Inflation Targeting, Capital Mobility and Macroeconomic Stability

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July 2003

Online at https://mpra.ub.uni-muenchen.de/13858/
MPRA Paper No. 13858, posted 08 Mar 2009 02:36 UTC
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May 2005

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Abstract: In this paper we examine the macroeconomic stability in a simple dynamic open economy model, in which monetary authorities adopt an flexible inflation-targeting regime in an environment with a liberalised capital account and flexible exchange rates. In this respect, inflation targeting is an essential part of a three-part policy (or trinity) that also includes flexible exchange rate and capital mobility. We show that a low degree of inflation targeting flexibility (i.e., central bank’s response is aggressive toward inflation) with a high degree of capital mobility implies a dynamically unstable solution in this simple rational expectations model. In contrast, when central bank adopts a high degree of inflation-targeting flexibility (accommodative central bank), stability can be ensured under any degree of capital mobility. Finally, under low degree of inflation targeting flexibility, it seems necessary to limit the degree of capital mobility in order to maintain stability in countries opening their economies to international capital flows, mainly in emerging market and transition economies.

JEL Classification: E52, E58, F32, F41

Keywords: inflation targeting, optimal monetary policy, capital account liberalisation.

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1. Introduction

An increasing number of countries have switched over the last 15 years to a new monetary policy regime that has become known as inflation targeting and can be seen as a potential remedy for persistent inflation. However, other remedies discussed and suggested in the literature include: the creation of an independent and conservative central bank, or the set up of an inflation performance contract for the central bank governor. Inflation targeting indeed can involve elements of all two remedies. By announcing a rather low inflation target and creating some degree of commitment to it, inflation targeting can help to reduce inflation and to maintain price stability.

In this context, inflation targeting has been adopted in the 1990s by many industrialised countries (New Zealand, Canada, the United Kingdom, Sweden, Finland, Australia and Switzerland). Also, it is now recognised that some emerging market economies (Brazil, Colombia, Chile, Korea, Mexico, Thailand, and South Africa) and transition economies (Czech Republic, Poland and Hungary) launched during the same period monetary policy strategies containing many of the elements of the inflation targeting adopted in more advanced economies. All countries happen to be open economies, with different degrees of restrictions, if any, on trade and capital mobility (Svensson, 1999). Thus, the adoption of explicit inflation targets in some industrialised countries was often a practical response to difficulties with their use of an exchange rate peg, which had failed to achieve low and stable inflation and had been subject to dramatic speculative attacks (i.e., the United Kingdom, Sweden, and Finland). Similarly, the Asian financial and currency crisis in the 1990s and its fallout in Latin America and Eastern Europe have also convinced many observers that dirty flexible or pegged exchange rate regimes are crisis prone (Eichengreen 2002). As a result, emerging market and transition economies opted for an inflation-targeting monetary policy
rule in combination with exchange rate flexibility providing thus a coherent alternative to exchange-rate based monetary policy strategies.

However, market conditions in emerging market and transition economies may require modifications of the typical inflation-targeting policy rule that has been recommended for industrialised economies with more developed financial markets. In particular, a country's size, openness, capital mobility, and degree of financial markets development would matter (Taylor, 2000). In this respect, a substantial body of literature has emphasised the advantages of the inflation targeting (Leiderman and Svensson, 1995; Bernanke and Mishkin, 1997, Bernanke et al., 1999; Mishkin and Schmidt-Hebbel, 2001; Neuman and von Hagen, 2002). Also, work by Ball (1999), Svensson (1999), and Battini, Harrison and Millard (2000) emphasized the importance of open economy considerations. Finally, a number of recent studies have analysed the initial effects of inflation targeting for emerging market and transition economies (Masson et al., 1997; Orlowski 2000; Corbo and Schmidt-Hebbel, 2001; Mishkin and Savastano, 2002; Amato and Gerlach, 2002; Jonas and Mishkin, 2003). Authors like Mishkin (2000), adopting a rather favourable position, argue that inflation targeting has the advantage to be easily understood by the public of these economies, which have often a past history of monetary mismanagement. On the other hand, Masson et al. (1997), taking a more cautious view, argue that the absence of reliable procedures for forecasting inflation, the difficulty of maintaining de facto independence for the central bank, and the lack of an anti-inflationary history may preclude the establishment of transparent framework of monetary policy and thus any attempt at inflation targeting.

Nevertheless, most of these studies failed to distinguish between open-economy and emerging market-country aspects of inflation targeting. And they have not yielded conclusive results on the viability of inflation-targeting regime in countries opening their economies to international capital flows under flexible exchange rates, such as in some emerging market
and transition economies. The purpose of this paper is to examine whether inflation targeting offers a viable alternative to an exchange-rate based monetary policy regime in countries opening their economies to capital flows, especially for financial and macroeconomic dynamic stability. According to Willett (2002), recent studies in open economy suggest that in general optimal rules for using interest rate adjustments to meet inflation targets will become more complicated in open economies and that simple inflation targeting can set up dangerous dynamic instabilities. Moreover, an especially serious and very relevant criticism of inflation targeting emphasises that the increasing capital account liberalisation with exchange rate flexibility, may cause financial instability in emerging market and transition economies with inflation targeting (Mishkin, 2001).

Indeed, the role played by capital account liberalisation in the 1990s financial crises, has lead many researchers to reassess the implications of capital mobility. A “bi-polar” solution for the choice of exchange rate regimes has been recommended by some authors suggesting adjustment of exchange arrangements to the new environment of increasing capital mobility (Fischer, 2001). Others argued against the excessive volatility in the financial markets associated with free capital movements suggesting capital controls to limit capital mobility (Williamson, 2000; Wyplosz, 2001). Both of these views rely on the so-called “impossible trinity” of fixed exchange rates, monetary independence, and free capital mobility. Inflation targeting being an alternative to exchange-rate based monetary policy regime, another way to think about inflation targeting is that it is an essential part of a three-part policy (or trinity) that also includes flexible exchange rate and free capital mobility. Nevertheless, an important question remains. Do inflation targeters have to use also capital controls against the excessive volatility in the financial markets under flexible exchange rates and free capital movements or they have to use exchange rate adjustments in the environment of increasing capital mobility for helping to maintain financial and macroeconomic stability? In this respect, the
weak response of exchange rate regime choices to capital account mobility, established empirically by von Hagen and Zhou (2005), suggests that policymakers have a tendency to utilise capital controls to help manage the exchange rate regimes. Then the question is: could this tendency to control international capital movements be justifiable in the inflation-targeting framework?

To answer to this question, we examine the stability conditions in a simple dynamic macro model with imperfect capital mobility, flexible exchange rates and flexible inflation targeting (Svensson, 1997 and 1999). Under flexible inflation targeting, the central bank aims at two objectives, hitting the inflation and output gap targets. A specific feature of this monetary strategy is that it takes the weight the central bank attaches to output stabilisation relative to inflation as given. This preference parameter determines how aggressive (or “conservative” in the sense of Rogoff’s classic 1985 article) or accommodative the response of the central bank is to deviations of inflation from its target value. In this framework, the degree of capital mobility and the degree of flexibility in inflation targeting are two critical features. We show that a low degree of flexibility in inflation targeting (i.e., central bank’s response is aggressive toward inflation) with a high degree of international capital mobility implies a dynamically unstable solution in this simple rational expectations model. However, it seems possible to maintain stability under a low degree of flexibility in inflation targeting when we limit the degree of capital mobility. Finally, when authorities adopt a high degree of flexibility in inflation targeting, a dynamically stable solution can be attained under any degree of international capital mobility. In this sense, capital controls might help manage inflation-targeting regime. However, their effectiveness depends without doubt on the development of domestic monetary and financial markets.

1 Evidence provided by von Hagen and Zhou (2005) on the relationship between capital mobility and exchange rate regime in transition economies, shows a strong influence from exchange rate regime choices on restrictions on capital mobility, while the feedback effects are absent.
The remainder of the paper is structured as follows. In Section 2, we give out the model. In Section 3 we derive the optimal monetary policy rule. In Section 4, we present the general solution of the dynamic system. In Section 5, we study the stability of the dynamic system. Section 6 summarises the main conclusions.

2. The Model

We consider an open economy model described by an aggregate supply equation, an aggregate spending relationship and two equilibrium conditions in asset markets (foreign exchange and domestic money). Aggregate supply is given by a conventional Lucas supply function:

\[ y = \pi - \pi' - \varepsilon_x, \]  

where \( y \) is the output gap, \( \pi \) the inflation rate, \( \pi' \) the expected inflation rate, and \( \varepsilon_x \) an inflationary (or supply side) shock. The aggregate demand depends on the expected real interest rate, \( (i - \pi') \), and the real exchange rate, \( s \), as follows:

\[ y = -\beta (i - \pi') + \gamma s + \varepsilon_d \quad \beta, \gamma > 0, \]  

where \( i \) is the nominal interest rate, \( s \equiv p^f + e - p \) the real exchange rate (\( p \) denoting the domestic price level, \( p^f \) the foreign price level in foreign currency and \( e \) the nominal exchange rate), and \( \varepsilon_d \) a demand shock. The foreign exchange market equilibrium is characterised by\(^2\)

\[ \eta_s s - \eta_y y + \nu (i - i' - \dot{\varepsilon}_e) + \varepsilon_f = 0, \quad \eta_s, \eta_y > 0, \]  

---

\(^2\) Equation (3) is derived from \( BP = BC + BK = BC(y, s) + BK(i - i' - \dot{\varepsilon}_e) = 0 \), which is the balance of payment equilibrium condition. Trade balance \( BC \) depends on income, \( y \), and real exchange rate, \( s \), while capital balance \( BK \) depends on the differential of national and foreign bonds yields \( (i - i' - \dot{\varepsilon}_e) \).
where \( \eta_s \) and \( \eta_y \) are respectively the real exchange rate-elasticity and the output-elasticity of the trade balance, \( i' \) the foreign interest rate, \( \epsilon_f \) shock affecting the foreign exchange market. The parameter \( \nu \) denotes the degree of capital mobility (or the degree of capital liberalisation), which is assumed to be influenced by different measures of capital control or by institutional rules prevailing on internal financial markets, which can be modified to limit the speed of capital movements. Finally, the money market equilibrium is characterised by

\[
m - p = y - \phi i + \epsilon_m, \quad \phi > 0, \tag{4}\]

where \( m \) is the money supply and \( \epsilon_m \) is a shock affecting money demand. Since money supply is considered to be endogenous in the inflation-targeting framework, equation (4) implies that money supply growth rate in the long run will be equal to long term current and expected inflation rates: \( \mu = \pi = \pi^e \) (with \( \mu = dm/ dt \)). If central bank searches for a credible inflation-targeting monetary policy, it must set a long-run money growth rate consistent with its inflation target: \( \mu = \pi^T \). Taking the time derivative and noting that \( \dot{m} = \mu \) and \( \dot{p} = \pi \), equation (4) can be rewritten as follows:

\[
\mu - \pi = \dot{y} - \phi \dot{i} + \epsilon_m. \tag{4'}
\]

Therefore, in order to satisfy long-term stationarity condition (i.e., \( \pi = \dot{s} = \dot{y} = \dot{i} = 0 \)), monetary authority is constrained to set in the short-run a money growth rate consistent with the inflation target: \( \mu = \Pi + \dot{\epsilon}_m = \pi^T + \dot{\epsilon}_m \). On the opposite, optimal monetary policy could not be credible because current inflation rate will be systematically different from the rationally expected inflation rate by private sector in the inflation-targeting monetary policy regime. In this context, private sector will expect an inflation rate different from the inflation target announced by the central bank and inflation targeting will not be a nominal anchor for inflation expectations as assumed in the inflation-targeting literature.
3. The optimal monetary policy

We interpret inflation targeting as stabilising around a given (long-run) inflation target $\pi^*$ (say 2 percent per year), as well as stabilising the output gap around an output gap target $y^*$. This can be represented by an intertemporal loss function for the central bank given by

$$E\int_0^\infty L(t)\exp(-\theta t)dt, \quad \text{with} \quad L(t) = \frac{1}{2} \left[ (\pi - \pi^*)^2 + \lambda (y - y^*)^2 \right], \quad \theta > 0. \quad (5)$$

The loss function of the central bank increases with deviations from output-gap target and also increases if inflation deviates from an inflation target. The parameter $\lambda$ measures the weight policymakers attach to output stabilisation relative to inflation stabilisation such that $0 < \lambda < \infty$. The case $\lambda = 0$ coincides with a regime that Svensson (1997) describes as strict inflation targeting, whereas $\lambda > 0$ describes flexible inflation targeting. The instrument of the monetary authority is the nominal interest rate and the central bank controls the nominal rate to affect output and inflation. For the purpose of this paper, we assume a zero output output-gap target, $y^* = 0$, which can be interpreted as a situation when there is no long-run output target, in the sense that the long-run output target is not subject to choice but given by the capacity level of output (Taylor 1996 and Svensson 1996, 1999). Since the target for the output gap is zero there is no incentive to generate an inflation bias\(^3\).

The optimal monetary policy is the solution to the sequence of single period decision problems of monetary authorities. These decision problems being independent, the central bank’s optimisation problem consists simply of minimising the one-period loss function, $L$, in (5). Thus, the first-order condition is given by

$$\lambda(y - y^*) \frac{\partial y}{\partial \pi} = -(\pi - \pi^*), \quad \Rightarrow \quad y = y^* - \frac{1}{\lambda} (\pi - \pi^*), \quad (6)$$

\(^3\) A positive output-gap target, $y^* > 0$, can be interpreted as a situation in which distortions in the economy (for example in the labour market) cause the socially preferred output level to exceed the natural output level. This introduces a benefit from inflation surprises and causes an inflation bias under discretion.
which leads to the following central bank’s optimal monetary policy rule (see, Appendix A):

\[
i = \pi^e + \frac{1}{\beta} \left[ \gamma + \frac{1}{\lambda} (\pi - \pi_T^r) + \epsilon_d \right],
\]

where the time-consistent expected inflation rate of private sector is equal to the central bank’s inflation target \( \pi^e = \pi_T^r \) in absence of persistent shocks (see equation A.4 in Appendix A). According to equation (7), it is optimal for the central bank to adjust the nominal interest rate upward to reflect expected inflation (to a full extent), the gap between current inflation and the inflation target, as well as increases in the real exchange rate and increases in the output gap due to a positive demand shock. In fact, this monetary policy rule, such as that proposed by Taylor (1993), expresses the central bank’s instrument, \( i \), as a function of inflation and output gap. This policy rule calls for increase in the nominal interest rate whenever current inflation rises above the central bank’s inflation target. This ensures that a rise in current inflation leads the central bank to boost the nominal interest rate enough to raise the real interest rate, thereby contracting the real economy when current inflation rises.

As noted by Svensson (2002), this type of simple interest rate rule has two advantages as a medium-term strategy for a monetary policy. First, it can easily be verified by outside observers and a commitment to the rule would therefore be technically feasible. Second, variants of the Taylor rule have been found to be relatively robust to different models, in the sense that they perform reasonably well in simulations with different models.

4. The general solution of the system

In order to determine the evolution of the economy over time, we must first solve the static equations of the optimal monetary rule (7) and equations (1) and (2) for the short-run solutions of expected inflation rate \( \pi^e \) and output \( y \). These solutions are given by the following expressions:
According to (8a) and (8b), a gap between current inflation and inflation target \((\pi \neq \pi^r)\), induces changes in inflation expectations and output. In addition, expected inflation rate could be revised as reaction to the observed inflationary shocks. The interaction between the expected inflation rate and the other variables \((\pi, i, y, \text{and} s)\) can generate a variety of dynamic configurations. Then the dynamic behaviour of the economy can be summarised in a two first-order differential equations in real exchange rate, \(s\), and inflation rate, \(\pi\). A linear approximation at the neighbourhood of the steady state \((\bar{s}, \bar{\pi})\) yields the following dynamic reduced form of the model (see Appendix A):

\[
\begin{pmatrix}
\dot{s} \\
\dot{\pi}
\end{pmatrix} = \begin{pmatrix}
\Phi_1 & \Phi_2 \\
\frac{\partial \gamma}{\Omega} \Phi_1 & -\frac{1}{\Omega} + \frac{\Phi \gamma}{\Omega} \Phi_2
\end{pmatrix} \begin{pmatrix}
s - \bar{s} \\
\pi - \bar{\pi}
\end{pmatrix},
\]

(9)

where \(\Phi_1 \equiv \frac{\gamma}{\beta} + \frac{\eta}{\nu} > 0\), \(\Phi_2 \equiv \left(\frac{1}{\lambda \beta} - 1\right) \left(1 + \frac{\beta \eta}{(1 + \beta)\nu}\right) \geq 0\), or 0,

and \(\Omega \equiv -\frac{\beta}{1 + \beta} \left(\frac{1}{\lambda \beta} - 1\right) - \frac{\phi}{\lambda \beta} \geq 0\).

The nominal exchange rate \(e\) (thus the real exchange rate, \(s\)) is a price clearing an efficient international financial market and it is not a predetermined state variable. It is free to make discrete jumps in response to “news” which includes all previously unanticipated current or future changes in exogenous variables and policy instruments. In contrast, \(\pi\) is assumed to be predetermined\(^4\), in the sense that it reacts to economic news slowly without jumps.

\(^4\) The intuition behind this assumption is that inflation rate adjusts more slowly than exchange rate. In fact prices as well the inflation rate adjust with lags in a low inflation environment due to menu costs or other rigidities.
Steady state is characterised by the condition \( \dot{s} = \pi = 0 \). Note by \( A \) the state matrix of the two-equation dynamic system (9). The paths taken by real exchange rate and inflation rate in their dynamic adjustment to steady state equilibrium depend on the signs of determinant and trace of \( A \):

\[
\det(A) = -\Phi_1 \Omega^{-1} \quad \text{and} \quad \text{tr}(A) = \Phi_1 - \frac{1}{\Omega} + \frac{\phi \gamma}{\Omega \beta} \Phi_2.
\]

Both signs depend on the degree of inflation-targeting flexibility, \( \lambda \), i.e. the weight policymakers attach to output stabilisation relative to inflation stabilisation. In this two-equation dynamic system with one predetermined and one non-predetermined variable, the existence of a unique convergent path therefore requires the presence of a stable and an unstable root. The transversality condition implies that rational agents will not choose unstable solutions and the jump variable will always attain the value required to put the system on the unique convergent trajectory. In the following, we examine the dynamic stability of the economy according to the choice of monetary authorities between different degrees of inflation-targeting flexibility. This leaves us with two qualitatively distinct phase diagram configurations in the \( \pi - s \) space, as shown in Figure 1.

5. The stability analysis of the dynamic system

a) High degree of flexibility in inflation targeting

We firstly consider the case of a high degree of inflation-targeting flexibility or the case of an accommodative response of central bank. In this case, the economy will be characterised by a local saddle-point configuration if the state matrix in (9) has a negative determinant, that is \( \det(A) = -\Phi_1 \Omega^{-1} < 0 \). To obtain a negative determinant, \( \Omega \) must be strictly positive. This involves a high degree of inflation-targeting flexibility defined by the following condition:

\[
\lambda > \frac{\beta + \phi(1 + \beta)}{\beta^2}.
\]
Smaller interest-elasticity of money demand \( \phi \), and higher interest-elasticity of aggregate demand \( \beta \) (more financial development) are stabilising factors in the sense that they extend the interval of \( \lambda \) values compatible with the economic stability. Hence, in a contemporary economy with financial markets well developed, we can put more accent on inflation target.

(a). The case of a high degree of flexibility in inflation targeting

\[ S = \pi_s \hfill s = \pi \hfill \]

(b). The case of low inflation-targeting flexibility with capital controls

\[ \dot{s} = 0 \]

**Figure 1**: Phase diagrams of alternative dynamic configurations

Under condition (10), the slope of \( \dot{s} = 0 \) locus in \( \pi - s \) space is \( -\Phi_1 / \Phi_2 > 0 \) (Figure 1a), because \( \Phi_1 > 0 \) and it is easily checked that \( \Phi_2 < 0 \). The slope of \( \dot{\pi} = 0 \) locus in \( \pi - s \) space is \( \Phi_1 / [(\beta / \phi \gamma) - \Phi_2] > 0 \). Note also that both \( \dot{s} = 0 \) and \( \dot{\pi} = 0 \) locus are upward
sloping, and the $\delta = 0$ locus is steeper than the $\pi = 0$ locus. The system has one stable eigenvalue and therefore saddle-point equilibrium.

The intuition behind this saddle-point stability is the following. Under a high degree of inflation-targeting flexibility or an accommodative response of central bank, nominal interest rate rises moderately in response to an initial increase in inflation rate due to an exogenous shock. Given inflation expectations, real interest rate will rise and involves a reduction in spending on consumption and investment, and so a decrease in aggregate demand. The fall in aggregate demand allows reducing smoothly inflation pressure and expectations. The differential between the domestic and foreign inflation rates makes domestic goods more expensive than foreign goods (appreciation of real exchange rate), implying a trade balance deficit that is partially compensated by the decrease in the domestic demand of foreign goods due to diminution of the home revenue. On the other hand, higher domestic nominal and real interest rates encourage capital inflows leading to a further appreciation of real exchange rate. The rise in the nominal interest rate will be further moderated as a reaction to the appreciation of real exchange rate. Smooth adjustments in home nominal interest rate and inflation rate will induce restrained reactions in nominal exchange and real exchange rates due to the capital movements. Consequently, the degree of capital mobility does not play a crucial role in determining the stabilising or destabilising features of this dynamic system. Thus, central bank adopting a high degree of flexible inflation targeting that puts relatively low weight on inflation stabilisation goal seems to be able to eliminate the risk of macroeconomic and currency instability. This theoretical result seems to be supported by empirical evidence provided recently by Collins and Siklos (2004), suggesting that countries with explicit inflation targets were not overly aggressive toward inflation and supports Svensson’s (1999) assertion that inflation-targeting central banks are flexible.
b) Stability under low inflation-targeting flexibility and low capital mobility

We consider now the case of a low degree of inflation-targeting flexibility or the case of an aggressive response of central bank toward inflation (or a “conservative” central banker in the sense of Rogoff). In this case central bank is more concerned about keeping inflation as close as possible to a given inflation target than about output target. In other words, the smaller the parameter $\lambda$ is, the less flexible will be the inflation-targeting regime. This case corresponds to the following condition:

$$\lambda < 1/ \beta. \quad (11)$$

Under condition (11), the slope of both $s = 0$ and $\pi = 0$ locus are downward sloping, and the $\pi = 0$ locus is steeper than the $s = 0$ locus (see Figure 1b). The system will have one equilibrium that can be stable or unstable. In this case, central bank’s nominal interest rate reacts aggressively to inflation rate. Thus, a sharp increase in nominal interest rate increasing operational costs of firms in terms of debt services could force some firms into insolvency, leading thus to a reduced competition in prices and higher inflation pressure. This inflationary pressure can be predictable by economic agents increasing probably their demand for loans, generating thus a further inflationary pressure. In this respect, many of emerging market economies (i.e., Latin American countries during the 1980s) and transition economies (i.e., Eastern European countries in 1990s) provide examples in which a sharp increase in nominal interest rate is not able to reduce inflation expectations and inflation rate. In fact, this analysis seems to be suitable in many of these countries with insufficiently developed and fragile monetary and financial markets. On the other hand, real exchange rate reactions depending on the degree of capital mobility can moderate or accentuate the optimal nominal interest rate reaction and the destabilising forces. With a low capital mobility, the effects of real exchange rate happen to moderate the inflation effects on the interest rate.
With a high capital mobility, the strong effects on the exchange rate are more than necessary in order to compensate the effects of inflation and make thus the economy unstable.

Consequently, the risk of an economic instability appears in the case of a high degree of capital mobility. In fact, persistent shocks leading to a continual gap between inflation and inflation target induce the doubt of private sector on the inflation target announced by the central bank. When interest rate and inflation rate react strongly to shocks under a low degree of flexibility in inflation targeting (i.e., an aggressive central bank toward inflation), private sector will adapt its expectations taking into consideration a predictable evolution of inflation (i.e., a lack of credibility in the inflation-targeting regime). A deviating path of expected inflation rate can be self-fulfilling in this model where central banks adopt a low degree of inflation-targeting flexibility attributing a high priority to the inflation stabilisation. In this respect, to stabilise the economy, it seems necessary to moderate the exchange rate reactions by limiting the degree of capital mobility. This result seems to be supported by evidence provided recently by von Hagen and Zhou (2005), suggesting that policymakers have a tendency to utilise capital controls to help manage the exchange rate regimes.

More specifically, the economy can become stable if some constraint on the degree of capital account liberalisation or capital mobility is introduced. In other words, under the condition (11), one has $\Omega < 0$, $\Phi_1 > 0$, $\Phi_2 > 0$, and it yields

$$\left. \frac{d\pi}{ds} \right|_{s=0} = -\frac{\Phi_1}{\Phi_2} < 0, \quad \left. \frac{d\pi}{dx} \right|_{s=0} = -\frac{\Phi_1}{-(\beta/\gamma) + \Phi_2} < 0 \quad \text{and} \quad \left. \frac{d\pi}{ds} \right|_{s=0} < \left. \frac{d\pi}{ds} \right|_{s=0}.$$

The state matrix in equation (9) has a positive determinant, i.e., $\det(A) = -\Phi_1\Omega^{-1} > 0$, and its trace is indefinite. Under a low degree of inflation-targeting flexibility and for reasonable values of parameter $\beta$, it can be shown that there is a case where the trace of the state matrix may be negative: $\Phi_1 - \frac{1}{\Omega} + \frac{\phi \gamma}{\Omega \beta} \Phi_2 < 0$. Consequently, the system will possess two stable
eigenvalues. This case can be worrying for economists in a framework with forward-looking variables. With two stable eigenvalues, the transversality condition can no longer determine a unique initial condition for the two variables. In this case, a decrease of capital mobility will reduce the reactivity of the exchange rate to a variety of news, and so its behaviour will be like that of a predetermined variable. Hence, if the decrease of the degree of capital mobility is not satisfactory, central bank must intervene in order to avoid the exchange rate jumps permitting the economy to find its unique adjustment path. Therefore, one can consider that there are two state predetermined variables and the behaviour of the economy is stable or cyclically stable. Thus, the trace condition can be rewritten as (see Appendix C):

$$\nu \Psi_1 + \eta_x - \Psi_2 \gamma \eta_y < 0,$$

(12)

where

$$\Psi_1 = [\gamma \beta (z-1) + (1 + \beta) \phi \gamma + (1 + \beta) \beta] \frac{[\beta (z-1) + (1 + \beta) \phi \phi \phi z] \beta}{\beta} > 0,$$

$$\Psi_2 = \phi (z-1) / \beta (z-1) + (1 + \beta) \phi \phi \phi z; \quad \text{with} \ z = 1/ \lambda \beta \ \text{and} \ 0 < \Psi_2 < 1.$$

To illustrate our proposition, we consider two extreme values of the degree of capital mobility (or capital account liberalisation). The first one is the case of perfect capital mobility ($\nu \to \infty$) and, the second one is the case of total absence of capital mobility ($\nu = 0$). In the first case ($\nu \to \infty$), it is straightforward to show that the condition of negative trace and therefore the dynamic stability conditions are not fulfilled and thus the economy could allow for any unstable path. In the second case ($\nu = 0$), the negative trace condition seems to be satisfied, but only under the following condition:

$$\eta_x < \Psi_2 \gamma \eta_y.$$

(13)

This condition can be satisfied, when in a context of exchange rate uncertainty under flexible exchange rates, the trade balance is not very sensitive to exchange rate changes (i.e., the parameter $\eta_x$ is small) and/or when the trade balance is very sensitive to revenue variations.
(i.e., the parameter $\eta_\gamma$ is high). Once condition (13) is accepted, condition (12) makes obligatory the afterwards restriction on the degree of capital mobility

$$\nu < (\gamma \Psi_2 \eta_\gamma - \eta_\gamma) / \Psi_1,$$

(14)
in order to guarantee the macroeconomic stability in this open economy. Therefore, under conditions (13) and (14), the economy can be dynamically stable and when the trace is near to zero, the system can adopt a cyclical behaviour. In addition, a small interest elasticity in money market ($\phi$) and a high interest rate elasticity in goods market ($\beta$) seems to be stabilising factors in the case of a low degree of flexibility in inflation targeting. In fact, the degree of capital mobility ($\nu$) plays its role through the impacts on nominal and real exchange rates. A higher degree of capital mobility will generate excessive short-run movements of nominal and real exchange rates. That can destabilise the economy through excessive reactions of current and expected inflation rates and nominal interest rate. Consequently, a further financial development (a greater $\beta$) and the reduction of interest elasticity in money market (a smaller $\phi$)$^5$ are favourable for a lower degree of flexibility in inflation targeting (conservative central bank) without destabilising the economy. Therefore, countries that have not liberalised and developed their internal monetary and financial markets are less able to apply a low degree of flexibility in inflation targeting. Otherwise, they must put up restrictions in their capital mobility. Nevertheless, restrictions in capital movements correspond to an economic distortion. It is an implication of the theory of second best that removing one distortion need not be welfare enhancing when other distortions are present. In this case, imposing stability conditions may be a better choice due to the gain resulting from macroeconomic stability than totally liberalising the capital account.

$^5$ Empirical studies on money demand following financial deregulation and innovation and the development of largely open money market for short-run monetary assets (in 1980-1990) confirm this hypothesis.
6. Conclusion

In this paper we examine the stability in a simple dynamic open economy macro model, in which monetary authorities adopt an inflation-targeting regime to guide the conduct of their policy in an environment with a liberalised capital account and flexible exchange rates. We examine the stability conditions in a framework where inflation targeting is an essential part of a three-part policy (or trinity) that also includes flexible exchange rate and capital mobility. Also, a rule governing growth path of money supply consistent with the inflation target is imposed to improve credibility. We show that stability conditions depend on the degree of flexibility in inflation targeting and the degree of capital mobility under monetary policy discretion. When central bank adopts a high degree of inflation-targeting flexibility (when central bank is principally concerned about output), the economy find a saddle-point equilibrium and a unique stable path can be determined for economic variables. In this case, stability could be consistent with any degree of capital mobility under flexible exchange rate regime. On the other hand, if central bank adopts a low degree of inflation targeting flexibility (if central bank is principally concerned about keeping inflation as close as possible to a given inflation target), a stable equilibrium is possible only under the condition of capital mobility restrictions. Consequently, it seems necessary to limit the degree of capital mobility in order to maintain stability in countries opening their economies to capital flows under flexible exchange rates, such as in emerging market and transition economies. According to this analysis, how an inflation-targeting regime can be adopted by emerging market and transition economies depends on their structural characteristics. The more developed domestic monetary and financial markets are, the higher weight the central bank can put on inflation target (i.e., the more aggressive inflation targeting) without destabilising the economy and imposing any restriction on capital mobility.
Appendix A. Derivation of the optimal monetary policy rule

The first order condition of central bank’s minimisation problem (5) is:

$$\frac{\partial L}{\partial \pi} = \lambda (y - y^r) \frac{\partial y}{\partial \pi} + (\pi - \pi^r) = 0.$$  \hspace{1cm} (A.1)

Using then equation (2) and the result $\frac{\partial y}{\partial \pi} = 1$ obtained from equation (1), we obtain:

$$\lambda \{ - \beta (i - \pi^e) + \gamma s - y^r + \epsilon_{d}\} + (\pi - \pi^r) = 0.$$  \hspace{1cm} (A.2)

Equation (A.2) gives, under the assumption $y^r = 0$, the following central bank’s optimal monetary policy rule:

$$i = \pi^e + \frac{1}{\beta} \left[ \gamma s + \frac{1}{\lambda} (\pi - \pi^r) + \epsilon_{d} \right].$$  \hspace{1cm} (A.3)

Under rational expectations and transitory shocks, one has $E(\epsilon_x) = 0$. Thus, we obtain from equation (1), $y^r = 0$. Using this result and solving (A.1) for $\pi^e$ yield:

$$\pi^e = \pi^r.$$  \hspace{1cm} (A.4)

According to (A.4), time-consistent inflation rate expectations of private sector are equal to the central bank’s inflation target. Thus, using equation (A.4), we can express the interest rate rule (7) in another way as follows:

$$i = \pi^r + \frac{1}{\beta} \left[ \gamma s + \frac{1}{\lambda} (\pi - \pi^r) + \epsilon_{d} \right].$$  \hspace{1cm} (A.5)

Appendix B. Derivation of the two differential equations of (9)

i) The differential equation for inflation rate ($\pi$):

Solving equations (1) and (2) to eliminate the expected inflation rate, $\pi^e$, yields:

$$y = - \left( \frac{\beta}{1 + \beta} \right) [i - \pi - \epsilon_{x}] + \left( \frac{\gamma}{1 + \beta} \right) s + \left( \frac{1}{1 + \beta} \right) \epsilon_{d}.$$  \hspace{1cm} (B.1)

Taking the derivative of (B.1) with respect to time, $t$, and using $\frac{dy}{dt} \equiv \dot{y}$, $\frac{dp}{dt} \equiv \dot{p} = \pi$, $\frac{dy}{dt} \equiv \pi$, $\frac{ds}{dt} \equiv \dot{s}$ and $\dot{\epsilon}_{y}, \dot{\epsilon}_{d} = 0$ (i.e., white noise shocks), we can write:

$$\dot{y} = - \frac{\beta}{1 + \beta} (i - \pi) + \frac{\gamma}{1 + \beta} \dot{s}.$$  \hspace{1cm} (B.2)
Using \( \frac{dm}{dt} \equiv \dot{m} \equiv \mu \) and taking the derivative of equations (4) and (A.5) with respect to time, \( t \), we get respectively:

\[
\mu - \pi = \gamma - \phi + \epsilon_m, \quad \text{(B.3)}
\]

and

\[
\dot{i} = \frac{\gamma}{\beta} \dot{s} + \frac{1}{\lambda \beta} \dot{\pi}. \quad \text{(B.4)}
\]

Combining (B.2) and (B.4), we obtain:

\[
\dot{y} = -\frac{\beta}{1 + \beta} \left( \frac{1}{\lambda \beta} - 1 \right) \pi - \phi \left( \frac{\gamma}{\beta} \dot{s} + \frac{1}{\lambda \beta} \pi - \pi \right) + \frac{\gamma}{1 + \beta} \dot{s} = -\left( \frac{\beta}{1 + \beta} \left( \frac{1}{\lambda \beta} - 1 \right) + \frac{\phi}{\lambda \beta} \right) \pi - \phi \frac{\gamma}{\beta} \dot{s} + \dot{\epsilon}_m, \quad \text{(B.5)}
\]

Finally substituting \( \dot{i} \) from (B.4) and \( \dot{y} \) from (B.5) into (B.3) yields:

\[
\mu - \pi = -\frac{\beta}{1 + \beta} \left( \frac{1}{\lambda \beta} - 1 \right) \pi - \phi \left( \frac{\gamma}{\beta} \dot{s} + \frac{1}{\lambda \beta} \pi - \pi \right) + \dot{\epsilon}_m = -\left( \frac{\beta}{1 + \beta} \left( \frac{1}{\lambda \beta} - 1 \right) + \frac{\phi}{\lambda \beta} \right) \pi - \phi \frac{\gamma}{\beta} \dot{s} + \dot{\epsilon}_m,
\]

which can be rewritten as

\[
\pi = \frac{1}{\Omega} \left( \mu - \pi + \phi \gamma \beta \dot{s} - \dot{\epsilon}_m \right), \quad \text{where} \quad \Omega = -\frac{\beta}{1 + \beta} \left( \frac{1}{\lambda \beta} - 1 \right) - \frac{\phi}{\lambda \beta}. \quad \text{(B.6)}
\]

Equation (B.6) implies that in steady state (i.e., \( \pi = \dot{y} = \dot{s} = 0 \)) money supply growth rate must be equal to inflation target: \( \pi = \pi^\tau \). The monetary authorities are assumed in the short run to adopt a money growth rule consistent with their inflation target and the long-run equilibrium: \( \mu = \pi^\tau + \dot{\epsilon}_m = \pi^\tau + \dot{\pi} \). The linear form of (B.6) around the steady state (\( \pi^\tau \), \( s^\tau \) and \( l^\tau \)) is:

\[
\pi = -\frac{1}{\Omega} (\pi - \pi) + \phi \gamma \beta \dot{s}.
\]

**ii) The differential equation for real exchange rate (\( s \))**

Using rational expectations (i.e., \( \dot{\epsilon}^s = \dot{\epsilon} \)), \( \dot{e} = \dot{s} - \pi^\tau = \dot{s} - \pi^\tau = \pi + \pi \), and equations (1) and (2), we rewrite equation (3) as follows:

\[
\dot{s} = i - i^\tau + \pi^\tau - \pi - \pi_s \frac{\beta (i - \pi + \pi)}{\nu (1 + \beta)} + \frac{\eta_s + \epsilon_f}{\nu}, \quad \text{(B.8)}
\]

The linear approximation of (B.8) at the steady state (\( \pi^\tau \), \( s^\tau \) and \( l^\tau \)) is given by

\[
\dot{s} = i - \pi^\tau - \pi \left( \frac{\pi_s}{\nu} \left( -\frac{\beta}{1 + \beta} (i - \pi + \pi) + \frac{\gamma}{1 + \beta} (s - \pi^\tau) \right) + \frac{\eta_s}{\nu} (s - \pi^\tau) \right).
\]
Using then the optimal interest rate rule (7), one has \( i - \bar{i} = \frac{\gamma}{\beta} (s - \bar{s}) + \frac{1}{\lambda \beta} (\pi - \bar{\pi}) \), we can rewrite the above equation as:

\[
\dot{s} = \left( \frac{\gamma}{\beta} \frac{\eta_s}{\nu} \right) (s - \bar{s}) + \left( 1 + \frac{\beta \eta_s}{(1 + \beta) \nu} \right) \left( \frac{1}{\lambda \beta} - 1 \right) (\pi - \bar{\pi}).
\]

(B.9)

The dynamic behaviour of the economy can be summarized by the two first-order differential equations (B.7) and (B.9) in matrix form:

\[
\begin{pmatrix}
\dot{\pi} \\
\pi
\end{pmatrix} =
\begin{pmatrix}
\Phi_1 & \Phi_2 \\
\frac{\phi \gamma}{\Omega \beta} \Phi_1 & \frac{1}{\omega} - \frac{\phi \gamma}{\Omega \beta} \Phi_2
\end{pmatrix}
\begin{pmatrix}
\dot{s} \\
(s - \bar{s})
\end{pmatrix}.
\]

(B.10)

where \( \Phi_1 = \frac{\gamma}{\beta} \frac{\eta_s}{\nu} > 0 \) and \( \Phi_2 = \left( \frac{1}{\lambda \beta} - 1 \right) \left( 1 + \frac{\beta \eta_s}{(1 + \beta) \nu} \right) \).

**Appendix C. Trace condition of the state matrix when \( \frac{1}{\lambda \beta} > 1 \)**

When \( \frac{1}{\lambda \beta} > 1 \), it is easy to show that \( \Omega = -\frac{\beta}{1 + \beta} \left( \frac{1}{\lambda \beta} - 1 \right) - \frac{\phi}{\lambda \beta} < 0 \).

Noting for simplicity \( z = \frac{1}{\lambda \beta} \), the trace condition, \( \Phi_1 - \frac{1}{\Omega} + \frac{\phi \gamma}{\beta \Omega} \Phi_2 < 0 \), is rewritten as:

\[
\frac{\gamma \beta (z - 1) + (1 + \beta) \phi \gamma + (1 + \beta) \beta}{[\beta (z - 1) + (1 + \beta) \phi z] \beta} + \frac{\eta_s}{\nu} - \frac{\phi (z - 1) \gamma \eta_s}{\nu [\beta (z - 1) + (1 + \beta) \phi z]} < 0.
\]

(C.1)

Multiplying both sides of (C.1) by \( \nu \) and regrouping, we get:

\[
\nu \Psi_1 + \eta_s - \Psi_2 \gamma \eta_s < 0
\]

(C.2)

where \( \Psi_1 = \frac{\gamma \beta (z - 1) + (1 + \beta) \phi \gamma + (1 + \beta) \beta}{[\beta (z - 1) + (1 + \beta) \phi z] \beta} > 0 \), and \( \Psi_2 = \frac{\phi (z - 1)}{\beta (z - 1) + (1 + \beta) \phi z} > 0 \).
References:
Bernanke, Ben S; Laubach, Thomas; Mishkin, Frederick S; Posen, Adam S. (1999), Inflation targeting. Lessons from the international experience, Princeton University Press.


