



Munich Personal RePEc Archive

Product variety and the demand for children

Weisdorf, Jacob and Guzmán, Ricardo Andrés

Department of Economics, University of Copenhagen, Escuela de Administración, Pontificia Universidad Católica de Chile

February 2009

Online at <https://mpra.ub.uni-muenchen.de/14228/>
MPRA Paper No. 14228, posted 24 Mar 2009 04:48 UTC

Product variety and the demand for children

Jacob Weisdorf* Ricardo Andrés Guzmán†

February 2009

Abstract

Why does the rate of population growth decline in the face of economic growth? This study shows that growing product variety, along with more expensive children, induce a permanent reduction in the demand for children, and a continuous rise in income and consumption.

Keywords: Demographic transition; demand for children; product variety; Malthus.

JEL classification: J13; N30; O10.

1 Introduction

Starting in the late 19th century, net reproduction rates in Western Europe have dropped from an average of three surviving children per woman to just below two children by the present day (Maddison 2001). This is known as the *Demographic Transition*. Yet, over the same period, income per capita has increased nine fold (*ibid.*). If we believe that children are normal, ordinary goods, then the fall in the demand for children needs to be explained by negative price effects that override the positive income effect. While more expensive children are certainly part of the explanation (e.g. Bergstrom 2007; Galor 2005; Galor and Weil 1999, 2000); De la Croix and Licandro 2008), this paper shows that, if children and consumption goods are all normal, ordinary, and gross substitutes, then more consumption goods variety will also depress the demand for children, and speed up the growth of income and consumption.

*Department of Economics, University of Copenhagen. Address: 6 Studiestræde, DK-1455 Copenhagen, Denmark. E-Mail: jacob.weisdorf@econ.ku.dk. (Corresponding author.)

†Escuela de Administración, Pontificia Universidad Católica de Chile. Address: Avenida Vicuña Mackenna 4860, Macul, Santiago, Chile; Zip code: 7820436 MACUL. E-mail: rnguzman@uc.cl.

2 The model

2.1 Setup

Consider a small, open economy. Time is continuous, indexed by $t \geq 0$. The number of adults alive in time t is $N(t) > 0$. Adults live for one period: those alive in time t will be dead in time $t + \Delta$, where Δ is any positive number. When the adults die, they are replaced by their children. All adults are identical.

A typical time- t adult maximizes a CES utility function:

$$u(t) = \left(\phi c(t)^{\frac{\sigma-1}{\sigma}} + (1-\phi) n(t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (1)$$

where $c(t)$ is his consumption of a composite good, and $n(t) > 0$ is the number of his children. Parameter $\phi \in (0, 1)$ is the weight of children on utility, and $\sigma > 1$ is the elasticity of substitution between consumption and children. Because $\sigma > 1$, consumption and children are gross substitutes.

The composite encompasses $G(t) \in \mathbb{R}_+$ different consumption goods in time t :

$$c(t) = \left(\int_0^{G(t)} x(g, t)^{\frac{\rho-1}{\rho}} dg \right)^{\frac{\rho}{\rho-1}} \quad (2)$$

where $x(g, t)$ is the adult's consumption of good $g \in [0, G(t)]$. We call $G(t)$ the *variety*. Parameter $\rho > 1$ represents the elasticity of substitution between the different types of goods. The fact that $\rho > 1$ implies that adults will want to diversify consumption.

Each time- t adult earns a nominal wage $w(t) > 0$. The typical adult faces the following budget constraint:

$$w(t) \geq p_c(t) c(t) + p_n(t) n(t),$$

where $p_c(t)$ denotes the price of the composite in time t , and $p_n(t) > 0$ denotes the price of a child. The economy is small and open, so all prices are exogenous.

Standard calculations yield the following Marshallian demands for consumption and children.

$$c(t) = \frac{\phi^\sigma p_c(t)^{-\sigma} w(t)}{\phi^\sigma p_c(t)^{1-\sigma} + (1-\phi)^\sigma p_n(t)^{1-\sigma}} \quad (3)$$

$$n(t) = \frac{(1-\phi)^\sigma p_n(t)^{-\sigma} w(t)}{\phi^\sigma p_c(t)^{1-\sigma} + (1-\phi)^\sigma p_n(t)^{1-\sigma}} \quad (4)$$

Supposing that all goods cost the same, so that they will be consumed on equal amounts:

$$x(0, t) = x(g, t), \text{ for all } g \in [0, G(t)]. \quad (5)$$

It follows that total expenditure in the composite is

$$p_c(t) c(t) = \int_0^{G(t)} p_g x(g, t) dg = p_g x(0, t) G(t), \quad (6)$$

where p_g is the price of each individual good type. Using equations (2) and (5) we obtain:

$$c(t) = x(0, t) G(t)^{\frac{\rho}{\rho-1}}. \quad (7)$$

And combining equations (6) and (7) we get the price of the composite:

$$p_c(t) = p_g G(t)^{-\frac{1}{\rho-1}}. \quad (8)$$

The economy produces on goods type, while the remaining types of goods are imported from abroad. Labour is immobile, and the domestic labor supply is inelastic and equal to $N(t)$. The nominal wage is given by

$$w(t) = p_g A(t) N(t)^{-\alpha}, \quad (9)$$

where $A(t) > 0$ is the total factor productivity (TFP) in time t , and $\alpha \in (0, 1)$. Because $\alpha \in (0, 1)$, the wage falls as population rises.

Finally, the following equation governs population dynamics:

$$\frac{d \ln N(t)}{dt} = n(t) - \bar{n}, \quad (10)$$

where \bar{n} is the replacement fertility rate. Equations (9) and (10) constitute the classical Malthusian assumptions.

2.2 Equilibrium

Assume that TFP, the price of children, and variety change at constant, non-negative rates:

$$\frac{d \ln A(t)}{dt} = \gamma_A, \quad (11)$$

$$\frac{d \ln p_n(t)}{dt} = \gamma_{p_n}, \quad (12)$$

$$\frac{d \ln G(t)}{dt} = \gamma_G, \quad (13)$$

where $\gamma_A, \gamma_{p_n}, \gamma_G \geq 0$.

Combining equations (4), (8), and (9), we obtain the demand for children:

$$n(t) = \frac{(1 - \phi)^\sigma p_n(t)^{-\sigma} p_g A(t) N(t)^{-\alpha}}{\phi^\sigma p_g^{1-\sigma} G(t)^{-\frac{1-\sigma}{\rho-1}} + (1 - \phi)^\sigma p_n(t)^{1-\sigma}}$$

Log-differentiating the above equation with respect to t , taking limits, and rearranging we get:

$$\begin{aligned} \lim_{t \rightarrow \infty} \alpha (n(t) - \bar{n}) &= \gamma_A - \lim_{t \rightarrow \infty} \frac{d \ln n(t)}{d t} - \sigma \gamma_{p_n} \\ &\quad - \lim_{t \rightarrow \infty} \frac{d \ln [\phi^\sigma p_g^{1-\sigma} G(t)^{-\frac{1-\sigma}{\rho-1}} + (1 - \phi)^\sigma p_n(t)^{1-\sigma}]}{d t}, \end{aligned} \quad (14)$$

where we have used equations (10)–(13).

In the long run, the demand for children is constant:

$$\lim_{t \rightarrow \infty} n(t) = n_{\text{LR}}, \quad (15)$$

$$\lim_{t \rightarrow \infty} \frac{d \ln n(t)}{d t} = 0, \quad (16)$$

where n_{LR} denotes the long-run demand for children. Inserting (15) and (16) into equation (14) we get an expression for the long-run demand for children becomes:

$$n_{\text{LR}} = \bar{n} + \frac{1}{\alpha} \left(\gamma_A - \sigma \gamma_{p_n} - \lim_{t \rightarrow \infty} \frac{d \ln [\phi^\sigma p_g^{1-\sigma} G(t)^{-\frac{1-\sigma}{\rho-1}} + (1 - \phi)^\sigma p_n(t)^{1-\sigma}]}{d t} \right).$$

It is straightforward to show that

$$\lim_{t \rightarrow \infty} \frac{d \ln [\phi^\sigma p_g^{1-\sigma} G(t)^{-\frac{1-\sigma}{\rho-1}} + (1 - \phi)^\sigma p_n(t)^{1-\sigma}]}{d t} = \frac{\sigma - 1}{\rho - 1} \gamma_G \quad (17)$$

Hence, the long-run demand for children can be expressed as

$$n_{\text{LR}} = \bar{n} + \frac{1}{\alpha} \left(\gamma_A - \sigma \gamma_{p_n} - \frac{\sigma - 1}{\rho - 1} \gamma_G \right) \quad (18)$$

Once we know n_{LR} , the rates of change in the real wage (w/p_g) and in consumption are easily obtained:

$$\gamma_{w/p_g} = \sigma \gamma_{p_n} + \frac{\sigma - 1}{\rho - 1} \gamma_G \quad (19)$$

$$\gamma_c = \sigma \left(\gamma_{p_n} + \frac{1}{\rho - 1} \gamma_G \right) \quad (20)$$

Two important results come out from equations (18) and (19). Firstly, consistent with the existing literature, equation (18) shows that a rising cost of children ($\gamma_{p_n} > 0$) will dampen the positive effect of technological progress ($\gamma_A > 0$) on the demand for children. Since more expensive children moderate the growth of population, equation (20) shows that this helps generating growth in real income per capita and consumption. Implicitly, this is caused by diminishing returns to labour in production. Secondly, if children and consumption goods are all gross substitutes (i.e. if $\sigma, \rho > 1$), then, in response to more product variety, adults will reduce their consumption of all other goods in their baskets, children included. Since more product variety moderates the growth of population, equation (20) shows that this, too, helps generating growth in real income per capita. Hence, more product variety, hand-in-hand with more expensive children, explains why income growth is consistent with population growth decline, even though children is a normal good.

Note that the existing literature overlooks the effect of more product variety on the demand for children because of a widespread use of Cobb-Douglas preferences. In the Cobb-Douglas case, the elasticity of substitution between consumption goods and children equals one ($\sigma = 1$), which eliminates change in product variety from equations (18) and (19).

References

- Bergstrom, T.C. (2007), “Some evolutionary economics of family partnerships”, *American Economic Review* 97, pp. 482–486.
- Boucekkine R., D. De la Croix and O. Licandro (2002), “Vintage human capital, demographic trends and growth”, *Journal of Economic Theory* 104, pp. 340–375.
- De la Croix, D. and O. Licandro (2008), “‘The child is father of the man’: Implications for the Demographic Transition”, Université Catholique de Louvain, mimeo.
- Galor, O. (2005), “From stagnation to growth: Unified growth theory”, in P. Aghion and S. Durlauf (Eds.), *Handbook of Economic Growth*, pp. 171–293.
- Galor, O., D.N. Weil (1999), “From Malthusian stagnation to modern growth”, *American Economic Review* 89, pp. 150–154.
- Galor, O., D.N. Weil (2000), “Population, technology, and growth: From Malthusian stagnation to the Demographic Transition and beyond”, *American Economic Review* 90, pp. 806–828
- Maddison, A. (2001). *The world economy—a millennial perspective*. Paris: Development Center Studies, OECD.