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Abstract

Using some special models, this study tries to quantify the size of underground economy in Romania. The exposition plan includes: 1) Models based on direct approaches; 2) Models based on indirect approaches; 3) A generalised model for the allocation of time; 4) A model based on May's logistic; 5) Conclusions.

Keywords: underground economy, tax evasion, fiscal policy, logistic function
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UNDERGROUND ECONOMY QUANTITATIVE MODELS. SOME APPLICATIONS TO ROMANIA'S CASE*

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1. MODELS BASED ON DIRECT APPROACHES

Many times, the estimations of underground economy are directly obtained by extrapolation of data collected from a limited number of households by surveys and samples. But, in recent years, the modern theoretical models on tax evasion, which started with the work of Allingham and Sandmo (1972) and continued with Cowell (1990) and other studies have been developed in a quantifying way. The support is provided by some rigorously organised empirical studies.

The main impediment is the difficulty to collect information on the number of hours worked by persons who illegally evade taxes, which makes it impossible to measure the effect of taxes on the allocation of time. To remedy these impediments, some studies analysed empirically labour-supply decisions in the underground economy using microdata from rigorously organised surveys such as that conducted in Québec by Fortin and Fréchette.[1]

Coming from surveys they identified some key empirical regularities about the work in the untaxed sector and then they built adequate quantifying models. However, their survey seems to be less accurate for tax evaders operating at the margin of being detected. In this case data based on extensive audits[2] is more revealing for this tax evades.[3]

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[1] See Fortin and Frechétte (1987) for a full description of the data set (Fortin and Frechétte, 1986). Related surveys carried out in Europe reported in Pestieau (1985) and Ginsburg *et al.* (1987) for Norway; and van Eck and Kazemir (1988) for the Netherlands.

[2] Most empiric studies in North America based on data from the Compliance Measurement Program (Witte and Woodbury, 1985; Feinstein, 1991; Smith, 1985; Portes *et al.*, 1989).

[3] The two measurement approaches, microsurveys and tax audits, should thus be viewed as complementing each other.

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The main conclusions deduced by Lemieux, Fortin and Fréchette from these empirical findings are the following: 1 – labour earnings in the underground sector are concentrated among workers with low earnings in the regular sector, while expenditures on goods and services produced in the underground sector are typically undertaken by people with high earnings in the regular sector; 2 – the wage rate in the regular sector and the wage rate in the underground sector are positively correlated with hours worked in the regular sector but negatively correlated with hours worked in the underground sector; 3 – earnings in the regular sector are a linear or slightly convex function of regular-sector hours, while earnings in the underground sector are a concave function of underground-sector hours (Lemieux *et al.*, 1994).

Then, they developed a model based on the idea that labour earnings in the underground sector are a convex function of hours of work, while in the regular sector labour earnings are a linear function of hours of work. The convexity of the earning function in the underground sector implies that the marginal revenues of the underground producers decrease as producers reach the limits of the informal markets on which they operate. By contrast, the wage rate of a worker in the regular sector does not vary with the number of hours worked.[4]

The results of Lemieux, Fortin and Fréchette's study suggest that the hours worked in the underground sector are quite responsive to changes of the net wage in the regular sector. Most important, relating to our interest in this paper, their model also provides a natural link between the slope of the relationship between tax revenues and tax rates (the "Laffer curve") and a more conventional measure of the marginal excess burden of taxes due to the misallocation of productive resources from the regular to the underground sector.[5]

Lemieux's model is based on a concave Cobb-Douglas earnings function in the underground sector

$$Y_1 = A_1 \cdot h_1^{\theta} \tag{1}$$

where $\theta < 1$,

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[4] A similar approach it used by Gronau (1977) to explain the allocation time among home production, market work, and leisure. He postulates that the value of home production is a concave function of hours worked at home, which explains why hours worked at home are negatively related to the market wage, just as hours in the underground sector are negatively related to the regular-sector wage in the Lemieux's article. One explanation for the concavity of the earning function in the underground sector is based on the principle that the informal nature of the economic activities in that sector imposes a limit on the scope of these activities. Because of these market limitations, the underground-sector workers face a downward-sloping demand for its output. As hours of work and output expand, the output price goes down, which tends to reduce the value of the marginal product of labour when there are constant (or decreasing) returns to scale in production. Labour earnings in the underground sector are thus typically a concave function of hours worked in that sector (Lemieux *et al.*, 1994).

[5] An interesting review of the debates surrounding the Laffer curve in Fullerton (1982).

and on a linear earnings function in the regular sector

$$Y_0 = W_0 \cdot h_0 \tag{2}$$

The variable h_0 represents hours of work in the regular sector; h_1 represents hours of work in the underground sector; W_0 is the wage in the regular sector; A_1 is a revenue-shifter in the underground sector.

To simplify the representation, the following quasi-linear utility function is considered:

$$U(l, C) = \gamma \cdot C + v(l) \tag{3}$$

where the function v(.) is a strictly concave utility function ($v' > 0$, $v'' < 0$). C represents a composite consumption good (the numeraire), while l is the number of hours of leisure that satisfies the time constraint

$$T = h_0 + h_1 + l \tag{4}$$

The budget constraint is given by

$$C = I + W_0 \cdot h_0 + A_1 \cdot h_1 \theta \tag{5}$$

where I represents nonlabour income, including any lump-sum transfer.

Under these conditions, the worker's problem is

$$\max_{(C,\, h_0,\, h_1)} [\gamma \cdot C + v \cdot (T - h_0 - h_1)] \tag{6}$$

subject to equation (3–5), $h_0 > 0$, and $h_1 > 0$. For workers who supply positive hours in both sectors (interior solution), the first order conditions for hours yield:

$$\delta U/\delta h_0 = -v' \cdot (T - h_0 - h_1) = -\gamma \cdot W_0 \tag{7}$$

$$\delta U/\delta h_1 = -v' \cdot (T - h_0 - h_1) = -\gamma \cdot \theta \cdot A_1 \cdot h_1^{\theta-1} \tag{8}$$

These two conditions imply the following equality for the marginal revenue of an hour of work in the two sectors:

$$W_0 = \theta \cdot A_1 \cdot h_1^{\theta-1} \tag{9}$$

Hours of work in the underground sector are thus determinated by setting the marginal revenue in the underground sector equal to the regular-sector wage. This is a very important result, which implies that, conditional on the regular-sector wage, underground-sector hours **do not** depend on preferences.

As Lemieux affirms, this "separation" between the determination of hours in the underground sector and preferences is similar to the separation result in the development literature on farm households which states that the labour demand of a farm is determined independently of the preferences of the household members. This separation result has several implications that are consistent with the empirical regularities. The main one considers the average wage rate in the underground sector:

$$W_1 = Y_1/h_1 = A_1 \cdot h_1^{\theta-1} \tag{10}$$

It can be demonstrated that $W_0 = \theta \cdot A_1$. Since $\theta < 1$, it follows that the regular-sector wage, W_0, is smaller than the underground-sector wage W_0. This prediction holds, on average, when the measure of W_0 used is the regular wage net of the tax rate. This result will be also incorporated within the hypothesis of our global model used to evaluate the dimension of underground economy in Romania.

Following the basic model of tax evasion and considering that net earnings in the regular sector, WN_0, are given by

$$WN_0 = (1 - t) \cdot W_0 \cdot h_0 \tag{11}$$

where t is flat tax rate, Lemieux postulates that workers who supply positive hours to the underground sector face a probability p of being detected by the authorities. Getting caught entails a penalty proportional to the amount of tax evaded. The penalty rate (tp) on underground income can be written as:

$$tp = n \cdot t \tag{12}$$

n being the penalty rate on evaded tax ($n > 1$). The expected rate of tax penalty on evaded income is thus given by $p \cdot n \cdot t$.

Again, in case of this model of tax evasion the result was that underground-sector hours do not depend on preferences conditional on W_0 and t. This is because of the fact that the utility function is linear in consumption, which implies risk-neutrality in consumption. The case in which workers are risk-averse but face a parametric wage in both the regular and the underground sector has been analysed by several authors (Sandmo, 1981; Cowell, 1985; Fluet, 1987). This case leads to a few interesting comparative-static results, however, even with strong restrictions on preferences for consumption and leisure. As noted by Cowell (1984), the basic reason for these ambiguities is that "in reaction to any perturbation, the individual can substitute across two margins (risk/no-risk and labour/leisure), so that in principle all sorts of behaviour could be consistent with rational expected utility maximisation". One main result of Lemieux's model developed in this way is that an increase in tax rates has a positive effect on the hours of work in the underground sector, but a negative effect on the hours of work in the regular sector.

To analyse the implications of tax policy, Lemieux considered an experiment in which the total hours worked in the two sectors does not depend on the marginal tax rate. In response to a change in the tax rate, people therefore reallocate a given hour of work from the regular to the underground sector, so that:

$$dh_0/dt = -dh_1/dt \tag{13}$$

The excess burden (EB) caused by the misallocation of hours between the two sectors is given by:

$$EB = W_0 \cdot [h_1(t) - h_1(0)] - A_1 \cdot \{[h_1(t)]\theta - [h_1(0)]\theta\} \tag{14}$$

where $h_1(t)$ and $h_1(0)$ are the hours worked in the underground sector with and without a tax rate t, respectively. The marginal excess burden (MEB) measures how much the excess burden has to increase in order to raise an additional currency unit (dollar) of taxes:

$$MEB = \delta EB/\delta T = (\delta EB/dt)\times(\delta t/\delta T) = [W_0 - \theta \cdot A_1 \cdot h_1^{\theta-1}]\cdot(dh_1/dt)\cdot(\delta t/\delta T) \tag{15}$$

where T represents total tax revenues ($T = t \cdot W_0 \cdot h_0$) and $\delta T/\delta t$ is the slope of the Laffer curve. This slope depends on how hours of work respond to a change in the tax rate.

After some transformations, and supposing

$$\theta \cdot A_1 \cdot h_1^{\theta-1} = (1 - t) \cdot W_0 \tag{16}$$

MEB can be written as:

$$MEB = t \cdot W0 \cdot (dh1/dt) \cdot (\delta t/\delta T) = (1 - \eta)/\eta \tag{17}$$

where η represents the elasticity of the Laffer curve, or $\delta ln(T)/\delta$).

Some results of the application of the model to empirical data collected in Québec City, Canada, by samples reported by the Lemieux's study are:

- the estimate value of θ is 0.67;
- since both η and MEB depend on the ratio of underground to regular income Y_1/Y_0 and on the marginal tax rate t, their respective values vary considerably across the different groups of the population;
- the estimates of the elasticity of the Laffer curve (η) range from 0.407 for workers who received some welfare payments during the year to 0.994 for workers aged 40 and more;
- these two extreme values of η yield values of the marginal excess burden in production (MEB) of 1.457 and 0.006, respectively;
- on the average, an increase in the tax rate does not substantially distort labour-market activities from the regular sector to the underground sector;
- on the other hand, the same increase in the tax rate, or in the tax-back rate embodied in social-welfare programs, has a very substantial effect on the allocation of time of the social-welfare claimants.

These conclusions only hold, however, when the probabilities of detection by the authorities and penalty rate are negligible ($p \cdot n = 0$). Although this might be true for some underground activities, the results of Lemieux's study suggest that it is not true in general. Government enforcement policies might thus be offsetting some of the distortion due to the presence of the tax and transfer system (Lemieux *et al.*, 1994).

Another model, which was calibrated so as to replicate observed labour supply in a random sample conducted in the metropolitan area of Québec City for 1985, is that of Fortin and Lacroix (1994). They used a simultaneous model of labour supply in the regular and irregular sectors which allows for the computation of the marginal cost of public funds associated with tax and enforcement instruments.

Also, there are some other interesting quantitative models in the existing literature which treat the behaviour of households relaying to the work in the underground sector and that are based on microdata samples, especially in the case of Northern European countries (Isachsen and Strom, 1981 and 1985), but they are beyond the goal of our study which focuses on the macro-data approach to the underground sector.

2. MODELS BASED ON INDIRECT APPROACH

2.1. MODELS BASED ON MONETARY APPROACH

As a rule, there are two categories of models, depending upon the methods of approach: the transaction approach and the currency demand approach, respectively.

The first category of models, developed first by Feige, comes from Fisher's quantity equation:

$$M/v = p/Tr \tag{18}$$

where M represents money; v – velocity; p – prices; Tr – total transactions.

This approach assumes that there is a constant relation over time between the volume of transactions and the official GDP. Assumptions have to be made about the velocity of money and about the relationship between the value of total transactions (p/Tr) and nominal GDP. Relating total nominal GDP to total transactions, GDP in the shadow economy can be derived by subtracting the official GDP from the total nominal GDP. To derive figures for the shadow economy, Feige had to assume a base year in which there was no shadow economy, and therefore the ratio of p/Tr to total nominal (official = total) GDP was "normal" and would have been constant over time if there had been no underground economy. This method has also several weaknesses: for instance, the assumption of a base year with no shadow economy, and the assumption of a "normal" ratio of transactions, constant over time. Moreover, to obtain reliable estimates, accurate figures for the total volume of transactions should be available. This availability might be especially difficult to achieve for cash transactions, because they depend, among other factors, on the quality of the paper used in the currency. Generally, although this approach is theoretically attractive, the empirical requirements, necessary to obtain reliable estimates, are so difficult to fulfil that its application may lead to doubtful results.

The second category of models based on currency demand approach was developed firstly by Cagan (1958) and Gutmann (1977). Then, important contributions based on econometric applications were made by Klovland (1980, 1984), Tanzi (1982) and Isachsen and Strom (1985).

In Klovland's approach, for instance, to model the behavior of the economic system the following variables are needed: C = currency held by the public; P = price index; Y = real GDP; i = rate of return on time deposits; π = rate of inflation; c = private consumption as a share of GDP; t = total taxes as a share of GDP; θ = stock adjustment parameter ($0 < \theta < 1$).

Klovland specified his model by the following two relations:

$$\ln C - \ln C - 1 = \theta \cdot (\ln C^* - \ln C - 1) \quad (19)$$

$$\ln(C^*/P) = \underset{}{\alpha_0} + \underset{+}{\alpha_1} \cdot \ln Y + \underset{-}{\alpha_2} \cdot i + \underset{-}{\alpha_3} \cdot \pi + \underset{+}{\alpha_4} \cdot c + \underset{+}{\alpha_5} \cdot t \quad (20)$$

where C^* is the long run demand for currency and $\alpha_0, \ldots, \alpha_5$ are coefficients.

The signs below the last equation give the expected signs of the coefficients. Isachsen and Strom (1985) used this model to estimate the size of the underground economy in Norway. Annual data were used, covering the period 1952-78.

The estimation produced result for ln(C/P). All the estimated coefficients, except the coefficient attached to the inflation rate, have the expected signs. Then, to proceed with the estimation of the size of the underground economy they rewrite (20) as

$$\ln C = \ln P + Z\alpha^* + \alpha_5^* t \quad (21)$$

where the term $Z\alpha^*$ includes the α's variables except for the tax rate. α^* and $\alpha_5^* t$ are estimated values. Then predicted currency holdings at time t is

$$C\tau^* = \exp(\ln P\tau + Z\tau\alpha^* + \alpha_5^* t_\tau) \quad (22)$$

In order to obtain an estimate of the size of the underground economy they selected 1952 as a base year. Thus, if the tax rate had remained at the 1952 level, the predicted value of currency holdings would have been

$$C_{52}, \alpha^* = \exp(\ln P\tau + Z\tau\alpha^* + \alpha_5^* t_{52}) \quad (23)$$

The difference, $\Delta C_\tau = C_\tau^* - C_{52,\tau}^*$, would then give the increase in the amount of currency needed to fuel the tax evasion part of the economy compared to the currency needs if tax rates remained at the 1952 levels. They next assume that no tax evasion took place in the base year or in years before that. Under these conditions, ΔC_τ would yield an estimate of all currency circulating in the underground economy at time τ. Then, according to Tanzi (1983) and others they assume that the income velocity of currency in the underground economy equals the velocity of M1 money in the official parts of the economy. Under such a hypothesis, for each year of the analyzed period the velocity rate, V_τ, and respectively the GDP contribution from the underground economy, $yu_?$, is given by the following relations:

$$V_\tau = Y_\tau/(M1_\tau - \Delta C_\tau) \tag{24}$$

$$yu_\tau = (Yu_\tau/Y_\tau) = \Delta C_\tau/\ (M1_\tau - \Delta C_\tau) \tag{25}$$

where Yu_τ ($Yu_\tau = V_\tau \times \Delta C_\tau$) and Yt are the GDP in the underground sector and the official GDP, respectively.

Using this model, Isachsen and Strom obtained the following values of the share of underground economy in Norway: 1.3% in 1971; 0.6% in 1973; 4.3% in 1976; 6.3% in 1978. Also, Klovland (1980) employed a narrow definition of the money stock and arrived at a higher velocity for the year 1978, that is 6.7. As part of the observed GDP, the underground economy then becomes 9.2%.

The first objection relates to the fact that not all transactions in the underground economy are paid in cash. Also, Isachsen and Strom (1981) used the survey method to discover that in Norway in 1980 roughly 80% of all transactions in the underground sector were paid in cash. The size of the total underground economy (including barter) may thus be even larger than previously estimated. Most studies consider only one particular factor, the tax burden, as a cause of the underground economy. Other reasons such as the impact of regulation, the complexity or visibility of the tax system, taxpayers' attitudes to the state, "tax morality", and so on are not considered because data for most countries are not available. If, as is most likely to happen, these other factors also have an impact on the extent of the underground economy, it might be larger than reported in most studies.[6]

A further weakness of this approach, at least when applied to the United States by Tanzi (1983), is discussed by Garcia (1978). They point out that increases in currency demand deposits are largely due to a slow-down in demand deposits

[6] One justification for the use of the tax variable only is that this variable has by far the strongest impact on the size of the underground economy in all studies known to us. An exception is the study by Frey and Weck-Hannemann (1984), where the variable "tax immorality" has a quantitatively larger and statistically higher influence in the model than the direct tax share. In a study on the U.S. underground economy, Pommerehne and Schneider (1985), showed that the tax variable has a dominating influence and contributes roughly 70–78% to the size of the underground economy.

rather than to an increase in currency caused by the activity in the underground economy. Blades criticises Tanzi's studies because the US-Dollar is used as an international currency, so that Tanzi should have considered (and compensated for) the amount of US-Dollars held in cash abroad. Finally, Frey and Pommerehne (1984) claim that Tanzi's parameter estimates are not very stable.[7]

Another weak point of this approach, as applied in most studies, is the assumption that the velocity in both types of economy is the same. As Klovland (1984) argues for the Scandinavian countries, there is already considerable uncertainty about the velocity of circulation of currency in the official economy; the velocity of currency in the underground sector is even more difficult to estimate. Without knowledge about the velocity of currency in the underground economy, one has to accept the assumption of an "equal" velocity of money in both sectors. Finally, the assumption of no underground economy in a base year is open to criticism. Relaxing this assumption would again imply an upward adjustment of the figures attained in the bulk of the studies already undertaken.

We also applied this model in the case of Romania's economy by considering the monetary aggregate M1 and as the origin of time series the year 1984. In this case, the figures representing the estimated size of the informal economy are the following: 42.7% in 1991, 32.0% in 1992, 71.3% in 1993, 56.7% in 1994, and 61.6% in 1995. When we appreciate the size of the informal sector in the present transition period in Romania we must consider some specific phenomena such as the corruption manifested within the incipient banking system or the illicit transfer of funds from the state enterprises to private-parasite enterprises.[8]

2.2. A GLOBAL MODEL BASED ON THE LABOUR SUPPLY METHOD

Firstly, we consider a national economy having only two sectors: a visible (or official) sector and an invisible (or underground) sector. In the case of a visible sector the registered GDP is supposed to have the following components:

$$Yv = A + Sv + Bv \tag{26}$$

where Yv is the GDP produced in the visible sector; A – consumption of fixed capital (only in the visible sector); Sv – wages of employees in the visible sector; Bv – profit of entrepreneurs (capitalists) in the visible sector. In the invisible sector the produced GDP will be:

[7] In studies for European countries, Kirchgassner (1983) and Schneider (1986) reach the conclusion that the estimation results for Germany, Denmark, Norway and Sweden are quite robust when using this approach.

[8] Moreover, in the first years of transition, at least in Romania's economy there was a "dollarization" phenomenon that evades from official statistics. Some estimations show that the stock of foreign currency held by the population is over one thousand million dollars, many transactions being made directly in dollars or other foreign currency (Dobrescu, 1994).

$$Ya = Sa + Ba \tag{27}$$

where Ya is the GDP produced in the underground sector; Sa – wages of employees in the invisible sector; Ba – profit of entrepreneurs (capitalists) in the invisible sector. In the case of the invisible sector it is supposed that there is only circulating capital.

Also, we consider that both the available time fund and the total number of labour force distribute among two components, as follows:

$$F = Fv + Fa \tag{28}$$

$$L = Lv + La \tag{29}$$

where F is the total available time fund for work within a calendar year; L – number of total potential working persons; Fv – time used for work in the visible sector by year; Fa – potential available time used by the persons having the status of employees in the visible sector for work as a second job in the underground sector; Lv – number of employees working in the visible sector; La – potential number of employees working in the underground sector.

We mention that the available working time is evaluated as the average number of hour values by a calendar year. For instance, they may be considered as an average number of hours worked by a person within a calendar year.

Now, we express the GDP created in the two sectors of a national economy as functions of productivity, which consider here as linear functions:

$$Yv = Lv \cdot Fv \cdot wv \tag{30}$$

$$Ya = (La \cdot F + Lv \cdot Fa) \cdot wa \tag{31}$$

where wv and wa are the average productivity by person by hour in the visible sector and, respectively, in the invisible sector.

To obtain the total number of hours worked by year in the underground sector, we considered two categories. One includes persons that work full-time in the underground sector (persons having the status of non-employees in the visible sector) (La·F). The other comes from the persons having the status of employees in the visible sector, but working also in the underground sector during the overtime work time as a second job (Lv·Fa).

What is interesting for the agents or people is the level of disposable income or available GDP. In the two sectors of the economy the available income or GDP will be given by the following relations:

$$Ydv = Yv - T = Yv \cdot (1 - t) \tag{32}$$

and, respectively,

$$Yda = Ya \tag{33}$$

which can also be written under the following forms:

$$Ydv = Lv \cdot Fv \cdot wv \cdot (1 - t) \quad (34)$$

$$Yda = (Lv \cdot Fa + La \cdot F) \cdot wa \quad (35)$$

where Ydv, Yda represent the available income in the visible sector and in the invisible sector, respectively; T is the total amount of taxes paid (or of obligatory levying); t – average tax rate relatively to Yv.

Taking into account the structural relations (28) and (29) we can write now the expression of the total available income as follows:

$$Yd = Lv \cdot Fv \cdot wv \cdot (1 - t) + [Lv \cdot (F - Fv) + (L - Lv) \cdot F] \cdot wa \quad (36)$$

or

$$Yd = (L - La) \cdot (F - Fa) \cdot wv \cdot (1 - t) + [(L - La) \cdot Fa + La \cdot F] \cdot wa \quad (37)$$

The first relation permits to analyse the impact of the number of persons working in the visible sector (Lv) and of their corresponding number of hours worked in this sector (Fv) on the total available income at the national level. The second makes the same but concerning the number of persons working in the invisible sector (La) and the number of hours worked in the invisible sector, respectively, by persons having the status of employees in the visible sector (Fa). We remember that in the case of persons actually having the status of non-employees in the visible sector (but having a potential to work by age and disposable free time criteria) it is supposed that they allocate their entire available working time to work in the invisible sector (F). At the same time, the persons actually having the status of employees in the visible sector are forced to divide the same entire disposable working time (F) between the work in the visible sector (Fv) and the work in the invisible sector (Fa).

Maybe this total available income, greater than the available income in the visible sector, is responsible for some unexplainable macroeconomic non-correlations registered between some "official" indicators.

Now, we define the yearly national potential by the following relation:

$$P = F \cdot L \cdot wv \quad (38)$$

where P is the maximum level of GDP by year.

Due to the productivity in the invisible sector being supposed as a rule to be weaker than the productivity in the visible sector, the following non-equality results:

$$Yv + Ya < P \quad (39)$$

Now, we can express the actual total available income by year as:

$$Yd = P \cdot [m + lv \cdot fv \cdot (1 - t - m)] \quad (40)$$

where m is the ratio between the productivity in the invisible sector and the productivity in the visible sector (wa/wv); lv – the share of employees in the visible sector in the total number of potential working persons (Lv/L); fv – the share of time used to work in the visible sector in the total available working time within a calendar year (Fv/F). On the other hand, if the entire activity were allocated in the visible sector, the maximum level of available GDP would be:

$$Yd^{*} = P \cdot (1 - t) \tag{41}$$

Now, we suppose that people have chosen the actual situation, that is the actual distribution of the total capacity to work between sectors. This produces an available income greater than or at least equal to that to be produced by the above hypothetical case. Therefore, there will be a restriction:

$$Yd > Yd^{*} \tag{42}$$

After some technical algebraic operations, the following equivalent restrictions are obtained:

$$m > 1 - t \tag{43}$$

$$wa > wv \cdot (1 - t) \tag{44}$$

We must now mention that our present model is conceived as a simulation model for the purpose to determine the variance interval thresholds of the underground sector coming from the existing statistical data at the macroeconomic level. In this case we have considered the absolute values both for the total potential number of working persons (L) and for the total number of potential working hours during a calendar year (F). This seems more exaggerated if compared with the real values registered in the case of the visible sector (Lv and Fv).

So, in the case of the invisible sector, we consider that the resulting levels of some indicators – productivity, profit rate – are smaller than in the real situation. We consider that this diminution must be considered in the following way. It is because besides the comprehension to obtain available income, for instance, in our model there is an implicit manner including the comprehension to leisure of the people having an available labour force. For instance, the actual available income computed by our simulation model is greater than the level that would be obtained in the case of a full-time work in the visible sector ($Yd > Yd^{*}$). The difference must be considered as implicitly including the satisfaction of the leisure comprehension problem.

Now, we consider it important to evaluate the variation interval of the underground sector dimension. Therefore, we write the share of the invisible sector in the national economy as:

$$ya(wa) = Ya(wa)/Y(wa) \tag{45}$$

where Ya(wa) is given by the relation (16) and Y is the total yearly GDP:

$$Y(wa) = Yv + Ya(wa) \tag{46}$$

For the productivity in the invisible sector we choose the following extreme values:

$$wamin = (1 - t)\cdot wv \quad (47)$$

$$wamax = wv \quad (48)$$

to which the following extreme values of the share of the invisible sector in the national economy correspond:

$$yamin = 1 - \{(lv\cdot fv)/[1 - t\cdot(1 - lv\cdot fv)]\} \quad (49)$$

$$yamax = 1 - lv\cdot fv \quad (50)$$

Within this interval we shall analyse the diverse alternatives regarding the average wages and profits in the underground sector relating to the situation registered in the visible sector.

In the case of the visible sector, to evaluate the average wage and the average rate of profit, we considered the following relations:

$$sv = [Sv/(Lv\cdot Fv)] = [Yv - (T + Bv + A)]/(Lv\cdot Fv) \quad (51)$$

$$bv = [Bv/(T + A + Sv)] = [Yv - (T + Sv + A)]/(T + A + Sv) \quad (52)$$

where sv is the average wage by person by hour of work in the visible sector; bv – the average profit rate in the visible sector; Sv – total amount of the yearly salaries in the visible sector; Bv – total amount of the yearly profits in visible sector.

On the other hand, for the invisible sector the corresponding relations are:

$$sa = [\, Sa/(La\cdot F + Lv\cdot Fa)] = (Ya - Ba)/(La\cdot F + Lv\cdot Fa) \quad (53)$$

$$ba = [Ba/Sa] = (Ya - Sa)/Sa = (wa - sa)/sa \quad (54)$$

where sa is the average wage by person by hour of work in the invisible sector; ba – the average profit rate in the invisible sector; Sa – total amount of the yearly salaries in the invisible sector; Ba – total amount of the yearly profits in the invisible sector.

In the case of the following applications we have replaced the consumption of fixed capital (A) by investment. Moreover the average wage in the invisible sector is considered as ranging between the values 0.5 and 2 relating to the average wage in the visible sector of the national economy. Here we present only some conclusions resulting from the model application in the case of Romania's economy.

For each year of the period 1989-1999 the number of employees in the visible sector (Yv) is that from the available official statistical publications. To evaluate the yearly time of work in the visible sector for each year we first diminished the number of days of the year by the weekend days and vacancies. Then we multiplied the result by the legal time working per day (8 hours) and then by an average coefficient that designs the average using degree of the total legal working time.

For both the total potential numbers of employees (L) and the total number of hours by year by person (F) we considered three versions denoted by maximum (I), intermediary (II), and minimum (III). Also, in the case of the maximal version, L included the employees in the visible sector (Lv), the official unemployed, the school population of age over 15 years, and the retired population of age under 70 years. In the case of the intermediary version, the same number as in the precedent case was added to Lv, but divided by two. In the case of the minimal version, only half of the unemployment number was added to Lv.

In the case of F, the maximal version was obtained by multiplying the calendar year number of days by eight (representing hours of work by day). To determine the intermediate value of F, we added 52 (representing the number of Saturdays by year) to the average number of days worked per year by a person in the visible sector (Fv). Moreover, we added half of the number of days used for vacancies in the visible sector. Then we multiplied the result by eight (number of hours worked by day). In the case of the minimal version we chose the same number of days worked in the visible sector but multiplied only by eight hours per day.

In the case of each variant (I, II, and III), we calculated the two threshold values – minimum and maximum – of the underground economy using formulas (49) and (50). Also, within this interval, we separated a subinterval. This covers, on the one hand, the intersection between the curve of salaries in the underground sector and the value of salaries in the official sector (inferior limit). On the other hand, it covers the intersection between the curve of profit rate in the underground sector and the value of profit rate in the official sector (superior limit). This subinterval represents, when it adopted one of the three variants for analysing, the set enclosing the most probable values of underground economy share.

The simulation output for recent years is shown synthetically in Table 1. Variant I, derived from the theoretical limits of the productive national potential, can suggest the maximum values to which the underground sector should extend. It is useful for long run forecasts. Versions II and III seem to produce estimates that are close to the figures commonly used in the literature to appoint the size of the underground economy. One of the most difficult problems remains the separation between the preference of people to use their available time for work in the underground economy and that to use the available time for leisure. In any case, Variant I can produce a satisfactory estimation of the total available time (including both components).

The estimated figures in the case of the hypotheses of V I seem to be similar to those obtained using the monetary method.

The underground sector can be seen as a homeostatic mechanism of an economy under *strain*. The latter, as a phenomenon, is caused either by the rules of the game – as in the command economy – or by dramatic changes in the parameters defining the functioning of an environment. Moreover, strain is enhanced by institutional fragility.

Table 1

Estimations of the underground economy size in Romania during 1989–1999

- in % -

	V I		V II		V III	
	Min	Max	Min	Max	Min	Max
1989	35.8	51.3	24.8	38.4	6.0	10.8
1990	48.4	59.4	33.3	43.7	10.7	15.8
1991	57.2	67.3	41.7	52.4	20.1	28.0
1992	62.3	71.3	47.4	57.6	26.7	35.4
1993	64.8	71.8	49.6	57.7	27.6	34.6
1994	64.4	71.7	48.9	57.2	26.9	34.0
1995	62.4	70.5	46.5	55.5	24.1	31.3
1996	63.0	70.0	46.0	53.8	24.2	30.4
1997	64.4	71.1	47.1	54.8	25.1	31.3
1998	64.8	71.8	47.5	55.6	25.4	32.0
1999	66.7	73.0	49.6	57.0	26.3	32.5

Whereas the underground sector is an outstanding structural feature of the command system (as a hyperregulated entity which ignores consumers' preferences) it continues to exist in market economies as well. For, as this paper argues, no real economy can escape strain. The issue, therefore, is the intensity of strain and its malign effects on the economy. An implication would be the need for policy-making to consider strain at both micro- and macrolevels. Reengineering enterprises and reforming economies can be scrutinised from this perspective.

Subject to dramatic changes in relative prices, transforming economies – as market economies under way – cannot be but under much *strain*. Therefore, it is not surprising that the transforming economies evince substantial underground sectors. As is our contention, the exceptional magnitude of the required resource reallocation and the fragility of the new institutions (the softness of the formal rules) stimulate the "development" of the hidden sectors.

The globalisation of trade and of the financial markets, as well as the "new information age" speed up the process of required change and add additional pressures on the transforming economies.

The formal model and the empiric analysis applied herein to the Romanian economy help gauging the potential size of the underground sector. It should be said that the fuzziness of property rights changes some of the assumptions of the models regarding the presumed relative productivities in the two sectors of the economy; this fact, however, does not invalidate the main results of the model.[9]

[9] The problems of the institutions, strain and underground economy were approached by us in a recent paper presented at the conference "The Importance of the Underground Economy in Economic

3. A GENERALISED MODEL FOR THE ALLOCATION OF TIME

In this section, we recall the two hypotheses of the model for the time allocation, the first two relations ((1) and (2)) from Section 1. Also, we have in Fig. 1 a graphical representation of this model.

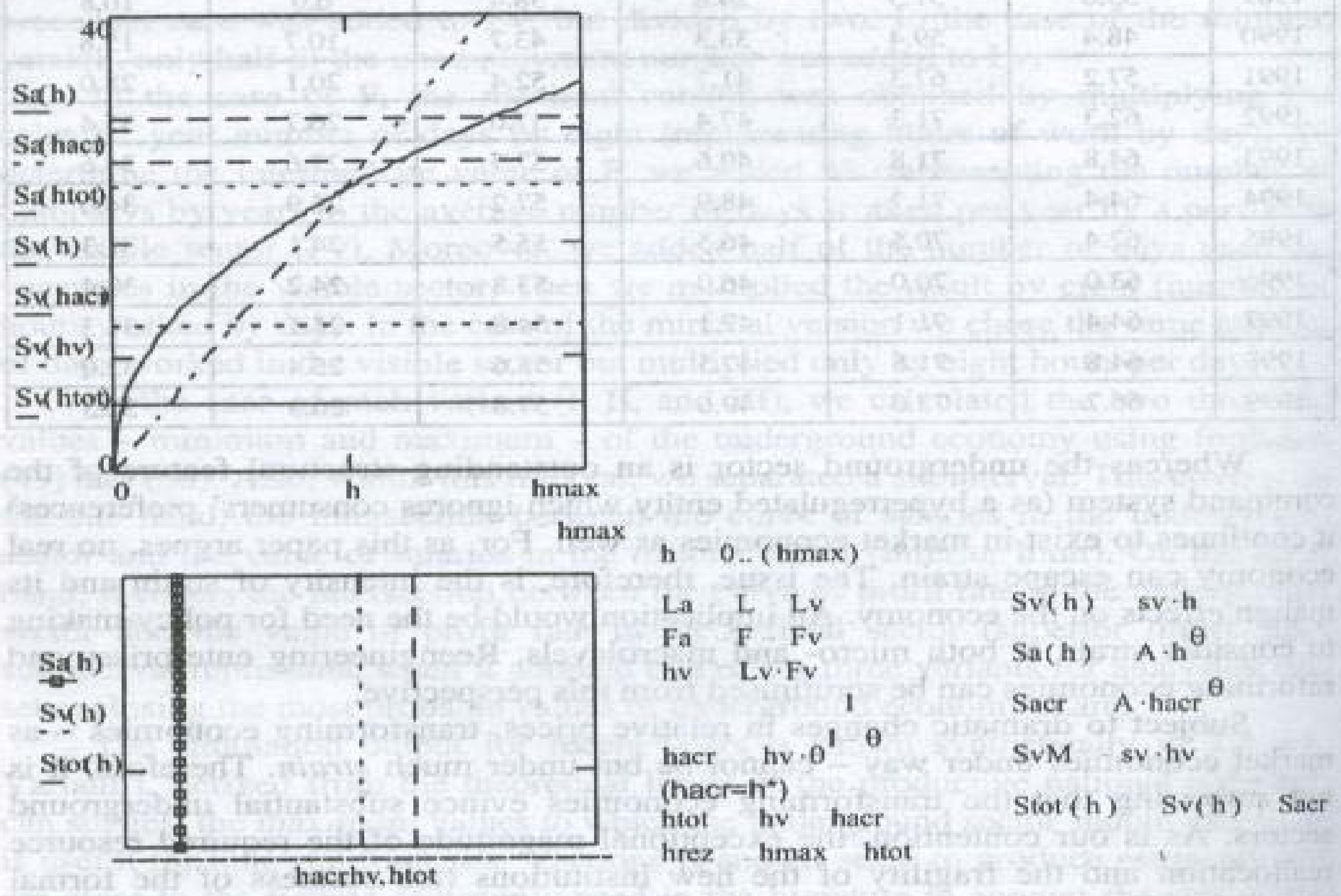

Fig. 1

The main implication of this model, also largely used in the literature, is that in point M of the graph the average salary in the underground sector equals that in the visible sector. Translated into terms of our model, it can be expressed by the following relations:

$$SaM = A1 \cdot hv\theta = SvM = sv \cdot hv \qquad (55)$$

Transition" held in Zagreb (Dăianu and Albu, 1997). There we benefited from the competent observations of Ivo Bicanic, Roger Bowles, Krassen Stanchev, John Tedstrom, and particularly Edgar Feige.

On the graph, also one may observe that the difference between Sa and Sv is maximal only in one point of abscise noted h^*, which represents the optimal number of hours worked in the underground sector.

To evaluate this optimal level of underground work, we write the function which must be maximised as:

$$G(ha) = A1 \cdot ha\theta - sv \cdot ha \tag{56}$$

and the first-order derivative as:

$$G'(ha) = \theta A1^* ha\theta - 1 - sv \tag{57}$$

However, at the national level, the function that concerns is:

$$H(ha) = S - S^* = (A1 \cdot ha\theta + sv \cdot ha) - sv \cdot (hv + ha) \tag{58}$$

Here, we can observe the equality between H and G. So, the two functions have the same solution for maxim.

After some algebraic operations, we obtained the following remarkable value for ha:

$$h^* = hv \cdot \theta 1[(1 - \theta)] \tag{59}$$

that then allowed us to express the optimal proportion of underground economy within the national economy by the following relation:

$$(h^*/h) = \{\theta[1/(1-\theta)]\}/\{1 + \theta[1/(1-\theta)]\} \tag{60}$$

where, h is total number of hours worked in a country in a year.

On the basis of this relation we calculated the share of underground economy for various values of parameter θ. An interesting result is that, at limit, the maximal share of underground economy (in shown optimal conditions) is only about 26.9% (*i.e.* 1/1+e).

Developing this model, we can obtain the map of the entire process of allocation of time, as it is shown in Fig. 1. Also, here the migration of some cohorts from the so-called "army of reserve" (leisure time) to the underground sector is represented. For instance, this may be provoked by an increase in number of hours worked in visible sector, which will move to a higher level the curve of wage in underground sector. This will make more attractive work in underground activity and it may lead to a re-allocation of available time of households. In these terms may be considered the potential labour force and the maximal values of underground sector in case of variant I from precedent section.

4. A MODEL BASED ON MAY'S LOGISTIC

As was shown in Section 2.2, an increase in the taxation degree creates conditions for an expansion of the underground economy. This phenomenon determines the public authorities react to it, generating a series of expenditures financed through taxes.

Under these conditions, variation (D) of the underground sector share in the total economic activity (X_t) may be written:

$$DX_t/DX_{t-1} = a + bT - 2(c + dT)X_{t-1} \tag{61}$$

where: a = the underground sector component which is not dependent on taxation; b = the underground sector component which is dependent on taxation; T = rate of taxation (values between 0 and 1); c = component that measures the reaction of the public authorities to the underground sector which is not dependent on the taxation degree; d = component that measures the reaction of the public authorities to the underground sector which is dependent on the taxation degree.

This is similar to:

$$X_t = (a + bT)X_{t-1} - (c + dT)(X_{t-1})^2 \tag{62}$$

If we note $\mu = a + bT$ and $\gamma = c + dT$ the equation (62) may be expressed as:

$$X_t = \mu X_{t-1} - \gamma(X_{t-1})^2 \tag{63}$$

equivalent with:

$$X_t = \mu X_{t-1}(1 - \beta X_{t-1}) \tag{64}$$

where $\beta = \gamma/\mu$.

In fact, this represents the generalised form of May's logistic.

The maximum value of X_t is $(\mu^2)/(4\gamma)$ and is achieved for of

$$X_t = (\mu)/(2\gamma). \tag{65}$$

In other words, the condition for the system not to explode is: $\mu < 2(\gamma)^{1/2}$.

In order to ensure that the weight of the underground sector in the whole economic activity is maintained constantly, respectively $X_t = X_{t-1}$, it is necessary that: $X_t = (1/\beta)[1 - (1/\mu)]$ That is:

$$X_t = (\mu - 1)/\beta \tag{66}$$

In modelling the economic system behaviour, an important role is played by the value of parameters β and μ. If we use the above-mentioned model in the analysis of the underground sector weight in the whole economic activity evolution, one may admit that if:

a) $\beta > 1$, social institutions are relatively efficient in the fight against the expansion of the underground sector;

b) $\beta = 1$, there is an equilibrium between the evolution of the underground sector and the social institutions;

c) $\beta < 1$, social institutions are relatively inefficient in the fight against the expansion of the underground sector.

Another condition for the economic system's operation is that the initial value of the underground sector weight (X_0) be smaller than $1/\beta$.

If we note $y_t = (1/\beta)X_t$ the values of y_t are comprised between 0 and 1 and the generalised form of May's logistic becomes:

$$X_t = (1/\beta)\mu y_t(1 - y_t) \tag{67}$$

Taking into account the behaviour of the economic system, equivalent to the famous canonical model of May's equation (parameter β is a scalar that permits to produce mappings on the ordinate axis) we are able to conclude in our case that:

a) if $0 < \mu < 1$, X_t moves monotonously towards the stationary solution $X_t = 0$;

b) if $1 < \mu < 2$, X_t moves monotonously towards the stationary solution $X_t = (\mu - 1)/\gamma$;

c) if $2 < \mu < 3$, X_t converges by a flattened oscillatory movement to the stationary solution $X_t = (\mu - 1)/\gamma$;

d) if $3 < \mu < 4$, X_t demonstrates a complex of permanent oscillations. For $\mu = 3.57\ldots$ an infinite number of fixed points with different periodicity and an infinite number of different periodical cycles may be observed; there also exist innumerable combinations of initial conditions from which completely μ-periodical, although bounded, trajectories begin.

As is shown in Fig. 2, an increase in the value of β determines a reduction of the limits between which the weight of the underground sector in the whole economic activity varies.

If $\beta < 1$, the system "explodes" for values of μ smaller than 4.

For $0.75 < \beta < 0.8925$, the model explodes directly from the permanent oscillation phase.

For $0.50 < \beta < 0.75$, the model explodes directly from the flattened oscillatory movement.

For $0 < \beta < 0.50$, the model explodes directly from the monotonous evolutions towards stationary values of X_t.

As a result, the impact of the taxation degree modifications on the size of the underground sector and the economic system's behaviour is very complex.

Because X_t may be expressed by the form:

$$X_t = a(1 + iT)X_{t-1}\{1 - [(1 + jT)X_{t-1}/(1 + iT)]\} \tag{68}$$

where $i = b/a$ and $j = d/c$, it can be deduced that an increase in the taxation degree determines an increase in the social institutions efficiency in the fight against the underground sector size growth only if j is larger than i.

In conclusion, when the influence of taxes on the underground sector size in conditions of a generalised form of May's logistic is analysed, one has to take into account the values of μ, γ, and β for $T = 0$ and $T = 1$, respectively.

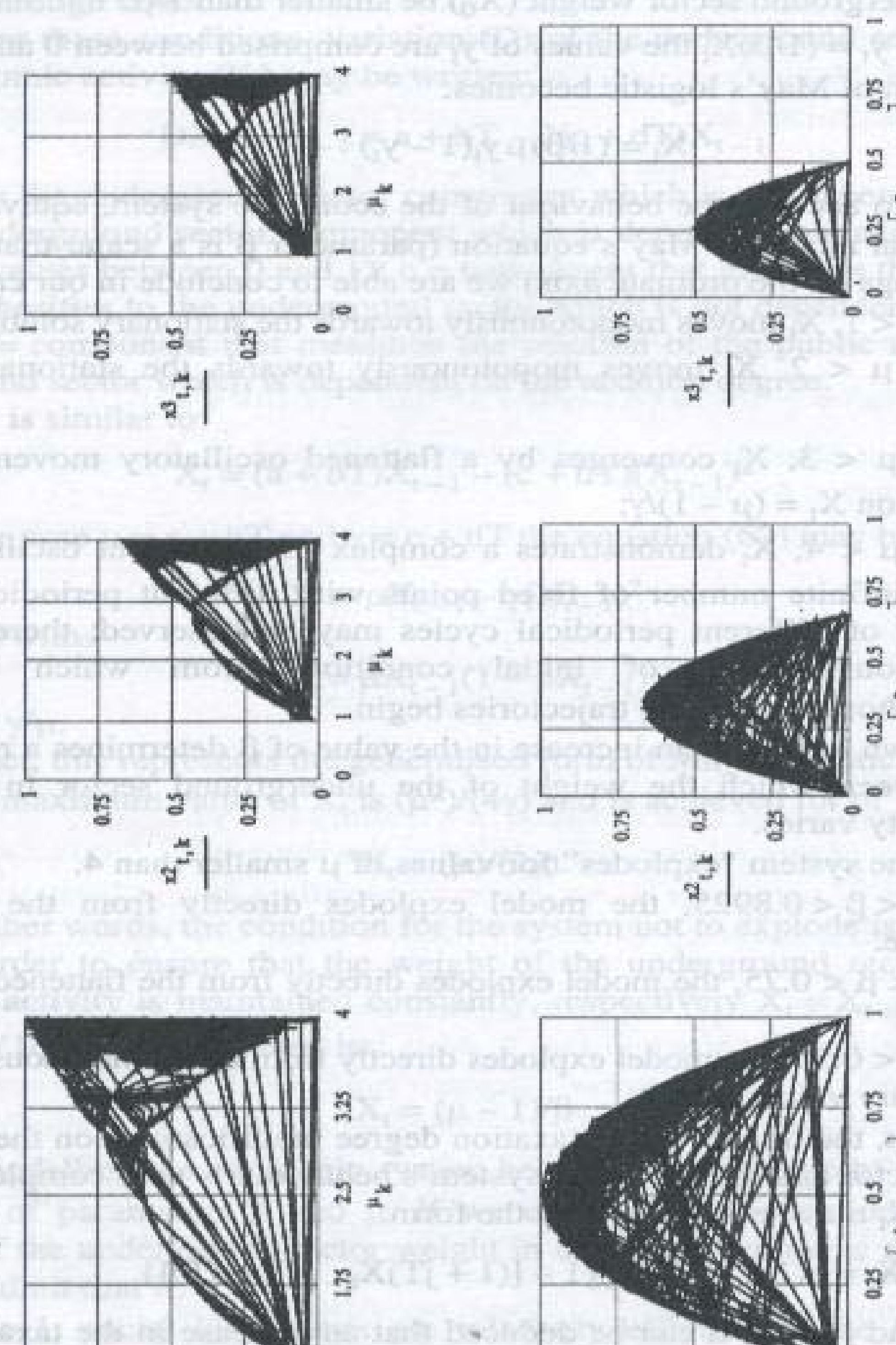

Fig. 2 – The X-μ Maps and Phase-Space Maps (β fixed).

When $T = 0$ we have $\mu_0 = a$, $\gamma_0 = c$, $\beta_0 = c/a$.

When $T = 1$ we have $\mu_1 = a + b$, $\gamma_1 = c + d$, $\beta_1 = (c + d)/(a + b)$.

So one can determine:

a) the sensitivity of parameter μ to taxes ($S\mu$) as:

$$S\mu = \mu_1 / \mu_0 = (a + b)/a = 1 + i$$

b) the sensitivity of parameter β to taxes ($S\beta$) as:

$$S\beta = (c + d)/c = 1 + j$$

The behaviour of the weight of the underground sector in the whole economic activity could be regarded through three aspects:

a) the domain of variation of factor μ (comprised between a and a + b). If $a + b > 4$ the system described by the model explodes and the value of T for which $a + bT = 4$ has to be calculated;
b) the values taken by factor β (comprised between (c/a) and $(c + d)/(a + b)$). In this case the modifications of the social institutions efficiency in correlation with the taxation degree can be observed and conclusions can be drawn in order to design fiscal policy;
c) the behaviour of the weight of the underground sector. For the critical points of μ the corresponding values of T have to be calculated and the behaviour of X_t evaluated.

For the economic policy designing, the model has two major implications:

a) If the social institutions are relatively not sensitive to the taxation variation ($j < i$), the level of taxation has to be as small as possible.
b) If the social institutions are relatively sensitive to the taxation variation ($j > i$), the public authorities have to choose between an increased but, in a way, controlled size of the underground sector, when taxes decrease and μ is smaller than 2, on the one hand, and a diminished, but, in a way, more difficult to control size of the underground sector when the taxes increase, on the other hand.

Using the data presented in the previous paragraph, we have estimated by OLS the values of parameters a, b, c, and d, in the case of Romania during the period 1989–1999.

The results are:

For V1min

$a = 1.529343$ (3.0164959)$b = 0.511305$ (0.475213)

$c = -0.843591$ (−1.1561546)$d = -0.701471$ (−0.416213)

$R^2 = 0.958653$ D − W =1.04569

For V1 max

a = 2.327760 (4.8321962)b = –1.686085 (–1.513812)

c = –1.896407 (–2.7648477)d = 2.493640 (1.518789)

R^2 = 0.9479941 D – W = 0.881358

For V2 min

a = 1.991626 (3.716607)b = –0.745782 (–0.576210)

c = –2.391435 (–2.054956)d = 2.684993 (0.901788)

R^2 = 0.948973 D – W = 0.927413

For V2 max

a = 1.581746 (9.050596)b = –0.1230846 (–0.774759)

c = –1.230846 (–4.366227)d = 0.859534 (1.009277)

R^2 = 0.841397 D – W = 1.694269

For V3 min

a = 1.217181 (2.3226131)b = 2.636076 (1.733179)

c = –2.095602 (–0.777951)d = –5.955063 (–0.722254)

R^2 = 0.94431 D – W = 1.783458

For V3 max

a = 4.143603 (4.8853421)b = –5.401661 (–2.479058)

c = –11.43375 (–4.085227)d = 22.46265 (3.054710)

R^2 = 0.951418 D – W = 2.268405

N.B. In the brackets there are the values of test Student. R^2 is the coefficient of determination. D – W is the value of the Durbin–Watson test. We mention here that the econometric computation has as a main goal to provide us only with the estimated values of parameters (a, b, c, and d) in the case of some variants presented in the section destined to the implicit labour supply method and not a very rigorous econometric analysis.

The results we have obtained illustrate the fact that the growing size of the underground sector in Romania is not mainly a consequence of the modifications operated in the taxation system. The primary cause of this evolution is the lack of an adequate institutional framework.

Within the estimations we have made in only two variants, the values of the parameters permit to apply our model of May's logistic type (V1min and V3 min). Between the two variants there are striking differences.

In Variant V1 min the social institutions appear to be relatively inefficient in their fight against the increase in the underground sector size ($\beta_0 = 0.551604$; $\beta_1 = 0.757143$). There is a small sensitivity to the taxation degree of the parameters μ and β (i = 0.33433; j = 0.83153).

The parameter μ takes values between 1.529343 and 2.040648. Under these conditions X converges monotonously towards (0.529343 + 0.511305T)/ /(0.843591 + 0.701471T). The convergence is monotonous if T takes values smaller than 0.9205 and with flattened oscillatory movements if T has values ranging between 0.9205 and 1.

It can be noticed that an increase in the taxation degree determines an increase in the underground sector size. For example, if T = 0, X_t converges towards 0.6275, and if T = 1, X_t converges towards 0.6735.

In Variant V3 min, at first sight, social institutions appear to be relatively efficient in their activity to maintain under control the size of the underground sector $\beta_0 = 1.7217$; $\beta_1 = 2.089314$). The values of the parameters μ and β are sensitive to the taxation degree (i = 20165722, j = 2.860078).

The parameter μ takes values between 1.217181 and 3.853257. Under these conditions X converges monotonously towards (0.2165722 + 2.636076T)/ /(2.095602 + 5.955063T) if T is smaller than 0.6763 . The convergence is monotonous if T takes values smaller than 0.2970 and with flattened oscillatory movements if T has values between 0.2970 and 0.6763. The increase in the taxation degree determines an increase in the underground sector size. If T = 0, X converges towards 0.1036 and if T = 0.6763, X converges towards 0.4959.

If T is greater than 0.6763, X shows permanent oscillations.

5. CONCLUSIONS

The transition towards a free market economic system in Central and East-European countries is doubled (followed) by a profound economic crisis. Generally, in these countries the production in the official sector has decreased rapidly after 1989 and official unemployment has grown to levels even greater than those currently registered in Western countries. Also, the income from the work in the official sector decreased dramatically during a historically short period. Under these conditions, an increasing share of the labour force supply moves to the black labour market. Contrary to the hypothesis of greater wages in the underground sector than those in the official sector, many suppliers of labour force on the black market accept wages smaller than in the official sector case (sa < sv).

On the other hand, the profit rate in the underground sector is always greater than the net profit rate in the visible sector (ba > bv). This is due to at least two reasons:

- in the underground sector there is no payment as taxes or other social duties;
- the expenditures on investments in fixed capital in the underground sector are minimal (near zero) as they use exclusively circulating capital and they operate under the rule of the profit maximisation in the short run, which become higher in a situation similar to the present one in the transition period, characterised by the scarcity of capital funds, doubled by an increasing inflation and a depreciation of national currency.

In the transition period, there is a relative scarcity of consumption goods and services and the production of heavy industries and generally of capital goods remains again monopoly of the state enterprises. Consequently, in the underground sector, if compared with the visible sector, the smaller amounts of disposable capital from small entrepreneurs go mainly to the production of consumption goods and services, where there is an uncovered demand for employment, the initial necessary amount of capital is smaller, being exclusively destined to ensure circulating capital funds, the velocity of the expended monetary funds and the rate of return to capital are greater.

Moreover, in the first years of transition when the inflation rate was greater than the interest rate (namely a negative interest rate in real terms) and the former fiscal legislation collapsed, there was a massive migration from the visible sector to the underground activities.

Also, in the conditions of an average small income per household, a supplementary supply of labour comes from some categories of population traditionally non-included in the labour force (such as: pupils, students, retired persons, and domestic women) and from the officially unemployed.

In the transition period some new differences have developed among groups of population: one is between employed people and unemployed people, other is between people employed only in the official sector and those employed in the underground sector or in both sectors. In conditions of a deepening crisis of production in the visible sector, the winners will be those employed in the underground sector (including persons employed in both sectors). Initially, the disposable income from such differences will be accumulated and then oriented (for reasons of efficiency) to underground activities. But then, when the amount of capital accumulated becomes greater than the power of absorption of the unofficial sector, there will be an inverse tendency: the newly rich people coming from underground activities will wish to officialise their new status and consequently they will try to "launder" their money and then to become "respectable" people, to enter legal activities. This last phase we may consider as the end of a cyclical sequence and the entire movement as a wave. After the stage of "primitive accumulation of capital", the Eastern economies will register a similar evolution as the Western economies did. On the general background of the restraint of the share of the underground economy, some fundamental re-structurations within the invisible sector will occur. The share of more sophisticated underground activities will increase if compared with the share of rudimentary and artisan underground activities.

If it is possible to enter a new spiraled cycle, it is a question for the future. Therefore, we consider that the description of the general process of economic development in the long run, based on a model of endogenous cycle, which explains the general economic fluctuations by a set of successive waves or overlapping cyclical phases relating to the report between underground economy and the visible economy, maybe as a plausible alternative to the standard models based only the dynamics of the official sector of economy in the short run. In this case, the historical economic evolution can be seen as a permanent but fluctuating overlapping transfer of cohorts of economic factors (labour, capital) from the official sector to the underground sector and the reverse. During instability periods, such as crises, wars or transitions, there is an intensification of transfers both ways between the two sectors of the economy, which corresponds to certain great structural re-adjustments of the economic system as well as the entire society.

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