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Abstract

In this paper I analyse a labour market where the wage is endogenously determined according to an Efficient Bargaining process between a firm and a labour union whose members are partitioned into two social groups: the old and the young. Furthermore, I exploit the Single-Mindedness theory, which considers the existence of a density function which endogenously depends on leisure. I demonstrate that, when preferences of one group for leisure are higher than those of the other group the latter suffers from higher tax rates and with lower level of wage rates and lower levels of leisure. Finally, since the former is more single-minded, it may exploit its greater political power in order to get a positive intergenerational transfer. Empirical evidence from the WERS 2004 survey confirms main results of the model.

JEL Classification: D71, J22, J26, J51

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1 Introduction

In recent years the interest of economists about trade unions behavior models has been gradually increasing. The earliest studies were principally oriented to the macroeconomic perspective and intended to explain the relationship between higher wages generated by the presence of labour unions and unemployment levels. From this point of view three models have been competitors in the attempt to explain this linkage: the Monopoly Union model, the Right-to-Manage (RTM) model and the Efficient Bargaining model.

The oldest Monopoly Union model was developed by Dunlop (1944) [14]; there, the union was seen as a monopolistic seller of labour which maximized its utility function by choosing the optimal level of wage, given the firm’s demand for labour. As a result, this model entailed more unemployment than it would be the case under perfect labour markets and a higher level of wage rates with respect to the competitive wage. Nowadays this model is gradually being abandoned, due to the distance between its hypothesis and what it is observed in reality (it never happens that employers leave the power to decide wage to labour unions).

The RTM model was originally proposed by Leontief (1946) [31] but it was only in the early 1980s with the British school (see Nickell and Andrews [42] or Manning [33] amongst the others) that it acquired its popularity. The model assumes that there exists a bargaining between a firm and a union over the real wage, subject to the labour demand unilaterally chosen by the firm. The wage derived by the bargaining process is lower and the employment level is higher than that generated by the monopoly union model and the RTM solution lies on the labour demand curve. The main achievement of the RTM model is that the equilibrium wage and the level of employment depend upon the bargaining power of the involved bargainers.

Finally, in the Efficient Bargaining model developed by McDonald and Solow (1981) [41] the union and the firm negotiate both upon the wage rate and the level of employment. The quite surprising result of this model is that the efficient solution lies not on the labour demand curve but on the Contract Curve, which in presence of risk-averse workers is positively sloped and stands at the right hand side of the labour demand curve. As a result both the level of employment and the wage rate are higher than the competitive solution.

Nevertheless, these three macroeconomic models do not take some relevant factors into account. First of all, they do not consider the distinction between “insiders”, whose preferences count and “outsiders”, whose preferences do not. Due to this distinction, the union indifference curves end up to be kinked at the point where the level of employment is equal to the membership (for a review of the insider-outsider models see Lindbeck and Snower [32]).
Secondly, the classical macroeconomic literature took the union size as exogenously given, whilst authors as Grossman [21] started to investigate this issue, considering the role of the seniority within the union and the voting mechanism which maximises the expected utility of the median worker.

From the early 1990s many economists such as Nickell & Wadhwni (1990) and McDonald & Suen (1992) started to analyse, from a microeconomic perspective, the role and the determination of trade union power. This concept of trade unions, according to McDonald and Suen [41] is “the ability of the trade union to divide up to its advantage the rents arising from the production process given other parameters, in particular the elasticity of revenue with respect to employment “.

In the last decade researchers have been moving toward new fields of research, most of them referring to the political economy area. One fertile field is represented by the study of how workers organize in social groups and the role of political insider mechanism; according to this literature (see Gilles Saint-Paul [20] “workers may be unable to coordinate in order to form a labour union, but by voting in favor of an institution that raises they are able to collectively achieve a higher wage level exactly as if they were organized in a union. Labour market rigidities allow insiders to monopolize the market at the economy wide level even though their bargaining power may be quite reduced at a firm level ”. Secondly, also the impact of unions on the voting behaviour of their members represent an interesting field which may provide interesting results in the future.

In this paper, I analyse the role of labour unions from a microeconomic perspective, exploiting the Single-Mindedness Theory. Wages are endogenously determined between a firm and a labour union, according to a typical Efficient Bargaining model; the labour union will be seen as a social institution of workers partitioned into two social groups (Young and Old). A peculiarity of the model is that the firm and the labour union negotiate also on the level of hours worked (instead of the employment as in the traditional McDonald and Solow model). This choice variable is very suitable for an Efficient Bargaining model, since the hours worked can be signed in a contract, representing that “manning agreement ” required by McDonald & Solow in their paper, in order to increase employment beyond the labour demand scheduling. Notice that the use of hours of work as a choice variable in bargaining literature is not frequent at all. In the paper I will briefly review the literature on this issue and I will conclude saying that too often studies have been using variable choices, such as the level of employment, which hardly are chosen in reality, instead of using hours of work which, as I will demonstrate in the empirical section of the paper, is the issue that together with the wage rate is bargained by labour unions with firms in the real world.

An important assumption I will state in the model is that preferences of
the old for leisure are different from preferences of the young. Under this condition, I will demonstrate that the equilibrium wage rate and the level of leisure of that group which has higher preferences for leisure and wage are higher than those of the other group. Since the single-mindedness of a group, which represents a proxy for the political power of that group, is captured by the density function, which is a monotonically increasing function of leisure [6], I conclude that there must exists one group who is more able to influence politicians and that this acquired power of influence enables it to get positive transfers. Since the Government must clear the budget, the burden of transfers is entirely carried by the other (less single-minded) group. Thus, with respect to previous work of mine, this study considers the mechanisms of labour unions, seen as an institution which represents the interests of different social groups and like every other institution take decisions. Again, in accordance with the SMT, the greater the ability of a single group to be focused on the minimum number of issues, the higher the probability that this group achieves its goals. The paper is organized as follows: section one introduces, section two makes a brief review of the literature about the bargaining over hours of work, section three explains the model, section four provides a possible extension, section five brings interesting empirical evidence about how bargaining takes place in U.K.’s workplaces, and section six concludes.

2 Bargaining over hours of work

At least until the 1990s the literature on unionism was principally concerned with the bargaining outcomes between unions and firms. The models which was developed on labour unions tried to find an answer to two questions: 1) does the bargaining take place exclusively on wage or it also involve the level of employment?; 2) what does the bargain power depend on? Nevertheless, at the beginning of the 1990s the research on unionism started to involve the problem of hours of work in the analysis (see Earle & Pencavel [15]). Indeed, it seemed clear that McDonald & Solow’s idea to left the bargaining take place over the employment was unealistic. At the same time, evidence on U.S. labour union history made clear that the reduction of hours of work was one of the main labour unions’ concern. The struggle for the reduction of working hours is narrated in depth by Hunnicut [25] There, many historical facts referring to the importance the labour unions attribute to working hours are reported. Two sentences above the others: 1) “Some of the most dramatic and significant events in the history of labor (...) and some of the most notable achievements (such as the ten hours per day and the eight hour day) were parts of labor’s century struggle for shorter hours”, and 2) “A large numbers of influential writers and social critics welcomed and promoted shorter hours, believing it was as natural and as good a result of technological
advances as higher wages”. And even though after the Second World War the interest for leisure in the U.S. has seemed to fade under the modern liberal theories which taught the best way to achieve the full employment, in Europe the struggle for the reduction of working hours is still one of the principal goals of workers. It became obvious then, to consider the hours of work into the labour union’s utility function. Earle & Pencavel studied some union-management bargaining models with an utility function which includes earnings, hours of work and the level of employment. Unfortunately this model seems to suffer from a mis-specification problem, since hours of work and the employment level are neither separable nor independent. Nevertheless, this paper had the merit to brought a comprehensive and systematic study over the negotiation about the hours of work which, as I demonstrate later on in the paper, are a typical choice variable in the real world.

3 The model

I consider an OLG model, where each generation lives only for two periods, the youth and old age. At any period of time, the generation of youths coexists with the generation of the elderly. At the beginning of the next period, the elderly die, the youths become elderly and a new generation of youths is born. As a consequence, there are two overlapping generations of people living at any one time. Generations are unlinked, meaning that there is no possibility to leave any bequest. Individuals consume all the available income earned at a given period of time; thus, it is not possible neither to save nor to borrow money.

Then, let a population of size equal to one be partitioned into two groups of workers, the young and the old, each of them endowed with a given amount of time (measured in hours). Thus, the space of groups is \( G = \{T - 1, T\} \), where \( T \) denotes the group of young workers and \( T - 1 \) the group of old workers. I will use index \( I \) to denote a social group, capital letters to indicate the group and small letters to indicate single individuals belonging to the \( I \)-th group. The size of a group does not change over time.

Each worker has to decide how to divide his total number of hours \( H \) between work \( (h_w) \) and leisure \( (h_l) \). I assume also that leisure can be employed to attend several activities, such as relaxing, taking care of family, participating in political activities and many others. Thus, leisure can be seen as a vector of \( N \) activities \( h_l = h_l(h_{l1}, ..., h_{lN}) \), where \( h_{ln} \geq 0 \).

Labour market is imperfect and this imperfection is due to the presence of a labour union which bargains the wage rate with the firm, according to a typical Efficient Bargaining model (McDonald & Solow, 1981). I assume that there exists only one union and that all the workers (old and young) are members of this union whose aim is to maximize its members’ both the
net-of-tax income and the level of employment. A classical achievement in the economic theory states that the presence of this market imperfection generates unemployment in equilibrium (for an introductory analysis on the effects of trade unions on the labour markets, see Oswald [44]).

I introduce one of the core assumptions of the model. I assume that the old and the young are identical in every respect except one: the intrinsic preference for leisure of a group is assumed to be greater than the preference for leisure of the other. I will not specify which group has the higher level of preferences for leisure, even though the empirical evidence seems to show that the old prefer leisure more than the young. Finally, Old workers’ preferences can be represented by a quasi-linear utility function. A representative old worker at time $t$ has the following lifetime utility function:

$$U_{t} = c_{t} + \psi \log h_{t}$$ (1)

∀ $t - 1 \in T - 1$

where $c_{t}$ is the consumption at time $t$, $h_{t}$ is the amount of hours of leisure at time $t$, and $\psi$ is a parameter representing the intrinsic preference of the old for leisure ($\psi \in [0, 1]$). The old worker consumes all his income:

$$c_{t} = w_{t}(1 - \tau_{Lt})(H - h_{t}) + b_{t} + r(S_{t})$$ (2)

where $w$ is the unitary wage per hour worked, $\tau_{Lt}$ is the tax rate on labour income, $H$ is the total amount of hours, $b_{t}$ is an intergenerational (lump-sum) transfer and $r(S_{t})$ represents the return which the old worker gains at the end of time $t$ over an amount of money he accumulated. I assume that $r(S_{t})$ represents mandatory savings. The last day of work, old workers withdraw the amount of money invested. Without loss of generality, I assume that the same day, they consume all their amount of money and die.

Similarly, the preferences of a representative young worker $y$ are given by the following lifetime utility function:

$$U_{t} = c_{t} + \psi \log h_{t}$$ (3)

∀ $t \in T$

where $c_{t}$ and $c_{t+1}$ represent the consumption at time $t$ and $t+1$, $h_{t}$ and $h_{t+1}$ leisure at time $t$ and $t+1$, $\beta$ is the time preference discount factor, and $\psi$ is the intrinsic preference of the young worker for leisure ($\psi \in [0, 1]$).

Finally, the parameter indicating the preferences for leisure of one group is assumed to be higher than the same parameter for the other group: $\psi^{I} >>

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2 for a survey on the factors which explain the difference in preferences for leisure among social groups, see Canegrati [6]

3 A quasi-linear utility function entails the non existence of the income effect.
ψ−I. Since the young know that at time $t + 1$ will be old, their utility function includes the leisure of the next period, weighted by a discount factor $β$. 

The young worker’s intertemporal budget constraint is given by:

$$c_τ^t + βτc_τ^{t+1} = w_τ^t(1 − τLt)(H − hLt) + b_τ^t$$

$$+ r(S_τ^t) + βτ(w_{τ+1}^t(1 − τLt+1) + b_{τ+1}^t + r(S_{τ+1}^t)) \quad (4)$$

Furthermore, I introduce the following budget constraints:

$$r(S_τ^{t-1}) = T_τ^{t-1} \quad (5)$$

$$r(S_τ^t) = T_τ^t \quad (6)$$

$$n_τ^{t-1}b_τ^{t-1} + n_τ^tb_τ^t + α \left| n_τ^{t-1}b_τ^{t-1} \right| \left| n_τ^tb_τ^t \right| = 0 \quad (7)$$

Since revenues are proportional to the amount of labour supplied, the taxation entails inefficiencies, since it distorts workers’ decisions on the amount of labour supplied and determines the mandatory savings. Furthermore, I assume that the wage rate does not change over time so that: $w_τ^t = w_{τ+1}^t$, and so I will drop the time index from the wage rates. $T_τ^{t-1}$ represents total revenues generated by the taxation of the old at time $t$ and it is equal to $n_τ^{t-1}r_{Lt-1}w^{t-1}(H − hLt)$. While $T_τ^t$ the total revenues generated by the taxation of the young at time $t$ and it is equal to $n_τ^tr_{Lt}w^t(1 − τLt)$. The condition $n_τ^{t-1}b_τ^{t-1} + n_τ^tb_τ^t + α \left| n_τ^{t-1}b_τ^{t-1} \right| \left| n_τ^tb_τ^t \right| = 0$ assures that an intergenerational transfer exists and that if one group is better off in the transfer process, the other one must be worse off. The term $α \left| n_τ^{t-1}b_τ^{t-1} \right| \left| n_τ^tb_τ^t \right|$ represents the efficiency loss which takes place via the redistribution process and can be measured by the amount of resources wasted during this process. For instance, one may think that this loss is due to the existence of bureaucracy costs or to rents grabbed by politicians. The parameter $α ∈ [0, 1]$ represents the measure of the loss which is quadratic in the transfers.

### 3.1 The Government

The literature has used different formulation for the Government’s objective function. A typical normative approach considers a benevolent Government which aims to maximize a Social Utility Function by choosing the optimal tax rate on labour, subject to a budget constraint where tax revenues are equal to public good expenditures. Otherwise, some authors such as Edwards and Keen considers a Leviathan model where, referring to the famous milestone paper by Brennan and Buchanan [4], they examine a Government which is concerned in part with maximizing the size of the public sector. Furthermore, the Edwards and Keen model assumes that the Government
retains some degree of benevolence, perhaps because it has re-election concerns. Nevertheless, this concerns were not formally modeled.

In this paper, I provide a possible explanation to this issue, introducing a political economy model where politicians act in order to maximize the probability of being re-elected.

A public policy vector is given by:

\[ \vec{q} = (\tau_{Lt}^{-1}, \tau_{Lt}^{T}, b_{T}^{-1}, b_{T}^{T}) \]

composed of two tax rates and two intergenerational transfers.

Finally, the Government is committed to clear the budget constraint; this means that it cannot transfer more resources than those collected by taxing individuals at every period of time. Thus, I assume that the Budget Surplus (Deficit) must be equal to zero. Since the Government cannot issue bonds to collect more financial resources and can only rely on taxation, the increase in a social group’s welfare entails the decrease in the welfare of the other social group, since the latter has to pay for the transfer to the former.

3.1.1 The labour union

Since the 1980s many labour economists have been trying to find a correct specification for the labour union utility function but still there is not an unambiguous consensus over it. Nevertheless, the idea that unions have indifference curves defined over wages and level of employment seems to be generally accepted nowadays. Oswald proposed an utilitarian union utility function, where the union utility is simply the sum of the utility of single groups. Thus, the utility function is a weighted average of they utility derived from the groups.

Denoting by \( n \) the total members of the union which coincide with the population (in the sense that I assume that every citizen belongs to the union, and denoting by \( n_{\tau}^{-1} \) the number of old workers, by \( n^{\tau} \) the number of young workers, by \( w_{\tau}^{-1} = w^{\tau-1}(1 - \tau_{Lt}^{-1}) \) the net-of-tax wage rate of the old, and by \( \tilde{w}_{\tau} = w^{\tau}(1 - \tau_{Lt}^{T}) \) the net-of-tax wage rate of the young, the objective function of the union can be written as follows:

\[
U(w_{\tau}^{-1}, w^{\tau}, H_{wt}^{T-1}, H_{wt}^{T}) = H_{wt}^{T-1}w_{\tau}^{-1} + H_{wt}^{T}w_{\tau}^{T}
\]  

(8)

where \( H_{wt}^{T-1} \) is the total amount of hours worked by the group of the old workers and \( H_{wt}^{T} \) is the total amount of hours worked by the group of young workers. An important assumption here is that in the last two stage of the game, the labour union, in order to bargain over the wage, acts with an univoque position when it has to face the firm, bargaining only one level of wage given only one level of employment. That is (8) becomes:

\[
U(\tilde{w}, H_{wt}) = H_{wt}^{-1}w(2 - \tau_{Lt}^{-1} - \tau_{Lt}^{T})
\]  

(9)
Nevertheless, even though the general aim of the labour union is maximizing the welfare of its workers in terms of wage and employment, we must consider that members are not all alike. A simply way to deal with this argument is dividing the workers in two groups: again, the old and the young. In particular, I assume that the group of the old has a relative preference for leisure and the group of the young has a relative preference for wage. If we denote with $\chi_{\tau-1}$ the parameter which synthesizes the preferences for the employment of the old (i.e. the average of the preferences of single old workers) and with $\chi_{\tau}$ the same parameter for the young, we may write the (Stoney-Gear) utility functions of the two group as follows:

\begin{align*}
U_{g\tau-1}(t) &= \chi_{\tau-1} \log H_{wt}^{\tau-1} + (1 - \chi_{\tau-1}) \log \tilde{w}_{\tau-1} \quad (10) \\
U_{g\tau}(t) &= \chi_{\tau} \log H_{wt}^{\tau} + (1 - \chi_{\tau}) \log \tilde{w}_{\tau} \quad (11)
\end{align*}

with $\chi_{\tau} > \chi_{\tau-1}$ where $w_{\tau-1}(\tilde{w}_{\tau})$ the wage rate of the old (young). Furthermore, I introduce two constraints:

\begin{align*}
H_{wt}^{\tau-1} + H_{wt}^{\tau} &= H_{wt} \quad (12) \\
\frac{n_{\tau-1}}{n} w_{\tau-1} + \frac{n_{\tau}}{n} \tilde{w}_{\tau} &= \bar{w} \quad (13)
\end{align*}

The first constraint simply assures the the sum of the employment of the two group is equal to the number of hours bargained by the firm and the labour union; in other words, the labour union has to decide how to divide the total hours of work amongst its members. The second constraint says that the average wage must be equal to the official bargained wage. This is important, since I assume that when, in the bargaining activity, the labour union acts as an unitary institution which represents all its members’ needs. The bargaining outcome is represented by only one wage rate (which I will call the official wage rate) and a total amount of hours worked. I assume also that the firm is only interested in the global level of employment and wage which maximize profits. Furthermore, the labour union takes its decision in a democratic way, according to a voting process which takes place between two candidates, which propose a policy to all the members and then workers vote for the proposal which maximizes their utility, according to a majoritarian principle. For instance, the two candidates may be seen as two streams of the labour union which support the young and old needs. Thus, the two candidates commit to a policy vector $\vec{p} = (H_{wt}^{\tau-1}, H_{wt}^{\tau}, w_{\tau-1}, \tilde{w}_{\tau})$ and then elections take place.

### 3.1.2 The firm

I assume that a monopolistic firm produces only one good, with a production function which only uses labour as input, and that it maximizes profits:
\[ \Pi = \Pi(\overline{w}, \overline{H}_{wt}, \tau_{Lt}^{-1}, \tau_{Lt}^T) \]  

Furthermore, I assume that the firm sells the good at a price \( p \), normalized to unity. The production function \( Y = F(H_{wt}) \) is represented by a twice differentiable, concave function, and I assume that \( y(0) = 0 \), that \( \frac{\partial p}{\partial H_{wt}} > 0 \), and that \( \frac{\partial^2 p}{\partial^2 H_{wt}} < 0 \). I assume also that the young and the old are equally productive and thus the firm demands an amount of hours \( \overline{H}_{wt} \).

### 3.1.3 A five-stage game

I consider a non-cooperative five-stage game among two political candidates, a trade union representing the two social groups and a firm. In the first stage the two political candidates announce their fiscal policy vector, by choosing the optimal level of labour tax rate \( \tau_{Lt} \) and the optimal transfers \( b_t \). In the second, elections take place. In the third stage labour union’s candidates choose their optimal policy vector by deciding how to split the total amount of hours worked and how to differentiate wages amongst the two groups. In the fourth stage elections within the labour union take place. In the last stage, the labour union and the firm bargain over wage and hours worked, according to a typical Efficient Bargaining model. The maximand is a weighted function \( \Omega \) of the firm and union objective functions:

\[ \Omega_t = \lambda \log \left[ U(\overline{w}, \overline{H}_{wt}) - x \right] + (1 - \lambda) \log \left[ \Pi(\overline{w}, \overline{H}_{wt}) - \overline{i} \right] \]  

where the parameter \( \lambda \) denotes the relative bargaining power of the trade union (\( 0 \leq \lambda \leq 1 \)), \( x \) the fall-back position of the labour union and \( \overline{i} \) the fall back position of the firm which without loss of generality is normalized to zero. To solve the model I use the backward induction. I start to solve the model from the \textit{fifth stage} where the firm and the union bargain over the wage rate, and the optimal value is determined by the maximization of the geometrically weighted average of the gains to the two parties. Differentiating (15) with respect to \( \overline{w} \) and \( \overline{H}_{wt} \), I obtain the First Order Condition of the problem:

\[ \frac{\partial \Omega_t}{\partial \overline{w}} = \lambda \frac{\partial U(\overline{w}, \overline{H}_{wt})}{\partial \overline{w}} \frac{\partial \overline{w}}{U(\overline{w}, \overline{H}_{wt})} - x + (1 - \lambda) \frac{\partial \Pi(\overline{w}, \overline{H}_{wt})}{\partial \overline{w}} \frac{\partial \overline{w}}{\Pi(\overline{w}, \overline{H}_{wt})} - \overline{i} = 0 \]

\[ \frac{\partial \Omega_t}{\partial \overline{H}_{wt}} = \lambda \frac{\partial U(\overline{w}, \overline{H}_{wt})}{\partial \overline{H}_{wt}} \frac{\partial \overline{H}_{wt}}{U(\overline{w}, \overline{H}_{wt})} - x + (1 - \lambda) \frac{\partial \Pi(\overline{w}, \overline{H}_{wt})}{\partial \overline{H}_{wt}} \frac{\partial \overline{H}_{wt}}{\Pi(\overline{w}, \overline{H}_{wt})} - \overline{i} = 0 \]

The resolution of the problem gives the optimal hours worked and the optimal wage:

\[ \overline{H}_{wt}^\ast = \overline{H}_{wt}(\tau_{Lt}^{-1}, \tau_{Lt}, x, \lambda) \]  

\[ \overline{w}^\ast = \overline{w}(\tau_{Lt}^{-1}, \tau_{Lt}, x, \lambda) \]
Comparative statics shows that $w^*_x < 0$, $w^*_\lambda > 0$, $w^*_{\tau_{Lt}^{-1}} < 0$ and $w^*_\tau < 0$. That is the optimal wage tends to increase the higher the bargaining power of the labour union and tend to decrease the higher is the fall-back position of the union and the higher the tax rates.

If the bargain is (Pareto) efficient the optimal solution $(w^*, H^*_w)$ must stands over the so called contract curve (CC), which represents the locus of tangency points between a union’s indifference curve and a firm’s isoprofit curve defined by the following condition: $(V(w) - \bar{u})/V'(w) = w (1 + \tau_{Lt}) - R'(H_w)$ (see equation 3 in MacDonald & Solow, 1981).

In the forth stage they have to decide how to divide the optimal number of hours worked chosen by the firm between the old and the young and how to differentiate the wages amongst the two cohorts. I suppose that every worker has an idiosyncratic preference toward one of the two candidate which is not observable ($\sigma^{i,I,u}$, where superscript $u$ stands for union) and that this random variable is normally distributed in a closed interval $[-\frac{1}{2}a^u, \frac{1}{2}a^u]$ and an initial reputation advantage for one of the two candidate ($\delta^u$) which again is a variable distributed in a closed interval $[-\frac{1}{2}d^u, \frac{1}{2}d^u]$. I will not into details about what it determines the idiosyncratic and candidate advantage parameters and I will take them as exogenously given. Nevertheless, one may imagine that the reputation is due to the mediatic exposure a candidate have or by the effort put in place by some active supporters. A worker $i$ in group $I$ votes for candidate $R$ if

$$V^I(p^R) > V^I(p^L) + \sigma^{i,I,u} + \delta^u$$

Furthermore, I assume that the two candidates are rent-seekers, and only aim to maximize the probability of winning (equivalently the share of votes) in order to win the elections. The probability of winning for candidate $R$ may be written as:

$$\pi^R = \frac{1}{2} + \frac{d^u}{s^u} \sum_{I=T-1,T} n^I s^{I,u}(U^{gi}(\vec{p}^R) - U^{gi}(\vec{p}^D))$$

where the utility function are represented by (10) and (11). Thus, the maximization problem for the labour union is:

$$\max \frac{1}{2} + \frac{d^u}{s^u} \sum_{I=T-1,T} n^I s^{I,u}(U^{gi}(\vec{p}^R) - U^{gi}(\vec{p}^D))$$

where $s \equiv \sum_I n^I s^{I,u}$.

$$s.t. H_{wt}^{\tau-1} + H_{wt}^\tau = \bar{H}_{wt}$$

$$n^{\tau-1}w^{\tau-1} + n^{\tau}w^\tau = \bar{n}w^\tau$$

**Proposition 1** In equilibrium both candidates’ policy vectors converge to the same platform; that is $\vec{p}^R = \vec{p}^D = \vec{p}^*$
Proof: $\vec{p}^*$ represents the policy which captures the highest number of swing voters. Suppose instead there exists other two policies $\vec{p}'$ and $\vec{p}''$; in moving from $\vec{p}^*$ to $\vec{q}'$ (or $\vec{q}''$) a candidate loses more swing voters than those it is able to gain. Thus, suppose a starting point where candidate R chooses $\vec{p}'$ and candidate D chooses $\vec{p}''$ such that by choosing $\vec{p}'$ and $\vec{p}''$ the elections outcome is a tie. If one candidate moved toward $\vec{p}^*$, it would be able to gain more swing voters than those it loses and thus, it would win the elections. So, choosing any policy but $\vec{p}^*$ cannot be an optimal answer. The only one policy which represents a Nash Equilibrium is $\vec{p}^*$ since it is the intersection between the optimal answers of the two candidates and no one candidate has an incentive to deviate. Since each candidate maximizes its share of votes, in equilibrium the two candidates receive both one half of votes; if one candidate should receive less than one half of votes it would always have the possibility to adopt the platform chosen by the other candidate and get the same number of votes.

**Corollary 1** The utility gained by the workers under the policy chosen by $R$ is equal to the utility gained under the policy chosen by $D$; that is: $V^i(\vec{p}^R) = V^i(\vec{p}^D)$.

**Proposition 2** In a PVM where social groups have to split a given amount of hours of work (or more in general a given amount of resources), the optimal quota of resources that any social group obtains is represented by a weighted average which is a function of the Numerosity, Density and Preferences of the group (which I will call as $NDP-WA_H$). A suitable expression for an $NDP-WA_H$ is represented by $NDP-WA_H = \frac{f^I(n^I_s^I\chi^I)}{\sum_I f^I(n^I_s^I\chi^I)}$.

Proof: The initial amount of resources that the young and the old have to split is represented by the total amount of hours chosen by the firm at stage six, $H^{*\tau}_{wt}$. Solving the PVM we obtain the two expressions which represent the optimal quota of hours for the two groups:

$$H^{\tau-1}_{wt} = \frac{s^{\tau-1}u^{\tau-1}n^{\tau-1}x^{\tau-1}}{s^{\tau-1}u^{\tau-1}n^{\tau-1}x^{\tau-1} + s^{\tau}u^{\tau}n^{\tau}x^{\tau}} H^{*\tau}_{wt}$$ (18)

$$H^*_{wt} = \frac{s^{\tau}u^{\tau}n^{\tau}x^{\tau}}{s^{\tau-1}u^{\tau-1}n^{\tau-1}x^{\tau-1} + s^{\tau}u^{\tau}n^{\tau}x^{\tau}} H^{*\tau}_{wt}$$ (19)

which (is easy to see) are an $NDP-WA = \frac{f^I(n^I_s^I\chi^I)}{\sum_I f^I(n^I_s^I\chi^I)}$.

**Corollary 2** The optimal quota of resources that any social group obtains is an increasing function of the Numerosity, Density and Preferences of the group and a decreasing function of the Numerosity, Density and Preferences of the other group.
The optimal wage that any social group obtains is an increasing function of the Numerosity, Density of the group and Preferences of the other group, and a decreasing function of the Numerosity and Density of the group:

\[
\frac{\partial NDP}{\partial n} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial n} > 0;
\]

\[
\frac{\partial NDP}{\partial \tau} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \tau} > 0;
\]

\[
\frac{\partial NDP}{\partial \chi} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \chi} > 0;
\]

\[
\frac{\partial NDP}{\partial n} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial n} < 0;
\]

\[
\frac{\partial NDP}{\partial \tau} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \tau} < 0;
\]

\[
\frac{\partial NDP}{\partial \chi} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \chi} < 0.
\]

**Proposition 3** In a PVM where social groups have to decide an optimal wage with an average wage equal to the population average wage, the optimal wage that any social group obtains is represented by a weighted average which is a function of the Numerosity, Density and Preferences of the group (which I will call as \( NDP - W_A_w \)). A suitable expression for the \( NDP - W_A_w \) is represented by \( NDP - W_A_w = \sum_i g_i(n_i s_i \chi_i) \), where \( g_i(n_i s_i \chi_i) = g_i(n_i s_i \chi_i) \).

**Proof:** Deriving the \( NDP - W_A_w \) of the group I we obtain:

\[
\frac{\partial NDP - W_A_w}{\partial n} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial n} > 0;
\]

\[
\frac{\partial NDP - W_A_w}{\partial \tau} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \tau} > 0;
\]

\[
\frac{\partial NDP - W_A_w}{\partial \chi} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \chi} > 0;
\]

\[
\frac{\partial NDP - W_A_w}{\partial n} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial n} < 0;
\]

\[
\frac{\partial NDP - W_A_w}{\partial \tau} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \tau} < 0;
\]

\[
\frac{\partial NDP - W_A_w}{\partial \chi} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \chi} < 0.
\]

**Corollary 3** The optimal wage that any social group obtains is an increasing function of the Numerosity and Density of the group and Preferences of the other group, and a decreasing function of the Numerosity, Density of the other group, and Preferences of the group.

**Proof:** Deriving the \( NDP - W_A_w \) of the group I we obtain:

\[
\frac{\partial NDP - W_A_w}{\partial n} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial n} > 0;
\]

\[
\frac{\partial NDP - W_A_w}{\partial \tau} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \tau} > 0;
\]

\[
\frac{\partial NDP - W_A_w}{\partial \chi} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \chi} > 0;
\]

\[
\frac{\partial NDP - W_A_w}{\partial n} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial n} < 0;
\]

\[
\frac{\partial NDP - W_A_w}{\partial \tau} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \tau} < 0;
\]

\[
\frac{\partial NDP - W_A_w}{\partial \chi} = \left( \frac{s^{\tau-1}u_{\tau-1}(\chi^{\tau-1} - 1) + s^{\tau}u_{\tau}w^\tau(\chi^{\tau-1} - 1)}{s^{\tau-1}u_{\tau}w^\tau(\chi^{\tau-1} - 1)} \right) \frac{\partial w}{\partial \chi} < 0.
\]
\[
\frac{\partial \text{NDP} - \text{WA}}{\partial \tau_i} = \left( -\frac{2s^{\tau_i-1}n^{\tau_i-1}n^{\tau_i-1}n^{\tau_i-1}(\chi^{\tau_i-1}-1)(\chi^{\tau_i-1})^2}{(s^{\tau_i-1}n^{\tau_i-1}(\chi^{\tau_i-1}-1)+s^{\tau_i-1}n^{\tau_i-1}(\chi^{\tau_i-1}-1))^2} \right) n_w^{\tau_i} < 0
\]

\[
\frac{\partial \text{NDP} - \text{WA}}{\partial \chi_i} = \left( \frac{s^{\tau_i-1}n^{\tau_i-1}n^{\tau_i-1}n^{\tau_i-1}(\chi^{\tau_i-1}-1)}{(s^{\tau_i-1}n^{\tau_i-1}(\chi^{\tau_i-1}-1)+s^{\tau_i-1}n^{\tau_i-1}(\chi^{\tau_i-1}-1))^2} \right) n_w^{\tau_i} > 0
\]

Figure 3 depicts the optimal policy vector in the labour union PVM. On the horizontal axis is located the total amount of hours $\bar{H}_{\text{wt}}$ and the amount of hours in equilibrium for the two groups $I$ and $-I$. On the vertical axis there is the wage rate. The intersection between the two axis gives the average wage rate for the equilibrium hours of work. The dashed line indicates the situation where the total amount of hours are equally divided amongst the two groups, $\bar{H}_{\text{wt}}/2$. $H_{\text{eq}}^{\tau_i}$ indicates the division of hours we have in equilibrium. The black point (E) indicates the situation where the two groups are identical as for numerosity, density and preferences. As a consequence, the amount of hours are equally splitted and the wage rate is the average population rate for both the groups. Otherwise, the blue point indicate the optimal policy for group $I$, whilst the red point the optimal policy for group $-I$. It can be seen that the blue point is located south-eastwards with respect to E, indicating the group $I$ gets a level of hours greater than $\bar{H}_{\text{wt}}/2$ and a wage rate lower than the average wage rate. Otherwise the red point is located north-eastwards with respect to E, indicating that the group $-I$ gets a level of hours lower than $\bar{H}_{\text{wt}}/2$ and a wage rate higher than the average wage rate. Note the important role played by the two weighted average means ($\text{NDP} - \text{WA}_H$ and $\text{NDP} - \text{WA}_H^I$): the distance of a the optimal policy point (i.e. the red point) from E is exactly equal to the scalar sum of vector $v_1 = H_{\text{eq}}^{\tau_i} - \bar{H}_{\text{wt}}^{\tau_i} = \bar{H}_{\text{wt}}^{\tau_i} \text{NDP} - AM_{HI} - \bar{H}_{\text{wt}}^{\tau_i}$ and vector $v_2 = \text{NDP} - AM_{HIw}^{\tau_i}$.

[FIGURE 3 HERE]

Effects on Welfare

an interesting problem which arises once we have solved the problem of the labour union is to evaluate the effects of the optimal policy on the welfare of the two groups. These are depicted in Figure 4.

[FIGURE 4 HERE]

From a geometrical point of view a policy which neither penalize nor advantage a group is one such that the the triangle which is formed by the two vectors representing the gain (loss) in hours worked and the gain (loss) in wage rate is an isosceles one. In Figure 4 the two isosceles triangles for the two groups are the white ones. Otherwise, if the triangle is not isosceles, it means that one dimension of the policy is out weighting the other dimension meaning that one group is better (worse) off. In the example the group $-I$ is the better off group, since the gain it derives from a higher wage with respect to the average population wage out weights the loss in the level of employment. Otherwise, the group $I$ is the worse off group since the gain it derives from an increase of employment is out weighted by
a strong reduction in the level of wage rate. Two more considerations are useful. First of all, being this a conflict game, if one group is better off the other one must be necessarily worse off; that is, a Pareto improvement is not achievable. Secondly, it is easy to see that it is possible to reach an equilibrium were both of group neither gain nor loss, but this only if and only if the following conditions hold:

\[ NDP - WA_H^I = -NDP - WA_w^I \]

\[ NDP - WA_w^I = -NDP - WA_H^I \]

Calculating the average number of hours per worker for the two groups, we obtain:

\[ h_{wt}^{\tau-1} = \frac{s^{\tau-1}}{s^{\tau-1} + s^{\tau} \chi_{w}^{\tau}} \]

\[ h_{wt}^{\tau} = \frac{s^{\tau}}{s^{\tau-1} \chi_{w}^{\tau} + s^{\tau} \chi_{w}^{\tau}} \]

Thus, I will suppose that workers do not chose the hours of leisure but simply work the average amount of hours worked calculated in (22) and (23). I substitute the two optimal expressions into the utility function of the workers and I write the Indirect Utility Function for the two groups:

\[ V^{\tau-1} = w^{\tau-1} (1 - \tau_{Lt}^{\tau-1}) (h_{wt}^{\tau-1}) + b_t^{\tau-1} + r(S_t^{\tau-1}) + \psi^{\tau-1} \log(\bar{H} - h_{wt}^{\tau-1}) \]

\[ V^{\tau} = w^{\tau} (1 - \tau_{Lt}^{\tau}) (h_{wt}^{\tau}) + b_t^{\tau} + r(S_t^{\tau}) + \psi^{\tau} \log(\bar{H} - h_{wt}^{\tau}) \]

\[ + \beta (w_{t+1}^{\tau} (h_{wt+1}^{\tau}) (1 - \tau_{Lt+1}^{\tau}) + b_{t+1}^{\tau} + r(S_{t+1}^{\tau}) + \psi^{\tau} \log(\bar{H} - h_{wt+1}^{\tau}) \]

In the second stage of the game elections take place. It is easy to verify that the elections’ outcome is a tie. The proof arises from the resolution of the first stage, where it will be demonstrated that in equilibrium, both parties choose an identical policy vector.

In the first stage, the two candidates, simultaneously and independently, announce a policy vector, \( q^A \) and \( q^B \). Every voter’s welfare depends on fiscal policies chosen by candidates which affect his consumption and which is known by both parties, and from another component, which derives from personal attributes of the candidates and which is only imperfectly observed by the parties. In other words, we are assuming that consumers’ preferences for consumption are perfectly visible, whilst other political aspects such as ideology are not. The two candidates face exactly the same optimization problem and maximize their share of votes or, equivalently, the probability of winning. The resolution is made for candidate A, but it also holds for candidate B.\(^4\)

\[ \max \pi^A = \frac{1}{2} + \frac{h}{s} \sum_{t=T-1}^{T} n^t s^t [V^i(q^A) - V^i(q^B)] \]

\(^4\)for a complete derivation of the probability of winning in a Probabilistic Voting Model see Persson and Tabellini (2000)
\[ s.t. \ T_1 \equiv r(S_t^{-1}) = T_t^{-1} \]
\[ T_2 \equiv r(S_c^t) = T_c^t \]
\[ T_3 \equiv n^{-1}b_t^\tau + n^\tau b_t^\tau + \alpha |n^{-1}b_t^\tau| \mid n^\tau b_t^\tau \mid = 0 \]

In the Appendix I provide a complete resolution to the problem.

**Proposition 4** In equilibrium both candidates’ policy vectors converge to the same platform; that is \( \bar{q}^A = \bar{q}^B = \bar{q}^* \).

Proof: same as Proposition 2.

**Corollary 4** The utility levels reached by workers are the same; that is: \( V_I^1(q^A) = V_I^1(q^B) \).

**Proposition 5** In equilibrium, the optimal tax rates for the groups are a function of the numerosity of groups, preferences of groups, preference of individuals, density of group, total amount of hours, bargaining power of the labour union and fall-back position of the labour union. That is: \( \tau_{lt}^{\tau-1} = \tau_{lt}^{\tau-1}(n^\tau-1, n^\tau, \chi^\tau, \psi^\tau, s^\tau, u, H, x, \lambda) \) and \( \tau_{lt}^{\tau*} = \tau_{lt}^{\tau*}(n^\tau-1, n^\tau, \chi^\tau, \psi^\tau, s^\tau, u, H, x, \lambda) \). The sign of the first derivatives depend on the parameter of the model.

**Proposition 6** Ceteris paribus, the group with lower preference for work obtains a higher number of leisure hours than the other group.

Proof: It derives by the analysis of (22) and (23), where it is easy to see that if the difference of numerosity and density are the same the group with the lower \( \chi \) obtains a greater number of leisure hours.

**Corollary 5** The group with the lower \( \chi \) is the more single-minded group.

Proof: by the definition of the SMT we know that a group is more single-minded than another group if \( s^I = s(h^I_{lt}) > s^{-I} = s(h^{-I}_{lt}) \) since we have assumed that the density is a monotonically increasing utility function in leisure. Since from the previous Proposition we know that the group with the lower \( \chi \) is the group which obtains the higher number of leisure hours, we have that that group has also a higher density. That is, it is more single-minded. For a sake of exposition, we assume from now on that the more-single minded group is that of the old.

Thus, the equilibrium levels of the transfers between the young and the old are the following:

\[ b^\tau_t = \frac{1 - \sqrt{s^{-1}}}{\alpha n^\tau} \tag{26} \]
\[ b^{-1}_t = \frac{1 - \sqrt{s^{-1}}}{\alpha n^\tau-1} \tag{27} \]
Proposition 7 There exists a social security transfer from the less single-minded group to the more single-minded group.

Proof: From the first order conditions with respect to \( b^\tau_t \) and \( b^{\tau-1}_t \), it is:

\[
\frac{s^\tau-1}{s^\tau} = \frac{1 - \alpha n^\tau b^\tau_t}{1 - \alpha n^\tau b^{\tau-1}_t}.
\]

From Corollary 2, \( s^\tau-1 = s(h^\tau_t - 1) > s^{\tau} = s(h^\tau_t) \) it must be \( 1 - \alpha n^\tau b^\tau_t > 1 - \alpha n^{\tau-1} b^{\tau-1}_t \) for the workers. Since \( \alpha n^{\tau-1} b^{\tau-1}_t > \alpha n^\tau b^\tau_t \), it can be seen the the group with the higher level of density obtains the higher transfers. Given the budget constraint:

\[
n^{\tau-1}b^{\tau-1}_t = -n^{\tau}b^{\tau}_t,
\]

taking into account the equilibrium conditions \( \frac{s^\tau-1}{s^\tau} = \frac{1 - \alpha n^\tau b^\tau_t}{1 - \alpha n^{\tau-1} b^{\tau-1}_t} \), it is

\[
\frac{s^\tau-1}{s^\tau} = \frac{1 - \alpha n^\tau b^\tau_t}{\alpha n^\tau b^\tau_t + 1} = (1 - \alpha n^\tau b^\tau_t)^2.
\]

Solving with respect to \( b^\tau_t \) and \( b^{\tau-1}_t \) we obtain the optimal values.

Notice that when densities of both groups are the same, transfers are equal to zero; that is if \( s^\tau-1 = s^\tau \), then \( b^{\tau-1} = b^\tau = 0 \).

Proposition 8 A transfer in the I-th group decreases with an increase in the amount of resources distorted by government and with an increase in the density of the other group, whilst it increases with an increase in the density of his own group.

Proof: Calculating the total differentials, we obtain \( \frac{\partial b^I_t}{\partial \alpha} < 0 \), \( \frac{\partial b^I_t}{\partial s^I} > 0 \), \( \frac{\partial b^I_t}{\partial n^I} < 0 \).

Proposition 7 spouses the SMT: the higher the homogeneity among a group, the higher the power of influence of that group on the Government and the higher the transfer that the group gets.

Proposition 9 The optimal Lagrange multipliers assume the following values:

\[
\lambda^* = \sqrt{s^\tau-1 s^\tau} \tag{28}
\]

Proof: \( \lambda = \frac{n^{\tau-1}s^\tau-1}{n^{\tau-1} - n^\tau a b^\tau_t} = \frac{s^\tau-1}{1 - n^\tau a b^\tau_t} = \frac{s^\tau}{1 - n^\tau a b^{\tau-1}_t} \)

Substituting the optimal intergenerational transfers value we obtain: \( \lambda^* \).

Notice that Proposition 9 respect the Lindbeck and Weibull result, according to that the Lagrange multiplier represents the increase in the probability of winning for a candidate, if it had an additional dollar available to spend on redistribution and must be always greater than zero.

4 An Intuitive Explanation of the decline in the Labour Unions’ membership

The decline in union membership around the world is a well-documented reality (ILO World Labour Report 1997-1998 [30]) but less known is the fact
that the decline is due to a rise in the percentage of employees who have never become members. Analysing the British situation, Bryson and Gomez [5] discovered that between 1983 and 2001, the percentage of employees who had never become a member of a labour union had risen by over two-thirds, from 28 to 48 per cent and that over the same period membership fell by a third, from 49 to 31 per cent. As a consequence, it seems quite natural to ask why the young have abandoned the willingness to joint unions. I will use the basic model I used in Section 2 to provide a possible explanation to the issue. For doing this I will only make some further necessary assumptions. Suppose that a third social group (generation) is introduced in the previous framework, say the teen-agers (T) born at time t. Thus, the space of generations is now \( G = \{T-2, T-1, T\} \). For the sake of simplicity, I assume that the teen-agers are all those who are not still in the labour market. Thus, the labour union membership is composed by the group of the young plus the group of the old. At every period of time \( t \), all the teen-agers become young and enter into the labour market (and join the labour union), all the young become old and all the old die. Joining the union has now a cost \( t^i_\tau \) which is different for every individual and it is uniformly distributed among the two cohorts. A young decides to join the union if his IUF is greater than the cost he has to bear; that is if

\[
IUF^\tau_\tau > t^i_\tau \tag{29}
\]

At first, let us assume that the numerosity of groups is the same; that is \( n^\tau = n^{\tau-1} = n^{\tau-2} \). This means the the flow of new labour union members is constant since the number of each social groups is exactly replaced by the group of the next generation. Suppose also that the density of the young and the old is the same and that (31) holds for every \( i \). These assumptions lead to two conclusions: every worker join the union and numerosity and density do not determine the optimal wage and hours of groups when the labour union has to split the optimal amount of hours and wage; thus, the optimal values are driven exclusively by the preferences of groups which lead to a desirable solution for both of them. Suppose instead that, at time \( t \) due to a demographic crisis, the numerosity of teen-agers decline over time at a steady rate equal to \( d \); that is \( n^\tau_\tau = n^{\tau-1}_\tau (1 - d), n^{\tau+1}_\tau = n^\tau_\tau (1 - d) = n^{\tau-1}_\tau (1 - d)^2, \ldots \). The group of teen-agers is now less numerous of the other two groups. At time \( t + 1 \) the teen-agers become young and the young become old. But now also \( n^{\tau-1} < n^{\tau-2} \) (whilst the density always remain equal across the cohorts). The power of the old is increased which drive the choice of the labour unions toward the elder needs. The expected IUF of the young workers decreases. At this point, suppose that, for some \( q_{t+1} \) young workers, (31) does not hold anymore, that is \( IUF^\tau_\tau < t^i_\tau \). This time, the young who have the higher costs to join the union decide not to join. The numerosity of the young members at time \( t + 1 \) will be now equal to \( n^\tau - q_{t+1} \). But this again reinforce the power of the old who obtain an even more favorable
policy. The expected IUF of the young decreases again and this leads a
greater amount of young workers not to join the union at \( t + 2 \) and so on
and so forth \textit{ad infinitum}. Assuming an infinite horizon the conclusion is
that, since \( q_{t+1} \leq q_{t+2} \leq \ldots \), the fraction of young members and the total
membership of the labour union will decline and the elder component will
obtain a relative higher IUF. The red area of figure 5 shows the percentage
of young workers which decide not to join the unions due to the high cost
they have to bear to join the union. The result of this process is that fewer
and fewer teen-agers will decide to join the union, the power of the old will
increase and the size of labour unions will become smaller and smaller.

5 Empirical Evidence from the Workplace Employment Relations Survey 2004

In this section I provide some empirical evidence about the bargaining be-
tween labour unions and firms in the United Kingdom. The source of data
is the Workplace Employment Relations Survey (WERS) 2004 (Department
of Trade and Industry, 2005 [13]), which is, to the best of my knowledge the
most comprehensive, existing dataset about industrial relations. A review
of the first findings was released in 2005 [26].

5.1 The Database

The WERS 2004 is a representation of the state of employment relations
and working life inside the British workplaces. Data was collected thanks to
face-to-face interviews with around 3,200 managers and almost 1,000 work-
ners representatives, whilst 20,000 employees answered to a questionnaire.
The scope of the survey covers 700,000 workplaces and 22.5 million employ-
ees which represents the 91 per cent of total employees. The structure of
the survey is made by both a cross-section and a panel component. The for-
mer component collects interviews with the senior manager responsible for
employment relations, whilst the latter was conducted using a random sub-
sample of workplaces that had participated in the 1998 WERS edition and
data was collected via interviews. The WERS 2004 is able to provide use-
ful information about many areas of study: Governance, Partnership, Skills,
Job Satisfaction and Stress, Performance and Technology, Small Workplaces
and Work Representation, Consultation and Communication. I will only
focus on the last issue, as it involves the role of labour unions and the bar-
gaining issues. In this area the WERS 2004 collected data on the union
membership density, the representative voice, the roles and activities of em-
ployee representatives and the role of direct communication. In evaluating
the main findings I will only focus on the representative voice.
5.2 Main Findings

Among the 36 per cent of workplaces which said to have members of labour unions, the WERS 2004 gives the possibility to analyse in depth how the relations between workers and management take place. First of all, as Table 1 shows, the attitude of management toward labour unions is most of the time positive: the 40.62 per cent of the managers said to be in favor of labour unions, the 47.33 per cent to be neutral and only the 8.96 to be not in favor. This may already be a significant indicator of the willingness of the management to negotiate and to assume a collaborative behaviour toward the labour unions. Furthermore, Table 2 shows that the 75.41 per cent of the labour unions deals with different issues such as the pay or conditions, whilst only the 11.89 per cent only deals with one issue suggesting the involvement of labour unions to different issues are one of the most clear reality; table 3 shows also that the negotiation takes place jointly between the management and labour unions in the 59.77 per cent of cases. Tables 4-15 inquire more in details the bargaining process over single issues, which are: rate of pay, hours of work, holiday entitlements, pension entitlements, recruitment and selections of employees, training of employees, grievance procedures, disciplinary procedures, staffing plans, equal opportunities, health and safety and performance appraisal. The second column of each table indicates the number of workplaces, the third the percentage of workplaces with respect to the number of workplaces which have said to have at least a labour union, and the fourth the percentage of the workplaces with respect to the number of total workplaces. As we expected, labour unions are involved in bargain over the wage, and this happens in the 57.11 per cent of cases, whilst it is consulted in the 10.33 per cent, only informed in the 12.51 per cent, and neither informed in the 18.10 per cent. The most striking result refers to the high percentage of labour unions which bargain over the hours of work; this happens in the 47.63 per cent of cases which is almost the same percentage which refers to the rate of pay; furthermore, over this issue, the 19.11 per cent of the unions are consulted, the 11.34 per cent informed, and 19.50 per cent non-informed. The same happens for the negotiation over holiday entitlements (in the 46.39 per cent of cases labour unions bargain, in the 13.21 per cent are consulted, in the 17.64 informed, and in the 20.36 non-informed). Once we analyse the results of the other variables, we may see that the degree of involvement of labour unions into the bargaining process strongly decreases. For instance, only in one-quarter of workplaces which have a labour union a bargaining over pension entitlements takes place; this percentage dramatically reduces if we consider the recruitment and selection of employees (the negotiation take place only in the 5.98 per cent of cases), the training of employees (6.60 per cent), grievance procedures (28.36 per cent), the disciplinary procedures (28.75 per cent), the staffing plans (6.68 per cent), the equal opportunities (11.34 per
cent), the health and safety (12.82 per cent) and the performance appraisal (10.88 per cent). From the overall analysis of this statistics, we can conclude that the labour unions are involved in negotiating the pay and the hours of work, whilst they do not seem to be involved in any issues which refer to employment, composition of workforce, and other worker benefits, such as pensions. Thus, the empirical evidence which refers to labour relations in the U.K. utterly sustain the Pencavel model of efficient bargaining over pay and hours work, whilst the hypothesis by the other three typical models of bargaining seem to be rejected, since the Monopoly Union model and the Right-to-Management model assume that the two parts exclusively bargain over wage, whilst the Efficient Bargaining model assume that they bargain over the level of employment and not on the hours of work; instead, we have seen that the WERS 2004 has shown that this does not happen in reality.

5.3 Pareto Efficiency?

At the time the survey was made, the U.K.’s economy was believed to run very close to its potential rate of growth, with a very low unemployment rate around the 5 per cent. After many years where Thatcher’s Government tried to weaken labour unions, with the victory of Blair’s Labour Party many reforms were passed in order to improve work relations in U.K.’s workplaces. The most remarkable example of law passed by the labourist Government is represented by the Employment Relations Act, passed in 1999 and based on the 1998 Fairness at Work White Paper, whose goal was to ensure that the UKs system of employment law was based on three main principles: fairness, flexibility and partnership. The labour unions were formally recognised, through the statutory recognition, as key partners at the workplace, working with employers for greater productivity and better work practices. According to a review conducted by the Department of Trade and Industry in 2003 which aimed to assess the effects the Act had brought in U.K. workplaces, “a thousand deals for recognition have been voluntarily agreed between employers and unions since 1998. Over 700 of these have been reached since the statutory procedure was introduced. (...) Where recognition has been awarded (or where it has been voluntarily agreed without using the statutory procedure), the employers and unions concerned appear to be moving forward into the bargaining process and establishing normal working relationships.” My model, the empirical evidence the WERS 2004 has brought, and the Review of the Employment Relations Act together demonstrate how it is possible to look at an economy where not necessarily the involvement of workers representatives is seen in a negative way. I am not only referring to recognized labour unions, since sometimes these institutions have more aggressive attitudes toward the bargaining issues and represent the more conflictual reality in industrial relations, but also to less institutionalized form of workers’ associations. As demonstrated by the
U.K. experience, a political and legislative environment which encourage the dialogue between employers and employees and where it is possible to negotiate over conditions and workers’ needs may increase the flexibility of the labour market and keep the economy growing at a sound pace.

6 Conclusions

In this model, I applied the SMT to the labour market, where wages are endogenously determined according to an Efficient Bargaining model over wage and hours of work. I analysed a society composed by two groups of workers (the old and the young) which belong to a labour union and a firm. I assumed also that the preferences of the old for leisure differ from the preferences of the young. Under these conditions, I demonstrated that in a PVM where social groups have to split the amount of hours worked, the optimal quota of resources that any social group obtains is represented by a weighted average which is a function of the Numerosity, Density and Preferences of the group, which I called $NDP – WA_H$. Since the single-mindedness of a group, which represents a proxy for the political power of that group, is captured by the density function which is a monotonically increasing function with respect to leisure, I conclude that the group with higher preferences for leisure (presumably the old) has a great power of influence onto politicians. This power enables them to get positive benefits in equilibrium, whose burden is entirely carried by the other group (presumably the young). Thus, with respect to the previous work, this study consider the mechanisms of labour unions, seen as an institution representing the interests of different social groups. Again, according to the SMT, the greater the ability of a single group to be oriented toward the minimum number of issues, the higher the probability that this group achieves its goals.

The empirical evidence provided by the WERS 2004 should definitely bring new support to models which too often were not sufficiently considered in the literature as they would deserve, such as Pencavel’s, and discourage the use of these models which only consider the bargaining over wage which seem to at odds with what happens in the real world.

Nevertheless, this work does not consider some aspects which would deserve to be analysed. First of all, it would be interesting to consider more in details, the mechanisms which are undertaken by unions to take their decisions (i.e. voting process, elections and so forth). Furthermore, since an imperfect labour market entails unemployment in equilibrium, it would be interesting to add some new social groups (i.e. the unemployed) to analyse the impact of labour unions on excluded workers. Finally, strange enough, the relation between hours of work and employment has not been studied in a systematic way.

I hope this suggestions will find a place in future researches.
7 Appendix

In this Appendix I provide a complete resolution to the candidates' problem. The two candidates face exactly the same optimization problem; they maximize their share of votes or, equivalently, the probability of winning. The resolution is made for candidate A, but it also holds for candidate B.

\[
\max \quad \pi^A = \frac{1}{2} + \frac{h}{s} \sum_{t=(T-1,T)} n^t s^t \left[ V^i(q^4) - V^i(q^D) \right]
\]

subject to:

\[
T^t = n^{-1} \tau^t w^{t-1} (\overline{H} - h^{t-1}) \tag{30}
\]

\[
T^t = n^{-1} \tau^t w^{t-1} (\overline{H} - h^{t-1}) \tag{31}
\]

\[
n^{-1} b^t + n^{-1} b^t + \alpha [n^{-1} b^t - n^t b^*_t] = 0 \tag{32}
\]

where: \( s^t = s^t (t, (\tau^t, w^t)) \)

Substituting \( w^t \) and \( h^{t-1} \) into equation (30), (31) and (32) I obtain:

\[
T^t = n^{-1} \tau^t w^{t-1} (\overline{H} - h^{t-1}) \tag{33}
\]

\[
n^{-1} b^t + n^{-1} b^t + \alpha [n^{-1} b^t - n^t b^*_t] = 0 \tag{34}
\]

and finally I may write the Lagrangian function:

\[
L = \frac{1}{2} + \frac{h}{s} \sum_{t=(T-1,T)} n^t s^t \left[ V^i(q^4) - V^i(q^D) \right] + \lambda \left( n^{-1} b^t - n^t b^*_t + \alpha [n^{-1} b^t - n^t b^*_t] \right)
\]

I write the First Order Conditions which can be seen as a modified version of the Lindbeck and Weibull First Order Conditions in a PVM modified for the exogenous density:

1. \( \frac{\partial L}{\partial \tau^t_L} = n^{-1} \left( \frac{\partial V^i(q^4)}{\partial \tau^t_L} - \frac{\partial V^i(q^D)}{\partial \tau^t_L} \right) = 0 \)

2. \( \frac{\partial L}{\partial \tau^t_L} = n^{-1} s^t \left( \frac{\partial V^i(q^4)}{\partial \tau^t_L} + s^t \frac{\partial V^i(q^4)}{\partial \tau^t_L} \right) = 0 \)

3. \( \frac{\partial L}{\partial \tau^t_L} = n^{-1} s^t + \mu [n^{-1} - n^t \alpha b^{t-1}] \)

4. \( \frac{\partial L}{\partial \tau^t_L} = n^t s^t + \mu [n^{-1} - n^{-1} \alpha b^{t-1}] \)

5. \( n^{-1} b^t - n^t b^*_t + \alpha [n^{-1} b^t - n^t b^*_t] = 0 \)

By Corollary 4 we know that in equilibrium \( V^i(q^4) = V^i(q^D) \) such that we obtain:

1. \( \frac{\partial L}{\partial \tau^t_L} = n^{-1} \left( s^t \frac{\partial V^i(q^4)}{\partial \tau^t_L} \right) = 0 \)

2. \( \frac{\partial L}{\partial \tau^t_L} = n^{-1} s^t \left( \frac{\partial V^i(q^4)}{\partial \tau^t_L} \right) = 0 \)

3. \( \frac{\partial L}{\partial \tau^t_L} = n^{-1} s^t - \mu [n^{-1} - n^{-1} \alpha b^{t-1}] \)

4. \( \frac{\partial L}{\partial \tau^t_L} = n^t s^t + \mu [n^{-1} - n^{-1} \alpha b^{t-1}] \)

5. \( n^{-1} b^t - n^t b^*_t + \alpha [n^{-1} b^t - n^t b^*_t] = 0 \)

and

1. \( \frac{\partial L}{\partial \tau^t_L} = \frac{\partial V^i(q^4)}{\partial \tau^t_L} = 0 \)

2. \( \frac{\partial L}{\partial \tau^t_L} = \frac{\partial V^i(q^4)}{\partial \tau^t_L} = 0 \)

3. \( \frac{\partial L}{\partial \tau^t_L} = s^t = \mu (n^{-1} \alpha b^{t-1}) \)

4. \( \frac{\partial L}{\partial \tau^t_L} = s^t = \mu (n^{-1} \alpha b^{t-1}) \)
\[(5)n^{\tau-1}b^{\tau-1} + n^\tau b^\tau + \alpha |n^{\tau-1}b^{\tau-1}||n^\tau b^\tau| = 0\]

Solving the equation we obtain the optimal tax rates:

\[\tau^{\tau-1*}_{Lt} = \tau^{\tau-1}_{Lt}(n^{\tau-1}, n^\tau, \chi^{\tau-1}, \chi^\tau, \psi^{\tau-1}, s^{\tau-1,u}, s^{\tau,u}, H, x, \lambda)\]

and

\[\tau^{\tau*}_{Lt} = \tau^{\tau}_{Lt}(n^{\tau-1}, n^\tau, \chi^{\tau-1}, \chi^\tau, \psi^{\tau-1}, s^{\tau-1,u}, s^{\tau,u}, H, x, \lambda).\]
References


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Table 1: How would you describe management's general attitude towards trade union membership among employees at this establishment. Is management...

<table>
<thead>
<tr>
<th>Attitude</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>In favour</td>
<td>381</td>
<td>40.62</td>
</tr>
<tr>
<td>Not in favour</td>
<td>84</td>
<td>8.96</td>
</tr>
<tr>
<td>Neutral</td>
<td>444</td>
<td>47.33</td>
</tr>
<tr>
<td>Other answer</td>
<td>29</td>
<td>3.09</td>
</tr>
<tr>
<td>Total</td>
<td>938</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2: Does the committee deal with a range of issues, or with a single topic such as health and safety?

<table>
<thead>
<tr>
<th>Approach</th>
<th>Frequency</th>
<th>Percentage (1)</th>
<th>Percentage (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse to answer</td>
<td>9</td>
<td>1.84</td>
<td>0.96</td>
</tr>
<tr>
<td>Range of issues</td>
<td>368</td>
<td>75.41</td>
<td>39.23</td>
</tr>
<tr>
<td>Single topic(s)</td>
<td>58</td>
<td>11.89</td>
<td>6.18</td>
</tr>
<tr>
<td>Both</td>
<td>53</td>
<td>10.86</td>
<td>5.65</td>
</tr>
<tr>
<td>Total</td>
<td>488</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases

Table 3: Does management negotiate jointly with the recognised unions, or are there separate negotiations?

<table>
<thead>
<tr>
<th>Approach</th>
<th>Frequency</th>
<th>Percentage (1)</th>
<th>Percentage (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse to answer</td>
<td>1</td>
<td>0.28</td>
<td>0.11</td>
</tr>
<tr>
<td>Don't know</td>
<td>16</td>
<td>4.53</td>
<td>1.71</td>
</tr>
<tr>
<td>Jointly (3)</td>
<td>211</td>
<td>59.77</td>
<td>22.49</td>
</tr>
<tr>
<td>Separately (4)</td>
<td>82</td>
<td>23.23</td>
<td>8.74</td>
</tr>
<tr>
<td>At least two (5)</td>
<td>43</td>
<td>12.18</td>
<td>4.58</td>
</tr>
<tr>
<td>Total</td>
<td>353</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases  
(3) all recognised unions negotiate over pay as one unit,  
(4) each recognised union negotiates independently over pay  
(5) At least two recognised unions jointly negotiate over...
<table>
<thead>
<tr>
<th>Table 4: Does management normally negotiate, consult, inform or not inform unions about the rate of pay?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Frequency</strong></td>
</tr>
<tr>
<td>Refuse to answer</td>
</tr>
<tr>
<td>Don't know</td>
</tr>
<tr>
<td>Negociates</td>
</tr>
<tr>
<td>Consults</td>
</tr>
<tr>
<td>Informs</td>
</tr>
<tr>
<td>Not inform</td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases

<table>
<thead>
<tr>
<th>Table 5: Does management normally negotiate, consult, inform or not inform unions about hours of work?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Frequency</strong></td>
</tr>
<tr>
<td>Refuse to answer</td>
</tr>
<tr>
<td>Don't know</td>
</tr>
<tr>
<td>Negociates</td>
</tr>
<tr>
<td>Consults</td>
</tr>
<tr>
<td>Informs</td>
</tr>
<tr>
<td>Not inform</td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases

<table>
<thead>
<tr>
<th>Table 6: Does management normally negotiate, consult, inform or not inform unions about holiday entitlements?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Frequency</strong></td>
</tr>
<tr>
<td>Refuse to answer</td>
</tr>
<tr>
<td>Don't know</td>
</tr>
<tr>
<td>Negociates</td>
</tr>
<tr>
<td>Consults</td>
</tr>
<tr>
<td>Informs</td>
</tr>
<tr>
<td>Not inform</td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases
Table 7: Does management normally negotiate, consult, inform or not inform unions about pension entitlements?

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percentage (1)</th>
<th>Percentage (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse to answer</td>
<td>7</td>
<td>0.54</td>
<td>0.31</td>
</tr>
<tr>
<td>Don't know</td>
<td>49</td>
<td>3.81</td>
<td>2.14</td>
</tr>
<tr>
<td>Negotiates</td>
<td>328</td>
<td>25.49</td>
<td>14.29</td>
</tr>
<tr>
<td>Consults</td>
<td>220</td>
<td>17.09</td>
<td>9.59</td>
</tr>
<tr>
<td>Informs</td>
<td>397</td>
<td>30.85</td>
<td>17.30</td>
</tr>
<tr>
<td>Not inform</td>
<td>286</td>
<td>22.22</td>
<td>12.46</td>
</tr>
<tr>
<td>Total</td>
<td>1,287</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases

Table 8: Does management normally negotiate, consult, inform or not inform unions about recruitment or selection of employees?

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percentage (1)</th>
<th>Percentage (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse to answer</td>
<td>7</td>
<td>0.54</td>
<td>0.31</td>
</tr>
<tr>
<td>Don't know</td>
<td>21</td>
<td>1.63</td>
<td>0.92</td>
</tr>
<tr>
<td>Negotiates</td>
<td>77</td>
<td>5.98</td>
<td>3.36</td>
</tr>
<tr>
<td>Consults</td>
<td>302</td>
<td>23.47</td>
<td>13.16</td>
</tr>
<tr>
<td>Informs</td>
<td>360</td>
<td>27.97</td>
<td>15.69</td>
</tr>
<tr>
<td>Not inform</td>
<td>520</td>
<td>40.40</td>
<td>22.66</td>
</tr>
<tr>
<td>Total</td>
<td>1,287</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases

Table 9: Does management normally negotiate, consult, inform or not inform unions about training of employees?

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percentage (1)</th>
<th>Percentage (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse to answer</td>
<td>7</td>
<td>0.54</td>
<td>0.31</td>
</tr>
<tr>
<td>Don't know</td>
<td>18</td>
<td>1.40</td>
<td>0.78</td>
</tr>
<tr>
<td>Negotiates</td>
<td>85</td>
<td>6.60</td>
<td>3.70</td>
</tr>
<tr>
<td>Consults</td>
<td>394</td>
<td>30.61</td>
<td>17.17</td>
</tr>
<tr>
<td>Informs</td>
<td>359</td>
<td>27.89</td>
<td>15.64</td>
</tr>
<tr>
<td>Not inform</td>
<td>424</td>
<td>32.94</td>
<td>18.47</td>
</tr>
<tr>
<td>Total</td>
<td>1,287</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases
**Table 10:** Does management normally negotiate, consult, inform or not inform unions about grievance procedures?

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percentage (1)</th>
<th>Percentage (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse to answer</td>
<td>6</td>
<td>0.47</td>
<td>0.26</td>
</tr>
<tr>
<td>Don't know</td>
<td>20</td>
<td>1.55</td>
<td>0.87</td>
</tr>
<tr>
<td>Negotiates</td>
<td>365</td>
<td>28.36</td>
<td>15.90</td>
</tr>
<tr>
<td>Consults</td>
<td>499</td>
<td>38.77</td>
<td>21.74</td>
</tr>
<tr>
<td>Informs</td>
<td>188</td>
<td>14.61</td>
<td>8.19</td>
</tr>
<tr>
<td>Not inform</td>
<td>209</td>
<td>16.24</td>
<td>9.11</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1,287</strong></td>
<td><strong>100</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases

**Table 11:** Does management normally negotiate, consult, inform or not inform unions about disciplinary procedures?

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percentage (1)</th>
<th>Percentage (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse to answer</td>
<td>6</td>
<td>0.47</td>
<td>0.26</td>
</tr>
<tr>
<td>Don't know</td>
<td>22</td>
<td>1.71</td>
<td>0.96</td>
</tr>
<tr>
<td>Negotiates</td>
<td>370</td>
<td>28.75</td>
<td>16.12</td>
</tr>
<tr>
<td>Consults</td>
<td>490</td>
<td>38.07</td>
<td>21.35</td>
</tr>
<tr>
<td>Informs</td>
<td>195</td>
<td>15.15</td>
<td>8.50</td>
</tr>
<tr>
<td>Not inform</td>
<td>204</td>
<td>15.85</td>
<td>8.89</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1,287</strong></td>
<td><strong>100</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases

**Table 12:** Does management normally negotiate, consult, inform or not inform unions about staffing plans?

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percentage (1)</th>
<th>Percentage (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse to answer</td>
<td>7</td>
<td>0.54</td>
<td>0.31</td>
</tr>
<tr>
<td>Don't know</td>
<td>27</td>
<td>2.10</td>
<td>1.18</td>
</tr>
<tr>
<td>Negotiates</td>
<td>86</td>
<td>6.68</td>
<td>3.75</td>
</tr>
<tr>
<td>Consults</td>
<td>466</td>
<td>36.21</td>
<td>20.31</td>
</tr>
<tr>
<td>Informs</td>
<td>347</td>
<td>26.96</td>
<td>15.12</td>
</tr>
<tr>
<td>Not inform</td>
<td>354</td>
<td>27.51</td>
<td>15.42</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1,287</strong></td>
<td><strong>100</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases
Table 13: Does management normally negotiate, consult, inform or not inform unions about equal opportunities?

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percentage (1)</th>
<th>Percentage (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse to answer</td>
<td>7</td>
<td>0.54</td>
<td>0.31</td>
</tr>
<tr>
<td>Don't know</td>
<td>27</td>
<td>2.10</td>
<td>1.18</td>
</tr>
<tr>
<td>Negotiates</td>
<td>146</td>
<td>11.34</td>
<td>6.36</td>
</tr>
<tr>
<td>Consults</td>
<td>561</td>
<td>43.59</td>
<td>24.44</td>
</tr>
<tr>
<td>Informs</td>
<td>270</td>
<td>20.98</td>
<td>11.76</td>
</tr>
<tr>
<td>Not inform</td>
<td>276</td>
<td>21.45</td>
<td>12.03</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1,287</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases

Table 14: Does management normally negotiate, consult, inform or not inform unions about health and safety?

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percentage (1)</th>
<th>Percentage (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse to answer</td>
<td>7</td>
<td>0.54</td>
<td>0.31</td>
</tr>
<tr>
<td>Don't know</td>
<td>22</td>
<td>1.71</td>
<td>0.96</td>
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<tr>
<td>Negotiates</td>
<td>165</td>
<td>12.82</td>
<td>7.19</td>
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<tr>
<td>Consults</td>
<td>679</td>
<td>52.76</td>
<td>29.59</td>
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<tr>
<td>Informs</td>
<td>200</td>
<td>15.54</td>
<td>8.71</td>
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<tr>
<td>Not inform</td>
<td>214</td>
<td>16.63</td>
<td>9.32</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1,287</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases

Table 15: Does management normally negotiate, consult, inform or not inform unions about performance appraisal?

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percentage (1)</th>
<th>Percentage (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refuse to answer</td>
<td>7</td>
<td>0.54</td>
<td>0.31</td>
</tr>
<tr>
<td>Don't know</td>
<td>30</td>
<td>2.33</td>
<td>1.31</td>
</tr>
<tr>
<td>Negotiates</td>
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<td>10.88</td>
<td>6.10</td>
</tr>
<tr>
<td>Consults</td>
<td>458</td>
<td>35.59</td>
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</tr>
<tr>
<td>Informs</td>
<td>256</td>
<td>19.89</td>
<td>11.15</td>
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<tr>
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<td>396</td>
<td>30.77</td>
<td>17.25</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1,287</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

(1) of the applicable cases  
(2) of both the applicable and non-applicable cases
FIGURE 3 – The optimal policy vector in the labour union PVM

FIGURE 4 – Effects on welfare
FIGURE 5 – The uniform distribution of the cost to join the labour union at time $t+1$ a) and $t+2$ b)