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Dai, Meixing and Sidiropoulos, Moïse

University of Strasbourg, BETA-Theme, France, LEAP, Department of Economics, Aristotle University of Thessaloniki, Greece

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Meixing Dai and Moïse Sidiropoulos
BETA and University Louis Pasteur of Strasbourg (France)

Abstract: Using an aggregate dynamic macroeconomic model, we study the macroeconomic and financial stability under flexible inflation-targeting regime associated with intermediate monetary growth target. Central banks, using the inflation target as a communication and strong nominal anchoring device, should also take into account the movements of asset prices in their optimal interest rate rule. They might react to changes in asset prices without introducing asset prices into the description of their policy objectives. We show that, the more flexible the inflation-targeting framework of monetary policy is, the more likely the monetary authorities are able to stabilise the economy around the long-term equilibrium. Therefore, achieving price stability under inflation-targeting regime with low flexibility can generate dynamic instability and will not be able to stabilise effectively the fluctuations of output and inflation. A commitment to a long run growth rate of money supply corresponding to the inflation target can reinforce the credibility of the central bank and the role of inflation target as strong and credible nominal anchor for private inflation expectations and allows the system to be more stability prone.

Key words: flexible inflation targeting, monetary targeting, optimal interest rate rule, stock price, financial development, financial markets, financial and macroeconomic instability.

JEL Classification: E4, E5, G1

Addresses of correspondence:
Meixing Dai, Université Louis Pasteur, BETA, 61, avenue de la Forêt Noire – 67085 Strasbourg Cedex – France. Phone: (33) 03 90 24 20 78; Fax: (+33) 03 90 24 20 71, e-mail: dai@cournot.u-strasbg.fr
Moïse Sidiropoulos, Université Louis Pasteur, BETA, 61, avenue de la Forêt Noire – 67085 Strasbourg Cedex – France; Phone: (33) 03 90 24 20 85; Fax: (+33) 03 90 24 20 71; e-mail: sidiro@cournot.u-strasbg.fr
1. Introduction

Over the last decade, many central banks have adopted a new framework for conducting monetary policy known as inflation targeting. Since New Zealand first adopted this framework in 1990, a growing number of industrial countries (including Canada, the United Kingdom, Sweden, Finland and Australia) as well as emerging countries (Chile, Brazil, Mexico, Israel, the Czech Republic, Poland, Hungary and the South Africa, etc.) have followed by anchoring their monetary policy to explicit quantitative inflation goals. Inflation targeting has been proposed as a tool to promote the central bank independence and to make obvious a deliberate attempt to improve their inflation performance. Namely, a credible announcement of inflation targeting would convince market participants to change their inflation expectations.

In this context, some members of the Federal Reserve (Fed) have also suggested introducing inflation targeting in the United States (Meyer, 2001; Bernanke et al., 1999) arguing that this is critical to secure price stability in the US in the post-Greenspan era. Goodfriend (2003), having described the features of inflation targeting practised implicitly by the Greenspan Fed, recommends the Fed to strictly and explicitly target its constant long run inflation objective over the business cycle. Even academic discussions have suggested the Bank of Japan, which targets monetary aggregates, to adopt inflation targeting in an effort to help the Japanese economy to recover and get out of the eight years old deflation (Svensson, 2003; Ito, 2004; Ito and Mishkin, 2004). Alesina et al (2001), in a discussion of the European Central Bank’s (ECB) monetary policy boldly claim that the ECB could improve its policy by adopting a version of IT. In fact, the new ECB is likely to have adopted a modified form of inflation targeting, although political considerations (the need to demonstrate continuity with the policies of the Bundesbank) apparently will dictate that the ECB pays attention to monetary aggregates as well in its two pillars strategy (Bernanke et al., 1999; Svensson, 2000b; Rudebusch and Svensson, 2002). It is comprehensible that some academics find IT intellectually attractive for the high transparency about central bank intentions and the increase in accountability implied by the announcement of an inflation target, crucial to constraining discretionary monetary policy so that it produces desirable long-run outcomes (Michkin, 1999). Nevertheless, others remain sceptical about the effectiveness of this regime. Both the ECB (ECB, 2001) and the Fed (Gramlich, 2000) have argued that they do not regard IT as an appropriate monetary policy framework. Economists in the Bank of Japan argue that there are

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1 For Michkin (1999), the Bundesbank’s monetary targeting is quite similar to inflation targeting as it announced inflation target and communicated transparently to the public and market participants.
no clear instruments to get out of deflation, and a simple announcement without instruments 
would not convince market participants to change their inflation expectations (Ito, 2004). The 
current debate over whether the ECB has to move to full-fledged inflation targeting reminds us 
that currently, no study accounts for implicit inflation targeting that the ECB practices with 
intermediate monetary growth goal. In fact, according to the Maastricht treaty, without 
prejudice to the objective of price stability, the ECB will also support the general economic 
policies in the Community with a view to contributing to the achievement of the objectives of 
the Community. These include a high level of employment and sustainable and non-
inflationary growth.

The essence of inflation targeting is that the central bank should strive to maintain inflation 
as close to a clearly specified target level as possible, while at the same time limiting 
fluctuations of real economic activity. That corresponds to the description of flexible inflation 
targeting. In practice, inflation-targeting central banks actually all pursue flexible inflation 
targeting since the consequences for the economy of strict inflation targeting are simply 
undesirable (Svensson, 1997, 2000a). Empirical evidence provided recently by Collins and 
Siklos (2004) suggests that countries with explicit inflation targets were not overly aggressive 
toward inflation. Flexible inflation targeting shows up in less policy activism, gradualism in 
returning the inflation back to target, and in aiming at the inflation target at a somewhat longer 
horizon.

During the process leading to the adoption of inflation-targeting regime, the policymaker 
must choose the relative weights to attach to inflation and output. This choice as well as the 
adoption of inflation-targeting regime might be narrowly linked to financial developments 
around the world in the 1980s and 1990s. In fact, the shift in the conduct of monetary policy in 
many countries from monetary targeting to inflation targeting follows important financial 
deregulation and innovations beginning in the 1980s, which have created ambiguity between 
different monetary aggregates. Precise definition and measurement of the money stock present 
some serious practical problems for policy makers who wish to use manipulation of the growth 
(or contraction) of the money stock as a tool of economic policy. Decreasing interest rate 
elasticity of money demand as well as changing velocity of money circulation has been 
reflected in the instability of the money demand function. It is now difficult to control money 
supply in order to reach inflation and output stability goals. Along with financial liberalisation, 
firms have found other financing channels than loans from commercial banks. That decreases 
central bank’s capability to affect firms’ investments by controlling credits through varying 
money supply.
Further insights for the role of financial markets development can be gained from comments on the adoption of inflation-targeting regime in emerging market economies. For Mishkin (2000) and Masson and al. (1997), inflation targeting is a framework that could be used to conduct monetary policy in some high-to-middle-income developing countries. As we may remark, it is not a pure coincidence that these countries have also more developed financial markets. However, financial markets are comparatively underdeveloped and over-regulated in most emerging market economies including these high-to-middle-income developing countries. Market conditions in these countries may require modifications of the typical policy rule that has been recommended for economies with more developed financial markets (Taylor, 2000).

In the same spirit, Cecchetti, Genberg and Wadhwani (2003) study whether there is any role of asset prices in the formulation of monetary policy in a flexible inflation-targeting framework. Like them, we ask whether there is any role of asset prices in the formulation of monetary policy in a flexible inflation-targeting framework associated with monetary targeting as in the case of ECB, with its two pillars monetary policy. In other words, can the monetary targeting or implicit inflation targeting, as practised by the Bundesbank, be successful in a context where financial markets are more liberalised and asset prices more volatile?

The large swings in asset prices and economic activity in the United States, the European Union, Japan, and other countries over the past several years have brought renewed focus on the role of asset prices in the transmission of monetary policy. Monetary policy has been viewed as both a possible cause of asset price booms and a tool for defusing booms before they can cause macroeconomic instability. Recent literature on the linkages between monetary policy and asset markets focuses on how monetary policy might cause an asset price boom and how monetary policy authorities should respond to asset price booms\(^2\). A traditional view focuses on the response of asset prices to a change in money supply, arguing that added liquidity causes asset prices to rise as a link in the transmission of monetary policy actions to the economy as a whole. A second view argues that asset price booms are more likely to arise in an environment of low and stable inflation, where monetary policy can encourage asset price booms simply by credibly stabilizing the price level\(^3\). According to Borio and Lowe (2002), low inflation can promote financial imbalances, regardless of the underlying cause of an asset price boom. For example, by generating optimism about the macroeconomic environment, low


\(^3\) This view is suggested by Austrian economists in the 1920s and more recently by economists of the Bank for International Settlements (BIS). See Bordo and Wheelock (2004).
inflation might cause asset prices to rise more in response to an increase in productivity growth than they otherwise would. A third view, coming from the dynamic general equilibrium macroeconomics literature, argues that asset price bubbles can result from failure of monetary policy to credibly stabilize the price level. In these models, poorly designed monetary policies, such as the use of interest rate rules without commitment to a steady long run inflation rate, can lead to self-fulfilling prophesies and asset price bubbles (Woodford, 2003).

Some researchers, addressing how monetary policy authorities should respond to asset prices, think that asset prices should be considered as a part of price stability that is the sole objective of many independent central banks. Cecchetti et al (2000) argued strongly to put asset prices as direct measure of the goal of monetary policy. Monetary policy should react when asset prices become misaligned with fundamentals. Similarly, Smets (1997) argues that monetary policy tightening is optimal in response to “irrational exuberance” in financial markets. Bernanke and Gertler (1999) examined monetary policy in the presence of asset price bubbles, with application to Japan. Their results indicate that the Japanese policy was too tight from 1985 to 1988 and too lax from 1988 to 1990, fuelling a stock bubble, and too tight, again, from 1992 until at least 1996. They argue that even without explicitly targeting the asset prices, the Bank of Japan should have tightened from 1998 to 1990, probably ending the bubble, much earlier.

On the other hand, discussions in Gertler et al (1998) argue that the monetary authority should not respond directly to asset price movements, but instead should monitor them for their informational properties. Indeed it is extremely difficult to identify asset price bubbles. Moreover, there is no reason to suppose that monetary authorities possess more information about fundamental asset prices than the market. This topic seems very crucial since responding to asset price fluctuations is likely to increase significantly macroeconomic stability only if bubbles are identified in their infancy, which is by definition the time when they are most difficult to identify. But, even if one could successfully identify bubbles, there are other reasons why a monetary authority might not react directly to asset prices. First, many financial prices are noisy and volatile making signal extraction difficult. Second, current asset prices reflect expectations about future monetary policy and risk premium integrated in asset prices tends to vary in time. Finally, if policy is explicitly guided by asset prices there is a risk of a potentially destabilising circularity. Bernanke and Gertler (1999, 2001) and Cecchetti et al (2000), using a dynamic New Keynesian model (Clarida, Gali and Gertler, 1999) evaluate the appropriateness of a policy response to asset prices by exploring the efficacy of a variety of

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4 For example, Cogely (1999) advances the existence of the bubbles and Goodfriend (1998), suggests that asset prices are too volatile and reflect little economic activity.
interest rate reaction functions in simple calibrated stochastic model economies in which asset prices play some explicit role. Bernanke and Gertler (1999) argue that the inflation targeting approach dictates that central banks should adjust monetary policy actively and pre-emptively to offset incipient inflationary and deflationary pressures. Consequently, monetary policy should not respond to changes in asset prices, except insofar as they signal changes in expected inflation. In the opposite, Cecchetti et al (2000) suggest that a central bank concerned with both hitting an inflation target at a given time horizon, and achieving as smooth a path as possible for inflation, is likely to achieve superior performance by adjusting its policy instruments not only to inflation (or its inflation forecast) and the output gap, but to asset prices as well.

In this paper we use an aggregate dynamic macroeconomic model in order to examine the macroeconomic stability under inflation-targeting regimes with different degrees of flexibility (strict and flexible inflation targeting in the sense of Svensson, 1997) in which asset prices play some explicit role on the conduct of the optimal monetary policy. For the inflation target to be an anchor in all circumstances (temporary or persistent shocks) for private inflation expectations, we show that credible commitment of central bank to a steady long run inflation rate is necessary. This is translated in our model by a commitment to a long run growth rate of money supply (two pillars strategy of the ECB). We show that, the more flexible the inflation targeting is, the more likely monetary authorities are able to stabilise the economy around the steady-state equilibrium. A flexible inflation targeting central bank should bear in mind those longer-run consequences of asset price bubbles and financial instability in the setting of current interest rates.

The remainder of the paper is organised as follows. In Section 1, we develop a theoretical model of the economy in which financial asset prices play a role. In Section 2, we characterise the optimal reaction function of the central bank in which the optimal monetary policy respond to asset prices movements, among other economic indicators. Section 3 concentrates on the study of the dynamic stability of the economy. Section 4 examines the effects of the various economic shocks. Section 5 concludes.

2. The Model

We consider an economy described by an inflation adjustment equation, an aggregate spending relationship linking output to real interest rate and stock prices, and two equilibrium conditions in financial markets (money, bonds and shares). Inflation is governed by an expectational Phillips curve of the form:
\[ \pi = \pi^e + \alpha (y - y^*) + \epsilon_x, \quad \alpha > 0, \]  \hspace{1cm} (1)

where \( \pi \) (\( \equiv dp/\)dt) is the inflation rate, \( \pi^e \) the expected inflation rate, \( y \) the current output (or growth rate), \( y^* \) the natural rate of output and \( \epsilon_x \) an inflationary (or supply) shock. The output depends negatively on the expected real interest rate, \( (i - \pi^e) \), and positively on the real value of shares in the stock market, \( q, \) as follows:

\[ y = -\beta(i - \pi^e) + \gamma q + \epsilon_d, \quad \beta, \gamma > 0. \]  \hspace{1cm} (2)

where \( i \) is the nominal interest rate and \( \epsilon_d \) is a demand shock. In equation (2), it is recognized that stock price plays an important role in determining aggregate demand. First of all, being part of net wealth, it can affect households’ consumption. Second, determining the value of the existing capital relative to its replacement cost (Tobin’s \( q \) theory of investment), it affects firms’ investment level. Third, being net worth and used as collateral, it affects the firms’ balance-sheet position and so the risk premium to accept for obtaining funds in the capital market. A fourth mechanism linking the stock market with aggregate demand could be the household liquidity effect: an increase in stock price can imply an increase in the net wealth of households, which in turn increases consumption spending. A final mechanism is referred to as the confidence channel. The confidences of consumers, even those who do not own any share, and that of entrepreneurs, even when their companies are not quoted in the stock market, are positively related to the level of stock price.

As bonds and shares are considered here as perfectly substitutes in the portfolios of private agents (i.e., absence of risk premium), the arbitrage between bonds and shares implies the same expected yield in the short run for these two assets:

\[ i - \pi^e = \frac{\dot{q}}{q} + \frac{\psi y}{q}. \]  \hspace{1cm} (3)

In equation (3), the expected yield for shares is composed by the expected rate of capital gains or losses \( \frac{\dot{q}}{q} \) and the rate of distributed dividends \( \frac{\psi y}{q} \), where the term \( \psi y \) represents the firms’ profits, which by assumption are entirely redistributed (\( \psi \), assumed to be constant, is the share of profits in national income). Equation (3) can be solved, under the transversality condition, in order to obtain the expression for stock price in terms of actual (present) value of the expected future profits:

\[ q(t) = \int_0^\infty \psi y(s) \exp \left\{ -\int_t^s [i(t) - \pi^e(t)] dt \right\} ds. \]  \hspace{1cm} (4)
Equation (4) can be written in the form of dynamic equation where one can introduce a variable representing exogenous shocks, $\varepsilon_q$, affecting shares yield and inducing difference between the rates of return of bonds and shares:

$$\dot{q} = (i - \pi^r)q - \psi y + \varepsilon_q.$$  

(5)

Various factors, such as the formation of “speculative bubbles” or/and an exogenous variation of risk premium associated to different assets\(^5\), can be at the origin of $\varepsilon_q$. Finally, the money market equilibrium is characterised by

$$m - p = l_1 y - l_2 i + \varepsilon_m, \quad l_1, l_2 > 0$$  

(6)

where $m$ represents the nominal money supply and $p$ the general price level. The real money demand, $m - p$, depends on the real income $y$, nominal interest rate, $i$, and an exogenous shock affecting the money market, $\varepsilon_m$. Thereafter, it is assumed that central bank uses an interest rate rule to conduct its monetary policy. Taking the time derivative and noting that $m = \mu$, $p = \pi$, equation (6) can be written as follows:

$$\mu - \pi = l_1 \dot{y} - l_2 \dot{i} + \dot{\varepsilon}_m.$$  

(6a)

Equation (6a) implies that, in order to satisfy long-term stationarity (with $\pi = \dot{q} = \dot{y} = \dot{i} = 0$), monetary authority is constrained to set a growth rate of money supply consistent with the inflation target, i.e. $\mu = \pi + \dot{\varepsilon}_m = \pi^r + \dot{\varepsilon}_m$, where $\pi$ is the long run money growth consistent with the inflation target $\pi = \pi^r$. On the opposite, optimal monetary policy could not be credible because current inflation rate will be systematically different from expected inflation rate due to the rational belief of private sector that monetary authority will always apply a monetary policy consistent with this inflation target. If this is the case, private agents will expect an inflation rate different from the inflation target announced by the central bank. Thus, inflation targeting will not offer the nominal anchor for private inflation expectations as assumed in the inflation-targeting literature.

3. The optimal monetary policy rule

We assume that monetary authorities act systematically to minimize fluctuations of output around the natural rate, $y^*$, and inflation around its inflation target, $\pi^r$. The nominal interest

rate is treated as the direct instrument of monetary policy. Central bank is assumed to minimize the present discounted value of the following intertemporal loss function:

\[ E \int_0^\infty L(t) \exp(-\theta t) dt, \quad \text{with} \quad L(t) = \frac{1}{2} [\lambda (y - y^*)^2 + \kappa (\pi - \pi^T)^2], \quad \lambda, \kappa, \theta > 0, \]  

where \( E \) is the expectation operator. Preference parameters \( \lambda \) and \( \kappa \) denote respectively the importance that authorities place on output and inflation targets. \( \theta \) is a discount factor. The optimal monetary policy is the solution to the sequence of single period decision problems of monetary authorities. These decision problems being independent, the central bank’s optimisation problem consists simply of minimizing the one-period loss function, \( L \), in (7). Thus, the first-order condition is given by

\[ \lambda \frac{\partial y}{\partial \pi} (y - y^*) = -\kappa (\pi - \pi^T) \Rightarrow y = y^* - \frac{\kappa \alpha}{\lambda} (\pi - \pi^T), \]  

which leads to the following central bank’s optimal monetary policy rule (see, Appendix A):

\[ i = \frac{1}{\beta} \left[ \beta y + \beta \epsilon_d - y^* + \frac{\kappa \alpha}{\lambda} (\pi - \pi^T) \right] + \pi^\epsilon \]  

where the time-consistent expected inflation rate of private sector are equal to the central bank’s inflation target

\[ \pi^\epsilon = \pi^T, \]  

in the absence of persistent shocks (see equation A.7 in Appendix A).

According to equation (9), it is optimal for the central bank to adjust the nominal interest rate upward to reflect expected inflation, the gap between current inflation and the inflation target, as well as increases in stock price and increases in output gap due to a positive demand shock. There is no major difficulty in including stock price in the central bank reaction function, since it is easily observable and observed in an instantaneous way. Stock price at every moment conveys information contained in a set of data provided by individual investors having a more upstream knowledge than the central bank, concerning the origin and the nature of the shocks. Generally, stock price tends to react quickly to any information. Reacting to the evolution of asset prices undoubtedly gives an advantage to the central bank in order to react quickly and to stay in tune with the evolution of the economy. Otherwise, it takes a risk while trying to base its monetary policy decision only on the base of useful information all collected by itself. The presence of stock price in optimal interest rate rule reveals that monetary

\[ \text{Some operational difficulties might result from the short-run volatility of stock price. One way to avoid the repercussion of stock price volatility into the interest rate is to use a moving average with stronger weights being given to more recent developments of stock price.} \]
authorities must react to asset price movements even they cannot judge on the presence of mania or speculative bubbles phenomena, and the modification of risk premium.

4. The stability analysis

The dynamic behaviour of the economy can be summarised in a two first-order differential equations in inflation rate $\pi$ and stock price $q$. A linear approximation at the neighbourhood of the steady state $(\bar{q}, \bar{\pi})$ yields the following dynamic reduced form of the model (Appendix B):

$$
\begin{bmatrix}
\dot{\pi} \\
\dot{q}
\end{bmatrix} =
\begin{bmatrix}
\eta \left(-1 + \frac{l_2 \gamma}{\beta} \Phi \right) & \eta \frac{l_2 \gamma}{\beta} \left( i - \pi^T + \frac{\gamma}{\beta} \bar{q} \right) \\
\Phi & i - \pi^T + \frac{\gamma}{\beta} \bar{q}
\end{bmatrix}
\begin{bmatrix}
\pi - \bar{\pi} \\
q - \bar{q}
\end{bmatrix},
$$

(11)

where $\eta = 1 \left( \frac{l_1}{\alpha} - \frac{l_2 \kappa \alpha}{\beta \lambda} \right)$ and $\Phi = \frac{\bar{q} \alpha \kappa}{\beta \lambda} - \frac{\psi}{\alpha}$.

Moreover, in this economy stock prices are considered to adjust more speedily than inflation rate. Stock prices, quoted in continuous time on a centralised market, are much more flexible than goods prices and wages. Thus they are free to make discrete instantaneous jumps in response to “news” concerning all previously unanticipated current or future changes in exogenous variables and policy instruments. On the other hand, inflation rate, resulting from a relatively slow adjustment of goods prices and wages due to the different costs of adjustment (e.g. menu costs), is considered as a predetermined variable. Therefore, the stock price $q$ is considered as a forward-looking variable (clearing an efficient financial market) and inflation rate, $\pi$ is a backward-looking variable\(^7\). This distinction is after all based on the relative speed of adjustment of these two variables.

Steady state is characterised by the condition $\dot{q} = \dot{\pi} = 0$. Denote by $A$ the state matrix of the two-equation dynamic system (11). The paths taken by inflation rate and stock price in their dynamic adjustment to steady state equilibrium depend on the signs of determinant and trace of $A$:

$$
\text{det}(A) = -\eta \left( i - \pi^T + \frac{\gamma}{\beta} \bar{q} \right) \quad \text{and} \quad \text{tr}(A) = \eta \left( -1 + \frac{l_2 \gamma}{l_1} \Phi \right) + i - \pi^T + \frac{\gamma}{\beta} \bar{q}.
$$

\(^7\) See e.g. Buitter and Panigirtzoglou (2003) for a similar assumption concerning stock price and inflation rate dynamics.
Both signs of determinant and trace depend on the relative inflation aversion of monetary authorities, \( \kappa / \lambda \), or in other words, the degree of flexibility of the inflation-targeting regime with respect to the competing objectives. This leaves us with qualitatively distinct phase diagram configurations in the \( \pi - q \) space. Consider the conditions necessary for the sign of \( \det(A) \) to be negative:

\[
\det(A) = -\eta \left( i - \pi^T + \frac{\gamma}{\beta} q \right) < 0, \quad \text{when} \quad \frac{\kappa}{\lambda} < \frac{l_1 \beta}{l_2 \alpha^2} \quad \text{and} \quad i + \frac{\gamma}{\beta} q > \pi^T.
\]

These conditions reveal that the monetary authorities preferences \((\kappa, \lambda)\) influence the system’s dynamic stability. If the central bank weights strongly enough the output target \((\lambda)\) relative to inflation target \((\kappa)\), one has \( \frac{\kappa}{\lambda} < \frac{l_1 \beta}{l_2 \alpha^2} \) or \( \eta > 0 \). On the contrary, one has \( \frac{\kappa}{\lambda} > \frac{l_1 \beta}{l_2 \alpha^2} \) or \( \eta < 0 \), when \( \kappa / \lambda \) is large enough. Moreover, condition \( i + \frac{\gamma}{\beta} q > \pi^T \) is verified if nominal interest rate is greater than the inflation target: \( i > \pi^T \). In fact, if monetary policy leads to a nominal interest rate lower than the inflation target \( i < \pi^T \), an inconsistency appears in the evaluation method of shares adopted in this model (namely in terms of the present value of the future profits discounted at real interest rate). Specifically, when \( i < \pi^T \), inconsistency appears in the no-arbitrage equation between shares and bonds. At steady state, for a negative real interest rate, equation (3) implies that stock price should be negative to validate the yields equality between shares and bonds. However, stock price cannot be negative by its nature (limited responsibility for shareholders) even if the price of an individual stock can fall to zero when a firm goes bankrupt. For a flow of positive aggregate profits, stock price must thus be positive. That would imply that the arbitrage condition is not checked at steady state equilibrium. In other words, stock price calculated according to the discount formula (4) tends towards an infinite value, which constitutes another anomaly. A theoretical solution would consist of including a significant risk premium in the discount rate of profits so that stock price becomes again positive according to (3) or finite according to (4). For these reasons, we propose in the following to limit our analysis to the case where real interest rate would be positive.

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8 The case where \( i < \pi^T \) occurred in particular during second half of years 1970 in USA and in Europe and corresponds to one period of macro-economic instability.

9 In the short-run, a fall in stock price allows validating the no-arbitrage condition.
a) Low inflation aversion

The case of low inflation aversion (or high flexibility in the inflation targeting) corresponds to the following condition:

\[ \frac{\kappa}{\lambda} < \frac{l_1 \beta}{l_2 \alpha}, \quad \text{or} \quad \eta > 0. \]  

(12)

Under this condition, the slope of the \( \dot{q} = 0 \) locus in \( \pi - q \) space (Figure 1) is:

\[ \left. \frac{d\pi}{dq} \right|_{\dot{q}=0} = -(i - \pi^r + \frac{\gamma}{\beta} \bar{q})/\Phi > 0 \]

where \( \Phi < 0 \). Indeed, \( \Phi < 0 \) if \( \frac{\kappa}{\lambda} < \frac{\psi \beta}{\bar{q} \alpha^2} \), or when monetary authorities assign a small weight to inflation target. Further, the slope of the \( \dot{\pi} = 0 \) locus in \( \pi - q \) space is

\[ \left. \frac{d\pi}{dq} \right|_{\dot{\pi}=0} = \frac{l_1 \gamma}{\beta} (i - \pi^r + \frac{\gamma}{\beta} \bar{q})/(1 - \frac{l_2 \gamma}{\beta} \Phi) = (i - \pi^r + \frac{\gamma}{\beta} \bar{q})/(\frac{\beta}{l_2 \gamma} - \Phi) > 0. \]

Both \( \dot{q} = 0 \) and \( \dot{\pi} = 0 \) lines are upward sloping, and the \( \dot{q} = 0 \) line is steeper than the \( \dot{\pi} = 0 \) line. In formal terms, the system (or matrix \( A \)) will have one stable eigenvalue and therefore saddle-point equilibrium. With one predetermined and one non-predicted variable, the presence of a stable and an unstable eigenvalues guarantees the existence of a unique convergent path. The transversality condition implies that rational agents will not choose unstable solutions. Economic agents are assumed to be able to find the unique convergent path thanks to their capacity of rational expectations. In other words, the jump variable (stock price) will always attain the value required to put the system on the unique convergent path.

Specifically, as parameter \( \eta \) is positive (so that, \( \det(A) < 0 \)), a sufficient condition for the long-run equilibrium to be saddle-point is a sufficiently small value for the ratio \( \kappa/\lambda \), that is,

\[ \frac{\kappa}{\lambda} < \frac{l_1 \beta}{l_2 \alpha}. \]

This ratio represents the relative weight assigned by monetary authorities to inflation target (or inflation stabilisation). In other words, \( \kappa/\lambda \) characterizes the monetary authority’s inflation aversion. In the extreme case, when \( \lambda = 0 \) (i.e., \( \kappa/\lambda \to \infty \)), we have the case of a strict inflation targeting. This means that monetary authority only cares about inflation. In the opposite, \( \lambda \to \infty \) (i.e., \( \kappa/\lambda = 0 \)), monetary authority has a very strong preference for output target. In this case, we have the extreme case of a flexible inflation targeting. Therefore, according to the condition (12), only a high degree of flexibility in
inflation targeting allows the steady state equilibrium to have this saddle-point configuration, as shown in Figure 1.

Under the inflation-targeting regime with high flexibility or an accommodative response of the central bank, nominal interest rate rises in a smoothing manner following an initial increase in inflation rate due to an exogenous shock. Given inflation expectations, real interest rate will rise and involve a reduction in aggregate demand due to its negative effect on consumption and investment. The fall in aggregate demand allows reducing smoothly inflation pressure and expectations.

The range of value for the ratio $\frac{\kappa}{\lambda}$ compatible with saddle-point equilibrium depends on parameters ($\beta$, $l_1$, $\alpha$ and $l_2$) reflecting the economic and financial characteristics of the underlying economy. If $\beta$ and $l_1$ have higher values and $\alpha$ and $l_2$ smaller values, the central bank can give greater relative weight to the inflation target. We notice that, higher $\beta$ corresponds to more important financial development and smaller $l_2$ to lesser interest rate elasticity of the money demand. More financial developments and decreasing interest rate elasticity of the money demand can give the central bank more liberty to define its inflation-targeting framework or the government to appoint a more conservative central banker without creating macro-economic and financial instability.

Figure 1. Phase diagram in the case of high flexibility in inflation targeting

b) High inflation aversion

We now switch to consider a monetary authority with a high inflation aversion (or a relatively low flexibility in the inflation targeting) in the sense that:

$$\frac{\kappa}{\lambda} > \frac{l_1 \beta}{l_2 \alpha^2}, \quad \text{or} \quad \eta < 0.$$ (13)
Under condition (13), the determinant is positive: \( \det(A) = -\eta \left( i - \pi^T + \frac{\gamma}{\beta} \right) > 0 \). Thus, three cases can be distinguished as follows:

\( i \) The system is locally unstable when \( \text{tr}(A) > 0 \). This is equivalent to the following condition

\[
\frac{\kappa}{\lambda} > \left( \frac{l_1 \beta}{l_2 \alpha} - \frac{\beta}{l_2 \alpha} - \frac{\psi \gamma}{\alpha^2 i} \right) \left( 1 - \frac{q_i}{\beta} \right).
\] (14)

where \( i = (i - \pi^T + \frac{\gamma}{\beta}) \). As we previously admit \( i > \pi^T \), it is easy to show that \( 1 - \frac{q_i}{\beta} > 0 \). We also assume that \( l_1 \beta - \frac{\beta}{l_2 \alpha} \frac{\psi \gamma}{\alpha^2 i} > 0 \). According to the condition \( i > \pi^T \), this assumption can be rewritten as \( \bar{q} > \frac{1}{\gamma} (\beta \alpha + \frac{\psi \gamma}{l_i}) \). In other terms, economy has a relatively important capitalisation (\( \bar{q} \)). As there are two positive real eigenvalues, the system can be simply explosive if \( \text{tr}(A)^2 - 4\det(A) > 0 \) or cyclically explosive if \( \text{tr}(A)^2 - 4\det(A) < 0 \) (i.e., there are two imaginary eigenvalues with positive real parts), as shown in Figure 2.

In Figure 2, we illustrate one particular case with \( \Phi < 0 \) (the alternative case with \( \Phi > 0 \) can be worked out easily). The slope of the \( \dot{q} = 0 \) locus in \( \pi - q \) space is
\[
\frac{d\pi}{dq}_{\dot{q}=0} = -(i - \pi^r + \frac{\gamma}{1 - \frac{\psi}{\lambda} l_2 l_3})/\Phi > 0.
\] The slope of the \(\dot{\pi} = 0\) locus in \(\pi - q\) space is
\[
\frac{d\pi}{dq}_{\dot{q}=0} = \frac{l_2 \gamma}{\beta} (i - \pi^r + \frac{\gamma}{1 - \frac{\psi}{\lambda} l_2 l_3})/\Phi > 0.
\]

ii) The system adjusts cyclically [\(\text{tr}(A) = 0\)]. This corresponds to the case:
\[
\frac{\kappa}{\lambda} = \left(\frac{l_1 l_2}{l_3} - \frac{\beta}{\alpha^2} - \frac{\psi}{\alpha^2} l_2 \right) > 0,
\]
Under condition (15), the trace is equal to zero. There are only two imaginary eigenvalues with zero real parts. The system generates cyclical movements and never converges automatically to the steady state after any shock. When \(\frac{\kappa}{\lambda} > \frac{l_1 l_2}{l_3^2}\), or equivalently \(\eta < 0\), \(\text{tr}(A) = 0\) is possible only if we have \(-1 + \frac{l_2 \gamma}{\beta} \Phi > 0\), that means \(\Phi > 0\) or equivalently \(\frac{\kappa}{\lambda} > \frac{\psi \beta}{\alpha^2}. Under these conditions, the slope of the \(\dot{q} = 0\) locus in \(\pi - q\) space (Figure 3) is:
\[
\frac{d\pi}{dq}_{\dot{q}=0} = -(i - \pi^r + \frac{\gamma}{1 - \frac{\psi}{\lambda} l_2 l_3})/\Phi < 0, \text{ as } \Phi > 0.
\]

The slope of the \(\dot{\pi} = 0\) locus in \(\pi - q\) space is:
\[
\frac{d\pi}{dq}_{\dot{q}=0} = \frac{l_2 \gamma}{\beta} (i - \pi^r + \frac{\gamma}{1 - \frac{\psi}{\lambda} l_2 l_3})/\Phi < 0, \text{ if } -1 + \frac{l_2 \gamma}{\beta} \Phi > 0.
\]

Therefore, the absolute value of \(\frac{d\pi}{dq}_{\pi=0}\) is superior to that of \(\frac{d\pi}{dq}_{\dot{q}=0}\), as shown in Figure 3.
iii) The system is saddle-point stable \([\text{tr}(A) < 0]\). This corresponds to the case where

\[
\frac{l_1 \beta}{l_2 \alpha^2} < \frac{\kappa}{\lambda} \left( \frac{l_1 \beta}{l_2 \alpha^2} - \frac{\beta}{l_2 \alpha^2} - \frac{\psi}{\alpha^2} \right) \left( 1 - \frac{\beta i}{\beta i} \right).
\]

(16)

The system can be simply saddle-point convergent, \([\text{tr}(A)^2 - 4\det(A) > 0]\) as there are one positive real eigenvalue and one negative real eigenvalue; or cyclically convergent if \(\text{tr}(A)^2 - 4\det(A) < 0\), as under this condition there are two eigenvalues with unreal parts of opposite signs. The condition \(-1 + \frac{l_2 \gamma}{\beta} \Phi > 0\) is imposed also here so the slopes of \(\dot{q} = 0\) locus and of \(\dot{\pi} = 0\) locus in \(\pi - q\) space (Figure 4) are negative. The slopes of these two phase-lines are smaller than in the previous case. The absolute value of \(\left. \frac{d \pi}{dq} \right|_{\pi=0} \) is always superior to that of \(\left. \frac{d \pi}{dq} \right|_{q=0} \), as shown in Figure 4.

We observe that for all these cases where the condition (13) is verified, the risk of macroeconomic instability is more and more important when one increases the ratio \(\kappa/\lambda\). As \(\kappa/\lambda\) increases, the economy moves from saddle-point equilibrium, to cyclical equilibrium and finally to totally unstable equilibrium. Thus the optimal monetary policy rule could involve a
serious risk of financial and macroeconomic instability when monetary authorities assign a higher and higher weight to inflation stabilisation or when, in the extreme case, they adopt a strict inflation-targeting framework of monetary policy. Under a strict inflation targeting, the monetary policy formulated only in terms of inflation target with an optimal interest rate rule could be ineffective to stabilise the economy although we suppose the central bank is credible. Comparing with the results of Dai and Sidiropoulos (2003), we remark that the introduction of a monetary growth rule compatible with the inflation target ameliorates the stability of the system. In fact, without the monetary growth rule, when the condition (13) is verified and when economic agents adjust endogenously their expectations using information extracting form financial markets data, we have financial and macroeconomic instability.

To understand why financial and macroeconomic instability may arise, one can imagine the undesirable effect resulting from an aggressive reaction of nominal interest rate to inflation rate. In fact, a sharp increase in nominal interest rate increasing operational costs of firms in terms of debt services could force some firms into insolvency, leading thus to a reduced competition in prices and higher inflation pressure. Economic agents could anticipate this inflationary pressure. They could then increase their demand for loans. That generates thus a further inflationary pressure. In this respect, many of emerging market economies (i.e., Latin American countries during the 1980s) and transition economies (i.e., Eastern European countries in 1990s) provide examples in which a sharp increase in nominal interest rate is not able to reduce inflation expectations and inflation rate. In fact, this analysis seems to be suitable in many of these countries with insufficiently developed and fragile monetary and financial markets. But, this does not mean that the Eurozone cannot be stoked by financial and macroeconomic instability in the case where the board of governors of the ECB named by European national governments are too conservative or choose itself a low degree of flexibility in the implicit inflation targeting regime.

5. Effects of exogenous shocks

A major motivation behind this work is to be able to understand what information asset prices can give us about the shocks affecting the economy. We now turn to an analysis of the impact and steady-state effects of three types of shocks affecting respectively aggregate supply, aggregate demand and the stock market equilibrium condition. We consider that shocks affecting money demand don’t have any impact on the steady-state equilibrium because by assumption they are characterised as shocks without trends. Such a shock may affect the price of goods and services but not long-term inflation rate and real stock price.
a) Inflationary shocks ($\varepsilon_\pi > 0$)

According to equation (1), the steady-state equilibrium conditions imply $\pi^e = \pi^T$ (see equation 10) and $\mu = \pi = \pi^T$ when the central bank has full credibility. Consequently, a supply shock has no steady-state effect on the inflation rate, $\Delta\pi = 0$. This is due to the fact that we assume that the money growth rate is controlled by the central bank at the level of inflation target. Its effect on the output is:

$$\Delta y = \frac{-\varepsilon_\pi}{\alpha}.$$  

(17)

This steady-state effects on the stock price is appreciated using equation (B.1) as follows:

$$\Delta q = \frac{-\psi \varepsilon_\pi}{\alpha (i_0 - \pi^T + \mu_0 / \beta)}.$$  

(18)

Finally, its steady-state effects on the nominal interest rate is given using (9), (10) and (18) as:

$$\Delta i = \frac{-\gamma \psi \varepsilon_\pi}{\alpha \beta (i_0 - \pi^T + \mu_0 / \beta)}.$$  

(19)

In order to examine the transitional dynamics of the model, it is convenient to consider initially the case of saddle-point equilibrium illustrated in Figure 1. The positive inflationary shock ($\varepsilon_\pi > 0$) affects inflation rate through its impact on the supply side. In the long run, both locus $\dot{q} = 0$ and $\dot{\pi} = 0$ shift to the left as is shown in Figure 5. Stock prices drop in an instantaneous way to their new steady state level, while inflation cannot change remaining in its initial steady state level. Indeed, the adjustment process is entirely realised by the stock prices and the interest rate. The fall in stock prices gives the possibility to the central bank to react immediately by lowering sharply the nominal interest rate in order to stabilise the economic slowdown pressures due to the supply shocks. This allows the central bank to react more quickly than if should only reacts to the current inflation rate without an additional response to asset price movements.
b) Demand shocks ($\varepsilon_d > 0$)

We now turn to an analysis of the impact and steady-state effects of shocks affecting aggregate demand. A demand shock does not produce any steady-state effects on the inflation rate and the output. Indeed its steady-state effects on these variables are neutralised by the response of monetary policy. The demand shocks only affect the stock prices and the nominal interest rate. Using (B.1) and (9), it yields:

\[
\Delta \bar{q} = \frac{-q_0 \varepsilon_d}{\beta(i_0 - \bar{\pi}) + \gamma q_0},
\]

(20)

\[
\Delta \bar{r} = \frac{(i_0 - \bar{\pi}) \varepsilon_d}{\beta(i_0 - \bar{\pi}) + \gamma q_0}.
\]

(21)

Consider the case of saddle-point equilibrium illustrated in Figure 1. A positive demand shock ($\varepsilon_d > 0$) is inflationary and leads monetary authorities to raise interest rate. Its impact on the dynamics of the economy is exerted through the optimised interest rate rule (see equation 9). The latter affects then the long-term values and the dynamics of inflation rate and stock prices. As inflation rate and real output remain unchanged at the steady-state equilibrium, the rise of interest rate involves a shift of the curves $\bar{\pi} = 0$ and $\bar{q} = 0$ to the left. The real stock price will lower following the increase of long-run real interest rate. The adjustment process could be illustrated as shown in Figure 5.

The inclusion of stock prices in the optimal interest rate rule allows the monetary authorities to moderate its interest rate reaction by taking into account the instantaneous fall of stock prices. Indeed, a positive demand shock implies a rise of interest rate. But its direct effect on interest rate will be partially counterbalanced by a decrease of stock prices.

c) Shocks affecting stock price ($\varepsilon_q > 0$)
Shocks affecting the stock price cannot affect inflation rate and output if there is an appropriate response of monetary policy. As the monetary authorities adopt an inflation-targeting framework to stabilise inflation and output, their effects on inflation rate and output are neutralised. In fact, through the reaction of the nominal interest rate to the level of stock price (but not directly to these shocks, see equation 9), monetary policy neutralises their effects on aggregate demand and so they are not transmitted to inflation rate and output. Consequently, these shocks have only effects on the stock prices and the nominal and real interest rates. Using (B.1) and (9), we obtain:

\[ \Delta q = -\frac{\beta e_q}{\beta(i_0 - \pi^T) + \gamma q_0}, \]  
\[ \Delta i = -\frac{\gamma e_q}{\beta(i_0 - \pi^T) + \gamma q_0}. \]  

According to equation (9), the monetary authorities need only to know the level of stock price, which is easily observable. They are not urged, when applying the optimal monetary policy rule, to know what is the nature of these shocks. In other words, central bankers have not to judge on the presence or not of mania or speculative bubbles phenomena, and the modification of risk premium or risk aversion of financial operators. Indeed, one of the arguments advanced by some academics and central bankers as to why a monetary authority might not react directly to asset prices is that monetary authority cannot judge the presence or not of speculative bubbles. In this respect, discussions in Gertler et al (1998) suggest that the monetary authority should not respond directly to asset price movements. This is because it is extremely difficult ex ante to identify asset price bubbles. Put another way, there is no reason to suppose monetary authorities possess more information about fundamental asset prices than the market. Goodfriend, in Gertler et al (1998), argues “central bankers have no particular expertise in pricing equities, which is a full-time job for armies of stock analysts and investors.” According to Cogley (1999), the miss-adjustment of asset prices are too difficult to detect and the errors due to reaction at asset prices can increase the variance of output. But even if one could successfully identify bubbles there are other reasons why a monetary authority might not react directly to asset prices. Many financial prices are noisy and volatile making signal extraction difficult. Further, current asset prices reflect expectations about future monetary policy. But if policy is explicitly guided by asset prices there is a risk of a potentially destabilising circularity. These arguments militate for prudence in the use of information conveyed by asset prices rather than to be unaware of them.
The adjustment and transmission mechanism is similar to the case of inflationary shock ($\varepsilon_\pi > 0$). The dynamic adjustments can be illustrated in Figure 5.

5. Conclusion

In this paper we have used an aggregate dynamic macroeconomic model in order to examine the macroeconomic stability under alternative inflation-targeting regimes (strict or flexible) in which asset prices play some explicit role on the conduct of the optimal monetary policy. We have examined the rationale of implicit inflation targeting like that of the ECB (two pillars strategy), as a communication and anchoring device, with a monetary policy responding also to asset price booms. Solving the central bank’s optimisation problem, we derive an optimal interest rule where nominal interest rate is set according not only to inflation and output, but also to asset prices. It is shown that achieving price stability under the regime of relatively strict inflation targeting, in the sense of Svensson, can generate financial and macroeconomic instability and will not be able to stabilise effectively the fluctuations of output and inflation. On the other hand, the more flexible the inflation targeting framework of monetary policy is, the more likely the ability of monetary authorities to reduce the risks of future financial and macroeconomic instability. A commitment to a long run growth rate of money supply identical to the inflation target, as part of the implicit inflation-targeting regime, can reinforce the credibility of the central bank and the role of inflation target as strong and credible nominal anchor for private inflation expectations and allows the system to be more stability prone.

One important implication of our results for governments nominating inflation-targeting central bankers is that they must not choose a too conservative board of governors. The choice depends of course on the economic and financial characteristics of the underlying economy. More developed financial markets and more flexible labour market allows the diminution of degree of flexibility in inflation targeting without introducing dynamic instability. For the central banks that practice explicit or implicit inflation targeting, more flexibility is without doubt a key to success as it reduces economic and financial instability and gains more support for their monetary policy.
Appendix A. Optimal interest rate rules of the central bank

The central bank minimises a loss function of the following form:

$$E_t \int_0^\infty L(t) \exp(-\theta t) dt,$$

where

$$L(t) = \frac{1}{2} \left[ \lambda (y - y^*)^2 + \kappa (\pi - \pi^*)^2 \right], \quad \lambda, \kappa, \theta > 0. \quad \text{(A.1)}$$

The decision of the central bank consists of solving its minimisation programme by taking account of subjacent economic model. The first-order condition is as follows:

$$\frac{\partial L}{\partial \pi} = 0, \Rightarrow \lambda (y - y^*) \frac{\partial y}{\partial \pi} + \kappa (\pi - \pi^*) = 0. \quad \text{(A.2)}$$

According to (1), one has\( \frac{\partial y}{\partial \pi} = \frac{1}{\alpha} \), and by using it in (A.2), one obtains:

$$\lambda (y - y^*) \frac{1}{\alpha} + \kappa (\pi - \pi^*) = 0. \quad \text{(A.3)}$$

That is to say: \( y = y^* - \frac{\kappa \alpha}{\lambda} (\pi - \pi^*) \). \quad \text{(A.4)}

Using (2) and (3) in the preceding equation, it follows:

$$\beta (i - \pi^*) = \gamma q + \varepsilon_d - y^* + \frac{\kappa \alpha}{\lambda} (\pi - \pi^*). \quad \text{(A.5)}$$

That brings us to the following interest rate rule:

$$i = \frac{1}{\beta} \left[ \gamma q + \varepsilon_d - y^* + \frac{\kappa \alpha}{\lambda} (\pi - \pi^*) \right] + \pi^*. \quad \text{(A.6)}$$

One can then calculate expected inflation rate by taking account of the assumption of rational expectations. According to (1), one has \( \pi^e = \pi^\pi + \alpha (y^\pi - y^*) \), that is to say \( y^\pi = y^* \), which implies with (A.3):

$$\pi^e = \pi^\pi. \quad \text{(A.7)}$$

By substituting the result of (A.7) in (A.6), one obtains:

$$i = \frac{1}{\beta} \left[ \gamma q + \varepsilon_d - y^* + \frac{\kappa \alpha}{\lambda} (\pi - \pi^*) \right] + \pi^\pi. \quad \text{(A.8)}$$
Appendix B. The dynamic equations of stock price and inflation rate

By taking account of the equations (1) and (10), the stock price dynamics can be characterised by the following equation:

\[
\dot{q} = (i - \pi^*) q - \frac{\psi}{\alpha} (\pi - \pi^* - \varepsilon) - \psi y^* + \varepsilon q. \tag{B1}
\]

In order to obtain the dynamic equation of inflation rate, one time derives the variables in (4):

\[
\dot{m} - \dot{\rho} = l_1 \dot{y} - l_2 \dot{i} + \dot{\varepsilon}_m, \quad l_1, l_2 > 0. \tag{B.2a}
\]

Knowing that \(\dot{m} = \mu, \dot{\rho} = \pi^*, \pi^* = \pi^T, \pi^e = 0\), that turns out to:

\[
\mu - \pi = l_1 \dot{y} - l_2 \dot{i} + \dot{\varepsilon}_m. \tag{B.2b}
\]

In using the interest rate rule (A.8), one has:

\[
\dot{i} = \frac{1}{\beta} \left[ \gamma \dot{q} + \dot{\varepsilon}_d - \dot{y}^* + \frac{\kappa \alpha}{\lambda} \pi \right]. \tag{B.3}
\]

According to (1) and (A.7), one has:

\[
\dot{y} = \frac{1}{\alpha} (\dot{\pi} - \dot{\varepsilon}_\pi). \tag{B.4}
\]

By using the results of (B.3) and (B.4), one obtains:

\[
\mu - \pi = \frac{l_1}{\alpha} (\dot{\pi} - \dot{\varepsilon}_\pi) - \frac{l_2}{\beta} \left[ \gamma \dot{q} + \dot{\varepsilon}_d - \dot{y}^* + \frac{\kappa \alpha}{\lambda} \pi \right] + \dot{\varepsilon}_m. \tag{B.5}
\]

In admitting that \(\dot{\varepsilon}_\pi, \dot{\varepsilon}_d = 0\), i.e. shocks without trend, one has:

\[
\mu - \pi = \frac{l_1}{\alpha} - \frac{l_2}{\beta} \frac{\kappa \alpha}{\lambda} \pi - \frac{l_2}{\beta} \dot{\gamma} + \dot{\varepsilon}_m. \tag{B.6}
\]

Using (B.1) in (B.5), and in admitting a money growth rule \(\mu = \pi + \dot{\varepsilon}_m = \pi^T + \dot{\varepsilon}_m\), one can present the differential equation of inflation rate as follows:

\[
\dot{\pi} = -\eta \pi + \frac{\eta l_2}{\beta} \dot{q} + \eta \pi, \quad \text{with } \eta = 1 / \left( \frac{l_1}{\alpha} - \frac{l_2}{\beta} \frac{\kappa \alpha}{\lambda} \right). \tag{B.7}
\]
References:


ECB (2001), The Monetary Policy of the ECB. Frankfurt: ECB.


