Education and selective vouchers

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Abstract

The literature on vouchers often concludes that a voucher-based system cannot be the outcome of a majority vote. This paper shows that it is possible to propose selective vouchers (of exogenous value) such that the majority of voters are in favour of selective vouchers. As long as the introduction of vouchers does not undermine the existence of public schools, introducing selective vouchers induces a Pareto improvement. Some agents use vouchers in equilibrium to buy private education, while the poorest agents continue attending public schools and enjoy an increase in per-capita expenditure.

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1 Introduction

Most Western countries publicly provide some private goods, such as education. These goods are financed through taxes and offered to all citizens at a lower than competitive price and possibly for free. Households can choose between the public and private supply.

For some of these goods, consumption of the publicly provided good can be implemented by private provision, while sometimes the two provisions are mutually exclusive. It is common in the literature to assume that education belongs to this second group of goods. In a first-best world, consumers choose the quality of instruction; this is a priori different for each agent. One drawback of the public school system is that, for equity reasons, all students at a particular public school (possibly also in a given area/region) receive the same service regardless of their preferences.

The public service can be congested: resources to finance education are limited as well as the variety in the educational offer.[1] Offering students incentives to move to the private sector reduces congestion and can relax public constraint and increase the decisional space of agents, but the political support for a high-quality public service might decrease.[2] The introduction of vouchers is one possible way to incentivise people to attend private schools.[3] Vouchers have been a topic of discussion in many Western countries in the last two decades and have been introduced in some of these countries (Chile

[1]Clearly, public schools are not always congested, but several empirical studies show that this is the case in several countries (e.g. Duncombe, Miner, and Ruggiero (1995), Kokkelenberg, Dillon, and Christy (2008), Ruggiero (1999), Smet (2001) and Wössmann and West (2006)). Other studies, e.g. Lenton (2008), instead find evidence of economies of scale and of scope in producing education. Often this result is obtained when transportation costs are not taken into account.

[2]If the quality of a public school increases, the intensity of its support by the people attending it also increases, but here we consider support in terms of number of voters. Those moving to the private sector do not have a direct interest in a high-quality public school.

[3]Vouchers are a sort of cheque exploitable only for the purchase of private education. They can be "universal" (everybody is entitled to receive them) or "selective" (only a subset of the population is eligible).

was one of the first countries to do so, while the Czech Republic one of the last). The introduction of vouchers has often encountered strong ideological opposition.[4] The aim of this work is to investigate (through a political economy model) whether rational, welfare-maximising citizens should agree on the introduction of vouchers. A voting model seems appropriate to forecast how a change in the level of taxation and the use of instruments such as vouchers would be perceived by voters.[5]

A broad part of the literature concludes that the introduction of vouchers do not improve welfare or that it encounters the opposition of the majority of citizens (unless additional concerns, such as peer effects, are introduced). On the contrary, I show that, at least under some combination of parameters, their introduction is always beneficial to the majority of society, for which reason an office-motivated politician should be in favour of their instauration. I also show that in many cases their introduction can lead to a Pareto improvement.

Even restricting attention to the political economy of education, the literature is extensive.[6] Part of the recent positive literature on education (including my contribution) builds on the notable paper by Epple and Romano

[4] Concerning the applied and empirical literature on education and on the introduction of vouchers, see, for instance, Card and Krueger (1992), Filer and Munich (2001), McEwan and Carnoy (2002) or Chakrabarti (2008).

[5] I consider people voting directly on the tax financing education, while most modern Western democracies are indirect. Even if a model of indirect political representation seems more realistic, I consider it appropriate to use a standard voting model mainly for one reason: even if citizens vote for a party (and not on policies), parties are elected by citizens according to their political programme, and, moreover, elected officials need the support of voters throughout their entire term and should make decisions in agreement with the majority of citizens. Of course the role of lobbies, pressure groups and citizens might vary from one country to another; still, I believe that concerning public subsidisation to education (normally a subject quite followed by the media), a classic voting model is appropriate. See also Budge (2006).

[6] Among the first attempts, Stiglitz (1974) is one of the most well known. Other important contributions come from Glomm and Ravikumar (1992), Blomquist and Christiansen (1998), Blomquist and Christiansen (1999), Chen and West (2000), De Fraja (2002), Gradstein and Justman (2002) and Epple, Romano, and Sieg (2006). The Handbook of the Economics of Education (2006), the International handbook on the economics of education (2004) and Gradstein, Justman, and Meier (2005) provide surveys of a consistent part of the recent literature on the field.

(1996). Education is considered mainly as a consumption good,[7] agents, whose type depends on their income, vote on the tax to finance public schools; differences in consumption are in terms of quality rather than quantity.

Epple and Romano (1996) show that a single crossing condition has to be imposed to ensure the existence of an equilibrium and identify two alternative conditions: Slope Rising in Income (SRI) and Slope Decreasing in Income (SDI).[8] Their main result (known as "ends against the middle") is that the richest and poorest households push to reduce the tax, while the middle class does the opposite. Vouchers are not considered. Under SDI, they conclude that the median voter is always decisive and that the poorest half of society forms a coalition facing the richest half.

Chen and West (2000) carry out a positive analysis of the school system, using the same model as Epple and Romano (1996). Their aim is to compare systems with universal, selective and no vouchers under SDI. The upper threshold for receiving selective vouchers is the median income, and the voucher value is equal to the (constant) marginal cost of education. The article concludes that the majority always prefer the no-voucher model to the universal one, while the *decisive* voter is indifferent between the selective and the no-voucher frameworks and there are no welfare differences.

Epple and Romano (1998) consider a universal vouchers model with students differing in income and ability. They conclude that the introduction of vouchers is supported by a majority of voters and followed by a fall in congestion; all their results rely on the presence of peer effects. The best students attend private schools together with the richest ones. Private schools are attended by either rich or skilled (or both) students. Only a minority of neither-rich-nor-skilled students remain in public schools, where the quality drops along with students' utility. The authors also develop a computational model, calibrated to existing empirical evidence.

Similarly to Chen and West (2000), I consider the introduction of selective vouchers as a possible way to reduce congestion (by reducing the price of

[7]See Dur and Glazer (2008).

[8]For more details on these conditions, see footnote 11 and also page 12.

private education, vouchers might allow some voters to consume it) and to increase the quality level in the public sector.[9] The differences in results with respect to Chen and West (2000) come from the attributes of the selective voucher: more people are entitled to use them and the value of vouchers is equal to the average cost of public school students. The market structure and the cost function in this model are similar to Epple and Romano (1998).

Agents vote on the tax to finance public school. My model is more general than that of Chen and West (2000): I do not require the cost of public and private education to be the same, nor do I require the marginal cost of a student to be constant. As in Chen and West (2000), I do not consider peer effects,[10] and I concentrate on the SDI condition; most results are not qualitatively affected by this choice and some of them would even be strengthened assuming SRI.[11] Absent vouchers, my model's results are identical with those

[9]The reduction of congestion is used to introduce the willingness of some agents to attend private school through its direct impact on the perceived quality of instruction. My model's results could be replicated using a model where public and private education are horizontally differentiated. Horizontal differentiation can be explained by the fact that private schools can propose special complementary services (such as religious instruction, flexible timetables, sport activities) that can be of interest to some agents.

[10]This assumption certainly simplifies results, but the main reason for excluding peer effects is different: it is not clear how peer effects operate.

Many (but not all) studies find evidence of positive peer effects among students. For instance, Zimmerman (2003) finds that peer effects "are not large but are statistically significant in many models", but not in all of them, and that the phenomenon is limited to verbal proficiency (but not for mathematical proficiency) and only affects some students of "medium ability". Burke and Sass (2008) find that the impact of peer effect on students is not always significant when accounting for peer effects among professors. McEwan (2003) finds that in Chile peer effects are not significant when adding mothers' level of education to the regression.

On top of that, it is unclear if peer effects have an impact on efficiency or if this is only a matter of redistribution; such as if the average level of instruction in a society increases when the best students are together instead of being split in classes of bad students.

Note that if ability is not correlated with income, there is no reason to suppose that students opting out from the public sector implies a change in the average quality of public school students.

[11]The SDI (Slope Decreasing in Income) monotonicity condition means that agents' preferred tax decreases with income.

Many empirical works have tried to determine which of the two assumptions is more realistic; at present it is still not clear which one is more likely to be so. Among others, in several papers Epple and Romano suggest that, for education, the hypothesis of Slope Rising in Income is more appropriated, while Justman supports the opposite assumption.

in Epple and Romano (1996), which I use as a benchmark. My model shows that in the extreme case, when the share of public school students attracted by vouchers exceeds a given threshold, the public sector collapses, and a minority of the population is worse off. Introducing vouchers always induces a Pareto improvement in the more realistic case in which public education continues to exist.

The paper is divided into four sections: Section 2 describes the basic attributes of the model; Section 3 illustrates the voting outcome without vouchers (benchmark case). Section 4 studies the effects of introducing vouchers, while in Section 5, I analyse the results of the vote over the tax and show under which conditions selective vouchers induce a Pareto improvement. The last Section concludes.

2 The model

I consider a model with two normal goods (the numeraire b and educational services X) whose basic setting is as follows:

1. Public and private schools are mutually exclusive. Subscript P indicates the public sector and R the private sector (e.g. X_P and X_R are respectively the qualities of public and private education).

2. The mass of voters is normalised to one. Each voter has a pupil attending school. Voters type depends solely on income ω (with density function f; always strictly positive on the support $[\omega_{\min}, \omega_{\max}]$). I assume the average income $\overline{\omega} = \int_{\omega_{\min}}^{\omega_{\max}} \omega f(\omega) d\omega$ to be greater than the median one (ω_{med}).[12]

The SDI assumption derives from a substitution effect that prevails on the income effect and vice versa for the SRI assumption. The SRI assumption is more reasonable for countries where the poorest's life conditions are dramatically worse than those of other groups (maybe children do not even attend school so as to work). Thus, education has a small impact on people's utility. On the other hand, the SDI assumption might be more appropriate for countries where poor people are sufficiently rich to be able to profit from education and consider it as an investment.

[12] Given the normalisation of population's size, average and aggregate income coincide.

3. Voters' utility function ($U(X, b)$) is separable and strictly concave in X.[13]

4. To incorporate congestion in the model, the school cost function is convex in the number of students n so that $C(X, n) = F + V(n)X$, with $V'(n) \geq 0$ and $V''(n) \geq 0$.[14] In particular, I assume $V(n) = c_1 n + c_2(n)^2$, where n is the number of students attending the school; thus, the cost function is $C(X, n) = F + \left(c_1 n + c_2(n)^2\right) X$.[15]

5. In the educational market, the public sector is the dominant firm, while the private sector is the competitive fringe. The shape of the cost function is the same for both the public and private sector.

6. Without loss of generality, I consider that only one public institute is present.[16] Each private school student can decide the level of educational quality to purchase. Low barriers to entry ensure that the

[13] This assumption is slightly more restrictive than the one ensuring the single crossing property in Epple and Romano (1996); the subsequent computations are simplified by this assumption, but the results and insights are not affected by it.

[14] Concerning congestion, see also footnote 9. The convexity assumption is controversial. Having an increasing marginal cost is reasonable when we consider the product sold by schools including complementary services (such as school bus, professors' office hours, sport facilities, etc.), buildings (close to each other), etc. because the cost of these services might be non-linear (especially in the case of capacity constraints). Empirical studies, e.g., Duncombe, Miner, and Ruggiero (1995), Epple and Romano (1998), Ruggiero (1999), Smet (2001), Wössmann and West (2006), Epple, Romano, and Sieg (2006) and Kokkelenberg, Dillon, and Christy (2008), seem to support this assumption. Lazear (2001) offers a survey of the literature on how class' size matters and under which condition it is correlated with the quality or cost of education.

[15] This form of the cost function, showing decreasing returns to scale, is supported by several empirical studies, e.g., Epple and Romano (1998), Epple, Romano, and Sieg (2006), Kokkelenberg, Dillon, and Christy (2008) and Duncombe, Miner, and Ruggiero (1995). It is important that the cost function not show increasing returns to scale in order to avoid the complicated scenario in which average cost of public schooling increases if some students opt out. Differences in results under constant and decreasing returns to scale are minor.

[16] This is equivalent to the assumption that all public schools have the same number of students and quality of service. This is possible with perfectly mobile students and the same budget for each institute (arbitrage effect), even with the presence of idiosyncratic heterogeneity (e.g., different average wealth) and peer effects (whose analysis is beyond the scope of this work). Passing from the general cost function $\Psi(X_P, \eta) = S + \left(c_1\eta + \psi\eta^2\right) X_P$ (with η the number of students per institute) with k institutes to this one would be simply a matter of renaming some variables.

number of students in each school always adjusts to what is the most efficient amount (i.e. for each firm i, $n_i = \arg\min(\frac{C(X,n_i)}{n_i})$). The quality of one unit of private education X_R is defined in order to normalise the private sector's price to one.[17]

7. Public education is financed via a proportional income tax t paid by all citizens and chosen through majority voting. Without loss of generality, I suppose that the government's budget constraint requires balancing only ordinary (variable) costs and the proportional income tax proceeds.[18]

8. I assume that tax proceeds are first used to finance vouchers. [19] The remaining resources are equally shared among public school students (thus, all students attending public school receive the same quality of education X_P).

9. The value v of vouchers and agents eligible to use them are exogenously chosen. With n_v denoting the number of people using vouchers in equilibrium, the public cost of vouchers is $n_v v$.[20]

[17]By the free entry assumption, q does not depend on the number of students in the private sector. Chen and West (2000) arrives at the same conclusion through a generic technology to produce education showing decreasing returns to scale. Epple and Romano (1996) do not need to specify the private sector market structure.

[18]In other words I suppose fixed costs to be covered by *ad hoc* lump sum taxes. This can be explained because, since fixed costs are infrequent and huge, thus they might have to be approved by specific procedures and financed through special public funds. This assumption does not qualitatively affect results.

[19]Fixing a minimal expenditure for public schools might imply a higher preferred tax, but it would not qualitatively affect the outcome. The alternative option (i.e., having total income shared between vouchers and public school) would imply a much less treatable model without adding special insights nor being a more realistic assumption.

[20]It might look arbitrary to have people voting over the tax to finance school but not over a) the subset of people entitled to receive vouchers or b) the vouchers' value. This is not a simplifying assumption; the voting mechanism is intended as a way to predict the attitude of an "office-motivated" politician. His or her political appeal depends on his or her decisions concerning policies that are of particular interest to voters and are not too technical. It is reasonable to suppose that voters are interested in general policies (such as the share of GDP devoted to education) while they do not have a defined position on technical problems (such as the value of the voucher) that require the collection of much information. Note, for instance, that the Swiss referendum was only on the introduction

By Assumption 7, total public (variable) expenditure on education $(c_1 + c_2 n_p) n_p X_P + n_v v$ must equal tax proceeds $t\varpi$. Rearranging the budget constraint, the quality of public schools is defined by:

$$X_P = \frac{t\varpi - n_v v}{g n_p} \tag{1}$$

where $g = (c_1 + c_2 n_p)$ is the per-pupil cost of one unit of public education. Clearly, since X_P cannot be negative, we must ensure that $t\varpi \geq n_v v$.

Households' behaviour can be summarised as follows:

- the problem of an agent choosing private school is[21]

$$\begin{cases} \max\limits_{X_R} U(X_R, b) \\ s.t.\;\; b = (1-t)\omega - \max\{X_R - v; 0\} \end{cases} \tag{2}$$

  His indirect utility can be written (in its reduced form) as $U^R(X_R^*, (1-t)\omega - \max\{X_R^* - v\})$, where X_R^* is the optimal level of consumption of private education. Clearly, since he does not profit from public education, if he uses vouchers his preferred level of taxation is $t = \frac{n_v v}{\varpi}$ (the minimum tax to finance them) and otherwise $t = 0$. His utility is strictly decreasing with the tax.

- the utility function of an agent of income ω attending public school is $U(X_P, b)$; replacing b with the after tax income and X_P with Equation 1, the indirect utility is:

$$U^P\left(\frac{t\varpi - n_v v}{g n_p}; (1-t)\omega\right) \tag{3}$$

The preferred level of taxation for this agent is the one that maximises his utility, i.e. $t^*(\omega) = \arg\max\limits_{t} U^P\left(\frac{t\varpi - n_v v}{g n_p}, (1-t)\omega\right)$. From the FOC it is

of vouchers; vouchers' value and who could profit from them were chosen by politicians. Future studies could allow us to find the optimal voucher value and the optimal threshold (in here, I exogenously chose them).

[21]Remember that the price of private education q has been normalised to 1.

possible to indirectly determine how the preferred tax of a voter attending public school changes with his type/income. Since the utility function is assumed to be separable in its two arguments, we have that $\frac{\partial t^*(\omega)}{\partial \omega} > 0$ (SRI) if and only if $-\omega(1-t)U^P_{22} > U^P_2$ and $\frac{\partial t^*(\omega)}{\partial \omega} < 0$ (SDI) if and only if $-\omega(1-t)U^P_{22} < U^P_2$.[22]

Both conditions are widely accepted in the literature; I assume that the SDI assumption holds. This assumption means that the marginal utility of education is much higher than that of the numeraire for low levels of consumption, while the opposite is true when an agent is consuming a richer bundle. As a consequence, richer people are less eager to substitute units of the numeraire for education.

Each agent chooses between public and private school by comparing the two possible levels of utility that he can attain. It is possible to identify the "indifferent voter(s)" $\widehat{\omega}$, i.e., the voter(s) having the same utility regardless of the type of school attended:

$$U^R\left(X^*_R, (1-t)\omega - \max\{X^*_R - v, 0\}\right) = U^P\left(\frac{t\overline{\omega} - n_v v}{g n_p}, (1-t)\omega\right). \qquad (4)$$

The identity of $\widehat{\omega}$ depends on public school quality and thus on the equilibrium tax t. Since the equilibrium tax depends on the identity of the pivotal voter $\underline{\omega}$, it is more precise to denote the indifferent voter by $\widehat{\omega}(\underline{\omega})$.

The two following lemmas allow us to conclude that, once we identify the indifferent voter, all richer agents attend private school and the others attend public school.[23]

Lemma 1 *In a given interval $\omega \in [\alpha, \varphi]$ and for $\alpha < \beta < \varphi$, if the agent $\omega = \beta$ prefers the private system so do all those richer than him (i.e., $\omega \in [\beta, \varphi]$).*

[22] Assuming one of the two monotonicity assumptions is necessary to ensure the existence of an equilibrium. For more details on the SRI and SDI assumptions, see footnote 11 and also Epple and Romano (1996).

[23] This is true as long as we compare agents all receiving the voucher or if none of them receive it.

Lemma 2 *Similarly to the previous lemma, if $\omega = \beta$ prefers the public system, so do all the poorer agents (i.e., $\omega \in [\alpha, \beta]$).*

The intuition behind those two lemmas is that the choice between public and private education depends on the marginal rate of substitution between education and the numeraire good, which is monotone in income. When an agent is sufficiently rich for private school to be preferable (because the reduction in consumption of b has minor effects), this is true *a fortiori* for all richer agents. Similarly, if an agent prefers public school, then poorer people prefer it too.

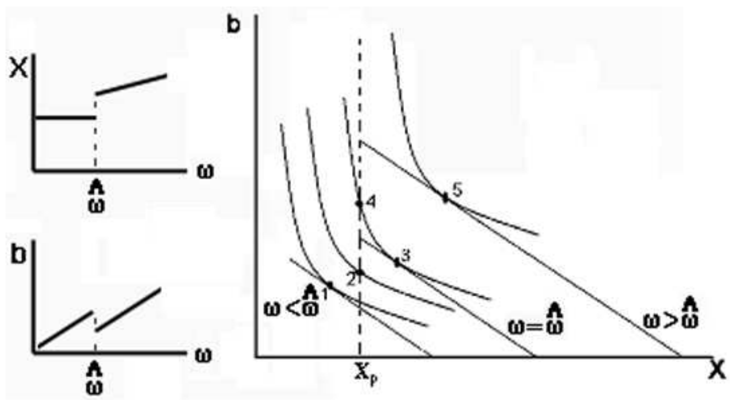

Figure 1: Moving from public to private school: Engel's and indifference curves

The indifferent agent can choose between two bundles: attending public school he can consume more of the numeraire but less education than what is desirable, and vice versa (see the left part of fig. 1), since even when attending private school, an agent has nevertheless to pay the tax financing the public school. From the sketch of the indifference curves,[24] one can see that for an agent with low income ($\omega < \hat{\omega}$) it is preferable to attend public school ($2 \succ 1$). Voter $\hat{\omega}$ is indifferent between public and private instruction

[24]The line represents the budget constraint and the vertical deviation in correspondence to the point X_p is due to the fact that agents can always attend public school, and in that case, he can consume all his disposable income to buy the numeraire; thus, there is a jump in his consumption.

($3 \sim_{\hat{\omega}} 4$). Finally, for those agents with sufficiently large income ($\omega > \hat{\omega}$), the point of tangency suggests that the private schooling is preferred.

Before considering the solution of the model, I consider the situation when vouchers are not available, which is considered as a benchmark to study the consequences of the introduction of vouchers.

3 The benchmark case (without vouchers)

When vouchers are not available, this model is the same as in Epple and Romano (1996), except that I do not set the cost function parameters to be the same for the public and private sectors.[25]

Equilibrium results for the no-voucher case are denoted by the superscript nv. Equation 1 becomes $X_p^{nv} = \frac{t\overline{\omega}}{g^{nv} n_p^{nv}}$ and 4 is $U^R\left(X_R^*, (1-t)\omega - X_R^*\right) = U^P\left(\frac{t\overline{\omega}}{g^{nv} n_p^{nv}}, (1-t)\omega\right)$.

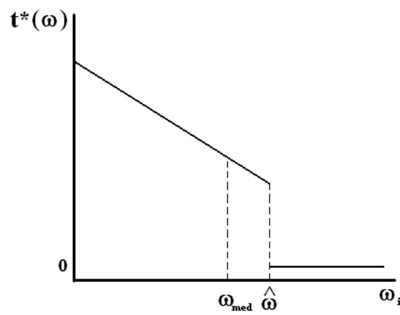

Figure 2: Preferred tax under SDI and no voucher (linear proxy)

Under the SDI assumption, the median voter is pivotal ($\underline{\omega} = \omega_{med}$); from Figure 2 it is intuitive to see why.[26] This means that the voting outcome in the no-voucher case is $t^{nv} = t(\omega_{med})$; all and only agents with income lower

[25]For more details and proofs of this section results, the reader can see Epple and Romano (1996) and Glomm and Ravikumar (1998).

[26]As the preferences over the tax are weakly decreasing in relation to income, the poorer households would prefer a higher level of taxation than would the richer ones. Any proposed $t < t(\omega_{med})$ can be an equilibrium because all households with income $\omega < \omega_{med}$ prefer $t(\omega_{med})$. Similarly, all tax proposals $t > t(\omega_{med})$ are defeated by a majority of voters composed of all agents with income $\omega \geq \omega_{med}$.

than the indifferent voter $\widehat{\omega}$ attend public school. The number of households attending public school is:[27]

$$n_p^{nv}(t^{nv}) = \int_{\omega_{\min}}^{\widehat{\omega}} f(\omega)d\omega = F\left(\widehat{\omega}\right) \tag{5}$$

The only difference compared to Epple and Romano (1996) is that in my article public and private school prices (respectively g and q) are not assumed to be the same. All their results concerning the SDI case hold here simply assuming $g = q$. On top of that, even when $g \neq q$, their results are still qualitatively applicable; only the identity of the indifferent voter will differ. In particular, with respect to Epple and Romano (1996), if $g > q$, the quality of public school is lower, and so is $\widehat{\omega}$ (i.e., the indifferent agent between public and private school is poorer), and the opposite is true for $g < p$.

In the next sections, these results are used as a benchmark to grasp the consequences of the introduction of vouchers.

4 Introducing vouchers

The policy maker proposes a voucher of magnitude $v = \frac{t^{nv}\overline{\omega}}{n_p^{nv}}$ to agents with income below $\omega_{\max}^{v} = \widehat{\omega}(t^{nv}(\omega_{med}))$ if they attend a private school.[28] These two values have been arbitrarily chosen: v is equal to the voucher-absent average cost of a public school student; the threshold for being eligible for the voucher program is equivalent to the income of the pivotal agent under no vouchers. This ensures not to subsidise agents who would attend private

[27] Agents, in choosing to attend (or not) public school, take n_p^{nv} (and thus the quality of public instruction) as given and consequently vote for the tax level. In equilibrium, the proportion of voters opting for public services coincides with agents' expectations. Glomm and Ravikumar (1998)'s proposition 2 (proving that a value for n_p^{nv} always exists that solves 4 and that this value is unique) holds in this framework. The conditions under which Glomm and Ravikumar (1998)'s proposition 2 holds are not restrictive: the cumulative density function $F(\omega)$ simply has to be a continuous function increasing in relation to ω.

[28] The value of the voucher (exogenously determined) is strictly smaller than the marginal cost of a student at equilibrium in the case without vouchers, i.e., $\frac{t^{nv}\overline{\omega}}{n_p^{nv}} < \frac{(c_1+2c_2n_p^{nv})t^{nv}\overline{\omega}}{(c_1+c_2n_p^{nv})n_p^{nv}}$. This depends on the non-decreasing returns to scale assumption.

schools without vouchers and it also simplifies comparisons with the no-voucher framework.

The public budget constraint 1 can be rewritten as

$$X_P = \left(t - \frac{n_v}{n_p^{nv}} t^{nv}\right) \frac{\overline{\omega}}{g n_p} \tag{6}$$

We can expect some agents entitled to receive a voucher to shift to the private sector. Moreover, this implies a reduction in congestion, so the quality of public school might increase, possibly attracting some students previously attending a private institute.

Since the price of private education is no longer the same for all agents, it is possible to identify up to two possible indifferent agents: one among voters receiving vouchers and another within the others.

It is preferable to consider separately the two different groups of agents $[\omega_{\min}, \omega_{\max}^v]$ and $[\omega_{\max}^v, \omega_{\max}]$. Lemmas 1 and 2 allow us to construct four (possibly empty) subsets:[29] in particular, for each of the two previous groups of agents, we can have some voters preferring public education and some preferring the privately provided counterpart.
$\widehat{\omega}_L(t) \in [\omega_{\min}, \omega_{\max}^v]$ is the income level for which

$$U^R\left((1-t)\omega - X_R^* + v\right) = U^P\left(\frac{t\overline{\omega} - n_v v}{g n_p}, (1-t)\omega\right) \tag{7}$$

while $\widehat{\omega}_R(t) \in [\omega_{\max}^v; \omega_{\max}]$ is such that

$$U^R\left((1-t)\omega - X_R^*\right) = U^P\left(\frac{t\overline{\omega} - n_v v}{g n_p}, (1-t)\omega\right) \tag{8}$$

The two previous equations mean that $\widehat{\omega}_L$ and $\widehat{\omega}_R$ are the income levels for which an agent is indifferent between private and public education. Clearly $\widehat{\omega}_L + v \leq \widehat{\omega} \leq \widehat{\omega}_R$.

The two critical levels of income $\widehat{\omega}_L$ and $\widehat{\omega}_R$ can also be seen in Figure

[29] Later on, I will give the existence conditions for the indifferent agents and thus the four subsets' bounds.

3, which shows qualitatively how utility changes with income for an agent attending private or public school, both with and without vouchers. The quality of public school in the graph is fixed and $X_p > X_p^{nv}$.

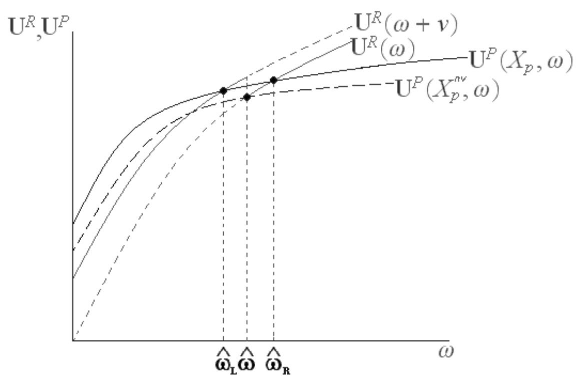

Figure 3: How utility changes with income

From Figure 3, $\widehat{\omega}_L < \widehat{\omega} < \widehat{\omega}_R$. If the intersection between $U^R(\omega + v)$ and $U^P(X_p, \omega)$ were to the right with respect to the one between $U^R(\omega)$ and $U^P(X_p^{nv}, \omega)$, from the graph it is clear that $\widehat{\omega}_L$ would be greater than $\widehat{\omega}$. Thus, it would not belong to the required interval and all agents in $[\omega_{\min}, \omega_{\max}^v]$ would attend public school. Likewise, all agents with income greater than $\widehat{\omega}$ prefer to consume private education when $U^R(\omega)$ and $U^P(X_p, \omega)$ do not cross to the right of $\widehat{\omega}$.

When both thresholds exist, there are four groups of agents, whose preferred choice is represented in Figure 4.

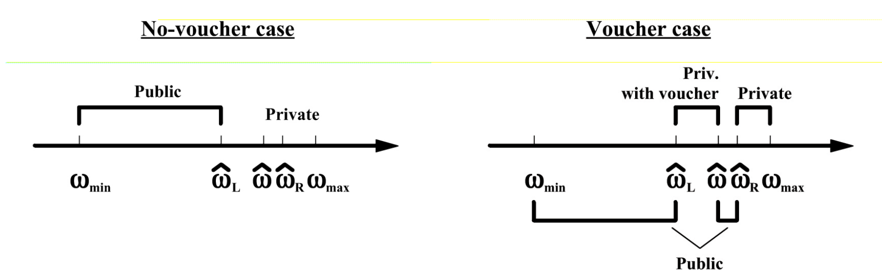

Figure 4: Intervals and choices

Having defined $\widehat{\omega}_L$ and $\widehat{\omega}_R$, it is now possible to precisely define n_v and n_p.[30] While n_v is the number of agents using the voucher at equilibrium, n_p is the number of agents attending public school; thus $n_v = \int_{\widehat{\omega}_L}^{\widehat{\omega}(t^{nv})} f(\omega)d\omega$ and

$$n_p = \int_{\omega_{\min}}^{\widehat{\omega}_L} f(\omega)d\omega + \int_{\widehat{\omega}(t^{nv})}^{\widehat{\omega}_R} f(\omega)d\omega \tag{9}$$

The following propositions and their corollaries prove that $\widehat{\omega}_L$ and $\widehat{\omega}_R$ exist; that is, $\widehat{\omega}_L \in [\omega_{\min}, \omega_{\max}^v]$ and $\widehat{\omega}_R \in [\omega_{\max}^v, \omega_{\max}]$.

Proposition 1 *For all $t \in (0,1)$ and $\overline{\omega} \in R_{++}$, there always exists a value for $n_p \in (0,1)$ for which the number of people willing to attend public school is equal to the value for n_p that agents anticipate to solve their maximisation problem.*

Proposition 1 guarantees the existence of an equilibrium.

Proposition 2 *If, ceteris paribus, the quality of public school increases, the preferred tax for a given level of income falls. Thus the same pivotal voter might choose different tax levels according to the framework.*

Corollary 1 *If $X_p > X_p^{nv}$, then $t(\omega_{med}) < t^{nv}(\omega_{med})$.*

This means that if the median voter is pivotal both with and without vouchers, the tax burden decreases if with the introduction of vouchers the quality of public schooling increases. This corollary has an important welfare implication, since a reduction in t generates an increase in welfare of all agents, including those who are not attending a public school.

Proposition 3 *If $\widehat{\omega}_L = \widehat{\omega}$, then $\widehat{\omega}_R = \widehat{\omega}$ and we are back to the case without vouchers. Moreover it cannot be that $\widehat{\omega}_L > \widehat{\omega}$.*

[30] Remember that n_p^{nv} is the number of people attending public school in the no-voucher case; defined by equation 5.

The idea is that an agent who (voucher-absent) prefers to attend a private school, wishes for a change in behaviour only if the quality of public school increases, otherwise there would be no reason to move to the public sector. The number of students attending public school when vouchers are available is always weakly smaller than in the no-voucher case, i.e., $gn_p \leq g^{nv} n_p^{nv}$ with strict inequality as long as $\widehat{\omega}_L < \widehat{\omega}$. As long as (and only when) $\widehat{\omega}_L < \widehat{\omega}$, the equilibrium is modified by the introduction of vouchers, and in particular, we observe a reduction in the number of students attending public school.

Proposition 4 *$\widehat{\omega}_R > \widehat{\omega}$ if and only if $X_P > Xp$: some agents move, after the introduction of vouchers, from the private to the public sector, only if the quality of public school increased as a consequence of the change. If $X_p \leq X_p^{nv}$, then $\widehat{\omega}_R = \widehat{\omega}$.*

The voting outcome tax can never be higher than the one preferred by the median voter (his preferred tax is the highest that can be supported by at least half of the population). Since $\widehat{\omega}_R > \widehat{\omega}$ only when $X_p > X_p^{nv}$, if we observe $\widehat{\omega}_R > \widehat{\omega}$, the total number of agents attending public school is necessarily smaller than in the case without vouchers ($n_p < n_p^{nv}$).

Proposition 5 *Public-school-quality at the equilibrium under vouchers is always greater or equal to the one without vouchers (for a given level of taxation), i.e., $X_p \geq X_p^{nv}$, with strict inequality when $\widehat{\omega}_L < \widehat{\omega}$.*

Proof. See the Appendix for the proofs of Propositions 1, 2, 3, 4 and 5. ■

Combining the result of these propositions, it is possible to conclude that there are two possible consequences of the introduction of vouchers: a) they are ineffective, that is $\widehat{\omega}_L = \widehat{\omega}_R = \omega^v_{\max}$ (nobody uses vouchers and the introduction does not affect agents in the economy); b) the richest people amongst those entitled to use vouchers and the poorest amongst those who are not eligible both adjust their behaviour: $\widehat{\omega}_L < \omega^v_{\max} < \widehat{\omega}_R$. Those propositions are used in the next sections in order to compare welfare under the different possible scenarios.

5 The vote over the tax

The tax to finance public school is chosen by households through a majority vote. Different scenarios are possible:[31]

- the preferred tax is decreasing in income for agents attending public school (SDI assumption).
- the preferred tax is $t = 0$ for private school students not using vouchers.
- $t = \frac{n_v v}{\overline{\omega}}$ is the preferred tax of private school students using vouchers; this is exactly the minimum tax to finance the voucher system.[32] With this level of taxation, strictly lower than the one preferred by any public school student, public education disappears.

The voting process outcome depends on the distribution of income and mainly on whether the median voter attends public school. I analyse separately the cases: i) (Subsection 5.1) where the median voter continues attending public school even after the introduction of vouchers ($\widehat{\omega}_L \geq \omega_{med}$) and introducing vouchers always induces a Pareto improvement; and ii) (Subsection 5.2) where the median voter uses the voucher to move to the private sector ($\widehat{\omega}_L < \omega_{med}$) and we observe a Pareto improvement only as long as the public school system does not collapse; otherwise, a minority of the population might be worse off.

[31] Recall that the preferred tax of an agent depends on his choice of public or private education but also on the opportunity to receive a voucher when choosing the private school.

[32] Remember that here voucher size is fixed, so voting for a higher tax would be useless for them.

5.1 The median voter attends a public school ($\widehat{\omega}_L \geq \omega_{med}$)

Restricting our attention to the case when $\widehat{\omega}_L \in [\omega_{med}, \widehat{\omega})$,[33] the outcome of the vote is precisely $t = t(\omega_{med})$. In fact, all agents with income $\omega < \omega_{med}$ (by definition half of the population) ask for a tax increase with respect to $t = t(\omega_{med})$, while all agents with income $\omega \in (\omega_{med}, \omega_{\max}]$ are favourable to a decrease in the equilibrium tax.[34] This means that the median voter is pivotal. Figure 5 represents agents' preferred tax in the case of vouchers when $\widehat{\omega}_L \geq \omega_{med}$.

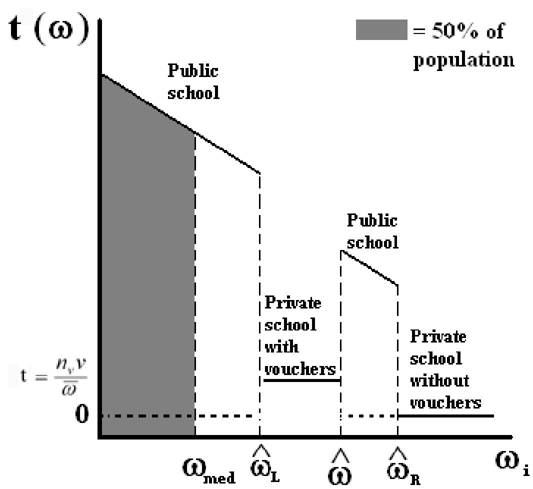

Figure 5: Agents' preferred tax

Note that, even though the median voter is again decisive, by Proposition 2 his preferred tax level is lower than in the no-voucher case: $t(\omega_{med}) < t^{nv}(\omega_{med})$. Moreover the public budget constraint is relaxed and the quality of public school necessarily increases.[35] Part of this effect is offset by the

[33] If $\widehat{\omega}_L = \widehat{\omega}$ (that is, given the utility function, vouchers are not sufficiently attractive, and in equilibrium, any agent may use them), we are back to the no-voucher, case and the introduction of vouchers is ineffective (see proposition 3).

[34] This is due to the SDI assumption and because a portion of the agents with income $\omega > \omega_{med}$ attend private school.

[35] Since voucher value is lower than the marginal cost of the most expensive public

arrival of some new students previously attending private school who are attracted by the higher public school quality; thus, the subset $\omega \in [\widehat{\omega}, \widehat{\omega}_R]$ is non-empty. By Proposition 4 and its corollary, we know that the number of agents moving from public school is higher than the number of students moving to it and that the final effect is an increase in the quality of the public service (financed through tax proceeds net of vouchers expenditure).

From a welfare standpoint, we observe a Pareto improvement. Intuitively, when vouchers are introduced, the quality of public schools increases, making public school students better off. Moreover, the tax burden falls, so all citizens are better off. By the Weak Axiom of Revealed Preferences (WARP), all agents changing behaviour while the previous bundle is still affordable must be better off.

To be more rigourous, for $\widehat{\omega}_L < \widehat{\omega}$, utility increases for all agents when introducing vouchers:

- $[\omega_{\min}, \widehat{\omega}_L]$: these agents always opt for public school. The quality of public school increases (Proposition 5). Since both their disposable income and the public school quality increase, their utility increases as well.

- $[\widehat{\omega}_L, \widehat{\omega}]$: they move from public to private education and use vouchers. If they stick to public education, they increase their utility (similarly to agents in $[\omega_{\min}; \widehat{\omega}_L]$). If they decide to opt out from public school, by WARP it must be that their utility from attending private school is even higher.

- $(\widehat{\omega}, \widehat{\omega}_R)$: The bundle previously consumed is still affordable. If they modify their choice, by WARP it means that the new bundle is preferred to the previous one.

- $[\widehat{\omega}_R, \omega_{\max}]$: all the agents in this interval attend private school in both cases. The price that they pay to attend private school is the same, and

school students, convincing them to consume private education makes the public budget constraint less binding and increases the quality of the public service.

the tax decreases. As a consequence, all these households are better off in the voucher case.

To sum up, when the introduction of vouchers is ineffective (i.e., $\widehat{\omega}_L = \widehat{\omega} = \widehat{\omega}_R$), agents are indifferent, and when $\widehat{\omega}_L \in [\omega_{med}, \widehat{\omega})$, the selective voucher system strictly Pareto dominates the no-voucher system.

5.2 The median voter attends a private school ($\widehat{\omega}_L < \omega_{med}$)

According to the density function $f(\omega)$, the number of people affected by the introduction of vouchers ($\omega \in [\widehat{\omega}_L, \widehat{\omega}_R]$) varies, and so does the number of agents willing to use vouchers in equilibrium. The consequences of introducing vouchers depend on how many agents want to move to private schooling.

If $\widehat{\omega}_L(\omega_{med}) < \omega_{med}$, the poorest part of the population (which is attending public school) cannot form a majority coalition. The shift from public to private induced by vouchers (*ceteris paribus*) increases the quality of public service, which attracts a group of voters ($\omega \in [\underline{\omega}, \widehat{\omega}_R]$) previously attending a private school. Two scenarios can occur depending on whether or not those willing to attend public school make up at least half of the population (i.e., $\int_{\omega_{\min}}^{\widehat{\omega}_L} f(\omega)d\omega + \int_{\widehat{\omega}}^{\widehat{\omega}_R} f(\omega)d\omega \geq 50\%$). The value of vouchers and $f(\omega)$ jointly determine which is the relevant scenario.[36]

Let us define the pivotal voter as

$$\underline{\omega} = \left\{ \omega \in (\widehat{\omega}, \widehat{\omega}_R] : \int_{\omega_{\min}}^{\widehat{\omega}_L} f(\omega)d\omega + \int_{\widehat{\omega}}^{\underline{\omega}} f(\omega)d\omega = 50\% \right\} \tag{10}$$

where we restrict the existence of $\underline{\omega}$ to the interval $(\widehat{\omega}, \widehat{\omega}_R]$, to ensure that

[36] Note that in all Western countries in which vouchers have been introduced, public school attendance exceeds half of the population. For instance, in Chile, where vouchers' value was set slightly below the average cost of students attending public school (OECD (1998)), private school attendance grew from 25% to 39% (Cox and Lemaitre (1999)).

he is attending a public school.[37] Intuitively the income $\underline{\omega}$ represents the agent whose preferred tax is the "median preferred-tax". Agents' preferred tax is summarised in Figure 6.

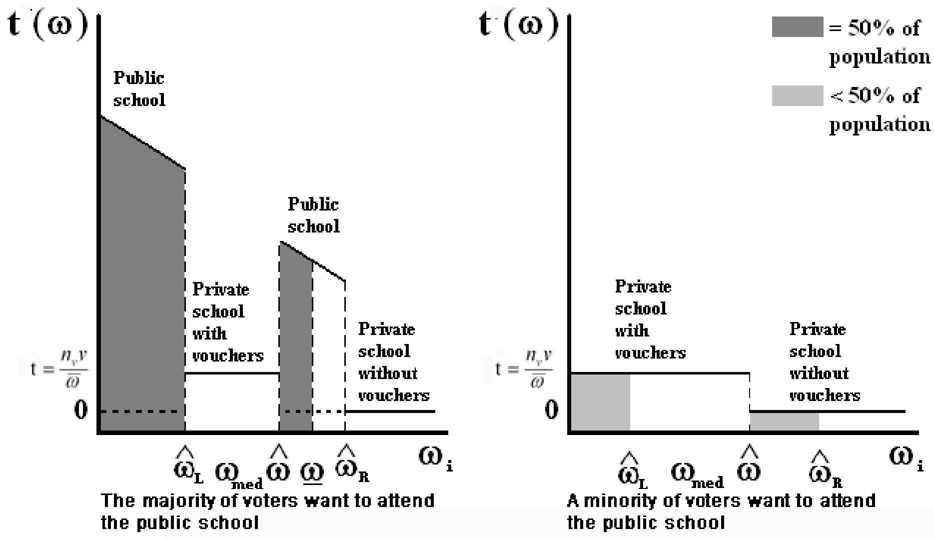

Figure 6: Agents' preferred tax

The existence of $\underline{\omega}$ implies that a coalition of public school students set the equilibrium tax and that the group in favour of having no tax does not influence the vote outcome alone. If any income fulfils the requirements in Equation 10, the tax is chosen by the group of agents attending private school and profiting from the voucher (and set to the minimum level to finance vouchers: $t = \frac{n_v v}{\overline{\omega}}$).

The equilibrium when the majority of voters attend public school

By construction, $\underline{\omega}$ is pivotal: all agents with income belonging to the interval $[\omega_{\min}, \widehat{\omega}_L] \cup [\widehat{\omega}, \underline{\omega}]$ (representing half of the population) prefer a tax rate larger than the one chosen by $\underline{\omega}$. Agents in $[\widehat{\omega}_L, \widehat{\omega}]$ and $[\underline{\omega}, \widehat{\omega}_R]$ ask for a lower but positive tax rate; the remaining $(\omega > \widehat{\omega}_R)$ ask for no tax at all. By the SDI

[37] $\int_{\omega_{\min}}^{\widehat{\omega}_L} f(\omega)d\omega + \int_{\widehat{\omega}}^{\underline{\omega}} f(\omega)d\omega = 50\%$ is equivalent to $\int_{\widehat{\omega}_L}^{\omega_{med}} f(\omega)d\omega = \int_{\widehat{\omega}}^{\underline{\omega}} f(\omega)d\omega$.

assumption, since $\underline{\omega} > \omega_{med}$, the equilibrium tax decreases with respect to equilibria in Sections 3 or 5.1.

From Equation 10, $\widehat{\omega} < \underline{\omega} \leq \widehat{\omega}_R$. By Proposition 4 we can conclude that quality of public education has necessarily increased and thus that a strict Pareto improvement occurred.

All agents' disposable income increases ($t(\underline{\omega}) < t^{nv}(\omega_{med})$); agents attending a private school (i.e., $\omega \in [\widehat{\omega}_L, \widehat{\omega}]$ and $\omega > \widehat{\omega}_R$) are better off with than without vouchers. The poorest agents ($\omega < \widehat{\omega}_L$) are also better off, since they pay less in taxes and receive a better public service.

People in $\omega \in [\widehat{\omega}, \widehat{\omega}_R]$ could stick to the private market and consume a better bundle with respect to the one consumed without vouchers (since the tax decreased); if they move to the public sector, by WARP we can conclude that they are better off.

All agents being strictly better off, we conclude that in this framework, the introduction of vouchers leads to a strict Pareto improvement.

The equilibrium when the majority of voters do not attend public school When agents willing to attend public school amount to less than 50% of the population, the decisive voter belongs to the group of people attending private school and profiting from the voucher. The minimum tax to finance vouchers for all agents entitled to receive them ($t = \frac{n_p^{nv} v}{\overline{\omega}}$) wins any pairwise comparison. Replacing v by its value, we obtain $t = \frac{n_p^{nv} v}{\overline{\omega}} = t^{nv}$. Every former student of the public school is receiving the average social cost of a public student in the no voucher case.

For this solution to be a stable equilibrium, at least half of the population has to be better off; otherwise, this level tax could not win against the proposal of having no vouchers. All people with income $\omega > \widehat{\omega}$ are indifferent, since the tax does not change with respect to the benchmark.

People with income $\omega \in [\widehat{\omega}_L, \widehat{\omega}]$ are always better off (by WARP).

Concerning people with income $\omega < \widehat{\omega}_L$, they all receive the same voucher to be spent for private education. Three frameworks are possible for them:

1. Private school market price (q) is lower than the average cost of producing public education ($AC(X_P)$) in the no-voucher case. It is socially optimal to dismantle the public school and distribute vouchers. Agents are better off; this solution is a Pareto improving equilibrium, and public schooling disappears.

2. $q = AC(X_P)$. They are indifferent (they consume the same amount of both goods). This equilibrium weakly Pareto dominates the no-voucher case and public schooling disappears.

3. $q > AC(X_P)$. They are strictly worse off (they consume the same quantity of numeraire but receive a worse educational service). Here, a minority of the population is worse off ($\omega < \widehat{\omega}_L$), another is better off ($\omega \in [\widehat{\omega}_L, \widehat{\omega}]$) and the remainder ($\omega > \widehat{\omega}$) is indifferent. For this framework to be an equilibrium (i.e., for voters to accept the introduction of vouchers), at least half of voters should agree on vouchers, which means that a substantial part of the richest agents has to form a coalition with the middle class against the lower class.

6 Conclusions

The aim of this work was to investigate the implications of introducing selective vouchers and in particular if this change would be accepted by the majority of voters. The main contributions of this work are to show:

1. that the usual conclusion that the median voter is always decisive under the assumption of SDI is not robust with regard to the introduction of vouchers.

2. that in addition to the known types of coalition ("lower class versus higher class" and "middle class versus the others"), we can have a third type of coalition where part of the bottom-middle class joins the coalition of the richest agents to ask for a reduction in taxes while the

top-middle class forms a coalition with the poorest voters to increase taxes.

3. that the introduction of vouchers always induces a Pareto improvement unless if, introducing them, the public sector collapses and meanwhile the market price of private education is higher than the average cost of producing public education. In this case the poorest subset of the population will be hurt by the introduction of vouchers.

4. that the introduction of vouchers should always be supported by a large majority (under the hypothesis of rational agents).[38]

5. that the middle class is the group that directly profits from vouchers; the poorest class is the one bearing their costs when a public school collapses. The richest class always weakly profits from the introduction of vouchers (through tax reductions).

My model is qualitatively robust to different specifications (such as Constant Return to Scale production functions) as long as we choose the value of the voucher that is smaller than the cost of public students but large enough to be attractive for some students.

From the results, it is possible to conclude that the introduction of vouchers should never be harmful for society (unless it produces the collapse of public schools) when its value is below or equal to the average per-student public expenditure and when it does not subsidise students that would have attended a private school in any case. Of course, this result depends on the initial specifications, which should be reiterated: 1) education is considered a horizontally differentiated good, and it is not harmful for society to have people attending the school of their choice.[39] 2) the role of peer effects is not significant; (alternatively, peer effects have a linear impact on instruction and the social welfare function is utilitarian).

[38] The majority of voters always profit from the introduction of vouchers; in most cases, all agents in the society do.

[39] If in a country, for instance, a school were less effective in the spread of knowledge, increasing its market share might have a negative impact on productivity, growth, etc.

Note that introducing selective vouchers of a fixed amount implies a jump in utility (thus a distortion) for agents having an income close to the threshold to be eligible. This is a structural problem of selective vouchers that can be avoided by introducing vouchers that are regressive in income. Further studies might find under which conditions such vouchers would be compatible with public budget constraint and allow an increase in welfare.

From these results, we also conclude that the introduction of vouchers would increase integration in a stratified society, increasing the variance in wealth amongst students in the same school (making private schools accessible to poorer people and public schools more attractive for richer people).

From these results, one might expect that the introduction of vouchers would be welcomed by voters; nevertheless in many Western countries (especially in Europe), vouchers are not very popular. In Switzerland, a referendum against vouchers was successful in the Canton Ticino region; in Italy the debate over vouchers was almost immediately halted because of the strong hostility shown by many political parties.

A combination of different factors may have generated this aversion towards vouchers: on the one hand, in many countries, private institutes have religious (and often even political) orientations, and vouchers are perceived as a way to subsidise a specific *credo* or as a way to diffuse specific cultures or principles.

Another reason for the failure of vouchers in Europe might be that generally only universal vouchers have been proposed and, which are more likely to decrease the quality of the public service and reduce redistribution.

Finally, a more substantial problem concerns the value of the voucher. A voucher of a small amount is ineffective, and a voucher that is too large implies that the public sector is no longer supported by the majority of the population. In my model, a benevolent social planner fixes the value of the voucher at a value at which the public budget constraint is relaxed when some students use them. If we let people decide the value of vouchers, we can expect to have different results compared to those of my model; in particular, it is possible for the vouchers' value to be larger than the public school student's social cost or so small that nobody would be interested in using them.

Appendix

A The effects of a change in the tax

Most variables are affected by changes in the income tax. Intuitively, if the tax rate falls, the first impact on the model is that on the one side the public investment in education ($t\overline{\omega}$) falls, and, on the other hand, the disposable income ($(1-t)\omega$) of all the agents increases. Both these effects imply that opting for private school becomes more attractive. Concerning the first effect, the reasons are obvious, while for the second one, they are slightly more subtle: an increase in the disposable income leads to an increase in the consumption of b for everybody, but since the quantity of b consumed by people attending public school is higher, by the concavity of the utility function, the increase in utility for people attending public school is lower than the utility for those preferring private education. Since private school becomes more attractive, a greater number of agents switch from the public to the private system (which means that the income of the two indifferent voters decreases). The number of voters using vouchers increases, tightening even more the public budget constraint. Simultaneously, with the number of people attending public school having fallen, the per-capita public expenditure increases (since gn_p drops), making public education more attractive.

To summarise, the impact on the quality of public school from a change in the tax is a priori undetermined. On the one side, a drop in the tax rate implies that the budget available for public school is lower. On the other side, this causes a decrease in the number of people attending public school (both because public school becomes less attractive and because agents' disposable income increases). When $\frac{\partial X_p}{\partial t} \geq 0$, it means that a reduction in the tax rate decreases public expenditure for education and the consequent shrinkage in the number of people attending public school is not enough to offset it (in other words, demand for public school is inelastic); thus, the per-capita expenditure will also plunge. The reverse is true for $\frac{\partial X_p}{\partial t} < 0$.

B Proof of Proposition 1

At equilibrium, n_p has to solve two equations. On one side, it is equal to the fraction of agents for whom the utility of attending a public school is larger than the utility of opting out of it, thus $n_p = \mu\{\omega : U^P(t, \omega, \overline{\omega}, n_p) \geq U^R(t, \omega, v)$, where μ is the probability measure associated with the distribution function. On the other hand (Equation 9), the number of agents with income in the interval $[\omega_{\min}, \widehat{\omega}_L] \cup [\widehat{\omega}, \widehat{\omega}_R]$ must be the same as the value for n_p used by agents to solve their maximisation problem.

Equating the two, we obtain $\mu\{\omega : U^P(t, \omega, \overline{\omega}, n_p) \geq U^R(t, \omega, v) = F(\widehat{\omega}_L) + (F(\widehat{\omega}_R) - F(\widehat{\omega}))$. Simple computations show that the left-hand side of the equation is decreasing in both $\widehat{\omega}_L$ and $\widehat{\omega}_R$ while the right-hand side is increasing. Since F is a continuous (and strictly increasing) function and since, for $n_p = 0$, the left-hand side is always larger than the right-hand side, a unique solution exists (fixed point theorem).

C Proof of Proposition 2

For a given revenue $\widetilde{\omega}$, the preferred tax $t(\widetilde{\omega}) = \arg\max_t U^P(\frac{t\overline{\omega} - n_v v}{g n_p}, (1-t)\widetilde{\omega})$. If, for any reason, the first argument (X_p) increases, its marginal utility of education (U_1^p) decreases. At equilibrium, the optimal tax by definition equalises the marginal utility of both arguments ($U_1^p = U_2^p$), which means that the marginal utility of the numeraire falls (thus, the numeraire consumption has to increase) and thus the tax drops.

D Proof of Proposition 3

If $\widehat{\omega}_L = \widehat{\omega}$, nobody uses the voucher, $n_v = 0$ and $X_p = \frac{\overline{\omega} t^{nv}}{g n_p}$. The number of students attending public school cannot be lower than in equilibrium in the no-voucher case, which implies that $g n_p \geq g^{nv} n_p^{nv}$. This makes public school (weakly) less attractive than in the no-voucher case, so all the households with income $\omega > \widehat{\omega}$ (who were already preferring the private system) confirm their choice. If $X_R \succ_\omega X_P$ for all $\omega > \widehat{\omega}$, then $g^{nv} n_p^{nv} = g n_p$ and thus $X_p^v = X_p$ and we are back to the equilibrium case without vouchers.

Finally, it cannot be that $\widehat{\omega}_L > \widehat{\omega}$. This would result in $n_v = 0$ and $g^v n_p^v = g n_p$; this would imply that $X_P^{nv} = X_P$ and thus that $\widehat{\omega}_L = \widehat{\omega}$, which is a contradiction. This proves that $\widehat{\omega}_L \leq \widehat{\omega}$ in all cases.

E Proof of Proposition 4

$\underline{X_P > X_P^{nv} \Leftarrow \widehat{\omega}_R > \widehat{\omega}}$: if $\widehat{\omega}_R > \widehat{\omega}$, agents in the interval $(\widehat{\omega}, \widehat{\omega}_R)$ are attending public school in the presence of vouchers while they were attending private schools before. The introduction of vouchers does not imply changes in the disposable income of agents with income above $\widehat{\omega}$; thus the original consumption bundle remains affordable. By the WARP, if we observe a change in this agents' behaviour, it must be that the new bundle is preferred. Since the numeraire consumption is constant, it must be that the quality of school consumed has increased, thus $X_P > X_R > X_P^{nv}$.

$\underline{X_P^{nv} = X_P \Rightarrow \widehat{\omega}_R = \widehat{\omega}}$: when $X_P^{nv} = X_P$, for agents in $(\widehat{\omega}, \omega_{\max}]$ nothing has changed. By simply replacing X_P by X_P^{nv} in equation 8, we are back to the condition in equation 4, and thus, by definition, the solution of the problem is $\widehat{\omega}$.

$\underline{X_P > X_P^{nv} \Rightarrow \widehat{\omega}_R > \widehat{\omega}}$: by definition, $\widehat{\omega}_R$ is the level of income for which the left- and right-hand sides of Equation 8 are equal. For $X_P^{nv} = X_P$, $\widehat{\omega}_R = \widehat{\omega}$. Increasing X_P, public school becomes more attractive (i.e., the right-hand side is bigger than the left-hand side). Only an increase in the level of income can re-establish the equality. Such an increase leads to a higher consumption of the numeraire both in the case of consumption of public school and that of private school; given the concavity of the utility function, the marginal increase is higher on the left-hand side than on the right-hand side, which ensures that for a sufficiently large increase in $\widehat{\omega}_R$, the equality holds once again.

F Proof of Proposition 5

By Proposition 3, $\widehat{\omega}_L$ cannot be greater than $\widehat{\omega}$. Two different scenarios are possible: $\widehat{\omega}_L = \widehat{\omega}$ or $\widehat{\omega}_L < \widehat{\omega}$.

Proof by contradiction. Suppose $\widehat{\omega}_L < \widehat{\omega}$ and $X_P \leq X_P^{nv}$: by Proposition 4, $\widehat{\omega}_R = \widehat{\omega}$ and thus a) $n_p = (n_p(t^{nv}) - n_v)$, b) ω_{med} is decisive, c) $t > t^{nv}$ and d) $g < g^{nv}$ (since $\widehat{\omega}_L < \widehat{\omega}$).

Then
$$\left[\frac{t^{nv}}{g^{nv}n_p^{nv}} - \left(\frac{t - \frac{n_v}{n_p^{nv}}t^{nv}}{gn_p}\right)\right]\overline{\omega} > 0$$

a necessary condition for that (since $t > t^{nv}$) is $n_p g + n_v g^{nv} > n_p^{nv} g^{nv}$. For this to be true it must be that $g > g^{nv}$ which is impossible.

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