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Abstract

This paper incorporates Rational Inattention as defined by Sims (2003a) to a traditional RBC model with multiple sources of uncertainty. Our model distinguishes between transitory and permanent labor and relative investment productivity shocks. The introduction of information frictions works as an endogenous adjustment cost: given the model parameters, the degree of sluggishness of endogenous variables in response to shocks is optimally determined. In practical terms, Rational Inattention increases the volatility and the contemporaneous correlations with output of consumption and decreases those of investment and hours. Moreover, it generates a trade-off between short-run and long-run shock variances. We believe these effects might have important welfare implications and can provide an analytical understanding on the links between business cycle fluctuations and the long-run performance of an economy.

1 Introduction

In recent years, a growing literature has stressed the importance of information frictions as an explanation for the characteristics displayed by macroeconomic time-series. In spite of different microeconomic foundations, those works assume in common that agents are unable to absorb all information available and, therefore, cannot promptly incorporate it into their decisions. Costs associated with processing information generate optimal levels of inattention, which takes shape either as infrequent or incomplete information updating.

Notably, Sims (2003a) proposed that Rational Inattention is a plausible mechanism for introducing sluggishness and delayed responses in economic models. In this framework, economic agents have limited capacity for processing information and, therefore, have to allocate their attention optimally to map different state variables. However, the mapping between economic conditions and decisions regarding, for instance, consumption levels or relative prices is imperfect: decisions take not into account the true state of the economy, but instead the perceived state.

Differently, however, from most examples in the literature of *signal extraction* or *imperfect information* the noise properties are endogenously determined. Information constrained decision-makers set the joint distribution of the perceived states in order to reduce as best as possible the uncertainty about the true states of the economy, subject to limited ability to process new information. Uncertainty is measured by the concept of *entropy*, as suggested by Shannon (1948). This formulation allows the possibility of allocating attention according to the relative importance attributed each variable in the decision-making process. Marćkowiak and Wiederholt (2008), for instance, use a general formulation for pricing decisions and show that firms tend to allocate a high degree of attention to map idiosyncratic variables and a small degree to aggregate ones.

In our formulation, the multiple sources of uncertainty come from aggregate productivity shocks. These shocks differ in the nature of their influence over production factors (whether neutral or factor specific) and on the extent in which effects take place (whether transitory or permanent shocks). Neutral technology shocks are given by simplistic labor productivity shocks, whose effects over the business cycle are well established in the literature. Factor specific shocks are innovations to relative investment productivity, whose relevance in short and long horizons has been extensively discussed by Greenwood *et al* (1998, 2000), Fisher (2006), Justiniano *et al* (2008), among others.

We allow both types of technology innovations to display a transitory and a permanent component. The first takes shape of a mean reverting AR(1) exogenous process and the second of a random walk with drift. decision-makers allocate their attention in order to identify what is the current state of the technology. But because the extent in which they can do it is limited by the information processing constraint, responses to long-term innovations are influenced by the structural parameters of short-term shocks. Higher variances for sort-term technology shocks make it harder to solve the information problem as a whole once it increases the uncertainty about all true states of the economy. In practical terms, we observe delayed responses in coping with long-term technological changes.

In the next Section, we present the complete model. Our benchmark economy consists of a typical RBC model with two different capital stocks: structures and equipment. We use Benigno and Woodfords's (2008) framework to reshape the central planner's problem into a purely linear-quadratic (LQ) one that correctly approximates the equilibrium conditions up to first order. The LQ formulation proves particularly convenient for the inclusion of information frictions because it reduces the dimensionality of the problem of allocating attention. If uncertainty is relatively small, linear approximations can deliver reasonable accurate results even under information frictions.

In Section 3, we present the theoretical results. In our benchmark calibration, the introduction of information frictions increases the volatility of consumption and its correlation with contemporary output. It also decreases the volatility and the contemporary correlation with output of hours and investment. Both features seem a desirable improvement over the benchmark model, resembling the effects obtained with the use of *ad hoc* adjustment costs. In this sense, we believe Rational Inattention and the inclusion of information frictions in a broader perspective can provide a justification for the use of convex adjustment costs to improve the performance of RBC models.

In the long-run, Rational Inattention imposes a trade-off between short-term and long-term volatilities. We suggest this result can impose relevant welfare losses due to short-term fluctuations, which stands as a counter-argument to Lucas (1987). It can also provide an analytical framework to study technology diffusion.

In Section 4, we conclude by pointing out further possibilities to extend our analysis.

2 Model

2.1 Agents

The model is a modified version of Greenwood *et al* (1998, 2000), that has been extensively used by many authors in recent contributions to business cycles research. The economy is populated by a representative household that maximizes the expected value of its infinite lifetime utility and a representative firm which establishes the production level using a technology based on labor hours and two different types of capital stock. The representative household chooses its consumption level (C) and the number of hours to supply the labor market (H) in order to maximize

$$U_{t_0} \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u(C_t, H_t), \tag{1}$$

where β is the discount factor. We assume that $u(.,.)$ is a log-utility function, such that:

$$u(C_t, H_t) = \ln C_t + \theta \ln(1 - H_t). \tag{2}$$

The production of the final good Y_t, in addition to labor hours, requires two types of capital: $K_{e,t}$ and $K_{s,t}$, respectively, equipment and structures, which are taken as given in the beginning of any date. Production obeys the following constant-returns to scale technology

$$Y_t = K_{e,t}^{\alpha_e} K_{s,t}^{\alpha_s} [A_t H_t]^{1-a_e-\alpha_s}, \tag{3}$$

where $\alpha_e, \alpha_s > 0$ and $\alpha_e + \alpha_s < 1$. A_t stands for a labor productivity factor, neutral in terms of the particular kind of capital stock. Aggregate demand and market clearing impose:

$$Y_t = C_t + I_{e,t} + I_{s,t}, \tag{4}$$

where $I_{e,t}$ and $I_{s,t}$ denote, respectively, the total amount invested in equipment and structures, measured in units of aggregate consumption.

Capital accumulation is specific to each type of investment. That is: firms decide how much they intend to invest in each type of capital on the basis of their relative productivities. Hence, the law of motion for capital holdings is given by

$$K_{s,t+1} = (1 - \delta^s)K_{s,t} + I_{s,t} \tag{5}$$

for investment in structures and

$$K_{e,t+1} = (1 - \delta^e)K_{e,t} + I_{e,t}Q_t \tag{6}$$

for investment in equipment, where Q denotes the productivity of investment in equipment relative to investment in structures. One possible interpretation is that Q denotes the current state of technology in equipment, an investment-specific technological factor affecting the incorporation of new equipment to production. Greenwood *et al* (1997) suggest that this kind of technological change accounts for 60% of the postwar growth of output per man-hour. Greenwood *et al* (2000), Fisher (2006) and Justiniano *et al* (2008) stress investment-specific shocks might have as well as important short-run implications, accounting for an equally high percentage of fluctuations in output and hours at business-cycle frequencies. Depreciation rates for capital holdings are such that $0 < \delta^s < \delta^e < 1$.

2.2 Balanced Growth

Consider the exogenous productivity variables, which are the sources of fluctuations in the model. For simplicity, we start by assuming that both variables have their initial levels normalized to unity. Both productivity factors are composed of a transitory (X_t^T) and a permanent (X_t^P) component , or $X_t = X_t^T . X_t^P$, for $X_t = A_{t,}\ Q_t$. The transitory component is an AR(1) mean-reverting stochastic process such that:

$$A_t^T = (A_{t-1}^T)^{\rho_a} \exp(\varepsilon_{a,t}^T), \quad Q_t^T = (Q_{t-1}^T)^{\rho_a} \exp(\varepsilon_{q,t}^T),$$

whereas the permanent component is a random walk with drift (respectively, γ_a and γ_q):

$$A_t^P = A_{t-1}^P \gamma_a \exp(\varepsilon_{a,t}^P); \quad Q_t^P = Q_{t-1}^P \gamma_q \exp(\varepsilon_{q,t}^P).$$

Forcing terms $\varepsilon_{x,t}^{T,P}$ are assumed to be independently and normaly distributed.

This generic formulation allows us to distinguish between short and long-term effects of productivity shocks. In addition, the inclusion of stochastic trends in the technological processes, as advocated by many authors, such as King and Rebelo (1999) and Galí (1999).

The permanent components of productivity shocks given above set the growth (stochastic) trend for model variables. We propose a transformation to make the problem stationary with regard to these stochastic trends. Because there is no labor force growth, the total amount of hours spent in production is stationary while other variables, such as output, consumption and investment, will grow at the same rate. Define $G_{x,t}$ as the growth rate of variable X_t under a stochastic growth path. From (4), for the balanced growth path to be consistent with common growth rates (and constant proportions of investment and consumption over GDP), it is clear that output, consumption, investment in structures and investment in equipment must grow at the same rate, G_t. From (5) and (6), respectively, $G_{k_s,t} = G_t$ and $G_{k_e,t} = Q_t^P G_t$. Using (3), it is possible to determine the value for G_t in terms of the growth trend for exogenous variables:

$$G_t \equiv A_t^P (Q_t^P)^{\alpha_e/1-a_e-\alpha_s}. \tag{7}$$

Consider the following transformation: $\hat{X}_t = X_t/G_t$, for the set of variables $X_t = Y_t$, C_t, $I_{e.t}$, $I_{s,t}$, $K_{s,t}$, and $\hat{K}_{e,t} = K_{e,t}/(G_t Q_t^P)$. Finally, redefine the notation for transitory components of both shocks as $\hat{A}_t$ and $\hat{Q}_t$. Capital hat-variables denote, therefore, deviations from their stochastic trends. The transformed (detrended) policy problem is given by maximizing:

$$U_{t_0} \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^t [\ln \hat{C}_t + \theta \ln(1 - H_t)], \tag{8}$$

subject to:

$$\begin{aligned} & \hat{K}_{e,t}^{\alpha_e} \hat{K}_{s,t}^{\alpha_s} [\hat{A}_t H_t]^{1-a_e-\alpha_s} \\ = \; & \hat{C}_t + g_{e,t+1} \frac{\hat{K}_{e,t+1}}{\hat{Q}_t} - (1-\delta^e) \frac{\hat{K}_{e,t}}{\hat{Q}_t} + g_{s,t+1} \hat{K}_{s,t+1} - (1-\delta^s) \hat{K}_{s,t}, \end{aligned} \tag{9}$$

$$\text{given } \hat{K}_{e,t_0} \text{ and } \hat{K}_{s,t_0}.$$

and where we have defined:

$$g_{s,t} \equiv \gamma_a \gamma_q^{\alpha_e/(1-a_e-\alpha_s)} \exp \left[\varepsilon_{a,t}^P + \frac{\alpha_e}{1-a_e-\alpha_s} \varepsilon_{q,t}^P \right],$$

$$g_{e,t} \equiv \gamma_a \gamma_q^{1+\alpha_e/(1-a_e-\alpha_s)} \exp \left[\varepsilon_{a,t}^P + (1 + \frac{\alpha_e}{1-a_e-\alpha_s}) \varepsilon_{q,t}^P \right].$$

Given the formulation above, Appendix A show that there is a deterministic steady state for the detrended problem characterized by constant values for all endogenous transformed variables and absence of shocks. The deterministic steady state for the detrended problem can be use to generate second order log-approximations for objective function and restrictions. By following the method proposed Benigno and Woodford (2008), it is then possible to redefine the problem described above as a purely quadratic objective function and purely linear restrictions. The new problem characterizes the policy function for endogenous variables that is equivalent to the solution to the original (detrended) problem obtained through a first order approximation of the set of equilibrium conditions. The LQ formulation proves convenient for the addition of information frictions.

Appendix B details the set of procedures involved in recasting policy objective (8) and restrictions expressed in (9) into a quadratic problem with linear restrictions of the form:

$$U_{t_0} = E_{t_0}\left[\sum_{t=0}^{\infty}\beta^{t-t_0}(S_t'AS_t + 2S_t'Dc_t + c_t'Bc_t)\right], \tag{10}$$

$$S_{t+1} = G_1S_t + G_2c_t + G_3\varepsilon_{t+1}. \tag{11}$$

where matrices A, B, D, G_1, G_2 and G_3 are all defined in terms of the structural coefficients from the original model in the Appendix B. The vector c_t includes the set of control variables for the policy problem given in log-deviations: aggregate consumption, labor hours and investment in equipment. S_t includes the state variables also given in log-deviations, such as labor productivity, relative productivity of equipment, and capital stocks of equipment and structures. ε_t amounts for the forcing terms of exogenous processes or innovations in productivities, including innovations to both transitory as well as permanent shocks. As noted above, when defining the evolution of the state variables in (11) we are explicitly making the assumption that transitory shocks follow a mean reverting AR(1) process.

2.3 Approximated Problem with Information Frictions

The standard RBC model as the one characterized in the previous Section assumes that decision-makers have unlimited information-processing capacity and, therefore, are able to timely characterize the state variables of the economy while making decisions. In a more realistic way, agents face limitations either to gather, select or evaluate the relevance of new information. As a consequence, they may fail to fully respond by adjusting their optimal plans to changes in state variables.

By following Sims (2003a), we use the concept of entropy from information theory to characterize the degree of uncertainty over a random variable. The idea initially proposed by Shannon (1948) consists of defining the information flow as the rate of uncertainty reduction (or entropy reduction[1]). Under limited information the social planner cannot observe the true state of the economy without error. Hence, the decision consists of choosing an information structure that reduces the uncertainty of the true state variables subject to the extent that entropy can be reduced. For an upper bound for information capacity given by κ, the decision-maker chooses an optimal signal that reduces the uncertainty of

[1] For any random variable X with pdf $p(X)$, entropy is given by $-E[\log(p(X))]$. If log is considered in its natural base, then the information capacity is given in "nats". If it is in base 2, then the information capacity is given in "bits".

the true state as much as possible. Higher κ imply a higher degree of attention. The formal expression is given by

$$\mathcal{H}(S_t|\mathcal{I}_{t-1}) - \mathcal{H}(S_t|\mathcal{I}_t) \leq \kappa, \tag{12}$$

where $\mathcal{H}(S_t|\mathcal{I}_{t-1})$ denotes the entropy of the state vector prior to observing the signal at t and $\mathcal{H}(S_t|\mathcal{I}_t)$ after observing it (*posterior*), where $\mathcal{I}_t$ denotes the information set available at that date. In the rational inattention theory, proposed by Sims (2003a, 2005), memory can be accessed without any cost so the entire history of past signals is assumed to be known.

The use of a representative agent in our model imply that uncertainty may be regarded as "aggregate uncertainty". As suggested by Sims (2005), a careful application of rational inattention at microestructure level would involve incorporating information constrained decisions to details such as bargainning, inventories, retailers, demand deposits, cash, and others elements, which could make the problem substantially more complex. One important consequence of this choice is the exclusion from the model of strategic complementarity or substitutibility of actions among private agents. As argued by Woodford (2001), Hellwig and Veldkamp (2009) or as stressed in the literature of global games, in a situation where decisions are not only subject to a private degree of information imprecision but also highly dependent of decisions made by other agents, higher order expectations would play a crucial role in determining the degree of sluggishness observed in the dynamics of aggregate economic variables.

Under information constraint the problem faced by the decision-maker can be stated as choosing a sequence of values for control variables and *posteriors* distributions for the true state variables given the observable signals in order to maximize the quadratic objective function in (10) subject to (11) and (12), where the expectation is now considered conditional on the information set $\mathcal{I}_t$. The problem, stated as it is, is infinite dimensional. However, we can use the properties of the LQ approach in order to make it more tractable: in this circumstances, the Rational Inattention approach becomes a tradicional problem of signal extraction.

Let us define $\hat{S}_t$ as the vector for the perceived state variables, which is composed by the sum of a vector of true state variables, S_t, and a vector of endogenous information-processing-induced measurement errors, ζ_t, which is also independent of the fundamental productivity shocks of the economy. By applying certainty equivalence, we first notice that the policy function that emerges from the full information problem is the same as the one from the information constrained problem. The only difference is that in the second case the policy function is linear in the *perceived* state variables $\hat{S}$. Effectively, it is then possible to break the problem into a deterministic part, which can be solved using conventional techniques, and into an information constrained part, which can be used to determine the *posterior* distributions for true state variables.

We assume the distribution of exogenous forcing terms is Gaussian, as well as the distribution of state variables in a distant past. In these circumstances, as shown in the Appendix C, the *posterior* distribution for state variables given the observed signals is also Gaussian, with mean $\hat{S}_t$ and variance Σ_t, where we hereby restrict our attention to the situation in which Σ is constant. The infinite dimension problem reduces to one in which it is just needed to characterize the second moments of the *posterior* distribution. Let us define the time-invariant variance for exogenous processes ε_t as Ω and the variance for *prior* state variables as Ψ[2]. The information constraint in (12) can then be expressed as[3]

$$-\log_2 |\Sigma| + \log_2 |\Psi| \leq 2\kappa. \tag{13}$$

The multiple sources of uncertainty present in the model impose an additional restriction: the difference between the prior and the posterior variance matrix be positive definite, or formally:

$$\Psi \succeq \Sigma. \tag{14}$$

This additional restriction rules out the possibility that information capacity can be trade-off by forgetting some existing information in order to increase the precision of more relevant dimensions of the state vector. As described in the Appendix C, the problem can then be restated as one that minimizes the expected loss from information constrained decisions by choosing Σ, subject to (13) and (14). This is a direct result that follows from the literature of rational inattention by using a quadratic objective function obtained from a second order approximation of the original objective function around a full information steady state.

The evolution of the economy as driven by the exogenous shocks ε_t and noise signals ζ_t is given by the following set of equations:

$$S_t = G_1 S_{t-1} + G_2 C_{t-1} + G_3 \varepsilon_t \tag{15}$$

$$\hat{S}_t = (I - \Sigma\Lambda^{-1})(G_1 + G_2 H_1)\hat{S}_{t-1} + \Sigma\Lambda^{-1}(S_t + \zeta_t) \tag{16}$$

$$C_t = H_1 \hat{S}_t \tag{17}$$

which characterize altogether the paths for state variables, perceived state variables and control variables as linear functions of perceived states. Equation (15) is the law of motion for state variables, (16) is a Kalman-filter-type equation that maps state variables into perceived states and (17) is a linear policy function that maps control variables as a function of perceived states. We have defined Λ as the covariance matrix of the vector of measurement errors induced by the information constraint. For a stationary *posterior* distribution, Λ is determined such that

[2] Defined from (11) as $\Psi = G_1 \Sigma G_1' + \hat{\Omega}$, where $\hat{\Omega} = G_3 \Omega G_3'$.

[3] Using the fact that entropy of a Gaussian distribution is half log of its variance plus some constant term.

$$\Lambda^{-1} = \Sigma^{-1} - \Psi^{-1}. \quad (18)$$

Matrix Λ can be decomposed such that $\Lambda = \Gamma'\hat{\Lambda}\Gamma$, where Γ is a matrix of eigenvectors and $\hat{\Lambda}$ a diagonal matrix of eigenvalues. Given a certain structure for the original variance of shocks Ω, an eigenvalue of proportionally smaller value implies a higher allocation of attention to the corresponding exogenous forcing term. In addition, because Λ is generally a non-diagonal covariance matrix, each element of the perceived state $\hat{S}_t$ is a combination of the true state plus a linear combination of endogenous noises, whose weights are given by the columns of Γ. In other words, the representative household chooses an optimal combination of measurement errors to reduce the uncertainty about each individual true state. *Reducing* the variance of the original shock expressed in Ω has the same effect of *increasing* the information processing capacity. This is because a smaller variance of, for example, labor productivity, implies a greater precision about that state and allows an addition allocation of capacity to increase the precision of other perceived states.

3 Theoretical Results

We start by noticing that the benchmark calibration assigns an uniform set of values for standard deviations of shocks. Trend stationary AR(1) shocks for labor and relative investment productivities, also refereed to, respectively, neutral technology and relative productivity of equipment shocks, have also the same autocorrelation coefficient. This approach allows us compare more directly the impulse response functions under rational inattention for different structural shocks. In particular, convergence towards full information paths occur under more or less the same range of given capacities. As usual, we assume there is no correlation between innovations of productivity shocks.

Appendix D displays the employed values for the benchmark calibration. Some of the parameter choices made depart from Greenwood *et al* (1998, 2000). Intertemporal discount rate takes into account a 4% year steady state interest rate. We set the parameters for capital share of income somewhat higher (35%) and the yearly depreciation rate of capital stock lower (5.5%) in order to obtain a more realistic proportion of steady state capital stock over quarterly GDP (approximately twelve times) without the use of distortive taxation. Preference parameter θ is set in order to obtain an amount of supplied labor hours in steady state of 24% of the total endowment. The share of consumption over GDP is set at 80%, while the remaining share is due to investment. Growth rate for new equipment productivity is set to an yearly increase of 2,5%.

3.1 Business Cycle Dynamics

Figure 1 allows a qualitative assessment on the effect of information constraints over the short-term dynamics of model variables. We explicitly consider several levels of information processing capability, along with the impulse-responses

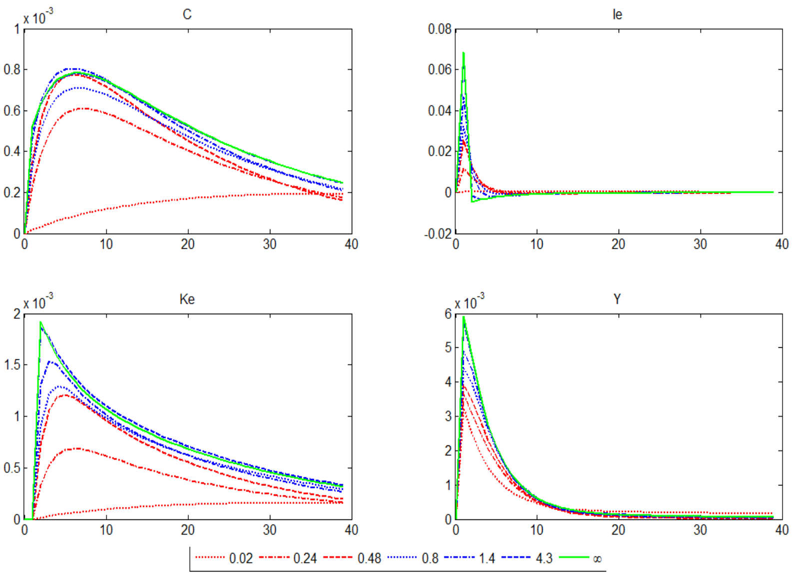

Figure 1: Response of Selected Variables to a one s.d. stationary Labor Productivity Shock.

generated by a one standard deviation shock in labor productivity under perfect information (in green). These effects are well established in the literature: a temporary increase in productivity leads to an increase in consumption and investment in line with the higher level in output. The introduction of information constraint alters the magnitude of such initial responses, while increasing the propagation of these shocks. For significantly small levels of information processing capacity, such as .02 bits, variables are general irresponsive to shocks. An increase in κ leads to convergence towards perfect information responses. For levels of κ higher than 1.4 bits, changes in capacity lead to almost imperceptible improvements.

Appendix E displays the entire set of impulse response functions to shocks considered in the model. It is noteworthy that only neutral technology shocks (labor productivity shocks) can affect output directly. Investment shocks affect output only by increasing the returns of investment and, as a consequence, the capital stock itself. Strategic complementarity between labor and capital compels labor supply to increase, driving the increase in output.

Both short-term shocks in labor and relative investment productivities present relatively similar dynamics: a higher productivity in equipment triggers an expansion in labor hours, investment in equipment and, as a consequence, higher fraction of equipment as a proportion of capital stock is observed. Consumption, however, responds differently: investment shocks are followed by an initial contraction, partially motivated by a higher level of investment in new equipment. In the subsequent periods, consumption increases due to a higher level of capital and hours employed in production. The abnormally higher productivity imposes a reallocation from investment in structures to equipment, which makes both types of capital stocks to move in opposite directions. In the following periods, however, higher equipment stock increases productivity in structures, which leads to a counterflow of investment in structures. Hence, the initial substitution of structures by equipment is followed by a shifts of resources towards the second.

Such features of the full information model may lead to undesirable characteristics. In particular, the initial decrease in consumption during an economic expansion can hardly describe the comovements between aggregate variables and output observed in the data. In addition, the opposite directions in which capital stocks move seems at odds with observed comovement in the level of activity among sectors. In order to cope with these apparent inconsistencies, some authors have explicitly modified the benchmark model with the inclusion of adjustment frictions. Greenwood *et al* (2000) include convex investment costs and obtain a significant improvement of business cycle statistics. Boldrin *et al* (2001) use habit preferences and a technology with limited intersectoral factor mobility to get similar improvements.

We believe the inclusion of information frictions can help mitigate these adverse features. One important effect of limited information is the reduction of initial responses of investment to a shock. In the full information model, this excess volatility of investment induces to swing-shaped responses of consumption to shocks as well as to the initial decrease in the stock of structures. As seen in Figure 1, by reducing the capacity of information processing it is then possible to reduce the excess volatility of investment and obtain smaller initial decreases in consumption, followed by hump-shaped increases. In the case of permanent shocks to investment productivity, the initial drop in consumption is completely reverted.

One important feature of RBC models is the way characteristics of underlying processes are reflected in the behavior of endogenous economic variables. Table 1 presents a set of statistics commonly reported by the RBC literature for model simulations under different degrees of inattentiveness. These results are compared with those derived from the full information model and with their empirical counterparts for US economy, reported by King and Rebelo (1999). Statistics are obtained using a 500 randomly simulated sequences of permanent and transitory shocks over 250 periods. The benchmark calibration of structural parameters are applied to all cases alike.

From the start, it is clear the current choice of parameters falls short in trying to replicate the volatility of empirical macroeconomic time-series. In addition, the full information model performs poorly in trying to replicate both relative volatilities and autocorrelations. In particular, the model displays a negative contemporaneous cross-correlation of consumption with output. When the economy is hit by a relative investment productivity shock, optimal policy precludes a contraction in consumption and a sharp increase in investment and labor hours. This feature is reverted as the information capacity constraint tightens: as uncertainty regarding the nature of the productivity shock becomes more relevant, less pronounced becomes the initial responses of investment and hours and, therefore, less significant becomes the initial drop in consumption.

Table 1: RBC Basic Statistics[a]

	K. R.[b]	Full Inf.	$\kappa = 4.3$	$\kappa = 1.4$	$\kappa = .80$	$\kappa = .24$
Relative S.D.						
σ_y^c	1.81	0.96	1.05	0.89	0.79	0.63
σ_c/σ_y	0.74	0.18	0.52	0.80	0.81	1.02
σ_h/σ_y	0.99	0.78	0.70	0.64	0.55	0.28
σ_i/σ_y	2.93	4.63	4.26	4.31	3.98	3.52
Cross-Correlation						
$\rho(c, y)$	0.88	$-.04$	0.36	0.40	0.50	0.65
$\rho(h, y)$	0.88	0.98	0.92	0.85	0.82	0.67
$\rho(i, y)$	0.80	0.99	0.91	0.79	0.78	0.62
Autocorrelation						
$\rho(\Delta y)$	0.84	0.63	0.66	0.70	0.71	0.68
$\rho(\Delta c)$	0.80	0.67	0.61	0.72	0.68	0.69
$\rho(\Delta h)$	0.88	0.67	0.69	0.77	0.81	0.87
$\rho(\Delta i)$	0.87	0.66	0.68	0.73	0.72	0.66

[a]Point estimates for model simulations using the benchmark calibration for structural shocks. Results are based on 500 replications of sample size 250.

[b]King and Rebelo (1999). Point estimates for per capita variables, using a HP filter over Stock and Watson (1999)'s data set.

[c]In percentage.

In essence, the addition of rational inattention to the standard model works in a similar way as introducing a generic adjustment cost. The difference being, of course, that it is endogenous: given a certain amount of information processing capacity, values for structural shocks and parameters of the economy, the degree the model dynamics depart from full information is determined as a result of a maximization routine.

In broad strokes, information frictions amplify the volatility of consumption and reduce the volatility of hours and investment. They also amplify contemporaneous correlation of consumption with output and reduces of hours and investment. These effects can be verified in Appendix F, where we present the standard errors for the statistics in Table 1. There is, however, no clear pattern emerging from the autocorrelation of model variables, except, perhaps in the case of labor hours. This is a well known caveat of RBC models, reflected is their apparent lack of internal propagation. We believe a more realistic calibration for structural shocks could deliver results closer to the ones displayed in the data. In the next Subsection, we turn our attention to the model's implication for the long-run dynamics of the economy.

3.2 Long-run Implications

3.2.1 Adjustment to Permanent Shocks

Our model provides a framework to study the adjustment dynamic of the economy in response to permanent non-stationary shocks. More interestingly, it enables us to see how short-term volatility, understood as the variances of transitory stationary shocks, may effect these adjustments. We believe these effects might have important welfare implications and can also provide an analytical understanding of links between business cycle fluctuations and the long-run performance of an economy.

In the current formulation, an increase in variances of transitory shocks implies that the economy takes longer to adjust to permanent shocks. The intuition is straightforward. The limited capacity to process information generates adjustment costs. These costs are endogenous, in the sense that they depend on other structural parameters. An increase in the variance of any type of shock makes the problem of processing information and allocating attention harder once any inference on the true state variables given the perceived states becomes more imprecise. Hence, the cost to adjust investment, hours and consumption to changes in technology is higher.

Figure 2 illustrates these ideas. The graph shows the transition path of the main variables in the model in response to a one standard deviation permanent increase in labor productivity. The solid green line stands for the full information case, while the others represent the same transition path under rational inattention for different volatility ratios between transitory and permanent shocks. At all cases, the information processing capacity is kept at constant level ($\kappa = 0.7$).

In the baseline calibration the standard deviations of transitory and permanent shocks are equal. Compared to the full information case, investment responds slowly to a technology shock and consumption is initially higher. As agents incorporate new information about technology into their decisions, they reduce consumption and increase investment. Hence, aggregate capital converges at a slower pace to its after-shock level.

The adjustment speed is even slower when we increase the volatility of the temporary shocks to a ratio of two, five and ten times the standard deviation of permanent shocks. Two factors contribute to the higher sluggishness. As short-term shock volatility increases, the overall information problem becomes harder to solve because of the higher degree of uncertainty about the true state variables. In addition, more attention is allocated to the transitory shocks relative to permanent shocks, increasing the relative uncertain of the second type when compared with the full information case.

Table 2 provides some measures of the importance of these effects. The first column shows that the welfare loss increases with higher variances, what highlights the overall increase in uncertainty mentioned above. The remaining columns demonstrate how different is the adjustment. The higher the variance of the transitory shocks, the longer the economy takes to adjust to the permanent change in productivity, as measured by the half-life and the convergence of

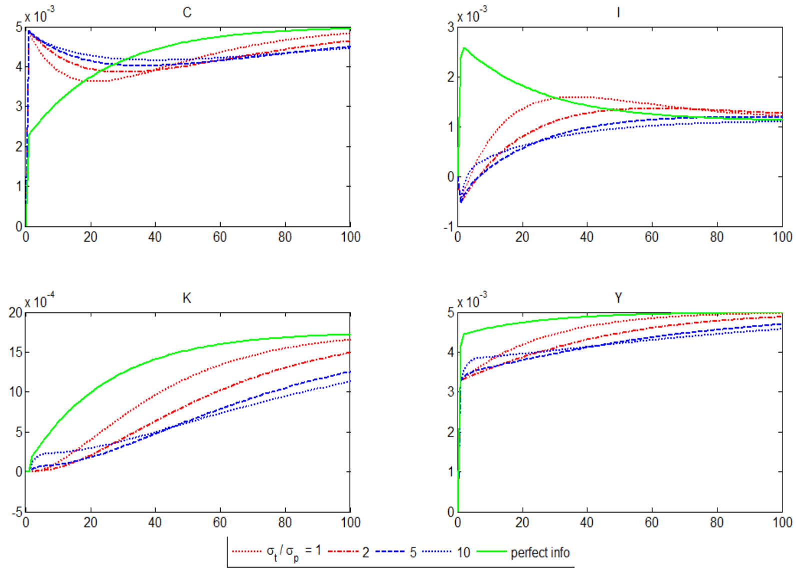

Figure 2: Response of Selected Variables to a one s.d. permanent Labor Productivity Shock.

capital stock to the after-shock capital level. Welfare loss along the transition path also tends to be higher.

We want to highlight the fact that the difference in this long-run adjustment comes from changes in the properties of *transitory shocks*. Clearly, similar effects would occur if we changed the properties of the permanent shocks. However, the former case is more interesting because it shows how the short-run behavior of the economy can affect its long-run dynamics.

Moreover, this property is not shared by other models that include *ad hoc* adjustment costs. This distinction is important: ignoring how adjustment costs may change as a function of other aspects of the economy may jeopardize the understanding on the importance of stabilization policies and the welfare cost of business cycles. Adam (2008) shows that in a model with rational inattention an increased focus by the central bank on stabilizing inflation can reduce variation of real output along the business cycle. Our model would further extend the benefit of price stabilization because the lower variance of these transitory shocks would also effect how the economy responds to permanent shocks. If instead of using rational inattention we included an *ad hoc* adjustment cost to investment, this second effect would not be present.

3.2.2 Technology Diffusion

Our model can also be used to understand the process of technology diffusion, in particular, from developed to emerging economies, which tend to have a more volatile macroeconomic and institutional environment. Consider the case of an economy where the government imposes legal barriers to import equipment, such as high import tariffs or other trade regulations. In addition, assume the investment specific technology within the country is not as well developed as the technology available abroad. Now assume there is a change in policy equivalent to a permanent increase in the investment specific technology. An interesting question is how long the economy takes to incorporate this jump in technology. In particular, what are the welfare implications of such a policy change.

Figure 3 shows the transition paths of consumption, investment in equipment, capital stock in equipment and output in the baseline calibration, for different values of information processing capacity. We choose the size of the shock to be two standard deviations to allow for a large shock while keeping it within a plausible range. Table 2 provides some related statistics.

Once again the economy takes longer to adjust the more strict are the information restrictions. In particular, we can interpret the results as showing that, although lowering trade barriers provides access to a higher technological standard, the diffusion of the new technology is not as fast because other factors demand attention from the agents. Therefore, investment in equipment increases less and capital adjusts at a slower pace.

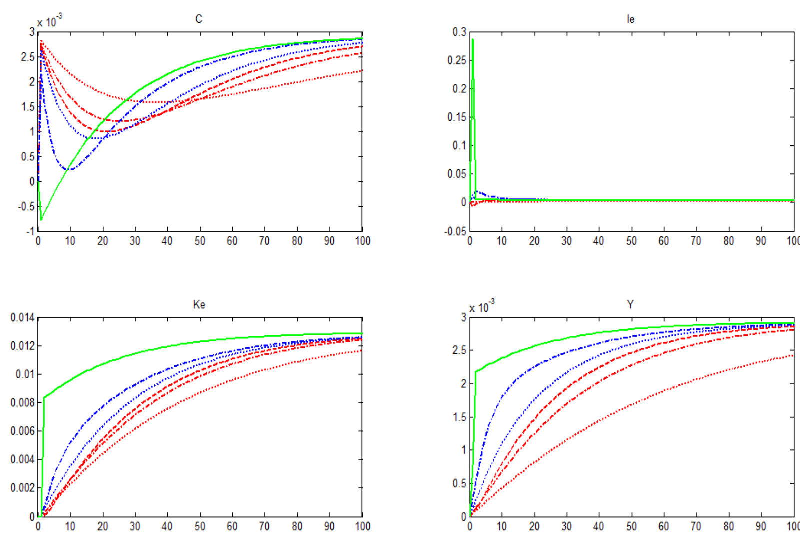

Figure 3: Response of Selected Variables to a one s.d. permanent Relative Investment Productivity Shock.

Why would households and firms not pay attention to technology productivity? Or why would the level of information process capacity devoted to technological aspects considered in our model vary? One possible explanation would be given by incorporating nominal aspects into the model. In this case, agents would allocate a certain amount of their attention to nominal state variables (consider the losses a firms may incur if they do not get their relative prices right, or wages in the case of consumers). In our example, if the country faces a high degree of volatility in nominal variables at the moment trade barriers are removed, such as inflation or nominal exchange rate, agents would allocate a greater amount of their attention to nominal aspects and respond at a slower pace to technological changes.

Although our model abstracts from nominal factors, we can simulate this effect by decreasing the amount of available information processing capacity agents have to allocate to technological processes. As can be seen in Table 2, impacts are significant. The more restrict agents are in their limit to process information, the longer the economy takes to converge to its after-shock level and the higher is the welfare loss in the adjustment path.

4 Final Remarks

This paper presented some preliminary results of a proposal that aims at combining two distinct research agendas: the RBC and the inattentiveness literatures. Our approach consider multiple sources of uncertainty regarding the current technological state of the economy and the effects of transitory and permanent shocks. We believed this set of assumptions delivered elements to consider the work ahead promising.

Our results could be extended in several possible dimensions. The most relevant would be the inclusion of nominal variables and pricing decisions. This approach could provide an assessment of long-term implications of monetary stabilization, an important discussion in macroeconomic research. In particular, it could provide a theoretical linkage between policy and incorporation of new technologies, underlining the effects upon welfare of short-term fluctuations.

An additional possibility is the inclusion of strategic complementarity between agent's decisions. This feature could substantially change our results, in particular, towards obtaining a higher degree in the propagation of shocks. As seen in Section 3, the incorporation of information frictions in the standard model seemed to produce no relevant effects in the autocorrelations of endogenous variables. Hence, exploring the inclusion of this additional characteristic seems desirable, in spite of the fact that it would prevent us from using the tractable representative-agent framework.

Finally, althought the inclusion of Rational Inattention improves some of the usual statistics of business cycle compared with the basic full infomation model considered here, better RBC resulting statistics could be obtained by using a benchmark model with more reasonable parameter values for exogenous shocks. The incorporation of other characteristics such as varying capital uti-

Table 2: Long Run Adjustment Dynamics

	Welfare Loss[a]	Welfare Loss in Transition[b]	Half-life[c]	Convergence[d]
Perm. Shock to Labor[e]				
$\sigma^T/\sigma^P = 1$	.0044	.00012	38	85
$\sigma^T/\sigma^P = 2$	.0170	.00019	53	115
$\sigma^T/\sigma^P = 5$	.1051	.00023	68	158
$\sigma^T/\sigma^P = 10$	.4137	.00018	74	194
Full Information	.	.	18	58
Perm. Shock to Investment[f]				
$\kappa = 0.02$	.0109	.0089	33	101
$\kappa = 0.29$	.0080	.0079	28	77
$\kappa = 0.70$	.0044	.0075	26	72
$\kappa = 1.40$	.0023	.0066	22	67
$\kappa = 4.30$	.0003	.0053	16	62
Full Information	.	.	3	35

[a]Unconditional expected loss, given the model parameters and values for variances of exogenous shocks.

[b]Expected loss relative to full information, conditional on the shock type.

[c]Number of quarters the capital stock take to cover half of the distance between the pre-shock and after-shock levels. For a permanent increase in labor productivity, we consider total capital stock. For a permanent increase in relative investment productivity, we consider capital stock in equipment.

[d]Number of quarters the capital stock take to cover 90% of the distance between the pre-shock and after-shock levels. For a permanent increase in labor productivity, we consider total capital stock. For a permanent increase in relative investment productivity, we consider capital stock in equipment.

[e]The presents statistics are for 1 (one) standard deviation permanent shock to labor productivity.

[f]The presents statistics are for 2 (two) standard deviation permanent shock to relative investment productivity.

lization, as in King and Rebelo (1999), or explicit modeling of labor market, as in Burnside *et al* (1993), could deliver more realistic values for volatilities or cross-correlations with output. In either case, information frictions seem an indispensable component for improving the models's performance.

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5 Appendix A - Definition of Steady State

From the main text, we take the suggested transformation to make the problem stationary. The representative consumer maximizes the transformed utility function:

$$U_{t_0} \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^t [\ln \hat{C}_t + \theta \ln(1 - H_t)],$$

subject to the following restriction set:

$$\begin{aligned} & \hat{K}_{e,t}^{\alpha_e} \hat{K}_{s,t}^{\alpha_s} [\hat{A}_t H_t]^{1-a_e-\alpha_s} \\ = \quad & \hat{C}_t + g_{e,t+1} \frac{\hat{K}_{e,t+1}}{\hat{Q}_t} - (1-\delta^e) \frac{\hat{K}_{e,t}}{\hat{Q}_t} + g_{s,t+1} \hat{K}_{s,t+1} - (1-\delta^s) \hat{K}_{s,t}, \end{aligned}$$

given $\hat{K}_{e,t}$ and $\hat{K}_{s,t}$.

The objective is to show that there is a deterministic steady state for the detrended system above, where all endogenous variables assume constant values. FOCs are given by:

- with respect to $\hat{C}_t$:

$$1/\hat{C}_t = \hat{\lambda}_t; \tag{19}$$

- with respect to H_t:

$$\theta \frac{H_t}{1 - H_t} = \hat{\lambda}_t (1 - a_e - \alpha_s) \hat{K}_{e,t}^{\alpha_e} \hat{K}_{s,t}^{\alpha_s} [\hat{A}_t H_t]^{1-a_e-\alpha_s}; \tag{20}$$

- with respect to $\hat{K}_{e,t+1}$:

$$\frac{\hat{\lambda}_t}{\hat{Q}_t} E_t(g_{e,t+1}) = E_t \hat{\lambda}_{t+1} \beta \{\alpha_e \hat{K}_{e,t+1}^{\alpha_e - 1} \hat{K}_{s,t+1}^{\alpha_s} [\hat{A}_{t+1} H_{t+1}]^{1-a_e-\alpha_s} + \frac{(1-\delta^e)}{\hat{Q}_{t+1}}\}; \tag{21}$$

- with respect to $\hat{K}_{s,t+1}$:

$$\hat{\lambda}_t E_t(g_{s,t+1}) = E_t \hat{\lambda}_{t+1} \beta \{\alpha_s \hat{K}_{e,t+1}^{\alpha_e} \hat{K}_{s,t+1}^{\alpha_s - 1} [\hat{A}_{t+1} H_{t+1}]^{1-a_e-\alpha_s} + (1-\delta^s)\}. \tag{22}$$

These equations, along with restrictions (3), (5), (6), (4) in the main text, can be use to determine the steady state values for endogenous variables. In order to do that, we need to show that FOCs are satisfied for time-invariant Lagrange multipliers. We start by notecing that, once stochastic terms are dropped out, there are no deviations from productivity factors from their (constant) growth trends. Also, given no population growth, $\hat{H} = H$. Other endogenous variables assume constant values. Therefore, we can drop the subscripts and expectation operators. From (19), $\hat{\lambda}$ is constant:

$$1/\hat{C} = \hat{\lambda};$$

We can use this fact over expressions (21) (22), yielding, respectively:

$$\frac{\hat{K}_e}{\hat{Y}} = \alpha_e[g_e\beta^{-1} - (1-\delta^e)]^{-1}, \tag{23}$$

$$\frac{\hat{K}_s}{\hat{Y}} = \alpha_s[g_s\beta^{-1} - (1-\delta^e)]^{-1}, \tag{24}$$

where $g_s = \gamma_a\gamma_q^{\alpha_e/(1-\alpha_e-\alpha_s)}$ and $g_e = \gamma_a\gamma_q^{1+\alpha_e/(1-\alpha_e-\alpha_s)}$.These establish the steady state level of captial stocks over GDP in terms of exogenous parameters, where we have used the relation in (3). Investment over GDP can be established from (5) and (6):

$$\frac{\hat{I}_e}{\hat{Y}} = [g_e - (1-\delta^e)]\frac{\hat{K}_e}{\hat{Y}}, \tag{25}$$

$$\frac{\hat{I}_s}{\hat{Y}} = [g_s - (1-\delta^s)]\frac{\hat{K}_s}{\hat{Y}}. \tag{26}$$

Using the previous results, we can use the demand equation (4) in order to determine consumption over GDP in terms of exogenous parameters:

$$1 = \frac{\hat{C}}{\hat{Y}} + \frac{\hat{I}_e}{\hat{Y}} + \frac{\hat{I}_s}{\hat{Y}}. \tag{27}$$

From (20), it is then possible to establish the steady state level of labor hours using the previous result:

$$\theta\frac{H}{1-H}\frac{\hat{C}}{\hat{Y}} = (1 - a_e - \alpha_s). \tag{28}$$

Finally, it is possible to recover the level of output in steady state using (3) and the previous results:

$$\hat{Y}^{1-a_e-\alpha_s} = \left(\frac{\hat{K}_e}{\hat{Y}}\right)^{\alpha_e}\left(\frac{\hat{K}_e}{\hat{Y}}\right)^{\alpha_s}[H]^{1-a_e-\alpha_s}. \tag{29}$$

6 Appendix B - The Quadratic Policy Problem

6.1 Second Order Approximation to Objective Function and Retrictions

From the previous section, we use the deterministic steady state for the detrended problem in order to establish an approximation point, hereby characterized by hat-variables *without* the subscript t. We follow Benigno and Woodford (2006, 2008) by applying a second order Taylor expansion for the objective function and restrictions. The objective is to define a purely quadratic approximation to the objective function and a set o linear restrictions that result on policy functions for the policy problem, equivalent to the ones produced by a second order approximation for both objective function and restrictions.

Second order approximation on objective function yields:

$$u(\hat{C}_t, H_t) = \tilde{C}_t - \theta\varphi\tilde{H}_t - \frac{1}{2}\tilde{C}^2 + \frac{\theta}{2}\varphi^2\tilde{H}_t^2 + tips + O_p^3,$$

where "*tips*" stands for "*terms independent of policy*" and φ is defined by $\varphi \equiv H/(1-H)$. Also, for any original variable X_t, denote:

$$\tilde{X}_t = \frac{(\hat{X}_t - \hat{X})}{\hat{X}},$$

where $\hat{X}$ (without t-subscript) denotes the steady state level for the detrended problem described in the previous section and $\hat{X}_t$ the detrended variable itself. The following relation applies up to second order:

$$\tilde{X}_t = \hat{x}_t + \frac{1}{2}\hat{x}_t^2,$$

where

$$\hat{x}_t = \ln(\hat{X}_t/\hat{X}).$$

Substitution on the original expression results:

$$u(\hat{C}_t, H_t) = c_t - \theta\varphi h_t - \frac{\theta\varphi}{2}(1+\varphi)h_t^2 + tips + O_p^3, \tag{30}$$

which give variables in terms of log-deviations from their steady state levels. We proceed by log-linearizing the restrictions to the policy problem.

- Technology:

$$\begin{aligned} \hat{y}_t + \frac{1}{2}\hat{y}_t^2 &= \alpha_e\hat{k}_{e,t} + \alpha_s\hat{k}_{s,t} + (1-\alpha_e-\alpha_s)(\hat{a}_t + \hat{h}_t) + \\ &+ \frac{1}{2}\left[\alpha_e\hat{k}_{e,t} + \alpha_s\hat{k}_{s,t} + (1-\alpha_e-\alpha_s)(\hat{a}_t + \hat{h}_t)\right]^2 + O_p^3. \end{aligned} \tag{31}$$

- Law of motion for stocks on equipment and structures:

$$
\begin{aligned}
\frac{\hat{I}_s}{\hat{K}_s}[\hat{\imath}_{s,t}+\frac{1}{2}\hat{\imath}_{s,t}^2] &= g_s(\hat{k}_{s,t+1}+\varepsilon_{a,t+1}^P+\frac{\alpha_e}{\alpha_h}\varepsilon_{q,t+1}^P)-(1-\delta^s)\hat{k}_{s,t} \\
&+\frac{1}{2}[g_s(\hat{k}_{s,t+1}+\varepsilon_{a,t+1}^P+\frac{\alpha_e}{\alpha_h}\varepsilon_{q,t+1}^P)^2+ \\
&-(1-\delta^s)\hat{k}_{s,t}^2]+O_p^3
\end{aligned} \tag{32}
$$

and

$$
\begin{aligned}
\frac{\hat{I}_e}{\hat{K}_e}[\hat{\imath}_{e,t}+\frac{1}{2}\hat{\imath}_{e,t}^2] &= g_e[\hat{k}_{e,t+1}-\hat{q}_t+\varepsilon_{a,t+1}^P+(1+\frac{\alpha_e}{\alpha_h})\varepsilon_{q,t+1}^P]+ \\
&-(1-\delta^e)\hat{k}_{e,t} \\
&+\frac{1}{2}\{g_e[\hat{k}_{e,t+1}-\hat{q}_t+\varepsilon_{a,t+1}^P+(1+\frac{\alpha_e}{\alpha_h})\varepsilon_{q,t+1}^P]^2+ \\
&-(1-\delta^e)(\hat{k}_{e,t}-\hat{q}_t)^2\}+O_p^3,
\end{aligned} \tag{33}
$$

where we have defined for notational convenience:

$$\alpha_h \equiv 1-\alpha_e-\alpha_s.$$

- Finally, second-order approximation on demand equation yields:

$$\hat{y}_t+\frac{1}{2}\hat{y}_t^2 = s_c\hat{c}_t+s_{i_s}\hat{\imath}_{s,t}+s_{i_e}\hat{\imath}_{e,t}+\frac{1}{2}\left[s_c\hat{c}_t^2+s_{i_s}\hat{\imath}_{s,t}^2+s_{i_e}\hat{\imath}_{e,t}^2\right]+O_p^3. \tag{34}$$

where:

$$s_c \equiv \frac{\hat{C}}{\hat{Y}},$$

$$s_{i_e} \equiv \frac{\hat{I}_e}{\hat{Y}},$$

$$s_{i_s} \equiv \frac{\hat{I}_s}{\hat{Y}}.$$

We can combine expressions in (31) and (34) with (32), and use (33) in order to obtain a set of two restrictions, respectively:

$$
\begin{aligned}
0 = & -\alpha_e \hat{k}_{e,t} - \alpha_s \hat{k}_{s,t} - \alpha_h(\hat{a}_t + \hat{h}_t) + s_c \hat{c}_t + s_{i_e} \hat{\imath}_{e,t} + \\
& + s_{k_s} g_s (\hat{k}_{s,t+1} + \varepsilon^P_{a,t+1} + \frac{\alpha_e}{\alpha_h} \varepsilon^P_{q,t+1}) - s_{k_s}(1-\delta^s)\hat{k}_{s,t} \\
& - \frac{1}{2}\left[\alpha_e \hat{k}_{e,t} + \alpha_s \hat{k}_{s,t} + \alpha_h(\hat{a}_t + \hat{h}_t)\right]^2 + \frac{s_c}{2}\hat{c}_t^2 + \frac{s_{i_e}}{2}\hat{\imath}_{e,t}^2 \\
& + \frac{s_{k_s}}{2}[g_s(\hat{k}_{s,t+1} + \varepsilon^P_{a,t+1} + \frac{\alpha_e}{\alpha_h}\varepsilon^P_{q,t+1})^2 - (1-\delta^s)\hat{k}_{s,t}^2] + O_p^3,
\end{aligned} \tag{35}
$$

and

$$
\begin{aligned}
0 \quad = \quad & -s_{i_e}(\hat{\imath}_{e,t} + \hat{q}_t) - \frac{s_{i_e}}{2}(\hat{\imath}_{e,t} + \hat{q}_t)^2 - s_{k_e}(1-\delta^e)\hat{k}_{e,t} + \\
& + s_{k_e} g_e (\hat{k}_{e,t+1} + \varepsilon^P_{a,t+1} + (1 + \frac{\alpha_e}{\alpha_h})\varepsilon^P_{q,t+1}) + \\
& + \frac{s_{k_e}}{2}[g_e(\hat{k}_{e,t+1} + \varepsilon^P_{a,t+1} + (1+\frac{\alpha_e}{\alpha_h})\varepsilon^P_{q,t+1})^2 - (1-\delta^e)\hat{k}_{e,t}^2] \\
& + O_p^3.
\end{aligned} \tag{36}
$$

where:

$$
s_{k_s} \equiv \frac{\hat{K}_s}{\hat{Y}},
$$

$$
s_{k_e} \equiv \frac{\hat{K}_e}{\hat{Y}}.
$$

By adding and subtracting the proper terms and using the definition for steady state variables, (35) can be written recursively, such that:

$$
V_{s,t} = F(\hat{c}_t, \hat{h}_t, \hat{\imath}_{e,t}, \hat{k}_{e,t}, \hat{k}_{s,t}, \boldsymbol{\xi}_t) + \beta V_{s,t+1}, \tag{37}
$$

where $F(.)$ is a linear-quadratic function of log-deviation of endogenous variables and the vector of exogenous shocks $\boldsymbol{\xi}_t$ at t, defined as:

$$
\begin{aligned}
F(.) \quad = \quad & \bar{f}\{-\alpha_e \hat{k}_{e,t} - \alpha_h \hat{h}_t + s_c[\hat{c}_t + \frac{1}{2}\hat{c}_t^2] + s_{i_e}[\hat{\imath}_{e,t} + \frac{1}{2}\hat{\imath}_{e,t}^2] + \\
& + \frac{\alpha_s}{2}\hat{k}_{s,t}^2 - \frac{1}{2}[\alpha_e \hat{k}_{e,t} + \alpha_s \hat{k}_{s,t} + \alpha_h(\hat{a}_t + \hat{h}_t)]^2\} + \\
& + tips + O_p^3
\end{aligned}
$$

where:

$$
\bar{f} \equiv [\alpha_s + s_{k_s}(1-\delta^s)]^{-1},
$$

and the pre-determined term $V_{s,t}$ is defined as

$$V_{s,t} \equiv \hat{k}_{s,t} + \frac{1}{2}\hat{k}_{s,t}^2.$$

One could notice that interactions between current capital stock and i.i.d innovations of permanent shocks have been included at "*tips*".

Proceeding in an analogous way for (36), we have

$$V_{e,t} = G(\hat{c}_t, \hat{h}_t, \hat{\imath}_{e,t}, \hat{k}_{e,t}, \hat{k}_{s,t}, \boldsymbol{\xi}_t) + \beta V_{e,t+1}. \tag{38}$$

where:

$$\begin{aligned} G(.) \quad = \quad & \bar{g}\{-s_{i_e}[(\hat{\imath}_{e,t} + q_t) + \frac{1}{2}(\hat{\imath}_{e,t} + q_t)^2] + \\ & +\alpha_e[\hat{k}_{e,t} + \frac{1}{2}\hat{k}_{e,t}^2]\} + tips + O_p^3, \end{aligned}$$

$$\bar{g} \equiv [\alpha_e + s_{k_e}(1-\delta^e)]^{-1},$$

$$V_{e,t} \equiv \hat{k}_{e,t} + \frac{1}{2}\hat{k}_{e,t}^2.$$

6.2 Elimination of Linear terms

By follow Benigno and Woodford (2008) we can use matrix notation in order to rewrite expressions (30), (37) and (38). We start by notecing the corresponding log-approximation of the (detrended) policy problem can be stated in the following way:

$$\max_{\boldsymbol{c}_t} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u(c_t) \tag{39}$$

subject to intertemporal restrictions

$$F_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} F(c_t, k_t, \boldsymbol{\xi}_t) \tag{40}$$

and

$$G_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} G(c_t, k_t, \boldsymbol{\xi}_t), \tag{41}$$

where F_{t_0} and G_{t_0} are predetermined terms at t_0 and therefore independent of policy from that date on. Notice that (40) and (41) are obtained by iterating forward expressions (37) and (38). Vector definitions are:

$$c_t = \begin{bmatrix} \hat{c}_t \\ \hat{h}_t \\ \hat{i}_{e,t} \end{bmatrix};\ k_t = \begin{bmatrix} \hat{k}_{e,t} \\ \hat{k}_{s,t} \end{bmatrix};\ \boldsymbol{\xi}_t = \begin{bmatrix} \hat{a}_t \\ \hat{q}_t \\ \varepsilon^P_{a,t} \\ \varepsilon^P_{q,t} \end{bmatrix}.$$

Equation (39) can be expressed in matrix notation as:

$$U_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [U_c.c_t + \frac{1}{2} c_t' U_{cc}.c_t] + tips + O_p^3. \tag{42}$$

The following definitions for the underlined terms apply:

$$U_c = \begin{bmatrix} 1 & -\theta\varphi & 0 \end{bmatrix};$$

$$U_{cc} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\theta\varphi[1-\varphi] & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

In the same fashion, restriction (40) is expressed as:

$$\begin{aligned} 0 \quad = \quad & \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ \bar{\lambda}(F_c.c_t + F_k.k_t) + \frac{1}{2} \bar{\lambda}[c_t' F_{cc}.c_t + \\ & +2c_t' F_{c\xi}.\boldsymbol{\xi}_t + 2k_t' F_{k\xi}.\boldsymbol{\xi}_t + k_t' F_{kk}.k_t + 2c_t' F_{ck}.k_t]\} \\ & -\bar{\lambda} F_{t_0} + tips + O_p^3 \end{aligned} \tag{43}$$

where $\bar{\lambda}$ is the associated Lagrange multiplier to be determined. Recall the definition of $\bar{f}$ as

$$\bar{f} \equiv [\alpha_s + s_{k_s}(1-\delta^s)]^{-1}.$$

Matrices can then be expressed as:

$$F_c = \bar{f}. \begin{bmatrix} s_c & -\alpha_h & s_{i_e} \end{bmatrix};$$

$$F_k = \bar{f}. \begin{bmatrix} -\alpha_e & 0 \end{bmatrix};$$

$$F_{cc} = \bar{f}. \begin{bmatrix} s_c & 0 & 0 \\ 0 & -\alpha_h^2 & 0 \\ 0 & 0 & s_{i_e} \end{bmatrix};$$

$$F_{kk} = \bar{f}. \begin{bmatrix} -\alpha_e^2 & -\alpha_e\alpha_s \\ -\alpha_e\alpha_s & -\alpha_s(\alpha_s - 1) \end{bmatrix};$$

$$F_{c\xi} = \bar{f}. \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\alpha_h^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$F_{k\xi} = \bar{f}.\begin{bmatrix} -\alpha_e\alpha_h & 0 & 0 & 0 \\ -\alpha_s\alpha_h & 0 & 0 & 0 \end{bmatrix};$$

$$F_{ck} = \bar{f}.\begin{bmatrix} 0 & 0 \\ -\alpha_e\alpha_h & -\alpha_s\alpha_h \\ 0 & 0 \end{bmatrix}.$$

Proceeding in the same way, restriction (41) can be expressed:

$$\begin{aligned} 0 \quad = \quad & \sum_{t=t_0}^{\infty} \beta^{t-t_0}\{\bar{\vartheta}(G_c.c_t + G_k.k_t) + \frac{1}{2}\bar{\vartheta}[c_t' G_{cc}.c_t + \\ & +2c_t' G_{c\xi}.\boldsymbol{\xi}_t + 2k_t' G_{k\xi}.\boldsymbol{\xi}_t + k_t' G_{kk}.k_t + 2c_t' G_{ck}.k_t]\} \\ & -\bar{\vartheta}G_{t_0} + tips + O_p^3 \end{aligned} \tag{44}$$

where notatinal choices are analogous as above and $\bar{\vartheta}$ is the associate Lagrange multiplier to be determined. Notecing that we have defined:

$$\bar{g} = [\alpha_e + s_{k_e}(1-\delta^e)]^{-1}.$$

Matrices can then be defined:

$$G_c = \bar{g}.\begin{bmatrix} 0 & 0 & -s_{i_e} \end{bmatrix};$$

$$G_k = \bar{g}.\begin{bmatrix} \alpha_e & 0 \end{bmatrix};$$

$$G_{cc} = \bar{g}.\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -s_{i_e} \end{bmatrix};$$

$$G_{kk} = \bar{g}.\begin{bmatrix} \alpha_e & 0 \\ 0 & 0 \end{bmatrix};$$

$$G_{c\xi} = \bar{g}.\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -s_{i_e} & 0 & 0 \end{bmatrix};$$

$$G_{k\xi} = \vec{0};$$

$$G_{ck} = \vec{0}.$$

Constants $\bar{\lambda}$ and $\bar{\vartheta}$ are then defined in such a way that the following holds:

$$\bar{\lambda}(F_c + F_k) + \bar{\vartheta}(G_c + G_k) = -U_c.$$

The solution is, therefore:

$$\bar{\lambda} = -\frac{1}{\bar{f}.s_c} \tag{45}$$

and

$$\bar{\vartheta} = -\frac{1}{\bar{g}.s_c} \tag{46}$$

By using the definitions for $\bar{\lambda}$ and $\bar{\vartheta}$, it is possible to show that the following relation holds up to second order:

$$\begin{aligned} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [U_c.c_t] \quad = \quad & \frac{1}{2} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [c_t' H_{cc}.c_t + k_t' H_{kk}.k_t + \\ & +2c_t' R.k_t + 2c_t' Z_{c\xi}.\boldsymbol{\xi}_t + 2k_t' Z_{k\xi}.\boldsymbol{\xi}_t] + T_0 + tips, \end{aligned}$$

while we have defined the new terms as:

$$H_{cc} \equiv \bar{\lambda} F_{cc} + \bar{\vartheta} G_{cc},$$

$$H_{kk} \equiv \bar{\lambda} F_{kk} + \bar{\vartheta} G_{kk},$$

$$R \equiv \bar{\lambda} F_{ck} + \bar{\vartheta} G_{ck},$$

$$Z_{c\xi} \equiv \bar{\lambda} F_{c\xi} + \bar{\vartheta} G_{c\xi},$$

$$Z_{k\xi} \equiv \bar{\lambda} F_{k\xi} + \bar{\vartheta} G_{k\xi},$$

and

$$T_0 = -(\bar{\lambda} F_{t_0} + \bar{\vartheta} G_{t_0}).$$

Plugging this last expression into (42), yields:

$$U_{t_0} = \frac{1}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [c_t' Q.c_t + k_t' H_{kk}.k_t + 2c_t' R.k_t + 2c_t' Z_{c\xi}.\boldsymbol{\xi}_t + 2k_t' Z_{k\xi}.\boldsymbol{\xi}_t], \tag{47}$$

where:

$$Q \equiv U_{cc} + H_{cc},$$

and Z_{xy} and R are defined elsewhere.

6.3 Additional Simplifications

Consider now the following definition for the vector of state variables S_t and control variables:

$$S_t \equiv \begin{bmatrix} \hat{k}_{e,t} \\ \hat{k}_{s,t} \\ \hat{a}_t \\ \hat{q}_t \end{bmatrix}; \qquad c_t \equiv \begin{bmatrix} \hat{c}_t \\ \hat{h}_t \\ \hat{\imath}_{e,t} \end{bmatrix}.$$

Expression (47) can finally be expresses as a quadratic objective function in terms of control and state variables as

$$U_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ c_t' B c_t + 2c_t' D S_t + S_t' A S_t \right\}, \tag{48}$$

where:

$$B \equiv Q$$

$$A \equiv \begin{bmatrix} H_{kk} & Z_{k\xi} \\ Z_{k\xi}' & tips \end{bmatrix}$$

and

$$D \equiv \begin{bmatrix} R & Z_{c\xi} \end{bmatrix}.$$

In addition to the objective function given in (48), the decisionmaker is subject to the following set of linear constraints,

$$\begin{aligned} \hat{k}_{e,t+1} &= \frac{(1-\delta^e)}{g_e}\hat{k}_{e,t} + \frac{s_{i_e}}{s_{k_e} g_e}\hat{\imath}_{e,t} + \\ &\quad +[1 - \frac{(1-\delta^e)}{g_e}]\hat{q}_t - \varepsilon^P_{a,t+1} - (1 + \frac{\alpha_e}{\alpha_h})\varepsilon^P_{q,t+1}, \end{aligned}$$

$$\begin{aligned} \hat{k}_{s,t+1} &= \frac{(1-\delta^s)}{g_s}\hat{k}_{s,t} + \frac{1}{g_s s_{k_s}}[\alpha_e \hat{k}_{e,t} + \alpha_s \hat{k}_{s,t} + \alpha_h(\hat{a}_t + \hat{h}_t) + \\ &\quad -s_c \hat{c}_t - s_{i_e}\hat{\imath}_{e,t}] - \varepsilon^P_{a,t+1} - \frac{\alpha_e}{\alpha_h}\varepsilon^P_{q,t+1}. \end{aligned}$$

and the following AR(1) processes for stationary component of exogenous shocks:

$$\hat{a}_{t+1} = \rho_a \hat{a}_t + \varepsilon^T_{a,t+1},$$

$$\hat{q}_{t+1} = \rho_q \hat{q}_t + \varepsilon^T_{q,t+1}.$$

Restrictions can then be writen in matrix notation according to the following:

$$S_{t+1} = G_1 S_t + G_2 c_t + G_3 \varepsilon_{t+1}. \tag{49}$$

where ε_t stands for a vector of i.i.d. innovations to shocks, or $\varepsilon_t = [\varepsilon^T_{a,t}, \varepsilon^T_{q,t}, \varepsilon^P_{a,t}, \varepsilon^P_{q,t}]'$. Expression above represents the law of motion for state variables in terms of its past values, present control variables and exogenous shocks. Matrices are defined by:

$$G_1 \equiv \begin{bmatrix} G^1_{11} & G^1_{12} \\ 0_{(2x2)} & G^1_{22} \end{bmatrix};$$

$$G_2 \equiv \begin{bmatrix} G^2_{11} & G^2_{12} \\ 0_{(2x2)} & 0_{(1x2)} \end{bmatrix};$$

$$G_3 \equiv \begin{bmatrix} 0_{(2x2)} & G^3_{12} \\ I_{(2x2)} & 0_{(2x2)} \end{bmatrix};$$

where I is an identity matrix. In particular, we have:

$$G^1_{11} = \begin{bmatrix} (1-\delta^e)/g_e & 0 \\ \alpha_e/(s_{k_s} g_s) & \alpha_s/(s_{k_s} g_s) + (1-\delta^s)/g_s \end{bmatrix};$$

$$G^1_{12} = \begin{bmatrix} 0 & 1-(1-\delta^e)/g_e \\ \alpha_h/(s_{k_s} g_s) & 0 \end{bmatrix};$$

$$G^1_{22} = \begin{bmatrix} \rho_a & 0 \\ 0 & \rho_q \end{bmatrix};$$

$$G^2_{11} = \begin{bmatrix} 0 & 0 \\ -s_c/(s_{k_s} g_s) & \alpha_h/(s_{k_s} g_s) \end{bmatrix};$$

$$G^2_{12} = \begin{bmatrix} s_{i_e}/(s_{k_e} g_e) \\ -s_{i_e}/(s_{k_s} g_s) \end{bmatrix};$$

$$G^3_{12} = \begin{bmatrix} -1 & -(1+\alpha_e/\alpha_h) \\ -1 & -\alpha_e/\alpha_h \end{bmatrix}.$$

The final problem reduces to maximize(48) subject to (49).

7 Appendix C - Derivation of Optimal Signal

7.1 Value Function

The problem with full information is given by:

$$\max_{c_t} E_{t_0} \left[\sum_{t=0}^{\infty} \beta^{t-t_0} (S'_t A S_t + 2 S'_t D c_t + c'_t B c_t) \right]$$

s.t.:

$$S_{t+1} = G_1 S_t + G_2 c_t + G_3 \varepsilon_{t+1}$$

where c_t is the vector of control variables and S_t the full vector of state variables. We assume further that ε_t has a multidymensional Normal distribution, such that $E(\varepsilon_t) = 0$ and $E(\varepsilon_t' \varepsilon_t) = \Omega$, all t. The LQ problem has some desired features: policy function is linear on state variables and, hence, certainty equivalence applies. Information frictions does not change the problem (Sims, 2003a). In addition, the value function is quadratic. This property is desireable to show that the optimal signal has a Gaussian distribution.

In order to do that, the first stage is to solve the deterministic problem. Writing the objective equation recursively and replacing the restriction in the objective function, we have:

$$V(S_t) = \max_{c_t} \{S_t' A S_t + 2 S_t' D c_t + c_t' B c_t + \beta E_t V(G_1 S_t + G_2 c_t + G_3 \varepsilon_{t+1})\}.$$

Conjecture 1 *The value function in quadratic in the state vector, or* $V(S_t) = S_t' P_1 S_t + P_2 S_t + d$, P_1 *following the properties described by Benigno and Woodford (2008) and* d *an unknown constant.*

Then, we can write the expression above as:

$$\begin{aligned} S_t' P_1 S_t + P_2 S_t + d &= \max_{c_t} \{S_t' A S_t + 2 S_t' D c_t + c_t' B c_t + \beta d + \\ &\quad + \beta S_t' G_1' P_1 G_1 S_t + \beta [S_t' G_1' P_1 G_2 c_t + c_t' G_2' P_1 G_1 S_t] + \\ &\quad + \beta c_t' G_2' P_1 G_2 c_t + \beta tr(P_1 G_3 \Omega G_3') + \\ &\quad + \beta P_2 G_1 S_t + \beta P_2 G_2 c_t\}, \end{aligned}$$

after evaluating conditional expectations and exploring the fact that ε_t is i.i.d. and $E(\varepsilon_t) = 0$. We next take FOC with respect to the control variable vector c_t. It is clear that the resulting policy function is indeed linear:

$$c_t = H_0 + H_1 S_t,$$

where

$$\begin{aligned} H_0 &= -2[B + \beta G_2' P_1 G_2]^{-1} [\beta G_2' P_2'], \\ H_1 &= -[B + \beta G_2' P_1 G_2]^{-1} [\beta G_2' P_1 G_1 + D'], \end{aligned}$$

a function both of P_1 and P_2. Replacing the policy function back to value function, it is possible to determine the values for P_1, P_2 and d. Define:

$$\hat{\Omega} \equiv G_3 \Omega G_3'. \tag{50}$$

For, d:

$$(1-\beta)d = H_0'BH_0 + \beta tr(P_1\hat{\Omega}) + \\ +\beta H_0'G_2'P_1G_2H_0 + \beta P_2G_2H_0,$$

For P_2:

$$P_2 = \vec{0}.$$

For P_1:

$$P_1 = A + 2DH_1 + H_1'BH_1 + \beta G_1'P_1G_1 + \\ +2\beta H_1'G_2'P_1G_1 + \beta H_1'G_2'P_1G_2H_1. \quad (51)$$

Equation (51) describes P_1 recursively, a matrix Riccati equation:

$$P_1(s+1) = A + \beta G_1'P_1(s)G_1 + \\ -(D' + \beta G_2'P_1(s)G_1)'(B + \beta G_2'P_1(s)G_2)^{-1}(D' + \beta G_2'P_1(s)G_1) \quad (52)$$

It can be solved by iterating the matrix difference equation starting from some initial value and converging to a fixed point or using a method based on eigenvalue-eigenvector decomposition (such as Blanchard-Quah). Finally, $d = \beta tr(P_1\hat{\Omega})/(1-\beta)$. Equations defining the value function are independent of S_t, which means that the value function given by the problem with information friction is analogous: $\hat{V}(\hat{S}_t) = \hat{S}_t'P_1\hat{S}_t + d$.

7.2 Gaussianity of Optimal Signal

Define the welfare loss in t due to imperfect information as $\Delta V_t = V(S_t) - \hat{V}(\hat{S}_t)$. The expected welfare loss is given by[4]:

$$E_t[\Delta V_t] = E_t[V(S_t) - \hat{V}(\hat{S}_t)].$$

Substituting for $V(S_t)$ and $\hat{V}(\hat{S}_t)$, and notecing that $E_t(S_t) = E_t(\hat{S}_t)$, yields:

$$E_t[\Delta V_t] = -E_t[(S_t - \hat{S}_t)'P_1(S_t - \hat{S}_t)] + 2E_t[S_t'P_1(S_t - \hat{S}_t)],$$

Considering that

$$E_t[(S_t - \hat{S}_t)] = 0,$$

by hypothesis, then it is clear that

$$E_t[S_t'P_1(S_t - \hat{S}_t)] = P_1 Cov[S_t, (S_t - \hat{S}_t)] = 0.$$

[4] For simplicity, we use $E_t[.]$ as short for $E_t[.|\mathcal{I}_t]$.

Then:

$$E_t[\Delta V_t] = -E_t[(S_t - \hat{S}_t)' P_1 (S_t - \hat{S}_t)].$$

The problem then becomes to choose a joint distribution of state variables and signals that minimize the loss function

$$\min_{q(S_t,\hat{S}_t)} -E_t[(S_t - \hat{S}_t)' P_1 (S_t - \hat{S}_t)],$$

subject to:

$$-\mathcal{H}(S_t,\hat{S}_t) + \mathcal{H}(\hat{S}_t) + \mathcal{H}(S_t) \leq 2\kappa$$

plus the conditions on $q(S_t,\hat{S}_t)$ being a pdf. $\mathcal{H}$ corresponds to the definition of entropy and κ is the channel capacity on the mutual information between S_t and $\hat{S}_t$. More explicitly, we can apply the definition of entropy to the problem above, yielding:

$$\min_{q(S_t,\hat{S}_t)} -\int\int (S_t - \hat{S}_t)' P_1 (S_t - \hat{S}_t) q(S_t,\hat{S}_t) dS_t d\hat{S}_t,$$

subject to:

$$\begin{aligned} & \int\int \log[q(S_t,\hat{S}_t)] q(S_t,\hat{S}_t) dS_t d\hat{S}_t - \int \log[q(S_t)] q(S_t) dS_t + \\ & -\int \left[\log\left[\int q(S,\hat{S}_t) dS_t\right] \int q(S,\hat{S}_t) dS_t\right] d\hat{S}_t \\ \leq \; & 2\kappa, \end{aligned}$$

$$\int q(S_t,\hat{S}_t) d\hat{S}_t = q(S_t),$$

and

$$q(S_t,\hat{S}_t) \geq 0.$$

We disregard this last restriction, assuming that it always holds. Following Sims (2003b) and Luo (2006), the maximization can be carried out pointwise by taking derivatives with respect to $q(S_t,\hat{S}_t)$. FOC yields:

$$-(S_t - \hat{S}_t)' P_1 (S_t - \hat{S}_t) - \lambda\{1 + \log[q(S_t,\hat{S}_t)] - 1 - \log\left[\int q(S,\hat{S}_t) dS_t\right]\} - \mu = 0.$$

Define:

$$q(S_t|\hat{S}_t) \equiv \frac{q(S_t,\hat{S}_t)}{\int q(S_t,\hat{S}_t) dS_t},$$

then:

$$q(S_t|\hat{S}_t) = \theta_0 e^{\theta_1 (S_t - \hat{S}_t)' P_1 (S_t - \hat{S}_t)}.$$

where θ_0 and can θ_1 be conveniently chosen so as the right-hand side is a multivariate normal distribution. The result implies that it is optimal to choose the joint distribution of S_t and $\hat{S}_t$ in such a way that the conditional distribution of the state variable given the signal is a multivariate normal distribution:

$$q(S_t|\hat{S}_t) \sim N(\hat{S}_t, \Sigma).$$

The infinite dymensional problem simplifies to one in which it is only necessary to establish the variance-covariance matrix of the posterior distribution of state variables given the signal, Σ.

7.3 Determination of Σ

Following Sims(2003a), the loss function can be written as:

$$\begin{aligned} E_t[V(S_t) - \hat{V}(\hat{S}_t)] &= -tr([A + DH_1 + H_1' B H_1]\Sigma) + \\ &\quad + \beta E_t[V(S^*_{t+1}) - V(S_{t+1}) + (S_{t+1}) - \hat{V}(\hat{S}_{t+1})] \end{aligned}$$

where $S^*_{t+1} = (G_1 + G_2 H_1) S_t + G_3 \varepsilon_t$ is the value of state variables that would emerge in the case where control variables are chosen optimally and without uncertainty upon the true state at t: S_t. Note that S_{t+1} is the true value of the state vector at $t+1$ when control variables are chosen under information capacity constraint, that is, the state at t is merely perceived: $\hat{S}_t$. Define

$$S^*_{t+1} - S_{t+1} = G_2 H_1 (S_t - \hat{S}_t)].$$

Because of the LQ structure, the left-hand side is constant. Expression simplifies to:

$$\begin{aligned} (1-\beta)M &= -tr([A + DH_1 + H_1' B H_1]\Sigma) + \\ &\quad -\beta E_t[(S^*_{t+1} - S_{t+1})' P_1 (S^*_{t+1} - S_{t+1}) + 2(S^*_{t+1} - S_{t+1})' P_1 S_{t+1}] \end{aligned}$$

By replacing the definition above, one gets:

$$(1-\beta)M = -tr(W\Sigma),$$

where W is given by:

$$W = A + DH_1 + H_1' B H_1 + \beta(H_1' G_2' P_1 G_2 H_1 + H_1' G_2' P_1 G_1 + G_1' P_1 G_2 H_1).$$

The optimization problem takes the following form:

$$\min_{\Sigma}\{tr(W\Sigma)\}$$

subject to the information capacity constraint:

$$-\log_2|\Sigma| + \log_2\left|G_1\Sigma G_1' + \hat{\Omega}\right| \leq 2\kappa$$

and an additional condition to ensure that $G_1\Sigma G_1' + \hat{\Omega} - \Sigma$ is a positive definite matrix:

$$\Sigma \preceq G_1\Sigma G_1' + \hat{\Omega}.$$

As shown by Sims (2003a), the problem is the one of maximizing a linear objective function subject ot a convex restriction set. In order to establish Σ numerically, we reparameterize the problem in terms of the upper triangular matrix ϕ^*, such that $\phi^{*\prime}\phi^* = \Lambda^*$ and $\Lambda^* = \Psi - \Sigma$. For an initial value of ϕ^* it is then possible to establish Σ by solving the Lyanpunov equation $\Lambda^* + \Sigma = \hat{\Omega} + G_1\Sigma G_1'$. For a given value of the Lagrangean multiplier, it is then possible to compute the value for the objective function subject to the information capacity constraint. Once the optimal ϕ^* has been found, it is possible to recover Σ by solving the same equation and then recovering the covariance matrix of the noise variables $\Lambda = var(\zeta_t)$ using the following expression:

$$\Lambda^{-1} = \Sigma^{-1} - \Psi^{-1},$$

which derives from the usual formula for the variance of a stationary Gaussian distribution updated based on a linear observation, according to Sims (2003a).

8 Appendix D - Parameter Choices

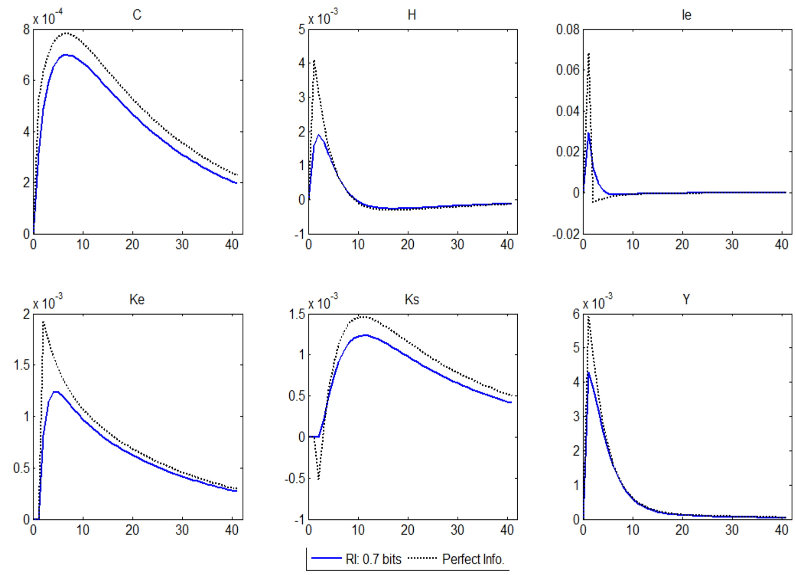
The following table presents the parameter values for the benchmark calibration, along with its definitions.

Table 3: Benchmark Calibration (Quarterly Data)

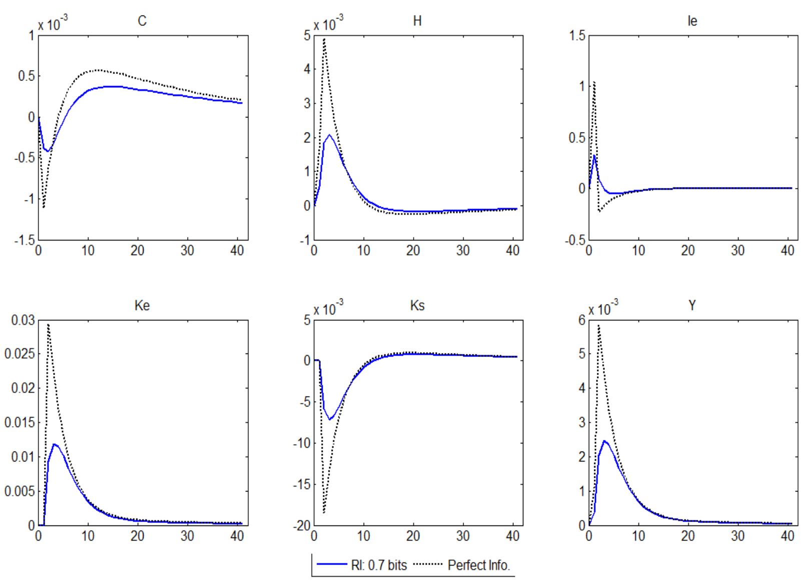
Symbol	Parameter Definition	Assigned Value
β	Intertemporal discount factor	$.96^{1/4}$
θ	Preference parameter on labor supply	2.74
α_e	Equipment share of income	.19
α_s	Structures share of income	.16
H	Steady state labor hours	.24
φ	Steady state labor-leisure hours ratio	.31
$1-\delta^e$	Gross depreciation rate of equipment	$(1-.035)^{1/4}$
$1-\delta^s$	Gross depreciation rate of structures	$(1-.075)^{1/4}$
ρ_a	AR(1) coeff. of neutral tech. shock	.75
ρ_q	AR(1) coeff. of investment tech. shock	.75
σ_a^T	Sd. of transitory neutral tech. shock	.0050
σ_q^T	Sd. of transitory relative investment shock	.0050
σ_a^P	Sd. of permanent neutral tech. shock	.0050
σ_q^P	Sd. of permanent relative investment shock	.0050
$\sigma_{a,q}^{T,P}$	Correlations among innovations	*zero*
γ_a	Gross growth trend on investment prod.	$(1+.004)^{1/4}$
γ_q	Gross growth trend on neutral prod.	$(1+.025)^{1/4}$
s_c	Steady state consumption over GDP	81%
s_{ie}	Steady state investment in equipment over GDP	12%
s_{is}	Steady state investment in structures over GDP	7%
s_{ke}	Steady state capital stock in equipment over GDP	480%
s_{ks}	Steady state capital stock in structures over GDP	770%

9 Appendix E - Model Dynamics

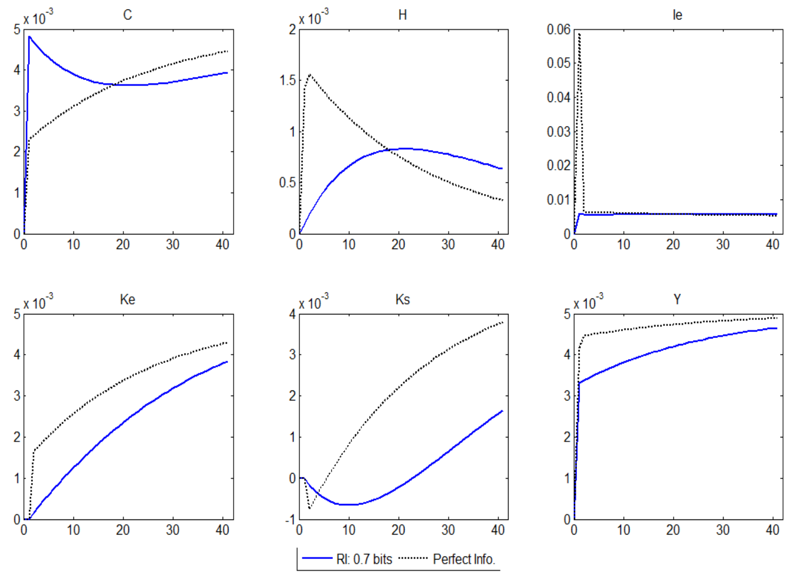
In this section, we present the model general dynamics in response to shocks. The perfect information case is contrasted with responses of endogenous variables under limited information. Capacity constraint is calibrated to 0.7 bits.



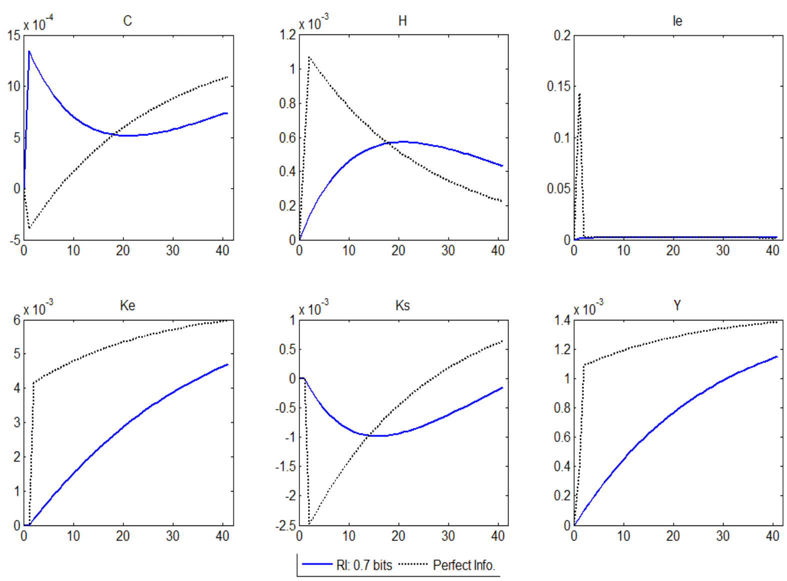
Impulse Response Functions to a one s.d. stationary shock on Labor Productivity (neutral technology shock).



Impulse Response Functions to a one s.d. stationary shock on Investment Relative Productivity.



Impulse Response Functions to a one s.d. permanent shock on Labor Productivity (neutral technology shock).



Impulse Response Functions to a one s.d. permanent shock on Investment Relative Productivity.

10 Appendix F - S.D.s for RBC Statistics

The following table presents the standard errors for simulations of RBC statistics, presented at Section 3.

Table 4: RBC Basic Statistics - Standard Errors

	Full Inf.	$\kappa = 4.3$	$\kappa = 1.4$	$\kappa = .80$	$\kappa = .24$
Relative S.D.					
σ_y^a	0.08	0.09	0.08	0.07	0.06
σ_c/σ_y	0.01	0.06	0.10	0.09	0.10
σ_h/σ_y	0.01	0.03	0.04	0.04	0.03
σ_i/σ_y	0.05	0.20	0.32	0.32	0.35
Cross-Correlation					
$\rho(c,y)$	0.08	0.10	0.11	0.10	0.07
$\rho(h,y)$	0.00	0.02	0.03	0.04	0.08
$\rho(i,y)$	0.00	0.02	0.05	0.05	0.08
Autocorrelation					
$\rho(\Delta y)$	0.05	0.05	0.04	0.05	0.05
$\rho(\Delta c)$	0.05	0.06	0.05	0.05	0.05
$\rho(\Delta h)$	0.04	0.04	0.03	0.03	0.03
$\rho(\Delta i)$	0.04	0.04	0.04	0.04	0.05

[a] In percentage.