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Deterrence of a criminal team: how to rely on its members’ shortcomings?

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Abstract

In this paper, we modelize a criminal organization as an agency where the Principal and the Agent have different sensibilities towards the risk of arrestation and punishment, and at the same time have different skills with respect to general organization tasks, crime realization or detection avoidance activities (i.e. allowing to reduce the probability of detection). In this set up, we first compare two regimes of exclusive sanctions (either the sanctions are borne by the Principal/beneficiary of the crime, or they are borne by the Agent/perpetrator of the crime), and we analyze the comparative efficiency of the various instruments which are at the disposal of public authorities to prevent corporation in criminal activities (frequency of control and level of monetary penalties). Finally, we study a case with joint liability.

JEL Classification: K13, K4.

Keywords: Criminal teams, corporate criminality, state dependent risk aversion, deterrence, monetary penalties versus detection.

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1 Introduction

A criminal organization may be seen as an agency where the Principal and the Agent have different sensibilities towards the risk of being caught and punished, and at the same time have different skills with respect to general organization tasks, crime realization or detection avoidance activities (i.e. allowing to reduce the probability of detection). We introduce a basic framework of criminal groups activity along this line in order to compare two regimes of exclusive sanctions (either the sanctions are borne by the Principal/beneficiary of the crime, or they are borne by the Agent/perpetrator of the crime) depending on their social benefits 1/ in terms of their deterrence effects of crime, 2/ in terms of the probability of detection. Then we analyze the comparative efficiency of the various instruments which are at the disposal of public authorities to prevent corporation in criminal activities (frequency of control and level of monetary penalties).

Concerning criminality team, it is often claimed in public opinion that sanctions which are inflicted to the sleeping partner in a crime, or to the beneficiary of a fraudulent act should be more heavy than the punishments applied to the perpetrator of the offences who may be sometimes entrapped in the crime. On the other hand, that public deterrence of criminal activity focuses at first on perpetrators may be explained by the existence of large costs of detection of Principals, as compared to the small monitoring costs of Agents. Our results suggest that, as far as there is no strict constraints on the resources allotted to the control and repression of criminal activities (public authorities may levy more and more resources to develop this activity), then the regime with exclusive sanction on the perpetrator always allows to obtain larger deterrence effects on Principals than the regime of exclusive sanction on the Principal, and that these advantages increase with the probability of public control and detection. On the other hand, the regime of Agent’s liability gives him more incentives to cheat, thus leading to a smaller probability of detection of criminal activities.

We also find that under mild conditions about the choice of fines, the regime of joint liability/mixed sanctions allows to obtain larger effects in terms of crime deterrence than the regime of exclusive sanctions upon the Principal; nevertheless, this also yields more difficulties in detecting the crime, since the Agent have more incentives to invest in activities avoiding public detection. In contrast, it yields (under reasonable conditions regarding the choice of fines) less crime deterrence but more detection than a regime of unilateral sanction upon the Agent.

Section 2 gives some related works on the topics. Section 3 describes our framework and the main behavioral assumptions of the model. Then,
in section 4, we characterize the optimal contract (effort, monetary transfer) between the Principal and the Agent, in each regime of sanction (exclusive sanction upon the Principal versus exclusive sanctions upon the Agent). We show that exclusive sanctions upon the Agent allow to obtain more compliance from the Principal, but induce higher efforts in cheating from the Agent. Section 5 displays the complete comparative statics of the model. Our results suggest that, as far as there is no strict constraints on the resources allotted to the control and repression of criminal activities (public authorities may levy more and more resources to develop this activity), the advantages in favour of a regime of exclusive sanctions on Agents increase with the probability of public control and detection. Section 6 studies the case for joint liability. We show that it implies more deterrence than the regime of exclusive sanction upon the leader, but at a cost since « joint liability » also reduces the effectiveness of public monitoring as compared to the second regime, once again. It means that it is more efficient to punish both partners in a criminal teams than the « stronger partner » (leader) alone, since the probability of crime is smaller when both bear a sanction - albeit, the frequency of detection of the team is smaller. In contrast, there is unfortunately no reason to believe that it is also more efficient to punish both partners than the « weaker » alone; maybe this is not the case: more conditions over basic assumptions regarding the technology of avoidance and individual preferences are required (not found here). Section 7 gives a brief conclusion.

2 Related literature

At least three kinds of literature are worth mentioning, since this work may be connected to each of them: corporate crime, gatekeepers and criminal organizations.

Corporate crime. At a basic level, the issue of corporate crime has been studied first with reference to employer-employee relationships. Arlen (1994) puts the distinction that corporate crime is not crime committed by corporations, but corresponds to illegal activities undertaken by individuals belonging to a corporation, but pursuing their own selfish interest - even though the offence incidentally benefits the corporation. Thus, the rational for corporate liability is that public monitoring of corporate crimes is most of the time difficult and costly to implement, enabling not enough (inefficient) deterrence, while in contrast, corporations have an advantage in detecting wrongdoings by their employees. As a matter of fact, vicarious criminal liability (shifting liability from
employees to the corporation) increases corporation’s enforcement expenditures, thus increases the probability of detection and then reduces the number of malicious employees who commit the offence. Nevertheless, Arlen’s analysis highlights the potentially perverse incentives of strict vicarious liability, coming from opposite forces created by the very increase in enforcement expenditures: as the probability of corporation’s employees detection increases, so is the probability of public detection, allowing the government to increase the corporation’s expected liability for these crimes. As a result, strict vicarious liability may finally lead a corporation to spend less in enforcement than it would absent of vicarious liability: this occurs soon as the benefits of the reduction in the number of offences committed by their employees fall well down the expected costs associated to enforcement and liability. Chu and Qian (1995) have argued that one solution to overcome the problem is to introduce vicarious liability under a negligence rule. Specifically, courts may provide enough incentives to induce from Principal honest reports either by lowering the due care level or by lowering the level of delegated liability.

Shavell (1997) observes that corporations have limited ability to penalize their wrongdoing employees. Although standard economic analysis of compensation concludes that optimal damages must be set to the value of harm, in practice corporations impose limited penalties on their employees for causing harm to third parties, the major sanction imposed being to be dismissed from their job. One way firms may remedy this problem is by paying supernormal (above-market) wages: this is because when the employee have more to lose when he commits an error or an offense and is dismissed. Thus supernormal wages induce more care and prevention of accidents. Nevertheless, private incentives of firm to the use of above-market wages deviate from social ones. On the one hand, this reflects that supernormal wages correspond to additional costs for a firm, whereas it is not for society. On the other, market price of firm product may exceed social costs, as they reflect the increase in the private costs following the adoption of supernormal wage. Thus the undesirable decrease in consumers’ purchases may render the setting of wage at a lower level than harm socially advantageous.

Gatekeepers’ liability: auditors and lawyers. Recent corporate scandals (Enron, Worldcom) since the begining of 2000s, and the even more recent subprime crisis which has been the departure of a major financial crisis at an international level, have made apparent the essential role of professional service providers, such as auditors, corporate lawyers, and securities analysts, in detecting and revealing corporate misconduct on the part of their clients and at the same time, the failure of the internal
control inside large diversified financial organizations. Kraakman (1986) has revised the classical gatekeeping theory, clarifying that a gatekeeping strategy requires gatekeepers "who can and will prevent misconduct reliably, regardless of the preferences and market alternatives of wrongdoers." After the historic debacle of Enron, there has been a renewal of the debate, beginning with Coffee (2002) who has argued that "Enron is a maddeningly idiosyncratic example of pathological corporate governance, which by itself cannot provide evidence of systematic governance failure" but properly understood it can explain "why and when reliance may not be justified on "reputational intermediaries," such as auditors, securities analysts, attorneys. His proposal is to convert gatekeepers into insurers, but cap their insurance obligations based on a multiple of the highest annual revenues the gatekeepers recently had received from their wrongdoing clients. Partnoy (2004) makes the point that the problem of gatekeeper liability is a shift in scholarship view which had more focused on reputation than on regulation or civil liability. Many scholars have argued that liability should not be imposed on gatekeepers and that reputation-related incentives alone would be sufficient. Partnoy compares various proposals and concludes that a contractual system based on a percentage of the issuer’s liability would be preferable to a regulatory system with caps based on a multiple of gatekeeper revenues. Schäfer (2004) argues that a wrong audit that causes damages to shareholders should be strictly regarded as a tort case. We also argue that a rule of gross negligence or of gross violation of professional standards in tort law can avoid the problems of underdeterrence as well as of overdeterrence in the compensation of pure financial loss in tort. However, we also argue that a wrong audit should lead to contractual liability, if it was made to prepare the sale of a company or parts of it from inside investors to outside investors or to prepare an initial public offering. Ganzua and Gomez (2007) consider a framework where they analyse the imposition of duties of care and reporting on gatekeepers, conditional on their having observed an underlying wrongdoing or misconduct of their clients. They make the assumption that the gatekeeper observes the state of the world affecting misconduct, and that the public authority (Courts or a regulator) are unable to verify whether misconduct had or not been observed by the lawyer or auditor. However it is costlessly verifiable ex post. The main results are that standards of professional behavior by auditors or lawyers may well be sufficient as incentives, and that the implications of the model tend to imply that the distinction between voluntary violation of duties and mere negligence is not very useful contrary to what is specified in existing Laws.

Criminal organization and corruption. A specific case of a crime in
a team is corruption. Marjit and Shi (1998) and Jacquemet (2006) provide two different surveys of the strategic approach of corruption. One of the main issue is not how to punish, but in contrast how to reward a corrupted official in order to better control crime. Chappe and At (2005,2008) develop two dynamic models of crime with and without information acquisition and study the conditions under which it is optimal for a criminal to delay commission of a crime rather than committing it immediately. They adress the issue of the optimal fine and level of deterrence. However, they do not consider the question of how liability may be allocated among the gang members. Garoupa (2000) provides a comparison of criminal organizations such as mafias with governments, and analyses the optimal contract between a gang member and the gang authority. The mafia extorts a rent to criminals when they commit a crime, which runs as a barrier to entry on the criminal market that enables public authorities to save on enforcement costs. Nevertheless, the models do not take into account the interactions within the gang (the maffia is a moral personn) and thus does not consider the issue of liability of the Principal.

A main reason explaining why the allocation of liability is neutral in these works, comes from the fact that this a pure transfer between both risk neutral parties, according to Shavell’s argument.

Privileggi and ali (2001) have focused on what occurs when one of the party is risk averse. They assume that the (risk neutral) Principal may delegate to the (risk averse) Agent the realization of the wrong-doings, thus leading to an agency problem. The Agent exerts an effort that negatively affects the probability of detection, but bears a cost in utility terms. The Principal has the opportunity to perfectly observe the decision of the Agent. In this framework, Privileggi and ali show that shifting the liability upon the Agent, all else equal (holding constant both the monetary sanctions and the probability of public detection), allows to obtain larger deterrence effects on the Principal than when sanctions only affect the Principal. Hence, a regime of exclusive sanctions (liability) on the Agent favors a better public monitoring of illegal activity. Nevertheless, they also show that the regime of exclusive sanction on the Agent may induce him to exert more effort in cheating, which leads to countervailing incentive effects on individuals who still find illegal behavior profitable: given that the probability of detection may be lower than in the other regime, public authorities may be faced with greater difficulties in repressing illegal activities. In the case where the Agent displays constant absolute risk aversion, Privileggi and ali show that there exists a kind of complementarity between the level of monetary sanctions and the level of probability of detection: a calibration of their
model exhibits that for any level of the CARA index, there always exists a combination consisting in a probability of public detection and a level of sanctions which allows to implement a small effort of cheating. Specifically, “strong” public enforcement policies based on large probability of detection and at the same time high level of monetary sanctions render the shift of the responsibility upon the Agent socially beneficial - that is, it allows more Principal to renounce illegal behavior, and facilitates public detection since Agents undertake less efforts in cheating.

Our papers takes a different view regarding two main behavioral assumptions.

One the one hand, a the main feature of the analysis of Privileggi and ali (2001) makes their framework close to the basic agency model of the employer-employee - the Agent’s cost of the effort is expressed in utility terms. In contrast, our paper introduces an alternative specification for the technology of effort. We assume here that the illegal activity is costly, i.e. individual who engage in it has to spend money coming from the use of productive factors, which may be scarce and specific. The rational for that is that when such offences are developed at a large scope, they become strictly speaking a parallel economy requiring at a basic level the coordination and cooperation between several individuals. Each may be endowed with specific human and non human assets which are worth for this activity: hence, corporate criminality is an industry. Remark that when the assumption about individuals’ risk neutrality is relaxed, the issue of the nature of the cost of effort matters, although it is irrelevant in the case where individuals are risk neutral. Is it a monetary cost, having the characteristics of the implicit technology of production to which the effort is associated; or is it only the desutility of effort - which may have monetary equivalent, but may be complementary to wealth? The case for the monetary cost of effort, which seems actually to be the widespread interpretation in the literature on Law & Economics, is attractive but not simply for its realism. As it will be seen, it has finally the main advantage to enable a complete characterization of the various regimes of sanction, and specifically non ambiguous effects when we proceed to a complete comparative statics analysis.

On the other hand, in this paper we allow a richer representation of Agent’s preferences; we introduce the state-dependent approach more usually used in the literature on safety and/or value of life analysis. The rational for such an assumption is that, although we do not address directly this issue here, these state-dependant preferences allow to take into account the fact that public authorities may use non monetary sanctions where criminal activities are detected and criminal individual are arrested, such as: imprisonment, full or partial loss of civic rights and
so on. The basic intuition says that individual even when engaged in illegal activities have an intrinsic preference for freedom (not being detected), and to the extent that going on with their wrongdoing is worth to them, all else equal, such that they are better off when they have the opportunity to cheat than when cannot. This is specifically what the state-dependent representation allows with a parsimonious model of individual preferences.

3 The model

Consider the case where the criminal activity or illegal behavior allows the Principal to obtain a payment equal to $B > 0$. The Principal is not the perpetrator of this malicious act, but he delegates\(^1\) it to an Agent, who bears a monetary cost given by $C(x) = x^2$. On the other hand, the Principal rewards Agent’s efforts with a payment equal to $w > 0$ in case of success, i.e. when the offences act has not been detected by the authority. Let the aggregate technology of monitoring be characterized by $p(x) = p_0 - \hat{p}(x)$, where $p_0$ denotes public monitoring such as the frequency of control by the public authority, and $\hat{p}(x)$ corresponds to the private efforts of concealing the illegal activity undertaken by the Agent, where $x$ is his effort or expenditures in wrongdoing.

Assumption 1:
1.1: $\hat{p}'(x) = -p'(x) > 0$, $\hat{p}''(x) = -p''(x) < 0$;
1.2: $\lim_{x \to \infty} \hat{p}(x) = p_0$

Assumption 1.1 says that even if the Agent has the opportunity to run counter to public efforts of detection, the marginal return of effort in cheating become smaller and smaller - there nevertheless exist decreasing returns to scale in this activity. Assumption 1.2 is introduced essentially for technical reasons. It may be argued that as a matter of fact, the Agent is not allowed to increase without any constraint his effort ($x$ may be only define on a subset $[0, x_{\text{max}}]$, with $x_{\text{max}} < \infty$): hence, once the maximum possible value is reached, public authorities may obtain any deterrence effect without the threat of induced effects on the probability of detection. Nevertheless, from a social point of view the issue is how much does it cost to reach the value $x_{\text{max}}$? The final section partly investigates this point. We do not introduce explicitly the social welfare

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\(^1\)Monitoring the Agent’s efforts may be a costly activity for the Principal - nevertheless, assuming it entails only fixed costs, under perfect information these may be seen as negligible (set to zero) and are ignored in the following analysis.

\(^2\)This assumption is made w.l.o.g.; alternatively, we could have had introduced a $C(x)$ satisfying $C'(x) > 0$ and $C''(x) \geq 0$. It can be verified that it adds nothing of specific importance.
objective, but we address the problem of the impact of the various instruments at the disposal of public authority, depending on whether or not public authorities have limited resources in the monitoring of illegal activities.

Let us focus on assumptions about individuals’ preferences. We assume that the Principal is a risk neutral individual. In contrast, the Agent is supposed to be a risk averse one, with a state-dependent representation of his preferences, where $u_1$ denotes his utility index when he is not detected, while $u_0$ denotes his utility index when he is detected. Both utility functions are supposed to be strictly increasing and strictly concave, and unique up to an affine transformation.

Although this is not the central topic of this paper, this representation enables to take into account various effects which are associated to the detection of illegal activities: the Agent is suffering a psychological penalty when he is caught, or more generally public authorities may apply non monetary sanctions in case where the fraudulent acts have been observed, such as imprisonment, full or partial loss of civic rights and so on. Hence all else equal, and to be short, we consider a case where the Agent is better off when he is not detected as compared to the case where he is. In order to represent this situation and be exhaustive about this issue, we require additional assumptions to hold:\footnote{They have been extensively discussed and justified in the literature on self-protection expenditures and/or willingness to pay for safety, health and life: see Dehez and Drèze (1987), Jones-Lee (1974) for example.}

**Assumption 2:** at all levels of Agent’s wealth:

1. $u_1 \geq u_0$,
2. $u_1' \geq u_0'$,
3. $-\frac{u_1''}{u_1'} \geq -\frac{u_0''}{u_0'}$.

While these would appear as very strong conditions at first glance, they have a great and intuitive appeal. The first one (2.1) implies generally that being caught is never beneficial for the Agent: whatever his wealth (accumulated through illegal activities), he is better off when he escapes from public detection. Relaxing such a condition would imply that the Agent has an incentive to give himself up\footnote{Remark that a less stringent condition may be obtained assuming only that $u_1(w_1) \geq u_0(w_0)$ at least whenever $w_1 \geq w_0$, i.e. soon as the Agent’s wealth when not detected is higher than when he is, which is always satisfied in the following analysis.}, which may be seem very strange on a priori grounds (up to pathological behaviors); more over, such an assumption would introduce a bias in the following analysis in favor of Agent’s liability. The second restriction (2.2) means that...
each additional Euro has more value in the state where the Agent is not detected than in the state where he is: the marginal contribution of additional units of wealth to his welfare is larger when he is not detected than when he is. Such a condition is required as far as we consider that the effort corresponds to a normal good$^5$.

Finally, (2.3) allows the Agent to be more sensible to risk when he is not detected than when he is: according to (2.3), $u_1$ displays more risk aversion than $u_2$ at all level of Agent’s wealth. While this last assumption proved to be more meaningful if we were to consider that public authorities may use the non monetary sanctions with different intensities but which cannot be precisely known by the Agent, it is sufficient to obtain that the Willingness to Pay for Safety is also a normal good; on the other hand, it is introduced here for practical motivations which will be clearer later on$^6$.

4 The consequences of exclusive sanctions

4.1 exclusive sanctions upon the Principal

The expected benefit obtained by the Principal when the illegal activity is performed is equal to:

$$
v_P \equiv \max_{w,x} (B - w - p(x)S), \text{ s.t. }$$

$$(1.1) : u_1(w - x) \geq k$$

Assuming that the Principal is a monopolist, we focus solely on the case where the participation constraint of the Agent binds. In a such a situation, the Principal demands an effort which is such that:

$$\frac{1}{\varphi'(x_P)} = S$$

and pays a reward to the Agent equal to:

$$w_P = u_1^{-1}[k] + x_P$$

By analogy with the literature on safety and/or the value of life, let us consider that condition (2) says that for the equilibrium value of the expenditures allotted to cheating and concealing the illegal activity (in

$^5$That is, the effort increases when the Agent becomes richer. While we do not explicitly address the issue here, the reader may refer to Dehez and Drèze (1987, proposition 3.2) for an explicit analysis in the case of individual safety expenditures.

$^6$Just remind that assumption 3 is usual in the literature when comparative statics analysis is performed.
terms of the level of Agent’s effort) then, the Social Marginal Cost of Safety (LHS of (2)) which is nothing but the ratio between the Marginal Cost of Cheating borne by the Agent to the marginal productivity of Cheating for the Principal, is just equal to the Willingness to Pay for Safety of the risk neutral Principal (RHS in (2)). Condition (3) means that the payment to the Agent has to cover the total cost of his expenditures in safety, up to some fixed cost (the constant $u_1^{-1}[k]$) which may be understood as representing the external opportunities of the Agent.

4.2 exclusive sanctions upon the Agent

Consider now that liability is delegated to the Agent, but that his choice corresponding to the level of effort can be perfectly monitored by the Principal. Now, the expected benefit obtained by the Principal when the illegal activity is performed is equal to:

$$v_A = \max_{x,w} (B - w), \ s.t.:$$

$$(4.1): U(x, w) \geq k$$

with $U(x, w) = p(x)u_0(w - x - S) + (1 - p(x))u_1(w - x) = Eu$.

In a such a situation, the Agent affords an effort such that$^7$:

$$\frac{1}{\tilde{p}'(x_A)} = \frac{u_1 - u_0}{Eu'}$$

where $Eu' = p(x_A)u'_0(w - x_A - S) + (1 - p(x_A))u'_1(w - x_A)$, and the Principal pays the associated reward which is:

$$w_A = u_1^{-1}[k] - ec_A$$

where $ec_A < 0$ denotes the certainty equivalent$^8$ of the risky prospect:

$$[(p(x_A), -x_A - S); (1-p(x_A), -x_A)]$$

at $w_A$.

Interestingly enough, condition (5) says now that the equilibrium value of the expenditures of safety (level of effort) chosen by the Agent when he is liable, is such that the Marginal Cost of Safety (LHS of (5)), is just equal to the Willingness to Pay for Safety of the risk averse Agent (RHS in (5)). Condition (6) means that the payment to the Agent has to cover the cost adjusted of the price of the risk associated to his

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$^7$See appendix 1 for SOC.

$^8$See appendix 2.
expenditure in cheating - and still up to some fixed cost (the constant $u_1^{-1}(k)$).

### 4.3 comparative analysis of deterrence effects

One of the main attractive features of the present model is that it allows a complete non ambiguous comparative statics analysis of the optimal contract. To see this, let us first consider the main result of the paper which is the following:

**Proposition 1** All else equal:

i) $v_A \leq v_P$.

ii) $w_A \geq w_P$.

iii) $x_A \geq x_P$.

**Proof.** i) By definition of $v_A$, $v_P$ and $ec_A$, and finally using (6) and (3), we have:

$$v_A = b - w_A$$

$$= b - [u_1^{-1}(k) + x_A + ec_A]$$

$$\leq b - u_1^{-1}(k) - x_A - p(x_A)S$$

$$\leq \max_x \left( b - u_1^{-1}(k) - x - p(x)S \right)$$

$$= v_P$$

Hence the result. ii) is a straightforward consequence of i), starting with $v_A = B - w_A \leq v_P \equiv B - w_P - p(x_P)S$ implies that $w_P + p(x_P)S \leq w_A$; hence the result, given that $p(x_P)S > 0$.

iii) Consider now conditions (2) and (5); if the Agent were risk neutral, his WTP for safety would also be equal to $S$; as a result, it is not so obvious on a priori ground whether $x_A \geq x_P$. However, remark that by concavity of function $u_1$, it comes that:

$$u_1(w - x_A) \geq S u_1'(w - x_A) + u_1(w - x_A - S)$$

Thus substracting each side with $u_0(w - x_A - S)$, then dividing by $Eu'$ and rearranging yields:

$$\frac{u_1(w - x_A) - u_0(w - x_A - S)}{Eu'}$$

$$\geq S \times \frac{u_1'(w - x_A)}{Eu'} + \frac{u_1(w - x_A - S) - u_0(w - x_A - S)}{Eu'}$$

$$\geq S \times \frac{u_1'(w - x_A)}{Eu'}$$

12
given that $u_1(w-x_A-S) \geq u_0(w-x_A-S)$. Once more, by concavity of $u_1$, it is easy to check that for any $p(x) > 0$, then $\frac{u'_1(w-x_A)}{E[u']}$ $\geq 1 \Leftrightarrow u'_1(w-x_A) \geq u'_0(w-x_A-S)$; hence:

$$\frac{u_1(w-x_A) - u_0(w-x_A-S)}{E[u']} \geq S$$

saying that any risk averse (with a state-dependent representation of preferences) decision maker will have a WTP for safety larger than a risk neutral decision maker. To conclude, just remark finally that $\left(\frac{1}{p(x)}\right)$ (LHS in (5)) is a increasing function of $x$: hence: $x_A \geq x_P$. ■

As it is easy to check, the way liability is allotted does not matter in the case where both the Principal and the Agent are risk neutral, since the level of effort and the reward paid to the Agent are equal to $x_P$ and $w_P$ respectively. In contrast, the way sanctions are allotted matters under risk aversion as far as efficiency in liability setting is concerned. Part i) of proposition 1 says that shifting liability from Principal to Agent entails more deterrence effects on sleeping partners in criminal and illegal activities: all else equal, the participation constraint of the Principal binds in the Agent’s liability system before it binds in the Principal’s liability one. But a straightforward consequence of part iii) of proposition 1 is the following:

**Corollary 2** All else equal, the probability of detection is smaller in the Agent’s liability regime than in the Principal’s liability regime.

The driving force behind this last result is the size of the willingness to pay of the team’s member who is liable. In each pure regime of sanction and punishment, the Agent’s expenditures in cheating and concealing the illegal activity basically reflects the willingness to pay of the party in the crime who will bear the sanction in case of detection. In fact, in a state-independent context, a risk neutral individual may have a willingness to pay higher than a risk averse one: this explains the ambiguous findings by Privileggi, Marchese and Cassone (2001). In contrast, in a state-dependent world, a risk neutral individual always has a willingness to pay smaller than a risk averse one. As a result, Agent’s efforts in avoiding detection and concealing the illegal activity are larger when he is liable than when the Principal is. Hence shifting liability from Principal to Agent may be socially worth according to deterrence effects

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9See Langlais (2005) for a general analysis.
on Principals. But on the second hand, it becomes less easy to detect illegal activities, since shifting the burden of the liability on the perpetuator of wrongdoings gives him more incentive to cheat. These countervailing effects are well known since Arlen (1994) and Shavell (1997). This also confirms the intuitions developed by Sanchirico (2005).

5 Comparative statics: probability versus penalty

In a second step, we compare the impact of the instruments which are available to public authorities namely the level of monetary sanctions and the frequency of control. An interesting point which deserves to be highlighted is that when the Principal is legally liable, thus the main instruments at the disposal of authorities to repress crimes and dishonest behaviors, namely $(p_0, S)$ have far different effects depending on the regime of liability.

**Proposition 3** All else equal, in the regime of Principal’s liability:

i) The analysis of the comparative statics gives:

<table>
<thead>
<tr>
<th></th>
<th>$w_P$</th>
<th>$x_P$</th>
<th>$v_P$</th>
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<tbody>
<tr>
<td>$p_0$</td>
<td>independent</td>
<td>independent</td>
<td>$-$</td>
</tr>
<tr>
<td>$S$</td>
<td>$+$</td>
<td>$+$</td>
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ii) Increasing the frequency of controls have more deterrence effects on Principals than raising the level of monetary sanctions.

**Proof.** i) To begin with, the frequency of control of illegal activities - i.e. the choice of $p_0$ by the authority - has no effect neither on the effort undertaken by the Agent, nor on the payment he obtains from the Principal. Further more, it has no effect of the Agent’s utility level, as far as his participation constraint always binds. An increase in $p_0$ simply reduces the expected outcome of the Principal in this case:

$$\frac{\partial v_P}{\partial p_0} = -S < 0$$

In contrast, an increase in the penalty $S$ paid by the Principal when he is detected induces effects, on the one hand, on the activity of the Agent and on the payment he receives, since the Agent increases his effort and receives a higher payment (the Principal gives him more incentives to
invest in effort):
\[
\frac{\partial x_P}{\partial S} = \frac{1}{p'(x_P)} > 0
\]
\[
\frac{\partial w_P}{\partial S} = \frac{\phi'(x_P)}{\phi'(x_P)} > 0
\]
and on the expected utility level of the Principal on the second:
\[
\frac{\partial v_P}{\partial S} = -p(x_P) = -(p_0 - \hat{p}(x_P)) < 0
\]
the Agent being not affected by the increase in \( S \).

ii) Soon as \( S > 1, S > p(x_P) \), hence the result. 

To conclude for the moment about this regime, the monetary sanction proved to be a less efficient instrument to deter crime as compared to the frequency of control, since the (direct) impact of the latter instrument on Principal’s satisfaction level is larger (in absolute value): hence, the reservation utility of Principal may be more easily reached, all else equal, without pervasive effects on the probability of detection - since the increase in \( p_0 \) has a one to one effect on the total probability of criminals’ detection \( p(x) = p_0 - \hat{p}(x) \), without any induced (additional) effects on \( \hat{p}(x) \) which may come from the protective measures undertaken by the Agent. To the contrary, given that any increase in the monetary sanction \( S \) leads the Agent to produce more effort, so that the total probability of detection decreases, this second instrument has effects which are more uncertain: the higher the sanction, the smaller the probability of detection and the decrease in the Principal’s utility.

When the Agent is liable, the comparative statics analysis provides richer results.

**Proposition 4** All else equal, in the regime of Agent’s liability:

i) The analysis of the comparative statics gives:

<table>
<thead>
<tr>
<th></th>
<th>( w_A )</th>
<th>( x_A )</th>
<th>( v_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_0 )</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( S )</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

ii) When \( x_A \) becomes large enough, raising the level of monetary sanctions entails more deterrence effects on Principal than increasing the frequency of controls.
Proof. i) An increase in the probability of control first affects the utility level of the Principal:

\[
\frac{\partial v_A}{\partial p_0} = -\frac{u_1 - u_0}{Eu'} < 0
\]

with additional (induced) effects on the terms of contract between the Principal and the Agent:

\[
\frac{\partial w_A}{\partial p_0} = \frac{u_1 - u_0}{Eu'} > 0
\]

\[
\frac{\partial x_A}{\partial p_0} = \frac{2}{\Omega} \left[ \frac{1}{\bar{p}(x_A)} \left( \frac{u'_1 - u'_0}{Eu''} \right) + \frac{1}{2} \frac{1}{\bar{p}(x_A)} \left( -\frac{Eu''}{Eu'} \right) \right] > 0
\]

where \( \Omega > 0 \) by the SOC (see appendix 1). An increase in the sanctions have the following impact:

\[
\frac{\partial v_A}{\partial S} = -p(x_A) \left( \frac{u'_0}{Eu'} \right) < 0
\]

and the induced effects on the contract are:

\[
\frac{\partial w_A}{\partial S} = p(x_A) \left( \frac{u'_0}{Eu'} \right) > 0
\]

\[
\frac{\partial x_A}{\partial S} = \frac{\left( \frac{u'_0}{Eu'} \right)}{\Omega} \left[ 1 + p(x) \left( \frac{C'(x_A)}{\bar{p}(x_A)} \right) \left( \frac{-Eu''}{Eu'} - \frac{-u''_0}{u'_0} \right) + p(x) \left( \frac{u'_1 - u'_0}{Eu''} \right) \right] > 0
\]

Remark that the numerator in \( \frac{\partial x_A}{\partial S} \) should be of any sign, in the absence of assumption 2.3 since, for any positive probability of being caught:

\[
\left( -\frac{Eu''}{Eu'} \right) \geq \left( -\frac{u''_0}{u'_0} \right) \iff \left( -\frac{u''_0}{u'_0} \right) \geq \left( -\frac{u'_1 - u'_0}{u'_0} \right)
\]

Hence, assumption 2.3 is sufficient to obtain the (intuitive) positive sign.

ii) It is easy to see that:

\[
-\frac{\partial v_A}{\partial p_0} \geq -\frac{\partial v_A}{\partial S} \iff p_0 \geq p(x_A) + \frac{u_1 - u_0}{Eu'}
\]

suggesting that as the Agent’s effort attains large values, \( p(x_A) \rightarrow p_0 \), and thus the monetary sanctions may become more efficient than the probability of detection to deter Principals to undertake the illegal activity since \( \frac{u_1 - u_0}{Eu''} > 0 \).
As compared to the previous case, both instruments have now pervasive effects, since they give more incentives to the Agent in cheating: the higher the intensity of public intervention, the harder the detection of wrongdoing. Moreover, comparing the sensitivity of the Principal to each of the instruments between regimes, we obtain:

**Corollary 5** All else equal:

i) The higher the frequency of controls, the larger the advantages of a Agent’s liability with respect to a Principal’s liability, in terms of Principal’s deterrence.

ii) The larger the level of monetary sanction, the smaller the advantages of a Agent’s liability with respect to a Principal’s liability, in terms of Principal’s deterrence.

**Proof.** i) The proof is direct given that:

\[-\frac{\partial v_A}{\partial p_0} = \frac{u_1 - u_0}{Ew} \geq -\frac{\partial v_P}{\partial p_0} = S : (A)\]

\[-\frac{\partial v_A}{\partial S} = (p_0 - \hat{p}(x_P)) \left( \frac{u'_0}{Ew'} \right) \leq -\frac{\partial v_P}{\partial S} = p_0 - \hat{p}(x_P) : (B)\]

ii) Using condition (B), it is direct that as \( x_P \to \infty \), then \( -\frac{\partial v_A}{\partial S} = 0 = -\frac{\partial v_P}{\partial S} \), and thus \( v_P \to v_A \).  

This last result means that the probability of detection is more efficient in the Agent’s liability system than in the Principal’s one, while to the converse the monetary sanctions have more effects in the Principal’s regime than in the Agent’s one such that for large value of the sanctions both regimes tend to reach the same results on the deterrence of Principal, roughly speaking.

6 Joint liability and sanctions

For practical purposes, a main criticism against the previous analysis is that generally speaking, the penal code imposes that all the members in a criminal teams will be punished in case of arrest. We now analyse in our framework the consequences of joint liability in terms of deterrence.

Assume now that for a given level of public expenditures in deterrence of criminal activities, the probability that the Agent be caught is \( p(x)q \) where \( q \in (0, 1) \), while the probability that the Principal be caught is
\((1 - \theta)p(x)\) where \(\theta \in (0, 1)\). Let us denote \(f_P\) and \(f_A\) the penalty inflicted respectively to the Principal and the Agent in case of arrest. The optimal contract \((\hat{x}, \hat{w})\) is the solution to:

\[
\max_{w, x} \{(b - w - (1 - \theta)p(x)f_P) \text{ s.t.: } U(x, w) \geq k\}
\]

where:

\[
U(x, w) = p(x)qu_0(w - f_A - x) + (1 - p(x)q)u_1(w - x)
\]

corresponding to an effort which satisfies:

\[
\frac{1}{-p'(\hat{x})} = q \times \frac{u_1(\hat{w} - \hat{x}) - u_0(\hat{w} - f_A - \hat{x})}{Ew'} + (1 - \theta) \times f_P \tag{7}
\]

with \(Ew' = p(\hat{x})qu_0'(w - f_A - \hat{x}) + (1 - p(\hat{x})q)u_1'(w - \hat{x})\), and a monetary transfer given by:

\[
\hat{w} = u_1^{-1} [k] + \hat{x} + p(\hat{x})qf_A + \pi_1(\hat{w} - \hat{x} - p(\hat{x})qf_A) \tag{8}
\]

The structure of the RHS in (7), which is defined roughly speaking as a weighted sum of two individual WTP, reflects that each one of the Principal and the Agent have now to bear a specific kind of risk: on the one hand, the risk of being detected; on the second, the risk of being arrested and punished. These risks are reallocated through the contract; as a consequence, the contractual effort is tailored to both the willingness to pay of the Agent (first term in the RHS of (7)) and the willingness to pay of the Principal (second term in the RHS of (7))\(^\text{10}\).

In the next proposition, we compare the regime of joint liability and sanction with the case of exclusive sanction upon the Principal.

**Proposition 6** Assume that \((1 - \theta)f_P \leq f \leq qf_A + (1 - \theta)f_P\); then:

i) \(\hat{x} \geq x_P\).

ii) \(\hat{v} \leq v_P\).

iii) \(\hat{w} \geq w_P\).

\(^{10}\text{Thus, condition (7) mimics the Bowen-Lindahl-Samuelson condition which characterizes the optimal production of public goods; detection avoidance may be seen as a club good, whose optimal level has to be tailored to the willingness to pay of the members of the club up to the specific investments in prosecution and punishment of enforcers.}\)
Proof. i) Let us compare the RHS in (7) and (2), for a given value of \((w, x)\); by assumption of concavity, the first term in the RHS of (7) satisfies: 
\[
\frac{\partial \varepsilon_{i, -w}^a}{\partial w} \geq f_A
\]
(see the proof of part iii) in proposition 1). Hence, under the assumption \(f \leq qf_A + (1 - \theta)fp\), we have:
\[
q \frac{u_1 - u_0}{Eu'} + (1 - \theta)fp \geq qf_A + (1 - \theta)fp \geq f
\]
According to (7) and (2), this implies that \(\frac{1}{-p'(x)} \geq \frac{1}{-p'(x)}\); hence the result follows.

ii) According to (8), we obtain:
\[
\dot{\varepsilon} \equiv b - \dot{w} - (1 - \theta)p(\hat{x})fp
\leq b - u_1^{-1}[k] - \hat{x} - p(\hat{x})\left(qf_A + (1 - \theta)fp\right)
\leq \max_x \left(b - u_1^{-1}[k] - x - p(x)\left(qf_A + (1 - \theta)fp\right)\right)
\]
which is nothing else but the definition of \(v_P\) where \(qf_A + (1 - \theta)fp\) has been substituted for \(f\); but as is easy to verify (applying the envelop theorem), \(v_P\) is a decreasing function of \(f\). Thus, it is straightforward to see that if: \(f \leq qf_A + (1 - \theta)fp\) then we also have:
\[
\dot{v} \leq \max_x \left(b - u_1^{-1}[k] - x - p(x)\left(qf_A + (1 - \theta)fp\right)\right)
\leq v_P = \max_x \left(b - u_1^{-1}[k] - x - p(x)f\right)
\]
Finally, part iii) is a straightforward consequence of part i) and ii): if \((1 - \theta)fp \leq f\), we have according to i):
\[
\dot{w} - w_P \geq p(x_P)f - (1 - \theta)p(\hat{x})fp \geq p(\hat{x})(f - (1 - \theta)fp) \geq 0
\]
given that \(p(\hat{x}) \leq p(x_P)\). ■

Proposition 3 implies that for mild conditions about the choice of fines, the regime of joint liability/mixed sanctions allows to obtain larger effects in terms of crime deterrence than the regime of exclusive sanctions upon the Principal; nevertheless, this also yields more difficulties in detecting the crime, since the Agent have more incentives to invest in activities avoiding public detection.

A direct comparison between (5) and (7) is quite intractable. Thus, in order to compare the regime of joint liability and sanction and the case of exclusive sanction upon the Agent, let us proceed in a different
way. Intuitively speaking, there is no discontinuity between the regime of mixed sanction/liability and the regime of exclusive sanction upon the Agent: it is easily seen that starting from the former and raising continuously the probabilities $\theta$ and $q$ up to their maximal value 1, we reach the second regime. Hence, we first study the comparative statics in the regime of joint sanction; then, we use these results in order identify the sufficient conditions which are required to compare both regimes. In appendix 3, it is shown that:

**Proposition 7** All else equal:

i) $\hat{x}$ is an increasing function of $q$ and a decreasing function of $\theta$; moreover, if $f_A \geq f_P$ then $\hat{x}$ is more sensible to $q$ than to $\theta$, i.e. $-\frac{\partial \hat{x}}{\partial q} > -\frac{\partial \hat{x}}{\partial \theta}$.

ii) $\hat{v}$ is a decreasing function of $q$ and an increasing function of $\theta$; moreover, if $f_A \geq f_P$ then $\hat{v}$ is more sensible to $q$ than to $\theta$, i.e. $-\frac{\partial \hat{v}}{\partial q} > -\frac{\partial \hat{v}}{\partial \theta}$.

The previous propositions implies that the level of avoidance activity which is the solution to (7) when the fine and the probabilities of detection and punishment are set to the level $(f_A, q, \theta)$ is larger than when those variables are equal to $(f_A, 1, 1)$; i.e. $\hat{x}(f_A, q, \theta) \leq \hat{x}(f_A, 1, 1)$ - and in contrast the corresponding expected profit for the Principal is smaller; i.e. $\hat{v}(f_A, q, \theta) \leq \hat{v}(f_A, 1, 1)$. Given that $\hat{x}(f_A, 1, 1)$ is by definition the solution to (5) where $f$ has been substituted by $f_A$, and that $x_A$ is an increasing function of $f$, a straightforward consequence is:

**Corollary 8** Assume that $f_P \leq f_A \leq f$; then:

i) $\hat{x} \leq x_A$.

ii) $\hat{v} \geq v_A$.

The regime of mixed sanction entails (under reasonable conditions regarding the choice of fines) less crime deterrence but more detection than a regime of unilateral sanction upon the Agent.

Remark that in order to compare our different regimes, we have obtain sufficient conditions which may deserve some comments.

First, let us consider the sufficient condition of proposition 3 (which implies the various values of the fines): $(1 - \theta)f_P \leq f \leq qf_A + (1 - \theta)f_P$; remark that the LHS inequality is satisfied (but not uniquely) soon as: $f_P \leq f$; if this inequality holds, then the RHS inequality may be written as: $f - f_P \leq qf_A - \theta f_P$, and it must be finally that $qf_A \geq \theta f_P$ holds.
On the other hand, the sufficient conditions of proposition 4 and corollary 5 together require:

\[ f_P \leq f_A \leq f \]

Once more, these conditions are only sufficient to guarantee the results; nevertheless they suggest that:

- as the enforcer focuses its monitoring efforts on one member of the criminal team (either the leader, or the active agent) rather than on both members, thus raising the corresponding specific probability of detection up to its potential maximal level, then it is recommended to also use larger fines \( f \) than the level which is individually applied in case of joint liability and sanction;

- at the same time, the total expected amount of fines raised by the enforcer in case of joint liability must be at least as large as the level applied in case of exclusive sanction;

- finally, it is suggested that in case of joint liability and sanction, the individual sanction/fine both in level and expected terms, which are applied to the active partner must be at least as large as those raised on the sleeping partner/leader of the team.

With respect to the state-dependent characteristics of Agent’s preferences, which enable to encompass cases where enforcers use non monetary sanctions: our results suggest that they are consistent the usual view on the use of fine and imprisonment; as the probability of convincing the Agent increases so is the level of the fine put on him, although prison sentencing or any other non monetary sanction be used at the same time.

7 Conclusion

The paper has addressed the issue of the choice of a regime of sanction in order to control criminal teams activities. It has followed a basic positive view according to previous works such as Privileggi, Marchese and Cassone (2001). But in contrast, the results are qualified here (almost all the time) in a general and non ambiguous way, allowing us to evaluate how the nature of the activity of detection avoidance together with the existence of a cooperative/non cooperative behavior between partners in a criminal team, facilitates the action of enforcers. We also have introduced (at least implicitly) more policy instruments than in previous studies: in addition to the monitoring activity of enforcers and the use of monetary sanctions in case of arrest, we have assumed that authorities have the opportunity to apply non monetary sanctions (prison sentence, loss of civil rights, incapacitation and so on); this enables us to consider
that (at least some) criminals have state-dependent preferences with different utility indexes depending on whether a criminal is or not arrested and punished.

The first main result of the paper is about the effects of liability shifting between the members of a criminal organization (we compare pure regimes of sanction in this set up) when they have specialized tasks in the team and a different sensibility to the risk of arrest and punishment. Specifically, we assume here that the perpetrator of the crime is a state-dependent risk averse individual, and that he bears a (linear) monetary cost due to the activity of detection avoidance, whereas the leader is risk neutral and bears only fixed costs (due to general management of the team and the monitoring of the members). In this set up, we have shown as in Privileggi, Marchese and Cassone (2001)’s paper that exclusive sanctions upon the Agent/perpetrator of the crime induces more deterrence of the Principal (leader of the team) than a regime of exclusive sanctions upon the Principal: it is more efficient to punish the « weaker » individual alone (more sensible to the risk of sanctions), in the sense that the frequency of crime is smaller. But the counterpart of this result (see also Sanchirico (2006)) is that it becomes more difficult for public authorities to detect illegal activities, in the sense that the frequency of detection and punishment becomes smaller in the former regime. We have shown that both results occur whatever the value of the fine, and the intensity of the public monitoring (likelihood of controls), which is in contrast with Privileggi and ali (2001)’s analysis for state-independent risk averse Agent having a desutility for efforts in the detection avoidance activity.

The second main result of the present paper is about the effects of a regime of «joint liability and sanction»: we have shown that it implies more deterrence than the regime of exclusive sanction upon the leader, but with a cost since « joint liability » also reduces the effectiveness of public monitoring as compared to the second regime, once again. It means that it is more efficient to punish both partners in a criminal teams than the «stronger partner» (leader) alone, since the probability of crime is smaller when both bear a sanction - albeit, the frequency of detection of the team is smaller. In contrast, there is unfortunately no reason to believe that it is also more efficient to punish both partners than the « weaker » alone; maybe this is not the case: more conditions over basic assumptions regarding the technology of avoidance and individual preferences are required (not found here).

In this paper, we have followed a descriptive view on the issue of criminal teams. This point of view has to be motivated, given that there exists a large body of literature focusing rather on the optimal en-
forcement of the penal code (see Garoupa (1997), Polinsky and Shavell (2000). On the one hand, it is well known that introducing risk aversion leads to puzzling results regarding the design of the optimal law enforce-
ment policy (see Polinsky and Shavell (1979), Neilson (1998)): under risk aversion, maximal fines and small probability of control may be or not optimal, depending on whether criminals are more or less sensible to the frequency than to the severity of the sanction (see also (Neilson and Winter (1997))). On the other hand, it must be reminded that there exist other goals of criminal law, notably incapacitation, rehabilitation, and retribution (see Shavell (1987)). Our paper may be understood as suggesting that there exist cases where these different goals (such as the incapacitation of all the gang members) may compet with the deterrence objective.
References


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Shavell S. (1997), The optimal level of corporate liability given the limited ability of corporations to penalize their employees, International Review of Law and Economics, 17, 203-213.


APPENDIX 1

Checking explicitly second order conditions of maximization may be useful there after, let us totally differentiate the FOC (5) written as $-Eu' + p'[u_0 - u_1] = 0$, the SOC require that:

$$Eu'' - Eu' + p''(u_0 - u_1) + 2(-p')' (u_0' - u_1') \leq 0$$

which may also be written after some straightforward but tedious manipulations as $\Omega \times Eu' \geq 0$, where:

$$\Omega = \left(\frac{1}{-p'(x_A)}\right) \left(\frac{p''(x_A)}{-p'(x_A)}\right) + 2 \left(\frac{u'_1 - u'_0}{Eu'}\right) + \frac{1}{2} \left(\frac{1}{-p'(x_A)}\right) \left(-\frac{Eu''}{Eu'}\right) \geq 0$$

where $-p'(x_A) = \dot{p}'(x_A)$ and $p''(x_A) = -\ddot{p}'(x_A)$. Remark that $\Omega$ is used when we turn to the analysis of comparative statics. Remark also that assumption 2.2 is a sufficient condition for SOC to hold.

APPENDIX 2
In our set up, state 0 corresponds to the case where the Agent is
detected and caught by the public authorities, while state 1 is the state
where he is not caught. Consider any gamble defined as: \((p(x), w_0; (1-
p(x)), w_1)\). Thus, the certainty equivalent at wealth \(w\) may be defined as
the amount of wealth which is accepted by the Agent to be not detected
by the authority and to have the same level of satisfaction as the gamble:

\[
p(x)u_0(w_0) + (1 - p(x))u_1(w_1)
= u_1(w + ec)
= u_1(p(x)w_0 + (1 - p(x))w_1 - 
\pi_1(w; p(x)w_0 + (1 - p(x))w_1))
\]

with \(\pi_1(.) > 0\) denoting the Arrow-Pratt absolute risk premium
associated to \(u_1\). Hence, defining \(w_0 = w - x - S\), and \(w_1 = w - x\), we
have: \(ec = -x - p(x)S - \pi_1(w; -x - p(x)S)\)

**APPENDIX 3**

Let us define the function:

\[
\lambda(f_A, q) = q\frac{u_1(w-x) - v_0(w-f_A-x)}{p(x)qu_0(w-f_A-x) + (1-p(x)q)u_1'(w-x)}
\]

which takes positive values on its domain \(\mathbb{R} \times [0, 1]\).

- Differentiating the system (7)-(8) in \(q\) and rearranging yields:

\[
D \frac{\partial x}{\partial q} = \Omega \frac{\partial w}{\partial q} + \frac{\lambda(f_A, q)}{q} \left(1 + p(x)q\frac{u_1' - u_0'}{p(x)qu_0' + (1-p(x)q)u_1'}\right)
\]

\[
(\lambda(f_A, q)(-p') - 1) \frac{\partial x}{\partial q} = -\frac{\partial w}{\partial q} + p(x)\frac{\lambda(f_A, q)}{q}
\]

where:

\[
D = \left(\frac{p''}{p'} + \Omega + + \lambda(f_A, q)(-p')q\frac{u_1' - u_0'}{p(x)qu_0' + (1-p(x)q)u_1'}\right) > 0, \text{ and:}
\]

\[
\Omega = q\frac{u_1' - u_0'}{p(x)qu_0' + (1-p(x)q)u_1'} + \lambda(f_A, q)\left(-\frac{Ew''}{Ew}\right) > 0, \text{ under assumption 2.}
\]

Solving for \(\frac{\partial x}{\partial q}\) leads to:

\[
\frac{\partial x}{\partial q} = \frac{\lambda(f_A, q)}{q} \times \frac{1 + p(x)\left(\Omega + q\frac{u_1' - u_0'}{p(x)qu_0' + (1-p(x)q)u_1'}\right)}{D + \Omega(\lambda(f_A, q)(-p') - 1)}
\]

The denominator is positive according to the SOC; the numerator is
also positive: this implies that \(\frac{\partial x}{\partial q} > 0\).

As a result, we also have the impact on the Principal’s expected
return is: \(\frac{\partial \pi}{\partial q} = -\frac{\lambda(f_A, q)}{q}p(x) < 0\).
• Differentiating the system (7)-(8) in $\theta$ and rearranging yields:

$$D \frac{\partial x}{\partial \theta} = \Omega \frac{\partial w}{\partial \theta} - f_P$$

$$(\lambda(f_A, q)(-p') - 1) \frac{\partial x}{\partial \theta} = - \frac{\partial w}{\partial \theta}$$

Solving for $\frac{\partial x}{\partial \theta}$ leads to:

$$\frac{\partial x}{\partial \theta} = \frac{-f_P}{D + \Omega(\lambda(f_A, q)(-p') - 1)}$$

hence implying that $\frac{\partial x}{\partial \theta} < 0$.

As a result, we also have: $\frac{\partial v}{\partial \theta} = p(x)f_P > 0$.

• Finally, remark that if $f_A \geq f_P$ then $\frac{\lambda(f_A, q)}{q} \geq f_A \geq f_P \implies \frac{\partial x}{\partial q} > -\frac{\partial x}{\partial \theta}$: when both $\theta$ and $q$ increase, then $x$ increases; moreover, if $f_A \geq f_P$ then $\frac{\lambda(f_A, q)}{q} \geq f_A \geq f_P \implies -\frac{\partial v}{\partial q} > \frac{\partial v}{\partial \theta}$: when both $\theta$ and $q$ increase, then $v$ decreases.