Trade and Variety-Skill Complementarity: A Simple Trade-Based Resolution of Wage Inequality Anomaly

Yoshinori Kurokawa
University of Minnesota

April 2006

Online at http://mpra.ub.uni-muenchen.de/14011/
MPRA Paper No. 14011, posted 12. March 2009 03:04 UTC
The Stolper-Samuelson theorem predicts the relative wage of high-skilled labor will increase in the U.S. but decrease in Mexico after trade, while data shows the skill premium began to rise \textit{in both countries} during the 1980s. This paper presents a \textit{simple trade-based} resolution of this “wage inequality anomaly.” The resolution is a straightforward application of well-known variety trade models. Intra-industry trade increases the variety of intermediate goods used by the high-skill intensive final good. If the varieties and high skill are “complements,” the skill premium rises \textit{in both countries}. Evidence supports this linking of intra-industry trade to wage inequality. (\textit{JEL} F12, F16)
One of the most well-documented empirical facts in recent U.S. economic history is that the relative wage of high-skilled to low-skilled labor began to rise during the 1980s, and this fact was observed in Mexico as well (Figure 1).1 As can be seen, these two countries showed a surprising similar timing of the rise in relative wage.2

One traditional explanation for this rising wage inequality is based on technological change. A sharp decline in equipment prices in the 1980s led to an increase in the demand for high-skilled workers, who were complements for this equipment, and a decline in the demand for low-skilled workers, who were substitutes (Per Krusell et al., 2000). This technology-based explanation is consistent with the decline in the price of high-tech goods and the increase in the wage inequality both in the U.S. and in Mexico.

A second explanation for the rising wage inequality is based on trade. The U.S. import of low-skill intensive goods from Mexico causes the relative demand for U.S. low-skilled workers to decline, and therefore the relative wage of low-skilled to high-skilled workers declines. This trade-based explanation has often been criticized due to the small volume of trade. Paul R. Krugman (1995, 2000) provides a theoretical argument to explain why the small volume of trade in the U.S. makes it unlikely that trade can account for the change in wages.

However, the movements of trade between the U.S. and Mexico are surprisingly similar to the movements of relative wage during the 1980s, and in particular, the trade between these two countries was dramatically increasing along with the rise in relative wage (Figure 2). This similarity in these movements cannot be explained only by

---

1 We calculated the U.S. relative wage during the period 1970-1990 on the basis of the Annual Survey of Manufactures (ASM). Due to data constraint, however, we cite the Mexican relative wage in 1970-1985 and 1988 calculated by Gordon Hanson and Anne Harrison (1985) on the basis of the Mexican Industrial Census. Prior to 1988, this Census was conducted at five-year intervals.
2 Hanson and Harrison (1995) show that it was around 1985 when Mexico announced it was joining the GATT that the wage inequality in Mexico began to rise.
technological change. Hence, we can no longer ignore the effect of trade on the recent increase in skill premium in wages. However, this poses a serious theoretical challenge. This is because the previous trade-based explanations, the standard H-O (Heckscher-Ohlin) model and its applications (Robert C. Feenstra and Gordon H. Hanson, 2001), demonstrate a discrepancy between the model and data.\(^3\)

The Stolper-Samuelson theorem of the H-O model predicts that the relative wage of high-skilled to low-skilled labor will increase in the high-skill abundant U.S. but decrease in low-skill abundant Mexico after trade. These models thus generate a positive relation between the trade and wage inequality in the U.S. but generate a negative relation in Mexico. On the other hand, as we have seen, the data shows that the trade and skill premium were rising in both countries during the 1980s, and thus generated a positive relation between the trade and wage inequality in both countries. This is a “wage inequality anomaly.”\(^4\)

Thus a new model of trade which can eliminate this anomaly should be considered. This new model will be a possible explanation for the rising wage inequality across countries as an alternative to the technology-based explanation.

In order to eliminate the wage inequality anomaly, a new model of trade needs to endogenize the technological term since the Stolper-Samuelson channel works as long as it is exogenous. Some economists are trying to explain the wage inequality on the basis of trade with endogenous technology. The major explanations are based on the

---

3 Feenstra and Hanson (2001) interpret the standard H-O model as the model of trade in two intermediate goods which are high-skill and low-skill intensive. It is shown that the decline in low-skill intensive imported input causes the fall in the relative wage of low skill. They define the import of low-skill intensive input as the “outsourcing.” Their model displays that the price of domestic final good relative to the price of imported input rises, which is consistent with U.S. data during the 1980s.

4 The focus of this paper is on the discrepancy between the standard H-O model and the data during the 1980s. We should also consider whether this discrepancy remained or not after the 1980s. Unfortunately, the movements of the Mexican skill premium during the 1990s are unclear as will be discussed in Section III. This problem is outside of the scope of this paper; however, further investigation is needed.
Schumpeterian mechanism. Elias Dinopoulos and Paul Segerstrom (1999) show that trade encourages R & D investment, resulting in innovation and skill-biased technological change in each country. They also show that this idea, linking innovation to skill premium, is supported by empirical evidence. Daron Acemoglu (2002) shows that trade “induces” skill-biased technological change in the U.S. and this improved technology can be transferred to other countries by spillover effects. Thus these explanations demonstrate the rise in the relative wage of high-skilled labor across the countries.

The main purpose of this paper is to present a much simpler resolution of the wage inequality anomaly on the basis of a trade model consistent with data. Our resolution is based on a straightforward application of well-known variety trade models. The standard variety trade models with monopolistic competition (Krugman, 1979; Avinash Dixit and Victor Norman, 1980; Wilfred J. Ethier, 1982) say that the variety of goods, which consumers can consume or producers can use, increases in both countries after trade, and therefore their utility or production increases. Let us emphasize again that they say something increases in both countries after trade.

Upon application of their logic, we show that the intra-industry trade in differentiated intermediate goods increases the variety of intermediate goods used by the high-skill intensive final good in both countries. This directly causes the marginal product of high skill, the demand for high skill by the final good, to shift upward in both countries.

---

5 Dinopoulos and Segerstrom (1999) show that a contemporaneous correlation between an index of the relative price of innovation and an index of the U.S. skill premium was 0.80 during the period 1963-1989.

6 Acemoglu (2002) might not be successful in explaining the fact that the U.S. and Mexico showed the surprisingly similar timing of the rise in skill premium around 1985. This is because the rise in skill premium in Mexico should be driven by the spillover effects in his model but this spillover process usually takes many years.
countries. Consequently, the real wage of high skill rises in both countries. Moreover, if the varieties and high skill are “complements,” the relative wage of high skill, the skill premium, rises in both countries. The intra-industry trade can stimulate the variety-skill complementarity. Thus the wage inequality anomaly is eliminated in our model of trade.

This linking of the intra-industry trade to the wage inequality is supported by available empirical evidence. The intra-industry trade between the U.S. and Mexico was extensive, and the intra-industry trade index between the U.S. and Mexico and the relative wage of high-skilled labor showed surprisingly similar movements in both U.S. and Mexico manufacturing industries during the 1980s. The variety-skill complementarity is an innocuous assumption as shown by the U.S. history of production organization, and the movements of the relative price of high-skill intensive final good and the relative wage of high skill are also consistent with the observations in the U.S.

The rest of this paper is organized as follows. In Section I, we formulate a very simple model of trade in differentiated intermediate goods, and we show that our model can eliminate the wage inequality anomaly. Section II shows that our model is supported by available empirical evidence. Finally, we summarize main results and mention future research in Section III.

I. Model

In this section, we first formulate our model. Second, we explicitly solve the model and show that trade can increase the skill premium in both countries, thus eliminating the wage inequality anomaly. Finally, we mention some economic reasons for the derived results.
A. Ingredients of the Model

Consider an economy with a final good sector and an intermediate goods sector. There are two types of skills: high-skilled and low-skilled labor. Their endowments are given by $H$ and $L$, respectively. These skills differ in that the high-skilled labor can do both high-skill and low-skill tasks while the low-skilled labor can do only a low-skill task. As will be shown later, this excludes the possibility that the relative wage of high-skilled to low-skilled labor is less than one in equilibrium.

The production side is as follows. The final good sector is perfectly competitive and non-traded. It uses the differentiated intermediate goods and high skill. Handling a variety of intermediate goods is a high-skill task, thus requiring the high skill. The technology is given by the following constant returns to scale production function:

$$y = \left[ \int_0^H x(j)^\rho dj \right]^{\varepsilon / \rho} + H^\varepsilon$$

where $y$ is the output of final good, $x(j)$ and $H$ are the demand for differentiated intermediate good $j$ and high skill, and the total number of variety is $n$. We assume that $\varepsilon < 1$ and $0 < \rho < 1$. The elasticity of substitution between the varieties and high skill is given by $\sigma = 1/(1-\varepsilon)$. As can be seen, the production function combines the Dixit-Stiglitz and CES production functions.

On the other hand, the differentiated intermediate goods sector is monopolistically competitive. Firms are symmetric and follow Cournot pricing rules. There is also free entry and exit. The intermediate goods can be traded. Each variety requires a low-skill task, and the technology of each variety is given by the following increasing returns to

---

7 Trade in these differentiated goods is interpreted as intra-industry trade (Dixit and Norman, 1980; Ethier, 1982). Thus in the following discussion, the word “trade” refers to intra-industry trade.
scale production function:

\[
x(j) = \left( \frac{1}{b} \right) \max \left[ l(j) - f, 0 \right], \quad \forall j
\]

where \( l(j) \) is the demand for low skill to produce each variety \( j \), \( f \) is the fixed cost in terms of low skill, and \( b \) is the unit low-skill requirement. Note that part of high skill can also do the low-skill task.

The demand side is as follows. For simplicity, we focus on a representative consumer who has the endowments of high skill and low skill: \( H \) and \( L \). He or she consumes the final good. His or her utility function is given by:

\[
u(c) = c
\]

where \( c \) is the quantity of the final good he or she consumes. His or her budget constraint is given by:

\[
p_y c = w^H H^S + w^L L^S
\]

where \( p_y \) is the price of the final good, \( w^H \) is the wage for the high skill, \( w^L \) is the wage for the low skill. \( H^S \) is the supply of high skill for the final sector, and \( L^S \) is the supply of low skill for the intermediate sector, which can include part of high skill. We assume that \( 0 \leq H^S \leq H \) and \( \bar{L} \leq L^S \leq \bar{L} + H \).

The feasible conditions for the high-skilled and low-skilled labor are:

\[
H = H^S \quad \text{and} \quad \int_0^H l(j) dj = L^S
\]

**B. Explicit Solutions and the Autarky Equilibrium**

We explicitly solve our model. First, we derive the solutions in the intermediate goods sector.\(^8\)

---

\(^8\) More detailed solutions in the intermediate goods sector are shown in Appendix A.
In the short run, given an arbitrary \( n \), each producer of varieties facing the indirect demand by the final good sector maximizes the profit \( p(j)x(j) - w^jbx(j) - w^jf \) where \( p(j) \) is the price of intermediate good \( j \). By setting \( w^j = 1 \) as numeraire and using the symmetry \( x(j) = x \), the short run output \( x \) and price \( \bar{p} \) of each variety can be given by:

\[
\bar{x} = \left[ \frac{b}{p_j n^{(\varepsilon / \rho - 1)}} - n^{\varepsilon / \rho} \right] H, \quad \forall j, \quad \text{and}
\]

\[
\bar{p} = \frac{b}{\rho}, \quad \forall j.
\]

Since the price does not depend on the number of varieties \( n \), the price in the long run, when the profit of each variety becomes zero by the free entry and exit, is also given by \( \bar{p} = b / \rho \), and the zero profit condition \( \bar{p}x - bx - f = 0 \) with \( \bar{p} = b / \rho \) gives the long run output \( \bar{x} \) of each variety. The equality of labor demand and supply in intermediate goods sector, \( \bar{n}(b\bar{x} + f) = L^S \), gives the long run number of varieties \( \bar{n} \). Thus in the long run, the price \( \bar{p} \) and output \( \bar{x} \) of each variety and the number of varieties \( \bar{n} \) are given by:

\[
\bar{p} = \frac{b}{\rho}, \quad \forall j,
\]

\[
\bar{x} = \frac{f\rho}{b(1 - \rho)} \left( \frac{1}{\rho} - 1 \right), \quad \forall j \quad \text{and}
\]

\[
\bar{n} = \frac{L^S(1 - \rho)}{f}.
\]

We next derive the solutions in the final good sector.\(^9\)

In our model with the CES production function, it is not difficult to obtain an explicit solution for the demand for each variety by the final good sector, but we solve the maximization problem for the final good sector by means of the following short-cut

\(^9\) More detailed solutions in the final good sector are shown in Appendix B.
method. Define a new good \( X = \left( \int_0^n x(j)^\rho \, dj \right)^{1/\rho} \) and its price \( p_X \), and we can show desired results more easily.

The profit of the final good sector now becomes:

\[
p_x \left( X^\varepsilon + H^\varepsilon \right)^{1/\varepsilon} - p_x X - w^H H.
\]

Since the new good \( X \) shows the constant returns to scale with varieties \( x(j) \), we have the following equality:

\[
p_x X = \int_0^n p(j)x(j) \, dj.
\]

First, by solving the cost minimization problem for the good \( X \), we find that the price of \( X \) is:

\[
p_x = \left( \int_0^n p(j)^{(\rho+1)/\rho} \, dj \right)^{(\rho-1)/\rho}.
\]

By symmetry \( p(j) = \bar{p} \), this \( p_x \) becomes:

(1) \[
p_x = n^{(\rho-1)/\rho} \bar{p}.
\]

Second, we solve for \( X \). Since the technology of the final good shows the constant returns to scale with \( X \) and \( H \), we have the following equality:

\[
y = \frac{p_x X + w^H H}{p_y}.
\]

On the other hand, the demand for the final good is given by:

\[
c = \frac{w^H H^S + L^S}{p_y}.
\]

The final good market clearing \( y = c \) and the feasible condition for the high skill \( H = H^S \) then give:
\[ p_X X = L^S. \]

Thus \( X \) is given by:

\begin{equation}
X = \frac{L^S}{p_X}
\end{equation}

where \( p_X = n^{(\rho-1)/\rho} \bar{p} \).

Third, we solve for \( w^H \). The first order conditions with respect to \( X \) and \( H \) for the final sector give:

\[ \left( \frac{X}{H} \right)^{\varepsilon-1} = \frac{p_X}{w^H}. \]

By solving for \( w^H \) with (2) and \( H = H^S \), in autarky equilibrium \( w^H \) is given by:

\begin{equation}
w^H = p_X \varepsilon \left( \frac{L^S}{H^S} \right)^{1-\varepsilon}
\end{equation}

where \( p_X = n^{(\rho-1)/\rho} \bar{p} \). Since we have normalized \( w^L = 1 \), the relative wage of high-skilled to low-skilled labor is simply given by \( w^H \).

This autarky equilibrium is represented in Figures 3-a and 3-b. The demand for high skill and low skill by the production side, \( H \) and \( L \), is represented by the iso-quant curve of the final good: \( y = \left[ \frac{L}{p_X} \right]^\varepsilon + H^\varepsilon \) which is given by \( y = \left( X^\varepsilon + H^\varepsilon \right)^{1/\varepsilon} \) and (2). On the other hand, the supply of labor for each sector, \( H^S \) and \( L^S \), is represented by AB. The autarky equilibrium is then achieved at A in Figure 3-a or C in Figure 3-b, and thus the relative wage of high skill \( w^H \), given by the slope of the iso-quant curve, is greater than or equal to one before trade.

Since the focus of this paper is on the skill premium, in the following main text we concentrate on the interesting case as shown in Figure 3-a, in which the relative wage of
high skill given by (3) is *greater than one*. Thus the high skill and low skill each do their own task, letting \( H^S = H \) and \( L^S = L \). In Appendix E, we briefly analyze the case as shown Figure 3-b, in which the relative wage of high skill given by (3) is *one* and the high skill is doing both high-skill and low-skill tasks.

**C. Trade Equilibrium and the Elimination of Wage Inequality Anomaly**

Consider two countries: country 1 and country 2. They have identical technologies and preferences. They can be different in their endowments of high-skilled and low-skilled labor. We assume that the relative wage of high-skilled to low-skilled labor is *greater than one in both countries* before trade as shown in Figure 3-a.

From the derived solutions in the intermediate goods, we easily get the following information. Here, let us focus only on the one country.

The output \( \bar{x} \) and price \( \bar{p} \) of each variety are not changed before and after the intra-industry trade in intermediate goods, and the supply of labor for the intermediate goods sector, which is given by \( L^S = \bar{L} \) before trade, cannot fall below this \( \bar{L} \) after trade. This implies that the number of varieties produced *within* each country, which is given by \( \bar{n} = \bar{L}(1 - \rho) / f \) before trade, does not decrease after trade. Thus the *total* number of varieties, which is available to the final sector after trade, surely *increases* since it is given by the sum of the number of varieties produced within each country after trade.

Given this information, we show the following results.

First, from (1) we see that \( p_x \) decreases after trade since the increase in \( n \) decreases \( n^{(\rho-1)/\rho} \) with \( \rho < 1 \) and \( p_j \) is constant at \( \bar{p} \).

Second, from (2) we see that \( X \) increases after trade since \( p_x \) decreases and \( L^S \),
which is $L$ before trade, does not decrease. This implies that the marginal product of high-skilled labor given by $MPH = (X^\varepsilon + H^\varepsilon)^{(1/\varepsilon)-1}H^{\varepsilon-1}$ increases for any $H$. That is, the demand for high skill by the final good shifts upward. Since the supply of high skill for the final good, which is $\overline{H}$ before trade, does not increase, this implies that the real wage of high skill $w^H / p_y$ increases.

Finally, from (3) we see that if $\varepsilon < 0$ ($\sigma < 1$), that is, if the varieties and high skill are “complements,” the relative wage of high skill $w^H$, the skill premium, increases after trade. This is because $p_X^\varepsilon$ increases and $(L^S / H^S)^{1-\varepsilon}$, which is $(\overline{L} / \overline{H})^{1-\varepsilon}$ before trade, does not decrease.

Thus it follows that the high skill and low skill each do their own task after trade as well. That is, the supply of labor for the final and intermediate sectors remains at $H^S = \overline{H}$ and $L^S = \overline{L}$, respectively. Hence, the number of varieties produced within each country after trade remains at the autarky level $\overline{n} = \overline{L}(1-\rho)/f$.

Note that the above results are also obtained in the other country. Hence, we get the following results.

The intra-industry trade in intermediate goods causes the total number of varieties available to the final good sector to simply increase from $\overline{n}_i$ to $\overline{n}_i + \overline{n}_2$, the sum of the autarky levels, in each country $i = 1$ and $2$. This causes the price $p_{X_i}$ to decline, and this decline in $p_{X_i}$ causes the output $X_i$ to increase in both countries. Consequently, the demand for high skill shifts upward, thus increasing the real wage of high skill in both countries (Figure 4). Moreover, if the varieties and high skill are complements, the decrease in the price $p_{X_i}$ also increases the relative wage of high skill $w_i^H$, the skill

---

10 In this paper we define the case $\varepsilon < 0$ ($\sigma < 1$) as the case where the varieties and the high skill are “complements.”
premium, in both countries. Thus the wage inequality anomaly has been eliminated in our model of trade.

Let us derive more implications from the above argument. First, since the number of varieties before trade is given by $\bar{n}_i = \bar{L}_i (1 - \rho)/f$ in each country $i = 1$ and 2, the ratio of the number of varieties produced within each country before trade is given by $\bar{n}_1/\bar{n}_2 = \bar{L}_1/\bar{L}_2$. This implies that the rate of increase in $\bar{n}_i$ is smaller in an absolutely low-skill abundant country and therefore the rate of decrease in $p_{Xi}$ is also smaller by (1). Hence, the rise in the relative wage of high skill $w^H$ is smaller in an absolutely low-skill abundant country by (3).

Second, if $\varepsilon = 0$ ($\sigma = 1$), that is, if the production function of the final good is given by the Cobb-Douglas function, from (3) we see that the relative wage of high skill $w^H$ is not affected by the decrease in $p_X$ and therefore does not change after trade.

**D. Economic Reasons for the Results**

Before going to Section II, we need to consider economic reasons for some of the results which have been shown in I-C on the basis of the explicit solutions to the model. First, we explain the economic reason why the good $X$ increases after trade, that is, why the $MPH$ increases after trade.

As we have seen, the activities in the intermediate goods sector never change at all in each country after trade. Some changes, however, do occur after trade. The number of varieties used by the final good sector increases, while the quantity of each variety used by the final good sector decreases in each country since each variety is shared by two countries.

Let us compare the marginal products of high-skilled labor before and after trade.
For each country $i = 1$ and $2$, they are given by:

$$MPH_i = \left[ (\bar{n}_i^{1/\rho} x_i^D)^\epsilon + \bar{H}_i^\epsilon \right]^{(1/\epsilon)-1} \bar{H}_i^{\epsilon-1} \quad \text{and}$$

$$MPH_i^* = \left[ (\bar{n}_1 + \bar{n}_2)^{1/\rho} x_i^{D_i^*})^\epsilon + \bar{H}_i^\epsilon \right]^{(1/\epsilon)-1} \bar{H}_i^{\epsilon-1}$$

where $MPH_i$ is the marginal product of high skill, $\bar{n}_i$ is the number of varieties, and $x_i^D$ is the demand for each variety by the final sector in each country $i = 1$ and $2$. Let the asterisk * denote the variables after trade.

According to Appendix C, the input of each variety decreases from $x_i^D = \bar{x}$ to $x_i^{D_i^*} = \left[ \bar{n}_i/(\bar{n}_1 + \bar{n}_2) \right] \times 100 \%$ of $\bar{x}$. On the other hand, as has been shown in I-C, the total number of varieties available to the final good increases simply from $\bar{n}_i$ to $\bar{n}_1 + \bar{n}_2$, so it expands to $\left[ (\bar{n}_1 + \bar{n}_2)/\bar{n}_1 \right] \times 100 \%$ of $\bar{n}_i$.

Can the effect of increase in the number of varieties be canceled by the effect of decrease in the demand for each variety? The answer is No. This is because $1/\rho > 1$. Hence, the effect of increase in the number of varieties is greater than the effect of decrease in the quantity of input. Thus the marginal products of high skill, that is, the demand for high skill, increases in both countries.

Recall that the result presented here relied on the crucial effect in the variety trade models which Ethier (1982) called the “international returns to scale.” That is, $n^{1/\rho} x$ displays that the increased number of inputs, $n$, translates into higher productivity since $1/\rho > 1$.

Next, we explain the economic reason why the relative wage of high skill can rise after trade. Note that the relative wage of high skill in the country $i = 1$ and $2$ is
simply given by \( w_i^H \).

Now the final good market clearings \( y = c \) in each country \( i = 1 \) and \( 2 \) before trade are given by:

\[
y_i = \frac{w_i^H H_i + L_i}{p_{yi}}.
\]

Since \( w_i^H / p_{yi} = MPH_i \), this becomes the following:

\[
y_i = MPH_i \cdot H_i + \frac{1}{p_{yi}} L_i.
\]

As we have seen, the marginal product of high skill increases in country \( i = 1 \) and \( 2 \) after trade. For the same reason, the output of final good also increases in each country \( i = 1 \) and \( 2 \) after trade.

Since \( MPH_i = \left[ \left( \bar{\pi}_i^{1/\rho x_i} \right)^{\varepsilon} + \bar{H}_i \right]^{\varepsilon-1} \bar{H}_i^{\varepsilon-1} \) and \( y_i = \left[ \left( \bar{\pi}_i^{1/\rho x_i} \right)^{\varepsilon} + \bar{H}_i^{\varepsilon} \right]^{1/\varepsilon} \), it can be shown that the rate of increase in \( MPH_i \) is greater than the rate of increase in \( y_i \) if \( \varepsilon < 0 \), that is, if the varieties and high skill are complements. This relation and the final good market clearing condition \( y_i = MPH_i \cdot H_i + \frac{1}{p_{yi}} \cdot L_i \) imply that the rate of increase in \( MPH_i \) should be greater than the rate of change in \( 1/p_{yi} \). In other words, the rate of increase in the real wage of high skill \( w_i^H / p_{yi} \) is greater than the rate of change in the real wage of low skill. Thus the relative wage of high skill can increase in each country \( i = 1 \) and \( 2 \).

**II. Evidence**

In this section, we show that the idea presented in this paper is supported by available empirical evidence and the movements of the relative price of high-skill
intensive final good and the relative wage of high-skilled labor are consistent with the observations.

**A. Intra-industry Trade and the Relative Wage of High-Skilled Labor**

Recall the idea presented in this paper. We have linked the intra-industry trade in differentiated varieties to the wage inequality through the variety-skill complementarity. Hence, the main implications of the model are that the intra-industry trade should be *extensive* and should be *positively* correlated with the relative wage of high-skilled to low-skilled labor, and that the varieties and high-skilled labor should be *complements*.

Table 1 lists the U.S. exports to and U.S. imports from Mexico in 1985 and 1990. The data is obtained from the International Trade Administration. As can be seen, in 1985 three SITC product categories (6, 7, and 8) appear in the top five on both lists, and machinery and transport equipment is 49 percent of U.S. export to and 29 percent of U.S. import from Mexico. As can also be seen, in 1990 four SITC product categories (0, 6, 7, and 8) appear in the top five on both lists, and machinery and transport equipment is 48 percent of U.S. export to and 45 percent of U.S. import from Mexico. This indicates that intra-industry trade between the U.S. and Mexico was *extensive* around 1985 when these two countries showed a surprisingly similar timing in the rise in skill premium. This extensiveness of intra-industry trade got stronger in 1990.

Next, Figures 5-a and 5-b plot the intra-industry trade (IIT) index between the U.S. and Mexico and the relative wage of high-skilled to low-skilled labor in both U.S. and Mexico manufacturing industries during the 1980s, respectively. This IIT index is a weighted average over SITC 3-digit manufacturing industries. IIT index for industry *i* is defined by the following Grubel-Lloyd index:
\[ IIT_i = 1 - \frac{|X_i - M_i|}{(X_i + M_i)} \]  

for industry \( i \)

where \( X_i \) and \( M_i \) represent export and import of industry \( i \). In order to find this index for a country, we compute a weighted average of the \( IIT_i \)'s as follows\(^{11}\):

\[ IIT = 1 - \frac{\sum_i |X_i - M_i|}{\sum_i (X_i + M_i)} \]

The data is obtained from the OECD International Trade by Commodities Statistics (ITCS).

On the other hand, the average annual wage of non-production relative to production workers is used as an index for this relative wage of high-skilled to low-skilled labor in U.S. and Mexico manufacturing industries. The source of data for the U.S. relative wage is the same as for Figure 1. The Mexican relative wage, on the other hand, is calculated by Hanson-Harrison (1995) on the basis of the Mexican Secretariat of Trade and Industrial Promotion (SECOFI).\(^{12}\) This SECOFI survey began in 1984, so we do not have data during the period 1980-1983. It is, however, fortunate for our purpose that the skill premium in Mexico began to rise after the sample period started.

As can be seen, the IIT index and the relative wage of high-skilled labor showed surprisingly similar movements both in the U.S. and in Mexico during the 1980s. In fact, the correlation between the IIT index and the relative wage of high skill was high: it was 0.80 in U.S. manufacturing industries during the 1980s, and in particular, it was 0.91

\(^{11}\) It is possible to relate our model to the work by Herbert G. Grubel and Peter J. Lloyd (1975). We can express their IIT index in terms of the solutions in our model. In fact, the IIT index in our model is simply one as is shown in Appendix D.

\(^{12}\) The Mexican Industrial Census and the SECOFI show almost identical movements (Hanson-Harrison, 1995). In fact, between 1985 and 1988, the relative wage given by the SECOFI rose from 1.917 to 2.116 as shown in Figure 5-b, which matches the movements the Industrial Census shows in Figure 1. Figure 5-b is based on the SECOFI, but we approximated the relative wage in 1980 from the relative wage in 1980 given by the Industrial Census. This approximated relative wage was adjusted so that the relative wage in 1988 is common to both data sources.
and 0.84 in U.S. and Mexico manufacturing industries after 1984, respectively.

Thus the linking of the intra-industry trade to the relative wage of high skill is supported by this evidence both in the U.S and in Mexico.

B. Variety-Skill Complementarity

It is plausible to assume that the increased variety of tasks translates into higher demand for high-skilled labor. In fact, the U.S. history of production organization supports this assumption of variety-skill complementarity.

During the first half of the 20th century, the spread of mass production, which is characterized by Ford’s factories, led to the larger size of manufacturing plants. On the other hand, during the second half of the century, flexible machine tools have allowed plants to operate at a smaller scale. The organization of production has changed from mass production with a traditional assembly line to smaller customized batches, thus making the size of plants smaller.13

Workers on the assembly line have a single routine task to perform; however, workers in each batch are no longer as highly specialized in a single routine task. Each batch is highly customizable and requires a worker who can handle a wide variety of tasks depending on the custom features of the batch. Thus the change in the production organization affected the number of tasks and therefore affected the importance of skills. As the tasks shifted from a single routine task to a wide variety of tasks, the required skill shifted from low skill to high skill. Thus the varieties and high skill have been complements in the history of U.S. production.14

13 Paul Milgrom and John Roberts (1990) show empirical facts on a change in the size of U.S. manufacturing plants.
14 Matthew F. Mitchell (2001) relates a plant size to skills, and he shows how much the change in the plant size can account for the movement in the skill premium over the century.
C. Relative Price of High-Skill Intensive Good

The standard H-O model predicts the same direction of movement of the relative price of high-skill intensive good and the relative wage of high-skilled labor since the rise in the relative wage of high skill should be driven by the rise in the relative price of high-skill intensive good in the high-skill abundant U.S. However, data shows that the relative price of high-skill intensive good was declining or constant during the 1980s while the relative wage of high skill was increasing in the U.S. (Robert Lawrence and Matthew J. Slaughter, 1993).

Our model demonstrates price movement consistent with this observed fact whereas the H-O model cannot. In I-D, it has been shown that the rate of change in $\frac{1}{p_{yi}}$ should be smaller than the rate of increase in $MPH_i$ if $\varepsilon < 0$. This implies that $\frac{1}{p_{yi}}$ can rise (but it should rise less than $MPH_i$) and therefore the price of final good $p_{yi}$ can decline. Here, recall that the price of the low-skill intensive variety is constant before and after trade. Hence, the relative price of high-skill intensive final good $p_{yi}$ can decline while the relative wage of high skill rises, letting $\varepsilon < 0$. Thus the rise in the relative wage of high skill can happen without the rise in the relative price of high-skill intensive final good.15

III. Conclusion and Future Research

The main purpose of this paper has been to eliminate the wage inequality anomaly with a much simpler trade model consistent with empirical evidence.

Section I has presented a simple trade-based resolution of the anomaly. We have

---

15 Note that the price of final good can be constant or increase if $\varepsilon << 0$. 18
shown that the intra-industry trade in differentiated intermediate goods increases the variety of intermediate goods used by the high-skill intensive final good in both countries; as a result, if the varieties and high skill are complements, then the skill premium rises in both countries after trade. Thus intra-industry trade can stimulate variety-skill complementarity.

Section II has shown that our model is consistent with empirical evidence. The linking of the intra-industry trade to the wage inequality is supported by available empirical evidence. The intra-industry trade between the U.S. and Mexico was extensive, and the IIT index between the U.S. and Mexico and the relative wage of high-skilled labor showed surprisingly similar movements in both U.S. and Mexico manufacturing industries during the 1980s. The assumption of variety-skill complementarity is also supported by the U.S. history of production organization, and the rise in the relative wage of high skill can happen without the rise in the relative price of high-skill intensive final good, which is also consistent with the observed fact in the U.S.

It is true that the standard H-O model and its applications are impossible explanations for the rising wage inequality across countries. However, we can no longer say that trade explanations in general are impossible because the model of trade presented in this paper provides a possible trade-based explanation for the rising wage inequality across countries as an alternative to the technology-based explanation. The result that trade can theoretically raise the wage inequality is not necessarily negative since our model shows that the real wage of both high skill and low skill can rise despite the increase in inequality.

Of course, there is still room for future research. First, we shall extend our model to a dynamic model and calibrate it to explain time series data. This data shows the rising
wage inequality over time both in the U.S. and in Mexico. This causes us to consider more the substitutive relation between high skill and low skill. Second, we can analyze the relation between the competition policies and wage inequality. In our model, the change in the number of varieties is related to wage inequality. This implies that government can affect wage inequality by entry policies which adjust the number of firms. Third, our model has been applied to the problems of trade between the U.S. and Mexico, but we can also directly apply it to the problems of intra-trade among EU nations.

Finally, the focus of this paper has been on the elimination of the discrepancy between the standard H-O model and the data during the 1980s. We should next consider whether this discrepancy remained or not after the 1980s. Unfortunately, the movements of the Mexican skill premium during the 1990s are unclear. Raymond Robertson (2004) argues that the skill premium in Mexico significantly declined from 1994 to 1998 on the basis of the Mexican Industrial Census. Unlike the observations during the 1980s, this finding seems consistent with the predictions of the Stolper-Samuelson theorem. However, it can also be shown on the basis of the Mexican Monthly Industrial Survey (EIM) that the Mexican skill premium actually increased over the same period. The Mexican Industrial Census and the EIM thus show movements in the opposite direction over the period 1994-1998, whereas both show a rising trend during the late 1980s. It would seem, therefore, that further investigation is needed in order to analyze the movements of the Mexican skill premium during the 1990s.
Appendix

A. Intermediate Goods

**Short run:** Given arbitrary $n$, derive the indirect demand, $MR = MC$, and find $p(j)$ and $x(j)$.

**Step 1:** Derive the indirect demand of intermediate good $j$.

From the problem of the final good, we get:

$$p(j) = p_y \left[ \frac{1}{\varepsilon} \left( \int_0^n x(j) \rho \, dj \right)^{\varepsilon/\rho} + H^\varepsilon \right]^{(1/\varepsilon)-1} \left( \int_0^n x(j) \rho \, dj \right)^{\varepsilon/\rho-1} \rho x(j)^{\rho-1}.$$

**Step 2:** $MR = MC$.

The differentiated intermediate goods sector is monopolistically competitive with Cournot pricing rules.

Solve the following problem:

$$\max_{x(j)} p(j)x(j) - w^L bx(j) - w^L f.$$

The first order condition w.r.t. $x(j)$ is given by:

$$p_y \left[ \int_0^n x(j) \rho \, dj \right]^{\varepsilon/\rho} + H^\varepsilon \left( \int_0^n x(j) \rho \, dj \right)^{\varepsilon/\rho-1} \rho x(j)^{\rho-1} = w^L b.$$

**Step 3:** Find $x(j)$ and $p(j)$.

By setting $w^L = 1$ as numeraire and using the symmetry $x(j) = \bar{x}$, we get:

$$\bar{x} = \left[ \frac{b}{p_y n^{(\varepsilon/\rho)-1} \rho} \right]^{\varepsilon/(1-\varepsilon)} \left( -n^{\varepsilon/\rho} \right)^{-1/\varepsilon} H, \ \forall j,$$

and

$$\bar{p} = \frac{b}{\rho}, \ \forall j.$$

**Long run:** Zero profit condition $\bar{p} \bar{x} - b \bar{x} - f = 0$ with $\bar{p} = b/\rho$ gives the following long-run output $\bar{x}$ of each variety:
\[ \bar{x} = \frac{f\rho}{b(1 - \rho)}, \quad \forall j. \]

With \( \bar{x} = f\rho / b(1 - \rho) \), the labor market clearing in the intermediate goods sector, \( \overline{n}(b\bar{x} + f) = L^S \), gives the long run number of varieties, \( \overline{n} : \)
\[ \overline{n} = \frac{L^S(1 - \rho)}{f}. \]

### B. Final Good

Define a new good \( X = \left( \int_0^n x(j)^\rho \, dj \right)^{1/\rho} \) and its price \( p_X \). Then the profit of the final good sector becomes:
\[ p_x \left( X^\varepsilon + H^\varepsilon \right)^{1/\varepsilon} - p_X X - w^H H. \]

This new good \( X \) shows the constant returns to scale with varieties \( x(j) \), and therefore we have the following equality:
\[ p_X X = \int_0^n p(j)x(j) \, dj. \]

**Step 1: \( p_X \).**

By solving the following cost minimization problem for the good \( X \), we can find that the price of \( X \) is
\[ p_X = \left( \int_0^n p(j)^{\rho/(\rho - 1)} \, dj \right)^{(\rho - 1)/\rho}. \]

\[ \min \int_0^n p(j)x(j) \, dj \]
\[ s.t., \left( \int_0^n x(j)^\rho \, dj \right)^{1/\rho} \geq X. \]

Define the Lagrangian \( L : \)
\[ L = \int_0^n p(j)x(j)\, dj + \lambda \left( \int_0^n x(j)^\rho \, dj \right)^{1/\rho} - X. \]

Then the first order condition w.r.t. \( x(j) \) gives:
\[ \lambda \left( \int_0^n x(j)^\rho \, dj \right)^{1/\rho} = \int_0^n p(j)x(j)\, dj. \]

Since \( p_x X = \int_0^n p(j)x(j)\, dj \), this implies \( \lambda \) is equivalent to \( p_x \). By solving for \( \lambda \), we get:
\[ \lambda = p_x = \left( \int_0^n p(j)^{\rho/(\rho-1)} \, dj \right)^{(\rho-1)/\rho}. \]

By the symmetry \( p(j) = \bar{p}, \, p_x \) is given by:
(1) \[ p_x = n^{(\rho-1)/\rho} \bar{p}. \]

**Step 2:** \( X \).

Since the technology of the final good shows the constant returns to scale with \( X \) and \( H \), we have the following equality:
\[ y = \frac{p_x X + w^H H}{p_y}. \]

On the other hand, the demand for the final good is given by:
\[ c = \frac{w^H H^s + L^s}{p_y}. \]

Hence, the final good market clearing \( y = c \) and \( H = H^s \) give:
\[ p_x X = L^s. \]

Thus \( X \) is given by:
(2) \[ X = \frac{L^s}{p_x}. \]
where \( p_X = n^{(\rho-1)/\rho} \bar{p} \).

**Step 3:** \( w^H \).

The first order conditions w.r.t. \( X \) and \( H \) for the final sector are given by:

\[
\begin{align*}
p_X \left( X^\varepsilon + H^\varepsilon \right)^{(1/\varepsilon)-1} eX^{\varepsilon-1} &= p_X \\
p_Y \left( X^\varepsilon + H^\varepsilon \right)^{(1/\varepsilon)-1} eH^{\varepsilon-1} &= w^H.
\end{align*}
\]

These give the following:

\[
\left( \frac{X}{H} \right)^{\varepsilon-1} = \frac{p_X}{w^H}.
\]

By solving for \( w^H \) with (2) and \( H = H^S \), \( w^H \) is given by:

\[
w^H = p_X \left( \frac{L^S}{H^S} \right)^{1-\varepsilon}
\]

where \( p_X = n^{(\rho-1)/\rho} \bar{p} \).

**C. Ratio of Demand for Each Variety**

The number of varieties used by the final sector increases only from \( \bar{n}_i \) to \( \bar{n}_1 + \bar{n}_2 \) in each country \( i = 1 \) and \( 2 \) after trade. On the other hand, the quantity of each variety used by the final sector decreases in each country after trade. We now need a balance of trade given by:

\[
\int_0^{\bar{n}_i} p(j)x_i(j)^{D*} \, dj = \int_0^{\bar{n}_1+\bar{n}_2} p(j)x_i(j)^{D*} \, dj
\]

where \( x_i(j)^{D*} \) is the demand for each variety \( j \) by the final sector in country \( i = 1 \) and \( 2 \). Let the asterisk * denote the variables after trade, and let the subscript \( i \) denote the country \( i \). This equality means that the total export value is equal to the total
import value. By symmetry this becomes:

\[ \bar{p}_1 x_2^* D^* = \bar{p}_2 x_1^* D^*. \]

Thus we get:

\[ \frac{x_1^*}{x_2^*} = \frac{\bar{n}_1}{\bar{n}_2}. \]

This means that the ratio of demand for each variety by each country is the ratio of
number of varieties produced in each country: \( \frac{x_1^*}{x_2^*} = \frac{\bar{n}_1}{\bar{n}_2}. \)

**D. IIT Index in Our Model**

In our model, there is only one tradable industry: the intermediate goods industry. We calculate the IIT index for country 1 as follows. The IIT index for country 1 is given by:

\[ IIT_1 = 1 - \frac{|X_1 - M_1|}{(X_1 + M_1)} \]

where \( X_1 = \bar{n}_1 x_1^* \) and \( M_1 = \bar{n}_2 x_1^* \).

Since \( x_1^* = \left[ \bar{n}_1 / (\bar{n}_1 + \bar{n}_2) \right] \bar{x} \) and \( x_2^* = \left[ \bar{n}_2 / (\bar{n}_1 + \bar{n}_2) \right] \bar{x} \), we get:

\[ X_1 - M_1 = \bar{n}_1 \left[ \bar{n}_2 / (\bar{n}_1 + \bar{n}_2) \right] \bar{x} - \bar{n}_2 \left[ \bar{n}_1 / (\bar{n}_1 + \bar{n}_2) \right] \bar{x} = 0. \]

Thus the IIT index for country 1 is simply given by \( IIT_1 = 1 \). Similarly, the IIT index for country 2 is simply given by \( IIT_2 = 1 \).

**E. The Movement of High-Skilled Labor**

In Section I, we have focused on the interesting case in which the relative wage \( w^H \) given by (3) is greater than one before trade. Thus the high skill and low skill each do their own task, letting \( H^S = \bar{H} \) and \( L^S = \bar{L} \). In this Appendix, we briefly analyze the
other case in which the relative wage $w''$ given by (3) is one and the high skill is doing both high-skill and low-skill tasks before trade.

In the autarky equilibrium as shown in Figure 3-b, the relative wage $w''$ given by (3) is one at C, and part of high skill is doing the low-skill task in the intermediate goods sector. This movement of high skill from A to C maximizes the output of final good, that is, the consumer’s utility.

As we have seen in I-C, the case as shown in Figure 3-a let us conclude that the skill premium rises after trade. On the other hand, if it is one before trade as shown in Figure 3-b, it can be shown that the relative wage $w''$ rises or remains after trade, and in any case, the number of varieties used by the final good surely increases.
REFERENCES


Table 1. U.S. Exports to and Imports from Mexico in 1985 and 1990

### 1985

<table>
<thead>
<tr>
<th>Rank</th>
<th>U.S. Exports to Mexico</th>
<th>Percent</th>
<th>U.S. Imports from Mexico</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7 - Machinery and Transport Equipment</td>
<td>49</td>
<td>3 - Mineral Fuels, Lubricants and Related Materials</td>
<td>41</td>
</tr>
<tr>
<td>2</td>
<td>5 - Chemicals and Related Products</td>
<td>11</td>
<td>7 - Machinery and Transport Equipment</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>6 - Manufactured Goods Classified Chiefly by Material</td>
<td>10</td>
<td>0 - Food and Live Animals</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>2 - Crude Materials, Inedible, except Fuels</td>
<td>9</td>
<td>6 - Manufactured Goods Classified Chiefly by Material</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>8 - Miscellaneous Manufactured Articles</td>
<td>7</td>
<td>8 - Miscellaneous Manufactured Articles</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0 - Food and Live Animals</td>
<td>7</td>
<td>5 - Chemicals and Related Products</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>3 - Mineral Fuels, Lubricants and Related Materials</td>
<td>4</td>
<td>9 - Commodities &amp; Transact not Class Elsewhere</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>9 - Commodities &amp; Transact not Class Elsewhere</td>
<td>3</td>
<td>2 - Crude Materials, Inedible, except Fuels</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>4 - Animal and Vegetable Oils, Fats and Waxes</td>
<td>1</td>
<td>1 - Beverages and Tobacco</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1 - Beverages and Tobacco</td>
<td>0</td>
<td>4 - Animal and Vegetable Oils, Fats and Waxes</td>
<td>0</td>
</tr>
</tbody>
</table>

Total 100

### 1990

<table>
<thead>
<tr>
<th>Rank</th>
<th>U.S. Exports to Mexico</th>
<th>Percent</th>
<th>U.S. Imports from Mexico</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7 - Machinery and Transport Equipment</td>
<td>48</td>
<td>7 - Machinery and Transport Equipment</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>6 - Manufactured Goods Classified Chiefly by Material</td>
<td>13</td>
<td>3 - Mineral Fuels, Lubricants and Related Materials</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>8 - Miscellaneous Manufactured Articles</td>
<td>11</td>
<td>8 - Miscellaneous Manufactured Articles</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>5 - Chemicals and Related Products</td>
<td>8</td>
<td>0 - Food and Live Animals</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>0 - Food and Live Animals</td>
<td>7</td>
<td>6 - Manufactured Goods Classified Chiefly by Material</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>9 - Commodities &amp; Transact not Class Elsewhere</td>
<td>6</td>
<td>9 - Commodities &amp; Transact not Class Elsewhere</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>2 - Crude Materials, Inedible, except Fuels</td>
<td>5</td>
<td>2 - Crude Materials, Inedible, except Fuels</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>3 - Mineral Fuels, Lubricants and Related Materials</td>
<td>3</td>
<td>5 - Chemicals and Related Products</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>4 - Animal and Vegetable Oils, Fats and Waxes</td>
<td>0</td>
<td>1 - Beverages and Tobacco</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1 - Beverages and Tobacco</td>
<td>0</td>
<td>4 - Animal and Vegetable Oils, Fats and Waxes</td>
<td>0</td>
</tr>
</tbody>
</table>

Total 100

Source: The International Trade Administration.
Figure 1. Relative Wage of High-Skilled to Low-Skilled Labor in U.S. and Mexico Manufacturing Industries

Note: The average annual wage of non-production relative to production workers is used as an index of the relative wage of high-skilled to low-skilled labor.

Source: Author’s calculations based on the ASM; Hanson and Harrison (1995).
Figure 2. Trade between U.S. and Mexico as Percent of U.S. GDP

Note: Trade is the sum of U.S. exports to and U.S. imports from Mexico.
Source: Author’s calculations based on the International Trade Administration and the Bureau of Economic Analysis.
Figure 3 – a. Autarky Equilibrium with \( w^H > 1 \)

The autarky equilibrium is achieved at A, and the slope at A is \( w^H > 1 \).
The high skill and low skill each do their own task.

Figure 3 – b. Autarky Equilibrium with \( w^H = 1 \)

The autarky equilibrium is achieved at C between A and B, and \( w^H = 1 \).
Part of the high skill denoted by \( H_x \) is doing the low-skill task.
Demand for high skill by final good
\[ = MPH \]
Supply of high skill for final good

Figure 4. Labor Market for High-Skilled Labor

In BOTH Countries
Figure 5-a. IIT Index between U.S. and Mexico and Relative Wage of High-Skilled Labor in U.S. (Manufacturing Industries)

Source: Author’s calculations based on the OECD ITCS and the ASM.

Figure 5-b. IIT Index between U.S. and Mexico and Relative Wage of High-Skilled Labor in Mexico (Manufacturing Industries)

Source: Author’s calculations based on the OECD ITCS; Hanson and Harrison (1995).