Preference for Income Taxation with Several Heterogeneous Consumers

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Abstract

The dominance of income over commodity taxation for the single consumer case, implies that if the consumer is asked about what tax she would pay to bear a given tax burden, she would choose income taxation. This paper provides a version of this preference for income taxation for the case of several heterogeneous consumers by means of a game where the government allows each consumer to choose between the two tax regimes.

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1 Introduction

The comparison between income (direct) and commodity (indirect) taxation is one of the oldest issues in Public Economics since Barone (1912) showed that, keeping constant the utility of the taxpayer, the Exchequer could obtain a larger revenue from an income tax as opposed to a commodity tax. Different versions of this income taxation dominance, such as Borgatta (1921), Joseph (1939), Hicks (1939), and Peacock and Berry (1951) among others, were subsequently published. Nowadays this dominance is taught in several microeconomics textbooks. In particular, Stigler (1987) and Varian (1992) show the version provided by Borgatta (1921) and Joseph (1939) for the single consumer and two goods case. In such a case the proposition asserts that a given tax revenue yield would leave the taxpayer better off under an income tax than under a commodity tax. The proof of this proposition is based on the fact that both taxes have to collect the same tax revenue k. Let us illustrate this proof by considering a single consumer with regular preferences on the amounts x, y of two commodities. Since $(p+t)x+qy = w$ and $px+qy = (1-T)w$ are the consumer's budget lines under commodity and income taxation respectively, the bundle chosen under commodity taxation $x(t), y(t)$ is affordable under income taxation as well, because commodity tax revenue equals income tax revenue, i. e. $tx(t) = Tw = k$.

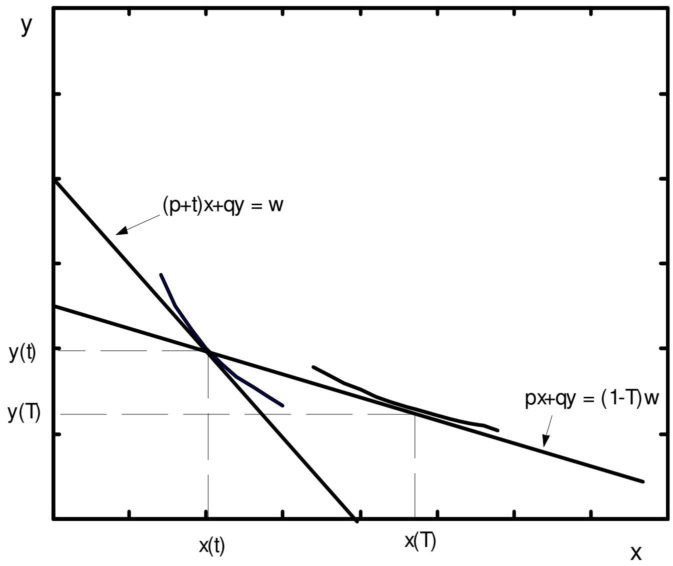

Figure 1

As can be seen in figure 1, a consequence of this feature is that the bundle chosen under commodity taxation $x(T), y(T)$ is directly revealed preferred to the bundle $x(t), y(t)$, that is, the weak axiom of revealed preference holds. This allows us to assert that, if the consumer is asked about what tax she would pay to bear a given tax burden, she would choose income taxation, that is, income taxation is preferred to commodity taxation.

This paper extends a version of this preference for income taxation for the case of several heterogeneous individuals. The model conceives of two goods, a fixed quantity k of numerarie which has to be taken by the government and n different heterogeneous consumers. This version of the preference for income taxation is a game where the government allows each consumer to choose between two tax regimes: a commodity taxation or an income taxation. In the case where the consumer chooses the income taxation, and parallel to what happens in the single consumer case, she has to bear a constant tax rate on her income given by the ratio between k and the total income of the economy; if she decides on the commodity taxation, she has to bear an excise tax added to the price of one of the goods. The tax revenue is the sum of both commodity and income taxation and the government keeps budgetary equilibrium all time. While the income tax rate is constant, the commodity tax depends on the number of consumers who are bearing it. As a result of that, strategic interdependence arises from the number of consumers who are paying the commodity tax. As we will see, in the unique pure strategy Nash equilibrium of this game everyone ends up choosing the income taxation regime.

2 The Model

Let $I = \{1, 2, ..., n\}$ be the set of heterogeneous consumers, denoted by $i \in I$, whose different preferences respect to the consumption of the goods X, Y are represented by a well-behaved utility function $u_i(x_i, y_i)$. Each consumer is endowed with a quantity w_i of observable income, let $W = \sum_I w_i$ the total income. An amount $0 < k < W$ of income has to be collected by a government; and (p, q) are the prices of goods X, Y, respectively.

For collecting the quantity k the government gives two options to consumers. On the one hand an excise tax $t \geq 0$ on the consumption of good X. In this case the i-th consumer budget constraint is

$$B_i(t) = \left\{(x_i, y_i) \in R_+^2 : w_i \geq (p+t)x_i + qy_i\right\}. \tag{1}$$

Let $(x_i(t), y_i(t))$ be the bundle that maximizes $u_i(x_i, y_i)$ subject to $B_i(t)$, and $v_i(t) = u_i\left(x_i(t), y_i(t)\right)$ her utility after the optimal decision.

On the other hand, the consumer can choose bearing a proportional tax rate $T \geq 0$ on her income. In this case her budget constraint is

$$B_i(T) = \left\{(x_i, y_i) \in R_+^2 : (1-T)w_i \geq px_i + qy_i\right\}. \tag{2}$$

Let $(x_i(T), y_i(T))$ be the bundle that maximizes $u_i(x_i, y_i)$ subject to $B_i(T)$, and $v_i(T) = u_i\left(x_i(T), y_i(T)\right)$ her utility after the optimal decision.

These two strategies are mutually exclusive, that is, the consumer who chooses to pay one tax is waived from paying the other.

Government's tax policy is as follows, it sets a constant income tax rate, given by $T^* = k/W$, to those consumers who choose *to bear income taxation*. Whereas for those consumers who choose *to bear commodity taxation* the tax rate is determined by fulfilling the budgetary equilibrium. More precisely, calling $D \subseteq I$ the set of consumers who choose *to bear commodity taxation* and $I \backslash D$ (complementary of D) the set of consumers who choose *to bear income taxation*, the government's budget constraint is given by

$$t \sum_{i \in D} x_i(t) + T^* \sum_{i \in I \backslash D} w_i = k. \tag{3}$$

The l. h. s of equation (3) represents total tax revenue. Its first part is the tax revenue collected from commodity taxation and its second one is that collected from income taxation. Notice that there may not exist a value of t which fulfills equation (3) for any value of k. In economic terms this means that indirect tax revenue is not enough to afford the fiscal debt after direct tax revenue $k - T^* \sum_{i \in I \backslash D} w_i$. To avoid this problem we assume throughout the paper that k is small enough to be affordable for each size of D. On the other hand, given that there is a solution for equation (3), it is possible that this can be multiple; in such a case, we assume that the government chooses the lowest positive one. Thus, taking into account that $W = \sum_{i \in D} w_i + \sum_{i \in I \backslash D} w_i$, we can write equation (3) as

$$t = T^* \frac{\sum_{i \in D} w_i}{\sum_{i \in D} x_i(t)}. \tag{4}$$

3 Game and equilibrium

Given the tax policy (T^*, t), our game is a one-shot game $\Gamma = \{I, S_i, \pi_i\}$, where I is the set of consumers, S_i =*{to bear income taxation, to bear commodity taxation}* is the set of strategies of each consumer, and

$$\pi_i(s) = \begin{cases} v_i(T^*), & \text{if } s_i = \textit{to bear income taxation} \\ v_i(t(s)), & \text{if } s_i = \textit{to bear commodity taxation} \end{cases}$$

is the payoff function of each consumer, where

$$s \in \prod_{i \in I} S_i = \{\textit{to bear income taxation, to bear commodity taxation}\}^n.$$

Therefore, in this game each consumer chooses between *to bear income taxation* or *to bear commodity taxation*. If the consumer chooses *to bear income taxation*, she has to pay a constant tax rate on her income given by $T^* = k/W$. The payoff of this action is independent of the others' actions. On the other hand, if she chooses *to bear commodity taxation*, she has to pay an excise tax t on the price of good X. This tax t is assessed accordingly with equation (3) and, in consequence, its value t depends on the number of consumers who are bearing it.

Hence, the payoff of this action depends on the others' actions, that is, strategic interdependence arises only from this tax.

It should be noted that choosing between *to bear commodity taxation* and *to bear income taxation*, is equivalent to choosing one of the budget constraints between (1) and (2). The following propositions lead us to the pure strategy Nash equilibrium of this game.

Proposition 1

Given the tax policy (T^{*}, t) and assuming that $D \neq \emptyset$, *to bear income taxation* is the dominant strategy at least for the consumer $h \in D$ with the largest ratio between consumption of good X and income.

Proof: Let $h \in D$ and $(x_h(t), y_h(t))$ be her consumption bundle which, due to the monotonicity, exhausts the bundle constraint given by (1), that is

$$w_h = (p+t)x_h(t) + qy_h(t). \tag{5}$$

Let us find out the conditions for which this consumption bundle is also affordable under the income tax regime. Thus, plugging $(x_h(t), y_h(t))$ into constraint (2) with $T = T^{*}$ and operating we can write

$$w_h \geq \left[p + T^{*}\frac{w_h}{x_h(t)}\right] x_h(t) + qy_h(t). \tag{6}$$

Comparing (5) and (6) $(x_h(t), y_h(t))$ is affordable under the income tax regime if and only if

$$p + t \geq p + T^{*}\frac{w_h}{x_h(t)},$$

taking into account (4) and clearing, this condition can be written as

$$\frac{x_h(t)}{w_h} \geq \frac{\sum_{i \in D} x_i(t)}{\sum_{i \in D} w_i}, \tag{7}$$

a condition which is held for at least the consumer in D with the largest ratio between her consumption of good X and her income. In fact, if $h \in D$ is the consumer so that $\frac{x_h(t)}{w_h} \geq \frac{x_i(t)}{w_i}$ $\forall i \in D$, we can write it as $w_i x_h(t) \geq w_h x_i(t)$ and adding with respect to $i \in D$ we have $x_h(t)\sum_{i \in D} w_i \geq w_h \sum_{i \in D} x_i(t)$, which is just the condition (7).■

Note that, on the one hand, proposition 1 states conditions for which the bundle chosen under commodity taxation is also affordable under income taxation. This means that the bundle chosen under income taxation is directly revealed preferred to the bundle chosen under commodity taxation. On the other hand, another drafting of proposition 1 is possible due to the fact that condition (7) can be fulfilled by other consumers without the largest ratio consumption of good X-income. Nevertheless, we have chosen the current draft for the sake of simplicity in exposition. Finally, since proposition 1 holds independently of the cardinality of D $(card(D))$, we can state the following proposition.

Proposition 2
To *bear income taxation* for every $i \in I$ is the unique pure strategy Nash equilibrium for the game Γ.

Proof: Let us suppose this is not true, that is, in equilibrium there is a number $k \geq 1$ of consumers who choose *to bear commodity taxation.* In such a case $D \neq \emptyset$ or $card(D) = k \in [1, n]$. Thus, according to proposition 1 this is not an equilibrium because there is at least one individual in D who prefers *to bear income taxation.* Moreover, proposition 1 applies for any size of set D, hence, the same argument works for $k-1, k-2, ..., 1$. Finally, the weak axiom of revealed preference ensures that when $D = \emptyset$ nobody has incentives *to bear commodity taxation* because, according to proposition 1, for every consumer the bundle chosen under income taxation is directly revealed preferred to the bundle chosen under commodity taxation.■

Proposition 2 is based on the fact that in our game there is always at least one consumer whose dominant strategy is *to bear income taxation.* According to proposition 1, this consumer is the one whose ratio consumption of good X-income is larger when every consumer bear commodity taxation. But this outcome occurs for each possible size of the set of consumers who bear commodity taxation. According to proposition 1, for each possible size of this set there is always at least one consumer whose dominant strategy is *to bear income taxation.* Thus, in equilibrium, the set of consumers who bear commodity taxation is empty. The following Cobb-Douglas example illustrates the result.

An example

Let us consider two different Cobb-Douglas consumers $u_i(x_i, y_i) = x_i^{\alpha_i} y_i^{1-\alpha_i}$ where $\alpha_1 = 1/3, w_1 = 1, \alpha_2 = 2/3, w_2 = 9$ and $k = p = q = 1$. The tax rate on income is $T^* = 1/10$, and the commodity tax rates for the pairs (*to bear commodity taxation, to bear commodity taxation*), (*to bear commodity taxation, to bear income taxation*) and (*to bear income taxation, to bear commodity taxation*) are $3/16, 3/7$ and $9/51$ respectively. The payoffs matrix is

	To bear commodity tax	*To bear income tax*
To bear commodity taxation	$0.4996, 4.2467$	$0.4698, 4.2859$
To bear income taxation	$0.4762, 4.2732$	$0.4762, 4.2859$

as we see, *to bear income taxation* is a dominant strategy for consumer 2 but not for consumer 1. According to proposition 1, this is because $\frac{x_2(t_S)}{w_2} > \frac{x_1(t_S)}{w_1}$ in case in which both consumers would be bearing commodity taxation. Given this dominant strategy for consumer 2 the best response for consumer 1 is *to bear income taxation.*

4 Pareto optimality

The previous sections have shown how n heterogeneous consumers prefer an income tax to a commodity tax in partial equilibrium. A brief reinterpretation of the same model allows us to illustrate a general equilibrium with production case. In fact, let us assume that our economy is formed by the same individuals as before but commodities are produced under constant returns to scale from the numerarie W, which is the primary input of the economy. That is, let x and y be the total production of commodities X and Y, $C_x(x) = px$ and $C_y(y) = qy$ represent the absorption of primary input for production of each commodity respectively. The total endowment W of primary input also has to finance the quantity k, yielding the following feasibility condition

$$W \geq k + px + qy,$$

introducing a non-convexity problem in the economy. In this trend, w_i is the quantity of primary input inelastically supplied by the i-th consumer and her income as well. Assuming that commodities X, Y are produced in competitive industries; profits in equilibrium are zero (due to the constant returns in production); and the aggregated supply of each good is perfectly elastic (allowing equilibrium output to be determined by total demand), the following proposition holds:

Proposition 3
For a constant returns economy with inelastic supply of primary input, the Nash equilibrium of the game Γ yields a Pareto optimal allocation.

Proof: If every consumer pays T^* the quantity k is financed through a non-distorting tax instrument. Thus, the equilibrium is just a Marginal Cost Pricing Equilibrium and, as is well-known, this is enough for Pareto optimality.■

This result was provided by Guesnerie (1975) for more general economies and further extended by several authors such as Bonnisseau and Cornet (1988) or Kahn and Vohra (1987). The insight is that for non-convex technologies any Pareto optimal allocation can be sustained as a marginal cost pricing equilibrium by means of a lump-sum redistribution of the losses.

Note that proposition 3 depends on the assumptions made about labor and technology. Because, as was pointed out by Little (1951) and Friedman (1953), the consideration of leisure and decreasing returns to scale (profits), make the labor supply elastic, allowing the income taxation to distort relative prices. In this case it is possible to reach the opposite results even for a single-consumer economy.

5 Conclusion

As we have seen, a version of the preference for income taxation has been proven for the case of several heterogeneous consumers. The proof is similar to that used for the single consumer case: the consumption bundle chosen under the commodity tax belongs to the feasible set defined by the income tax. The fact that while the income tax rate is constant (the quotient between the fixed cost and total income) and that the commodity tax rate adjusts depending on the number of individuals who choose to pay it, means that strategic interdependence arises only from commodity taxation. In this trend, we show that an equilibrium with consumers bearing the commodity taxation is not possible because, if this were the case, those consumers whose ratio between consumption of the good assessed for the commodity tax relative to its income is larger would have incentives to change to the income tax. This effect prevails independently of the size of the set of individuals who are bearing the commodity tax whenever it is non empty. Finally, if all the individuals except one decide to pay the income tax the best option left is also to pay that tax. Therefore the unique pure strategy Nash equilibrium is that in which all consumers bear income taxation. Finally, in terms of a general equilibrium framework with constant returns in production and inelastic supply of primary input this feature yields a Pareto optimal allocation by means of a marginal cost pricing equilibrium.

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