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Abstract

This paper constructs a two-country (Home and Foreign) general equilibrium model of Schumpeterian growth without scale effects. The scale effects property is removed by introducing two distinct specifications in the knowledge production function: the permanent effect on growth (PEG) specification, which allows policy effects on long-run growth; and the temporary effects on growth (TEG) specification, which generates semi-endogenous long-run economic growth. In the present model, the direction of the effect of the size of innovations on the pattern of trade and Home's relative wage depends on the way in which the scale effects property is removed. Under the PEG specification, changes in the size of innovations increase Home's comparative advantage and its relative wage, while under the TEG specification, an increase in the size of innovations increases Home's relative wage but with an ambiguous effect on its comparative advantage.

JEL Classification: F10, O3, O4
Key words: Comparative advantage, Trade, Schumpeterian growth, Scale effects, R&D races.

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1 Introduction

Many models of endogenous growth and trade emphasize the role of continual product innovation based on R&D investment in determining the pattern of trade between countries. Grossman and Helpman (1991*a*, *b*, *c*) have developed models where innovations lead to either improvements in the quality of existing products ("quality ladders" models) or increase in the variety of the goods ("love for variety" models). Taylor (1993) has extended the continuum Ricardian model of Dornbusch et al. (1977) based on the "quality ladders" approach by Grossman and Helpman. All these studies exhibit the scale effect property: if one incorporates population growth in these models, then the size of the economy (scale) increases exponentially over time, R&D resources grow exponentially, and so does the long-run growth rate of per-capita real output.

The scale effects property is a consequence of the assumption that the growth rate of knowledge is directly proportional to the level of resources devoted to R&D. Jones (1995*a*) has argued that the scale effects property of earlier endogenous growth models is inconsistent with post-war time series evidence from all major advanced countries that shows an exponential increase in R&D resources and a more-or-less constant rate of per-capita GDP growth. Jones's criticism has stimulated the development of a new class of models that generate growth without scale effects.[1] However, the theoretical literature on trade and growth without scale effects has focused either on closed economy models or on structurally identical economies engaging in trade with each other.[2] This paper develops a two-country general equilibrium framework without scale effects to determine the equilibrium relative wages and the pattern of trade between countries.

My approach borrows from Taylor's work (1993) in that industries differ in research technologies and in the set of technological opportunities available for each industry. In the presence of heterogeneous research technologies (captured by different productivity in R&D services), the pattern of R&D production and the pattern of goods production within each country can differ. As a result, there is a case for trade between countries in R&D services.

In the present model, there are two countries that may differ in relative size: Home and Foreign. The population in each country grows at a common positive and exogenously given rate and labor is the only factor of production. There is a continuum of industries producing final consumption goods. Labor in each industry can be allocated between the two economic activities, manufacturing of high-quality

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[1] See Dinopoulos and Thompson (1999) for more details on this issue.

[2] Dinopoulos and Syropoulos (2001) have recently developed a two-country general equilibrium model of endogenous Schumpeterian (R&D based) growth without scale effects to examine the effect of globalization on economic growth when countries differ in population size and relative factor endowments.

goods and R&D services, which are used to discover new products of higher quality. As in Grossman and Helpman 's (1991*c*) version of the quality-ladders growth model, the quality of each final good can be improved through endogenous innovation. The arrival of innovations in each industry is governed by a memoryless Poisson process whose intensity depends positively on R&D investments and negatively on the rate of difficulty of conducting R&D. I consider two alternative specifications regarding the difficulty of conducting R&D in order to remove the scale effects property. The first specification is called the permanent effects of growth (PEG) and it has been proposed by Dinopoulos and Thompson (1996). According to this specification, R&D becomes more difficult over time and the degree of R&D difficulty is proportional to the size of the world market. The second specification is called the temporary effects of growth (TEG) and it has been proposed by Segerstrom (1998). With this specification, R&D also becomes more difficult over time but the degree of difficulty is an increasing function of cumulative R&D effort in each industry.

Several comparative-steady-state results in Taylor's (1993) model change with the removal of the scale effects property. First, in his model, the direction of the effect of the size of innovations (which can vary across industries) on the pattern of goods production, R&D production, the pattern of trade, and the relative wage depends on the assumption that the size of innovations is heterogeneous. Under the heterogeneity assumption, the increase in the inventive step creates a deficit in the balance of payments for Home because it raises the royalties' payments that Home has to pay for using the front-line technology.[3] Balance of payments is maintained through two adjustments; Home raises its goods trade balance by increasing the range of goods produced at Home and it reduces its reliance on imported R&D by conducting more itself. Removing part of this heterogeneity in his model, by eliminating Home's relative advantage in goods versus R&D, results in zero trade in R&D and no effect of the size of innovations on the pattern of trade and Home's relative wage.[4] On contrast, in the present model, the direction of the effect of the size of innovations on the pattern of trade and Home's relative wage depends on the way in which the scale effects property is removed. Under the PEG specification, changes in the size of innovations increase Home's comparative advantage and its relative wage, while under the TEG

[3] Taylor (1993) divides the world's available technologies into two sets: the set of front line technologies and the set of backward technologies. Frontline technologies are those that are minimum cost given the prevailing wage rate. He further assumes that when an innovator located in Foreign succeeds in the global R&D races and discovers the front line technology, it has two options: it can either implement this improvement on the foreign technology or it can go multinational and carry the innovation abroad to a wholly owned subsidiary. This subsidiary would then pay the foreign firm a royalty.

[4] Eliminating the across country heterogeneity in his model, results in factor price equalization and indeterminate pattern of trade in both goods and R&D.

specification, an increase in the size of innovations increases Home's relative wage but with an ambiguous effect on its comparative advantage.

The analysis in the present model generates new additional findings. Under both specifications, the model generates a unique steady-state equilibrium in which there is complete specialization in both goods and R&D production within each industry. Trade between the two countries occurs only in goods and not in R&D services. In contrast to the work of Grossman and Helpman (1991*c*), factor price equalization does not hold in the steady-state equilibrium under either specification (Propositions 1 and 4).

Under the PEG specification, Home's relative wage depends positively on the Foreign's relative size, the consumer's subjective discount rate, and the population growth rate. It depends negatively on the R&D difficulty parameter, and the Home's relative size, (Proposition 2). In contrast to previous models (Grossman and Helpman (1991*c*), and Taylor (1993)), these results highlight the effects of population growth and the R&D difficulty on relative wages.

Under the PEG specification, the range of goods produced in Home and exported depends positively on the R&D difficulty parameter, Home's relative size, and the size of innovations. It depends negatively on Foreign's relative size, the consumer's subjective discount rate, and the population growth rate (Proposition 3). These results also highlight the importance of population growth and the R&D difficulty parameter on the pattern of goods and R&D production, and the pattern of trade between the two countries.

The remaining paper is organized as follows. Section 2 outlines the features of the model. Section 3 describes the steady state equilibrium of the model under the PEG specification and section 4 presents the comparative steady state results under the PEG specification. Section 5 analyzes the steady state equilibrium of the model under the TEG specification and section 6 presents the comparative steady state results under the TEG specification. Section 7 concludes this paper by summarizing the key findings and suggesting possible extensions. The algebraic details and proofs of propositions in this paper are relegated to Appendix A.

2 The Model

This section develops a two-country, dynamic, general-equilibrium model with the following features. Each country engages in two activities: the production of final consumption goods and research and development. Each of the two economies is populated by a continuum of industries indexed by $\theta \in [0, 1]$. A single primary factor, labor, is used in both goods and R&D production for any industry. In each industry θ firms are distinguished by the quality j of the products they produce. Higher values of j denote higher quality and j is restricted to taking on integer values. At time $t=0$, the state-of-the-art quality product in each industry is $j=0$, that is, some firm in each industry knows how to produce a $j=0$ quality product and no firm knows how to produce any higher quality product. The firm that knows how to produce the state-of-the-art quality product in each industry is the global leader for that particular industry. At the same time, challengers in both countries engage in R&D to discover the next higher-quality product that would replace the global leader in each industry. If the state-of-the-art quality in an industry is j, then the next winner of an R&D race becomes the sole global producer of a j+1 quality product. Thus, over time, products improve as innovations push each industry up its "quality ladder," as in Grossman and Helpman (1991*c*).

I assume for simplicity, that all firms in the global economy know how to produce all products that are at least one step below the state-of-the-art quality product in each industry. This assumption, which is standard in most quality-ladders growth models, prevents the incumbent monopolist from engaging in further R&D, which is standard assumption in most quality-ladder models.

For clarity, I adopt the following conventions regarding notation. Henceforth, superscripts "h" and "f" identify functions and variables of "Home" and "Foreign" countries, respectively. Functions and variables without superscripts are related to the global economy, while functions and variables with subscripts are related to activities and firms within an industry.

2.1 Household Behavior

Let $N^i(t)$ be country i's population at time t. I assume that each country' s population is growing at a common constant, exogenously given rate $g_N = \dot{N}^i(t)/N^i(t)$ > 0. In each country there is a continuum of identical dynastic families that provide labor services in exchange for wages, and save by holding assets of firms engaged in R&D. Each individual member of a household is endowed with one unit of labor, which is inelastically supplied. I normalize the measure of families in each country at time 0 to equal unity. Thus, the population of workers at time t in country i is $N^i(t) = N_0^i e^{g_N t}$.

Each household in country i maximizes the discounted utility[5]

$$U = \int_0^\infty e^{-(\rho - g_N)t} \log u(t) dt, \quad (1)$$

[5] Barro and Sala-i-Martin (1995 Ch.2) provide more details on this formulation of the household's behavior within the context of the Ramsey model of growth.

where $\rho > 0$ is the constant subjective discount rate. In order for U to be bounded, I assume that the effective discount rate is positive (i.e., $\rho - g_N > 0$). Expression log u(t) captures the per capita utility at time *t*, which is defined as follows:

$$\log u(t) \equiv \int_0^1 \log[\sum_j \lambda^j q(j,\theta,t)]d\theta . \tag{2}$$

In equation (2), $q(j,\theta,t)$ denotes the quantity consumed of a final product of quality *j* (i.e., the product that has experienced j quality improvements) in industry $\theta \in [0,1]$ at time t. Parameter $\lambda > 1$ measures the size of quality improvements (i.e., the size of innovations).

At each point in time t, each household allocates its income to maximize (2) given the prevailing market prices. Solving this optimal control problem yields a unit elastic demand function for the product in each industry with the lowest quality-adjusted price

$$q^i(j,\theta,t) = \frac{c^i(t)N^i(t)}{p^i(j,\theta,t)}, \tag{3}$$

where $c^i(t)$ is country *i*'s per capita consumption expenditure, and $p^i(j,\theta,t)$ is the market price of the good considered in country *i*. Because goods within each industry adjusted for quality are by assumption identical, only the good with the lowest quality-adjusted price in each industry is consumed. The quantity demanded of all other goods is zero. The global demand for a particular product is given by aggregating equation (3) across the two countries to obtain

$$q(j,\theta,t) = \sum_{i=h,f} q^i(j,\theta,t). \tag{4}$$

Given this static demand behavior, the intertemporal maximization problem of country *i*'s representative household is equivalent to

$$\max_{c^i(t)} \int_0^\infty e^{-(\rho - g_N)t} \log c^i(t) dt , \tag{5}$$

subject to the intertemporal budget constraint $\dot{a}^i(t) = r^i(t)a(t) + w^i(t) - c^i(t) - g_N a^i$, where $a^i(t)$ denotes the per capita financial assets in country $i, w^i(t)$ is the wage income of the representative household member in country *i*, and $r^i(t)$ is country *i*'s instantaneous rate of return at time *t*. The solution to this maximization problem obeys the well-known differential equation

$$\frac{\dot{c}^i(t)}{c^i(t)} = r^i(t) - \rho , \tag{6}$$

Equation (6) implies that a constant per-capita consumption expenditure is optimal when the instantaneous interest rate in each country equals the consumer's subjective discount rate ρ.

2.2 Product Markets

In each country firms can hire labor to produce any final consumption good $\theta \in [0,1]$. Let $L^i(\theta, t)$ and $Q^i(\theta, t)$ respectively denote the amounts of labor devoted in manufacturing of final consumption good θ in country *i* and the output of final consumption good θ in country *i*. Then the production function of the final consumption good θ in country *i* is given by the following equation

$$Q^i(\theta,t) = \frac{L^i(\theta,t)}{\alpha_Q}, \qquad (7)$$

where α_Q is the unit labor requirement associated with the final consumption good θ. For simplicity, I assume that the unit labor requirement is equal to 1, which implies that one unit of labor is required to manufacture one unit of the good. I also assume that each vertically differentiated good must be manufactured in the country in which the most recent product improvement has taken place. That is, I rule out international licensing and multinational corporations.[6]

The assumptions that goods within an industry are identical when adjusted for quality and Bertrand price competition in product markets imply that the monopolist in each industry engages in limit pricing. The assumption that the technology of all inferior quality products is public knowledge imply that the quality leader charges a single price, which is λ times the lowest manufacturing cost between the two countries:

$$p = \lambda \min\{w^h, w^f\}. \qquad (8)$$

I choose the wage of foreign labor, w^f, as the numeraire of the model by setting:

$$w^f \equiv 1. \qquad (9)$$

I also assume that the wage of home labor, w^h, which is also Home's relative wage, ω, is greater than one[7]

$$w^h = \omega > 1. \qquad (10)$$

Assumption (10) implies that the price of every top quality good is equal to

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[6] Taylor (1993) incorporates multinational corporations in a model of endogenous growth and trade. In his model, innovations are always implemented on front line production technologies (i.e, that is technologies that are minimum cost given the prevailing wage rates) and when innovation and implementation occur at different countries, the resulting transactions are considered as imports and exports of R&D.

[7] In propositions 1 and 4, I provide sufficient conditions under which this assumption holds.

$$p=\lambda . \tag{11}$$

It follows that the stream of profits of the incumbent monopolist that produces the state-of-the-art quality product in Home will be equal to

$$\pi^{h}(\theta,t)=(\lambda-\omega)q=\left(\frac{\lambda-\omega}{\lambda}\right)E(t), \tag{12}$$

while the stream of profits of the incumbent monopolist that produces the state-of-the-art quality product in Foreign will be equal to

$$\pi^{f}(\theta,t)=(\lambda-1)q=\frac{(\lambda-1)}{\lambda}E(t), \tag{13}$$

where $E(t)=[c^{h}(t)N^{h}(t)+c^{f}(t)N^{f}(t)]$ is the world expenditure on final consumption goods.

2.3 R&D Races

Labor is the only input engaged in R&D in any industry $\theta\in[0,1]$. Let $L_{R}^{i}(\theta,t)$ and $R^{i}(\theta,t)$ respectively denote the amounts of labor devoted in R&D services in industry θ in country *i* and the output of R&D services in industry θ in country *i*. The production function of R&D services in industry θ in country *i* exhibits constant returns and is given by the following equation[8]

$$R^{i}(\theta,t)=\frac{L_{R}^{i}(\theta,t)}{\alpha_{R}^{i}(\theta)}, \tag{14}$$

where $\alpha_{R}^{i}(\theta)$ is the unit labor requirement in the production of R&D services. The presence of heterogeneous research technologies in the present model allows us to determine the pattern of R&D services first and then the pattern in the trade of manufacturing goods.[9]

The continuum of products $\theta\in[0,\ 1]$ is indexed by decreasing home relative unit labor requirement in R&D. If $\theta_2>\theta_1$ for any θ_1 and $\theta_2\in[0,1]$, then $\frac{\alpha_{R}^{f}(\theta_1)}{\alpha_{R}^{h}(\theta_1)}>\frac{\alpha_{R}^{f}(\theta_2)}{\alpha_{R}^{h}(\theta_2)}$ should hold.

[8] The empirical evidence on returns to scale of R&D expenditure is inconclusive. Segerstrom and Zolnierek (1999) among others developed a model where they allow for diminishing returns to R&D effort at the firm level and industry leaders have R&D cost advantages over follower firms. In their model, when there are diminishing returns to R&D and the government does not intervene both industry leaders and follower firms invest in R&D.

[9] Taylor (1993) has introduced heterogeneity in the research technologies and in the technological opportunity for improvements in technologies. The presence of heterogeneous research technologies makes trade in R&D services between countries possible.

Following Dornbusch et al. (1977), the continuous and decreasing relative unit labor requirement in R&D for each good θ is defined as follows

$$A(\theta) = \frac{\alpha_R^f(\theta)}{\alpha_R^h(\theta)} \text{ and } A'(\theta) < 0 . \tag{A.1}$$

In each industry θ there are global, sequential and stochastic R&D races that result in the discovery of higher-quality final products. A challenger firm k that is located in country $i \in \{h, f\}$ targeting a quality leader in country $i \in \{h, f\}$ engages in R&D in industry θ and discovers the next higher-quality product with instantaneous probability $I_k^i(\theta,t)dt$, where dt is an infinitesimal interval of time and

$$I_k{}^i(\theta, t) = \frac{R_k^i(\theta, t)}{X(t)}, \tag{15}$$

where $R_k^i(\theta, t)$ denotes firm *k*'s R&D outlays and *X(t)* captures the difficulty of R&D in industry θ at time *t*. I assume that the returns to R&D investments are independently distributed across challengers, countries, industries, and over time. Therefore, the industry-wide probability of innovation can be obtained from equation (14) by summing up the levels of R&D across all challengers in that country. That is,

$$I^i(\theta, t) = \sum_k I_k^i(\theta, t) = \frac{R^i(\theta, t)}{X(t)}, \tag{16}$$

where $R^i(\theta, t)$ denotes total R&D services in industry θ in country *i*. Variable $I^i(\theta, t)$ is the effective R&D.[10] The arrival of innovations in each industry follows a memoryless Poisson process with intensity $I(\theta,t) = \sum_i R^i(\theta,t)/X(t)$ which equals the global rate of innovation in a typical industry. The function *X(t)* has been introduced in the endogenous growth literature after Jone's (1995*a*) empirical criticism of R&D based growth models generating scale effects.

[10] The variable $I^i(\theta, t)$ is the intensity of the Poisson process that governs the arrivals of innovations in industry θ in country *i*. Dinopoulos and Syropoulos (2001) model the strategic interactions between a typical incumbent and its challengers as a differential game for Poisson jump processes and derive the equilibrium conditions that govern the solution to a typical R&D contest. They also provide an informal and intuitive derivation of these conditions. In the present model, I follow their informal derivation to derive my results.

A recent body of theoretical literature has developed models of Schumpeterian growth without scale effects.[11] Two alternative specifications have offered possible solutions to the scale-effects property. The first specification proposed by Dinopoulos and Thompson (1996) removes the scale-effects property by assuming that the level of R&D difficulty is proportional to the market size measured by the level of population,

$$X(t) = kN(t), \tag{17}$$

where $k > 0$ is a parameter.

This specification captures the notion that it is more difficult to introduce new products and replace old ones in a larger market. The model that results form this specification is called the permanent effects of growth (PEG) model because policies such as an R&D subsidy and tariffs can alter the per-capita long-run growth rate.[12]

The second specification proposed by Segerstrom (1998) removes the scale effects property by assuming that R&D becomes more difficult over time because "the most obvious ideas are discovered first." The model that results from this specification is called the temporary effects of growth (TEG) model. In this model, the long-run growth rate is proportional to the exogenous rate of population growth and it is not affected by any standard policy instruments. Under the TEG specification, R&D starts being equally difficult in all industries ($X(\theta,0)=1$ for all θ), and the level of R&D difficulty grows according to

$$\frac{\dot{X}(t)}{X(t)}=\mu[I^{h}(\theta,t)+I^{f}(\theta,t)]=\mu I(\theta,t), \tag{18}$$

where $\mu > 0$ is a constant.

In subsequent sections I will consider each specification separately to analyze the steady-state equilibria and to derive their comparative-static properties.

Consider now the stock-market valuation of temporary monopoly profits. Consumer savings are channeled to firms engaging in R&D through the stock market. The assumption of a continuum of industries allows consumers to diversify the industry-specific risk completely and earn the market interest rate. At each instant in time, each challenger issues a flow of securities that promise to pay the flow of

[11] See Dinopoulos and Thompson (1999) for an overview of these models.

[12] Dinopoulos and Thompson (1998) provide micro foundations for this specification in the context of a model with horizontal and vertical product differentiation.

monopoly profits if the firm wins the R&D race and zero otherwise.[13] Consider now the stock-market valuation of the incumbent firm in each industry. Let $V^i(t)$ denote the expected global discounted profits of a successful innovator at time t in country *i*, when the global monopolist charges a price p for the state-of-the-art quality product. Because each global quality leader is targeted by challengers from both countries who engage in R&D to discover the next higher-quality product, a shareholder faces a capital loss $V^i(t)$ if further innovation occurs. The event that the next innovation will arrive occurs with instantaneous probability *Idt*, whereas the event that no innovation will arrive occurs with instantaneous probability 1-*Idt*. Over a time interval dt, the shareholder of an incumbent's stock receives a dividend π(t)dt and the value of the incumbent appreciates by $dV^i(t) = [\partial V^i(t)/\partial t]dt = \dot{V}^i(t)dt$. Perfect international capital mobility implies that $r^h = r^f = r$. The absence of profitable arbitrage opportunities requires the expected rate of return on stock issued by a successful innovator to be equal to the riskless rate of return *r*; that is,

$$\frac{\dot{V}^i(\theta,t)}{V^i(\theta,t)}[1 - I(\theta,t)dt]dt + \frac{\pi^i(\theta,t)}{V^i(\theta,t)}dt - \frac{[V^i(\theta,t) - 0]}{V^i(\theta,t)}I(\theta,t)dt = rdt\,. \tag{19}$$

Taking limits in equation (19) as $dt \to 0$ and rearranging terms appropriately gives the following expression for the value of monopoly profits

$$V^i(\theta,t) = \frac{\pi^i(\theta,t)}{r(t) + I(\theta,t) - \dfrac{\dot{V}^i(\theta,t)}{V^i(\theta,t)}}\,. \tag{20}$$

A typical challenger *k* located in country *i* chooses the level of R&D investment $R_k^i(\theta,t)$ to maximize the expected discounted profits

$$V^i(\theta,t)\frac{R_k^i(\theta,t)}{X(t)}dt - w^i\alpha_R R_k^i(\theta,t)dt\,, \tag{21}$$

where $I_k^i dt = [R_k^i(\theta,t)/X(t)]dt$ is the instantaneous probability of discovering the next higher-quality product and $w^i\alpha_R R_k^i(\theta,t)$ is the R&D cost of challenger *k* located in country *i*.

Free entry into each R&D race drives the expected discounted profits of each challenger down to zero and yields the following zero profit condition:

$$V^i(t) = w^i\alpha_R X(t)\,. \tag{22}$$

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[13] If the monopolist is located in Home, the monopoly profits are define by equation (12) and if the monopolist is located in Foreign the monopoly profits are defined by equation (13).

The pattern of R&D production across the two countries can be determined by utilizing equations (20) and (22). Evaluating these equations on the competitive margin in R&D production, $\tilde{\theta}$, I can obtain the R&D schedule (i.e., the schedule of relative labor productivities in goods) as follows

$$\omega = RD(\tilde{\theta}) = \frac{\lambda A(\tilde{\theta})}{\lambda + A(\tilde{\theta}) - 1}, \tag{23}$$

where $RD(\tilde{\theta})$ is continuous and decreasing in $\tilde{\theta}$. For low values of θ, Home has higher relative labor productivity than Foreign, and thus it earns higher wage. Therefore, Home has comparative advantage in producing and conducting R&D the final goods with lower θ and Foreign has comparative advantage in producing and conducting R&D the final goods with higher θ. The R&D schedule can be depicted in Figure 1.

Lemma 1. *Under assumption* (A.1) *and for any value of the relative wage,* $\omega < \lambda$*, there exists an industry* $\tilde{\theta}$ *defined by equation* (23) *such that*

(a) $\omega = RD(\tilde{\theta})$ *schedule is downward sloping, i.e.,* $RD'(\tilde{\theta}) < 0$,

(b) *firms are indifferent between conducting R&D in Foreign or in Home*,

(c) *for each industry* $\theta \in [0, \tilde{\theta})$*, only Home conducts R&D*,

(d) *for each industry* $\theta \in (\tilde{\theta}, 1]$*, only Foreign conducts R&D.*

Proof. See Appendix A.

One can find the results from Lemma 1 in Dornbusch et al. (1977). However, the derivation of Lemma 1 differs between the present model and the one in Dornbusch et al. (1977). In their model, the results from Lemma 1 come from the assumption of perfect competition in all markets. In the present model, the intuition behind Lemma 1 results from the zero profit conditions regarding R&D. If in industry θ, R&D is undertaken by Home, then the zero profit conditions for R&D imply that Foreign has negative profits in this particular industry (see equations (20) and (22)). The larger the range that home exports, the lower the home's comparative advantage in R&D. The decreasing mutual R&D condition suggests that Home firms have higher discount profits than foreign firms for the goods in the range $\theta \in [0, \tilde{\theta})$. Foreign challengers would not be able to finance their R&D costs in the range of industries $\theta \in [0, \tilde{\theta})$ and choose not to engage in R&D since this would yield negative profits. The reverse is true for those industries that Foreign undertakes R&D. Home has negative profits in the industries $\theta \in (\tilde{\theta}, 1]$, so it does not engage in R&D in those industries. Thus, both countries sustain their comparative advantage.

2.4 Labor Markets

Consider first the Home labor market. All workers are employed by firms in either production or R&D activities. Taking into account that each industry leader charges the same price p and that consumers only buy goods from industry leaders in equilibrium, it follows from (7) that total employment of labor in production in Home is $\int_0^{\tilde{\theta}} Q^h(\theta,t)d\theta$. Solving equation (14) for each industry leader's R&D employment $L_R^h(\theta,t)$ and then integrating across industries, total R&D employment by industry leaders in Home is $\int_0^{\tilde{\theta}} R^h(\theta,t)\alpha_R^h(\theta)d\theta$. Thus, the full employment of labor condition for Home at time t is given by

$$N^h(t) = \int_0^{\tilde{\theta}} Q^h(\theta,t)d\theta + \int_0^{\tilde{\theta}} R^h(\theta,t)\alpha_R^h(\theta)d\theta. \tag{24}$$

I can derive in a similar way the full employment of labor condition for Foreign at time t and obtain

$$N^f(t) = \int_{\tilde{\theta}}^1 Q^f(\theta,t)d\theta + \int_{\tilde{\theta}}^1 R^f(\theta,t)\alpha_R^f(\theta)d\theta. \tag{25}$$

Equations (24) and (25) complete the description of the model.

3 Steady-State Equilibrium Under the PEG Specification

In this section I derive the steady-state equilibrium under the PEG specification proposed by Dinopoulos and Thompson (1996), which is described according to equation (17).

Assuming that the relative wage, ω, is constant over time at the steady-state equilibrium, equation (22) implies that $\dot{V}^i(\theta,t)/V^i(\theta,t) = \dot{X}(t)/X(t) = g_N$. That is, the expected global discounted profits of a successful innovator at time *t* in country *i*, $V^i(t)$, and the level of R&D difficulty, *X(t)*, grow at the constant rate of population growth, g_N. In the steady-state equilibrium, the market interest rate, *r*, must be equal to the subjective discount rate, ρ.[14] Combining equations (20) and (22) after taking into account equation (17), I obtain the following zero profit conditions for Home and Foreign respectively for each industry:

$$\frac{\left(\frac{\lambda-\omega}{\lambda}\right)E(t)}{(\rho + I(\theta,t) - g_N)} = \omega\alpha_R^h(\theta)kN(t), \qquad \forall\theta \in [0,\tilde{\theta}) \tag{26}$$

[14] This property is shared by all Schumpeterian models where growth is generated by the introduction of final consumption (as opposed to intermediate production) goods.

$$\frac{\frac{(\lambda-1)}{\lambda}E(t)}{(\rho+I(\theta,t)-g_N)}=\alpha_R^f(\theta)kN(t), \qquad \forall\theta\in(\widetilde{\theta},1]. \tag{27}$$

Integrating equation (26) over $[0,\widetilde{\theta})$ and equation (27) over $(\widetilde{\theta},1]$, I obtain the following zero profit conditions for home and foreign, respectively at the economy-wide level:

$$\frac{\widetilde{\theta}(\lambda-\omega)E(t)}{\lambda}= \omega X(t)\left[(\rho-g_N)A_1^h(\widetilde{\theta})+A_2^h(\widetilde{\theta})\right], \tag{28}$$

$$\frac{(1-\widetilde{\theta})(\lambda-1)E(t)}{\lambda}=X(t)\left[(\rho-g_N)A_1^f(\widetilde{\theta})+A_2^f(\widetilde{\theta})\right], \tag{29}$$

where $A_1^h(\widetilde{\theta})=\int_0^{\widetilde{\theta}}\alpha_R^h(\theta)d\theta$, $A_2^h(\widetilde{\theta})=\int_0^{\widetilde{\theta}}I(\theta,t)\alpha_R^h(\theta)d\theta$, $A_1^f(\widetilde{\theta})=\int_{\widetilde{\theta}}^1\alpha_R^f(\theta)d\theta$, and

$A_2^f(\widetilde{\theta})=\int_{\widetilde{\theta}}^1 I(\theta,t)\alpha_R^f(\theta)d\theta$.

Substituting equations (3), (9), and (10) into Home's full employment of labor condition (equation 24) yields the resource condition

$$N^h(t)=\frac{E(t)\widetilde{\theta}}{\lambda}+X(t)A_2^h(\widetilde{\theta}), \tag{30}$$

Similar substitutions yield the resource condition for Foreign:

$$N^f(t)=\frac{(1-\widetilde{\theta})E(t)}{\lambda}+X(t)A_2^f(\widetilde{\theta}). \tag{31}$$

The above resource conditions described by equations (30) and (31) hold at each instant in time because, by assumption, factor markets clear instantaneously in both countries.

Equations (28), (29), (30), and (31) represent a system of four equations in four unknowns $\widetilde{\theta}$, ω, I, and E. Manipulating these four equations yields a second schedule in (θ,ω) space, the mutual resource schedule[15]

$$\omega=MR(\widetilde{\theta})=\frac{[\overline{N}^f+k(\rho-g_N)A_1^f(\widetilde{\theta})]\widetilde{\theta}}{[\overline{N}^h+k(\rho-g_N)A_1^h(\widetilde{\theta})](1-\widetilde{\theta})}, \tag{32}$$

where $\overline{N}^h(t)=N^h(t)/N(t)$ is Home's population relative to the world population and $\overline{N}^f(t)=N^f(t)/N(t)$ is Foreign' s population relative to the world population.

[15] In Appendix A, I derive the mutual resource schedule and show that it is upward-sloping in (θ,ω) space.

The mutual resource schedule states that the relative wage, which clears labor markets in both countries, is an increasing function of the range of goods $\widetilde{\theta}$ produced in Home. If the range of goods produced by Home increases, Home's relative demand for labor (both ω in manufacturing and R&D) increases. The excess demand for labor drives the level of the relative wage higher. The mutual resource condition *(MR)* can be depicted in Figure 1. The vertical axis measures Home's relative wage, ω, and the horizontal axis reflects the measure of industries, θ. The intersection of the downward sloping $RD(\widetilde{\theta})$ schedule and the upward sloping $MR(\widetilde{\theta})$ schedule at point E determines the steady-state equilibrium relative wage, ω^*, and the marginal industry $\widetilde{\theta}^*$ in which both countries undertake production in goods and R&D services.[16]

Therefore, I arrive at:

Proposition 1. *For sufficiently large* $\frac{A_1^f(\widetilde{\theta})'}{A_1^h(\widetilde{\theta})'} > \frac{\left[\overline{N}^f(t) + k(\rho - g_N)A_1^f(\widetilde{\theta})\right]}{\left[\overline{N}^h(t) + k(\rho - g_N)A_1^h(\widetilde{\theta})\right]}$*, and if* $\omega < \lambda$ *and for any* $\theta \in [0, 1]$, $\alpha_R^f(\theta) > \alpha_R^h(\theta)$*, then there exists a unique steady-state equilibrium such that*

(a) *Home's relative wage,* ω^**, is greater than one*,

(b) *Home has a sustained comparative advantage in the range of industries* $\theta \in [0, \widetilde{\theta}^*)$. *In each industry* $\theta \in [0, \widetilde{\theta}^*)$*, only Home conducts R&D, produces, and exports the state of the-art product*,

(c) *Foreign has a sustained comparative advantage in the range of industries* $\theta \in (\widetilde{\theta}^*, 1]$. *In each industry* $\theta \in (\widetilde{\theta}^*, 1]$*, only Foreign conducts R&D, produces, and exports the state of the-art product*.

Proof. See Appendix A.

The results from this proposition can be found in other models. The static continuum Ricardian model developed by Dornbusch et al. (1977) and the dynamic learning-by-doing model introduced by Krugman (1987) produce similar features with the equilibrium depicted in Figure 1. Proposition 1 identifies the unique steady-state equilibrium level of Home's relative wage and the marginal industry by utilizing the mutual R&D and resource conditions. The pattern of trade in goods is determined by comparative advantage across industries since no multinational firms and trade in R&D sector are

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[16] In appendix, I provide the sufficient condition for which the $MR(\widetilde{\theta})$ schedule intersects the $RD(\widetilde{\theta})$ schedule at a point above the 45° line, such as $\omega > 1$.

allowed[17]. In addition and in contrast to earlier work, the model predicts that the pattern of trade is determined by additional factors such as population growth and the R&D difficulty parameter. Finally, factor price equalization is not a property of the equilibrium depicted in Figure 1.

4 Comparative Steady-State Analysis Under the PEG Specification

In this section I examine the comparative static properties of the steady-state equilibrium presented in Figure 1. By totally differentiating the equilibrium conditions (23) and (35), one can obtain propositions 2 and 3[18]:

Proposition 2. If $\overline{N}^f(t)/\overline{N}^h(t) > A_1^f(\tilde{\theta})/A_1^h(\tilde{\theta})$

and $\overline{N}^f(t) + k(\rho - g_N)A_1^f(\tilde{\theta})/\overline{N}^h(t) + k(\rho - g_N)A_1^h(\tilde{\theta}) > A_1^f(\tilde{\theta})/A_1^h(\tilde{\theta})$. *Then under the PEG specification, Home's relative wage,* ω^**,*

(b) (a) *depends positively on Foreign's relative size,* $\overline{N}^f(t)$*, the size of innovations,* λ*, the consumer's subjective discount rate,* ρ *and the population growth rate,* g_N*,*

(c) *depends negatively on Home's relative size,* $\overline{N}^h(t)$*, and the R&D difficulty parameter, k.*

Proof. See Appendix A.

The comparative steady-state properties of the PEG specification analysis are novel and differ from those in Taylor (1993). For example, in Taylor (1993), Home's relative wage depends on Foreign's relative size, Foreign's unit labor requirement in manufacturing, and Home's unit labor requirement. Proposition 2 identifies population growth and R&D difficulty parameter as important factors that affect the equilibrium relative wage. The comparative steady-state results from the PEG specification can be illustrated graphically. Figure 2 shows that an increase in Foreign's relative size, $\overline{N}^f(t)$, shifts the MR schedule upward. At the initial relative wage equilibrium, ω^*, the increase in $\overline{N}^f(t)$ would create an excess supply of labor abroad and an excess demand for labor at home. The increase in relative wage restores the equilibrium at point E'. An increase in the population growth, g_N, operates through a decrease in the effective discount rate, $\rho - g_N$. It does not affect the RD schedule in Figure 2, but shifts the mutual resource schedule MR to the left, thus resulting in a higher relative wage, ω^{**}. The increase in

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[17] Taylor (1993) developed a model a model where there is heterogeneity in research technologies and allowed for trade in R&D services as well.

[18] The results presented in this section are derived algebraically in Appendix A.

the population growth rate, g_N, increases the value of the expected discounted profits of a successful innovator in both countries. At the same time, both countries become more innovative (the global R&D investment, $I(\theta, t)$, has to increase to maintain the equality in equation (27)). For a given increase in the global R&D investment, the relative wage has to increase in order to restore the equilibrium condition in equation (26).

An increase on Home's relative size, $\overline{N}^h(t)$, in the R&D difficulty parameter, *k*, or in the consumer's subjective discount rate, ρ, each shifts the MR schedule downward and decreases the relative wage (see Figure 3).

Finally, an increase in the size of innovations shifts the RD schedule upward and thus it raises Home's relative wages. The upward shift of the RD curve can be seen from the *RD* condition (equation 23), which can be rewritten as $\frac{\lambda - \omega^*}{\lambda - 1} = \frac{\omega \alpha_R^h(\widetilde{\theta}^*)}{\alpha_R^f(\widetilde{\theta}^*)}$. From this last expression it is obvious that as the size of innovations increases, Home's relative profit from manufacturing (the left-hand side) increases, while its relative R&D labor cost (the right-hand side) remains the same. At the marginal industry $\widetilde{\theta}^*$, Home firms have (compared to Foreign firms) higher profits from manufacturing than before. As a result, the relative wage should increase to offset the increase the labor cost. These results can be seen in Figure 4.

Proposition 3. *Assume that* $\overline{N}^f(t) / \overline{N}^h(t) > A_1^f(\widetilde{\theta}) / A_1^h(\widetilde{\theta})$ *and*
$\overline{N}^f(t) + k(\rho - g_N)A_1^f(\widetilde{\theta}) / \overline{N}^h(t) + k(\rho - g_N)A_1^h(\widetilde{\theta}) > A_1^f(\widetilde{\theta}) / A_1^h(\widetilde{\theta})$. *Then under the PEG specification, the range of goods Home produces, conducts R&D, and exports,* $\widetilde{\theta}^*$,

(a) *depends positively on Home's relative size,* $\overline{N}^h(t)$, *the R&D difficulty parameter, k, and on the size of innovations,* λ.

(b) *depends negatively on Foreign's relative size,* $\overline{N}^f(t)$, *the consumer's subjective discount rate,* ρ, *and the population growth rate,* g_N.

Proof. See Appendix A.

These comparative steady-state properties can also be illustrated graphically by utilizing Figures 2 through 4. In Figure 2 an increase in Foreign's relative size, $\overline{N}^f(t)$, shifts the *MR* schedule upward and reduces Home's comparative and absolute (if any) advantage in both goods and R&D production. In contrast to previous work, the pattern of goods and R&D production as well as the pattern of trade are also determined by population growth and the R&D difficulty parameter. An increase in the population growth rate, g_N, shifts the *MR* schedule upward in Figure 2, and increases the relative wage. As a result, it reduces Home's comparative and absolute (if any) advantage in both goods and R&D production. An increase in the R&D difficulty parameter, *k*, or a decrease in the consumer's subjective discount rate, ρ, each shifts the MR schedule downward and increase Home's comparative and absolute (if any) advantage in both goods and R&D production (see Figure 3).

Finally, an increase in the size of innovations shifts the RD schedule upward and thus it raises Home's comparative advantage. The upward shift of the RD curve can be seen from the RD condition (equation 23). As the size of innovations increases it becomes more profitable for Home firms to engage in R&D and thus produce and export more goods. These results can be seen in Figure 4.

5 Steady-State Equilibrium Under the TEG Specification

In this section I derive the steady-state equilibrium under the TEG specification proposed by Segerstrom (1998), which is described according to equation (18).
It is easy to show that under the TEG specification, equation (23) still holds.[19]

Combining equations (20) and (22), I obtain the following zero profit conditions for Home and Foreign respectively:

$$\frac{\left(\frac{\lambda-\omega}{\lambda}\right)E(t)}{(\rho+I(\theta,t)-g_N)}=\omega\alpha_R^h(\theta)X(t), \qquad \forall\theta\in[0,\tilde{\theta}), \tag{33}$$

$$\frac{\frac{(\lambda-1)}{\lambda}E(t)}{(\rho+I(\theta,t)-g_N)}=\alpha_R^f(\theta)X(t), \qquad \forall\theta\in(\tilde{\theta},1], \tag{34}$$

In a steady-state equilibrium all per capita variables are constant. Therefore, the level of R&D difficulty grows at the same rate of population growth, $\dot{X}(t)/X(t)=\dot{N}(t)/N(t)=g_N$. This last result, combined with equation (16) yields

$$I=\frac{g_N}{\mu}. \tag{35}$$

Integrating equation (33) over $[0,\tilde{\theta})$ and equation (34) over $(\tilde{\theta},1]$ (after taking into account equation (32)), I obtain the following zero profit conditions for Home and Foreign, respectively

[19] In Appendix A, I provide the proof for the validation of equation (23) under the TEG specification.

$$\frac{(\lambda-\omega)E(t)\widetilde{\theta}}{\lambda}=S\omega X(t)A_1^h(\widetilde{\theta}), \tag{36}$$

$$\frac{(\lambda-1)E(t)}{\lambda}=SX(t)A_1^f(\widetilde{\theta}) \ . \tag{37}$$

where $S=(\rho+g_N/\mu-g_N)$.

The full employment of labor conditions in Home and Foreign are given respectively by equations (30) and (31).

Equations (30), (31), (36), and (37) represent a system of four equations in four unknowns $\widetilde{\theta}$, ω, X, and E. Manipulating these four equations yields a second schedule in (θ,ω) space, the mutual resource schedule[20]

$$\omega=MR(\widetilde{\theta})=\frac{\overline{N}^f(t)A_1^f(\widetilde{\theta})A_2^h(\widetilde{\theta})\lambda\widetilde{\theta}}{\overline{N}^h(t)A_1^h(\widetilde{\theta})[(1-\widetilde{\theta})A_1^f(\widetilde{\theta})S+(\lambda-1)A_2^f(\widetilde{\theta})]+\overline{N}^f(t)A_1^f(\widetilde{\theta})\widetilde{\theta}[A_2^h(\widetilde{\theta})-SA_1^h(\widetilde{\theta})]}.$$

This last equation is referred to as equation (38).

The mutual resource schedule states that the relative wage ω, which clears labor markets in both countries, is an increasing function of the range of goods $\widetilde{\theta}$ produced in Home. If the range of goods produced by Home increases, Home's relative demand for labor (both in manufacturing and R&D) increases. The excess demand for labor drives the level of the relative wage higher. The mutual resource condition can be depicted in Figure 1. The intersection of the downward sloping *RD*($\widetilde{\theta}$) schedule and the upward sloping *MR*($\widetilde{\theta}$) schedule at point E determines the steady-state equilibrium relative wage, ω^*, and the marginal industry $\widetilde{\theta}^*$ in which both countries undertake production in goods and R&D services. Therefore, I arrive at:

Proposition 4. *If* $A_2^f(\widetilde{\theta})'<0$, $A_2^h(\widetilde{\theta})< SA_1^h(\widetilde{\theta})$, $A_2^h(\widetilde{\theta})'<SA_1^h(\widetilde{\theta})'$ and *if* $\omega<\lambda$ *and for any* $\theta\in[0,1]$, $\alpha_R^f(\theta)>\alpha_R^h(\theta)$, *then there exists a unique steady-state equilibrium such that*

(a) *Home's relative wage,* ω^**, is greater than one*,

(b) *Home has a sustained comparative advantage in the range of industries* $\theta\in[0,\widetilde{\theta}^*)$. *In each industry* $\theta\in[0,\widetilde{\theta}^*)$, *only Home conducts R&D, produces, and exports the state of the-art product*,

[20] In Appendix A, it is shown how the mutual resource condition is derived and that it is upward-sloping in (θ,ω) space.

(c) *Foreign has a sustained comparative advantage in the range of industries* $\theta \in (\tilde{\theta}^*, 1]$. *In each industry* $\theta \in (\tilde{\theta}^*, 1]$, *only Foreign conducts R&D, produces, and exports the state of the-art product*.

Proof. See Appendix A.

As in the PEG specification analysis, the pattern of trade in goods is determined by comparative advantage across industries. Factor price equalization is also not a property of the equilibrium depicted in Figure 1. Finally, in contrast to the work of Taylor (1993), trade in R&D services does not occur.

6 Comparative Steady-State Analysis Under the TEG Specification

In this section I examine the comparative static properties of the steady-state equilibrium presented in Figure 1.

By totally differentiating the equilibrium conditions (23) and (32) one can obtain propositions 6 and 7[21]:

Proposition 5. If $\alpha_R^f(\theta) > \alpha_R^h(\theta)$ for all $\theta \in [0,1]$ and $\overline{N}^f / \overline{N}^h > (1-\tilde{\theta})/\tilde{\theta}$, *then under the TEG specification, Home's relative wage,* ω^*, *depends*

(a) *positively on Foreign's relative size,* $\overline{N}^f(t)$, *the consumer's subjective discount rate,* ρ, *the size of innovations,* λ,

(b) *negatively on the Home's relative size,* $\overline{N}^h(t)$, *the R&D difficulty growth parameter* μ, *and the population growth rate,* g_N.

Proof. See Appendix A.

These comparative steady-state properties can be illustrated graphically. Figure 2 shows that an increase in Foreign's relative size, $\overline{N}^f(t)$, shifts the MR schedule upward. The increase in relative wages works as a mechanism to restore the equilibrium. These results are identical with those derived under the PEG specification. Under the TEG model, an increase in the population growth rate, g_N, shifts the MR schedule downward in Figure 3 and decreases the relative wage from ω^* to ω^{**}. The increase in the population growth rate, g_N, has two effects on the value of the expected discounted profits of a successful innovator in both countries (see equations (33) and (34)). First, the increase in the population growth rate, g_N, has a positive direct effect on the discounted expected global profits. Second, it has a

[21] The results presented in this section are derived algebraically in Appendix A.

negative indirect effect through the global innovation rate, I, (see equation (35). Thus, if the later negative effect dominates the positive direct effect, then the relative wage must decrease to restore the equilibrium. An increase in the consumer's subjective discount rate, ρ, or an increase in the R&D difficulty growth parameter μ, shifts the *MR* schedule upward and increases the relative wage as seen in Figure 2.

Finally, an increase in the size of innovations shifts upward both the RD schedule and the MR schedule. Thus, the increase in the size of innovation will raise Home's relative wages. Similar to the PEG specification, the upward shift of the RD curve can be seen from the RD condition (equation 23). As the size of innovations increases, Home's relative profit from manufacturing increases, while its relative R&D labor cost remains the same. At the marginal industry $\tilde{\theta}^*$, Home firms have (compared to Foreign firms) higher profits from manufacturing than before. As a result, the relative wage should increase to offset the increase the labor cost. On contrast to the PEG specification, the increase in the size of innovations will also affect the labor conditions, causing an upward shift in the MR condition. These results can be seen in Figure 5, where the increase in the size of innovations increases the equilibrium wage to ω^{**}, while the effect on $\tilde{\theta}^*$ is ambiguous.

Proposition 6. If $\alpha_R^f(\theta) > \alpha_R^h(\theta)$ for all $\theta \in [0,1]$ *and* $\overline{N}^f / \overline{N}^h > (1-\tilde{\theta})/\tilde{\theta}$ *, then under the TEG specification the range of goods Home produces, conducts R&D, and exports,* $\tilde{\theta}^*$*, depends*

(a) *positively on Home's relative size,* $\overline{N}^h(t)$*, the R&D difficulty growth parameter* μ*, and the population growth rate,* g_N*,*

(b) *negatively on Foreign's relative size,* $\overline{N}^f(t)$*, the consumer's subjective discount rate,* ρ*.*

(c) *ambiguously on the size of innovations,* λ*.*

Proof. See Appendix A.

These comparative steady-state properties can also be illustrated graphically by utilizing Figures 2 through 5 and are identical with those derived under the PEG specification. On contrast, an increase in the population growth rate, g_N, shifts the MR schedule downward in Figure 3, and decreases the relative wage. As a result, it increases Home's comparative and absolute (if any) advantage in both goods and R&D production. An increase in the R&D difficulty parameter μ, or an increase in the consumer's subjective discount rate, ρ, each shifts the MR schedule upward and decreases Home's comparative and absolute (if any) advantage in both goods and R&D production (see Figure 2).

Proposition 7. *Under the TEG specification, the global R&D investment,* I*, depends positively on the R&D difficulty growth parameter,* μ*, and negatively on the population growth rate,* g_N*.*

Proof. If follows from equation (35).

The level of R&D investment, I, is completely determined by the exogenous rate of population growth $g_N > 0$ and the R&D difficulty growth parameter $\mu > 0$. The balanced-growth innovation rate is higher when the population of consumers grows more rapidly or when R&D difficulty increases more slowly over time. These results are standard in the endogenous growth literature without scale effects. [22]

7 Conclusions

The previous literature on "quality ladders" framework that analyzed Ricardian models of trade exhibits the scale effects property. In this paper, I have developed a model of trade based on "quality-ladders" growth without scale effects to analyze how the pattern of trade and the relative wage are determined in steady-state equilibrium. The model explores its comparative steady state properties of equilibria under two alternative specifications regarding the R&D difficulty, the PEG and the TEG specification. The absence of scale effects generates novel and interesting results. Several comparative-steady-state results in Taylor's (1993) model change with the removal of the scale effects property. In the present model, the direction of the effect of the size of innovations on the pattern of trade and Home's relative wage depend on the way in which the scale effects property is removed. The analysis in the present model generates new additional findings. Under both specifications, the model generates a unique steady-state equilibrium in which there is complete specialization in both goods and R&D production within each industry. In contrast to previous models (Grossman and Helpman (1991*c*), and Taylor (1993)), the comparative steady state exercises in the present model highlight the effects of population growth and the R&D difficulty on relative wages.

Given the relatively simplicity of the model, this dynamic formulation provides a useful framework to examine other issues. For example, the introduction of trade instruments and their effect on the pattern of trade between countries can be examined under the two alternative models. Alternatively, a North-South model of trade might yield interesting implications.

[22] See Segerstrom (1998) for more details on this.

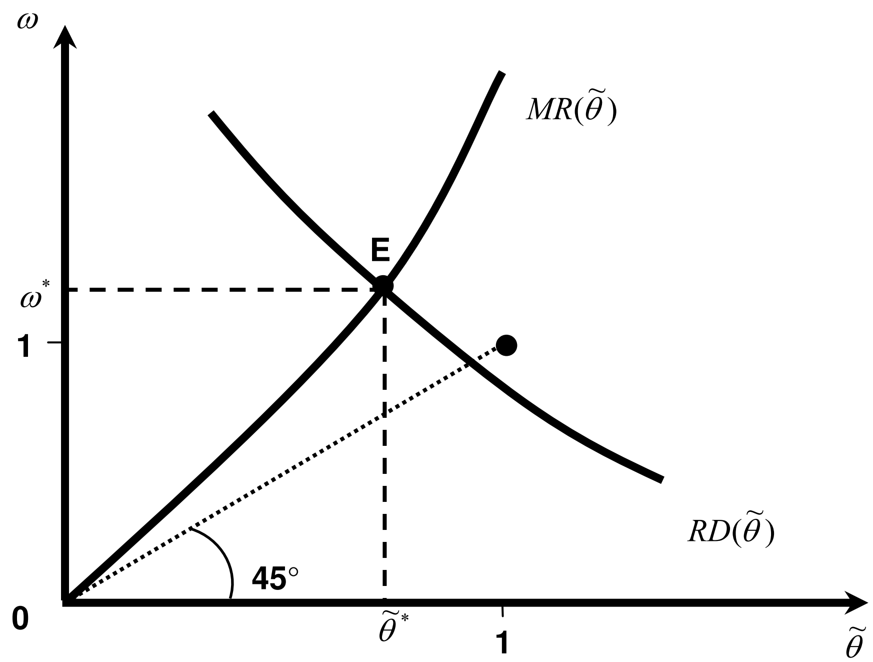

Figure 1. Steady-State Equilibrium Under the PEG and the TEG Specifications.

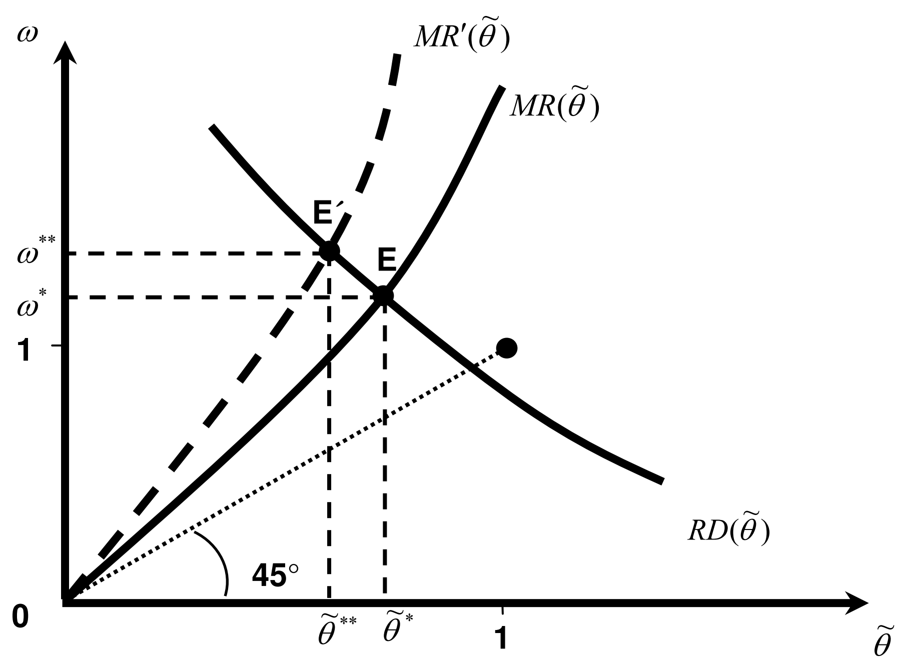

Figure 2. Comparative Steady-State Analysis Under the PEG and the TEG Specifications.

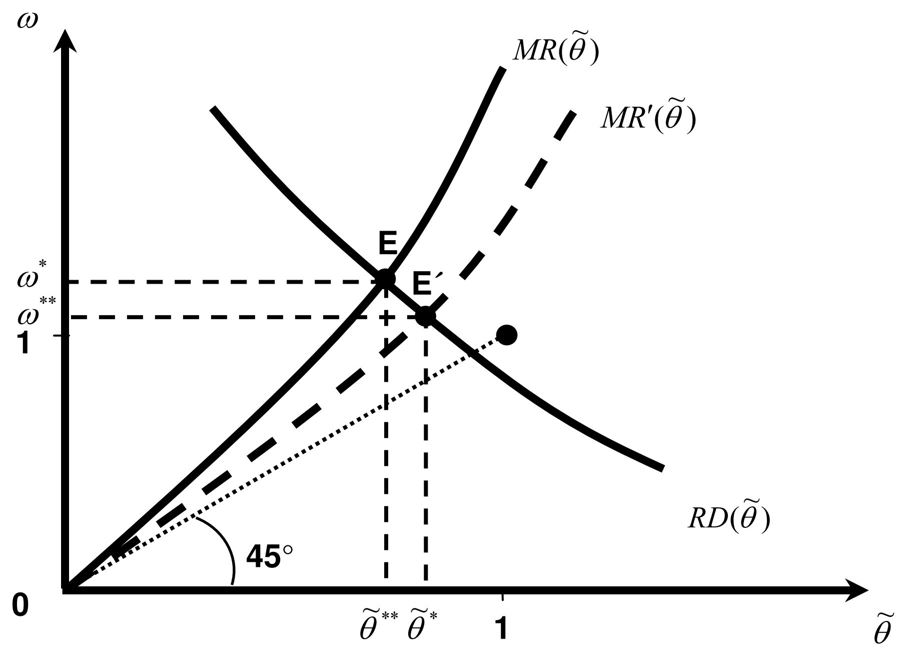

Figure 3. Comparative Steady-State Analysis Under the PEG and the TEG Specifications.

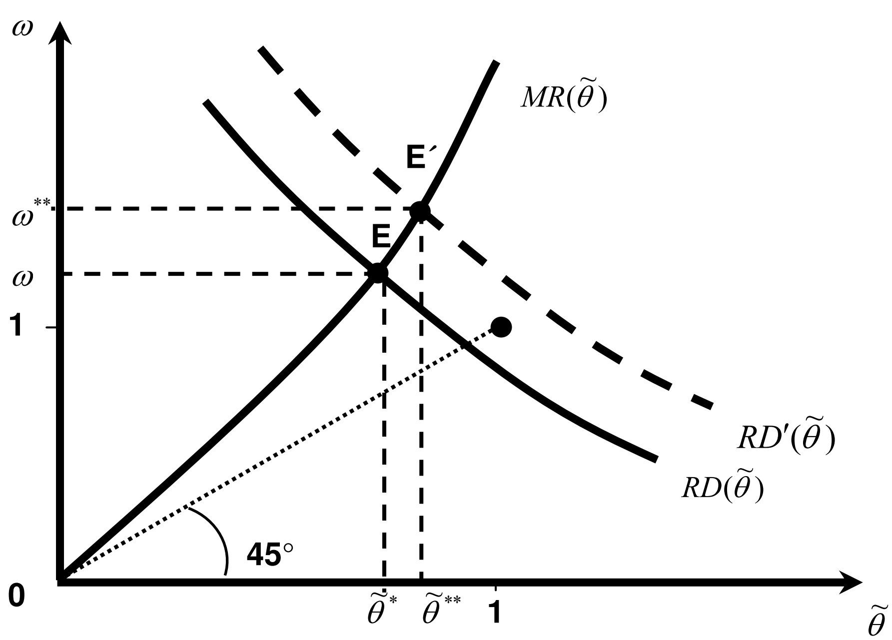

Figure 4. Comparative Steady-State Analysis Under the PEG and the TEG Specifications.

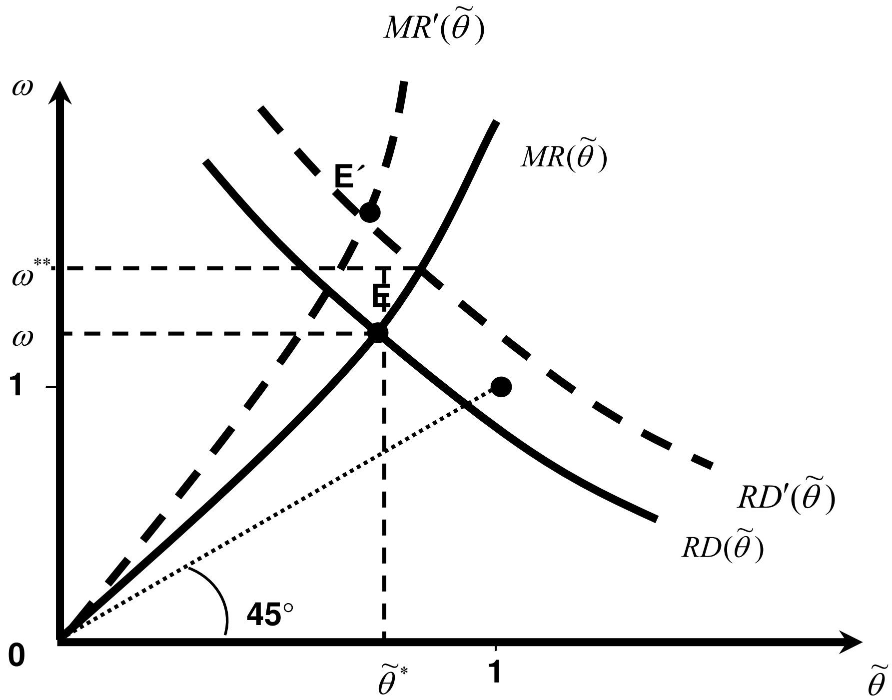

Figure 5. Comparative Steady-State Analysis Under the TEG Specification.

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APPENDIX

PROOFS OF PROPOSITIONS

(AVAILABLE UPON REQUEST BY THE AUTHOR)

A.1 Proof of Lemma 1

A.1.1 Lemma 1

Lemma 1 results from equations (20) and (22) (after taking into account equations (12) and (13)). Then, from the zero profit conditions, one can obtain the mutual R&D condition:

$$\omega = RD(\tilde{\theta}) = \frac{\lambda A(\tilde{\theta})}{\lambda + A(\tilde{\theta}) - 1} \tag{A.1}$$

The slope of the mutual R&D condition is given by

$$\frac{d\omega}{d\theta} = RD'(\tilde{\theta}) = \frac{\overbrace{\lambda(\lambda-1)}^{+}\overbrace{A'(\tilde{\theta})}^{-}}{\underbrace{(\lambda + A(\tilde{\theta}) - 1)^2}_{+}} > 0 \ . \tag{A.2}$$

$$\frac{d\omega}{d\lambda} = \frac{A(\tilde{\theta})[A(\tilde{\theta}) - 1]}{[\lambda + A(\tilde{\theta}) - 1]^2} ><0?\mathrm{d}$$

If $A(\tilde{\theta})$>1, which implies that $\alpha_R^f(\tilde{\theta}) > \alpha_R^h(\tilde{\theta})$, then $d\omega / d\lambda$.

A.2 Proofs of Propositions 1, 2, and 3 Under the PEG Specification

A.2.1 Proposition 1

First, I derive the mutual resource schedule (equation (32)) for the PEG specification. I solve for $A_2^h(\tilde{\theta})$ of the right hand side in equation (28) and substitute it into equation (30) and then solve for E(t) and obtain the following:

$$E(t) = \frac{[N^h(t) + X(\rho - g_N)A_1^h(\tilde{\theta})]\omega}{\tilde{\theta}} \ . \tag{A.3}$$

Then, from equation (29), I solve for $A_2^f(\tilde{\theta})$ and substitute it into equation (31) and obtain the following:

$$N^f(t) = E(t)[1 - \tilde{\theta}] - X(\rho - g_N)A_1^f(\tilde{\theta}) \qquad . \tag{A.4}$$

Substituting equation (A.3) into equation (A.4), I obtain the following mutual resource condition (equation (32) in the main text):

$$\omega = MR(\tilde{\theta}) = \frac{[\overline{N}^f + k(\rho - g_N)A_1^f(\tilde{\theta})]\tilde{\theta}}{[\overline{N}^h + k(\rho - g_N)A_1^h(\tilde{\theta})](1-\tilde{\theta})}. \qquad (A.5)$$

The sign of the slope of the mutual resource condition curve is found to be positive:

$$\frac{d\omega}{d\tilde{\theta}} = \frac{[\overline{N}^f + k(\rho - g_N)A_1^f(\tilde{\theta})][\overline{N}^h + k(\rho - g_N)A_1^h(\tilde{\theta})]}{\left\{[\overline{N}^h + k(\rho - g_N)A_1^h(\tilde{\theta})](1-\tilde{\theta})\right\}^2} + \qquad (A.6)$$

$$+ \frac{\tilde{\theta}(1-\tilde{\theta})k(\rho - g_N)\left\{A_1^f(\tilde{\theta})'[\overline{N}^h + k(\rho - g_N)A_1^h(\tilde{\theta})] - A_1^h(\tilde{\theta})'[\overline{N}^h + k(\rho - g_N)A_1^h(\tilde{\theta})]\right\}}{\left\{[\overline{N}^h + k(\rho - g_N)A_1^h(\tilde{\theta})](1-\tilde{\theta})\right\}^2}$$

If $A_1^f(\tilde{\theta})'[\overline{N}^h + k(\rho - g_N)A_1^h(\tilde{\theta})] - A_1^h(\tilde{\theta})'[\overline{N}^h + k(\rho - g_N)A_1^h(\tilde{\theta})] > 0$, then $d\omega/d\tilde{\theta} > 0$. Therefore, the relative wage, ω, increases in $\tilde{\theta}$.

Thus, for sufficiently large $\frac{A_1^f(\tilde{\theta})'}{A_1^h(\tilde{\theta})'} > \frac{[\overline{N}^f(t) + k(\rho - g_N)A_1^f(\tilde{\theta})]}{[\overline{N}^h(t) + k(\rho - g_N)A_1^h(\tilde{\theta})]}$, the MR schedule defined by equation (A.5) intersects the RD schedule such that a unique steady-state equilibrium exists and $\omega > 1$. The intersection of the downward sloping RD($\tilde{\theta}$) schedule and the upward sloping MR($\tilde{\theta}$) schedule at point E determines the equilibrium relative wage, ω, and the marginal industry $\tilde{\theta}$ in which both countries undertake production in goods and R&D services.

First, I will prove equation (10) that is assumed in Proposition 1. A sufficient condition for home relative wage to be strictly greater than 1 is to have the mutual R&D curve lies above the $\omega = 1$ line. This is true if $A(\theta) = \frac{\alpha_R^f(\theta)}{\alpha_R^h(\theta)} > 1$ (that is if $\alpha_R^f(\theta) > \alpha_R^h(\theta)$), then $\omega(\theta) = RD(\theta) > 1$.

A.2.2 Proposition 2

I can write the two equilibrium relationships governing Figure 1 in a more general form as follows:

$\omega \equiv RD(\tilde{\theta}, \lambda)$, where $RD_1 < 0$, $RD_2 > 0$, (A.7)

$\omega \equiv MR(\tilde{\theta}, \overline{N}^f, \overline{N}^h, k, \rho, g_N)$, where $MR_1 > 0$, $MR_2 > 0$, $MR_3 < 0$, (A.8)

$MR_4 < 0$, $MR_5 > 0$, $MR_6 > 0$.

The following conditions have to hold in order to sign the above expressions:

$MR_1 > 0$ if and only if $\frac{A_1^f(\tilde{\theta})'}{A_1^h(\tilde{\theta})'} > \frac{[\overline{N}^f(t) + k(\rho - g_N)A_1^f(\tilde{\theta})]}{[\overline{N}^h(t) + k(\rho - g_N)A_1^h(\tilde{\theta})]}$

$MR_4 < 0$ and $MR_6 > 0$ if and only if $\dfrac{A_1^f(\widetilde{\theta})}{A_1^h(\widetilde{\theta})} < \dfrac{[\overline{N}^f(t) + k(\rho - g_N)A_1^f(\widetilde{\theta})]}{[\overline{N}^h(t) + k(\rho - g_N)A_1^h(\widetilde{\theta})]}$

I totally differentiate equations (23) and (32) in the main text and obtain the following system of two equations in the differentials of two endogenous variables as follows:

$$d\omega - RD_1 d\widetilde{\theta} = RD_2 d\lambda$$

$$d\omega - MR_1 d\widetilde{\theta} = MR_2 d\overline{N}^f + MR_3 d\overline{N}^h + MR_4 dk + MR_5 d\rho + MR_6 dg_N \qquad \text{(A.9)}$$

I can write the system (A.29) in the reduced form as follows:

$$\begin{bmatrix} 1 & -RD_1 \\ 1 & -MR_1 \end{bmatrix} \begin{bmatrix} d\omega \\ d\widetilde{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & RD_2 \\ MR_2 & MR_3 & MR_4 & MR_5 & MR_6 & 0 \end{bmatrix} \begin{bmatrix} d\overline{N}^f \\ d\overline{N}^h \\ dk \\ d\rho \\ dg_N \\ d\lambda \end{bmatrix}, \qquad \text{(A.10)}$$

I calculate the determinant of the matrix of the endogenous variables (which I denote with Δ) as follows:

$$\Delta = \begin{vmatrix} 1 & -RD_1 \\ 1 & -MR_1 \end{vmatrix} = RD_1 - MR_1 < 0. \qquad \text{(A.11)}$$

Using the system of equations given by (A.10) and by employing the Cramer's rule, I establish the comparative steady-state results for the PEG specification regarding ω. I calculate the determinant of the matrix formed by replacing the second column of the matrix of the endogenous variables in (A.10) with the corresponding column vector of the exogenous variable in consideration. Thus, I obtain the following results:

$$\frac{d\omega}{d\overline{N}^f} = \frac{RD_1 MR_2}{\Delta} > 0, \qquad \text{(A.12)}$$

$$\frac{d\omega}{d\overline{N}^h} = \frac{RD_1 MR_3}{\Delta} < 0, \qquad \text{(A.13)}$$

$$\frac{d\omega}{dk} = \frac{RD_1 MR_4}{\Delta} < 0, \qquad \text{(A.14)}$$

$$\frac{d\omega}{d\rho} = \frac{RD_1 MR_5}{\Delta} > 0, \qquad \text{(A.15)}$$

$$\frac{d\omega}{dg_N} = \frac{RD_1 MR_6}{\Delta} > 0, \qquad \text{(A.16)}$$

$$\frac{d\omega}{d\lambda} = \frac{-RD_2 MR_1}{\Delta} > 0. \qquad \text{(A.17)}$$

The signs of equations (A.12) through (A.17) prove Proposition 2.

A.2.3 Proposition 3

Using the system of equations given by (A.10), I establish the comparative steady-state results for the PEG specification regarding $\widetilde{\theta}$. I calculate the determinant of the matrix formed by replacing the second column of the matrix of the endogenous variables in (A.10) with the corresponding column vector of the exogenous variable in consideration. Thus, I obtain the following results:

$$\frac{d\widetilde{\theta}}{d\overline{N}^f} = \frac{MR_2}{\Delta} < 0, \qquad \text{(A.18)}$$

$$\frac{d\widetilde{\theta}}{d\overline{N}^h} = \frac{MR_3}{\Delta} > 0, \qquad \text{(A.19)}$$

$$\frac{d\widetilde{\theta}}{dk} = \frac{MR_4}{\Delta} > 0, \qquad \text{(A.20)}$$

$$\frac{d\widetilde{\theta}}{d\rho} = \frac{MR_5}{\Delta} < 0, \qquad \text{(A.21)}$$

$$\frac{d\widetilde{\theta}}{dg_N} = \frac{MR_6}{\Delta} < 0, \qquad \text{(A.22)}$$

$$\frac{d\widetilde{\theta}}{d\lambda} = -\frac{-RD_2}{\Delta} > 0. \qquad \text{(A.23)}$$

The signs of equations (A.18) through (A.23) prove Proposition 3.

A.3 Proofs of Propositions 4, 5, and 6 Under the TEG Specification

A.3.1 Proposition 4

The RD schedule under the TEG specification is given by equation (23).

Next, I derive the mutual resource schedule (equation (38)) under the TEG specification. First, I solve for X in equation (36), substitute it into equation (30), and solve for E(t) to obtain the following:

$$E(t) = \frac{N^h(t)\lambda S\omega A_1^h(\widetilde{\theta})}{\widetilde{\theta}[S\omega A_1^h(\widetilde{\theta}) + (\lambda - \omega)A_2^h(\widetilde{\theta})]}. \qquad \text{(A.24)}$$

Then, from equation (37), I solve for X and substitute it into equation (31) and obtain the following:

$$N^f(t) = \frac{[(1-\widetilde{\theta})SA_1^f(\widetilde{\theta}) + (\lambda - 1)A_2^f(\widetilde{\theta})]E(t)}{\lambda SA_1^f(\widetilde{\theta})}, \qquad \text{(A.25)}$$

where $S = (\rho + \frac{g_N}{\mu} - g_N)$.

Substituting equation (A.24) into equation (A.25), I obtain the following mutual resource schedule (equation (38)):

$$\omega = MR(\tilde{\theta}) = \frac{\overline{N}^f(t) A_1^f(\tilde{\theta}) A_2^h(\tilde{\theta}) \lambda \tilde{\theta}}{\overline{N}^h(t) A_1^h(\tilde{\theta})[(1-\tilde{\theta}) A_1^f(\tilde{\theta}) S + (\lambda - 1) A_2^f(\tilde{\theta})] + \overline{N}^f(t) A_1^f(\tilde{\theta}) \tilde{\theta} [A_2^h(\tilde{\theta}) - S A_1^h(\tilde{\theta})]}, \text{(A.26)}$$

For the following expression, I use D for the denominator of equation (A.26).

$$\frac{d\omega}{d\tilde{\theta}} = \frac{\overline{N}^f(t)\lambda[A_1^f A_2^h + \tilde{\theta} A_1^f A_2^h(\tilde{\theta})' + \tilde{\theta} A_2^h A_1^f(\tilde{\theta})']D}{D^2} -$$

$$-\frac{\overline{N}^f(t) A_1^f A_2^h \lambda \tilde{\theta} \{\overline{N}^h A_1^h(\tilde{\theta})'[(1-\tilde{\theta}) A_1^f S + (\lambda - 1) A_2^f] - \overline{N}^h(t) A_1^h A_1^f S + \overline{N}^h(t) A_1^h A_2^f(\tilde{\theta})'(\lambda - 1) + \overline{N}^f A_1^f (A_2^h - S A_1^h)\}}{D^2} -$$

$$-\frac{\overline{N}^f(t) A_1^f A_2^h \lambda \tilde{\theta} \{\overline{N}^f(t) \tilde{\theta} A_1^f(\tilde{\theta})'(A_2^h - S A_1^h) + \overline{N}^f(t) A_1^f \tilde{\theta} (A_2^h(\tilde{\theta})' - S A_1^h(\tilde{\theta})'\}}{D^2}$$

This last expression refers to equation (A.27).

The following conditions have to hold in order to sign the above expressions:

$A_2^f(\tilde{\theta})' < 0$, $A_2^h(\tilde{\theta}) < S A_1^h(\tilde{\theta})$ and $A_2^h(\tilde{\theta})' < S A_1^h(\tilde{\theta})'$.

Under the above conditions, the mutual resource condition is upward sloping (i.e., $d\omega / d\tilde{\theta} > 0$).

A.3.2 Proposition 5

I can write the two equilibrium relationships governing Figure 1 in a more general form as follows:

$\omega \equiv RD(\tilde{\theta}, \lambda)$, where $RD_1 < 0$, $RD_2 > 0$, (A.28)

$\omega \equiv MR(\tilde{\theta}, \overline{N}^f, \overline{N}^h, \mu, \lambda, \rho, g_N)$, where $MR_1 > 0$, $MR_2 > 0$, $MR_3 < 0$, (A.29)

$MR_4 < 0$, $MR_5 > 0$, $MR_6 > 0$, $MR_7 > 0$.

The following conditions have to hold in order to sign the above expressions:

$MR_1 > 0$ if and only if $A_2^f(\tilde{\theta})' < 0$, $A_2^h(\tilde{\theta}) < S A_1^h(\tilde{\theta})$ and $A_2^h(\tilde{\theta})' < S A_1^h(\tilde{\theta})'$,

$MR_7 < 0$ if and only if $\frac{\overline{N}^f(t)}{\overline{N}^h(t)} < \frac{(1-\tilde{\theta})}{\tilde{\theta}}$.

I totally differentiate equations (23) and (32) in the main text and obtain the following system of two equations in the differentials of two endogenous variables as follows:

$$d\omega - RD_1 d\tilde{\theta} = RD_2 d\lambda$$

$$d\omega - MR_1 d\tilde{\theta} = MR_2 d\overline{N}^f + MR_3 d\overline{N}^h + MR_4 d\mu + MR_5 d\lambda + MR_6 d\rho + MR_7 dg_N \quad \text{(A.30)}$$

I can write the system (A.41) in the reduced form as follows:

$$\begin{bmatrix} 1 & -RD_1 \\ 1 & -MR_1 \end{bmatrix} \begin{bmatrix} d\omega \\ d\tilde{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & RD_2 & 0 & 0 \\ MR_2 & MR_3 & MR_4 & MR_5 & MR_6 & MR_7 \end{bmatrix} \begin{bmatrix} d\overline{N}^f \\ d\overline{N}^h \\ d\mu \\ d\lambda \\ d\rho \\ dg_N \end{bmatrix}, \tag{A.31}$$

I calculate the determinant of the matrix of the endogenous variables (which I denote with Δ) as follows:

$$\Delta = \begin{vmatrix} 1 & -RD_1 \\ 1 & -MR_1 \end{vmatrix} = RD_1 - MR_1 < 0. \tag{A.32}$$

Using the system of equations given by (A.31) and by employing the Cramer's rule, I establish the comparative steady-state results for the PEG specification regarding ω. I calculate the determinant of the matrix formed by replacing the second column of the matrix of the endogenous variables in (A.31) with the corresponding column vector of the exogenous variable in consideration. Thus, I obtain the following results:

$$\frac{d\omega}{d\overline{N}^f} = \frac{RD_1 MR_2}{\Delta} > 0, \tag{A.33}$$

$$\frac{d\omega}{d\overline{N}^h} = \frac{RD_1 MR_3}{\Delta} < 0, \tag{A.34}$$

$$\frac{d\omega}{d\mu} = \frac{RD_1 MR_4}{\Delta} < 0, \tag{A.35}$$

$$\frac{d\omega}{d\lambda} = \frac{RD_1 MR_5 - MR_1 RD_2}{\Delta} > 0, \tag{A.36}$$

$$\frac{d\omega}{d\rho} = \frac{RD_1 MR_{61}}{\Delta} > 0, \tag{A.37}$$

$$\frac{d\omega}{dg_N} = \frac{RD_1 MR_7}{\Delta} < 0. \tag{A.38}$$

The signs of equations (A.33) through (A.38) prove Proposition 5.

A.2.3 Proposition 6

Using the system of equations given by (A.31), I establish the comparative steady-state results for the PEG specification regarding $\tilde{\theta}$. I calculate the determinant of the matrix formed by replacing the second column of the matrix of the endogenous variables in (A.31) with the corresponding column vector of the exogenous variable in consideration. Thus, I obtain the following results:

$$\frac{d\tilde{\theta}}{d\bar{N}^f} = \frac{MR_2}{\Delta} < 0 \qquad , \tag{A.39}$$

$$\frac{d\tilde{\theta}}{d\bar{N}^h} = \frac{MR_3}{\Delta} > 0, \tag{A.40}$$

$$\frac{d\tilde{\theta}}{d\mu} = \frac{MR_4}{\Delta} > 0, \tag{A.41}$$

$$\frac{d\tilde{\theta}}{d\lambda} = \frac{MR_5 - RD_2}{\Delta} >< 0? \tag{A.42}$$

$$\frac{d\tilde{\theta}}{d\rho} = \frac{MR_6}{\Delta} < 0, \tag{A.43}$$

$$\frac{d\tilde{\theta}}{dg_N} = -\frac{MR_7}{\Delta} > 0. \tag{A.44}$$

The signs of equations (A.39) through (A.44) prove Proposition 6.