Technological dependence and budgetary policy in an uncertain horizon model of small open economy

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Abstract It is set up in this paper a formulation which endogeneizes the choice of investment inputs in open economy context where a small country may depend on foreign technology. Then it is integrated into a macroeconomic model of small specialized country with Blanchard-Yaari type uncertain horizon consumers. Contrary to some models with ad hoc formulation, it is found in this model that generally the budgetary policy has effects on the short- and long-run capital stock. The corresponding intertemporal spill-over effects of the budgetary policy, through the supply side of economy on external position, aggregate consumption, real exchange rate and capital evaluation ratio, are also examined.

Keywords: technological dependence, choice of investment inputs, capital stock, capital evaluation ratio, real exchange rate and external position.

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Recently, economists try to build some open economy macroeconomic models based on solid microeconomic foundations. Particular interests are given to the building of models in the line of intertemporal approaches. These models can be interpreted as theories of determination of external position of a country as they can explain it in term of the optimal behavior of consumers under intertemporal budgetary constraints and that of firms subject to capital accumulation constraint and costs of adjustment linked to investment. It is possible to study the intertemporal spill-over effects of budgetary policy in using these models although the conclusions obtained in different models are quite contradictory. [1]

There is a great potential for the development of these approaches. But our aim here is to provide a supply-side theory of open economy, which is not well treated in these existing models, especially in the case of a specialized country. This theory concerns the use of domestic and foreign goods in the investment process with costs of adjustment. The use of a goods (foreign or domestic) in the investment process has an importance in the determination of the short- and long-run effects of the budgetary policy in a small economy. Meanwhile, in the precedent theoretical works, it was often assumed arbitrarily for reason of simplicity the use of one or another as investment input. Giovannini (1988) and Wyplosz (1991) have adopted the idea that the domestic goods is used in the investment process. Turnovsky and Sen(1991) also treat the domestic goods as the unique input of domestic investment, but to avoid a non-desirable result such as the absence of the long-run effect of budgetary policy on the the capital stock, they have to introduce an endogenous supply of labor. In the same spirit, Buiter (1989) has considered alternatively the two possible cases in his two-country models: one country can use the domestic or foreign goods in the investment process. The problematic – the absence of long-run effect of the budgetary policy on capital – does not appear here in the two-country models due to the fact the real interest rate reacts to the budgetary policy shocks in the long-run so does the capital stock.

As noted by Giovannini (1988), one can generate long-run budgetary policy effect on capital stock in considering the case where the foreign goods is used as the investment goods and the domestic goods as the adjustment goods. The symmetric case where the role of the two goods is inverted can be

[1] *See* Giavazzi and Wyplosz (1984), Blanchard (1985), Pitchford (1990), and Obstfeld (1990) and Weil (1989) for the basic models of intertemporal approaches. See Dai (1992) for a synthesis of these models.

also envisaged; one can easily imagine that the permanent budgetary policy produces equally in this case some effects on capital stock so that on other variables in the short- and long-run. In fact, the question is not to replace the ad hoc formulation by some new formulations no less restrictive. The question is how can we justify this clear division of roles between different goods in the capital stock formation.

The different ad hoc formulations can be reinterpreted in terms of dependence of the small open economy on foreign technology. This interpretation will offer us a new perspective in the formulation of the use of goods in the capital accumulation process.

In terms of technological dependence, the known formulations of the demand of goods for investment and adjustment correspond to the following simple situations. The total dependence: the less advanced economies import in general the equipment and the auxiliaries from the foreign countries (or furnish themselves some auxiliaries). The total independence: one small country manages to accumulate and maintain capital stock in using uniquely the national product. Through these interpretations, one can see how the formulations of the precedent works are arbitrary, as the reality is much more complex and there are infinite intermediary situations.

The usage of this or that goods in the capital accumulation process is dictated by a certain technological structure that the country has managed to adopt. The choice problem does exist once the different technologies are incorporated in the foreign and domestic goods. The decision procedure as regard to the choice of investment inputs implies first the study of feasibility, and secondly a comparison of relative advantages in adopting different goods under a certain degree of technological dependence. [2]

The complex technological dependence can not be resumed in postulating a few extreme cases. In a more flexible and general formulation, it is necessary to take account of the situations such as the strategy of substitution to importation, the partial dependence on foreign technology, and the spe-

[2] *We can often observe some strategical behaviors which can not be reduced to the simple economic problem of comparing the apparent advantages of the adoption of this or that equipment in the very short-run. A less advanced country could sacrifice the short-run advantage resulting from the adoption of the matured techniques that an advanced country can furnish it, and would try to develop itself the needed equipment with a supplementary costs in expecting that it would be able to compete on the same ground the advanced country. This kind of strategic behaviors can not, unluckily, be easily modeled here.*

cialization and cooperation between industrial countries in the production of investment goods.

We propose here to model the economic aspect of the problem. Given the technical proprieties of the goods, the small open economy is confronted to a problem of economic decision. How will it choose the investment inputs so that to minimize investment costs relating to the formation of optimal productive capital stock? In knowing the marginal efficiency of every goods in the formation of capital stock, the representative firm will integrate this decision into the general program of maximization of its market value.

In order not to complicate further the model, I choose to neglect the role of labor in the investment process. In reality, the investment activity absorbs an important quantity of labor either in a explicit manner, as shown by the existence of a specialized sector in the installation of new firms, or in implicit manner, as illustrated by the existence of qualified persons, in a firm, in charge of the good functioning of equipment.

In this paper, a new formulation of choice of investment inputs is set up in the first section ; it is integrated in the second section into the Blanchard-Yaari-type uncertain horizon macroeconomic model of a specialized open economy [3]; the stability propriety of the model is then analyzed in the third section; the long-run effects of budgetary policy are discussed in the fourth section; some discussions about the dynamic adjustment paths of different variables are given in the fifth section; and I conclude in the last section.

[3] *The* BLANCHARD-YAARI-type uncertain horizon macroeconomic model is often used to study open economy policy issues as we can observe in the works of MATSUYAMA (1987, 90), BUITER (1989) and PLOEG (1991). The main properties of the model are that there is no debt neutrality and the net external position of a country is well determined. VAROUDAKIS uses also this framework to study the effect of budgetary policy in context of endogenous growth. For the discrete version of BLANCHARD-YAARI-type uncertain horizon open economy macroeconomic model, we can refer to FRENKEL and RAZIN (1987).

1 The formulation of firm's problem with choice of investment inputs

1.1 The process of transformation of investment inputs into productive capital

The net investment in every period, $I(t)$, is obtained in transforming the domestic goods and the foreign goods according to a function of the following form: [4]

$$I = \Psi(I^D, I^F), \tag{1}$$

with

$$\Psi_1 > 0,\ \Psi_{11} < 0,\ \Psi_2 > 0,\ \Psi_{22} < 0,$$
$$\Psi_{12} = \Psi_{21} >, < 0,\ \Psi_{11}\Psi_{22} - \Psi_{12}^2 > 0.$$

where I, I^D and I^F do not represent homogeneous concepts. As net investment, I can be added to the existing capital stock to increase the production capacity. Meanwhile, I^D and I^F are the respective quantities of domestic and foreign goods employed in the investment process.

The partial derivatives, $\Psi_1, \Psi_2 > 0$, postulate that the marginal efficiency of transformation of every goods in the investment process is positive. One unity of whatever goods can always be transformed into a certain quantity of the capital stock. The fact that one has $\Psi_{11}, \Psi_{22} < 0$ signifies that this process of transformation is subject to decreasing return to scale. In terms of costs, it needs more and more of one goods to obtain the same quantity of net investment for a given period if the quantity of the other goods stays fixed. The value of Ψ_{11} and Ψ_{22} represent the evolution of the marginal efficiency of every goods as its quantity increases in this process. To take account of the relation between the two goods in the process, one has to examine the cross derivatives, Ψ_{12}, Ψ_{21}, which can be positive or negative. The negative or positive sign of these two derivatives means that there exists a certain substitutability or complementarity between the two goods, respectively.

[4] *This formulation is similar to that adopted by* Uzawa (1969) and Hayashi (1982), as they consider also the net investment as result of a process of transformation of the gross investment represented by the total quantity of the goods used. The difference is that I consider a problem of optimal combination of different inputs when there exists a multiplicity of goods which can enter into the process of transformation. But, for these two authors, this problem is anteriorly resolved.

1.2 The choice of investment inputs in the optimization problem of the firm

The firm behaves in a competitive manner on labor and goods markets. It uses the capital K and the labor L to produce according to a production technology characterized by the function $f(K, L)$ which satisfies the following proprieties:

$$f_1 > 0,\ f_{11} < 0,\ f_2 > 0,\ f_{22} < 0,\ f_{11}f_{22} - f_{12}f_{21} > 0.$$

In noting p as the relative price of domestic goods in terms of foreign goods, [5] and w as the wage rate determined by the labor market equilibrium condition, the firm's objective is to maximize its market value:

$$\max_{L,I,K} \int_t^{\infty} \left[f(K, L) - wL - I^D - \frac{I^F}{p} \right] e^{-\int_t^s r(u)du} ds. \tag{2}$$

under the capital accumulation constraint:

$$\dot{K} = I - \delta K. \tag{3}$$

The profit of every period is equal to the total revenue, which comes from the sale of the production, minus the wage cost and the costs of investment in terms of domestic goods. The constraint of capital accumulation means that the variation of the capital stock is determined by the net investment and the depreciation of the existing capital stock at rate δ.

The first-order conditions of this optimization problem are obtained as following:

$$f_2 = w, \tag{4}$$

$$q\Psi_1 - 1 = 0, \tag{5}$$

$$q\Psi_2 - \frac{1}{p} = 0, \tag{6}$$

$$\dot{q} = (r + \delta)q - f_1, \tag{7}$$

$$\dot{K} = \Psi(I^D, I^F) - \delta K, \tag{8}$$

$$\lim_{s \to \infty} q(s)K(s)e^{-\int_t^s r(u)du} = 0. \tag{9}$$

The condition (4) postulates a static efficiency relation between the marginal productivity of the labor and the wage rate. By admitting that the wage is

[5] *It is also interpreted as the real exchange rate in this model, so it is a measure of national competitiveness.*

perfectly flexible, the full employment will always be guaranteed. The conditions (5), (6) reflect the efficiency rule which must be followed when an investment is carried out: what brought by a unity of gross investment in the future must be sufficient to compensate the initial cost. They imply also that $p = \Psi_1/\Psi_2$, that means the marginal rate of substitution of the two goods in the investment process is equal to their relative price. The condition (7) describes the evolution of the capital evaluation ratio. Finally, the conditions (8) and (9) represent respectively the capital accumulation constraint and the transversality condition.

The demand of the inputs of investment, I^D and I^F, can be determined by the conditions (5) and (6). The total differentiation of these two equations gives:

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} \begin{bmatrix} dI^D \\ dI^F \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{q^2} \\ -\frac{1}{qp^2} & -\frac{1}{pq^2} \end{bmatrix} \begin{bmatrix} dp & dq \end{bmatrix} \tag{10}$$

The resolution of the system (10) gives:

$$\begin{bmatrix} dI^D \\ dI^F \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \frac{\Psi_{21}}{p^2q} & \frac{1}{pq^2}(\Psi_{21} - p\Psi_{22}) \\ -\frac{\Psi_{11}}{p^2q} & \frac{1}{pq^2}(\Psi_{11} - p\Psi_{21}) \end{bmatrix} \begin{bmatrix} dp & dq \end{bmatrix} \tag{11}$$

with $\Delta = \Psi_{11}\Psi_{22} - \Psi_{12}\Psi_{21} > 0$, by the concavity of the function $\Psi(.,.)$.

The partial derivatives of I^D, I^F and I are given as following:

$$\frac{dI^D}{dp} = \frac{\Psi_{21}}{\Delta p^2 q} >< \ si \ \Psi_{21} >< 0, \tag{12}$$

$$\frac{dI^D}{dq} = \frac{1}{\Delta p q^2}(\Psi_{21} - p\Psi_{22}) > 0, \tag{13}$$

$$\frac{dI^D}{dp} = -\frac{\Psi_{11}}{\Delta p^2 q} > 0, \tag{14}$$

$$\frac{dI^D}{dp} = \frac{1}{\Delta p q^2}(p\Psi_{21} - \Psi_{11}) > 0, \tag{15}$$

$$\frac{dI}{dp} = \frac{1}{\Delta p^2 q}(\Psi_1\Psi_{21} - \Psi_2\Psi_{11}) > 0, \tag{16}$$

$$\frac{dI}{dq} = (\Psi_1 I_q^D + \Psi_2 I_q^F) > 0. \tag{17}$$

These results are obtained in using the relations $(\Psi_1\Psi_{22} - \Psi_2\Psi_{12}) < 0$ and $(-\Psi_1\Psi_{12} + \Psi_2\Psi_{11}) < 0$. The last two inequalities come from the assumption of the normality of the two goods in the formation of the new capital stock. The normality of the two goods specified here implies that as the net investment I increases, the demand of every goods rises. That can be seen more clearly in deriving (5), (6) and (1):

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_1 \\ \Psi_{21} & \Psi_{22} & \Psi_2 \\ \Psi_1 & \Psi_2 & 0 \end{bmatrix} \begin{bmatrix} dI^D \\ dI^F \\ dq \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} dp \\ dI \end{bmatrix} \tag{18}$$

To obtain $dI^D/dI > 0$ and $dI^F/dI > 0$, it needs $(\Psi_1\Psi_{22} - \Psi_2\Psi_{12}) < 0$ and $(-\Psi_1\Psi_{12} + \Psi_2\Psi_{11}) < 0$.

In writing the demand of investment inputs, I^D and I^F, and the net investment I in functional form, one obtains:

$$I^D = I^D(p, q),\ I_p^D >, < 0,\ I_q^D > 0; \tag{19}$$

$$I^F = I^F(p, q),\ I_p^F > 0,\ I_q^F > 0; \tag{20}$$

$$I = I(p, q),\ I_p > 0,\ I_q > 0; \tag{21}$$

The demand of the domestic goods for the investment is a positive function of the marginal value of the capital. While, the effect of a rise of p can be positive or negative according to the relation between the two goods in the investment process. If they are complementary, a rise of p induces necessarily an augmentation of I^D. When they are substitutable, a rise of p implies a diminution of I^D accompanied by an augmentation of I^F. The effect of the augmentation of p and of q on I^F is always positive. From the two demand functions, one deduces easily that the net investment will be a positive function of the real exchange rate (p) and of the marginal value of the stock of the capital (q).

One remark about this formulation is that the choice of investment is part of the firm's decision. As the two goods are demanded, then the real exchange rate intervenes in the function of the net investment. Meanwhile, the net investment depends only on capital evaluation ratio in the model where only one type of goods is used in the investment process. Even the case, where the two goods share the roles in the investment process so that the net investment depends also on the real exchange rate. But even this case makes sense in terms of policy effect, the sharing itself is arbitrary. This is not the case in the new formulation of the choice of investment inputs.

As the full employment is guaranteed by the flexibility of the wages, the supply of the domestic goods y and the internal return of the capital stock R are both function of K. Using $f(K, L)$ and $f'(K, L)$, we obtain respectively

$$y = y(K), \;\; y'(K) > 0;$$

$$R = R(K), \;\; R'(K) < 0;$$

which show that the supply of the domestic goods and the internal return of the capital stock are respectively an increasing and decreasing function of K.

2 The short- and long-run macroeconomic equilibrium in an uncertain horizon model

2.1 The short-run macroeconomic equilibrium

The above analysis about the demand of investment inputs and the firm's decision of the production and capital accumulation can be integrated into a general uncertain horizon model of a small open economy. The demand side of this economy is characterized by the existence of cohorts of consumers who have an uncertain horizon in the sense of Blanchard (1984, 85) and Yaari (1965). [6] The short-run macroeconomic relations are as following:

$$\dot{C} = (r - \rho)C - \theta(\rho + \theta)(A + B), \tag{22}$$

$$\dot{A} = r(A + B) + X(p) - (1 - \alpha)C - \frac{I^F(p, q)}{p} - \frac{G^F}{p}, \tag{23}$$

$$\dot{q} = (r + \delta)q - R(K), \tag{24}$$

$$\dot{K} = I(p, q) - \delta K, \tag{25}$$

$$\dot{p} = (r^* - r)p, \tag{26}$$

$$y(K) - \alpha C - I^D(p, q) - G^D - X(p) = 0, \tag{27}$$

$$rB + G^D + \frac{G^F}{p} = T. \tag{28}$$

The equation (22) describes the dynamics of the aggregate consumption of the domestic consumers and it does not necessitate supplementary remarks,

[6] *See appendix 7 for derivation details of aggregate demand of domestic and foreign goods for consumption.*

excepting that the capital stock does not enter in the definition of the accumulated wealth. That is due to the fact that I assume that the profits realized by the firm are distributed in egalitarian fashion to all living consumers independent of their ages – the assumption made for facilitating the analysis. If the capital stock is owned by the old, the question becomes then how to guarantee that the capital brings the same rate of return as the bond in every period? As the rate of return of bond is determined according to the condition of arbitrage – equal to the real interest rate on the international financial market in adjusting the variation rate of the real exchange rate –, it can be different from the rate of return of shares which depends on the dynamics of the capital stock and the capital evaluation ratio. It is then possible that the rate of return of the shares is higher than that of the bonds. [7]

The equation (23) describes the evolution of the external position of the small economy in terms of the interests payments and the trade surplus or deficit.

The equations (24) and (25) are the simple reproduction of the equations (7) and (8), the only difference is that f_1 is replaced by R. The equation (26) is the condition of arbitrage on the international financial market. It remains to define the short-run domestic real interest rate, which is influenced by more factors as a result of the introduction of a mechanism of capital accumulation with choice of investment inputs than in absence of it. The equation (27) postulates the equilibrium condition on the domestic goods market. An interactive mechanism between demand and supply on this market is introduced in formulating a supply function which depends on the process of capital accumulation. One can see that the demands of domestic goods by the domestic firm for the investment are added to the domestic and foreign demand of the this goods.

[7] *It is surely possible to assume that the shares are remunerated as the bonds so that it brings the same rate of return fixed at the level of the exogenous real interest rate. The net profits (or losses) are distributed to workers in a egalitarian manner. This formulation can be adopted, but the entry of capital stock in the definition of accumulated private wealth will modify significantly the model. It is equally difficult to justify the position taken in this formulation as it can not be explained why the net profits (or losses) are distributed to workers not capitalists.*

2.2 The long-run macroeconomic equilibrium

The steady state of the economy is described by obtained from the equations obtained by imposing to the dynamic equations the steady-state condition, i.e. $\dot{C} = \dot{A} = \dot{q} = \dot{K} = \dot{p} = 0$:

$$(r-\rho)C - \theta(\rho+\theta)(A+B) = 0, \tag{29}$$

$$r(A+B) + X(p) - (1-\alpha)C - \frac{I^F(p,q)}{p} - \frac{G^F}{p} = 0, \tag{30}$$

$$(r+\delta)q - R(K) = 0, \tag{31}$$

$$I(p,q) - \delta K = 0, \tag{32}$$

$$r = r^*, \tag{33}$$

$$y(K) - \alpha C - I^D(p,q) - G^D - X(p) = 0, \tag{34}$$

$$rB + G^D + \frac{G^F}{p} = T. \tag{35}$$

Before commenting some of these equations, it is noted that the presence of the long-run public debt, B, if it does not vary, will not influence the effect of the budgetary policy. As its presence complicates only the presentation of the model in increasing the number of possible cases, only the balanced budgetary policy will be studied in imposing in the following $B = 0$.

The equation (30) reflects the fact that the movement of the balance of payments stops in the long-run. Comparing to the model where only domestic goods is demanded in the investment process, a new factor intervenes in this equation, which is the demand of the foreign goods for the investment, which influences the trade balance – an important factor of the external equilibrium. The equation (31) postulates that the capital evaluation ratio is uniquely determined, in the long-run, by the optimal capital stock, the world real interest rate and the depreciation rate of the capital stock. The equation (32) tells us that the replacement investment is proportional to optimal capital stock. If the first depends on p and q, then that the long-run optimal capital stock is influenced by the real exchange rate. Finally, the equation (34), which is the equilibrium condition on the domestic goods market, experiences two modifications in the same sense as the short-run equilibrium condition on this market.

3 The short-run real interest rate and the stability analysis of the dynamic system

3.1 The determination of the short-run dynamic real interest rate

As the determination of the short-run domestic real interest rate is important in the dynamic analysis of the economy, it will be discussed in detail. In using the fact the consumption is a function of total wealth, it can be defined as a function of A, K, r, p and q. That gives: [8]

$$C = C(r, A, K, p, q, T). \; C_r < 0, \; C_A, C_K > 0, \; C_p, C_q, C_T < 0. \tag{36}$$

A rise of current interest rate will reduce the actualized value of total wealth, that explains the diminution of the current consumption. An increase of external position will increase total wealth so that the current consumption level. An increase of the capital stock will induce generally an augmentation of the human wealth ceteris paribus, and stimulate consequently the consumption of the current period. The influence of other variables can be interpreted in the same line as capital.

In order to determine the short-run domestic real interest rate, the function $C(.)$ given by (36) is substituted in the instantaneous equilibrium condition on the domestic goods market, the equation (34), to obtain:

$$y(K) - \alpha C(r, A, K, p, q, T) - I^D(p, q) - G^D - X(p) = 0. \tag{37}$$

The differentiation of (37) and the rearrangement of the terms give:

$$dr = \frac{1}{\alpha C_1} X dZ, \tag{38}$$

with $X = [y'_K - \alpha C_K, -I^D_p - X' - \alpha C_p, -\alpha C_A, -I^D_q - \alpha C_q, -1, \alpha C_T]$ and $dZ = [dK, dp, dA, dq, dG^D, dT]'$.

It is admitted that $y'_K - \alpha C_K > 0$. In economic term, it can be justified in assuming that an augmentation of the capital stock K will induce a surplus of production which will not be absorbed totally by the increase of the demand due to the revenue effect that it generates (propensity to consume

[8] *As it can be seen from the appendix 7,* $C = (\rho+\theta)(A+H)$ *with* $H = \int_t^\infty [y(K) - I^D - I^F/p - T] e^{-\int_t^s [r(u)+\theta] du} ds$, *one can use the variational derivation to define the current consumption as a function of the current values of some other variables.*

inferior to one). This explains the negative effect of an augmentation of K on r, because in case of excess supply of the goods, one has to a reduce r to stimulate the current private demand for consumption.

The variation of the real exchange rate influences domestic real interest rate by three channels. The variation of p exerts an influence on domestic goods market, first, through the demand for the investment expressed by the firm; secondly, through the demand addressed to the domestic firm by foreigners – the adoption of a particular instantaneous utility function (Cobb-Douglas) induces the disappearance of the effect of the real exchange rate on the demand of the domestic goods by domestic residents; thirdly, by the intermediary of the effect of human wealth (C_p).

At the first view, the impact of the variation of p on r is not defined, even the mechanism of transmission is quite simple. It depends in fact on the sign of I_p^D and on the importance of X' and C_p. When I_p^D is negative, i.e. the domestic and foreign goods are substitutable in the investment process, the term $-I_p^D - X'$ will be positive. If this price effect surpasses the revenue effect, so that $-I_p^D - X' - \alpha C_p > 0$, that means that a rise of p induces an excess supply on domestic goods market, whose reequilibrium necessitates a reduction of r. In the case where the two goods are complementary in the investment process, the positive sign of the expression of $-I_p^D - X' - \alpha C_p$ is more difficult to be assumed.

Given that $C_r < 0, C_A > 0, C_T < 0$ and $I_q^D > 0$, $C_q < 0$, the effects of an augmentation of A, G^D and T, except q, can be determined easily in using the argument of pressure on the domestic goods market. One obtains,

$$r = r(K, p, A, q, G^D, T). \;\; r_K < 0, \; r_p < (?)0, \; r_A, r_q, r_{G^D} > 0, \; r_T < 0; \quad (39)$$

with the relation

$$r_K + r_p + r_T + r_q + r_{G^D} + r_T = 0 \quad (40)$$

which is verified in the long-run as r aligns to r^*, the world interest rate.

The function defined by (39) plays a important role in the dynamic analysis. The dynamics of the real exchange rate disappears if the domestic real interest rate aligns in the short-run as in the long-run on the rate on the international financial market.

3.2 The stability analysis of the dynamic system

The dynamic system is constituted of the equations (22)-(26). The complexity of the system does not permit us to take a rigorous analysis in specifying the necessary and sufficient conditions of local stability for the system. Even this is the case, I can clear out the possibility of stability of the system facing policy or parametric shocks.

In using the definition of r given by (39), the system of the equations (22) – (26) can be linearized around the steady-state equilibrium as following:

$$\begin{bmatrix} \dot{C} \\ \dot{A} \\ \dot{q} \\ \dot{K} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} r^* - \rho & r_A C - \theta(\rho+\theta) & r_q C & r_K C & r_p C \\ -(1-\alpha) & r_A A + r^* & r_q A - \frac{I_q^F}{p} & r_K A & \phi_1 \\ 0 & r_A q & r_q q + r^* + \delta & r_K q - R_K & r_p q \\ 0 & 0 & I_q & -\delta & I_p \\ 0 & -r_A p & -r_q p & -r_K p & -r_p p \end{bmatrix} \begin{bmatrix} C - \bar{C} \\ A - \bar{A} \\ q - \bar{q} \\ K - \bar{K} \\ p - \bar{p} \end{bmatrix} \tag{41}$$

where $\phi_1 = r_p A + X' + \frac{I^F}{p^2} - \frac{I_p^F}{p}$, which corresponds to the case where the government controls $\frac{G^F}{p}$. There may be different control modes of the public spending. Implicitly, this differentiation is taken under the assumption that the government fixes the spending on foreign goods in terms of domestic goods.

The determinant of the stability matrix of the system (41) is:

$$\begin{aligned} \Delta &= [(r^* - \rho)r^* - (1-\alpha)\theta(\rho+\theta)][(r^*+\delta)(r_p\delta p + I_p r_K p) - f_{11}(I_q r_p p - I_p r_q p)] \\ &\quad +(r^* - \rho) r_A p \{I_q^F I_p f_{11}/p - \phi_1[(r^*+\delta)\delta + R_K I_q]\}. \end{aligned}$$

Given that there are two predetermined variables, A and K, and three variables which adjust instantaneously, the stability matrix must has at least two stable roots (the eigenvalue has negative real part) for the system to be able to converge, under the assumption of perfect foresight, to the steady-state equilibrium after a shock without imposing supplementary terminal conditions. [9]

The existence of two stable eigenvalues implies that the stability matrix of the system (41) has a positive determinant, $\Delta > 0$, but the inverse is not necessarily true. The fact that $\Delta > 0$ can correspond to zero, two or four stable eigenvalues. Given only the information disposable on the partial

[9] *See* Buiter (1984).

derivatives, the trace and the sum of the cofactors of the elements on the trace, we can not affirm which is the true situation. Without excluding the possibility of a system totally unstable, consider now the case where $r_A = 0$, i.e. the effect of the non-human wealth on r is so feeble that it can be neglected.

When $r_A = 0$, the determinant becomes:

$$\Delta = [(r^* - \rho)r^* - (1-\alpha)\theta(\rho+\theta)][(r^* + \delta)(r_p\delta p + I_p r_K p) - f_{11}(I_q r_p p - I_p r_q p)].$$

It will be positive if $(r^* - \rho)r^* - (1-\alpha)\theta(\rho+\theta) < 0$ and $[(r_p\delta p + I_p r_K p) - f_{11}(I_q r_p p - I_p r_q p)] < 0$. The fact that $\Delta > 0$ is a very general information which does not permit us to know about the exact number of stable eigenvalues in this particular case. In fact, in imposing $r_A = 0$, the system is decomposable into two independent sub-systems, C-A and q-K-p.

For the sub-system C-A to have a stable eigenvalue corresponding to the existence of a predetermined variable, the condition $\Delta_1 = (r^* - \rho)r^* - (1 - \alpha)\theta(\rho+\theta) < 0$ is sufficient. As regard to the sub-system q-K-p, I admit that $\Delta_2 = (r_p\delta p + I_p r_K p) - R_K(I_q r_p p - I_p r_q p) < 0$. It can alternatively admitted that $r_p < 0$ and $r_q > 0$ which are more restrictive conditions comparing with the precedent condition ($\Delta_2 < 0$). They are equivalent to assume that $I_p^D + X' + \alpha C_p < 0$, which means that an appreciation of the real exchange rate generates a net diminution of the demand for the domestic goods, and that $I_q^D + \alpha C_q > 0$. These conditions combined with the positive sign of the trace of the sub-system, $Tr. = r_q q + r^* - r_p p > 0$, imply that there exists only one stable root. In consequence, under these assumptions I have for the total system two stable eigenvalues corresponding to the number of predetermined variables.

Although the assumption $r_A = 0$ facilitates the analysis of the stability, we can not make it when we want. In absence of capital accumulation, $r_A \neq 0$ is a necessary and sufficient condition for the dynamics of p not to degenerate in the sense that p aligns instantaneously at its value at the new steady-state equilibrium. Similarly, $r_A \neq 0$ guarantees that the domestic real interest rate can deviate from the fixed level of the world interest rate. In the present case, the assumption, $r_A = 0$, can be made without serious consequence. The dynamics of p does not degenerate as the investment process demands the two goods to achieve the capital accumulation. The gradual adjustment of the capital stock will induce that of the real exchange rate and will guarantee the possible temporary deviation of the domestic real interest rate from its long-run level.

If the eigenvalues λ_1, ..., λ_5 are far from being zero, the fact that the eigenvalues vary continuously (in the sense of the complex roots) in terms of of the coefficients of the stability matrix let us thinking that the real parts of the five eigenvalues of the system will not change sign when r_A becomes a little positive. This marge is perhaps quite great for one to generalize the result of the saddle-point stability.

We remark in reexamining the determinant of the sub-system q-K-p, $\Delta_2 = (r_p \delta p + I_p r_K p) - f_{11}(I_q r_p p - I_p r_q p)$, that as there are complementarity between the investment inputs and the effects of p and of q through the human wealth (so that the derivatives r_p and r_q change sign and value), Δ_2 can become positive. If this is the case, the sub-system can then be unstable. In general, we can not generalize the (of saddle-point) stability of the system in question.

4 The analysis of the long-run effects of the budgetary policy

The budgetary policy is limited to the case of non-anticipated permanent variation of the public spending in terms of the domestic goods financed by of the lump-sum taxes. The long-run effect of the budgetary policy on the endogenous variable such as C, A, q, K and p can be examined in using the equations (29)-(32) and (34). This system of equations is impossible to be resolved analytically. We have to proceed a total differentiation around the steady-state equilibrium in order to study the incidence of the budgetary policy in the long-run.

The total differentiation of equations (29)-(32) and (34) around the steady-state equilibrium gives:

$$\begin{bmatrix} M \end{bmatrix} \begin{bmatrix} dC \\ dA \\ dq \\ dK \\ dp \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} dG^D \\ d(\frac{G^F}{p}) \end{bmatrix} \tag{42}$$

with

$$\left[\ M\ \right]=\begin{bmatrix} r^*-\rho & -\theta(\rho+\theta) & 0 & 0 & 0 \\ \alpha-1 & r^* & -\frac{I_q^F}{\bar{p}} & 0 & X'-\frac{I_p^F}{\bar{p}}+\frac{I^F}{\bar{p}^2} \\ 0 & 0 & r^*+\delta & -R_K & 0 \\ 0 & 0 & I_q & -\delta & I_p \\ -\alpha & 0 & -I_q^D & y_k & -I_p^D-X' \end{bmatrix}.$$

The determinant of the matrix at the left side of the system (42) can be represented as following:

$$\Delta=[(r^*-\rho)r^*-(1-\alpha)\theta(\rho+\theta)]\begin{bmatrix} r^*+\delta & -R_K & 0 \\ I_q & -\delta & I_p \\ -I_q^D & y_K & -I_p^D-X' \end{bmatrix}$$

$$+\alpha\theta(\rho+\theta)\begin{bmatrix} -\frac{I_q^F}{\bar{p}} & 0 & X'-\frac{I_p^F}{\bar{p}}+\frac{I^F}{\bar{p}^2} \\ r^*+\delta & -R_K & 0 \\ I_q & -\delta & I_p \end{bmatrix} \tag{43}$$

Given that $R_K<0$, I_q, $I_q^D>0$, $y_K>0$, $I_p>,<0$, the first term in the expression Δ (43) is positive if I admit that $[(r^*-\rho)r^*-(1-\alpha)\theta(\rho+\theta)]<0$ and $I_p^D+X'<0$. The first inequality is absolutely necessary to guarantee the stability of the dynamic system. The second is not restrictive comparing to the sign of the expression $I_p^D+X'+\alpha C_p$ in the stability analysis. These conditions are more than sufficient to guarantee the positive sign of the first term of Δ, because there exists numerous factors which act in the inverse sense, so that I can think that they compensate the effect of the negative sign of I_p^D+X'.

As $I_q^F, I_q^F>0$, the sign of the second term of Δ depends on that of the included matrix. It needs then examine attentively the expression $X'-\frac{I_p^F}{\bar{p}}+\frac{I^F}{\bar{p}^2}$ which represents the direct effect of an appreciation of the real exchange rate over the trade balance. There exists meanwhile an indirect effect exerted through the influence of the real exchange rate on the capital evaluation ratio q, which in turn influences the demand of the foreign goods in the investment process, represented by the expression $-\frac{I_q^F}{\bar{p}}$. To justify this comment, it is sufficient to give the following demonstration: in deriving the trade balance $X(p)-(1-\alpha)C-\frac{I^F(p,q)}{p}$ with regard to the real exchange rate, I obtain

$$X'-\frac{I_p^F}{p}-\frac{I_q^F}{p}\frac{dq}{dp}+\frac{I^F}{p^2}. \tag{44}$$

In using the equations (31) and (32), the relation in the long-run between q and p can be deduced as follows:

$$\frac{dq}{dp} = \frac{I_p R_K}{-I_q R_K + \delta(r+\delta)}. \tag{45}$$

The substitution of (45) into (44) gives justly the determinant of the matrix in the second term of (43). If I admit that the appreciation of the real exchange rate will induce a trade deficit in the long-run accounting for its direct and indirect effect, the second term of Δ is then incontestably positive.

Given the conditions imposed above, the determinant of the matrix at the left side of (42), Δ, will be positive. We can now proceed to the resolution of the system in order to find out the long-run effect of the budgetary policy.

The solution of (42) according to the habitual procedure gives:

$$\begin{bmatrix} dC \\ dA \\ dq \\ dK \\ dp \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} A_{15} & A_{12} \\ A_{25} & A_{22} \\ A_{35} & A_{32} \\ A_{45} & A_{42} \\ A_{55} & A_{52} \end{bmatrix} \begin{bmatrix} dG^D \\ d(\frac{G^F}{p}) \end{bmatrix} \tag{46}$$

with

$$\begin{aligned}
A_{15} &= -\theta(\rho+\theta)\{I_p R_K \frac{I_q^F}{\bar{p}} - (X' - \frac{I_p^F}{\bar{p}} + \frac{I^F}{\bar{p}^2})[\delta(r^*+\delta) - R_K I_q)]\} < 0, \\
A_{25} &= -(r^*-\rho)\{I_p R_K \frac{I_q^F}{\bar{p}} - (X' - \frac{I_p^F}{\bar{p}} + \frac{I^F}{\bar{p}^2}) \\
&\quad [\delta(r^*+\delta) - R_K I_q)]\} >, < 0, \text{if } r^* <, > \rho, \\
A_{35} &= -R_K I_p[(r^*-\rho)r^* - (1-\alpha)\theta(\rho+\theta)] < 0, \\
A_{45} &= -(r^*+\delta)I_p[(r^*-\rho)r^* - (1-\alpha)\theta(\rho+\theta)] > 0, \\
A_{55} &= -[\delta(r^*+\delta) - R_K I_q)][(r^*-\rho)r^* - (1-\alpha)\theta(\rho+\theta)] > 0; \\
A_{12} &= \theta(\rho+\theta)\{\delta(r^*+\delta)(I_p^D - X') - R_K[I_q(I_p^D + X') - I_q^D I_p]\} < 0, \\
A_{22} &= (r^*-\rho)\{\delta(r^*+\delta)(I_p^D - X') \\
&\quad -R_K[I_q(I_p^D + X') - I_q^D I_p]\} >, < 0, \text{ if } r^* <, > \rho, \\
A_{32} &= -\alpha\theta(\rho+\theta)R_K I_p > 0, \ A_{42} = -\alpha\theta(\rho+\theta)\delta(r^*+\delta)I_p < 0, \\
A_{52} &= -\alpha\theta(\rho+\theta)[\delta(r^*+\delta) - R_K I_q] < 0.
\end{aligned}$$

The effects of the domestic budgetary policy can be represented in the table 1.

	C	A	q	K	p
G^D	$-$	$\pm$	$-$	$+$	$+$
$\frac{G^F}{p}$	$-$	$\pm$	$+$	$-$	$-$

Table 1: *The effect of the budgetary policy in a model with choice of investment inputs.*

The effects of the budgetary policy on the aggregate total consumption C and the external position A will not be fundamentally different from the results obtained in the case where there is no capital accumulation, if it is admitted that the appreciation of the real exchange rate creates a trade deficit and that the increase of the price of the domestic goods reduces the demand on this market. We do not doubt that the greatness of the effect of the policy on these two variables are modified following the introduction of the capital accumulation process specified in this paper. This modification goes generally in the sense of a diminution, relatively more feeble, of C in the case of a permanent augmentation of G^D. The effect of a permanent augmentation of G^F/p reinforces in contrast the diminution of C. The reason for this is that the supply effect comes to add to the revenue effect corresponding to the tax collected to finance the spending. This supply effect does not exist in a model where the investment process demands only the domestic goods or the foreign goods. In the present case, the augmentation of G^D reduces the human wealth by the channel of the taxation, but increases it by the channel of the capital accumulation because the rise of p induced by the increase of the spending will bring a augmentation of the capital stock, which implies in turn an increase of the human wealth. An inverse mechanism will exert in the case of an augmentation of G^F/p.

The interesting results are that the effects of the budgetary policy on the capital evaluation ratio, q, and the capital stock, K, can be transmitted through the real exchange rate, p. We note that the permanent augmentation of G^D and of G^F/p has respectively a negative and positive effect on the capital evaluation ratio. We want to give here an intuitive explanation of these results. If a permanent augmentation of G^D can not be absorbed by an increase of p, which stimulates the supply and reduces the demand, one has then an increase of the capital stock and a decrease of the capital evaluation ratio. This result is interesting, because in the models where only one goods is used in investment process, the effect of the budgetary policy on the capital stock is often unexisting in the long-run due to the fact that the marginal efficiency of the capital must be equal to the exogenous world real interest rate. In the present model, the increase of p stimulates the investment in the short-run and raises then the optimal capital stock in the long-run. This price effect has meanwhile a negative impact on q. That is evident, because by the equation (31), one has $q = R(K)/(r + \delta)$. In using assumption of decreasing return to scale, the augmentation of K signifies necessarily the diminution of q, as R diminishes when K augments.

5 The simulation of dynamic adjustment path of the endogenous variables following the policy shocks

The dynamic adjustment paths of the endogenous variables C, A, q, K, p are quite difficult, if not impossible, to be characterized analytically. To illustrate graphically the theoretical results, I must make here two numerical simulations in giving the values of the parameters and the partial derivatives.

5.1 Simulation 1: the case of a creditor country

It is imposed here $r = 5\%$, $\rho = 4\%$, $\theta = 3\%$, $\delta = 5\%$, $I_q^D = 1.2$, $I_q^F = 1.3$, $R = 0.12$, $R_K = -0.5$, $I_p^D = 0.5$, $I_p^F = 0.7$, $I^F = 2$, $p = 1.5$, $q = 1.2$, $X' = -0.5$, $\alpha = 0.6$, $r_A = 0.0001$, $r_q = 0.003$, $r_K = -0.005$, $r_p = -0.002$, $I_q = 2.8$, $I_p = 0.7$, $A = 10$, $B = 0$, $C = 2.1$. This case corresponds to a country which is creditor at steady-state. The value of r_A is crucial to guarantee the stability of the system. For $r_A \leq 0.0003$, we have generally a stable system with two stable eigenvalues.

The eigenvalues of the stability matrix are respectively $\lambda_1 = -1.15478$, $\lambda_2 = -0.00019683$, $\lambda_3 = 0.00562425$, $\lambda_4 = 0.060043$, $\lambda_5 = 1.20683$. There are then two stable eigenvalues corresponding to the number of predetermined variables.

Given the complexity of the dynamic effects of the two predetermined variables on the initial adjustment of the non-predetermined variables, there may be an over-shooting as well as under-shooting of the non-predetermined variables according to the combination of the contradictory mechanisms.

Figure 1: *The internal transmission of the effect of a non-anticipated permanent augmentation of the public spending on the domestic goods: the case of a creditor country.*

Figure 1 illustrates the case of a non-anticipated permanent augmentation of the spending on the domestic goods ($dG^D > 0$). Take for example $(A_0 - \bar{A}) = 1$ and $(K_0 - \bar{K}) = -1.5$. In this case, the capital accumulation is a dominant force, as it influences strongly the adjustment path of C, A, q and p, particularly in the short-run.

It is interesting to observe that, if the absolute variation of A and K are not so different, C, q and p over-shoot their value at the steady state. These three variables adjust quite differently. The consumption C decreases strongly at first and rises then progressively. Concerning the capital evaluation ratio, q, it varies in the short-run in the inverse sense of its long-run variation (path indexed 1), that is quite particular. It can be easily understood that the initial augmentation of q is generated by the reevaluation of the capital stock at the moment of the announcement of the budgetary policy. This particular case can be excluded if the variation of long-run is superior to that of short-run. In consequence, there may be a case of under-shooting of q (illustrated by the path indexed 2). The real exchange rate over-shoots its long-run value in the initial period, its path will decreases in the following periods. The path of A is only feebly influenced by the adjustment of the capital stock. In contrast, the capital stock, K, is strongly influenced by the dynamics of consumption and saving (here A) such that it first increases and then decreases. In some alternative combinations of the parameters, we can expect that C and p can take other forms of adjustment paths, given that the dynamics due to the capital accumulation and the wealth accumulation do not work in the same sense.

The role of r_A can be examined particularly here. In the case of a creditor economy, the value r_A is very subtle to guarantee the stability of the dynamic system. The above numeric simulation corresponds to a situation where $r_A = 0.0001$, that is very feeble. The augmentation of r_A makes the system more pro-unstable. When I tries to simulate the model with $r_A = 0.0002$, in maintaining unchanged the value of the other parameters, the system comprises only one stable eigenvalues which is $\lambda_1 = -1.15476$. The other eigenvalues are $\lambda_2 = 1.20679$, $\lambda_3 = 0.0590322$, $\lambda_4 = 0.0070776$, $\lambda_5 = 0.000457561$.

5.2 Simulation 2: the case of an indebted country

It is imposed here $r = 5\%$, $\rho = 6\%$, $\theta = 3\%$, $\delta = 5\%$, $I_q^D = 1.2$, $I_q^F = 1.3$, $R = 0.12$, $R_K = -0.5$, $I_p^D = 0.5$, $I_p^F = 0.7$, $I^F = 2$, $p = 1.5$, $q = 1.2$, $X' = -0.5$, $\alpha = 0.6$, $r_A = 0.0003$, $r_q = 0.003$, $r_K = -0.005$, $r_p = -0.002$, $I_q = 2.8$, $I_p = 0.7$, $A = -10$, $B = 0$, $C = 2.7$. This case corresponds to an indebted country. The value of r_A is not any more crucial to guarantee the stability of the system. For $r_A \geq 0$, there may be generally a stable system with two, or four stable eigenvalues.

For the particular case here, the eigenvalues of the stability matrix are respectively $\lambda_1 = -1.19982$, $\lambda_2 = -0.0168863$, $\lambda_3 = 0.00701107$, $\lambda_4 = 0.0515631$, $\lambda_5 = 1.15817$. We have then two stable roots corresponding to number of predetermined variables.

Figure 2 illustrates the case where the non-anticipated permanent augmentation of the public spending on the domestic goods ($dG^D > 0$) generated a long-run augmentation of the external position, A, as its sign is negative (debt owned to the rest of the world). In this case, one has generally $A_0 - \bar{A} < 0$ and $K_0 - \bar{K} < 0$. One can take for example $A_0 - \bar{A} = -1$ and $K_0 - \bar{K} = -1.5$. One can remark that C over-shoots its long-run value in reducing more strongly at the moment of policy shock. The predetermined variable A decreases first and then rises. This shows that the influence of the

Figure 2: *The internal transmission of the effects of a non-anticipated permanent augmentation of the public spending on the domestic goods: case of an indebted country.*

dynamics of the capital accumulation on A is visible in the present case. In contrast, the adjustment of K is characterized by a increasing path. The ratio of capital evaluation takes a decreasing path in over-shooting its long-run value. The real exchange rate p over-shoots its long-run value, but it follows a path which descends first and rises ulteriorly.

Contrary to the case of an creditor economy, the relative bigger value of r_A do not make the system more unstable. Given $r_A = 0.04$, without modifying the other parameters, [10] the system comprises now four stable eigenvalues which are respectively $\lambda_1 = -1.13817$, $\lambda_2 = -0.00149745$, $\lambda_3 = -0.181203 + 0.14748i$, $\lambda_4 = -0.181203 - 0.14748i$, and one unstable root which is $\lambda_5 = 1.19867$.

[10] *Note that the precedent simulation corresponds to* $r_A = 0.0003$ *which is much smaller than* $r_A = 0.04$.

6 Conclusion

In this model, the decision about which inputs to be demanded in the process of capital accumulation is taken by the firms. This point is generally neglected by the existing intertemporal approaches of open economy macroeconomics based on microeconomic foundations. The incorporation of this point, in formulating choice of investment inputs in a small specialized economy with uncertain horizon consumers, permits us to examine the effects of budgetary policies on the capital stock and its intertemporal spill-over effects on the other variables such as the external position, the domestic consumption, the real exchange rate etc.. The introduction of this new formulation is based on a new conception – the degree of technological dependence. In integrating the choice of inputs to the general intertemporal optimization problem of the firms, I can model the different limit and intermediary situations of demand of investment inputs.

In comparing with the ad hoc formulations, the one set up in this paper is more general and rigorous. First, the different formulations already adopted by the anterior works can be easily conciliated with this one in assuming different degree of efficiency of a goods in the investment process. Secondly, this formulation sums up in fact the stage of development of a country, the degree of dependence of this country on the foreign technology, the spirit of the leading class and the industrial policies of the country. These different figures correspond to a particular specification of the marginal efficiency, the complementarity and the substitutability of the goods in the formation of the capital stock. Thirdly, this new formulation permits the budgetary policy to generate the long-run effects on the capital stock without the introduction of the labor in the utility function, or assuming arbitrarily the division of roles between goods.

The most interesting result is, with the formulation of the choice of investment inputs, the optimal capital stock and the capital evaluation ratio depend generally on the real exchange rate. If one admits that a permanent augmentation of the public spending on the domestic goods has a positive effect on the real exchange rate, that will induce a diminution of the capital evaluation ratio and an augmentation of the capital stock. When this spending is destined to buy the foreign goods, the negative effect of the policy on the reel exchange rate induces the inverse reactions of the capital evaluation ratio and the optimal capital stock. To get rid of ambiguities concerning the effects of the budgetary policy, three conditions are imposed. The first condition is on the level of the real interest rate, which postulates that the real interest rate must be inferior to a certain value determined by parameters such as the rate of time preference, the rate of death (and/or birth) and the marginal propensity to consume the domestic goods. This condition is otherwise a condition of stability of the dynamic system. The second condition is that the sum of the direct and indirect effect of an appreciation of the real exchange rate on the trade balance is negative in the long-run. The third condition guarantees the negative effect exerted in the long-run on the demand of the domestic goods by an appreciation of the real exchange rate.

The dynamic system is stable under a number of assumptions despite the great dimensionality and the interdependence between different variables. Under the assumption that the effect of the wealth accumulation is very feeble on the domestic real interest rate, it is sufficient then to postulate two assumptions quite reasonable to guarantee the stability of the system. The first is that an increase of the price of the domestic goods reduces the demand of the domestic goods. The second is that the rise of the ratio of capital evaluation generates an excess demand of the domestic goods due in great part to the demand of inputs for the investment.

In using the numeric simulation, it is shown that in the case of a small creditor economy, the important effect of the external position on the domestic real interest rate may make the dynamic system more unstable as the consumers want to save more and more facing a higher and higher domestic real interest rate. In the case of a small indebted economy, the stability is more general. More wealth effect on the domestic real interest rate makes in contrast the equilibrium more stable. For a stronger reaction of the real interest rate to the accumulated wealth, the system may have more stable eigenvalues (four) than the number of predetermined variables (two), even though the non-oscillatory stability became problematic.

The dynamic adjustment of the non-predetermined and predetermined variables, is governed by the interaction of the two forces, one is the adjustment of the external position, and the other the capital accumulation. According to the interaction between the external position and the productive capital, the adjustment paths of different variables can take divers forms, often non-monotone. There are numerous possibilities of dynamic adjustment paths for the real exchange rate in this model. In presence of the effect of the external position and that of the capital accumulation, there may be, under the budgetary policy shock, at least three possibilities: the initial over-shooting, the retarded over-shooting and the under-shooting. As regard to the dynamic adjustment paths of the aggregate consumption, they can equally take the forms similar to these of the real exchange rate.

Appendix

7 The behavior of the consumers et the aggregation

The individual consumer has a uncertain life as defined in Blanchard (1984, 85). He consumes the domestic and foreign goods. To modify his consumption path, he can use the international financial market. He will receive on one hand the interest payments on the accumulated wealth and on the other hand the wages. He shares, at equal right, with the other consumers, the profit or the loss of the firm[11]. The consumer pays equally a lump-sum tax to the government which uses it to finance the public expenditures. The consumer, conscious of his negligible influence on goods and labor markets, behaves in a competitive manner on these two markets. The intertemporal optimization problem of the individual consumer is the following:

$$\max_{\{c^D(s),c^F(s)\}} \int_t^{\infty} \ln{}^{\alpha 1-\alpha} e^{-[\rho+\theta](s-t)} ds \tag{47}$$

under the constraints:

$$\dot{a}(t) = [r(t)+\theta]\omega(t) + w(t)l(t) + \zeta\{Y(t) - w(t)L(t) - I^D(t) \tag{48}$$

$$-\frac{I^F(t)}{p(t)}\} - c^D(t) - \frac{c^F(t)}{p(t)} - \tau(t),$$

$$\lim_{s\to\infty} a(s)e^{-\int_t^s r(u)du} \geq 0, \tag{49}$$

$$c^D(t), c^F(t) \geq 0. \tag{50}$$

Here is considered the case of a Cobb-Douglas instantaneous utility function. The subjective discount rate of the consumer is represented by ρ which is invariable. The value of his accumulated wealth is noted by $a(t)$, which corresponds to his part in the country's external position. The terms between the braces in the equation (48) represent the total profit realized by the firms. The coefficient ζ is the part of profit distributed to a individual consumer. Given that every individual offers the same quantity of labor, ζ will be equal to l/L with l and L design respectively the level of individual and aggregate levels of employment. The sum $I^D(t)+I^F(t)/p(t)$ represents the total costs of investment.

The equation (49) is the condition of transversality. It means that a consumer can borrow for his consumption, but he has ultimately to reimburse his debt. The interest rate $r(t)$ is determined by the domestic financial market which is connected

[11] *This point is a little troubling in the sense that in a capitalist economy the profits are distributed to the entrepreneurs and the owners of equities of the firm. This assumption is adopted for not to complicate the model in making the bonds and shares not perfectly substitutable.*

closely to this on the international financial market.

In resolving the program of consumer, the following relations result:

$$c^D = \alpha c, \tag{51}$$

$$c^F = (1-\alpha)pc, \tag{52}$$

$$c = c^D + pc^F = [\rho + \theta](a+h), \tag{53}$$

$$\dot{c} = (r-\rho)c, \tag{54}$$

$$h = \int_t^\infty [w(s)l(s) + \zeta\{Y(s) - w(s)L(s) - I^D(t) \tag{55}$$

$$-\frac{I^F(t)}{p(t)} - \tau(s)]e^{-\int_t^s [r(u)+\theta]du}ds,$$

where h is the human wealth.

In noting that C, C^D, C^F, A, T and H as the aggregate variables corresponding respectively to c, c^D, c^F, a, τ and h, the macroeconomic relations can be deduced easily in following the procedure of Blanchard (1985).

$$C^D = \alpha C, \tag{56}$$

$$C^F = (1-\alpha)pC, \tag{57}$$

$$C = C^D + \frac{C^F}{p} = (\rho+\theta)(A+H), \tag{58}$$

$$\dot{C} = (r-\rho)C - \theta(\rho+\theta)A, \tag{59}$$

$$H = \int_t^\infty [Y(s) - I^D - \frac{I^F}{p} - T(s)]e^{-\int_t^s [r(u)+\theta]du}ds, \tag{60}$$

$$\dot{A} = rA + f(K,L) - I^D - \frac{I^F}{p} - C - T, \tag{61}$$

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