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Abstract

An essential part of local fiscal policy is the choice of the tax base. In this paper, we take four criteria to evaluate tax bases, namely: efficiency, simplicity, flexibility, and fairness. The results of such an evaluation highly depend on how we describe the involved agents. We construct a two stage model of a local economy with three types of agents: Leviathans, households, and housing firms. Each Leviathan seeks to maximize the surplus of his local fiscal budget. Each household seeks to maximize its life-time utility from three types of goods: composite private goods, housing, and local public goods. Each housing firm seeks to maximize its profits. In this model, we analyze the characteristics of four distinct tax bases: land rent, housing capital rent, housing sales, and housing property. In particular, we analyze the responses of the households, the housing firms, and the housing prices on a change of a specific tax rate. The results are used to evaluate each tax base with respect to our four criteria.

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JEL Classification: H11, H21, R51

1 The Evaluation of Tax Bases

In a strict sense, any governmental activity implies taxation. In the real world, there is nothing without costs. The essential feature of a government is that it has a monopoly on the legal use of physical force. Hence, it may use its physical force to cover its costs. Taxation means that a government forces some group to pay a contribution, without offering a specific service in return. The contributing group is defined via a certain tax base. In theory, the tax base could be anything. Normally, it belongs to one of these classes: income, wealth, or consumption. The size of a contribution is defined via the respective tax tariff. In theory, the tariff could have any structure. Basically, three classes are distinguished: progressive, proportional, or regressive.

A government is supposed to use its power to tax, in the general interest. Economists have contrived various criteria to evaluate taxes.[1] Now, there exists a wide consensus that a tax ought to have the following features:

1. Efficiency: A tax is efficient, if it supports the optimal allocation of resources.

2. Simplicity: In general, the more simple a tax is, the less it incurs compliance or administration costs.

3. Flexibility: Economic conditions may change. The more flexible a tax is, the less adaptation costs arise.

4. Fairness: A tax is considered as fair, if it treats every citizen according to some general procedure and to her relative economic condition.

But, it seems hardly possible to make a final judgement on whether a specific tax is in the general interest, or not. For instance: Is a (proportional) sales tax better than a (progressive) property tax? - We cannot really answer this. There are at least three factors which make a tax evaluation more difficult:

1. Interdependence of criteria: There may exist connections between certain criteria: positive or negative ones.

2. Motivation of tax payers: Because the tax payers are forced, their responses do not reveal their true preferences.

3. Motivation of the government: The impact of a tax may depend on the government's motivation.

The next two subsections present two different approaches on the government's motivation. Section 2 recapitulates some major arguments about efficiency in the provision of local public goods. In Section 3, a two stage

[1] One salient proposition was made by Smith (1776). He set up the following four criteria: 1) equity; 2) certainty; 3) convenience; 4) administrative economy. For more about the respective discussion, see Musgrave (1986, 2000).

model is developped which shall help to understand and to evalute four local tax bases. First, a general framework is entroduced, then the framework is adapted to each of the four tax bases. In the last section, a brief evaluation of each tax bases is made.

1.1 The Social Welfare Maximizer Approach

In the social welfare maximizer approach, the government is conceived as monolithic and benevolent. In principal, it is asked which tax a government should choose to improve the total market outcomes. The government is normally supposed to have full knowledge. There are no obstructions in the tax implementation. Nevertheless, it is recognized that taxes can hardly generate perfect solutions. The reason can be depicted as follows: The only tax which does not generate deadweight losses is the lump-sum tax. However, such a tax can hardly be considered as fair. Thus, the social welfare maximizer approach adopts a second best approach. The respective theory explains why it may be welfare improving to deviate from a first best condition, if another first best condition cannot be met. Together, these two approaches coincide with a third one, called 'optimal taxation'.

A fundamental contribution to the optimal taxation approach was made by Ramsey (1927). Ramsey asked how a government could minimize the deadweight loss, if it wants to generate a certain amount of revenue by commodity taxes. He set up a model in which a representative (producer-consumer) household maximizes its net utility, given a finite set of commodities. The net prices that this households pays correspond to the commodity related taxes. The government has all the relevant information. Its optimization exhibits the following rule: Each commodity tax rate has to be stipulated in such a manner that a marginal change of any single tax rate would lead to the same marginal change in the (compensated) demand for all commodities ('Ramsey rule'). If the demand for each commodity is independent of the prices for others, then another rule appears: Each tax rate has to be inversely proportional to the (compensated) price elasticity of the demand for the related commodity ('inverse-elasticity rule').[2]

The optimal taxation approach has been further developed in various directions. Further aspects have been examined, such as: pre-existing distortions, externatlities, the impact of public expenditure, or risk.[3] In many models, the optimal tax is derived with the help of a social welfare function. Then, the specific function implicitly determines the importance of the the

[2]For an introduction into the optimal taxation approach, see Stiglitz (2000), chapter 20; for a more formal treatment, see Auerbach (1985).

[3]One salient contribution was made by Diamond and Mirrlees (1971).

different tax criteria. The central issue thus becomes how to justify a specific choice. Depending on the form of the function, any tax could turn out to be optimal. Hence, there has been an intensive discussion over the 'right' form of the social welfare function. Today, some consensus might include the following assertions:

- The marginal utility of the individual income is decreasing.
- The marginal cost of an individual good is decreasing.
- The marginal deadweight loss of substitution activity is increasing.
- The aggregation of the utility should follow some (adapted) maximin principle.

However, since this approach considers the government as benevolent and fully informed, tax criteria such as simplicity of flexibility become irrelevant. Moreover, there appears no reason why the power of the government should be divided. Optimal taxation normally takes place in a centralized state. Anyway, as Arrow (1951) showed, it is impossible to derive a social welfare function from a relevant set of axioms. Thus, the respective results of the optimal taxation approach can hardly be robust.

1.2 The Leviathan Approach

In the Leviathan approach, the government is conceived as monolithic and self-interested. The assumption of the self-interest can be justified by at least three reasons: First, it bases on the 'homo economicus' approach. It is thus consistent with the standard economic models of households or firms. Second, the assumption is rather pessimistic. It conforms to a risk-averse social policy. And third, evolutionary models in general suggest that a self-interested government would dominate benevolent ones. Thus, the Leviathan approach mainly deals with two simultaneous problems: the provision of public goods and the constraints on governments. To provide public goods gets problematic due to the free-rider option. To constrain governments gets problematic due to the monopoly on the legal use of physical force.

Hence, private and public agents differ less in their general motivation but more in their general options. In particular, a government can decide how to tax; and a citizen can decide how to react on different taxes. It may be stipulated by a (fiscal) constitution which modes of taxation and which modes of reaction are permissible. On the one hand, the modes of taxation have three major elements: the base, the tariff, and the amount of revenue. A constitution directly constrains a government by restricting these elements. On the other hand, two modes of reaction can be divided: 'voice' and 'exit'. A constitution indirectly constrains a government by arranging and protecting these modes. Thus, the Leviathan approach coincides with the constitutional approach.

A fundamental analytical framework for a constitutional approach was set up by Brennan and Buchanan (1980). They asked how the power to tax could be best constrained by constitutional rules. Brennan and Buchanan specified Leviathan as a tax revenue maximizer. They analyzed his behavior in various tax settings. In a setting with income taxes, Leviathan is allowed to choose his optimal tariff. As the two authors could show, he would choose a regressive tariff because it counters the incentives for the citizens to substitute leisure for labor. He would transfer then all of the social income (beyond some subsistence level) to his own budget. Nothing of the tax revenue would be spent on public goods. Therefore, Brennan and Buchanan recommended to stipulate a certain income tax tariff and a certain yardstick for public expenditure into the constitution. Overall, they found support for constraints, such as: narrowing of tax bases, proportional tax rates, ear-marking, or decentralization.[4]

The Leviathan approach has been further developed in some directions. One can already discern some consensus on the treatment of the tax criteria. Central features of this treatment are: First, efficiency: The Leviathan approach measures efficiency by both, deadweight loss and tax misuse. Basically, there exists a trade-off between the two. The citizens can constrain tax misuse by voice or by exit. Each of these modes incurs a specific type of competition; each has its specific costs and benefits. The efficiency of a tax may depend on the specific supply of public goods. Second, simplicity: Information and transaction costs are taken into consideration. Such costs tend to induce asymmetries between the Leviathan and the citizens. The Leviathan can use such asymmetries to increase his tax misuse. The simpler a tax is, the less information and transaction costs are induced. Third, flexibility: Constitutional constraints are stipulated in a long-run perspective. In order to adapt to short-run requirements, they need to be flexible. In a favorable sense, the constraints are supposed to work as automatic stabilizers. And fourth, fairness: The Leviathan approach regards consent as the highest norm of fairness. Consent may be reached, if the citizens stand behind a 'veil of ignorance'. Behind such a veil, they do not know their social position in a future, postconstitutional period. Thus, any citizen orientates his vote towards the common interest.[5]

[4]See also Brennan and Buchanan (1977, 1978, 1979).

[5]See also Findlay/ Wilson (1987).

2 Local Fiscal Policy

The costs of reaction to fiscal policy depend on the government structure. Generally, these costs will be the lower, the more decentralized the public sector is. This can be explained by the existence of region specific capital. Region specific capital may improve the efficiency of any region specific decision. The costs of such capital negatively depends on the size of the respective region. The capital gets lost, if the owner exits. One may ask, subsequently: What are the gains of reaction to fiscal policy?

Tiebout (1956) examined an extreme case in which the exit costs are zero. He claimed that the exit mode could incur efficiency in the provision of public goods. In his local setting, each jurisdiction is governed by a 'city manager'. Each city manager seeks to maximize the profits of his jurisdiction. He offers a special, fixed package of local public goods and charges a head tax. A citizen can choose a special package by moving to the respective jurisdiction. She 'votes with her feet' and thus reveals her true preferences. Depending on the actual number of local residents, a city manager promotes entry or exit. The competition between the city managers puts their profits under pressure. In the total equilibrium, they all have zero profits. Further equilibrium conditions are:

- perfect market transparency;
- no scale economies;
- no spillovers;
- local independence of private income; and
- the range of public offerings covers the range of the citizens' preferences.

Tiebout admitted that the equilibrium conditions are altogether enormously demanding. Nonetheless, he believed that his setting allows to derive the right policy rules. These should in particular help to lower the exit costs.[6]

Buchanan and Goetz (1972), however, found fault with the Tiebout setting as a reference for public policy. In particular, they argued that it neglects inherent inefficiencies caused by the 'fact of location'. Thus, it rather describes an adjustment process in a non-spatial world of voluntary clubs. In this setting, nothing prevents the citizens from optimizing both at the same time: the private and the public goods consumption. In a spatial world, by contrast, a citizen's move from one jurisdiction to another affects social

[6]For a general discussion, see Mueller (2003), chapter 9.

benefits and average costs, in both jurisdictions. The choice of local public goods induces externalities.[7]

Hamilton (1975) regarded the criticism by Buchanan and Goetz as rather empirical. As he showed, a spatial Tiebout model may have an efficient outcome, if some more specific assumptions on the cost and the utility functions are made. But, not to depend on such specific assumptions, Hamilton suggested to extend the spatial Tiebout model by a constructed price system for local public goods. This price system has two basic elements: First, the local government imposes a proportional property tax. Second, it imposes a minimum of housing consumption. The system roughly works as follows: If a citizen lives in a jurisdiction where the legal minimum of housing consumption is below her personal optimum, then she will move to a jurisdiction with a higher minimum. The reason is that the average tax base in the latter jurisdiction will become broader. The respective government gets the option to increase its provision of public goods or to decrease its property tax rate. At the end of the adjustment process, the local economy reaches an equilibrium in which:

- each jurisdiction is internally homogeneous;
- the capitalization rates of the property taxes are all zero;
- the local governments do not redistribute any income;
- the consumption structure is Pareto efficient.[8]

Thus, the exit mode generates efficiency gains in the provision of local public goods. But, it also incurs externalities, since it is connected with land and housing. Under general conditions, such externalities might lead to distortions in the use of land, housing, and local public goods. To avoid these distortions, a government can impose zoning measures. Zoning measures restrict the use of land and housing. Externalities can thus be internalized. But, zoning measures increase a government's power. How this power affects the total efficiency, depends on the government's objective.

Epple and Zelenitz (1981) modified the Tiebout setting to examine whether the exit mode by itself can be sufficient to generate a Pareto efficient outcome. In their model, the jurisdictions have fixed boundaries and there are no exit costs. Each government seeks to maximize its profits. It decides on its amount of local public goods and its property tax rate. The local public goods are in fact publicly provided private goods. Housing is supplied by

[7]Foundations for a theory of clubs were laid in Buchanan (1965).

[8]See also Hamilton (1976).

competitive firms. All citizens have identical incomes and preferences. Utility is a function of a composite private good, housing, and the local public goods. The local economy reaches an equilibrium, if:

- each citizen is housed in exactly one jurisdiction;
- each local housing market clears; and
- the utility level is equal in each jurisdiction.

Epple and Zelenitz analyzed the sensitivity of such an equilibrium. They found out the following: If a government raises its property tax rate, then local residents exit, the net price of housing falls. However, the impact of an increase in local public goods is not totally clear, due to the complementary relationship between local public goods and housing. Under special conditions, the housing price can rise and the number of residents fall. These results turn out to be independent of a government's objective. However, this does not hold for the distribution of income. Epple and Zelenitz found out that a Leviathan is able to misuse some tax revenue. The main reason is that the jurisdictionary boundaries are fixed. A Leviathan thus usurps the part of the land rent which stems from the elasticity of the housing supply. The higher the number of jurisdictions in the economy is, the lower will be the misuse. However, it never can be totally prevented by exit.

Thus, the exit mode takes up two important tasks: First, it serves as a mechanism for revealing preferences. If a citizen moves from one jurisdiction to another which is only different in its fiscal package, then she reveals that she prefers this package. Second, it serves as a constraint for self-interested governments. If a government offers a comparatively bad fiscal package, then the exit mode will induce negative consequences for this government. - Generally, the exit mode will the better execute these tasks, the less centralized the public sector is. However, its gains depend on further factors. Some major insights about these factors are:

- Henderson (1985): Interjurisdictional land markets may promote efficiency in the total economy.
- Hoyt (1991): Due to congestion of local public goods, it may be less efficient to tax land than to tax property.
- Henderson (1995): To fincance local public goods, homeowners would choose a head or a land tax rather than a property tax.
- Hoyt (1999): In smaller jurisdictions, citizens have stronger incentives to constrain Leviathan, due to higher capitalization rates.

- Caplan (2001): If the capitalization rate of a tax equals 1, then a land- or a homeowner cannot constrain Leviathan by exit. She either pays the tax or loses the respective amount in a sale.

3 A Two Stage Model of Local Taxation

The characteristics of a local economy are mainly based on the fact of location. The fact of location means that the economy is connected with land; which is neither changeable nor replicable. These two characteristics of land impose constraints on the whole economy. The constraints are particularly effective with respect to two types of goods: housing and local public goods. The utility that a housing unit offers depends on its specific location. Each housing market thus becomes a heterogeneous monopoly. Due to connection with land, there arises rivalness in the use of local public goods. Thus, some mechanism is needed to coordinate the demand for such goods.

Altogether, a model of a local economy shall help to describe and explain the relationships between land, housing, and fiscal policy. In order to reach this, strong simplifications are needed. We may argue that earlier models have not taken the right choice of simplifications for our specific purposes. Hence, we shall construct a new one.[9]

3.1 The Framework

Our local economy consists of $o = 1, 2, ..., \bar{J}$ regions. We take $\bar{J}$ as being large. Each region has the same land size; which is denoted by $\bar{L}_o$. The land size of the total economy is thus: $\bar{L} = \bar{J}\bar{L}_o$. There are three different types of agents: Leviathans, households, and housing firms. Each type is homogeneous. Each region is governed by a Leviathan. In the total economy, there are as many households as housing firms, namely $\bar{N}$, with $\bar{N} > \bar{J}$. The agents interact in two stages:

In stage 1, each household gets endowed with a parcel of land of the same size:

$$\bar{l}_o = \frac{\bar{L}_o}{\bar{N}_o};$$

where $\bar{N}_o$ is the number of land owners in region o. Thus:

$$\bar{N}_o = \frac{\bar{N}}{\bar{J}}.$$

[9]Similar model structures can be found in Epple/ Zelenitz (1981); Bucovetsky/ Wilson (1991); Hoyt (1991, 1999). For the equilbrium concept, see Epple/ Filimon/ Romer (1993). For the technique of the sensitivity analysis, see Silberberg (1990).

Each Leviathan announces his local fiscal package; which consists of: a land rent tax rate (τ_{lo}), a housing capital rent tax rate (τ_{ko}), a housing sales tax rate (τ_{sj}), a property tax rate (τ_{vjt}), and the amount of local public goods ($G_{o=j}$). Then, each household demands housing for its own parcel of land (h_o^d). The housing is supplied by the housing firms (h_o^s). They use the land ($\bar{l}_o$) and housing capital (k_{ho}) as inputs. The capital is demanded on a global market. The land rents (r_{lo}) are left to the respective owners.

Stage 2 is divided into two periods ($t = 1, 2$). In the first period, the Leviathans implement their fiscal policies. The households are free to migrate. They can choose the offering of local public goods that they prefer. But, they can only consume where they reside; which we call region j. In the second period, migration is not possible, anymore. In both periods, each household earns a wage (w_t). Its consumption consists of the following elements: a non-durable composite private good (x_{jt}); a durable housing good (h_j); and a durable local public good (G_j).

Thus, the three types of agents encounter the following optimization problems in our local economy:

A housing firm seeks to maximize its profit (π_{ho}). It offers its product on a competitive, intraregional market. The price on this market is p_{ho}; which depends on τ_{lo}, τ_{ko}, and G_o. In its production, the firm uses k_{ho} and $\bar{l}_{ho}$. The market for housing capital is exogenous. The respective rent (r_k) thus is taken as constant. Since the land input comes from the demander of the product, the land rent (r_{lo}) is determined as a residual value. Both inputs can be subject to taxation. Hence, the general profit function is:

$$\pi_{ho} = \rho_{ho} - c_{ho} \quad \text{with} \tag{1}$$

$$\rho_{ho} = p_{ho}(\tau_{lo}, \tau_{ko}, G_o) h_o(k_{ho}, \bar{l}_{ho});$$
$$c_{ho} = \bar{r}_k(1 + \tau_{ko}) k_{ho} + r_{lo}(1 + \tau_{lo}) \bar{l}_{ho}.$$

A household seeks to maximize its life-time utility (u_j). It consumes three types of goods (x_{jt}, h_j, G_j), in two periods (t= 1, 2), in one region (j). The utility that it gets from the local public good (u_g) does not depend on the consumed amounts of the other two goods. With the choice of x_{jt}, the household introduces a time preference factor (β) into its calculation. Hence, the general utility function is:

$$u_j = \sum_t \beta^{t-1} u_t(x_{jt}, h_j) + u_g(G_j). \tag{2}$$

The household receives incomes from two sources: land and labor. Its relevant land rent (r_{lo}) is determined in its region of origin (o), in stage 1. Its wage (w_t) is determined exogenously, in each period of stage 2. The household can vary the disposability of its life-time income (y_o) via an exogenous financial capital market, at an interest rate $\bar{r}_y$. It spends its income on x_{jt} and h_j. The prices that it pays are p_{xt} and p_{hj}. The latter price depends on the whole market for local public goods. Hence, the general income function is described by:

$$y_o = \sum_t y_{ot} = r_{lo}\bar{l}_o + \sum_t (1+\bar{r}_y)^{-(t-1)} w_t \quad \text{with} \tag{3}$$

$$y_{o1} = \gamma_1 y_o = p_{x1}x_{j1} + p_{hj}(\vec{\tau}_{sj}, [\tau_{vjt}], \vec{G}_j)(1+\tau_{sj})(1+\tau_{vj1})h_j;$$
$$y_{o2} = (1-\gamma_1)(1+\bar{r}_y)y_o = p_{x2}x_{j2} + p_{hj}(\vec{\tau}_{sj}, [\tau_{vjt}], \vec{G}_j)\tau_{vj2}h_j.$$

A Leviathan seeks to maximize his fiscal budget surplus ($S_{o=j}$). There are four different local tax bases (z). Each tax generates a specific amount of revenue ($R_{zo=zj}$). Depending on the tax institutions, the Leviathan spends a specific share of the revenue for local public goods (α_z). Hence, the fiscal budget surplus is formally described by:

$$S_{o=j} = \sum_z (1-\alpha_z)R_{zo=zj} \quad \text{with} \tag{4}$$

$$R_{lo} = \bar{N}_o \tau_{lo} r_{lo} \bar{l}_{ho};$$
$$R_{ko} = \bar{N}_o \tau_{ko} \bar{r}_k k_{ho};$$
$$R_{sj} = N_j \tau_{sj} p_{hj} h_j;$$
$$R_{vj} = N_j \sum_t \tau_{vjt} (1+\bar{r}_y)^{(t-1)} p_{hj} h_j.$$

In our local economy, interaction takes place in two stages. Each stage has its own equilibrium.

In stage 1, the regional boundaries are closed. Every agent is merely informed about her own region. A region o reaches an (internal) equilibrium under two conditions:

First, the households satisfy their individual housing demands:

$$h_o^d = h_o^s. \tag{5}$$

Second, the housing firms make no profits:

$$\pi_o = 0. \tag{6}$$

In stage 2, the regional boundaries are opened. Thus, every agent gets informed about the relevant variables and parameters in the total economy. The total economy reaches an equilibrium under three conditions:

First, in each region, the total demand for housing equals the total housing stock:

$$H_j^d = H_{o=j}^s. \tag{7}$$

Second, every household lives in one and only one region:

$$\sum_j N_j = \bar{J}\bar{N}_o = \bar{N}. \tag{8}$$

Third, every household reaches the same and highest possible utility level:

$$u_j = u^*. \tag{9}$$

For the shapes of u_j and c_{ho}, the standard assumptions are made. To simplify, we can assume without any greater loss that: $w_t = \bar{w}$ and $\tau_{vj1} = \tau_{vj2}$. Table 1 gives a summary of the model's elements.

3.2 Taxation of Land Rent

In stage 1, each household gets endowed with a parcel of land. The land is used as an input for housing; which generates a rent. Thus, the household receives a land rent without any previous investment. The extent of this gain positively depends on the provision of local public goods. The costs of these goods are taken by the local Leviathan. He incurs positive externalities to the local land owners. Hence, it may be expedient to internalize these externalities. One instrument for the internalization could be a land rent tax. Next, we shall look at the effects of such a tax, in our model.

A housing firm operates on a competitive market. It chooses its capital input such that its profit can reach zero. The land input is given. The land rent arises as a residual value. This value is shared between the land owner and the local Leviathan.

Formally, the optimization problem of a housing firm looks as follows:

$$\max_{k_{ho}} \pi_{ho} = p_{ho}(\tau_{lo}, G_o)h_o(k_{ho}, \bar{l}_{ho}) - \bar{r}_k k_{ho} - r_{lo}(1+\tau_{lo})\bar{l}_{ho}. \tag{10}$$

The solution to this problem is:

$$\bar{r}_k = p_{ho}\left(\frac{\partial h_o}{\partial k_{ho}}\right). \tag{11}$$

name	*interpretation*
c_{ho}	cost of a housing
G_j	local public good in region j
h_j	individual consumption of housing in region j
H_j^d	total housing demand in region j
H_o^s	total housing supply in o
$\bar{J}$	number of regions in the economy
k_{ho}	capital input for housing
$\bar{l}_o$	land size of parcel in o
$\bar{L}_o$	land size of region o
$\bar{N}_o$	number of households from region o (*origin*)
$\bar{N}_j$	number of households in region j (*consumption*)
p_{hj}	external net price of housing
p_{ho}	internal net price of housing
$\bar{p}_{xt}$	price of the composite private good
r_{lo}	land rent
$\bar{r}_k$	housing capital rent
$\bar{r}_y$	interest rate
R_z	public revenue from tax (base) z
$S_{o=j}$	fiscal budget surplus
u_j	individual utility in region j
w_t	wage in period t
x_{tj}	individual consumption of composite private good in period t and region j
y_o	individual life-time income from region o
y_{ot}	disposable income in period t
α_z	share of specific tax revenue spent on local public goods
β	time preference factor
γ_{jt}	share of disposable income in period t
π_{ho}	profit from a single housing production contract
ρ_{ho}	revenue from housing contract
τ_{ko}	tax on capital rent
τ_{lo}	tax on land rent
τ_{sj}	sales tax rate on housing in j
τ_{vjt}	property tax rate in period t and region j

Table 1: Elements of the two stage model: summary

Thus, the housing firm increases its capital input until the value of its marginal product equals the capital rent. From this optimum condition, we can derive the following sensitivity result:

$$\frac{dk_{ho}}{d\tau_{lo}} = -\frac{\frac{\partial p_{ho}}{\partial \tau_{lo}}\frac{\partial h_o}{\partial k_{ho}}}{p_{ho}\frac{\partial^2 h_o}{\partial k_{ho}^2}} < 0. \tag{12}$$

It is assumed that:

$$\frac{\partial p_{ho}}{\partial \tau_{lo}} < 0; \quad \frac{\partial h_o}{\partial k_{ho}} > 0; \quad \frac{\partial^2 h_o}{\partial k_{ho}^2} < 0.$$

Thus, we can state the following: A housing firm decreases (increases) its capital input as a response to a marginal increase (decrease) of the land rent tax.

A household maximizes its life-time utility, constrained by its life-time income. It gets utility from the consumption in region j, in stage 2. The land rent tax enters its calculus via its income from the region o, in stage 1. Formally, its optimization problem looks as follows:

$$\max_{x_{j1},x_{j2},h_j,\gamma_1} u_j = u_1(x_{j1}, h_j) + \beta u_2(x_{j2}, h_j) + u_g(G_j) \tag{13}$$

subject to

$$y_o = y_{o1} + y_{o2} = \bar{w}\left(1 + \frac{1}{1+\bar{r}_y}\right) + r_{lo}\bar{l}_{ho};$$

where

$$y_{1o} = \gamma_1 y_o = \bar{p}_x x_{j1} + p_{hj}(\vec{G}_j)h_j;$$
$$y_{2o} = (1+\bar{r}_y)(1-\gamma_1)y_o = \bar{p}_x x_{j2};$$
$$r_{lo}\bar{l}_{ho} = \frac{p_{ho}(\tau_{lo}, G_o)h_o(k_{ho}(p_{ho}(\tau_{lo}, G_o))) - \bar{r}_k k_{ho}(p_{ho}(\tau_{lo}, G_o))}{1+\tau_{lo}}.$$

In general, such an optimization problem can be solved in a Lagrangian approach. In our case, however, we would have to specify the utility function. Otherwise, it cannot be solved for x_1, x_2 and h_j. Anyway, we are rather interested in the household's sensitivity towards tax rate changes. To derive the respective results, we can do without any specification. As results, we get:

$$\frac{dx_{j1}}{d\tau_{lo}} = 0; \tag{14}$$

$$\frac{dx_{j2}}{d\tau_{lo}} = 0; \tag{15}$$

$$\frac{dh_j}{d\tau_{lo}} = \frac{\bar{r}_k k_{ho} - h_o\left(p_{ho} - (1+\tau_{lo})\frac{\partial p_{ho}}{\partial \tau_{lo}}\right)}{p_{hj}(1+\tau_{lo})^2} < 0; \tag{16}$$

$$\frac{d\gamma_1}{d\tau_{lo}} = (1-\gamma_1)\frac{\bar{r}_k k_{ho} - h_o\left(p_{ho} - (1+\tau_{lo})\frac{\partial p_{ho}}{\partial \tau_{lo}}\right)}{y_o(1+\tau_{lo})^2} < 0. \tag{17}$$

This holds, since:

$$p_{ho}h_o > \bar{r}_k k_{ho} + h_o(1+\tau_{lo})\frac{\partial p_{ho}}{\partial \tau_{lo}}.$$

Thus, we can state the following:

A change of the land rent tax does not lead to a change of a household's consumption of the composite private good. But, the household changes its housing demand and its share of first-period expenditure. These responses are always negative. The housing demand responds the more, the higher the capital rent is. The share of first-period expenditure responds the more, the lower the wages are.

A Leviathan chooses the land rent tax rates which maximize his fiscal budget surplus. As he takes into account, the tax base depends directly and indirectly on his own policy. His optimization problem looks exactly as follows:

$$\max_{\tau_{lo}} S_{lo} = \tau_{lo} r_{lo}(p_{ho}(\tau_{lo}, G_o), \tau_{lo})\bar{L}_{ho}(1-\bar{\alpha}_{lo}). \tag{18}$$

The solution is:

$$\tau_{lo} = -\frac{r_{lo}}{\frac{\partial r_{lo}}{\partial p_{ho}}\frac{\partial p_{ho}}{\partial \tau_{lo}} + \frac{\partial r_{lo}}{\partial \tau_{lo}}} \quad \Rightarrow \quad \epsilon_{lo} = -1. \tag{19}$$

Thus, a Leviathan fixes the tax rate to the point where the elasticitiy of the land rent is (minus) unity.

The total equilibrium with land rent taxation can be described by two conditions. These combine and specify the five conditions of the general framework.

First:

$$\bar{N} = \sum_j \frac{\sum_o h_j^d(y_o(r_{lo}(p_{ho}(\tau_{lo}, G_o), \tau_{lo})), p_{hj}(\vec{G}_j))}{h_{o=j}^s(k_h(p_{ho}(\tau_{lo}, G_o)))}. \tag{20}$$

This means: The total number of households in the local economy must equal the sum of all housing consumers. The individual housing demand in region j depends on the household's land rent from region o and the housing price in region j. The number of households in j (N_j) must equal the total housing demand divided by the individual housing supply. The housing supply is determined by the optimal capital input, determined in stage 1.

Second:

$$u^* = v_j(y_o(r_{lo}(p_{ho}(\tau_{lo}, G_o), \tau_{lo})), p_{hj}(\vec{G}_j), G_j). \tag{21}$$

Every household in the economy must reach the same, maximum level of utility. The indirect utility depends on the household's land rent from region o, the housing price in j, and the supply of local public goods in j.

From these two conditions, we get the following sensitivity results:

$$\frac{dp_{ho}}{d\tau_{lo}} = -\frac{\partial p_{ho}}{\partial \tau_{lo}} > 0; \tag{22}$$

$$\frac{dp_{hj}}{d\tau_{lo}} = \frac{p_{ho}h_o - \bar{r}_k k_{ho}}{(1+\tau_{lo})^2} \frac{\frac{\partial v_j}{\partial y_o}}{\frac{\partial v_j}{\partial p_{hj}}} < 0. \tag{23}$$

Thus: Since we assume that the partial response of the internal housing price on a land rent tax change is negative, the total response must be positive. The respective total response of the external housing price is, by contrast, negative. The extent of this response negatively depends on the wages and on the capital rent.

3.3 Taxation of Housing Capital Rent

In stage 1, housing is produced, namely by two factors: land and housing capital. The latter factor is traded on a global market. For our local economy, its supply is perfectly elastic. A local housing firm can thus demand any amount at a fixed price. Because the local supply of land is fixed, the amount of housing capital input completely determines the housing supply. Because a citizen can only consume local public goods where she resides, the housing capital input influences the demand for these goods. Due to congestion, it may be necessary to regulate this demand. Hence, it could be expedient to impose a tax on the capital rent. Such a tax forms a link between housing supply, local public goods consumption, and fiscal revenue. Next, we shall examine the effects of the capital rent tax, in more details.

A housing firm chooses the amount of capital input which maximizes its profit. For this input, it pays an exogenously determined price plus the local tax. Since capital is its only variable input, the firm cannot avoid the tax payment by factor substitution. In formal terms, its optimization problem and the solution are described as follows:

$$\max_{k_{ho}} \pi_{ho} = p_{ho}(\tau_{ko}, G_o)h_o(k_{ho}, \bar{l}_{ho}) - \bar{r}_k(1+\tau_{ko})k_{ho} - r_{lo}\bar{l}_{ho}; \tag{24}$$

$$\bar{r}_k(1+\tau_{ko}) = p_{ho}\left(\frac{\partial h_o}{\partial k_{ho}}\right). \tag{25}$$

Thus, the housing firm increases its capital input until its value of the marginal product equals the gross capital rent. This optimum leads to the following sensitivity on tax rate changes:

$$\frac{dk_{ho}}{d\tau_{ko}} = \frac{\bar{r}_k - \frac{\partial p_{ho}}{\partial \tau_{ko}}\frac{\partial h_o}{\partial k_{ho}}}{p_{ho}\frac{\partial^2 h_o}{\partial k_{ho}^2}} < 0. \tag{26}$$

This means: A housing firm negatively responds to a change of the capital rent tax. The response will be the higher, the higher the price level on the global capital market is.

A capital rent tax affects a household's behavior via its income constraint. Here, its income from land ownership is:

$$r_{lo}\bar{l}_{ho} = p_{ho}(\tau_{ko}, G_o)h_o(k_{ho}(p_{ho}(\tau_{ko}, G_o))) - \bar{r}_k(1+\tau_{ko})k_{ho}(p_{ho}(\tau_{ko}, G_o)).$$

The sensitivity results thus become:

$$\frac{dx_{j1}}{d\tau_{ko}} = 0; \tag{27}$$

$$\frac{dx_{j2}}{d\tau_{ko}} = 0; \tag{28}$$

$$\frac{dh_j}{d\tau_{ko}} = -\frac{\bar{r}_k k_{ho} - h_o\frac{\partial p_{ho}}{\partial \tau_{ko}}}{p_{hj}} < 0; \tag{29}$$

$$\frac{d\gamma_1}{d\tau_{ko}} = -(1-\gamma_1)\frac{\bar{r}_k k_{ho} - h_o\frac{\partial p_{ho}}{\partial \tau_{ko}}}{y_o} < 0. \tag{30}$$

We find out the following: A change of the capital rent tax leaves the composite private goods consumption unaffected. By contrast, such a change affects the consumption of housing and the share of first-period expenditure negatively. The impact on the housing consumption positively depends on the capital rent level. The impact on the share of first-period expenditure positively depends on this level, and negatively on the wage level.

If a Leviathan seeks to maximize his budget surplus from a capital rent tax, then he has to consider the responses of the capital input. His optimization problem is:

$$\max_{\tau_{ko}} S_{ko} = \tau_{ko}\bar{r}_{ko}k_{ho}(p_{ho}(\tau_{ko}, G_o), \tau_{ko})N_o(1-\bar{\alpha}_{ko}). \tag{31}$$

In his optimum:

$$\tau_{ko} = -\frac{k_{ho}}{\frac{\partial k_{ho}}{\partial p_{ho}}\frac{\partial p_{ho}}{\partial \tau_{ko}} + \frac{\partial k_{ho}}{\partial \tau_{ko}}} \quad \Rightarrow \quad \epsilon_{ko} = -1. \tag{32}$$

Hence, the fiscal budget surplus reaches its maximum at a point where the elasticity of the capital with respect to the capital rent tax rate is (minus) unity. A response to a tax rate change may come from two directions: first, the housing firms adapt the capital input; second, the households adapt their housing demand.

Our local economy with capital rent taxes reaches a total equilibrium, if the following two conditions are satisfied:

$$\bar{N} = \sum_j \frac{\sum_o h_j^d(y_o(r_{lo}(p_{ho}(\tau_{ko}, G_o), \tau_{ko})), p_{hj}(\vec{G}_j))}{h_{o=j}(k_h(p_{ho}(\tau_{ko}, G_o)))}; \tag{33}$$

$$u^* = v_j(y_o(r_{lo}(p_{ho}(\tau_{ko}, G_o), \tau_{ko})), p_{hj}(\vec{G}_j), G_j). \tag{34}$$

Formally, the sensitivity of the intraregional and of the interregional housing markets is described by:

$$\frac{dp_{ho}}{d\tau_{ko}} = -\frac{\partial p_{ho}}{\partial \tau_{ko}} > 0; \tag{35}$$

$$\frac{dp_{hj}}{d\tau_{ko}} = \bar{r}_k k_{ho} \frac{\frac{\partial v_j}{\partial y_o}}{\frac{\partial v_j}{\partial p_{hj}}} < 0. \tag{36}$$

The internal housing price positively responds to a change of the capital rent tax. The total response just reverses the sign of the partial response. The external housing price, by contrast, negatively responds to such a change. This response will be the higher, the higher the capital rent and the lower the wage level is.

3.4 Taxation of Housing Sales

In stage 2, the regional boundaries are opened. The households are free to choose the fiscal package which they find best. But, such a choice induces a corresponding demand for housing. Thus, each interregional housing price depends on the whole set of local fiscal packages. We can say that the local fiscal packages 'capitalize' into the housing prices. To improve the interplay between the housing markets and the market for local public goods, it seems expedient to create a link between capitalization and fiscal revenue. One possible instrument for this is a sales tax. A sales tax directs a share of the housing price to the fiscal budget. It serves as an entry or exit fee. Next, we shall examine how this particularly affects decisions and equilibria in our local economy.

A housing firm makes its decisions in stage 1, based on the intraregional housing price. Since a sales tax is imposed only in stage 2, based on the interregional housing price, the housing firm remains unaffected.

When a household migrates to a region j, it demands a certain amount of housing and pays the gross interregional housing price. After that, it can start its consumption. The household optimizes its consumption in the following way:

$$\max_{x_{j1},x_{j2},h_j,\gamma_1} u_j = u_1(x_{j1}, h_j) + \beta u_2(x_{j2}, h_j) + u_g(G_j) \tag{37}$$

subject to

$$y_o = y_{o1} + y_{o2} = \bar{w}\left(1 + \frac{1}{1+\bar{r}_y}\right) + p_{ho}h_o - \bar{r}_k k_{ho} \tag{38}$$

where

$$y_{o1} = \gamma_1 y_o = \bar{p}_x x_{j1} + p_{hj}(\vec{\tau}_{sj}, \vec{G}_j)(1+\tau_{sj})h_j; \tag{39}$$

$$y_{o2} = (1-\gamma_1)(1+\bar{r}_y)y_o = \bar{p}_x x_{j2}. \tag{40}$$

It responds to a change of the sales tax rate as follows: If

$$p_{hj} > -(1+\tau_{sj})\frac{\partial p_{hj}}{\partial \tau_{sj}}; \tag{41}$$

then

$$\frac{dx_{j1}}{d\tau_{sj}} = \frac{p_{hj} + (1+\tau_{sj})\frac{\partial p_{hj}}{\partial \tau_{sj}}}{\bar{p}_x \frac{\partial u_{j1}}{\partial h_j}} > 0; \tag{42}$$

$$\frac{dh_j}{d\tau_{sj}} = -\frac{2 + h_j\frac{\partial u_{j1}}{\partial h_j}}{1+\tau_{sj}} \frac{p_{hj} + (1+\tau_{sj})\frac{\partial p_{hj}}{\partial \tau_{sj}}}{p_{hj}\frac{\partial u_{j1}}{\partial h_j}} < 0; \tag{43}$$

$$\frac{d\gamma_1}{d\tau_{sj}} = -\frac{p_{hj} + (1+\tau_{sj})\frac{\partial p_{hj}}{\partial \tau_{sj}}}{y_o \frac{\partial u_{j1}}{\partial h_j}} < 0. \tag{44}$$

Moreover,

$$\frac{dx_{j2}}{d\tau_{sj}} = (1+\bar{r}_y)\frac{dx_{1j}}{d\tau_{sj}}. \tag{45}$$

Hence, under the condition that the net external housing price is greater than the partial derivative of the gross price with respect to the tax rate, the household's responses to an increase of the sales tax rate (et vice versa) are: First, it increases the first-period consumption of the composite private good. The extent of this response negatively depends on the price for this good. Second, it increases the second-period consumption of the composite private good. The extent of this response negatively depends on the respective price, and positively on the interest rate. Third, it decreases the housing consumption. And finally, the household decreases the share of the first-period expenditure. The extent of this response negatively depends on the wage level.

In order to maximize his fiscal budget surplus from a sales tax, a Leviathan must take into account each fiscal package in the whole economy. Only then, he is able to predict the relevant housing price. His optimization calculus with a sales tax formally looks as follows:

$$\max_{\tau_{sj}} S_{sj} = \tau_{sj} p_{hj}(\vec{\tau}_{sj}, \vec{G}_j) H_j (1 - \bar{\alpha}_{sj}); \tag{46}$$

$$\tau_{sj} = -\frac{p_{hj}}{\frac{\partial p_{hj}}{\partial \tau_{sj}}} \quad \Rightarrow \quad \epsilon_{sj} = -1. \tag{47}$$

Again, a Leviathan searches for the point where the relevant tax elasticity equals -1. Here, the relevant tax elasticity refers to the interregional housing price. Hence, it also arises from the coordination of all housing markets in our local economy.

Our local economy with housing sales taxes reaches a total equilibrium under two specified conditions. The sales taxes are imposed on the interregional housing prices in stage 2. The intraregional prices are determined in stage 1. It thus seems plausible that a change of a sales tax has no effect on the latter prices. From our formal analysis, we get:

$$\bar{N} = \sum_j \frac{\sum_o h_j^d(y_o(r_{lo}(p_{ho}(G_o))), p_{hj}(\vec{\tau}_{sj}, \vec{G}_j)(1 + \tau_{sj}))}{h_o^s(k_h(p_{ho}(G_o)))} \tag{48}$$

and

$$u^* = v_j(y_o(r_{lo}(p_{ho}(G_o))), p_{hj}(\vec{\tau}_{sj}, \vec{G}_j)(1 + \tau_{sj}), G_j). \tag{49}$$

$$\frac{dp_{ho}}{d\tau_{sj}} = 0; \tag{50}$$

$$\frac{dp_{hj}}{d\tau_{sj}} = -\frac{p_{hj}}{1 + \tau_{sj}} - \frac{\partial p_{hj}}{\partial \tau_{sj}} < 0 \quad \text{if} \tag{51}$$

$$p_{hj} > -(1+\tau_{sj})\frac{\partial p_{hj}}{\partial \tau_{sj}}.$$

In fact, an intraregional housing price does not respond, if a Leviathan announces a change of his sales tax rate. There is no anticipation. The response of an interregional housing price depends on a condition. If the net price is greater than the relevant partial derivative of the gross price, then the net price responds negatively.

3.5 Taxation of Housing Property

In stage 2, the households distribute their life-time incomes over two periods. In order to better conform to the distribution, it could be expedient to impose a tax in each period. One type of a multi-period tax is a (housing) property tax. Such a tax relates to the housing value in each period. It works similar to a member fee. It creates a link between the consumption of and the financing of local public goods. However, the households are not totally free in their consumption. The housing that they consume in the second period must be the same as in the first one. Therefore, it seems necessary that the government commits to the second period taxation, already in the first one. For simplicity, we assume that the tax rates in both periods are identical. Next, we shall see which sensitivity is engendered by such a property tax.

Based on the intraregional housing price, a housing firm makes its decisions in stage 1. Thus, it does not respond to a change of the property tax rate.

A household now integrates a second period tax payment into its consumption plans. This tax payment also determines its migration decision in the first period. The consumption plans get the follwing form:

$$\max_{x_{j1},x_{j2},h_j,\gamma_1} u_j = u_1(x_{j1},h_j) + \beta u_2(x_{j2},h_j) + u_g(G_j) \tag{52}$$

subject to

$$y_o = y_{o1} + y_{o2} = \bar{w}\left(1+\frac{1}{1+\bar{r}_y}\right) + p_{ho}h_o - \bar{r}_k k_{ho};$$

$$y_{1o} = \gamma_1 y_o = \bar{p}_x x_{j1} + p_{hj}(\vec{\tau}_{vj},\vec{G}_j)(1+\tau_{vj})h_j;$$

$$y_{2o} = (1-\gamma_1)(1+\bar{r}_y)y_o = \bar{p}_x x_{j2} + p_{hj}(\vec{\tau}_{vj},\vec{G}_j)\tau_{vj}h_j.$$

The sensitivity results are: If

$$p_{hj} > -\frac{1+\bar{r}_y}{2+\bar{r}_y}\left(1+\tau_{vj}+\frac{\tau_{vj}}{1+\bar{r}_y}\right)\frac{\partial p_{hj}}{\partial \tau_{vj}};$$

then

$$\frac{dx_{j1}}{d\tau_{vj}} = \frac{\left(\left(\frac{2+\bar{r}_y}{1+\bar{r}_y}p_{hj}\right) + \left(1+\tau_{vj}+\frac{\tau_{vj}}{1+\bar{r}_y}\right)\frac{\partial p_{hj}}{\partial \tau_{vj}}\right)\Omega_{v1}}{\bar{p}_x \frac{\partial u_1}{\partial h_j}} > 0; \tag{53}$$

and

$$\frac{dh_j}{d\tau_{vj}} = -\frac{\left(\left(\frac{2+\bar{r}_y}{1+\bar{r}_y}p_{hj}\right) + \left(1+\tau_{vj}+\frac{\tau_{vj}}{1+\bar{r}_y}\right)\frac{\partial p_{hj}}{\partial \tau_{vj}}\right)\Omega_{v2}}{p_{hj} \frac{\partial u_1}{\partial h_j}} < 0. \tag{54}$$

Moreover,

$$\frac{dx_{j2}}{d\tau_{vj}} = (1+\bar{r}_y)\frac{dx_{j1}}{d\tau_{vj}}; \tag{55}$$

and

$$\frac{d\gamma_1}{d\tau_{vj}} = -\frac{\Omega_{v3}p_{hj} + \Omega_{v4}\frac{\partial p_{hj}}{\partial \tau_{vj}} + \Omega_{v5}}{y_o \frac{\partial u_{j1}}{\partial h_j}} \leq 0 \quad \text{or} \quad > 0. \tag{56}$$

This is because, all $\Omega_v > 0$. As we can see, a property tax induces more complicated responses by a household. In relation to the distribuion of its life time income, we even cannot predict the direction of its response, anymore. Under the condition that the net housing price is greater than the time-weighted partial derivative of the gross price with respect to the tax rate, a change of the tax rate leads to: first, a positive response of the first-period consumption of the composite private good; which is negatively related to its price level; second, a positive response of the second-period consumption of this good; which is negatively related to its price level and positively to the interest rate. and third, a negative response of the housing consumption.

In our local economy, a Leviathan provides his local public goods as one 'shot', at the beginning of the first period. For this shot, he spends a constant share of his total fiscal revenue. Thus, the Leviathan needs to pre-finance the revenue from the second period property tax. He can do this on the global financial market. He maximizes his property tax revenue in the following way:

$$\max_{\tau_{vj}} S_{vj} = \left(\tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_y}\right) p_{hj}(\vec{\tau}_{vj}, \vec{G}_j)H_j(1-\bar{\alpha}_{vj}); \tag{57}$$

$$\tau_{vj} = -\frac{p_{hj}}{\frac{\partial p_{ho}}{\partial \tau_{vj}}} \quad \Rightarrow \quad \epsilon_{vj} = -1. \tag{58}$$

Again, the Leviathan takes the point where the relevant tax elasticity equals -1. Despite of the multi-period character of the property tax, the optimum point does not directly depend on the interest rate.

Our local economy with property taxation reaches an equilibrium, if:

$$\bar{N} = \sum_j \frac{\sum_o h_j \left(y_o(r_{lo}(p_{ho}(G_o))), p_{hj}(\vec{\tau}_{vj}, \vec{G}_j) \left(1 + \tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_y}\right)\right)}{h_o(k_h^s(p_{ho}(\vec{\tau}_{vj}, \vec{G}_o)))}; \tag{59}$$

$$u^* = v_j \left(y_o(r_{lo}(p_{ho}(G_o))), p_{hj}(\vec{\tau}_{vj}, \vec{G}_j) \left(1 + \tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_y}\right), G_j \right); \tag{60}$$

The relevant sensitivity result is:

$$\frac{dp_{hj}}{d\tau_{vj}} = -\frac{2+\bar{r}_y}{1+\bar{r}_y} \frac{p_{hj}}{1+\tau_{vj}+\frac{\tau_{vj}}{1+\bar{r}_y}} - \frac{\partial p_{hj}}{\partial \tau_{vj}} < 0 \quad \text{if} \tag{61}$$

$$p_{hj} > -\frac{1+\bar{r}_y}{2+\bar{r}_y}\left(1+\tau_{vj}+\frac{\tau_{vj}}{1+\bar{r}_y}\right)\frac{\partial p_{hj}}{\partial \tau_{vj}}.$$

Thus, we encounter the same condition as for the households' sensitivity. If the net housing price is greater than the time-weighted partial derivative of the gross housing price with respect to the tax rate, then this price responds negatively to a property tax rate change.

4 Evaluation Results

What we search for is an appropriate tax base for local Leviathans. We evaluate the appropriateness according to four criteria: efficiency, simplicity, flexibilty, and fairness. Based on a discussion of the citizens' modes of reaction, the Tiebout mechanism, the fact of location, and externalities, we led our attention to four distinct local tax bases: land rent, housing capital rent, housing sales, and housing properties. To better understand the characteristics of these tax bases, we constructed a two stage model of local taxation. This model describes three types of agents in a local economy. The interaction is structured into two stages and two periods. Some simplifying assumptions are further made on: wages, private consumption, tax tariffs, and local public expenditure. Thus, the model allows to analyze the sensitivity of households, housing firms, and housing markets on tax rate changes. Based on this model, we finally seek to evalutate the chosen local tax bases.

First, the land rent: A land rent tax incurs inefficiencies in the input of housing capital, the housing demand, and the distribution of the private income over time. The consumption of other private goods remains unaffected. Since the land rent negatively depends on the internal tax rate and positively on the internal public goods, the share of revenue spent on local public goods may tend to be high. As a residual value, a land rent cannot be directly observed. In reality, it could be possible that a housing firm manipulates its value. A land rent tax is rather flexible. It influences the intra- and the interregional housing price. The extent of the influence negatively depends on the housing capital rent and the wage. A land rent tax can be judged as being fair. It puts a burden on the households according to their initial endowments and their incomes. Thus, it could be accepted by all citizens behind a veil of ignorance.

Second, the housing capital rent: A housing capital rent tax incurs inefficiencies in the housing production and demand, the consumption structure over time; but not in the composite private goods consumption. While the capital rent is taken as exogenous, a Leviathan sets every endogenous variable for the determination of the capital input. Thus, the tax waste may tend to be low. The term 'housing capital' comprises any sort of input, except land. In reality, the inputs may be highly heterogeneous and thus difficult to capture. A capital rent tax takes influence on both types of housing prices; which depends on the capital rent and the wage, in a straight forward manner. Thus, this tax can be judged as particularly flexible. Moreover, it can be judged as being rather fair. Its burden is taken by the households according to their housing ownership.

Third, the housing sales: A housing sales tax distorts each consumption decision. The direction in which the decisions change is dependent on the shape of the housing price function. However, the housing production is not affected. This is because consumption and production base on different housing prices. Since a Leviathan competes in the period of taxation with all the others, there will be no room for tax waste. The housing sales tax appears as a simple wedge between the gross and the net interregional housing price. The extent to which a change of the tax rate influences the housing price (in a positive or negative direction), depends on the parameters of the housing price function, alone. The sales tax can be judged as fair in the sense that each household contributes to the local public goods which it actually consumes.

And finally, the housing property: A housing property tax distorts the consumption of private composite goods and of housing. The sign of these distortions depend on the housing price function and the interest rate. Thus, the distortion in the distribution of the income over time tends to remain small but difficult to predict. Due to the competition among the Leviathans, it will be impossible to waste tax revenue. However, in reality, there may arise a discrepancy between the housing price and its value in later periods. If the housing is not sold, its value can only be estimated. The extent to which the housing price responds to a tax rate change depends on the parameters of the housing price function and the interest rate, but not explicitly on the wages or other prices. Behind a veil of ignorance, a housing property may appear as a rather uncertain tax base. Whether it will turn out to be fair depends on the efficiency of the financial capital market, time structure of the local public goods, and the commitment of the Leivathans.

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Appendix

Computation for 3.2

$$F_{l1} = p_{ho}(\tau_{lo}, G_o)\frac{\partial h_o}{\partial k_{ho}} - \bar{r}_k = 0$$

$$\frac{dk_{ho}}{d\tau_{lo}} = -\frac{\frac{\partial F_{l1}}{\partial \tau_{lo}}}{\frac{\partial F_{l1}}{\partial k_{ho}}}$$

$$A_{l1} d_{l1} = z_{l1} \quad \Rightarrow \quad A_{l1}^{-1} z_{l1} = d_{l1}$$

$$A_{l1} = \begin{bmatrix} \frac{\partial^2 u_1}{\partial x_{1j}^2} & 0 & \frac{\partial^2 u_1}{\partial x_{1j} \partial h_j} & 0 \\ -\bar{p}_x & 0 & & \\ 0 & \beta \frac{\partial^2 u_2}{\partial x_{2j}^2} & \beta \frac{\partial^2 u_2}{\partial x_{2j} \partial h_j} & 0 \\ 0 & -\bar{p}_x & & \\ \frac{\partial^2 u_1}{\partial h_j \partial x_{1j}} & \beta \frac{\partial^2 u_2}{\partial h_j \partial x_{2j}} & \frac{\partial^2 u_1}{\partial h_j^2} + \beta \frac{\partial^2 u_2}{\partial h_j^2} & 0 \\ -p_{hj} & 0 & & \\ 0 & 0 & 0 & 0 \\ 1 & -(1+\bar{r}_y) & & \\ -\bar{p}_x & 0 & -p_{hj} & y_o \\ 0 & 0 & & \\ 0 & -\bar{p}_x & 0 & -(1+\bar{r}_y) y_o \\ 0 & 0 & & \end{bmatrix}$$

$$d_{l1} = \begin{bmatrix} dx_{1j}/d\tau_{lo} \\ dx_{2j}/d\tau_{lo} \\ dh_j/d\tau_{lo} \\ d\gamma_1/d\tau_{lo} \\ d\lambda_1/d\tau_{lo} \\ d\lambda_2/d\tau_{lo} \end{bmatrix}$$

$$z_{l1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\gamma_1 \left[\frac{\partial p_{ho}}{\partial \tau_{lo}} \frac{h_o}{1+\tau_{lo}} - \frac{p_{ho} h_o - \bar{r}_k k_{ho}}{(1+\tau_{lo})^2} \right) \\ -(1-\gamma_1)(1+\bar{r}_y) \left(\frac{\partial p_{ho}}{\partial \tau_{lo}} \frac{h_o}{1+\tau_{lo}} - \frac{p_{ho} h_o - \bar{r}_k k_{ho}}{(1+\tau_{lo})^2} \right) \end{bmatrix}$$

$$\frac{\partial^2 u_1}{\partial x_{1j}^2} = \frac{\bar{p}_x}{p_{hj}} \frac{\partial^2 u_1}{\partial h_j \partial x_{1j}}$$

$$\frac{\partial^2 u_2}{\partial x_{2j}^2} = \frac{\bar{p}_x}{p_{hj}(1+\bar{r}_y)} \frac{\partial^2 u_2}{\partial h_j \partial x_{2j}}$$

$$\frac{\partial^2 u_1}{\partial x_{1j} \partial h_j} = (1+\bar{r}_y) \beta \frac{\partial^2 u_2}{\partial h_j \partial x_{2j}}$$

$$\frac{\partial^2 u_1}{\partial h_j^2} = \frac{p_{hj}}{\bar{p}_x} \frac{\partial^2 u_1}{\partial x_{1j} \partial h_j} - \beta \frac{\partial^2 u_2}{\partial h_j^2}$$

$$A_{l2} d_{l2} = z_{l2} \quad \Rightarrow \quad A_{l2}^{-1} z_{l2} = d_{l2}$$

$$A_{l2}=\begin{bmatrix} \frac{\partial v_j}{\partial y_o}\frac{h_o}{1+\tau_{lo}} & \frac{\partial v_j}{\partial p_{hj}} \\ \frac{\sum_o \frac{\partial h_j}{\partial y_o}}{1+\tau_{lo}} - \frac{\sum_o h_j \frac{\partial h_o}{\partial k_{ho}}\frac{\partial k_{ho}}{\partial p_{ho}}}{h_o^2} & \frac{\sum_o \frac{\partial h_j}{\partial p_{hj}}}{h_o} \end{bmatrix}$$

$$d_{l2}=\begin{bmatrix} dp_{ho}/d\tau_{lo} \\ dp_{hj}/d\tau_{lo} \end{bmatrix}$$

$$z_{l2}=\begin{bmatrix} -\frac{\partial v_j}{\partial y_o}\left(\frac{h_o\frac{\partial p_{ho}}{\partial \tau_{lo}}}{1+\tau_{lo}} - \frac{p_{ho}h_o-\bar{r}_k k_{ho}}{(1+\tau_{lo})^2}\right) \\ -\frac{1}{h_o}\sum_o \frac{\partial h_j}{\partial y_o}\left(\frac{h_o\frac{\partial p_{ho}}{\partial \tau_{lo}}}{1+\tau_{lo}} - \frac{p_{ho}h_o-\bar{r}_k k_{ho}}{(1+\tau_{lo})^2}\right) + \frac{\sum_o h_j \frac{\partial h_o}{\partial k_{ho}}\frac{\partial k_{ho}}{\partial p_{ho}}\frac{\partial p_{ho}}{\partial \tau_{lo}}}{h_{lo}^2} \end{bmatrix}$$

Computations for 3.3

$$F_{k1}=p_{ho}(\tau_{lo},G_o)\frac{\partial h_o}{\partial k_{ho}}-\bar{r}_k(1+\tau_{ko})=0$$

$$\frac{dk_{ho}}{d\tau_{ko}}=-\frac{\frac{\partial F_{k1}}{\partial \tau_{ko}}}{\frac{\partial F_{k1}}{\partial k_{ho}}}$$

$$A_{k1}d_{k1}=z_{k1} \quad\Rightarrow\quad A_{k1}^{-1}z_{k1}=d_{k1}$$

$$A_{k1}=A_{l1}$$

$$d_{k1}=\begin{bmatrix} dx_{1j}/d\tau_{ko} \\ dx_{2j}/d\tau_{ko} \\ dh_j/d\tau_{ko} \\ d\gamma_1/d\tau_{ko} \\ d\lambda_1/d\tau_{ko} \\ d\lambda_2/d\tau_{ko} \end{bmatrix}$$

$$z_{k1}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \gamma_1\bar{r}_y k_{ho} \\ (1-\gamma_1)(1+\bar{r}_y)\bar{r}_k k_{ho} \end{bmatrix}$$

$$A_{k2}d_{k2}=z_{k2} \quad\Rightarrow\quad A_{k2}^{-1}z_{k2}=d_{k2}$$

$$A_{k2}=\begin{bmatrix} \frac{\partial v_j}{\partial y_o}h_o & \frac{\partial v_j}{\partial p_{hj}} \\ \sum_o \frac{\partial h_j}{\partial y_o} - \frac{\sum_o h_j \frac{\partial h_o}{\partial k_{ho}}\frac{\partial k_{ho}}{\partial p_{ho}}}{h_o^2} & \frac{\sum_o \frac{\partial h_j}{\partial p_{hj}}}{h_o} \end{bmatrix}$$

$$d_{l2} = \begin{bmatrix} dp_{ho}/d\tau_{ko} \\ dp_{hj}/d\tau_{ko} \end{bmatrix}$$

$$z_{l2} = \begin{bmatrix} -\frac{\partial v_j}{\partial y_o}\left(h_o \frac{\partial p_{ho}}{\partial \tau_{ko}} - \bar{r}_k k_{ho}\right) \\ \frac{1}{h_o^2}\sum_o h_j \frac{\partial h_o}{\partial k_{ho}}\left(\frac{\partial k_{ho}}{\partial \tau_{ko}} + \frac{\partial k_{ho}}{\partial p_{ho}}\frac{\partial p_{ho}}{\partial \tau_{ko}}\right) - \frac{1}{h_o}\sum_o \frac{\partial h_j}{\partial y_o}\left(h_o \frac{\partial p_{ho}}{\partial \tau_{ko}} - \bar{r}_k k_{ho}\right) \end{bmatrix}$$

Computations for 3.4

$$A_{s1} d_{s1} = z_{s1} \quad \Rightarrow \quad A_{s1}^{-1} z_{s1} = d_{s1}$$

$$A_{s1} = \left[\begin{array}{cccc} \frac{\partial^2 u_1}{\partial x_{1j}^2} & 0 & \frac{\partial^2 u_1}{\partial x_{1j}\partial h_j} & 0 \\ -\bar{p}_x & 0 & & \\ 0 & \beta\frac{\partial^2 u_2}{\partial x_{2j}^2} & \beta\frac{\partial^2 u_2}{\partial x_{2j}\partial h_j} & 0 \\ 0 & -\bar{p}_x & & \\ \frac{\partial^2 u_1}{\partial h_j \partial x_{1j}} & \beta\frac{\partial^2 u_2}{\partial h_j \partial x_{2j}} & \frac{\partial^2 u_1}{\partial h_j^2} + \beta\frac{\partial^2 u_2}{\partial h_j^2} & 0 \\ -p_{hj}(1+\tau_{sj}) & 0 & & \\ 0 & 0 & 0 & 0 \\ 1 & -(1+\bar{r}_y) & & \\ -\bar{p}_x & 0 & -p_{hj}(1+\tau_{sj}) & y_o \\ 0 & 0 & & \\ 0 & -\bar{p}_x & 0 & -(1+\bar{r}_y)y_o \\ 0 & 0 & & \end{array}\right]$$

$$d_{s1} = \begin{bmatrix} dx_{1j}/d\tau_{sj} \\ dx_{2j}/d\tau_{sj} \\ dh_j/d\tau_{sj} \\ d\gamma_1/d\tau_{sj} \\ d\lambda_1/d\tau_{sj} \\ d\lambda_2/d\tau_{sj} \end{bmatrix}$$

$$z_{s1} = \begin{bmatrix} 0 \\ 0 \\ \lambda_1\left(\frac{\partial p_{hj}}{\partial \tau_{sj}}(1+\tau_{sj}) + p_{hj}\right) \\ 0 \\ h_j\left(\frac{\partial p_{hj}}{\partial \tau_{sj}}(1+\tau_{sj}) + p_{hj}\right) \\ 0 \end{bmatrix}$$

$$\frac{\partial^2 u_1}{\partial x_{1j}^2} = \frac{\bar{p}_x}{p_{hj}(1+\tau_{sj})}\frac{\partial^2 u_1}{\partial h_j \partial x_{1j}}$$

$$\frac{\partial^2 u_2}{\partial x_{2j}^2} = \frac{\bar{p}_x}{p_{hj}(1+\tau_{sj})(1+\bar{r}_y)}\frac{\partial^2 u_2}{\partial h_j \partial x_{2j}}$$

$$\frac{\partial^2 u_1}{\partial h_j^2} = \frac{p_{hj}(1+\tau_{sj})}{\bar{p}_x}\frac{\partial^2 u_1}{\partial x_{1j}\partial h_j} - \beta\frac{\partial^2 u_2}{\partial h_j^2}$$

$$A_{s2}d_{s2} = z_{s2} \quad \Rightarrow \quad A_{s2}^{-1}z_{s2} = d_{s2}$$

$$A_{s2} = \begin{bmatrix} \frac{\partial v_j}{\partial y_o}h_o & (1+\tau_{sj})\frac{\partial v_j}{\partial p_{hj}} \\ \sum_o \frac{\partial h_j}{\partial y_o} - \frac{\sum_o h_j \frac{\partial h_o}{\partial k_{ho}}\frac{\partial k_{ho}}{\partial p_{ho}}}{h_o^2} & (1+\tau_{sj})\frac{\sum_o \frac{\partial h_j}{\partial p_{hj}}}{h_o} \end{bmatrix}$$

$$d_{s2} = \begin{bmatrix} dp_{ho}/d\tau_{sj} \\ dp_{hj}/d\tau_{sj} \end{bmatrix}$$

$$z_{s2} = \begin{bmatrix} -\frac{\partial v_j}{\partial p_{hj}}\left(p_{hj} + (1+\tau_{sj})\frac{\partial p_{hj}}{\partial \tau_{ko}}\right) \\ -\sum_o \frac{\partial h_o}{\partial p_{hj}}\frac{1}{h_o}\left(p_{hj} + (1+\tau_{sj})\frac{\partial p_{hj}}{\partial \tau_{ko}}\right) \end{bmatrix}$$

Computations for 3.5

$$A_{v1}d_{v1} = z_{v1} \quad \Rightarrow \quad A_{v1}^{-1}z_{v1} = d_{v1}$$

$$A_{v1} = \begin{bmatrix} \frac{\partial^2 u_1}{\partial x_{1j}^2} & 0 & \frac{\partial^2 u_1}{\partial x_{1j}\partial h_j} & 0 \\ -\bar{p}_x & 0 & & \\ 0 & \beta\frac{\partial^2 u_2}{\partial x_{2j}^2} & \beta\frac{\partial^2 u_2}{\partial x_{2j}\partial h_j} & 0 \\ 0 & -\bar{p}_x & & \\ \frac{\partial^2 u_1}{\partial h_j \partial x_{1j}} & \beta\frac{\partial^2 u_2}{\partial h_j \partial x_{2j}} & \frac{\partial^2 u_1}{\partial h_j^2} + \beta\frac{\partial^2 u_2}{\partial h_j^2} & 0 \\ -p_{hj}(1+\tau_{vj}) & 0 & & \\ 0 & 0 & 0 & 0 \\ 1 & -(1+\bar{r}_y) & & \\ -\bar{p}_x & 0 & -p_{hj}(1+\tau_{vj}) & y_o \\ 0 & 0 & & \\ 0 & -\bar{p}_x & -p_{hj}\tau_{vj} & -(1+\bar{r}_y)y_o \\ 0 & 0 & & \end{bmatrix}$$

$$d_{v1} = \begin{bmatrix} dx_{1j}/d\tau_{vj} \\ dx_{2j}/d\tau_{vj} \\ dh_j/d\tau_{vj} \\ d\gamma_1/d\tau_{vj} \\ d\lambda_1/d\tau_{vj} \\ d\lambda_2/d\tau_{vj} \end{bmatrix}$$

$$z_{v1} = \begin{bmatrix} 0 \\ 0 \\ \lambda_1 \left(\frac{\partial p_{hj}}{\partial \tau_{vj}} (1+\tau_{vj}) + p_{hj} \right) + \lambda_2 \left(\frac{\partial p_{hj}}{\partial \tau_{vj}} \tau_{vj} + p_{hj} \right) \\ 0 \\ h_j \left(\frac{\partial p_{hj}}{\partial \tau_{vj}} (1+\tau_{vj}) + p_{hj} \right) \\ h_j \left(\frac{\partial p_{hj}}{\partial \tau_{vj}} \tau_{vj} + p_{hj} \right) \end{bmatrix}$$

$$\frac{\partial^2 u_1}{\partial x_{1j}^2} = \frac{\bar{p}_x}{p_{hj} \left(1 + \tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_k} \right)} \frac{\partial^2 u_1}{\partial h_j \partial x_{1j}}$$

$$\frac{\partial^2 u_2}{\partial x_{2j}^2} = \frac{\bar{p}_x}{p_{hj} \left(1 + \tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_k} \right)} \frac{\partial^2 u_2}{\partial h_j \partial x_{2j}}$$

$$\frac{\partial^2 u_1}{\partial h_j^2} = \frac{p_{hj} \left(1 + \tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_k} \right)}{\bar{p}_x} \frac{\partial^2 u_1}{\partial x_{1j} \partial h_j} - \beta \frac{\partial^2 u_2}{\partial h_j^2}$$

$$\Omega_{v1} = \frac{\left(1 + \tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_k} \right) + \frac{\tau_{vj}}{1+\bar{r}_k} h_j \frac{\partial u_{1j}}{\partial h_j}}{1+\tau_{vj}}$$

$$\Omega_{v2} = \frac{2 + h_j \frac{\partial u_{1j}}{\partial h_j}}{1+\tau_{vj}} + \frac{\tau_{vj} h_j \frac{\partial u_{1j}}{\partial h_j}}{(1+\tau_{vj}) \left(1 + \tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_k} \right) (1+\bar{r}_k)}$$

$$\Omega_{v3} = \frac{(2+\bar{r}_y) \left(1 + 2\tau_{vj} + (2\bar{r}_y + \bar{r}_y^2)(1+\tau_{vj})^2 \right)}{(1+\bar{r}_y^2)(1+\bar{r}_y + 2\tau_{vj} + \bar{r}_y \tau_{vj})(1+\tau_{vj})}$$

$$\begin{aligned} \Omega_{v4} = & \frac{1 + \bar{r}_y + 2\tau_{vj} + \bar{r}_y \tau_{vj} + \tau_{vj} h_j \frac{\partial u_{1j}}{\partial h_j}}{(1+\bar{r}_k^2)(1+\bar{r}_y + 2\tau_{vj} + \bar{r}_y \tau_{vj})(1+\tau_{vj})} * \\ & * \frac{1 + 2\tau_{vj} + (2\bar{r}_k + \bar{r}_y^2)(1+\tau_{vj})^2}{(1+\bar{y}_k^2)(1+\bar{r}_y + 2\tau_{vj} + \bar{r}_y \tau_{vj})(1+\tau_{vj})} \end{aligned}$$

$$\Omega_{v5} = \frac{h_j \left((1 + 3\tau_{vj} + \bar{r}_y^2 (1+\tau_{vj})^2 + \bar{r}_k (2 + 5\tau_{vj} + 2\tau_{vj}^2) \right) \frac{\partial u_{1j}}{\partial h_j}}{(1+\bar{r}_y^2)(1+\bar{r}_y + 2\tau_{vj} + \bar{r}_y \tau_{vj})(1+\tau_{vj})}$$

$$A_{v2} d_{v2} = z_{v2} \quad \Rightarrow \quad A_{v2}^{-1} z_{v2} = d_{v2}$$

$$A_{v2} = \begin{bmatrix} \frac{\partial v_j}{\partial y_o} h_o & \left(1 + \tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_k}\right) \frac{\partial v_j}{\partial p_{hj}} \\ \sum_o \frac{\partial h_j}{\partial y_o} - \frac{\sum_o h_j \frac{\partial h_o}{\partial k_{ho}} \frac{\partial k_{ho}}{\partial p_{ho}}}{h_o^2} & \left(1 + \tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_k}\right) \frac{\sum_o \frac{\partial h_j}{\partial p_{hj}}}{h_o} \end{bmatrix}$$

$$d_{l2} = \begin{bmatrix} dp_{ho}/d\tau_{vj} \\ dp_{hj}/d\tau_{vj} \end{bmatrix}$$

$$z_{v2} = \begin{bmatrix} -\frac{\partial v_j}{\partial p_{hj}} \left(p_{hj} \left(1 + \frac{1}{1+\bar{r}_y}\right) + \left(1 + \tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_y}\right) \frac{\partial p_{hj}}{\partial \tau_{vj}}\right) \\ -\sum_o \frac{\partial h_o}{\partial p_{hj}} \frac{1}{h_o} \left(p_{hj} \left(1 + \frac{1}{1+\bar{r}_y}\right) + \left(1 + \tau_{vj} + \frac{\tau_{vj}}{1+\bar{r}_y}\right) \frac{\partial p_{hj}}{\partial \tau_{vj}}\right) \end{bmatrix}$$