose the following hypotheses.

Assumption A1. *For any $\xi\in D$, $W^i_\xi\gg 0$.*

Assumption A2. *Given $z=(z_\xi)\in\mathbb{R}^{L\times D}_+$, define $U^i(z)=\sum_{\xi\in D}u^i_\xi(z_\xi)$, where for any $\xi\in D$, the function $u^i_\xi:\mathbb{R}^L_+\to\mathbb{R}_+$ is concave, continuous and strictly increasing. Also, $U^i(W)$ is finite.*

Given $\eta\in D$, let $\Omega(\eta)$ be the set of assets $j\in J(\eta)$ on which additional enforcement mechanisms are effective at some node $\mu\in\eta^+$. Note that, given a subtree $D(\xi)$ in which additional enforcement mechanisms are persistently effective, $\Omega(\eta)\neq\emptyset,\ \forall\,\eta\in D(\xi)$.

Theorem. *Under Assumptions A1-A2, suppose that additional enforcement mechanisms are persistently effective in a subtree $D(\xi)$. Independently of the prices $(p,q)\in\Pi$, there are strictly positive upper bounds $(\Psi_\eta)_{\eta\in D(\xi)}$ such that, if collateral bundles satisfy*

$$\min_{j\in\Omega(\eta)}\|C_{(\eta,j)}\|_\Sigma<\Psi_\eta,\qquad\forall\;\eta\in D(\xi),$$

then agent i's problem does not have a physically feasible solution.

Proof. To shorten the notation, given $z=(z_1,\ldots,z_m)\in\mathbb{R}^m_+$, let $\|z\|_\Sigma:=\sum_{s=1}^m z_s$ and $\|z\|_{\max}:=\max_{1\leq s\leq m}\ z_s$. Fix $\sigma>1$. Given $\eta\geq\xi$, define the number

$$\overline{\pi}_\eta:=\frac{U^i(W)}{\min_{l\in L}W^i_{(\eta,l)}},\qquad\text{and}\qquad\underline{\pi}_\eta:=\frac{u^i_\eta(\sigma W_\eta)-u^i_\eta(W_\eta)}{\sigma\|W_\eta\|_{\max}}.$$

Thus, for each $\eta\in D(\xi)$,

$$\Upsilon_\eta:=\min_{j\in\Omega(\eta)}\sum_{\mu\in\eta^+}\lambda_{(\mu,j)}\underline{\pi}_\mu\min_{l\in L}A_{(\mu,j,l)}$$

is strictly positive, where $A_{(\mu,j,l)}$ denotes the l-th coordinate of $A_{(\mu,j)}$.

Suppose that, at each $\eta \in D(\xi)$,

$$\min_{j\in\Omega(\eta)} \|C_{(\eta,j)}\|_\Sigma < \Psi_\eta := \frac{\Upsilon_\eta}{\overline{\pi}_\eta}.$$

Assume that, for some $(p,q) \in \Pi$, there is an optimal budget and physically-feasible solution $(x^i,\theta^i,\varphi^i) \in \mathbb{E}$ for agent i's problem. It follows from Lemma 1 (see Appendix) that there are, for every $\eta \in D$, multipliers $\gamma^i_\eta \in \mathbb{R}_{++}$ and non-pecuniary returns (super-gradients) $v^i_\eta \in \partial u^i_\eta\left(x^i_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi^i_{(\eta,j)}\right)$ such that,[2] for each $j \in J(\eta)$,

$$\gamma^i_\eta p_\eta \geq v^i_\eta + \sum_{\mu\in\eta^+} \gamma^i_\mu p_\mu Y_\mu, \tag{3}$$

$$\gamma^i_\eta q_{(\eta,j)} \geq \sum_{\mu\in\eta^+} \gamma^i_\mu F_{(\mu,j)}(p_\mu). \tag{4}$$

Also, the family of multipliers $(\gamma^i_\eta)_{\eta\in D}$ can always be constructed to satisfy (see Lemma 1)

$$\gamma^i_\eta p_\eta W^i_\eta \leq \sum_{\eta\in D} u^i_\eta\left(x^i_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi^i_{(\eta,j)}\right) \leq \sum_{\eta\in D} u^i_\eta(W_\eta), \tag{5}$$

where the last inequality follows from Assumption A2 jointly with the physical feasibility of agent i's consumption. Moreover, it is possible to find lower and upper bounds for $\gamma^i_\eta p_\eta$ at each $\eta \in D$. Assumption A1 and equation (5) ensure that $\gamma^i_\eta\|p_\eta\|_\Sigma \leq \overline{\pi}_\eta$. Given $\eta \in D$, let $c^i_\eta = x^i_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi^i_\eta$ be the consumption bundle chosen by agent i at η. Using equation (3), we have that

$$\gamma^i_\eta p_\eta(\sigma W_\eta - c^i_\eta) \geq v^i_\eta(\sigma W_\eta - c^i_\eta) \geq u^i_\eta(\sigma W_\eta) - u^i_\eta(c^i_\eta) \geq u^i_\eta(\sigma W_\eta) - u^i_\eta(W_\eta) > 0.$$

Therefore, $\gamma^i_\eta\|p_\eta\|_\Sigma \geq \underline{\pi}_\eta$. Since $\Omega(\eta) \neq \emptyset$ and $\min_{j\in\Omega(\eta)} \|C_{(\eta,j)}\|_\Sigma < \Psi_\eta$, at every node $\eta \in D(\xi)$ there exists $j \in \Omega(\eta)$ such that

$$\gamma^i_\eta(p_\eta C_{(\eta,j)} - q_{(\eta,j)}) \leq \gamma^i_\eta p_\eta C_{(\eta,j)} - \sum_{\mu\in\eta^+} \gamma^i_\mu F_{(\mu,j)}(p_\mu) < 0, \tag{6}$$

where the last inequality follows from the definition of the upper bound of collateral requirements. Finally, using the Lemma 2 in the Appendix, we conclude that agent i's problem does not have a solution, contradicting the optimality of $(x^i,\theta^i,\varphi^i) \in \mathbb{E}$ under prices $(p,q) \in \Pi$. $\square$

Note that, by construction, upper bounds on collateral requirements, $(\Psi_\eta)_{\eta\in D(\xi)}$, depend only on the primitives of the economy and, for computational objectives, can be easily found.

[2] Given a concave function $f : X \subset \mathbb{R}^n \to \mathbb{R} \cup \{-\infty\}$ and $x \in X$, the super-differential of f, $\partial f(x)$, is defined as the set of points $p \in X$, called super-gradients, such that $f(y) - f(x) \leq p(y - x)$, $\forall y \in X$.

Additionally, given collateral requirements, we can find lower bounds for the effectiveness on $D(\xi)$, $(\underline{\lambda}_\mu)_{\mu>\xi}$, such that, if $(\lambda_{(\mu,j)})_{\mu>\xi,j\in\Omega(\mu^-)}$ satisfy

$$\min_{j\in\Omega(\mu^-)} \lambda_{(\mu,j)} > \underline{\lambda}_\mu, \quad \forall \mu > \xi,$$

then, independently of prices, and for *any enforcement mechanism* inducing such effectiveness, there is no physical feasible solution for the agents' problem.[3] These lower bounds are informative, i.e. $\underline{\lambda}_\mu \in (0,1)$, only for collateral requirements that are not high enough. In fact, for larger collateral requirements there is no default and, therefore, the market price of collateral requirements is always greater that the loan value.

4. On endogenous effectiveness

A key feature of our model is that we assume the amount of payments besides the collateral guarantees is independent of the borrowers and does not depend on the history of default. This assumption allowed us to identify sold with purchased assets. Implicitly, we pool the debt contracts into derivatives following a trivial securitization, that is, by identifying prices and payments of debt markets with those of investment markets. However, our analysis may be extended for equilibrium models in which the effectiveness of payment enforcement mechanisms is an endogenous and personalized variable.

For instance, we may suppose that the access to credit markets depend on previous payments. That is, consider a dynamic infinite horizon general equilibrium model in which, at any state of nature, and for every agent, the access to credit securities depend on the history of default. Thus, in this new framework, for default penalties sufficiently restrictive on the access of credit markets, there may be endogenous incentives inducing borrowers to deliver payments larger than the depreciated value of collateral requirements. Also, suppose that financial markets still preserve some features from our original model. That is, each type of credit contract is securitized into only one derivative, primitive and derivative prices are equal, and lenders perfectly foresee the payments of derivatives. Specifically for this last feature, suppose that, in case of default, agents advance any payment in addition to the depreciated collateral as a *percentage* of the remaining debt, facing payment functions with an analogous specification of our $(F_{(\mu,j)})_{(\mu,j)\in J^+(D)}$.

In this new context, under hypotheses on individual' characteristics analogous to Assumptions A1-A2, there are two conditions under which our Theorem still holds:

[3] Using the same arguments in the proof of the Theorem, it is sufficient to take

$$\underline{\lambda}_\mu := \frac{\overline{\pi}_{\mu^-} \min_{j\in\Omega(\mu^-)} \|C_{(\mu^-,j)}\|_\Sigma}{\min_{j\in\Omega(\mu^-)} \sum_{\eta\in\mu} \underline{\pi}_\eta \min_{l\in L} A(\eta,j,l)}, \quad \forall \mu > \xi.$$

(i) For any plan of prices, individual optimal allocations satisfy inequalities analogous to (3)-(5);

(ii) At the moment of the credit operation, borrowers are only required to constitute the associated collateral requirements.

In fact, assume that additional enforcement mechanisms are persistently effective in a sub-event tree. If there is a physical feasible optimal allocation, using the same arguments of the proof of our theorem, condition (i) implies that if collateral requirements are not high enough, then unitary loan prices persistently exceed the associated collateral value. Thus, by condition (ii), the agent may improve his respective utility increasing borrowing along the event-tree. A contradiction.

Therefore, a natural question arises. When does an economy satisfy conditions (i) and (ii)? Regarding condition (i), it follows from Lemma 2 that any convex model satisfies it.[4] However, some enforcement mechanisms may induce non-convex budget sets. Even in these cases, condition (i) still holds if these non-convexities involve only the borrowers' deliveries.[5] On the other hand, condition (ii) holds unless there is some restriction on the short sales in addition to collateral requirements.

Appendix

Lemma 1. *Let $(p,q)\in\Pi$ and fix a budget and physically feasible plan $z^i := (x^i,\theta^i,\varphi^i)\in\mathbb{E}$. Under Assumptions A1 and A2, if z^i is an optimal allocation for agent i's problem at prices (p,q), then for every $\eta\in D$, the function u^i_η is super-differentiable at the point $c^i_\eta := x^i_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi^i_{(\eta,j)}$, there are multipliers $\gamma^i_\eta\in\mathbb{R}_{++}$ and super-gradients $v^i_\eta\in\partial u^i_\eta\left(c^i_\eta\right)$ such that, for each $j\in J(\eta)$,*

$$\gamma^i_\eta p_\eta \geq v^i_\eta + \sum_{\mu\in\eta^+}\gamma^i_\mu p_\mu Y_\mu, \tag{7}$$

$$\gamma^i_\eta q_{(\eta,j)} \geq \sum_{\mu\in\eta^+}\gamma^i_\mu F_{(\mu,j)}(p_\mu). \tag{8}$$

Also, the plan of multipliers $(\gamma^i_\eta)_{\eta\in D}$ satisfy

$$\gamma^i_\eta p_\eta W^i_\eta \leq \sum_{\eta\in D} u^i_\eta(c^i_\eta). \tag{9}$$

Proof. Given $T\in\mathbb{N}$, define $D_T=\{\eta\in D : t(\eta)=T\}$ and $D^T=\{\eta\in D:\eta\in\bigcup_{k=0}^T D_k\}$. For any $\eta\in D$, let $Z(\eta)=\mathbb{R}^L_+\times\mathbb{R}^{J(\eta)}_+\times\mathbb{R}^{J(\eta)}_+$. For convenience of notations, let $z_{\xi_0^-} := 0\in Z(\xi_0^-)$, where

[4] We mean that a model is *convex* when agents' objective functions are concave and, for each vector of prices, budget sets are convex.

[5] Technically, in this case, the arguments in the proof of Lemma 1 can be remade by redefining the truncated problem $(P^{i,T})$ in such form that, for any $\eta\in D^T$, variables φ_η are fixed and equal to the optimal choices φ^i_η.

$Z(\xi_0^-) := \mathbb{R}_+^L$. Consider the optimization problem:

$$
(P^{i,T}) \qquad \begin{array}{ll} \max & \sum_{\eta \in D^T} u_\eta^i \left(x_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi_{(\eta,j)} \right) \\ \text{s.t.} & \begin{cases} z_\eta := (x_\eta,\theta_\eta,\varphi_\eta) \in Z(\eta) & & \forall\, \eta \in D^T, \\ g_\eta^i(z_\eta, z_{\eta^-}; p,q) & \le & 0, \quad \forall\, \eta \in D^T, \\ x_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi_{(\eta,j)} & \le & 2W_\eta, \quad \forall\, \eta \in D^T, \\ z_\eta & \in & [0, z_\eta^i], \ \forall\, \eta \in D_T. \end{cases} \end{array}
$$

where the inequality $g_\eta^i(z_\eta, z_{\eta^-}; p, q) \le 0$ represents the budget constraint at node η, that is, inequality (1) or (2), and given $(x, y) \in \mathbb{R}^m \times \mathbb{R}^m$, the interval $[x, y] := \{z \in \mathbb{R}^m : \exists a \in [0, 1],\ z = ax + (1 - a)y\}$. It follows from the existence of an optimal individual plan at prices (p, q) that there exists a solution for $(P^{i,T})$, namely $(z_\eta^{i,T})_{\eta\in D^T}$.[6]

Given $\eta \in D$, define the concave function $\nu_\eta^i : \mathbb{R}^L \times \mathbb{R}^{J(\eta)} \times \mathbb{R}^{J(\eta)} \to \mathbb{R} \cup \{-\infty\}$ as

$$
\nu_\eta^i(z_\eta) = \begin{cases} u_\eta^i \left(x_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi_{(\eta,j)} \right) & \text{if } x_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi_{(\eta,j)} \ge 0; \\ -\infty & \text{otherwise.} \end{cases} ,
$$

where $z_\eta = (x_\eta, \theta_\eta, \varphi_\eta)$. It follows that, for any $T \ge 1$, $\sum_{\eta\in D^T} \nu_\eta^i(z_\eta^{i,T}) \le \sum_{\eta\in D} \nu_\eta^i(z_\eta^i)$.[7]

For each $\eta \in D$ and $\gamma_\eta \in \mathbb{R}_+$, define $\mathcal{L}_\eta^i(\cdot, \gamma; p, q) : \mathbb{Z}(\eta) \times \mathbb{Z}(\eta^-) \to \mathbb{R}$ as

$$
\mathcal{L}_\eta^i(z_\eta, z_{\eta^-}, \gamma_\eta;\, p, q) = \nu_\eta^i(z_\eta) - \gamma_\eta\, g_\eta^i(z_\eta, z_{\eta^-}; p, q).
$$

Given $T \in \mathbb{N}$, for each $\eta \in D^{T-1}$, define the set $\Xi^T(\eta)$ as the family of allocations $(x_\eta, \theta_\eta, \varphi_\eta) \in Z(\eta)$ that satisfies $x_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi_{(\eta,j)} \le 2W_\eta$. Also, for any $\eta \in D_T$, let $\Xi^T(\eta)$ be the set of

---

[6] In fact, define a new problem $(\tilde{P}^{i,T})$,

$$
(\tilde{P}^{i,T}) \qquad \begin{array}{ll} \max & \sum_{\eta \in D^T} u_\eta^i \left(x_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi_{(\eta,j)} \right) \\ \text{s.t.} & \begin{cases} z_\eta := (x_\eta,\theta_\eta,\varphi_\eta) \in Z(\eta) & & \forall\, \eta \in D^T, \\ g_\eta^i(z_\eta, z_{\eta^-}; p,q) & \le & 0, \quad \forall\, \eta \in D^T, \\ x_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi_{(\eta,j)} & \le & 2W_\eta, \quad \forall\, \eta \in D^T, \\ z_\eta & \in & [0, z_\eta^i], \ \forall\, \eta \in D_T, \\ \text{If} \quad q_{(\eta,j)} = 0 & \text{then} & \theta_{(\eta,j)} = 0. \end{cases} \end{array}
$$

Under Assumption A2 the objective function on $(\tilde{P}^{i,T})$ is continuous, and the set of admissible allocations is compact in $\prod_{\eta\in D^T} Z(\eta)$. Note that, to ensure this it is necessary to have non-zero collateral requirements, otherwise, long and short positions are unbounded.

Thus, there is a solution $(z_\eta^{i,T})_{\eta\in D^T}$. Moreover, this solution for $(\tilde{P}^{i,T})$ is also an optimal choice for $(P^{i,T})$. Essentially, the existence of a *finite* optimum at prices (p, q) for the agent i's problem ensure that, when $q_{(\eta,j)} = 0$, the payments $F_{(\mu,j)}(p_\mu)$ must be equal zero, for each $\mu \in \eta^+$. Thus, when $q_{(\eta,j)} = 0$, choosing positives amounts of $\theta_{(\eta,j)}$ does not induce any gains.

[7] Note that, otherwise, agent i improve his utility in D choosing the allocation $(z_\eta^{i,T})_{\eta\in D^T}$ in the sub-tree D^T, without making any (physical or financial) trade after the nodes with date T.

allocations $(x_\eta,\theta_\eta,\varphi_\eta)\in Z(\eta)$ that satisfies both $x_\eta+\sum_{j\in J(\eta)} C_{(\eta,j)}\varphi_{(\eta,j)}\leq 2W_\eta$ and $(x_\eta,\theta_\eta,\varphi_\eta)\in[0,z^i_\eta]$. Let $\Xi^T:=\prod_{\eta\in D^T}\Xi^T(\eta)$.

It follows from Rockafellar (1997, Theorem 28.3), that there exist non-negative multipliers $(\gamma^{i,T}_\eta)_{\eta\in D^T}$ such that the following saddle point property holds,

$$\sum_{\eta\in D^T}\mathcal{L}^i_\eta(z_\eta,z_{\eta_-},\gamma^{i,T}_\eta;\,p,q)\leq\sum_{\eta\in D^T}\mathcal{L}^i_\eta(z^{i,T}_\eta,z^{i,T}_{\eta_-},\gamma^{i,T}_\eta;\,p,q),\quad \forall (z_\eta)_{\eta\in D^T}\in\Xi^T, \tag{10}$$

and $\gamma^{i,T}_\eta g^i_\eta(z^{i,T}_\eta,z^{i,T}_{\eta_-};p,q)=0$.

Claim A. *For each $\mu\in D$, the sequence $(\gamma^{i,T}_\mu)_{T\geq t(\mu)}$ is bounded. Moreover, given $T>t(\mu)$,*

$$\nu^i_\mu(a_\mu)-\nu^i_\mu(z^i_\mu)\leq\left(\gamma^{i,T}_\mu\nabla_1 g^i_\mu(p,q)+\sum_{\eta\in\mu^+}\gamma^{i,T}_\eta\nabla_2 g^i_\eta(p,q)\right)\cdot(a_\eta-z^i_\eta)+\sum_{\xi\in D\setminus D^{T-1}}\nu^i_\xi(z^i_\xi),\quad\forall\, a_\mu\in\Xi^T(\mu),$$

where, for any $\eta\in D$, the vector $(\nabla_1 g^i_\eta(p,q),\nabla_2 g^i_\eta(p,q))$ is defined by

$$\begin{aligned}\nabla_1 g^i_\eta(p,q)&=\left(p_\eta,q_\eta,(p_\eta C_{(\eta,j)}-q_{(\eta,j)})_{j\in J(\eta)}\right),\\ \nabla_2 g^i_\eta(p,q)&=\left(-p_\eta Y_\eta,(F_{(\eta,j)})_{j\in J(\eta)},(p_\eta Y_\eta C_{(\eta,j)}-F_{(\eta,j)})_{j\in J(\eta)}\right).\end{aligned}$$

Proof. Given $t\leq T$, substitute the following allocation in inequality (10)

$$z_\eta=\begin{cases}(W^i_\eta,0,0), & \forall\,\eta\in D^{t-1},\\ (0,0,0), & \forall\,\eta\in D^T\setminus D^{t-1}.\end{cases}$$

We have:

$$\sum_{\eta\in D_t}\gamma^{i,T}_\eta p_\eta W^i_\eta\leq\sum_{\eta\in D^T}\nu^i_\eta(z^{i,T}_\eta)\leq\sum_{\eta\in D}\nu^i_\eta(z^i_\eta). \tag{11}$$

Assumptions A1 ensure that, for each $\eta\in D$, $\min_{l\in L}W^i_{(\eta,l)}>0$. Also, Assumption A2 implies that $\|p_\eta\|_\Sigma>0$, guaranteing that, for each $\mu\in D$, the sequence $(\gamma^{i,T}_\mu)_{T>t(\mu)}$ is bounded.

On the other hand, given $(z_\eta)_{\eta\in D^T}\in\Xi^T$, using (10), we have that

$$\sum_{\eta\in D^T}\mathcal{L}^i_\eta(z_\eta,z_{\eta^-},\gamma^{i,T}_\eta;p,q)\leq\sum_{\eta\in D}\nu^i_\eta(z^i_\eta).$$

Thus, fix $\mu\in D^{T-1}$ and $a_\mu\in\Xi^T(\mu)$. If we evaluate inequality above in

$$z_\eta=\begin{cases}z^i_\eta, & \forall\,\eta\neq\mu,\\ a_\mu, & \text{for }\eta=\mu,\end{cases}$$

we obtain

$$\nu^i_\mu(a_\mu)-\gamma^{i,T}_\mu g^i_\mu(a_\mu,z^i_{\mu^-};p,q)-\sum_{\eta\in\mu^+}\gamma^{i,T}_\eta g^i_\eta(z^i_\eta,a_\mu;p,q)\leq\nu^i_\mu(z^i_\mu)+\sum_{\eta\in D\setminus D^T}\nu^i_\eta(z^i_\eta). \tag{12}$$

Since functions $(g^i_\xi(\cdot\,;p,q);\xi\in D)$ are affine, we have

$$\begin{aligned} g^i_\mu(a_\mu,z^i_{\mu^-};p,q) &= \nabla_1 g^i_\mu(p,q)\cdot a_\mu - p_\mu\omega^i_\mu + \nabla_2 g^i_\mu(p,q)\cdot z^i_{\mu^-}\\ g^i_\eta(z^i_\eta,a_\mu;p,q) &= \nabla_1 g^i_\eta(p,q)\cdot z^i_\eta - p_\eta\omega^i_\eta + \nabla_2 g^i_\eta(p,q)\cdot a_\mu,\quad \forall\eta\in\mu^+. \end{aligned}$$

Also, budget feasibility of $(z^i_\eta)_{\eta\in D}$ at prices (p,q), jointly with monotonicity of preferences, ensure that,

$$\begin{aligned} -p_\mu\omega^i_\mu + \nabla_2 g^i_\mu(p,q)\cdot z^i_{\mu^-} &= -\nabla_1 g^i_\mu(p,q)\cdot z^i_\mu,\\ \nabla_1 g^i_\eta(p,q)\cdot z^i_\eta - p_\eta\omega^i_\eta &= -\nabla_2 g^i_\eta(p,q)\cdot z^i_\mu,\quad \forall\,\eta\in\mu^+. \end{aligned}$$

Therefore,

$$\gamma^{i,T}_\mu g^i_\mu(a_\mu,z^i_{\mu^-};p,q)+\sum_{\eta\in\mu^+}\gamma^{i,T}_\eta g^i_\eta(z^i_\eta,a_\mu;p,q)=\left(\gamma^{i,T}_\mu\nabla_1 g^i_\mu(p,q)+\sum_{\eta\in\mu^+}\gamma^{i,T}_\eta\nabla_2 g^i_\eta(p,q)\right)\cdot(a_\mu-z^i_\mu).$$

Using (12), we conclude the proof. ⊠

Since D is countable and, for any node η, the sequence $(\gamma^{i,T}_\eta)_{T\geq t(\eta)}$ is bounded, using Tychonoff Theorem (see Aliprantis and Border (1999, Theorem 2.57)), there is a common subsequence $(T_k)_{k\in\mathbb{N}}\subset\mathbb{N}$ and non-negative multipliers, $(\gamma^i_\eta)_{\eta\in D}$, such that, for each $\eta\in D$, $\lim_{k\to\infty}\gamma^{i,T_k}_\eta=\gamma^i_\eta$ and

$$\gamma^i_\eta\, g^i_\eta(z^i_\eta,z^i_{\eta_-};p,q)=0,$$

where, as we said above, the last equation follows from the strictly monotonicity of u^i_η. Moreover, taking the limit as T goes to infinity in inequality (13) we obtain that

$$\sum_{\eta\in D_t}\gamma^i_\eta p_\eta W^i_\eta \;\leq\; \sum_{\eta\in D}\nu^i_\eta(z^i_\eta),\quad \forall t\geq 0. \tag{13}$$

Therefore, equation (9) follows.

Since for any $\eta\in D$, $\Xi^{s_1}(\eta)=\Xi^{s_2}(\eta)$ when $\min\{s_1,s_2\}>t(\eta)$, it follows from the inequality in the statement of Claim above, taking the limit as T goes to infinity, that

$$\nu^i_\eta(a_\eta)-\nu^i_\eta(z^i_\eta) \;\leq\; \left(\gamma^i_\eta\nabla_1 g^i_\eta(p,q)+\sum_{\mu\in\eta^+}\gamma^i_\mu\nabla_2 g^i_\mu(p,q)\right)\cdot(a_\eta-z^i_\eta),\quad \forall\; a_\eta\in\Xi^{t(\eta)+1}(\eta). \tag{14}$$

Thus,

$$\left(\gamma^i_\eta\nabla_1 g^i_\eta(p,q)+\sum_{\mu\in\eta^+}\gamma^i_\mu\nabla_2 g^i_\mu(p,q)\right)\in\partial\left(\nu^i_\eta+\delta_{Z_1(\eta)}+\delta_{Z_2(\eta)}\right)(z^i_\eta),$$

where the functions $\delta_{Z_h(\eta)} : \mathbb{R}^L \times \mathbb{R}^{J(\eta)} \times \mathbb{R}^{J(\eta)} \to \mathbb{R} \cup \{-\infty\}$, $h \in \{1,2\}$, satisfy

$$\delta_{Z_1(\eta)}(x_\eta, \theta_\eta, \varphi_\eta) = \begin{cases} 0, & \text{if } (x_\eta, \theta_\eta, \varphi_\eta) \in Z(\eta), \\ -\infty, & \text{otherwise.} \end{cases}$$

$$\delta_{Z_2(\eta)}(x_\eta, \theta_\eta, \varphi_\eta) = \begin{cases} 0, & \text{if } x_\eta + \sum_{j\in J(\eta)} C_{(\eta,j)}\varphi_{(\eta,j)} \leq 2W_\eta, \\ -\infty, & \text{otherwise.} \end{cases}$$

where $z_\eta = (x_\eta, \theta_\eta, \varphi_\eta) \in \mathbb{R}^L \times \mathbb{R}^{J(\eta)} \times \mathbb{R}^{J(\eta)}$. Since the plan $(z^i_\eta)_{\eta\in D}$ is physically feasible, there exists a neighborhood V of z^i_η such that $\delta_{Z_2(\eta)}(b) = 0$ for every $b \in V$. Then, we have that $\partial\delta_{Z_2(\eta)}(z^i_\eta) = \{0\}$. Also, it follows by Theorem 23.8 and 23.9 in Rockafellar (1997), that there exists $v^i_\eta \in \partial u^i_\eta(c^i_\eta)$ and $\kappa^i_\eta \in \partial\delta_{Z(\eta)}(x^i_\eta, \theta^i_\eta, \varphi^i_\eta)$ such that

$$\gamma^i_\eta \nabla_1 g^i_\eta(p,q) + \sum_{\mu\in\eta^+} \gamma^i_\mu \nabla_2 g^i_\mu(p,q) = (v^i_\eta, 0, (C_{(\eta,j)} v^i_\eta)_{j\in J(\eta)}) + \kappa^i_\eta. \tag{15}$$

Notice that, by definition, for each $z_\eta \geq 0, \kappa \in \partial\delta_{Z(\eta)}(z_\eta) \Leftrightarrow 0 \leq \kappa(y - z_\eta),\ \forall\ y \geq 0$, therefore, $\kappa^i_\eta \geq 0$. Thus, the inequalities stated in the lemma hold from equation (15). On the other hand, strictly monotonicity of function u^i_η, ensure that $v^i_\eta \gg 0$ and, therefore, it follows from (7), that γ^i_η is *strictly positive*. □

In a context of collateralized assets and *linear utility penalties for default*, Páscoa and Seghir (2007) show that Ponzi schemes could be implemented if there exists a subtree $D(\xi)$ such that, for every node $\mu \geq \xi$, there is always some asset $j \in J(\mu)$ whose price exceeds the respective collateral value, $p_\mu C_{(\mu,j)} - q_{(\mu,j)} < 0$ (see Remark 3.1 in Páscoa and Seghir (2007)). In such event, the individual's problem does not have a finite solution. In our context, the same result follows by analogous arguments.

LEMMA 2. *Assume that, given $x \in \mathbb{R}^{L\times D}_+$, if $U^i(x)$ is finite, then $U^i(y) > U^i(x)$ for any $y > x$. Also, suppose that additional enforcement mechanisms are persistently effective in a subtree $D(\xi)$ such that, for any $\eta \in D(\xi)$, there exists $j \in J(\eta)$ for which $p_\eta C_{(\eta,j)} - q_{(\eta,j)} < 0$. Then, agent i's individual problem does not have a finite solution, otherwise, Ponzi schemes could be implemented.*

PROOF. Assume there is a budget feasible plan for agent i, $(x^i, \theta^i, \varphi^i)$, that gives a finite optimum. Under the monotonicity condition stated in the Lemma, $p_\eta \gg 0$ for every node $\eta \in D(\xi)$. For each $\eta \in D(\xi)$, let $J^1(\eta) = \{j \in J(\eta) : p_\eta C_{(\eta,j)} - q_{(\eta,j)} < 0\}$. Now, consider the allocation $(x_\xi, \theta_\xi, \varphi_\xi)_{\xi\in D}$, with

$$\big((x_\mu, \theta_\mu, \varphi_\mu);\ (\theta_\eta, \varphi_{(\eta,j)})\big)_{\mu\notin D(\xi),\, \eta\in D(\xi)} = \Big((x^i_\mu, \theta^i_\mu, \varphi^i_\mu);\ (\theta^i_\eta, \varphi^i_{(\eta,j)})\Big)_{\mu\notin D(\xi),\, \eta\in D(\xi)},\ \ \forall j \in J(\eta)\backslash J^1(\eta)$$

and

$$\begin{aligned}
\varphi_{(\eta,j)} &= \varphi^i_{(\eta,j)} + \delta_\eta, \quad \forall \eta \in D(\xi), \forall j \in J^1(\eta), \\
x_{(\eta,l)} &= x^i_{(\eta,l)} + \frac{1}{(\#L)\, p_{(\eta,l)}} \sum_{j \in J^1(\eta)} \left(q_{(\eta,j)} - p_\eta C_{(\eta,j)} \right) \delta_\eta, \quad \forall l \in L, \text{ if the node } \eta = \xi, \\
x_{(\eta,l)} &= x^i_{(\eta,l)} + \frac{1}{(\#L)\, p_{(\eta,l)}} \sum_{j \in J^1(\eta)} \left(q_{(\eta,j)} - p_\eta C_{(\eta,j)} \right) \delta_\eta \\
&\qquad - \frac{1}{(\#L)\, p_{(\eta,l)}} \sum_{j \in J^1(\eta^-)} p_\eta A_{(\eta,j)}\, \delta_{\eta^-}, \;\; \forall \eta > \xi, \forall l \in L,
\end{aligned}$$

where the plan $(\delta_\eta)_{\eta \in D(\xi)}$ is chosen in such form that the following conditions hold,

$$\sum_{j \in J^1(\xi)} \left(q_{(\xi,j)} - p_\xi C_{(\xi,j)} \right) \delta_\xi \quad > \quad 0, \tag{16}$$

$$\sum_{j \in J^1(\eta)} \left(q_{(\eta,j)} - p_\eta C_{(\eta,j)} \right) \delta_\eta \quad > \quad \sum_{j \in J^1(\eta^-)} p_\eta A_{(\eta,j)}\, \delta_{\eta^-}, \quad \forall \eta > \xi. \tag{17}$$

It follows that $(x_\xi, \theta_\xi, \varphi_\xi)_{\xi \in D}$ is budget feasible at prices (p, q). Moreover, equations above show that Ponzi schemes are possible at prices (p, q). In fact, agent i increases his borrowing at ξ and pays his future commitments by using new credit. It follows that $(x_\xi, \theta_\xi, \varphi_\xi)_{\xi \in D}$ improves the utility level of agent i, contradicting the optimality of $(x^i, \theta^i, \varphi^i)$. $\square$

References

[1] Aliprantis, C.D., K.C. Border (1999): "Infinite dimensional analysis: a hitchhiker's guide," 2nd edn.. Berlin Heidelberg New York: Springer.

[2] Araujo, A., M. Páscoa, and J.P. Torres-Martínez (2002): "Collateral avoids Ponzi schemes in incomplete markets," Econometrica, 70, 1613-1638.

[3] Páscoa, M. and A. Seghir (2007): "Harsh default penalties lead to Ponzi schemes," Games and Economic Behaviour, doi:10.1016/j.geb.2007.10.002.

[4] Rockafellar, R.T. (1997): "Convex analysis," *Princeton University Press*, Princeton, New Jersey, USA.

DEPARTMENT OF ECONOMICS, NORTHWESTERN UNIVERSITY
2001 SHERIDAN ROAD, EVANSTON, IL, USA, 60208-2600
E-mail address: `revilth@u.northwestern.edu`

DEPARTMENT OF ECONOMICS, UNIVERSITY OF CHILE
DIAGONAL PARAGUAY 257 OFFICE 1604, SANTIAGO, CHILE.
E-mail address: `jutorres@econ.uchile.cl`