From Duty to Right: The Role of Public Education in the Transition to Aging Societies

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From Duty to Right: The Role of Public Education in the Transition to Aging Societies

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Abstract

This paper argues that the introduction of compulsory schooling in early industrialization promoted the growth process that eventually led to a vicious cycle of population aging and negative pressure on education policy. In the early phases of industrialization, public education was undesirable for the young poor who relied on child labor. Compulsory schooling therefore discouraged childbirth, while the accompanying industrialization stimulated their demand for education. The subsequent rise in the share of the old population, however, limited government resources for education, placing heavier financial burdens on the young. This induced further fertility decline and population aging, and the resulting cycle may have delayed the growth of advanced economies in the last few decades.

Keywords: Compulsory Education; Fertility; Generational Conflict; Growth.

JEL Classification: D70; H50; J10; J20; O40.

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1 Introduction

This paper argues that the introduction of compulsory schooling in early industrialization promoted the growth process that eventually led to a vicious cycle of fertility decline, population aging, and negative pressure on public education policy. This cyclical process may have decelerated the growth of advanced economies in the last few decades and possibly in the future.

A broad picture of the proposed hypothesis is represented by the flow chart in Figure 1. Public education was perceived as a duty in the early phases of industrialization, where households relied on the labor of their children. Imposing compulsory schooling discouraged childbirth by regulating child labor and, more indirectly, by promoting technological progress. This eventually led to a major education reform (i.e., the extension of public education), when child education rather than child labor became desirable for most young households. However, the accompanying fertility decline, which skewed the age distribution toward the old, enhanced their political power and then squeezed the government budget for education. This induced higher spending on private education, lower fertility, and further population aging. Since the positive reaction of private education reflects the widespread demand for skills, this cyclical process is a phenomenon indigenous to the industrialized stage.

This socioeconomic structure sheds light on the historical relationship between public education and demographic change in the era of modern growth. Table 1 presents the long-term trend in the related variables for major advanced countries. Although the timing varies across those countries, the share of public education in total government expenditure, $E/G$, reached a peak after the birth rate, $B$, began steadily decreasing.$^1$ That is, these countries had a period in which public education and fertility evolved inversely, and no monotonic relationship is observed between demographic change and the budget allocation to public education.

In light of historical evidence presented later, we argue that these asymmetry trends reflect the decline in child labor under the half-time system in education, followed by the switch of parental strategies toward investment in child education. On the other hand, the subsequent parallel trends—monotonic decline in public education and fertility followed by population aging—are

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$^1$ As for France, only central government expenditure is considered in both $E$ and $G$ in Table 1. Thus, the ratio $E/G$ does not take into account financial transfers for public education between central and local governments. This deficiency might be associated with its sharp rise between the years 1954 and 1975.
explained by the vicious cycle in Figure 1. Indeed, consistent with the positive reaction of private education in the figure, the last decade witnessed several OECD countries raising the relative burden of private to public education spending, as presented by Table 2. The underlying government failure to meet the households’ growing demand for schooling would have discouraged childbirth and accelerated population aging in those countries.\textsuperscript{2}

In order to encompass the dynamic process shown above, this paper develops a growth theory that incorporates five key elements. First, parents face a trade-off between the quantity and quality of children, as formulated in standard models of fertility and education, such as Becker et al. (1990) and Galor and Weil (2000).\textsuperscript{3} Second, public and private education are substitutes for each other in terms of skill acquisition. Those two properties together generate a potential link between education policy and fertility, as shown by Figure 1. In response to a decline in public education, households in industrialized stages spend more on private education by having fewer children.

Third, public education imperatively takes away the time children have for paid work. Their forgone wages therefore equal the opportunity costs of child education. Fourth, technological progress is assumed to be age-biased. This improves the productivity of adult workers over that of children, and thus makes sending children to school more beneficial than having them work.\textsuperscript{4} Fifth and finally, old generations have limited altruism; they basically prefer public services that directly and immediately benefit themselves, such as pensions and health care, over public education. This leads to age-cohort differences in the stances toward education policy.\textsuperscript{5}

This research plays a complementary role in the recent and growing literature on demographic change and macroeconomics in political-economy frameworks. It appears that no established theory

\textsuperscript{2}Population shrinking is one of the upcoming issues for some advanced countries. As for Japan, the natural increase rate of the population became negative in 2005 for the first time after 1920 (Statistic Bureau, Ministry of Internal Affairs and Communications, 2008, p. 4). A French demographer Bourgeois-Pichat estimates that the population of European countries would die out within 300 years if their fertility rates were and remained at the West German level (Johnson et al., 1989, p. 3).

\textsuperscript{3}This paper advances the unified growth theories developed by Galor and Weil (2000) and Galor and Moav (2002), who explore the transition from Malthusian stagnation to modern growth through the demographic transition. In contrast to their approach, this article incorporates endogenously determined public education to analyze its relationship with fertility. The other four properties discussed are also added in the present model.

\textsuperscript{4}A similar sector-biased technological progress is employed by Hazan and Berdugo (2002), who explore the departure from the poverty trap where child labor is prevalent. One of the differences from their research is that the present paper emphasizes the role of compulsory and universal investment in education, rather than population growth, as a driving force for the abandonment of child labor.

\textsuperscript{5}Using the American National Election Survey for 1988, Vinovskis (1995, pp. 192–209) finds that additional federal assistance for public schools is supported by 77.1% of interviewees aged through 18–29 and only by 46.9% of those aged 70 and above. Furthermore, her multiple classification analysis verifies this trend by controlling for the effects of other factors such as sex, race, education, family income, and so on.
has examined the interrelationship between public education, private education and demographic changes, despite its importance for the assessment of education policy in aging societies. As discussed below, while there are three related seminal theories, these can be interpreted as partial theories from the perspective of this research.

First, Galor and Moav (2006) demonstrate the mechanism of the birth of public schooling on the course of industrialization in the West. They highlight the profit rates of capitalists in forming their policy preferences, in the presence of capital-skill complementarity. Unlike the present paper, their growth theory implies the monotonic evolution of education policy, excluding the possibility of its aggravation in industrialized stages. Furthermore, in their scenario, education reform occurs without any political conflict, in sharp contrast to our focus on compulsory education enforced on the poor against their will.6

Second, Doepke and Zilibotti (2005) explore the mechanism of child labor legislation. By noting the lock-in effect of fertility decisions on policy preferences as well as the eroding effect of child labor on the unskilled wage, they reveal the possibility of multiple equilibria such that child labor may or may not be abolished. While their attention is directed at the early stages of industrialization where child labor prevails, the present study builds on a longer-term perspective to encompass subsequent population aging that provokes political confrontation between generations.

Finally, Holtz-Eakin et al. (2004) develop a growth model where the age composition of the economy is a prime determinant of the allocation of public resources.7 They demonstrate that a fertility decline alters the policy preferences of the median voter in favor of the elderly, who have a low priority for education. However, unlike the present article, their analysis leaves unexplored an opposing causality: the effect of education policy on demographic factors. This is because their model abstracts from parents' child-rearing strategies by assuming exogenous fertility and no private education.

Those added features build on various omitted elements such as physical capital, agricultural land, pecuniary externalities, and international trade. Our theory predicts their potential impacts, which are presumably country-specific, on the progress of early education reforms and industrial-

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6 This difference is largely owing to the fact that the present article highlights the role of compulsory education as a child labor regulation. As evidenced in Section 2, early factory legislation required education attainments for child workers while limiting their work hours.

7 Similar demographic approaches are employed in Kemnitz (1999), Pecchenino and Utendorf (1999), and Gradstein and Kaganovich (2004). All of these, however, assume exogenous population growth.
ization. Furthermore, it is shown that the depth of the downward spiral in the developed stage would depend on how to allocate the government budget for education between various types of skills. Hence, this research offers theoretical implications for the observed cross-country variations in public education, population aging, and growth performance.

The remainder of the paper is organized as follows. Section 2 presents historical evidence supporting the central thesis of the paper. Section 3 describes the basic structure of the growth model and solves optimization problems. Section 4, as the main part of the paper, analyzes the process of industrialization that encompasses the transition into the aforementioned vicious cycle. Section 5 discusses the underlying assumptions and the robustness of the model, and considers the implications for comparative economic development. Section 6 concludes the discussion and addresses a direction of future research. The proofs of technical results and the description of the data used are provided in the Appendix.

2 Historical and Empirical Evidence

This section presents historical and empirical evidence supporting the central thesis of the paper. The focus here is placed on the experiences of advanced economies—Japan, the United States, and Western Europe—over the last two centuries. Consistent with the theory developed below, it shows that industrialization forces triggered the decisive shift of individuals’ attitudes toward child labor and education. The last part of the section summarizes the empirical evidence for the relationship between population aging and gray power.

2.1 Public Education as a Duty

The notion of compulsory education as a duty has been embraced by the poor, who live from hand to mouth and need the earnings of their children. In the early stages of their industrial revolutions, these countries employed children, and their respective child labor regulations and education reforms were disagreeable for the lower class. As a result of compromise, early education reforms were designed to provide special classes for working children. On the basis of this historical fact, the growth theory that is developed later encompasses the coexistence of child labor and public schooling.
2.1.1 Japan

It is widely recognized that the Japanese industrial revolution took place after the Meiji Restoration in 1868. Ohkawa and Rosovsky (1965, p. 66; 1978, p. 142) argue that modern economic growth began in the mid-1880s. Similarly, Rostow (1978, p. 425) dates the years 1885–1905 as the period of the take-off.

Japan’s modern education reform was launched with the proclamation of the Educational System Order in 1872, which aimed for egalitarian education by establishing school districts throughout the country.\(^8\) The law made four-year elementary education compulsory, along with the charging of school fees (Taira, 1978, p. 196). An educational system in the early stages of development such as this placed a heavy, unbearable financial burden on the lower classes at that time. As described by Taira (ibid.), “To poor farmers, compulsory education appeared as an encouragement to the children to loaf in school when they could be helping on the farm. In some poorer parts of Japan there were a number of riots against compulsory education, in which hundreds of school buildings were destroyed.”\(^9\)

In fact, statistical records indicate that not many children were sent to primary school in the early years of industrialization. The classroom attendance rate was approximately 20.44% in 1873 and 31.24% in 1890, and exceeded 50% for the first time in 1900 (Japanese National Commission for Unesco [sic], 1966, p. 64).\(^10\) According to the estimation by Umemura et al. (1988, pp. 80–83), the employment rate of male children aged between 10 and 14 was approximately 60% in 1872, reaching nearly 50% in 1894, 40% in 1899, and 30% in 1909.\(^11\) They also show that the corresponding figure for female children was initially lower (52.2% between 1872 and 1880) and less variable over time. This appears to be plausible because many girls in the Meiji period were perhaps supposed to engage in babysitting or housekeeping.

In order to encourage education for child workers on or off labor markets, some forms of half-

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\(^8\) Although there were many small schools for reading, writing, and arithmetic before the Meiji Restoration, they were typically run privately with no subsidies from the government (Dore, 1964, p. 176). For a comprehensive English survey of the development of Japan’s modern educational system, see the webpage of the Ministry of Education (http://www.mext.go.jp/b_menu/hakusho/html/hpbz198103/index.html).

\(^9\) See Taira (1971, p. 373) for similar discussions.

\(^10\) While the commission also reports the rate of school attendance, they believe that the rate of classroom attendance provides more accurate information about actual attendance; the former is defined as “an official ratio of the number of children attending school to the total number of the school-age children,” whereas the latter is “the ratio of average daily attendance to the total number of the school-age children (ibid., pp. 64–65).”

\(^11\) These figures would be underestimated because of the assumption that all registered students were unemployed.
time education were introduced in the late 19th century. In 1875, the city of Tokyo permitted night classes opened by private elementary schools and, in 1876, permitted night classes by public elementary schools (Ishii, 1992, p. 25). In 1894, the Ministry of Education ordered local governments to develop special curriculums for children who could not manage to attend regular classes (Saito, 1996, p. 83). Between 1876 and 1930, no less than 300 schools for childminders were established across 36 prefectures (Osada, 1995, Table 16, pp. 214–215).

2.1.2 The United States

As is well known, there were regional differences in the timing of industrialization in the United States. The economic take-off of New England occurred in the period 1815–1850, whereas that of the American North occurred in 1843–1870 (Rostow, 1978, p. 392). In the north-eastern region, the share of child workers employed in manufacturing was 23.1% in 1820 (Goldin and Sokoloff, 1982, p. 748). It is estimated that in 1832, children amounted to about 40% of factory workers in New England (Weiner, 1991, p. 142). The proportion of child labor would have been much higher in agriculture, a dominant sector in the early 19th century.\(^{12}\) At the national level, about one-sixth of children between the ages 10 to 15 were gainfully employed even in 1880 (Sanderson, 1974, p. 297; Weiner, 1991, p. 145).

One of the principal obstacles to universal education conceivably was how to bring children out of their workplaces to school. Hence, early education reforms were presumably a compromise between the need for child labor and that for child education. In the late 1830s, industrial states passed laws, which required factory children to go to school three months every year (Church, 1976, p. 59). In 1836, Massachusetts legislated that (at least) three months of school attendance was required, in advance, for the employment of children under the age of 15 in manufacturing (Weiner, 1991, pp. 142–143). Under the Massachusetts’ labor-attendance law in 1842, it was the duty of the local school committees to secure the school attendance of children, while all factory children under the age of 12 were prohibited from working more than 10 hours a day (Ensign, 1969, p. 49).

\(^{12}\) In 1820, the proportion of workers employed in agriculture was 73% in New England and 74% in the Middle Atlantic (Goldin and Sokoloff, 1982, p. 748).
2.1.3 Britain

Child labor played an important role even in early 19th century Britain, one of the most advanced economies at that time. In 1816, workers under the age of 18 accounted for 51.2% of workers in the British cotton industry, and 60.2% in the Scottish flax industry (Nardinelli, 1990, p. 109). The Census of England and Wales records that at least 36.6% of boys aged 10–14 were working in 1851 (Cunningham, 1996, p. 42). The earnings of children were an important income source of the working class. In the period from 1817 to 1839, records show that the children of mining, factory, and outworking workers earned 23.9%, 28.2%, and 27.5% of household income, respectively (Horrell and Humphries, 1995, p. 491).

These figures indicate parents’ hesitation in sending their children to school at that time. For instance, the records of the Mitcham National School show that during the years 1830–1939, the average length of stay of pupils was 34.5 months, and 57% of boys left school to start work (Madoc-Jones, 1977, pp. 45, 47). Even in 1889, the London School Board issued as many as 96,450 initial notices to parents failing to send their children to school (Rubinstein, 1977, p. 245). Furthermore, there is a view that the London poor, estimated to be 30.7% of the population in 1891, opposed education for their children, in part because those poor needed the earnings of their children (ibid., pp. 235–236). It is conjectured from these figures that the first half of the 19th century saw widespread, intense opposition to compulsory education.

In these circumstances, early factory legislation made a reconciliation between regulating child labor and promoting child education. As in the other nations concerned here, compulsory education became gradually effective through legislative amendments. The Factories Act of 1833 imposed two-hour schooling on six days a week on working children aged 9 to 11 in the main textile industries, although its implementation was tough for factory inspectors (Silver, 1977, p. 141). The performance of the current law and countermeasures were reported to the government periodically by four factory inspectors, and their opinions were reflected in the Factory Act of 1844 (Hutchins and Harrison, 1926, pp. 71, 85). As stated by Silver (1977, p. 141), the law marked a turning point: “The half-time system, foreshadowed in the early Factory Acts and effective from 1845, began as a

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13 Hurt (1979, p. 155) introduces two court cases, from The Times in November 1875, in which the London School Board was involved. They show that compulsory school attendance was a big burden on the poor.

14 Hopkins (1994, p. 142) presents an anecdotal episode in the mid-19th century Black Country: A prevailing proverb at that time was “The father went to the pit and made a fortune; the son went to school and lost it.”
strategy for combating excessive child labour and became, in the 1850s and 1860s, an educational theory."

2.1.4 France

As in Britain's case, the progress of French education reform was gradual. The education law in 1833 made primary education universally available (Weissbach, 1989, p. 3). Nevertheless, that was not fully made use of by the impoverished, who needed child labor for their daily life. In 1840, more than a quarter of the children aged 6 to 12 in France did not attend school in part because their labor was needed at home (ibid.).

Evidence shows that children were working for their families out of necessity, at least before the mid-19th century. According to a study published in 1840, it cost 960 francs a year for a four-member family to live in Melhouse, whereas a laborer or dyer in the local cotton industry could earn at most 450 francs a year (Heywood, 1988, p. 108). As for handloom weavers in Rouen, the annual cost of living, 912 francs, was greater than the standard parental income, 861 francs (ibid., pp. 108–109). In the mid-1840s, several school inspectors found that school attendance in rural areas declined when bread became expensive (ibid., p. 88). This indicates a low priority of education among household expenditures.

Given the economic importance of children, it is not surprising that France began its education reform while accepting child employment. Evening classes were held by the municipality in Mulhouse in the early 1830s, and in Lille in the early 1840s, although they were not very successful (ibid., pp. 244–245). The child labor law passed in 1841 provided a certain level of education for children under 12 years, while regulating the minimum age of employment to 8 (Ogg and Sharp, 1926, p. 382). Faced with working class opposition, this first child labor law was ineffective, and the child labor law passed in 1874 was more effectively enforced by a special inspectorate (Nardinelli, 1990, pp. 126–127, 137).

2.1.5 Germany

In Prussia, the first school edict was issued as the General-Landschul-Reglement of 1763, with the aim of providing compulsory education for all children aged 5 to 13 (Melton, 1988, p. 174).

15In 1840, 756,464 boys were in communal primary schools in winter; however, the number of boys attending declined to 463,464 in summer, possibly a season when their labor was especially needed at home (ibid.).
However, such an early attempt was fruitless. The four schools in Berlin, Kloster Berg, Stettin, and Königsberg “produced far too few graduates to meet the goals of the reform (ibid., p. 175–176).” One of the reasons for the failure was the need for child labor for the poor. Indeed, summer school was impossible to run because rural families, especially in summer, needed their children to be working (ibid., p. 176).

Similarly, 40 out of about 100 petty schools in Bremen were closed between 1788 and 1810, during which children found new work opportunities in tobacco processing (Cunningham, 1995, p. 103). This indicates that a large proportion of households in Bremen gave priority to child labor over child education at that time and probably in earlier periods as well. During the period 1800–1846, children under the age of 14 accounted for 10%–20% of factory workers (Lee, 1978, p. 466). In particular, until the 1850s, children were of great assistance in textile factories, where their hands were suitable for picking up the threads (ibid.). Even around the end of the century, a rather huge share of children was engaged in part-time work. A survey by the Prussian government at that time reports that 40% of all school children spent at least 18 hours of the week working (Berghahn, 1994, p. 81).

Under these circumstances, early education reform encountered difficulties in its enforcement. The Prussian law of 1839 obliged children under the age of 16 to attend five hours of school a day, while limiting their work time to ten hours a day (Ogg and Sharp, 1926, p. 389). However, the execution was unpleasant and impossible for authorities such as the local police, teachers, and clergymen and, accordingly, the schooling time was curtailed to three hours a day in 1853 with limited enforcement (ibid., pp. 389–390). One can imagine from these events that the local authorities had sympathy for poor children who were working for their families. Similar results were observed in other German states such as Bavaria, Baden, Württemberg, and Hesse; they failed to restrict child labor and spread public schooling (ibid., p. 390).

2.2 Public Education as a Right

More recently, compulsory education is widely perceived to be essential for securing the minimum living standard in advanced countries. In fact, children in OECD countries attend 11.9 years of schooling on average (OECD, 2006, p. 41), exceeding or very close to the duration of compulsory
education in most cases.\footnote{International data for the duration of compulsory schooling are provided online by the UNESCO Institute for Statistics (http://www.uis.unesco.org/).} This fact, along with the evidence given in Section 2.1, suggests that the notion of compulsory education for their citizens had shifted from a duty to a right.\footnote{This point has been emphasized by many scholars. See Shibuya (1988, p. 167) for Japan’s case, and Weiner (1991) for a more comprehensive analysis.}

The acceptance of compulsory education became widespread around the turn of the 20th century, although the timing varied across countries. In Japan, the aforementioned classroom attendance rate at primary level rose to 89.63\% in 1913 (Japanese National Commission for Unesco [sic], 1966, p. 64). This implies that most school-age children kept going to primary schools after being enrolled. In the United States, 32.3\% of children aged 14 to 17 were enrolled in secondary schools in 1920, a sharp increase from 6.7\% in 1890 (Church, 1976, p. 289). In \textit{Secondary Education for All}, published in 1922, the British Labour Party suggests the need for post-elementary education for working-class children (Hopkins, 1994, p. 321).\footnote{In 1950, the share of secondary school pupils in the 10–19 age group reached 33.7\% in England and Wales (Flora et al., 1983, pp. 626).}

This hypothesis is supported by Nardinelli (1990), who investigates the historical evidence for Britain, France, Germany, Japan, and the United States. He argues that “As working-class incomes rose, parents kept children out of the labor force until later and later ages. The production of well-educated children became a realistic and desirable alternative for the working-class family in the late nineteenth and twentieth centuries. The desire of parents to improve the quality of their children would have been sufficient, in the absence of child labor laws, to remove children from factories in the long run (p. 149).” The minor effect of child labor laws on school levels is also found by Landes and Solmon’s (1972) regression analysis for the U.S. states in the late 19th century.

While there were various industrialization forces underlying the increased demand for public education, two of them are emphasized as particularly important in this paper. The first force is the rise in the net return on education investment; it motivated parents who have far-sighted views on child-rearing. By 1880, Félix Pécaut, the apostle of progressive education in France, reported that families began to notice the advantage of obtaining a school certificate for finding jobs, inducing parents to permit longer school attendance by their children (Weber, 1976, pp. 73, 328). Likewise, high school education in the United States began to be regarded as a means to secure white-collar occupations emerging at the end of the 19th century (Church, 1976, p. 290).
In Britain at that time, secondary and university education were increasing their importance in achieving or maintaining social status (Glass and Grebenik, 1965, p. 117). In terms of the cost, technological progress reduced the forgone wages of children attending school. Lee (1978, p. 467) argues that upgraded machines made a significant contribution to the 30% decline in child labor of German textile factories between 1846 and 1853, a period when child labor legislation was poorly enforced.

The second force is the improvement in parental incomes; it abated households’ demand for child labor to secure current subsistence and possibly allowed them to be concerned about the future well-being of their children. In the United States, the aforementioned spread of secondary education was in part because of rapid, widespread growth in per capita income after 1900, which would have allowed more families to keep sending their children to school without relying on their forgone wages (Church, 1976, pp. 289–290). Similarly in France, fewer children were in the labor force in the second half of the 19th century, during which real wages kept rising (Heywood, 1988, p. 109; Nardinelli, 1990, p. 146). In Britain, the average real wage over the period 1900–1909 increased to almost twice the level of the period 1850–1859 (Polland, 1978, p. 171). The improved living standard is consistent with judicial records in the late 19th century. Although truancy was the second most common violation between 1888 and 1916, the number of prosecutions reduced by half between 1883 and 1910 (Hurt, 1979, p. 203). Finally, in Japan, the real GDP per capita more than doubled between 1900 and 1955 (Maddison, 2001, p. 206), and the entrance rate for higher secondary schools exceeded 50% in 1955 (Japanese National Commission for Unesco [sic], 1966, Table 27, p. 78).

2.3 Population Aging and Gray Power

It is not difficult to imagine that old generations have an influential political voice in aging democracies, as they would command a large share among all the voters. For fear of excessive gray power in the political process, there is a debate in Japan about the lowering of the voting age from 20 to 18 (cf. Nishida, 2008; Nihon Keizai Shimbun, 2008). According to the data presented by Preston (1984), the postwar aging of the U.S. society is accompanied by improvements in the relative well-being of the elderly to children.\footnote{This would be to some extent the result of the lobbying activities by some demographic groups such as the American Association of Retired Persons (AARP) and the National Council on Aging (NCOA). They expanded their...}
One of the premises of this paper is that population aging hinders the allocation of public resources from meeting the young’s growing demand for education. The premise does not necessarily entail the rise in a public spending bias toward the old. Even if the bias improved, the budget allocated to public education may be insufficient to shorten the gap with the private demand for education. Nevertheless, it would be reasonable to consider that the generational bias such as this is likely to raise the burden of private education.\(^{20}\)

By looking at social spending growth of ten OECD countries between 1960 and about 1980, O’Higgins (1988, pp. 218–220) finds the general lack or loss of priority of public education over pensions and health care, apart from the change in age structure. Poterba (1997, 1998), using U.S. panel data between 1961 and 1991, suggests that state and local spending on education per child is inversely correlated with the fraction of the elderly population within states. Employing Poterba’s approach, Ohtake and Sano (forthcoming) find similar patterns across Japanese prefectures after the 1990s. Neither Poterba nor Ohtake and Sano considers higher education, which would gain less support from the elderly than would basic education would (cf. Vinovskis, 1995, p. 208). Perhaps government expenditure on higher education would be more vulnerable to the pressure from population aging. For earlier periods, the regression results of Lindert (1994), based on the 1880–1930 panel of 21 (mostly currently advanced) countries, indicate that aged countries tend to have large public spending on social transfers including pension and health subsidies, and low primary- and secondary-school enrollment rates.

3 The Model

Consider a one-sector, closed economy operating over an infinite discrete-time horizon. In perfectly competitive environments, producers generate a single homogeneous final good by employing labor. Growth in output per worker is driven by investment in human capital and technological progress.

The economy has an overlapping-generations structure. A new generation, consisting of a continuum of \(N_t\) individuals, is born at the beginning of every period \(t\) and lives for three periods. Thus, political power in the 1960s (Longman, 1987, p. 234), and the AARP, in particular, has grown to be the largest politically active group, comprising 35 million members (The Economist, 2005, p. 40).

\(^{20}\)Pampel (1994) examines age inequality in public spending, which, however, does not include educational expenditures. Based on annual data of 18 advanced industrial democracies between 1959 and 1986, the regression analyses reveal the following political conditions under which population aging raises a public spending bias to the old: weak leftist parties and the lack of class-based corporatism.
there are three generations (children, adult, and elderly) at each point in time. The government imposes a tax on the adult generation to support the other members of the population.

### 3.1 Firms

The final good is produced in one sector, where competitive firms employ adult and child workers. The economy has no access to physical capital and land.\(^{21}\)

Let \(H_t\) and \(L_t\) be the aggregate levels of adult labor in efficiency units and of child labor, respectively, employed in period \(t\). The aggregate level of output produced in period \(t\), \(Y_t\), is generated by the production function

\[
Y_t = A_t(H_t + \phi_t L_t),
\]

where \(A_t > 0\) is the level of technology, and \(\phi_t\) is the marginal productivity of child relative to adult labor in period \(t\). For simplicity, it is assumed that \(\phi_t A_t = \bar{w}\), where \(\bar{w} > 0\) is a fixed value; that is, the marginal productivity of child labor in absolute terms is stationary at \(\bar{w}\) over time.\(^{22}\)

Without loss of generality, the price of the final good is normalized to unity. Standard profit maximization reveals that the wage rate per unit of efficiency labor in period \(t\), denoted as \(w_t\), equals the marginal labor productivity; i.e., \(w_t = A_t\). On the other hand, \(\bar{w}\) equals the competitive wage rate per unit of child labor.

### 3.2 The Government

As a form of taxation, the government imposes public service obligations on adult individuals. Each of them is obliged to serve a fixed time fraction \(\tau \in (0,1)\). The resulting government revenue in period \(t\), \(\tau N_t\), is spent entirely on education for children and the welfare of the elderly in the same period.

The level of public (compulsory) education in period \(t\), \(e_t^g\), is defined in terms of the government expenditure per child-rearing household in the period.\(^{23}\) This implies that \(e_t^g \in [0, \tau]\). On the other hand, the old-age related service in period \(t\), \(x_t\), is defined as the government budget allocated to

\(^{21}\)Section 5 considers how the basic results would be affected by relaxing the assumptions on production environments.

\(^{22}\)Without the stationarity of \(\phi_t A_t\), the basic results would be retained as long as the relative marginal productivity of child labor, \(\phi_t\), is eroded by technological progress. As will be shown later, the decline in \(\phi_t\) induces parents to shift from the use of child labor to the education of children.

\(^{23}\)The case with per child expenditure, an alternative formulation, is considered later in Footnote 40.
each old individual in the period. It follows that

\[ x_t = (\tau - e_t^2)n_{t-1}, \]  

where \( n_{t-1} = N_t/N_{t-1} \) denotes the ratio of the adult to the old population in period \( t \). A rise in \( e_t^2 \) shifts the government spending from the elderly to children, leading to a reduction in \( x_t \). A rise in \( n_{t-1} \) increases the number of taxpayers per old individual, and thereby increases \( x_t \).

### 3.3 Households

#### 3.3.1 The Environment

Consider the life stream of an individual \( i \ (= a, b) \) of generation \( t \) (born in period \( t - 1 \)). In the first period (childhood), the individual works and/or receives education. These decisions are made by his/her single parent, who uses child labor income, and also by the government.

In the second period (adulthood or parenthood), the individual serves for the government and allocates the “disposal” time \( 1 - \tau \equiv T \) between working and child-rearing. Depending on the education in childhood, the individual acquires \( h_t^i \) units of skills in this period. Meanwhile, the individual raises \( n_t^i \) units of children by spending a time fraction \( \delta + e_t^1 \) per child, where \( \delta > 0 \) is the fixed cost and \( e_t^1 \) is the level of private education. Each of these children equally supplies \( l_t^i \) units of raw labor to earn \( \bar{w}l_t^i \). These wage incomes are spent on their family consumption, \( c_t^i \), with no savings. It follows that the budget constraint is

\[ c_t^i \leq z_t^i[T - n_t^i(\delta + e_t^i - \omega_t^i)], \]  

where \( z_t^i \equiv w_t h_t^i \) denotes parental potential income, and \( \omega_t^i \equiv \bar{w}l_t^i / z_t^i \) is the child labor income in terms of the parent’s time.

In the third period (elderhood), the individual retires and consumes \( x_{t+1} \) units of the old-aged related service for free of charge. The old individual is economically independent of his/her descendents, and no direct transfers occur between them. There are no heterogeneities among the retired within generations.

Preferences are defined over consumption in adulthood, the quantity of children, the average of their future potential incomes, and the old-aged related service provided in elderhood. The lifetime utility function of an individual \( i \) of generation \( t \) is given by

\[ u_t^i = (1 - \alpha) \ln c_t^i + \alpha [\ln n_t^i + \ln(\bar{w}l_{t+1}h_{t+1}^i)] + \min(x_{t+1}, \gamma), \]

\( \text{(4)} \)
where \( \alpha \in (0, 1) \), and \( \gamma > 0 \) is a saturation point above which \( x_{t+1} \) has no effect on \( u_t^i \). Thus, abundant public resources incline the elderly to bear the costs of other services, and in this sense \( \gamma \) measures the degree of selfishness. The elderly with an extremely large \( \gamma \) essentially have a linear preference over \( x_{t+1} \), implying no altruism.

### 3.3.2 The Formation of Child Labor and Human Capital

Once determined through a political process, the level of public education \( e^g_t \) is enforceable on any child in period \( t \). On the ground that skill acquisition is a laborious process, child labor supply \( l_t^i \) is formulated by

\[
l_t^i = l(e_t^i, e_t^q) \begin{cases} 
  > 0 & \text{if } e_t^i \in [0, \hat{e}_t); \\
  = 0 & \text{if } e_t^i \geq \hat{e}_t,
\end{cases}
\]

where \( l_1(e_t^i, e_t^q) < 0 \) and \( l_2(e_t^i, e_t^q) < 0 \) in the first case, and \( l(0, 0) = 1.24 \). The critical value \( \hat{e}_t \) is set to be nonnegative and is given by a single-valued, decreasing function \( \hat{e}(e_t^q) \).\(^{25}\) The properties of the function are represented by the Child Labor Frontier, on which \( e_t^i = \hat{e}(e_t^q) \), in the lower right panel of Figure 4. Any pair \((e_t^i, e_t^q)\) on and outside the frontier exhausts the time of children, leading to \( l(e_t^i, e_t^q) = 0 \).

It follows that the earnings of a child \( i \) in period \( t \), \( \omega_t^i \equiv \bar{w}l_t^i/z_t^i \), depend on the levels of education as well as of parental potential income:

\[
\omega_t^i = \frac{\bar{w}l(e_t^i, e_t^q)}{z_t^i} \equiv \omega(e_t^i, e_t^q, z_t^i),
\]

where the function \( \omega \) is nonincreasing in each argument. In light of (3), this shows that the forgone wages of children are the costs of schooling for their parents and that growth in parental potential income \( z_t^i \) makes child labor less valuable for them.

Education in childhood is the only means to acquire advanced skills in adulthood. The level of efficiency labor of an adult individual \( i \) in period \( t + 1 \) is determined by

\[
h_{t+1}^i = h(e_t^i, e_t^q),
\]

where \( h(0, 0) = 1 \), \( \lim_{e_t^i \to \infty} h_1(e_t^i, 0) = 0 \), \( h_j(e_t^i, e_t^q) > 0 \) and \( h_{jk}(e_t^i, e_t^q) < 0 \) for any \((e_t^i, e_t^q) \geq 0\) and \( j, k = 1, 2 \). These properties have the following meanings. First, no education investment

\(^{24}\) For example, if \( l(e_t^i, e_t^q) = \max(1 - e_t^i - \epsilon e_t^q, 0) \), where \( \epsilon > 0 \), then \( \hat{e}(e_t^q) = \max(1 - \epsilon e_t^q, 0) \). Thus, the set \([0, \hat{e}_t] \) in (5) may be empty depending on \( e_t^q \).
results in one unit of efficiency labor. Second, the function is increasing and strictly concave with respect to each input. Third, the marginal productivity of private education decreases with the level of public education. These properties generate a substitutional relationship between private and public education.

In what follows, it is further assumed that the marginal productivity of private education, evaluated on the Child Labor Frontier, is sufficiently large. That is,

\[
\frac{\partial \ln h(e^i_t, e^g_t)}{\partial e^i_t} \bigg|_{e^i_t = \hat{e}(e^g_t)} > \delta^{-1} \quad \forall e^g_t \in [0, \tau].
\]

As will become clear, this condition assures that high-income parents do not use child labor regardless of the degree of the child labor regulation.

### 3.3.3 Optimization

Adult individuals maximize their own utilities by taking market prices and public policies as given. Substituting (6) into (3) and then the result and (7) into (4), an adult individual \( i \) in period \( t \) chooses the number of children to bear, \( n^i_t \), and the level of private education, \( e^i_t \), in such a way that

\[
\{n^i_t, e^i_t\} = \arg \max \left\{ (1 - \alpha) \ln [T - n^i_t(\delta + e^i_t - \omega(e^i_t, e^g_t, z^i_t))] + \alpha \ln [n^i_t h(e^i_t, e^g_t)] \right\},
\]

subject to \( T \geq n^i_t(\delta + e^i_t) \) and \( e^i_t \geq 0 \). The first inequality is the nonnegativity constraint on the time for his/her labor market participation.

The objective function is strictly concave with respect to \( n^i_t \). The first-order condition for an interior solution reveals that the net cost of child-rearing is

\[
n^i_t(\delta + e^i_t - \omega^i_t) = \alpha T.
\]

The relationship between \( n^i_t \) and \( e^i_t \) indicates that the individual faces a trade-off between the quantity and the quality of children. Equally important, an increase in the relative wage of child labor, \( \omega^i_t \), induces him/her to have more children.

Given (9), the constraint for the labor market participation is unbinding for any \( e^i_t \geq 0 \) as long as \( z^i_t \geq \bar{w}/[\delta(1 - \alpha)] \equiv z^{\min} \). It should be emphasized that under this condition, the relative potential income of child labor, \( \bar{w}/z^i_t \), is lower than the fixed cost of child-rearing, \( \delta \). That is, child-rearing is more or less costly in net, and child labor income alone does not motivate parents to have children.
In order to analyze the optimization with respect to \( e_t^i \), substitute (9) for \( n_t^i \) in (8). The solid line of Figure 2 depicts the resulting objective function \( V(e_t^i, e_t^g, z_t^i) \) on the space \( e_t^i \geq 0 \), for given \( e_t^g \in [0, \tau] \) and \( z_t^i \geq z_{\text{min}} \). The function exhibits strict concavity above the kinked point \( \hat{e}(e_t^g) \), where child labor income \( \omega(e_t^i, e_t^g, z_t^i) \) is just zero. The locally-optimal level \( e(e_t^g) \) has the following properties.

**Lemma 1** Under (A1), the optimal value of \( e_t^i \) on the interval \([\hat{e}(e_t^g), \infty)\) is given by a single-valued function \( e(e_t^g) \) such that \( e(e_t^g) > \hat{e}(e_t^g) \) and \( e'(e_t^g) < 0 \) \( \forall e_t^g \in [0, \tau] \).

*Proof.* See Appendix 1. \( \square \)

Thus, \( e(e_t^g) \) is the best choice among the levels of private education that do not allow child labor. The lemma does not ensure the optimality of \( e(e_t^g) \) on \( \mathbb{R}_+ \) when the complement set \([0, \hat{e}(e_t^g)]\), where child labor is used, is non-empty (i.e., \( \hat{e}(e_t^g) > 0 \)). This situation is considered in Figure 2, which graphically represents how private education is induced by growth in \( z_t^i \) with \( e_t^g \) kept constant. As \( z_t^i \) increases, the opportunity cost of schooling, \( \omega(e_t^i, e_t^g, z_t^i) \), decline on \([0, \hat{e}(e_t^g)]\) and accordingly the function \( V \) shifts downward on this interval. Thus, sufficiently large \( z_t^i \) makes the level \( e(e_t^g) \) optimal on \( \mathbb{R}_+ \). This is the rationale for the lemma below.

**Lemma 2** If \( z_t^i \) is sufficiently large under (A1), the function \( e(e_t^g) \) yields the globally optimal level of private education, \( e_t^i \), for each \( e_t^g \in [0, \tau] \).

*Proof.* See Appendix 1. \( \square \)

This result, along with Lemma 1, suggests that \( e(e_t^g) \) is the reaction function of private education for high-income households. In response to a supply of public education, their optimal strategy is to spend \( e(e_t^g) \) units of time per child on education, with no use of child labor.

On the other hand, optimization is more complicated for low-income households with small \( z_t^i \). They may use child labor while investing in private education since a globally optimal solution may emerge somewhere on \([0, \hat{e}(e_t^g)]\) in Figure 2.\(^{26}\) In order to simplify the analysis in this situation, suppose that there is an income-dependent discount factor by which the poor do not care about

\(^{26}\)For example, consider the case with no public education provided (i.e. \( e_t^g = 0 \)). Equation (35) in Appendix 1 shows that if \( \delta - \bar{u}/z_t^i \) is positive but sufficiently small, then \( V(0, 0, z_t^i) > V(e(0), 0, z_t^i) \), noting that \( e(0) > \hat{e}(0) \geq 0 \). In this case, no private education is at least more optimal than choosing \( e(0) \). However, \( e_t^i = 0 \) may be sub-optimal on \([0, \hat{e}(e_t^g)]\), where the slope \( V_t(e_t^i, e_t^g, z_t^i) \) depends on the quantitative properties of \( l(e_t^i, e_t^g) \) and \( h(e_t^i, e_t^g) \).
the future incomes of their children. Namely, let $\ln(w_{t+1}h^i_{t+1})$ in (4) be multiplied by

$$\beta^i_t = \beta(z^i_t) \equiv \begin{cases} 0 & \text{if } z^i_t \leq z^*; \\ 1 & \text{if } z^i_t > z^*, \end{cases}$$

(A2)

where $z^* > z^{\min}$ is a large value of parental potential income for which Lemma 2 holds.\textsuperscript{27} It follows that the second term of the utility function is modified to $\alpha[\ln n^i_t + \beta^i_t \ln(w_{t+1}h^i_{t+1})]$. This rules out the case in which children engage in both work and private education.

It follows that the optimal decision on private education is summarized as

$$e^i_t = \begin{cases} 0 & \text{if } z^i_t \in [z^{\min}, z^*]; \\ e^g_t & \text{if } z^i_t > z^*, \end{cases}$$

(10)

where $e(e^g_t) > \dot{e}(e^g_t)$ and $e'(e^g_t) < 0 \forall e^g_t \in [0, \tau]$. The properties of the function $e(e^g_t)$ are graphically represented by the lower-right panel of Figure 4. Under Assumption (A1), the solid curve $e^i_t = e(e^g_t)$ lies outside the Child Labor Frontier on the range where $e^g_t \in [0, \tau]$. This implies that choosing a level $e(e^g_t)$ for private education leaves no room for child labor, regardless of the degree of compulsory schooling. Equally important, the positive slope of the curve indicates a substitutional relationship between private and public education.\textsuperscript{28}

Substituting (6) and (10) into (9),

$$n^i_t = \begin{cases} \frac{\alpha T}{\delta-\omega(0; e^g_t, z^i_t)} = n^p(e^g_t, z^i_t) & \text{if } z^i_t \in [z^{\min}, z^*]; \\ \frac{\alpha T}{\delta+e(e^g_t)} = n^r(e^g_t) & \text{if } z^i_t > z^*, \end{cases}$$

(11)

where the effect of public education on fertility changes qualitatively between the two cases. As depicted by Figure 3, the function $n^p(e^g_t, z^i_t)$, which yields the fertility rate for low-income households, is decreasing in $e^g_t$. An increase in $e^g_t$ reduces child labor, $l^i_t = l(0, e^g_t)$, and thus child labor income, $\omega^i_t = \tilde{w}l^i_t/z^i_t$. The resulting rise in the child-rearing cost induces fertility decline (upper left panel).\textsuperscript{29} On the other hand, Figure 4 shows that the function $n^r(e^g_t)$, which yields the fertility rate for high-income households, is increasing in $e^g_t$. An increase in $e^g_t$ reduces the household spending

\textsuperscript{27}Assumption (A2) incorporates the insight of Fisher (1930, pp. 73–74), who argues that poverty is likely to induce impatience. Since the realization of the quantity of children precedes that of the quality, it appears plausible that low-income households attach greater importance on the former.

\textsuperscript{28}Recall that the unbinding child labor regulation is consistent with the words of Nardinelli (1990, p. 149), quoted in Section 2.2. The second derivative of $e(e^g_t)$ is generally ambiguous because it depends on the third derivatives of the function $h$. See (36) in Appendix 1 for details.

\textsuperscript{29}$\dot{e}^g_t$ in Figure 3 is the minimum value of $e^g_t$ for which $l(0, e^g_t) = 0$. Thus, for any $e^g_t \geq \dot{e}^g_t$, no child labor is used, and $n^p(e^g_t, z^i_t)$ is constant at $\alpha T/\delta$. 

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on education, \( e_i = e(e_i^9) \), and promotes fertility through the quality-quantity trade-off (upper left panel). Public education in this case works as education subsidies or a child care service.

### 3.4 Demographic Structure and Technological Change

Let \( N_t^i \) be the size of the adult population belonging to group \( i \) (= \( a, b \)). Since there is no within-group heterogeneity,

\[
N_{t+1}^i = \int_0^{N_t^i} n_t^i dv = n_t^i N_t^i,
\]

where the initial value \( N_t^i > 0 \) is historically given. The entire size of the adult population in period \( t \), \( N_t \), is the sum of \( N_t^a \) and \( N_t^b \). Hence, the average number of children per parent in period \( t \), \( n_t \), is

\[
n_t = \frac{N_{t+1}^i}{N_t} = q_t n_t^a + (1 - q_t) n_t^b,
\]

where \( q_t = N_t^a / N_t \) is the share group \( a \) in the adult population in period \( t \). Then, it follows that one may write

\[
q_{t+1} = \frac{n_t^a}{n_t} q_t,
\]

showing that \( q_t \) decreases over time as long as group \( a \) is less fertile than group \( b \).

In light of (7), the average level of efficiency units of adult labor in period \( t + 1 \), denoted as \( h_{t+1} \), is

\[
h_{t+1} = q_{t+1}h(e_t^a, e_t^9) + (1 - q_{t+1})h(e_t^b, e_t^9).
\]

Thus, \( h_{t+1} \) is determined fully by private and public investment in education, \( e_t^i \) and \( e_t^9 \) respectively, and the share of group \( a \) among children in period \( t \), \( q_{t+1} \). Suppose that \( h_{t+1} \) fully governs technological progress between periods \( t \) and \( t + 1 \). That is,

\[
g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = g(h_{t+1}),
\]

where \( g \) is a continuous function such that \( g(0) = 0 \) and \( g'(h_{t+1}) > 0 \ \forall h_{t+1} > 0 \). This formulation implies no depreciation in accumulated knowledge and that technological progress is driven by the quality rather than the quantity of the population. The scale effect of the population, argued by Kuznets (1960) and Kremer (1993) among others, would be appropriate for analyzing the development process in the pre-modern period, and its omission appears to be plausible for the periods after the Industrial Revolution—a focus of this paper.
As follows from (14) and (15), the potential income of an adult individual \( i \) in period \( t + 1 \), \( z_{i,t+1} \), is given by

\[
z_{i,t+1} = (1 + g_{t+1})A_t h_{i,t+1} = A_t \eta(e_{i,t}^i, e_{i,t}^q, e_{i,t}^j, q_{t+1}),
\]

where \( i, j = a, b \) and \( i \neq j \). The function \( \eta \) incorporates the positive externalities of private and public education on the growth rate of technology \( g_{t+1} \) as well as their direct impact on individual human capital \( h_{i,t+1} \). The individual takes into account these two channels in forming his/her attitude toward public education.

As shown below, endogenously introduced public education promotes technological progress and thereby accelerates the decline in the marginal productivity of child relative to adult labor, \( \phi_t = \bar{w}/A_t \), triggering the shift from the use of child labor to the education of children.

### 3.5 The Political System

#### 3.5.1 Political Groups

The allocation of government resources in period \( t \) reflects the preferences of three interest groups: the young, divided into groups \( a \) and \( b \), and the old.\(^{30}\) Their desirable levels of public education are denoted as \( e_{t}^{ga} \), \( e_{t}^{gb} \), and \( e_{t}^{go} \), respectively. The provided level of public education is determined by a weighted average such that

\[
e_{t}^{g} = \lambda_t[\theta e_{t}^{ga} + (1 - \theta)e_{t}^{gb}] + (1 - \lambda_t)e_{t}^{go},
\]

where \( \lambda_t \) denotes the political power of the young in period \( t \), and \( \theta \in (0, 1) \) indicates the political power of group \( a \) in the adult generation. Once determined, the level of public education \( e_{t}^{g} \) is enforceable on any children in period \( t \).

Along the line with the discussions in Sections 1–2, \( \lambda_t \) is assumed to increase with the fraction of the young to the old population in period \( t \), \( n_{t-1} \). This is formulated by a function such that

\[
\lambda_t = \lambda(n_{t-1}),
\]

where \( n_{t-1} \) is the population of children in period \( t \), and the function \( \lambda \) is defined as

\[
\lambda(n_{t-1}) = \begin{cases} 
\lambda'(n_{t-1}) > 0 & \forall n_{t-1} > 0, \\
\lim_{n_{t-1} \to 0^+} \lambda(n_{t-1}) = 0 & \lim_{n_{t-1} \to \infty} \lambda(n_{t-1}) = 1.
\end{cases}
\]

In light of Pampel (1994) mentioned in Section 2.3, the quantitative properties of the function \( \lambda \) would depend

\[^{30}\text{In what follows, adult individuals are sometimes referred to as “young households” or “the young.” Despite the economic structure with three overlapping generations, this way of bipolarization makes sense because individuals in childhood make neither economic nor political decisions.}\]
on country-dependent institutional conditions such as the degrees of corporatism and leftist parties. This point will be taken up later in Section 5.

Furthermore, \( \theta \) is limited on the range

\[
0 < \theta < \min(\hat{\theta}^g / \tau, 1),
\]

where \( \hat{\theta}^g \) is, as shown by Figure 3, the critical level of public education above which no room is left for the poor to use child labor; i.e., \( l(0, e^g) = 0 \) if and only if \( e^g \geq \hat{\theta}^g \). The restriction on \( \theta \) indicates some degree of political influence exerted by the poorer group \( b \). The rationale for this postulation comes from the possibility that the strict enforcement of compulsory schooling provokes either boycotts or riots by the poor. Faced with these fears, group \( a \) finds better to make a compromise between child labor and child education so as to promote universal education. This way of political process appears to be consistent with the historical evidence presented in Section 2.1. In Japan, early education reforms were sometimes accompanied by riots against compulsory schooling. In Western Europe, the opposition from the poor was behind the half-time system in education. Most of those escapees or opponents, who needed child labor rather than child education, were not enfranchised in the era of the early education reforms.\(^{31} \)

On the basis of these facts, the political power of group \( b \) is assumed to be effective regardless of their voting status.

### 3.5.2 Individuals’ Policy Preferences

Adult individuals in period \( t \) choose their preferred education policy so as to maximize the utility specified by (4) and (A2). They take into account the effects of public policies on market prices, so that the wage rate \( w_{t+1} = A_{t+1} \) as well as the discount factor \( \beta^i_t \) is added to (8). Therefore, it follows from (16) that

\[
e^q_i = \arg \max \left\{ (1 - \alpha) \ln \left[ T - n^i_t (\delta + e^i_t - \omega (e^i_t, e^q_t, z^i_t)) \right] + \alpha \beta(z^i_t) \ln \eta(e^i_t, e^q_t, e^j_t, q_{t+1}) \right\},
\]

subject to \( e^q_i \in [0, \tau] \). The function \( \eta \) incorporates the positive externality of public education on the wage rate \( w_{t+1} \) as well as their direct impact on individual human capital \( h^i_{t+1} \). It is assumed that those parents can choose the level of private education after the provided level of public education

\(^{31} \)As for Japan in 1902, the enfranchised people, who had to pay some amount of taxes, accounted for only 2.2% of the entire population (Soma, 1986, p. 31). In Western Europe, the franchise extended towards the turn of the 19th century (Flora et al., 1983, Ch. 3).
\( e_t^g \) is revealed. By contrast, they must decide the number of children in advance, according to (11), with perfect foresight into \( e_t^g \).\(^{32}\)

Next, turn to the policy preferences of old individuals in period \( t \). Substituting (2) into (4), their desirable level of public education is,

\[
e_t^{g_0} = \arg \max \left\{ \min \left[ (\tau - e_t^{g_0})n_{t-1}, \gamma \right] \right\}
\]

subject to \( e_t^{g_0} \in [0, \tau] \). Hence, there exists a critical level of public education, \( \tilde{e}_t^g \geq 0 \), below which the elderly’s utility is constant at \( \gamma \). That is,

\[
\tilde{e}_t^g = \max \left( \tau - \frac{\gamma}{n_{t-1}}, 0 \right) \equiv \tilde{e}^g(n_{t-1}).
\]

(21)

The aged group, when indifferent, is assumed to accept the policy proposed by the young. Thus, \( \tilde{e}_t^g \) is the maximum amount of public education budgets acceptable for the old and, in view of (17),

\[
e_t^{g_0} = \min \left\{ \theta e_t^{g_0a} + (1 - \theta) e_t^{g_0b}, \tilde{e}_t^g \right\}.
\]

(22)

Since \( \tilde{e}_t^g \) is a nondecreasing function of \( n_{t-1} \), equation (22) shows that the higher the number of taxpayers per aged person, the more generous the old tend to be toward education policy. Furthermore, since \( \tilde{e}^g(n_{t-1}) < \tau \forall n_{t-1} > 0 \), the old always claim a certain amount of public resources to enjoy their own consumption.

4 The Process of Industrialization

This section demonstrates that the described economy undergoes the process of industrialization consistent with the evidence in Sections 1–2. In particular, the underlying evolution of public education and fertility is depicted in Figure 5 and is basically consistent with the historical records in Table 1.\(^{33}\)

Suppose that in period 0, adult individuals of group \( a \) are endowed with non-basic skills such that \( h_0^a > h_0^b = 1 \), where \( h_0^a \) is the minimum value of \( h(e_t^g) \) on the interval \([0, \tau]\). The economy starts out with a share of group \( a \) in the adult generation and a level of technology such that

\[
q_0 \in (0, \varepsilon); \quad A_0 \in (A^{\min}_t, z^*).
\]

(A3)

\(^{32}\)As will be discussed later in Footnote 34, the timing of childbirth changes the reason for the young poor to oppose child labor regulations and compulsory education.

\(^{33}\)The ratio \( e_t^g/\tau = e_t^g N_t/\tau N_t \) in Figure 5 corresponds to \( E/G \) in Table 1 (i.e., the share of public education in total government expenditure).
where $\varepsilon > 0$ is a sufficiently small value, and $A_{\text{min}} \equiv \max(z_{ \text{min}}, z^*/h_0^0)$. Then it follows that $z_0^a > z^* > z_0^b$, where $z_t^i = A_t h_t^i$. This initial income inequality, along with (10) and (15), reveals that

$$z_t^a \geq \max(z^*, z_t^b) > z_{\text{min}} \quad \forall t \geq 0,$$

(23)

where $z_0^a > z^*$. Thus, children of group $a$ never work and, instead, engage in skill-acquisition in any period. Furthermore, adult individuals in any period devote a positive time fraction in the labor market (i.e., no complete specialization in child-rearing).

Under these circumstances, the process of industrialization is divided into two stages, between which group $b$, majority of the young, alters its child-rearing strategies. In Stage I ($z_t^b \leq z^*$), public education is undesired by members of group $b$, who need child labor to secure current consumption. The enforcement of compulsory schooling regulates child labor to some extent and also promotes technological progress that reduces the relative productivity of child to adult labor. These two forces together adversely affect the return on child labor and thus discourage average fertility. The accelerated technological progress expedites the departure from Stage I.

In Stage II ($z_t^b > z^*$), public education is desired by all young households, who are wealthy enough to abandon child labor. Their positive attitudes toward human capital investment bring about a major education reform and fertility decline—an event corresponding to the change in $e_t^2/\tau$ and $n_t$ between periods $t^* - 1$ and $t^*$ in Figure 5. The increased share of the elderly population, however, squeezes the flow of government resources to education. To make up for the reduced public support, young households spend more on private education by having fewer children. This process leads to a vicious cycle of population aging and a decline in public education, through which growth performance may be deteriorated.

4.1 Stage I: Public Education as a Duty

The economy develops in Stage I on the time interval $[0, t^*)$, where $t^*$ is the critical period in which $z_t^b$ exceeds $z^*$ for the first time. That is, in view of (23),

$$z_t^a > z^* \geq z_t^b > z_{\text{min}} \quad \forall t \in [0, t^*).$$

(24)

As shown below, in this circumstance, compulsory schooling works as a child labor regulation that reduces consumption of the low-income group $b$ and thereby discourages childbirth. The poor’s
negative stance toward education reform leads to a half-time system that allows child labor to some extent.

4.1.1 Private Education, Child Labor, and Policy Preferences

Since (A2) reveals that in this stage $\beta_t^a = 1 > \beta_t^b = 0 \forall e_t^a \geq 0$, only adult individuals of group $a$ have an incentive to invest in child education. Furthermore, it follows from (5) and (10) that the chosen levels of private education and child labor are, $\forall e_t^a \in [0, \tau]$,

$$e_t^a = e(e_t^a) > e_t^b = 0;$$

$$l_t^b = l(0, e_t^b) \geq l_t^a = l(e(e_t^a), e_t^a) = 0.$$  \hspace{1cm} (25)

Thus, adult individuals of group $a$ adjust spending on private education in response to the government subsidies for education. They do not send their children to work regardless of child labor regulation. Since $z_t^a$ is greater than $z^*$ in this stage, sending their children to work, rather than to school, does not compensate for their future loss in human capital. By contrast, the low-income group $b$ cannot afford education and rely on child labor income.

Therefore, it follows from (20) that their desirable levels of public education are

$$e_t^{qa} = \arg\max (e_t^a, e_t^{qa}, e_t^b, n_{t+1}) = \tau;$$

$$e_t^{qb} = \arg\max \left\{ T - n_t^b(\delta - \omega(0, e_t^b, z_t^b)) \right\} = 0.$$  

The asymmetric stances above indicate the existence of between-group conflict (class conflict) over education policy; public service for education is desired by adult members of group $a$. It raises the future potential income of their children $w_t+1 h_t^{aq}$ with no harm because the child labor regulation is not binding for them. By contrast, this is not the case for adult members of group $b$, who care only about current consumption and the quantity of their children, $n_t^b$. Since $n_t^b$ is determined in advance, a rise in $e_t^b$ merely diminishes their child labor income $\omega_t^b$ and current consumption $c_t^b = z_t^b[T - n_t^b(\delta - \omega_t^b)]$ with no positive welfare effect. This is the lock-in effect of fertility decisions, emphasized by Deopke and Zilibotti (2005), prompts the unwealthy group to oppose compulsory education.\footnote{The reason for their opposition changes depending on the timing of childbirth. If parents choose $n_t^b$ after the education policy for period $t$ was determined, $n_t^b$ is adjusted so that (9) holds. Then, it follows that consumption is $c_t^b = (1 - \alpha)Tz_t^b$ regardless of the implemented education policy. In this case, it is the adverse effect on childbirth, rather than on consumption, that induces the opposition of group $b$ against compulsory education.}
4.1.2 The Evolution of Average Fertility and Public Education

Conditional Dynamics Equations (17), (18), and (22) reveal that, in the presence of the aforementioned between-group conflict, the provided level of public education in period \( t + 1 \) is

\[
e_{t+1}^g = \lambda(n_t)\theta\tau + [1 - \lambda(n_t)] \min[\theta\tau, \tilde{e}^g(n_t)] \equiv e^g(n_t; \theta\tau),
\]

(26)

where \( \theta\tau \) is viewed as the representative policy preference of the young. This indicates that whether the elderly accept the education level or not depends on \( n_t \), which becomes the number of adult individuals (taxpayers) per aged person in period \( t + 1 \). As depicted by Figure 6, \( e^g(n_t; \theta\tau) \) is constant at \( \theta\tau \) as long as \( n_t \) is above a certain level, below which the function is strictly monotonically decreasing towards the origin.

It follows from (11), (12), and (24) that average fertility in Stage I is,

\[
n_t = q_t n^r(e_t^g) + (1 - q_t) n^p(e_t^g, z_t^b),
\]

(27)

where, as shown by Figures 3–4, a rise in \( e_t^g \) has two opposing forces. While it promotes the fertility of group \( a \) by subsidizing private education, it decreases the fertility of group \( b \) by regulating child labor. The former positive effect is negligible if the share of group \( a \) in the adult population, \( q_t \), is sufficiently small (i.e., if child labor is needed by most young households). On the other hand, the latter negative effect completely dissipates for any \( e_t^g \in (\tilde{e}^g, \infty) \), where the complete regulation on child labor reduces \( n_t^b \) to a minimum level \( \alpha T/\delta \) (cf. Footnote 29). Therefore, given a sufficiently small \( q_t \), average fertility in (27) decreases with small \( e_t^g \)—a situation depicted in Figure 6.

Based on those results, Figure 6 represents the evolution of average fertility and education policy in Stage I for a given pair \( q \in (0, \varepsilon) \) and \( z^b \in (z^{\min}, z^*) \). The average number of children per adult in period \( t \), \( n_t \), determines public support for education in the subsequent period, \( e_{t+1}^g \), which in turn affects average fertility in the same period, \( n_{t+1} \). Thus, the intersection of the two solid curves generates a conditional steady-state equilibrium, in which \( n_{t+1} = n_t \). The arrows indicate that \( n_t \) nonmonotonically converges to the steady-state level. However, the stability is not always the case because the slopes of the two curves are generally ambiguous in their degrees. The central result here is that the limited but positive supply of public education lowers the growth path of \( n_t \) (compared with the case in which \( e_t^g \) is fixed at zero) while keeping \( n_t \) above \( \alpha T/\delta \).
Unconditional Dynamics Assumption (A3) and historically-determined factors yield the initial set \((z_0^g, z_0^b, N_0, N_0^o, q_0, A_0)\), where \(N_0^o > 0\) is the population size of the old generation in period 0. Then, the initial level of public education is \(e_0^g = e^g(N_0^o/N_0; \theta \tau)\).

Note that the set \((e_t^g, z_t^g, z_t^b, N_t, q_t, A_t)\) fully determines the counterpart for the subsequent period (i.e., forward-looking solutions). Given the pair \((e_t^g, z_t^g)\), each young household makes the optimal decision on fertility \(n_t^i\), private education \(e_t^i\), and thus \(h_{t+1}^i\). These choices, along with \(N_t\) and \(q_t\), determine \((n_t, N_{t+1}, q_{t+1}, g_{t+1})\), noting (12)–(15). These outcomes in turn yield parental potential income \(z_{t+1}^g \equiv A_{t+1}h_{t+1}^g\) and the level of public education \(e_{t+1}^g = e^g(n_t; \theta \tau)\).

Equation (27) implies that in Stage I, the fertility rate of group \(a\) is less than the average as long as \(q_t \in (0, 1)\). Then, it follows from (13) and \(q_0 < \varepsilon\) from (A3) that

\[
0 < q_{t+1} < q_t \quad \forall t \in [0, t^*),
\]

indicating the growing share of group \(b\) in the adult population. Accordingly, \(q_t\) remains on a small interval \((0, \varepsilon)\) for all \(t \in [0, t^*]\), implying that average fertility and human capital are affected significantly by the behaviors of the majority group \(b\).\(^{35}\)

Then, (26) and (27) reveal that, \(\forall t \in [0, t^*),\)

\[
0 < e_t^g \leq \theta \tau; \quad n_t > \frac{\alpha T}{\delta}.
\]

The limited supply of public education, along with (19) and (25), yields \(b_t^b = l(0, e_t^g) > 0\) and \(h_{t+1}^b = h(0, e_t^g) > 1\), permitting children of group \(b\) to engage in both work and skill acquisition. This situation, caused by the poor’s negative pressure on public education, corresponds to the half-time system mentioned in Section 2.1. The partial enforcement of universal schooling enhances average human capital and thereby advances technological progress. This is the intuition of the proposition below.

**Proposition 1** Under (A1)–(A3), compulsory schooling enforced over Stage I raises the growth rates of technology and lowers the growth path of average fertility in the stage, compared to the case with no public education.

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\(^{35}\) More precisely, \(\varepsilon\) is defined as a small value such that, for any \(e_t^g \in [0, \theta \tau]\) and \(z_t^g \in (z_{\min}, z^*)\), (a) a weighted average of human capital \(eh(e_t^g, e_t^i) + (1 - \varepsilon)h(0, e_t^g)\) increases with \(e_t^i\) and (b) that of fertility \(en^r(e_t^g) + (1 - \varepsilon)n^r(e_t^g, z_t^g)\) decreases with \(e_t^g\) and is greater than \(\alpha T/\delta\). The condition (a) is necessary because, due to the negative reaction of private education, a rise in \(e_t^g\) has an ambiguous effect on \(h(e_t^g, e_t^i)\). Such an indirect adverse effect is negligible as long as \(\varepsilon\) is sufficiently small.
Proof. As follows from (28), (29), and Footnote 35, public education enforced in period \( t \in [0, t^*] \) enhances average human capital \( h_{t+1} \) in (14) for a given \( q_{t+1} \). In view of (13) and (27), it also reduces average fertility \( n_t \) and raises the share of group \( a \) in the adult generation \( q_{t+1} \) (compared to the case with \( \epsilon_t^a = 0 \)). Since, as implied by (25) and (27), group \( a \) has a higher level of private education and a lower fertility rate than those of group \( b \), the rise in \( q_{t+1} \) has a positive and a negative impact on \( h_{t+1} \) and \( n_{t+1} \), respectively. These results along with (15) establish the proposition.

Therefore, the enforcement of compulsory education expedites the departure from Stage I, by promoting growth in potential income of group \( b \), \( z_t^b = A_t h(0, \epsilon_t^b) \). Note that the evolution of average fertility \( n_t \) is under the influence of two opposing dynamic forces. On the one hand, growth in \( z_t^b \) reduces \( n_t^b \) by making child labor less and less productive than adult labor. On the other hand, as shown by (28), the more fertile group \( b \) increases its population share \( 1 - q_t \) over time. Since \( q_t \) in Stage I is bounded by a small value \( \varepsilon \), however, average fertility is likely to exhibit a decreasing trend over time. When \( n_t \) and thus \( \epsilon_t^a \) evolve nonmonotonically, growth in \( h_t^b \) and \( z_t^b \) may be nonmonotonic as well.

4.2 Stage II: Public Education as a Right

In Stage II, which begins in period \( t^* \), investment in human capital becomes advantageous for all young households, a change that induces decisive fertility decline. While they unanimously support extensive education policy, the elderly, who prefer the other public service, gradually expand their population share and political influence. Their negative pressure on public education raises the cost of private education and thereby discourages fertility. This leads to a vicious cycle of population aging and a decline in public education.

4.2.1 Private Education, Child Labor, and Policy Preferences

Consider a period in which the relationship \((z_t^a, z_t^b) \gg z^* \) holds as in period \( t^* \). Then, (A2) shows that \( \beta_t^a = \beta_t^b = 1 \) for any \( \epsilon_t^a \geq 0 \), implying the incentive to care about the future incomes of their children. Furthermore, it follows from (5) and (10) that for any \( \epsilon_t^a \in [0, \tau] \),

\[
\epsilon_t^i = c(\epsilon_t^a) > \hat{c}(\epsilon_t^a);
\]

\[
l_t^i = l(\epsilon_t^a, \epsilon_t^a) = 0.
\]
That is, parents uniformly decide how much to invest privately in education, depending on the
government support for education. Furthermore, the chosen level of private education does not
allow child labor regardless of the regulation on it.\footnote{Recall that the unbinding child labor regulation is consistent with the words of Nardinelli (1990, p. 149), quoted in Section 2.2.} This is because, in view of (A2), \( z_t^i > z^* \) exceeding \( z^* \) sufficiently reduces the values of child labor (in terms of parental time), \( \omega_t^i \).

Substituting these results into (20), the desirable level of public education for an adult individual \( i \) is

\[
e_t^{gi} = \arg \max \eta(e_t^i, e_t^{gi}, c_t^i, q_{t+1}) = \tau. \tag{31}
\]

Thus, public education is now viewed as a desirable service for all adult individuals, and no political conflict arises among them. Since child labor regulation is no longer binding for any household, compulsory education is, given \( n_t^i \), expected to promote the future potential income of children \( w_{t+1} h_{t+1}^i \) without reducing current consumption \( e_t^i = z_t^i [T - n_t^i (\delta + e_t^i)] \).

Since, as shown above, households are identical with respect to child-rearing, the resulting levels of parental potential incomes are\footnote{The inequality is because the restriction on \( A_0 \) in (A3) implies that \( A_{t+1} h(e_t^i, e^{gi}_t) > z^* \) for any \( e_t^i \in [0, \tau] \) and \( A_{t+1} \geq A_0 \). That is, the potential income of an adult individual \( i \), \( z_{t+1}^i \), is greater than \( z^* \) whenever the individual receives private education in period \( t \).}

\[
z_t^{a} = z_t^{b} > z^* \quad \forall t \geq t^*.
\]

That is, after period \( t^* \), there is no income inequality between the two young groups, and the retreat to Stage I (\( z_t^b \leq z^* \)) does not occur. Accordingly (30)–(31) hold for any \( t \geq t^* \).

4.2.2 The Evolution of Average Fertility and Public Education

The increased demand for public education encounters opposition from the elderly, who more or less need the other service \( x_t \). This is implied by (21), in which the desirable level of public education for the young, \( \tau \), inevitably exceeds the acceptable level for the elderly, \( e_t^g \). Thus, in light of (26), the level of public education in period \( t + 1 (\geq t^*) \) is given by a single-valued function such that

\[
e_{t+1}^{g} = e^g(n_t; \tau) = \lambda(n_t) \tau + [1 - \lambda(n_t)] e^g(n_t), \tag{32}
\]

where \( \lim_{n_t \to 0} e^g(n_t; \tau) = 0 \) and \( \lim_{n_t \to \infty} e^g(n_t; \tau) = \tau \). Note that a decline in \( n_t \) (the ratio of the adult to old population) strictly monotonically decreases \( e^g(n_t; \tau) \) through two channels, \( \lambda(n_t) \) and
\( \bar{e}^\theta(n_t) \). First, it weakens the relative political power of adult individuals, who claim \( \tau \) units of public education. Second, since it implies a decline in the number of taxpayers per aged person, more budgets tend to be necessary for the elderly to be saturated at \( x_{t+1} = \gamma \).

Equation (11) reveals that in Stage II, there is no between-group difference in fertility rates \( (n_t^n = n_t^b) \) and thus their average is

\[
  n_t = n^r(\bar{e}_t^\theta) = \frac{\alpha T}{\delta + c(\bar{e}_t^\theta)},
\]

where \( n^r(\bar{e}_t^\theta) < \alpha T/\delta \) and \( n^{fr}(\bar{e}_t^\theta) > 0 \ \forall \bar{e}_t^\theta \in [0, \tau] \), as shown by Figure 4. Thus, recalling (29), the levels of average fertility in this stage are lower than those in Stage I. This reflects the shift of group \( b \) from the use of child labor to the education of children. Furthermore, the positive effect on fertility of public education arises from its substitutional relationship with private education.

Based on those properties, Figure 7 depicts the evolution of average fertility and public education. The arrows reveal that a value \( n_t \) determines the education policy in the subsequent period, \( e^\theta_{t+1} = e^\theta(n_t; \tau) \), and then the average number of children per adult, \( n_{t+1} = n^r(e^\theta_{t+1}) \). Given the initial value \( n_{t-1}^r > \alpha T/\delta \) from (29), \( n_t \) and \( e^\theta_t \) monotonically decline towards a non-trivial stationary-state equilibrium, which occurs on the intersection of the two solid curves.\(^{38}\) The equilibrium is characterized by the positive levels of average fertility and of public education.

The analysis here rules out the possibility that public education goes back to a level in Stage I. For that purpose, suppose that

\[
  e^\theta(n^r(0); \tau) \geq \theta \tau,
\]

where \( \theta \tau \) is, as shown by (29), the upper limit of public education in Stage I.\(^{39}\) Since \( e^\theta(n_t; \tau) \) is strictly increasing in \( n_t \), the figure shows that \( e^\theta_t > \theta \tau \ \forall t \geq t^* \). These results establish the following key proposition.

**Proposition 2** Under (A1)–(A4),

(a) \( n_{t+1} < n_t < n^*_{t-1} \) and \( e^\theta_{t-1} < e^\theta_{t+1} < e^\theta_t \ \forall t \geq t^* \);

(b) \( n_t \) converges to a nontrivial stationary-state equilibrium where \( n_t = n^r(e^\theta(n_t; \tau)) \).

\(^{38}\) The derived properties of \( n^r(e_t^\theta) \) and \( e^\theta(n_t; \tau) \) do not ensure the uniqueness of the steady-state equilibrium. In the case of multiple equilibria, \( n_t \) converges to the largest steady-state level, which exists on \( (0, \alpha T/\delta) \).

\(^{39}\) In view of (21) and (32), this assumption is satisfied if, for instance, the selfishness parameter of the old, \( \gamma \), is sufficiently small and the political power of the young, \( \lambda(n_t) \), is sufficiently large for \( n_t = n^r(0) \).
Proposition 2(a) asserts a monotonic fertility decline and a rise and fall of public education on the time interval $[t^*-1, \infty)$. In particular, the initial fertility decline from $n_{t^*-1}$ to $n_{t^*}$ as opposed to the rise in public education from $e^g_{t^*-1}$ to $e^g_t$ is consistent with the historical evidence in Table 1. This asymmetry reflects the shift in child rearing strategies of group $b$, from the use of child labor to the education of children. The key assumption here is the restriction on $q_0$ in (A3): Under the condition, group $b$ maintains a sufficiently large share in the adult population over Stage I.

The proposition also suggests the subsequent decline in average fertility and in public education. The underlying mechanism is found in a cyclical interaction between them. As a result of a fertility decline, the old generation expands its population share and becomes more influential in the intergenerational allocation of government resources. The resulting squeeze on the budget for public education imposes higher education costs on the young, and thereby discourages childbirth. As will become apparent, the diminishing public investment in education may retard technological progress and growth in output per worker.

A few remarks deserve special attention at this point. First, Proposition 2 does not indicate a nominal decline in public education expenditures, as $e^g_t$ is measured in the time of adult workers. Second, a reduction in $e^g_t$ necessitates larger household spending on education because, in this stage, human capital investment is beneficial for the young. Such a substitutional relationship between private and public education is not observed in Stage I and, hence, the downward spiral of fertility and public education is the phenomenon peculiar to Stage II. Third and finally, the steady-state levels of fertility and public education depend on the degrees of social and political factors such as selfishness of the old, their political power in a given age structure, and the subsidiary effect of public education on private education. These are reflected in the level of $\gamma$ and the quantitative properties of the functions $\lambda$ and $e$, respectively.

Figure 8 shows the dynamic effects of exogenously augmenting the subsidiary effect of public education. This improvement lightens financial burdens of private education for each $e^g_t > 0$, thus shifting the fertility curve $n_t = n^*(e^g_t)$ rightward. As a result, the economy in the lower steady-state levels of fertility and public education allocate to each child. Since the right hand side of the equation is decreasing in $n_t$, it implies a one-to-one, positive relationship between $n_t$ and $e^g_t$. Thus, the dynamical system becomes essentially the same as the one in Figure 7, and $n_t$, $e^g_t$, and $e^g_t/n_t$ decrease over Stage II.

\[40\text{As defined in Section 3.2, } e^g_t \text{ denotes public education expenditure per child-rearing household in period } t. \] Although per-child expenditure appears to be plausible for the level of public education, using this alternative measure would not change the main results qualitatively. In such a case, (33) is replaced with $n_t = n^*(e^g_t/n_t)$, where $e^g_t/n_t$ is the time of public education allocated to each child. Since the right hand side of the equation is decreasing in $n_t$, it implies a one-to-one, positive relationship between $n_t$ and $e^g_t$. Thus, the dynamical system becomes essentially the same as the one in Figure 7, and $n_t$, $e^g_t$, and $e^g_t/n_t$ decrease over Stage II.
state equilibrium converges toward the new one through an upward spiral between $n_t$ and $e^q_t$. What causes this structural change? As shown by (36) in Appendix 1, $e'(e^q_t)$ depends on properties of the function $h$ such as, (a) how much public education $e^q_t$ erodes the marginal return to private education $h_1(e^i_t, e^q_t)$, and (b) how much $e^q_t$ directly enhances human capital $h(e^i_t, e^q_t)$. In reality, these would be associated with how to allocate the public education budget among various opportunities that foster individual human capital. If the budget $e^q_t$ is used intensively for the instruction of particular skills, for instance, high returns to investment in other skills would be left to households. This could undermine the subsidiary effect of public education and thus induce relatively low fertility. Therefore, the focus of public education is a potential factor for the cross-country variations in the speed of population aging and the decline in public education.

4.2.3 Economic Growth Slowdown

As formulated by (15), the growth rate of technology, $g_{t+1}$, has a monotonic relationship with the average level of efficiency units of adult labor in period $t + 1$, $h_{t+1}$. Substituting (30) into (14),

$$h_{t+1} = h(e(e^q_t), e^q_t) \quad \forall t \geq t^*, \quad (34)$$

where the effect of public education is generally ambiguous. In addition to its direct positive effect, a rise in $e^q_t$ adversely affects individual human capital by discouraging private education. Therefore, a decline in $e^q_t$ deteriorates growth performance of the economy as long as the direct effect is dominant.

**Proposition 3** Under $(A1)$–$(A4)$, the decline in public education lowers the growth rate of technology and that of parental potential income over Stage II, unless the positive response of private education, $e'(e^q_t)$, is significant.

As discussed above, the slope of $e(e^q_t)$ depends on the quantitative properties of the production function of human capital, $h$. For instance, if public education has a limited impact on the marginal productivity of private education (i.e., the cross derivative $h_{12}(e^i_t, e^q_t)$ is sufficiently small in absolute value), these two types of education have a weak substitutability with each other. Under this circumstance, it is likely that the decline in public education over Stage II stimulates private education insufficiently, leading to lower average human capital and delayed technological progress.
5 Discussions of the Model

This section considers the workings of the basic assumptions adopted in the model and examines how the main results are affected by relaxing them. Furthermore, it shows that extending the basic model yields notable insights for the development of education reforms and the speed of fertility decline.

5.1 Basic Assumptions

Assumptions (A1) and (A2) together generate parents’ discrete choice between private education and child labor in any situation. Namely, there is no such case that working children receive private (rather than public) education. This substantially simplifies their policy preferences and facilitates the dynamic analysis. Without (A1), many children may work even in developed stages—a case not supported by historical evidence (cf. Section 2.2). As for (A2), if the weight parameter on the quality of children, $\beta^i_t$, is either stationary or a continuous increasing function of $z^i_t$, the desirable level of public education for group $b$, $c^gb_t$, might rise from zero in Stage I. This permits a gradual development of public schooling in Stage I—a more realistic result—yet at the cost of exposition.

The initial conditions in Assumption (A3) play a crucial role for the described scenario of industrialization. First, the share of group $a$ (higher income class) in the adult generation, $q_0$, is limited so that average human capital and fertility in Stage I primarily reflect the child-rearing strategies of the poorer group $b$. This excludes the possibility that average human capital decreases in Stage I (cf. Footnote 35) and also makes $n_{t+1}$ greater than $\alpha T/\delta$ so that average fertility declines between periods $t^* - 1$ and $t^*$.

Second, the restriction on the technology level ensures that the economy starts out with Stage I, where child education is a burden only for group $b$. As mentioned in Footnote 37, it also ensures that those who received private education in their childhood obtain potential income $z^i_{t+1}$ greater than $z^*$. Even if this condition is violated, technological progress eventually keeps $z^i_{t+1}$ above $z^*$ regardless of the skill level, and thus Propositions 2–3 hold in the end.

5.2 Omitted Elements

The model abstracts from various elements primarily for tractability. It is expected that they affect the political system, given by (17), not only through the policy preferences of the three groups
represented by \( e_t^{qa}, e_t^{qb}, \) and \( e_t^{q0} \) but also through their political powers weighted by \( \lambda_t \) and \( \theta \). In general, both the function \( \lambda \) and the parameter \( \theta \) would depend on the degree of the eagerness for (or aversion to) public education, as it motivates the establishment of political institutions including nonprofit organizations and unions, which foster corporatism among individuals sharing the same interests (cf. Section 2.3).

5.2.1 Inequality

The relative political power of group \( a \) in the adult population, represented by \( \theta \), should vary with some endogenous variables such as the ratio of group \( a \) in the adult population, \( q_t \), and the level of parental potential income of group \( b \), \( z_t^b \). This is because poorer households will have a stronger incentive to combat against compulsory education, so as to secure subsistence consumption by utilizing child labor. According to the dynamic analysis above, \( q_t \) decreases and \( z_t^b \) grows (not necessarily monotonically though) in Stage I. These changes would reduce \( \theta \) and thus the provision of public education in this stage. On the other hand, as just mentioned in relation to Assumption (A2), there is a potential wealth effect that induces group \( b \)'s desire for public education in Stage I. The former negative force would be dominant in the early phases of industrialization if both \( q_0 \) and \( z_0^b \) are sufficiently small (i.e., high degree of between-group inequality in period 0). Therefore, the theory predicts the delaying effect of inequality on the extension of public education and fertility decline.

5.2.2 Factor Markets

The evolution of the economy would be under the influence of omitted production factors—physical capital and agricultural land—in the presence of international trade and pecuniary externalities.\(^{41}\) To begin with, suppose that the final good can be either consumed or stored as capital that can be rented out to producers in the subsequent period. Members of group \( a \) would save a fraction of their incomes for consumption in their old age. Equally important, the interest rate increases with the aggregate stock of human capital, provided that the modified production function exhibits complementarity between physical and human capital. Such a pecuniary externality would reinforce their desire for universal education, whereas the positive interest accrued to the old would moderate their desire for compulsory education.

\(^{41}\)The discussions in the first two paragraphs are along the line of Galor (2005).
their opposition (compared with the case in the basic model). These effects would translate into larger values of $\theta$ and $e^g_t$, promoting public education over Stage I at the cost of fertility.\textsuperscript{42}

In contrast to capitalists, landowners would seek to block education reforms. Suppose that the economy has another production sector, which employs agricultural land and raw labor (either child or uneducated adult labor). Due to the lack of complementarity between land and skills, imposing compulsory education would erode the productivity of land by absorbing labor supply in agriculture. Under these circumstances, several economic factors promoting agricultural profits, such as land abundance/inequality and international trade, would spur the landed class into opposition against public schooling. Provided that landownership is under the old generation, their rent-seeking activities would have downward pressure on the relative political power of the young, $\lambda_t$. This leads to the delay in education reforms and fertility decline.

A potential pecuniary externality exists in the labor markets as well. In the basic model, the relative supply of child labor has no impact on the wage rate of adult labor because, given the production function in (1), there is no substitutability between them.\textsuperscript{43} In the presence of substitutability, labor market participation by children is expected to undermine the demand for adult labor.\textsuperscript{44} This adverse effect would be relatively minor in the early stages of industrialization, where the wage discrepancy between child and adult labor is small; the earnings of children would be more important from the viewpoint of the poor. In the process of industrialization, however, the expansion of the wage discrepancy would intensify the adverse effect and accelerate the reversal of group $b$’s policy preferences. Furthermore, it motivates group $a$’s desire for child labor regulations because they do not use child labor in any case. These foster the development of public schooling.

5.2.3 Preferences

As formulated by (4), parents in the described economy do not derive utility from the leisure of children. Relaxing this assumption would alter their child-rearing quantitatively. That is, low- and high-income households would reduce child labor and private education, respectively, in order to allow their children some amount of leisure. As a result, the curve $l^i_t = l(0, e^g_t)$ in Figure 3

\textsuperscript{42}As shown by (22), $e^g_t$ is the maximum level of public education acceptable to the old generation.

\textsuperscript{43}Nardinelli (1990, pp. 131–143) provides some historical examples in which adult workers did not perceive child workers as their competitors. For example, in mid-19th century France, where the family labor system was pervasive in both agriculture and industry, child labor legislation did not gain support from the working class (p. 137).

\textsuperscript{44}Deopke and Zilibotti (2005) argue that the labor market participation of child labor erodes the wage rate of unskilled labor. For a general model of pecuniary externalities across occupations, see Mookherjee and Ray (2003).
shifts toward the origin, leading to the decline in child labor income $\omega_t$ and in the fertility of the poor. Although the curve $e_t^{t+1} = e_t^g$ in Figure 4 shifts to the same direction, the resulting impact on fertility is opposite, as the lighter expenditure for private education allows the rich to have more children. Hence, as depicted by Figure 8, the fertility curve swings rightward. The levels of fertility and public education would be higher in the new steady-state equilibrium, noting that the relaxation is unlikely to affect the young’s demand for public education.\(^{45}\)

The parameter $\gamma$, denoting the degree of the selfishness of the old generation, would depend on whether the elderly live with their offsprings. For example, in Japan, only in a decade from 1995 did the proportion of old people living alone in all the households rise from 17.2% to 22.5% (Statistic Bureau, Ministry of Internal Affairs and Communications, 2008, p. 68). Such a change in family structure would weaken intergenerational ties within dynasties and raise $\gamma$ over Stage II. The resulting downward shift in the curve $e_t^{t+1} = e_t^g(n_t, \tau)$ leads economies to lower steady-state levels of average fertility and public education.

6 Concluding Remarks

This research has advanced the following thesis about the role of public education in the industrialization process experienced by advanced economies: Mandatory education, embodied in early factory legislation, promoted industrialization accompanied with the decline in child labor. The subsequent shift in the young majority’s attitude to public education, from a duty to a right, paved the way for major education reforms, yet then leading to a vicious cycle of fertility decline, population aging and negative pressure on public education. The diminishing government support for human capital investment was and will be a potential obstacle to economic growth in this century.

In order to avoid falling deeply into this spiral, advanced economies need to tackle some policy issues. First, the budget for public education should be used to efficiently lighten financial burdens of education on households. In other words, public education needs to be an effective substitute for private education. This is the channel through which the governments can keep relatively high fertility. Note that in reality, this subsidiary effect could be undermined by an improper education policy. If public schools focus intensively on the acquisition of particular skills, for instance, high

\(^{45}\) Recall that, as shown by (31), the desirable level of public education for the young, $\tau$, is a corner solution because public schooling in Stage II benefits them with no harm in the basic model. Therefore, unless the leisure of children is highly important for their parents, the demand for public education would be unchanged.
returns to investment in other skills would be left to households. This might necessitate their large
spending on education at the cost of fertility, accelerating the aforementioned cyclical process.

Second, the generational conflict over government resources should be mitigated to prevent
a disproportionate budget allocation to current consumption. Although one of the solutions is
to foster intergenerational altruism (represented by the inverse of $\gamma$ in the model), it will take a
long time to be effective. Perhaps a more practical, fast-working remedy is to design incentive-
based mechanisms that benefit all groups concerned. The key point here is how to motivate social
contributions by beneficiaries of public services, and examples would include university fellowships
for students looking after old people. Further explorations in this issue are left for future research.

The final remark is directed at the implications for comparative economic development. The
proposed theory demonstrates the development of public schooling and the accompanying fertility
decline observed in the modern period. This dynamic process is shaped by various elements omitted
in the described economy. In particular, the process would be promoted by complementarity
between physical and human capital or substitutability between adult and child labor. On the
other hand, it would be delayed by a high degree of inequality or a low degree of complementarity
between agricultural land and human capital. Since their quantitative impacts depend on country-
specific nature such as the levels and distributions of those production factors and the access to
international trade, a deeper theoretical and empirical research in these aspects would provide
additional insights for the observed variations in the evolution of public education and the speed
of population aging across countries.

**Appendix 1 Technical Discussions**

**Proof of Lemma 1.** Substituting (9) into the objective function (8) yields

$$
\xi - \alpha \ln[\delta + e_t^i - \omega(e_t^q, e_t^q, z_t^i)] + \alpha \ln h(e_t^i, e_t^q) \equiv V(e_t^i, e_t^q, z_t^i),
$$

where $\xi \equiv \ln[\alpha^\alpha (1 - \alpha)^{1-\alpha} T]$ and $\omega(e_t^q, e_t^q, z_t^i) = 0$ if $e_t^i \geq \dot{e}(e_t^q)$. It follows from (6) that if $e_t^i > \dot{e}(e_t^q)$,

$$
V_1(e_t^i, e_t^q, z_t^i) = \frac{\alpha G(e_t^i, e_t^q)}{(\delta + e_t^i) h(e_t^i, e_t^q)},
$$

37
where \( G(e^i_t, e^g_t) \equiv h_1(e^i_t, e^g_t)(\delta + e^i_t) - h(e^i_t, e^g_t) \). Given the conditions of \( h \) in (7), \( G(e^i_t, e^g_t) \) exhibits the following properties for \( e^i_t \). First, \( G_1(e^i_t, e^g_t) < 0 \forall (e^i_t, e^g_t) \geq 0 \). Second, \( \forall e^g_t \geq 0 \),

\[
\lim_{e^i_t \to \infty} G(e^i_t, e^g_t) < 0,
\]

where the inequality is because \( \forall (e^i_t, e^g_t) \geq 0 \), the term \( [h_1(e^i_t, e^g_t) - h(e^i_t, e^g_t)] \) is negative (by concavity) and decreases with \( e^i_t \). Third, (A1) implies that \( G(\bar{e}(e^g_t), e^g_t) > 0 \forall e^g_t \in [0, \tau] \). Therefore, the Implicit Function Theorem reveals that there exists a single-valued function \( e(e^g_t) \) such that \( \forall e^g_t \in [0, \tau], G(e(e^g_t), e^g_t) = 0 \), where \( e(e^g_t) > \bar{e}(e^g_t) \), and

\[
e'(e^g_t) = -\frac{G_2}{G_1} = \frac{1}{h_{11}} \left( \frac{h_{12}}{h} - h_{12} \right) < 0, \tag{36}
\]

where each function is evaluated at \( e^i_t = e(e^g_t) \). Note that \( e^i_t = e(e^g_t) \) is the optimal solution on \( [\bar{e}(e^g_t), \infty) \), as the function \( V \) is strictly concave for all \( e^i_t > \bar{e}(e^g_t) \) and is continuous at \( e^i_t = \bar{e}(e^g_t) \).

**Proof of Lemma 2.** Using (35), one finds that for any \( e^i_t \in [0, \bar{e}(e^g_t)) \) and \( e^g_t \geq 0 \),

\[
\lim_{z^i_t \to \infty} V_1(e^i_t, e^g_t, z^i_t) = \frac{\alpha G(e^i_t, e^g_t)}{(\delta + e^i_t)h(e^i_t, e^g_t)} > 0,
\]

where the inequality is shown above. This result and Lemma 1 reveal that for each \( e^g_t \in [0, \tau] \), there exists a large value of \( z^i_t \) above which \( e^i_t = e(e^g_t) \) is the optimal solution on \( \mathbb{R}_+ \). Hence, the lemma follows.
Appendix 2  Data Sources for Table 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coverage</th>
<th>Source</th>
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</thead>
<tbody>
<tr>
<td>Japan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>All</td>
<td>NIPSSR (2008, Table 3.1, p. 17).</td>
</tr>
<tr>
<td>65+</td>
<td>All</td>
<td>NIPSSR (2008, Table 2.6, p. 14).</td>
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<td>United States</td>
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<td></td>
</tr>
<tr>
<td>65+</td>
<td>All</td>
<td>Carter et al. (2006, Vol. 1, Aa125, and Aa139, pp. 44–47).</td>
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<tr>
<td></td>
<td></td>
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<tr>
<td>Britain</td>
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</tr>
<tr>
<td>B</td>
<td>All</td>
<td>Mitchell (2007, Table A6, pp. 94–120).</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2001</td>
<td>Calculated as ((AB + CD)/F), where (A) = Public expenditure on primary, secondary and post-secondary non-tertiary education as percentage of GDP; (B) = Proportion of central government expenditure to public expenditure on primary, secondary and post-secondary non-tertiary education; (C) = Public expenditure on tertiary education as percentage of GDP; (D) = Proportion of central government expenditure to public expenditure on tertiary education (OECD, 2004, Table B4.1 on p. 249 and Tables 4.2a–4.2b available only online at <a href="http://www.oecd.org/edu/eag2004">www.oecd.org/edu/eag2004</a>); (F) = Central government expenditure as percentage of GDP (Statistical Office of the European Communities, <a href="http://epp.eurostat.ec.europa.eu/">http://epp.eurostat.ec.europa.eu/</a>).</td>
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<tr>
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<td>Mitchell (2007, Table A6, pp. 94–120).</td>
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<tr>
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<td>All</td>
<td>Mitchell (2007, Table A2, pp. 12–48).</td>
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<tr>
<td>B</td>
<td>All</td>
<td>Mitchell (2007, Table A6, pp. 94–120).</td>
</tr>
<tr>
<td>60+</td>
<td>All</td>
<td>Mitchell (2007, Table A2, pp. 12–48).</td>
</tr>
</tbody>
</table>
References


Notes: The flow chart illustrates the transition to a vicious cycle of population aging and a decline in public education. In the early phases of industrialization, compulsory schooling regulated child labor and promoted technological progress, thereby discouraging fertility. A major education reform was eventually triggered by the rise in the young’s demand for skill acquisition. However, the increased population share of the old enlarged their political power to curtail the government budget for education. The resulting positive response of private education induced further fertility decline.
Table 1. Public Education, Birth Rates, and the Elderly Share

<table>
<thead>
<tr>
<th></th>
<th>Japan</th>
<th>United States</th>
<th>United Kingdom</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E/G$</td>
<td>$B$</td>
<td>65+</td>
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<tr>
<td>1890</td>
<td>8.0</td>
<td>28.7</td>
<td>5.5$^a$</td>
</tr>
<tr>
<td>1920</td>
<td>12.3</td>
<td>36.2</td>
<td>5.3</td>
</tr>
<tr>
<td>1960</td>
<td>21.2</td>
<td>17.3</td>
<td>5.7</td>
</tr>
<tr>
<td>1980</td>
<td>19.7</td>
<td>13.5</td>
<td>9.1</td>
</tr>
<tr>
<td>2005</td>
<td>15.6</td>
<td>8.4</td>
<td>20.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E/G$</td>
<td>B</td>
</tr>
<tr>
<td>1852</td>
<td>2.5</td>
<td>26.8</td>
</tr>
<tr>
<td>1906</td>
<td>8.6</td>
<td>20.5</td>
</tr>
<tr>
<td>1954</td>
<td>9.1</td>
<td>18.7</td>
</tr>
<tr>
<td>1975</td>
<td>25.3</td>
<td>14.1</td>
</tr>
<tr>
<td>2001</td>
<td>20.8</td>
<td>13.0</td>
</tr>
</tbody>
</table>

Notes: $E/G$ = Public expenditure on education as percentage of total public expenditure. $B$ = Crude birth rate per 1000. 65(60)+ = Number of population aged 65(60) and over as percentage of total population. $^a$Year of reference 1888; $^b$Excluding expenditure on military service; $^c$Including only England and Wales; $^d$Including only central government expenditure; $^e$Year of reference 1851; $^f$Including only West Germany; $^g$Year of reference 1871; $^h$Year of reference 1890.

Sources: See Appendix 2.
Table 2. Proportions of Private Expenditure on Educational Institutions (All Levels of Education)

<table>
<thead>
<tr>
<th>Country</th>
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<th>2005</th>
</tr>
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<tr>
<td>Canada</td>
<td>17.7</td>
<td>24.5</td>
</tr>
<tr>
<td>France</td>
<td>8.6</td>
<td>9.2</td>
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<td>Germany</td>
<td>22.2</td>
<td>18.0</td>
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<td>Italy</td>
<td>2.9</td>
<td>9.5</td>
</tr>
<tr>
<td>Japan</td>
<td>24.8a</td>
<td>31.4</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>8.5</td>
<td>20.0</td>
</tr>
<tr>
<td>United States</td>
<td>25.0a</td>
<td>32.7</td>
</tr>
<tr>
<td>OECD average</td>
<td>13.4a</td>
<td>14.5</td>
</tr>
</tbody>
</table>

Notes: Public subsidies are included. *Year of reference 1998.
Sources: OECD (2001, Table B3.1, p. 93; 2008, Table B3.1, p. 251).
Figure 2. The Decision of Private Education ($\beta^2_t = 1$)

Notes: The diagram shows the choice of private education $e_t^1$ by an adult individual who cares about the quality of his/her children. $e(e_t^0)$ is the unique value that is optimal on $[\hat{e}(e_t^0), \infty)$, where the children do not work. Whether $e_t^1 = e(e_t^0)$ is preferred to any $e_t^1 \in [0, \hat{e}(e_t^0))$ depends on the level of child labor income $\omega_t^1 \equiv \bar{w}l(e_t^1, e_t^0)/z_t^1$ in the range. As the downward arrow indicates, growth in parental potential income $z_t^1$ lowers ($\omega_t^1$ and thus) the objective function $V$ on $[0, \hat{e}(e_t^0))$, ultimately making $e(e_t^0)$ globally optimal. On the other hand, a value below $\hat{e}(e_t^0)$ may be more optimal when $z^1_t$ is small.
Figure 3. Public Education for the Poor

Notes: The diagram depicts how compulsory schooling affects the fertility of a low-income household \((z_t^i \leq z^*)\). Imposing a higher level of public education \(e_t^q\) limits the supply of child labor \(l_t^i\) and thus child labor income \(\omega_t^i = \bar{w}l(0, e_t^q)/z_t^i\). The resulting rise in the cost of child-rearing induces fertility decline (upper left panel).
Figure 4. Public Education for the Rich

Notes: The diagram depicts how education policy affects the fertility of a high-income household ($z^*_i > z^*$). Increasing public education $e^q_i$ eases the financial burden of private education $e^q_i$ (i.e., subsidiary effect) and promotes fertility $n^*_i$ through the quality-quantity trade-off (upper left panel). The reaction curve $e^q_i = e(e^q_i)$ lies outside the Child Labor Frontier, on which $l(e^q_i, e^q_i) = 0$, so that the household does not use child labor for any $e^q_i \in [0, \tau]$. 
Figure 5. The Evolution of Public Education and Fertility over Stages I–II

Notes: The figure depicts the evolution of average fertility, $n_t$, and the share of public education in total government spending, $e_t^g / \tau$, over the two stages. $e_t^g / \tau$ is initially limited by the poor opposing against compulsory education, which discourages child labor and fertility while promoting technological progress. The sharp rise in public education between periods $t^* - 1$ and $t^*$, accompanied by fertility decline, was triggered by their shift from the use of child labor to the investment in child education. The subsequent decline in public education and fertility results from the vicious cycle in Figure 1.
Figure 6. The Conditional Evolution of Average Fertility and Public Education in Stage I

Notes: The figure depicts the evolution of average fertility $n_t$ and public education $e_l^g$ in Stage I for a constant small pair $(q, z^b)$. The curve $e_{t+1}^g = e^g(n_t; \theta \tau)$ indicates that a rise in $n_t$, which becomes the ratio of the adult to the old population in period $t + 1$, enlarges the government budget for education, yet only up to $\theta \tau$ due to the opposition of the low-income, majority group $b$. The other curve, derived from Figures 3–4, indicates the negative effect of compulsory schooling on average fertility. The figure shows a case in which the pair $(e_l^g, n_t)$ converges to a conditional steady-state equilibrium. Although the dynamic behavior is generally ambiguous, it is certain that a positive level of $e_l^g$ in each period lowers the growth path of $n_t$. 

\[ n_t = qn^e(e_t^g) + (1 - q)n^p(e_t^g, z^b) \]
Notes: The diagram depicts a downward spiral between population aging and the decline in public education $e_t^g$ in Stage II, where all young households desire public education. As shown by the curve $e_{t+1}^g = e^g(n_t; \tau)$, a rise in average fertility $n_t$, which becomes the ratio of the adult to the old population in period $t + 1$, raises the government budget for education towards $\tau$. The curve $n_t = n^r(e_t^g)$, based on Figure 4, implies that public support for education promotes fertility. The initial decline from $n_{t^*-1}$ to $n_{t^*}$ reflects the switch of group $b$ from using child labor to investing in the quality of children. The resulting population aging brings down public education from $e_t^g$ to $e_{t^*+1}^g$, and correspondingly average fertility drops from $n_{t^*}$ to $n_{t^*+1}$. The economy converges toward the steady-state equilibrium, which occurs at the largest intersection of the two curves.
Figure 8. Augmentation of the Subsidiary Effect of Public Education

Notes: The figure shows the dynamic effects of exogenously augmenting the subsidiary effect of public education $e_t^g$. This structural change swings the fertility curve $n_t = n^r(e_t^g)$ rightward by reducing private education for each $e_t^g > 0$ (cf. Figure 4). As a result, the economy in the lower steady-state equilibrium converges toward the new one through an upward spiral between fertility and public education.