General Purpose Technologies and their Implications for International Trade

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for Schumpeterian Growth and Trade

by

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Abstract

General purpose technologies (GPTs) are drastic innovations, such as electrification, the transistor, and the Internet, that are characterized by the pervasiveness in use, innovational complementarities, and technological dynamism. The model develops a two-country (Home and Foreign) dynamic general equilibrium framework and incorporates general purpose technology diffusion within Home that exhibits endogenous Schumpeterian growth. The model studies the effects of the diffusion of the general purpose technology on the pattern of trade and Home’s relative wage. Based on specific assumptions, the adoption of a GPT by a particular industry generates an increase in the productivity of manufacturing workers at Home. By assumption, the diffusion of a GPT across industries is governed by S-curve dynamics, and the diffusion of the GPT within an industry at Home is considered exogenous. The model analyzes the long-run and transitional dynamic effects of a new GPT on trade patterns, product cycles and (transitional) divergence in per-capita growth rates between the two countries.

Key words: General purpose technologies, Schumpeterian growth, comparative advantage, scale effects, R&D races.

JEL Classification: F1, O3, O4, L1, L2

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1. Introduction

In any given economic “era” there are major technological innovations, such as electricity, the transistor, and the Internet, that have far-reaching and prolonged impact. These drastic innovations induce a series of secondary incremental innovations. The introduction of the transistor, for example, triggered a sequence of secondary innovations, such as the development of the integrated circuit and the microprocessor, which are also considered drastic innovations. These main technological innovations are used in a wide range of different sectors inducing further innovations. For example, microprocessors are now used in many everyday products like telephones, cars, personal computers, and so forth.

Even though the distinction between a drastic innovation and the incremental one is quite important to understand the proper roles of technological innovations as engines of growth, economists have paid relatively less attention to the former. Bresnahan and Trajtenberg (1995) christened these types of drastic innovations “General Purpose Technologies” (GPTs henceforth). A GPT is a certain type of drastic innovation which is characterized by the pervasiveness in use (generality of purpose), innovational complementarities, and inherent potential for technical improvement.

Several empirical studies have documented the cross-industry pattern of diffusion for a number of GPTs.¹ In addition, a strand of empirical literature has established that the cross-industry diffusion pattern of GPTs is similar to the diffusion process of product-

¹ For example, Helpman and Trajtenberg (1998b) provide evidence for the diffusion of the transistor. They state that transistors were first adopted by the hearing aids industry. Later, transistors were used in radios followed by the computer industry. These three industries are known as early adapters of the transistor GPT. The fourth sector to adopt the transistor was the automobile industry, followed by the telecommunication sector.
Specific innovations and it is governed by standard S-curve dynamics. In other words, the internal-influence epidemic model can provide an empirically-relevant framework to analyze the dynamic effects of a GPT. During this diffusion process, these drastic innovations could generate growth fluctuations and even business cycles.

Second, the dynamic effects of these GPTs take a long period of time to materialize. For instance, David (1990) argues that it may take several decades before major technological innovations can have significant impact on macroeconomic activity. Third, these GPTs act as “engines of growth”. As a better GPT becomes available, it gets adopted by an increasing number of user industries and fosters complementary advances that raise the industry’s productivity growth. As the use of a GPT spreads throughout the economy, its effects become significant at the aggregate level, thus affecting overall productivity growth. In his presidential address to the American Economic Association, Jorgenson (2001) documents the role of information technology in the resurgence of U.S. growth in the late 1990s. There is plenty of evidence that the rise in structural innovations and it is governed by standard S-curve dynamics. In other words, the internal-influence epidemic model can provide an empirically-relevant framework to analyze the dynamic effects of a GPT. During this diffusion process, these drastic innovations could generate growth fluctuations and even business cycles.

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4 David (1990) describes a phase of twenty-five years in the case of the electric dynamo. He argues that the observed productivity slowdown in the earlier stage of electrification and computerization was due to the adjustment process associated with the adoption of a new GPT.

5 At the aggregate level, information technology is identified with the output of computers, communications equipment, and software. These products appear in the GDP as investments by businesses, households, and governments along with net exports to the rest of the world.
productivity growth in the late 1990s can be traced to the introduction of personal computers and the acceleration in the price reduction of semiconductors, which constituted the necessary building blocks for the information technology revolution.6

The growth effects of GPTs have been analyzed formally by Helpman and Trajtenberg (1998a). They emphasize the lost output that occurs because the GPT does not arrive ready to use but requires the invention of a set of complementary components. During the period when components are being developed, the new GPT will not yet be in use. Meanwhile the labor that is drawn into developing new components will be drawn out of producing final output. The result will be a fall in the overall level of output.7

Petsas (2003) analyzes the dynamic effects of GPTs within a quality-ladders model of scale-invariant Schumpeterian growth. He discusses the transitional dynamics and the long-run equilibrium. Along the transition path, the measure of industries that adopt the new GPT increases, consumption per capita falls, and the interest rate rises.

All these models use a closed economy setup that does not allow them to explain the effects of a drastic innovation technological innovation on international aspects of the economy, such as the pattern of trade, relative wages, and economic growth differences across countries.

6 Another study from OECD documents that U.S. investment in information processing equipment and software increased from 29% in 1987 to 52% in 1999. The diffusion of information and communication equipment accelerated after 1995 as a new wave of information and communication equipment, based on applications such as the World Wide Web and the browser, spread rapidly throughout the economy.

7 Others have pointed out a variety of additional channels though which the cost of adjusting to a new GPT can show up at the macroeconomic level. Greenwood and Yorukoglu (1997) argue that real resources are used up in learning to use the new GPT. Aghion and Howitt (1998b)
Chung (2000) examined the open-economy implications of a GPT using the framework developed by Helpman and Trajtenberg (1998a). He showed that with the introduction of GPTs into the world economy, the developing countries temporarily gain competitiveness in marginal final good industries but end up losing those industries again as a sufficient number of intermediate goods for the new GPT are created in the developed countries.

However, Chung’s model (2000) exhibits the scale effect property: if one incorporates population growth in these models, then the size of the economy (scale) increases exponentially over time, R&D resources grow exponentially, and so does the long-run growth rate of per-capita real output.

The scale effects property is a consequence of the assumption that the growth rate of knowledge is directly proportional to the level of resources devoted to R&D. Jones (1995a) has argued that the scale effects property of earlier endogenous growth models is inconsistent with post-war time series evidence from all major advanced countries that shows an exponential increase in R&D resources and a more-or-less constant rate of per-capita GDP growth. Jones’s criticism has stimulated the development of a new class of models that generate growth without scale effects. However, the theoretical literature on trade and growth without scale effects has focused either on closed economy models or on structurally identical economies engaging in trade with each other. This paper point out that the process of reallocating labor from sectors using older technologies to those using the new GPT may involve a rise in unemployment. See Dinopoulos and Thompson (1999) for more details on this issue.

Dinopoulos and Syropoulos (2001) have recently developed a two-country general equilibrium model of endogenous Schumpeterian (R&D based) growth without scale effects to examine the effect of globalization on economic growth when countries differ in population size and relative factor endowments.
develops a two-country general equilibrium framework without scale effects to determine the effect of the introduction of a GPT on the equilibrium relative wages and the pattern of trade between countries.

My approach borrows from Taylor’s work (1993) in that industries differ in production technologies. In his model, industries also differ in research technologies and in the set of technological opportunities available for each industry. In the presence of heterogeneous research technologies (captured by different productivity in R&D services), the pattern of R&D production and the pattern of goods production within each country can differ. As a result, there is a case for trade between countries in R&D services. The absence of heterogeneity in research technologies in my model makes the removal of scale effects more tractable, but eliminates the need for trade in R&D services between countries.

In the present model, there are two countries that may differ in relative size: Home and Foreign. The population in each country grows at a common positive and exogenously given rate and labor is the only factor of production. There is a continuum of industries producing final consumption goods. Labor in each industry can be allocated between the two economic activities, manufacturing of high-quality goods and R&D services, which are used to discover new products of higher quality. As in Grossman and Helpman’s (1991a) version of the quality-ladders growth model, the quality of each final good can be improved through endogenous innovation.

The arrival of innovations in each industry is governed by a memoryless Poisson process whose intensity depends positively on R&D investments and negatively on the rate of difficulty of conducting R&D. In order to remove the scale effects property, I
consider the permanent effects of growth (PEG) specification that it has been proposed by Dinopoulos and Thompson (1996). According to this specification, R&D becomes more difficult over time and the degree of R&D difficulty is proportional to the size of the world market.

This paper is organized as follows. Section 2 outlines the features of the model. Section 3 describes the steady state equilibrium of the model and section 4 presents the comparative steady state results. Section 5 concludes this paper by summarizing the key findings and suggesting possible extensions.

2. The Model

This section develops a two-country, dynamic, general-equilibrium model with the following features. Each country engages in two activities: the production of final consumption goods and research and development. Each of the two economies is populated by a continuum of industries indexed by $\theta \in [0,1]$. A single primary factor, labor, is used in both goods and R&D production for any industry. In each industry $\theta$ firms are distinguished by the quality $j$ of the products they produce. Higher values of $j$ denote higher quality and $j$ is restricted to taking on integer values. At time $t=0$, the state-of-the-art quality product in each industry is $j=0$, that is, some firm in each industry knows how to produce a $j=0$ quality product and no firm knows how to produce any higher quality product. The firm that knows how to produce the state-of-the-art quality product in each industry is the global leader for that particular industry. At the same time, challengers in both countries engage in R&D to discover the next higher-quality product that would replace the global leader in each industry. If the state-of-the-art quality in an industry is $j$, then the next winner of an R&D race becomes the sole
global producer of a j+1 quality product. Thus, over time, products improve as innovations such as push each industry up its “quality ladder,” as in Grossman and Helpman (1991a).

I assume for simplicity, that all firms in the global economy know how to produce all products that are at least one step below the state-of-the-art quality product in each industry. This assumption, which is standard in most quality-ladders growth models, prevents the incumbent monopolist from engaging in further R&D, which is standard assumption in most quality-ladder models.

For clarity, I adopt the following conventions regarding notation. Henceforth, superscripts “h” and “f” identify functions and variables of “Home” and Foreign” countries, respectively. Functions and variables without superscripts are related to the global economy, while functions and variables with subscripts are related to activities and firms within an industry.

2.1. Introduction and Diffusion of a New GPT

The introduction of a new GPT and its diffusion path is modeled as follows: The world economy has achieved a steady-state equilibrium, manufacturing final consumption goods with an old GPT. I begin the analysis at time \( t = t_0 \), when a new GPT arrives at Home unexpectedly. Firms in each industry start adopting the new GPT at an exogenous rate.

At Home, at each point in time, a fraction of industries, \( \eta \), uses the new GPT and a fraction of industries, \((1-\eta)\), does not use the new GPT. For example, if the old GPT is
the steam power and the new GPT is electricity, $\eta$ industries use electricity in their production and $(1-\eta)$ industries use steam power in their production.\footnote{Devine (1983) provides an excellent historical perspective on electrification where he documents the transformation from shafts to wires. He states: “Until late in the nineteenth century, production machines were connected by a direct mechanical link to the power sources that drove them. In most factories, a single centrally located prime mover, such as a water wheel or steam engine, turned iron or steel “line shafts” via pulleys and leather belts….By the early 1890s then, direct current motors had become common in manufacturing, but were far from universal. Mechanical drive was first electrified in industries such as clothing and textile manufacturing and printing, where cleanliness, steady power and speed, and ease of control were critical”. Helpman and Trajtenberg (1997) explore the adoption of the transistor, an important semiconductor GPT by a number of industries. As they state: The early user sectors were hearing aids and computers. The prominent laggards were telecommunications and automobiles. These examples indicate that the timing of adopting a new GPT differs across industries.}

I use the epidemic model to describe the diffusion of a new GPT across the continuum of industries.\footnote{See Thirtly and Ruttan (1987, pp.77-89) for various applications of the epidemic model to the diffusion of technology.} Its form can be described by the following differential equation,

\[
\frac{d\eta}{\eta} = \delta(1-\eta),
\]

where $\dot{\eta} = \frac{d\eta}{dt}$ denotes the rate of change in the fraction of industries that use the new GPT and $\delta > 0$ is the rate of diffusion. Equation (1) states that the number of new adoptions during the time interval $dt$, $\dot{\eta}$, is equal to the number of remaining potential adopters, $(1-\eta)$, multiplied by the probability of adoption, which is the product of the fraction of industries that have already adopted the new GPT, $\eta$, and the parameter $\delta$, which depends upon factors such as the attractiveness of the innovation and the frequency of adoption, both of which are assumed to be exogenous.
The solution to equation (1) expresses the measure of industries that have adopted the new GPT as a function of time and yields the equation of the sigmoid (S-shaped) logistic curve:

\[
\eta = \frac{1}{[1 + e^{-(\varphi + \delta t)}]}, \tag{2}
\]

where \( \varphi \) is the constant of integration. Notice that for \( t \to \infty \), equation (2) implies that all industries located at Home have adopted the new GPT.\(^{12}\)

2.2. Household Behavior

Let \( N_i(t) \) be country i’s population at time \( t \). I assume that each country’s population is growing at a common constant, exogenously given rate \( g_N = \dot{N}_i(t)/N_i(t) > 0 \). In each country there is a continuum of identical dynastic families that provide labor services in exchange for wages, and save by holding assets of firms engaged in R&D. Each individual member of a household is endowed with one unit of labor, which is inelastically supplied. I normalize the measure of families in each country at time 0 to equal unity. Thus, the population of workers at time \( t \) in country \( i \) is \( N(t)^i = e^{g_N t} \).

Each household in country \( i \) maximizes the discounted utility\(^{13}\)

\[
U = \int_0^\infty e^{-(\rho - g_N) t} \log u(t) dt, \tag{3}
\]

\(^{12}\) When \( t \to -\infty \), then \( \eta = 0 \). If one assumes that the new GPT arrives at Home at time \( t = 0 \), then \( \eta > 0 \). That is, the new GPT arrives at Home by a given fraction of industries \( \eta \) (i.e., the industry or industries that developed this particular GPT).

\(^{13}\) Barro and Sala-i-Martin (1995 Ch.2) provide more details on this formulation of the household’s behavior within the context of the Ramsey model of growth.
where $\rho > 0$ is the constant subjective discount rate. In order for $U$ to be bounded, I assume that the effective discount rate is positive (i.e., $\rho - g_N > 0$). Expression $\log u(t)$ captures the per capita utility at time $t$, which is defined as follows:

$$
\log u(t) = \int_0^1 \log[\sum_j \lambda^j q(j, \theta, t)] d\theta.
$$

(4)

In equation (4), $q(j, \theta, t)$ denotes the quantity consumed of a final product of quality $j$ (i.e., the product that has experienced $j$ quality improvements) in industry $\theta \in [0,1]$ at time $t$. Parameter $\lambda > 1$ measures the size of quality improvements (i.e., the size of innovations).

At each point in time $t$, each household allocates its income to maximize (4) given the prevailing market prices. Solving this optimal control problem yields a unit elastic demand function for the product in each industry with the lowest quality-adjusted price

$$
q^i(j, \theta, t) = \frac{c^i(t) N^i(t)}{p^i(t)},
$$

(5)

where $c^i(t)$ is country $i$’s per capita consumption expenditure, and $p(t)$ is the market price of the good considered. Because goods within each industry adjusted for quality are by assumption identical, only the good with the lowest quality-adjusted price in each industry is consumed. The quantity demanded of all other goods is zero. The global demand for a particular product is given by aggregating equation (5) across the two countries to obtain

$$
q(j, \theta, t) = \sum_{i=h,f} q^i(j, \theta, t).
$$

(6)

Given this static demand behavior, the intertemporal maximization problem of country $i$’s representative household is equivalent to
\[
\max_{c(t)} \int_0^\infty e^{-(\rho-g_N)t} \log c^i(t) \, dt,
\]
subject to the intertemporal budget constraint
\[\dot{a}^i(t) = r^i(t)a(t) + w^i(t) - c^i(t) - g_N a^i,\]
where \(a^i(t)\) denotes the per capita financial assets in country \(i\), \(w^i(t)\) is the wage income of the representative household member in country \(i\), and \(r^i(t)\) is country \(i\)'s instantaneous rate of return at time \(t\). The solution to this maximization problem obeys the well-known differential equation
\[
\frac{\dot{c}^i(t)}{c^i(t)} = r^i(t) - \rho,
\]
Equation (8) implies that a constant per-capita consumption expenditure is optimal when the instantaneous interest rate in each country equals the consumer’s subjective discount rate \(\rho\). I normalize total consumer spending to equal a constant value \(E\) at each point in time, and I choose for simplicity
\[E = 1.\]
This normalization implies that the value of output equals one at each point in time and combined with the differential equation for consumption expenditure per capita implies that the nominal interest rate \(r\) equals the subjective discount rate
\[r(t) = \rho \quad \text{for all } t.\]

2.3. Product Markets

In each country firms can hire labor to produce any final consumption good \(\theta \in [0,1]\). Let \(L^i(\theta, t)\) and \(Q^i_\zeta(\theta, t)\) respectively denote the amounts of labor devoted in manufacturing of final consumption good \(\theta\) in country \(i\) and the output of final consumption good \(\theta\) in country \(i\) produced with the aid of the \(\zeta \in [0,1]\) GPT. Then the
production function of the final consumption good $\theta$ in country $i$ is given by the following equation

$$Q^i_{\varsigma}(\theta, t) = \frac{L^i(\theta, t)}{\alpha_Q^i(\theta) \gamma_{\varsigma}},$$  \hspace{1cm} (11)$$

where $\alpha_Q^i(\theta)$ is the unit labor requirement associated with the final consumption good $\theta$ in country $i$ and $\gamma_{\varsigma}$ captures the productivity gains associated with the new GPT and equals to

$$\gamma_{\varsigma} = \begin{cases} \gamma_1 = \gamma < 1 & \forall \theta \in [0, \eta] \\ \gamma_0 = 1 & \forall \theta \in [\eta, 1] \end{cases}.$$  \hspace{1cm} (12)$$

I assume that each vertically differentiated good must be manufactured in the country in which the most recent product improvement has taken place. That is, I rule out international licensing and multinational corporations.\(^{14}\)

Following Dornbusch et al. (1977), the relative labor unit requirement for each good $\theta$ is given by

$$A(\theta) \equiv \frac{\alpha_Q^i(\theta)}{\alpha_Q^j(\theta)} \quad A'(\theta) < 0$$  \hspace{1cm} (A.1)$$

The relative unit labor requirement function in (A.1) is by assumption continuous, and decreasing in $\theta$.

The assumptions that goods within an industry are identical when adjusted for quality and Bertrand price competition in product markets imply that the monopolist in

\(^{14}\) Taylor (1993) incorporates multinational corporations in a model of endogenous growth and trade. In his model, innovations are always implemented on front line production technologies (i.e., that is technologies that are minimum cost given the prevailing wage rates) and when innovation and implementation occur at different countries, the resulting transactions are considered as imports and exports of R&D.
each industry engages in limit pricing. The assumption that the technology of all inferior
good products is public knowledge imply that the quality leader charges a single price,
which is \( \lambda \) times the lowest manufacturing cost between the two countries:

\[
p = \lambda \min \{ \alpha^h_Q(\theta)w^h, \alpha^f_Q(\theta)w^f \}.
\]

(13)

I assume that if for any industry \( \theta \), its manufacturing unit cost is lower in Foreign than in
Home, \( w^f \alpha^f_Q(\theta) < w^h \alpha^h_Q(\theta) \), then \( w^f \alpha^f_Q(\theta) < w^h \alpha^h_Q(\theta) \gamma \) also holds.

I also assume that the wage of home labor, \( w^h \), is greater than the wage of foreign labor,
\( w^f \). That is, the home relative wage, \( \omega \), is greater than one\(^{15}\)

\[
\omega = \frac{w^h}{w^f} > 1.
\]

(14)

The last two assumptions imply that the price of every top quality good is equal to

\[
p = \lambda \alpha^f_Q(\theta)w^f.
\]

(15)

It follows that the stream of profits of the incumbent monopolist that uses the \( \zeta \in \{0,1\} \)
GPT and produces the state-of-the-art quality product in Home will be equal to

\[
\pi^h_\zeta(\theta, t) = [\lambda w^f \alpha^f_Q(\theta) - w^h \alpha^h_Q(\theta) \gamma \zeta] q = \left(1 - \frac{\omega \alpha^h_Q(\theta) \gamma \zeta}{\lambda \alpha^f_Q(\theta)}\right) N(t),
\]

\]

(16)

while the stream of profits of the incumbent monopolist that produces the state-of-the-art
quality product in Foreign will be equal to

\[
\pi^f(\theta, t) = [\lambda \alpha^f_Q(\theta) - w^f \alpha^f_Q(\theta)] q = \frac{(\lambda - 1)}{\lambda} N(t),
\]

(17)

where \( N(t) = [N^h(t) + N^f(t)] \) is the size of world population and the world expenditure
on final consumption goods.

\(^{15}\) In proposition 1 I provide sufficient conditions under which this assumption holds.
2.4. R&D Races

Labor is the only input engaged in R&D in any industry $\theta \in [0,1]$. Let $L^i_R(\theta, t)$ and $R^i(\theta, t)$ respectively denote the amounts of labor devoted in R&D services in industry $\theta$ in country $i$ and the output of R&D services in industry $\theta$ in country $i$. The production function of R&D services in industry $\theta$ in country $i$ exhibits constant returns and is given by the following equation\(^\text{16}\)

$$R^i(\theta, t) = \frac{L^i_R(\theta, t)}{\alpha_R},$$

where $\alpha_R$ is the unit labor requirement in the production of R&D services. Note that the absence of a superscript and the absence of the industry index $\theta$ in the unit labor requirement imply that they are the same across countries, industries and goods of different quality levels. The absence of heterogeneous research technologies allows me to focus on the implications of assumption (A.1) on the properties of the model.\(^\text{17}\)

In each industry $\theta$ there are global, sequential and stochastic R&D races that result in the discovery of higher-quality final products. A challenger firm $k$ that is located in country $i \in \{h, f\}$ targeting a quality leader in country $i \in \{h, f\}$ engages in R&D in

\(^{16}\) The empirical evidence on returns to scale of R&D expenditure is inconclusive. Segerstrom and Zolnierek (1999) among others developed a model where they allow for diminishing returns to R&D effort at the firm level and industry leaders have R&D cost advantages over follower firms. In their model, when there are diminishing returns to R&D and the government does not intervene both industry leaders and follower firms invest in R&D.

\(^{17}\) Taylor (1993) has introduced heterogeneity in the research technologies and in the technological opportunity for improvements in technologies. The presence of heterogeneous research technologies makes trade in R&D services between countries possible. The absence of heterogeneous research technologies in the present model, makes the removal of scale effects more tractable, but eliminates the possibility of trade in R&D services between the two countries.
industry $\theta$ and discovers the next higher-quality product with instantaneous probability $I_k^I(\theta, t)dt$, where $dt$ is an infinitesimal interval of time and

$$I_k^I(\theta, t) = \frac{R_k^I(\theta, t)}{X(t)}, \quad (19)$$

where $R_k^I(\theta, t)$ denotes firm k’s R&D outlays and $X(t)$ captures the difficulty of R&D in industry $\theta$ at time $t$. I assume that the returns to R&D investments are independently distributed across challengers, countries, industries, and over time. Therefore, the industry-wide probability of innovation can be obtained from equation (19) by summing up the levels of R&D across all challengers in that country. That is,

$$I^I(\theta, t) = \sum_k I_k^I(\theta, t) = \frac{R^I(\theta, t)}{X(t)}, \quad (20)$$

where $R^I(\theta, t)$ denotes total R&D services in industry $\theta$ in country $i$. Variable $I^I(\theta, t)$ is the effective R&D.$^{18}$ The arrival of innovations in each industry follows a memoryless Poisson process with intensity $I(\theta, t) = \sum_i R^I(\theta, t)/X(t)$ which equals the global rate of innovation in a typical industry. The function $X(t)$ has been introduced in the endogenous growth literature after Jone’s (1995a) empirical criticism of R&D based growth models generating scale effects.

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$^{18}$ The variable $I^I(\theta, t)$ is the intensity of the Poisson process that governs the arrivals of innovations in industry $\theta$ in country $i$. Dinopoulos and Syropoulos (2001) model the strategic interactions between a typical incumbent and its challengers as a differential game for Poisson jump processes and derive the equilibrium conditions that govern the solution to a typical R&D contest. They also provide an informal and intuitive derivation of these conditions. In the present model, I follow their informal derivation to derive my results.
A recent body of theoretical literature has developed models of Schumpeterian growth without scale effects.\textsuperscript{19} Two alternative specifications have offered possible solutions to the scale-effects property. The first specification proposed by Dinopoulos and Thompson (1996) removes the scale-effects property by assuming that the level of R&D difficulty is proportional to the market size measured by the level of population,

\[ X(t) = kN(t), \quad (21) \]

where \( k > 0 \) is a parameter.

This specification captures the notion that it is more difficult to introduce new products and replace old ones in a larger market. The model that results form this specification is called the permanent effects of growth (PEG) model because policies such as an R&D subsidy and tariffs can alter the per-capita long-run growth rate.\textsuperscript{20}

Consider now the stock-market valuation of temporary monopoly profits. Consumer savings are channeled to firms engaging in R&D through the stock market. The assumption of a continuum of industries allows consumers to diversify the industry-specific risk completely and earn the market interest rate. At each instant in time, each challenger issues a flow of securities that promise to pay the flow of monopoly profits if the firm wins the R&D race and zero otherwise.\textsuperscript{21}

Consider now the stock-market valuation of the incumbent firm in each industry. Let \( V^i_s(t) \) denote the expected global discounted profits of a successful innovator at time \( t \) in country \( i \), when the global

\textsuperscript{19} See Dinopoulos and Thompson (1999) for an overview of these models.

\textsuperscript{20} Dinopoulos and Thompson (1998) provide micro foundations for this specification in the context of a model with horizontal and vertical product differentiation.

\textsuperscript{21} If the monopolist is located in Home, the monopoly profits are define by equation (16) and if the monopolist is located in Foreign, the monopoly profits are defined by equation (17).
monopolist uses the $\zeta \in \{0,1\}$ GPT and charges a price $p$ for the state-of-the-art quality product. Because each global quality leader is targeted by challengers from both countries who engage in R&D to discover the next higher-quality product, a shareholder faces a capital loss $V_i^1(t)$ if further innovation occurs. The event that the next innovation will arrive occurs with instantaneous probability $I dt$, whereas the event that no innovation will arrive occurs with instantaneous probability $1 - I dt$. Over a time interval $dt$, the shareholder of an incumbent’s stock receives a dividend $\pi(t) dt$ and the value of the incumbent appreciates by $dV_i^1(t) = [\partial V_i^1(t)/\partial t] dt = \dot{V}_i^1(t) dt$. Perfect international capital mobility implies that $r^h = r^f = r$. The absence of profitable arbitrage opportunities requires the expected rate of return on stock issued by a successful innovator to be equal to the riskless rate of return $r$; that is,

$$\frac{\dot{V}_i^1(\theta, t)}{V_i^1(\theta, t)}[1 - I(\theta, t) dt] dt + \frac{\pi_i^1(\theta, t)}{V_i^1(\theta, t)} dt = \frac{[V_i^1(\theta, t) - 0]}{V_i^1(\theta, t)} I(\theta, t) dt = r dt. \quad (22)$$

Taking limits in equation (22) as $dt \to 0$ and rearranging terms appropriately gives the following expression for the value of monopoly profits

$$V_i^1(\theta, t) = \frac{\pi_i^1(\theta, t)}{\rho + I(\theta, t) - \frac{V_i^1(\theta, t)}{V_i^1(\theta, t)}}. \quad (23)$$

A typical challenger $k$ located in country $i$ chooses the level of R&D investment $R_i^k(\theta, t)$ to maximize the expected discounted profits

$$V_i^1(\theta, t) \frac{R_i^k(\theta, t)}{X(t)} dt - w^i R_i^k(\theta, t) dt. \quad (24)$$
where \( I_k^i \text{d}t = [R_k^i(t, t)/X(t)]\text{d}t \) is the instantaneous probability of discovering the next higher-quality product and \( w^i a_R R_k^i(t, t) \) is the R&D cost of challenger \( k \) located in country \( i \).

Free entry into each R&D race drives the expected discounted profits of each challenger down to zero and yields the following zero profit condition:

\[
V_j^i(t) = w^i a_R X(t) .
\]  

(25)

The pattern of R&D production across the two countries can be determined by utilizing equations (23) and (25). Combining these equations and evaluating them at the margin I can obtain the R&D schedule (i.e., the schedule of relative labor productivities in goods) as follows

\[
\omega = \text{RD}(\theta) = \frac{a_f^Q(\theta)}{a_Q(\theta)\gamma},
\]

(26)

where \( \text{RD}(\theta) \) is continuous and decreasing in \( \theta \). For low values of \( \theta \), Home has higher relative labor productivity than Foreign, and thus it earns higher wage. Therefore, Home has comparative advantage in producing and conducting R&D the final goods with lower \( \theta \) and Foreign has comparative advantage in producing and conducting R&D the final goods with higher \( \theta \). The R&D schedule can be depicted in Figure 5.1.

**Lemma 1:** Under assumption (A.1) and for any given value of the relative wage, \( \omega \in \{a_f^Q(1)/a_Q^h(1), a_f^Q(0)/a_Q^h(0)\} \), there exists an industry \( \tilde{\theta} \) defined by equation (26) such that

(a) firms are indifferent between conducting R&D in Foreign or in Home,

(b) for each industry \( \theta \in [0, \tilde{\theta}] \), only Home conducts R&D.
for each industry \( \theta \in [\bar{\theta}, 1] \), only Foreign conducts R&D.

Proof. See Appendix.

One can find the results from Lemma 1 in Dornbusch et al. (1977). However, the derivation of Lemma 1 differs between the present model and the one in Dornbusch et al. (1977). In their model, the results from Lemma 1 come from the assumption of perfect competition in all markets. In the present model, the intuition behind Lemma 1 results from the zero profit conditions regarding R&D. If in industry \( \theta \), R&D is undertaken by Home, then the zero profit conditions for R&D imply that Foreign has negative profits in this particular industry (see equations (23) and (25)). Thus, for all industries that Home undertakes R&D, Foreign has negative profits and does not engage in R&D in these industries. The reverse is true for those industries that Foreign undertakes R&D. Home has negative profits in these industries, so it does not engage in R&D in those industries. Thus, both countries sustain their comparative advantage.

2.5. Labor Markets

Consider first the Home labor market. All workers are employed by firms in either production or R&D activities. Taking into account that each industry leader charges the same price \( p \) and that consumers only buy goods from industry leaders in equilibrium, it follows from (11) that employment of labor in the production of goods using the new GPT in Home is 
\[
\int_0^\bar{\theta} Q^h(\theta, t)a^h_{\partial}d\theta,
\]
while employment of labor in the production of goods using the old GPT is 
\[
\int_{\eta}^\bar{\theta} Q^h(\theta, t)a^o_{\partial}d\theta.
\]
Solving equation (18) for each industry leader’s R&D employment \( L^h_R(\theta, t) \) and then integrating across industries,
total R&D employment by industry leaders in the home country is $\int_0^\bar{\theta} R^h(\theta, t)\alpha_R d\theta$.

Thus, the full employment of labor condition for the home country at time $t$ is given by

$$N^h(t) = \int_0^\eta Q^h(\theta, t)\alpha_Q^h(\theta)\gamma d\theta + \int_\eta^\bar{\theta} Q^h(\theta, t)\alpha_Q^h d\theta + \int_0^\bar{\theta} R^h(\theta, t)\alpha_R d\theta.$$  \hspace{1cm} (27)

The full employment of labor condition for the foreign country at time $t$ is given by

$$N^f(t) = \int_0^1 Q^f(\theta, t)\alpha_Q^f(\theta)d\theta + \int_0^1 R^f(\theta, t)\alpha_R d\theta.$$  \hspace{1cm} (28)

Equations (27) and (28) complete the description of the model.

3. Steady-State Equilibrium

In this section I derive the steady-state equilibrium. Assuming that the relative wage, $\omega$, is constant over time at the steady-state equilibrium, equation (25) implies that $\dot{V}_i^i(t)/\gamma^V_i(t) = \dot{X}(t)/\gamma^X(t) = g_N$. That is, the expected global discounted profits of a successful innovator at time $t$ in country $i$, $V_i^i(t)$, and the level of R&D difficulty, $X(t)$, grow at the constant rate of population growth, $g_N$. Combining equations (23) and (25) after taking into account equations (16) and (17), I obtain the following zero profit conditions for Home and Foreign respectively for each industry:

$$\left(1 - \frac{\omega \alpha_Q^h(\theta) \gamma}{\lambda \alpha_Q^f(\theta)}\right) \frac{\rho + l(\theta, t) - g_N}{\lambda} = w^h \alpha_R k \quad \forall \theta \in [0, \eta]$$  \hspace{1cm} (29)

$$\left(1 - \frac{\omega \alpha_Q^h(\theta)}{\lambda \alpha_Q^f(\theta)}\right) \frac{\rho + l(\theta, t) - g_N}{\lambda} = w^h \alpha_R k \quad \forall \theta \in [\eta, \bar{\theta}]$$  \hspace{1cm} (30)

$$\frac{(\lambda - 1)}{\lambda} \frac{\rho + l(t) - g_N}{\lambda} = w^f \alpha_R k. \quad \forall \theta \in [\bar{\theta}, 1]$$  \hspace{1cm} (31)
Utilizing equations (29)-(31), one can rank the level of global R&D investment between Home and Foreign. Notice that the level of global R&D investment, \( I \), does not depend on \( \theta \) for \( \theta \in [\bar{\theta}, 1] \). On contrast, the level of global R&D investment, \( I \), depends on \( \theta \) for \( \theta \in [0, \bar{\theta}] \). As Home conducts R&D in more industries, the level of global R&D investment increases.

Integrating equation (29) over \([0, \eta]\), equation (30) over \([\eta, \bar{\theta}]\), and equation (31) over \([\bar{\theta}, 1]\) I obtain the following zero profit conditions for Home and Foreign, respectively at the economy-wide level:

\[
\left( \eta - \frac{\omega \gamma}{\lambda} \right) \alpha_Q^h (\theta) \left( \frac{\alpha^b}{\alpha_Q}(\theta) \right) = w^h \omega \alpha^b_{ct} k [\rho + I(\theta, t) - g_N] \eta \tag{32}
\]

\[
\left( \bar{\theta} - \eta - \frac{\omega}{\lambda} \right) \alpha_Q^h (\theta) \left( \frac{\alpha^b}{\alpha_Q}(\theta) \right) = w^h \omega \alpha^b_{ct} k [\rho + I(\theta, t) - g_N] (\bar{\theta} - \eta) \tag{33}
\]

\[
\frac{(\lambda - 1)}{\lambda} = w^f \alpha_{ct} k [\rho + I(\theta, t) - g_N] \tag{34}
\]

Substitution of equations (5) and (13) into the first integral of equation (27) yields the demand for manufacturing labor in Home

\[
\frac{\gamma N(t)}{\lambda w^f} \int_{0}^{\eta} \frac{\alpha_Q^h (\theta)}{\alpha_Q^f (\theta)} d\theta.
\tag{35}
\]

Combining equations (18), (20), and (21), one can obtain the demand for R&D labor in Home

\[
kN(t) \alpha_{ct} \int_{0}^{\bar{\theta}} I^h (\theta, t) d\theta. \tag{36}
\]

Given that there is a large number of independent industries, the law of large numbers implies that the integral in equation (33) can be written as follows:
\[\int_0^\theta I(\theta, t)d\theta = \tilde{\theta}I(t). \quad (37)\]

where \(I(t) = \int_0^1 I(\theta, t)d\theta\) is the average “effective-R&D” of the world economy.

Substituting equations (35) and (37) (after taking into account equation (34)) into Home’s full employment of labor condition (equation 27) yields the resource condition

\[
\overline{N}_h^b(t) = \frac{1}{\lambda w^f} \left( \gamma \int_0^n \frac{a_Q^h(\theta)}{a_Q^f(\theta)}d\theta + \int^n_\theta \frac{a_Q^h(\theta)}{a_Q^f(\theta)}d\theta \right) + k\alpha_R \tilde{\theta}I(t), \quad (38)
\]

where \(\overline{N}_h^b(t) \equiv N_h^b(t)/N(t)\) is Home’s population size relative to the world population size.

Similar substitutions yield the resource condition for the foreign country:

\[
\overline{N}_f^f(t) = \frac{(1 - \tilde{\theta})}{\lambda w^f} + k\alpha_R (1 - \tilde{\theta})I(t), \quad (39)
\]

where \(\overline{N}_f^f(t) \equiv N_f^f(t)/N(t)\) is the foreign country’s population relative to the world population.

The above resource conditions described by equations (38) and (39) hold at each instant in time because, by assumption, factor markets clear instantaneously in both countries.

Equations (32), (33), (34), (38), and (39) represent a system of four equations in four unknowns \(\tilde{\theta}, w_h, w_f,\) and \(I\.\) Manipulating these four equations yields a second schedule in \((\theta, \omega)\) space, the mutual resource schedule\(^{22}\)

\(^{22}\) In Appendix D, I derive the mutual resource schedule and show that it is upward-sloping in \((\theta, \omega)\) space.
The mutual resource schedule states that the relative wage $\omega$, which clears labor markets in both countries, is an increasing function of the range of goods $\tilde{\theta}$ produced in Home. If the range of goods produced by Home increases, Home’s relative demand for labor (both in manufacturing and R&D) increases. The excess demand for labor drives the level of the relative wage higher.

The mutual resource condition (MR) can be depicted in Figure 5.1. The vertical axis measures the home country’s relative wage, $\omega$, and the horizontal axis reflects the measure of industries, $\theta$. The intersection of the downward sloping $RD(\tilde{\theta})$ schedule and the upward sloping $MR(\tilde{\theta})$ schedule at point E determines the steady-state equilibrium relative wage, $\omega$, and the marginal industry $\tilde{\theta}$ in which both countries undertake production in goods and R&D services.\(^{23}\)

Therefore, I arrive at:

**Proposition 1:** For sufficiently large $\frac{N^f}{N^b}$, there exists a unique steady-state equilibrium such that

(a) Home’s relative wage, $\omega$, is greater than one,

(b) Home has a sustained comparative advantage in the range of industries $\theta \in [0, \tilde{\theta}]$. In each industry $\theta \in [0, \tilde{\theta}]$, only Home conducts R&D, produces, and exports the state of the-art product.

\(^{23}\) For sufficiently large $\frac{N^f}{N^b}$, the $MR(\tilde{\theta})$ schedule intersects the $RD(\tilde{\theta})$ schedule at a point above the 45° line, such as $\omega > 1$. 

\[\omega = MR(\tilde{\theta}) = \left( \frac{N^f + k\alpha_R (\rho - g_N)}{(1 - \tilde{\theta}) N^b + \tilde{\theta}k\alpha_R (\rho - g_N)} \right)^{\tilde{\theta}} \]  

(40)
(c) Foreign has a sustained comparative advantage in the range of industries \( \theta \in [\bar{\theta}, 1] \). In each industry \( \theta \in [\bar{\theta}, 1] \), only Foreign conducts R&D, produces, and exports the state-of-the-art product.

(d) Home uses the new GPT in the range of industries \( \theta \in [0, \eta] \) and uses the old GPT in the range of industries \( \theta \in [\eta, \bar{\theta}] \).

Proof. See Appendix.

The results from this proposition can be found in other models. The static continuum Ricardian model developed by Dornbusch et al. (1977) and the dynamic learning-by-doing model introduced by Krugman (1987) produce similar features with the equilibrium depicted in Figure 1.

Figure 1 illustrates the steady-state equilibrium in the presence of the new GPT. The pattern of trade in goods is determined by comparative advantage across industries. In addition and in contrast to earlier work, the model predicts that the pattern of trade is determined by additional factors such as population growth and the R&D difficulty parameter. Moreover, the absence of heterogeneity in research technologies results in no trade in R&D services. Taylor (1993), has introduced heterogeneity in research technologies and result in an equilibrium with trade in R&D services. Finally, factor price equalization is not a property of the equilibrium depicted in Figure 1.

Proposition 2: If \( \eta \) is governed by S-curve dynamics, there are two steady-state equilibria: the initial steady-state equilibrium arises before the adoption of the new GPT, where \( \eta = 0 \), and the final steady-state equilibrium is reached after the diffusion process of the new GPT has been completed, where \( \eta = 1 \). At the final steady-state equilibrium:
*Home produces, conducts R&D, and exports more goods, \( \tilde{\theta}(1) > \tilde{\theta}(0) \), Home’s relative wage is higher, \( \omega(1) > \omega(0) \)*

**Proof.** See Appendix

These comparative steady-state properties can be illustrated with the help of Figure 2. Before the introduction of the new GPT in Home, the world economy is in a steady state (point A) where \( \eta = 0 \), with Home exporting the range of goods \( \tilde{\theta}(0) \), and with its relative wage given by \( \omega(0) \). The new GPT arrives in the world economy at time \( t = 0 \) with a given measure of industries \( \eta > 0 \). Thus, at time \( t = 0 \), a portion of the RD schedule jumps upward for those industries that are using the new GPT, since these industries are now more productive due to new GPT. An increase in the measure of industries that adopt the new GPT makes the RD schedule in Figure 2 shift upward from RD (where \( \eta = 0 \)) to RD’ (where \( \eta = 1 \)) resulting in higher relative wage and in higher comparative advantage for Home. In other words, when all industries at Home have adopted the new GPT, final goods producers in Home gain competitiveness. The new steady state is at point B, where \( \eta = 1 \), with Home exporting the range of goods \( \tilde{\theta}(1) \), and with its relative wage given by \( \omega(1) \).
Figure 1: Steady-State Equilibrium Before the Introduction of the new GPT.

Figure 2: Steady-State Equilibria
Point A: No industry has adopted the new GPT
Point B: All industries have adopted the new GPT.
4. Transitional Dynamics

In this section I analyze the transitional dynamics of the model. The equation that governs the GPT diffusion (1) together with equations (20) and (22) (which hold at each instant of time) enable me to construct a system of two differential equations that govern the evolution of $\tilde{\theta}$ and $\eta$. By proper substitutions, I obtain:

\[
\frac{\left(1 - \frac{\omega a_h^b(\theta)\gamma}{\lambda a_Q^r(\theta)}\right)}{\rho + I(\theta, t) - (g_{w^h} + g_N)} = w^h a_{r_k} \forall \theta \in [0, \eta] \tag{41}
\]

\[
\frac{\left(1 - \frac{\omega a_h^b(\theta)}{\lambda a_Q^r(\theta)}\right)}{\rho + I(\theta, t) - (g_{w^h} + g_N)} = w^h a_{r_k} \forall \theta \in [\eta, \tilde{\theta}] \tag{42}
\]

\[
\frac{(\lambda - 1)}{\lambda} \frac{1}{\rho + I(\theta, t) - (g_{w^r} + g_N)} = w^r a_{r_k} \forall \theta \in [\tilde{\theta}, 1] \tag{43}
\]

Integrating equations (41)-(43), I obtain:

\[
\left(\eta - \frac{\omega \gamma}{\lambda} \int_0^\eta \frac{a_h^b(\theta)}{a_Q^r(\theta)} d\theta\right) = w^h \omega a_{r_k} [\rho + I(\theta, t) - g_{w^h} + g_N] \eta \tag{44}
\]

\[
\left(\tilde{\theta} - \eta - \frac{\omega}{\lambda} \int_{\eta}^{\tilde{\theta}} \frac{a_h^b(\theta)}{a_Q^r(\theta)} d\theta\right) = w^h \omega a_{r_k} [\rho + I(\theta, t) - g_{w^h} - g_N] (\tilde{\theta} - \eta) \tag{45}
\]

\[
\frac{(\lambda - 1)}{\lambda} = w^r a_{r_k} [\rho + I(\theta, t) - g_{w^r} - g_N] , \tag{46}
\]

Manipulating equations (44) and (45), yields:

\[
\int_{\eta}^{\tilde{\theta}} \frac{a_h^b(\theta)}{a_Q^r(\theta)} d\theta = \frac{(\tilde{\theta} - \eta)\gamma}{\eta} \int_{0}^{\eta} \frac{a_h^b(\theta)}{a_Q^r(\theta)} d\theta \tag{47}
\]

Taking logs and differentiate in equation (47) yields:
\[
\hat{\theta} = \frac{\left(\eta a^b_n(\eta)(\tilde{\theta} - \eta)(A_1(\eta) + A_2(\tilde{\theta}, \eta)) - A_1(\eta)A_2(\tilde{\theta}, \eta)\alpha^f_n(\eta)\tilde{\theta}\right)\alpha^f_n(\tilde{\theta})\delta(1 - \eta)}{\tilde{\theta} A_1(\eta)\alpha^f_n(\eta)\left[\alpha^b_n(\tilde{\theta})(\tilde{\theta} - \eta) - \alpha^f_n(\tilde{\theta}) A_2(\tilde{\theta}, \eta)\right]} \tag{48}
\]

where

\[
A_1(\eta) = \int_0^\eta \frac{a^b_n(y)}{\alpha^f_n(y)} \, dy \quad \text{and} \quad A_2(\tilde{\theta}, \eta) = \int_\eta^{\tilde{\theta}} \frac{a^b_n(y)}{\alpha^f_n(y)} \, dy
\]

Equations (48) and (1) determine the evolution of the two endogenous variables of the model, the range of goods Home exports, \(\tilde{\theta}\), and the number of industries at Home that have adopted the new GPT, \(\eta\).

Equation (48) is equal to zero, when \(\eta = 1\), or when the following condition is satisfied:

\[
\left(\eta a^b_n(\eta)(\tilde{\theta} - \eta)(A_1(\eta) + A_2(\tilde{\theta}, \eta)) - A_1(\eta)A_2(\tilde{\theta}, \eta)\alpha^f_n(\eta)\tilde{\theta}\right) = 0 \tag{49}
\]

By totally differentiating equation (49), after rearranging I obtain \(d\tilde{\theta}/d\eta < 0\).

Thus, \(\hat{\theta} = 0\) defines a downward-sloping curve in Figure 5.3. Starting from any point on this curve, an increase in \(\eta\) leads to \(\hat{\theta} > 0\) and a decrease in \(\eta\) leads to \(\hat{\theta} < 0\).

The right-hand side of equation (1) is independent of \(\tilde{\theta}\), and therefore the \(\dot{\eta} = 0\) locus is a vertical line. Starting from any point on this line, decrease in \(\eta\) leads to \(\dot{\eta} > 0\). The area to the left of the vertical line (i.e., locus \(\dot{\eta} = 0\)) identifies a region in which the potential number of adopters is greater than one. Therefore, this region is not feasible. There exists a downward-sloping saddle path going through the balanced-growth equilibrium point B. Thus, I arrive at:

**Proposition 3.** Assume that \(\partial \hat{\theta}/\partial \tilde{\theta} < -\delta\). Then, there exists a negative-sloping globally stable-saddle-path going through the final balanced-growth equilibrium point B.
Along the saddle path, the measure of industries that adopt the new GPT, \( \eta \), increases, the relative wage, \( \omega \), increases, and the market interest rate, \( r \), is equal to the subjective discount rate, \( \rho \).

Proof. See Appendix.

The analysis is predicated on the assumption of perfect foresight.\(^{24}\) When the new GPT arrives, the range of goods Home exports, \( \tilde{\theta} \), jumps upward instantaneously to \( \tilde{\theta}' \) (point A’ in Figure 3). This jump increases Home’s relative wage since there is more intense competition for workers at Home in order to produce more goods. The higher values of \( \tilde{\theta} \) and \( \omega \) lower the discounted expected monopoly profits at Home (see assumption (A.1) and equations (41)-(43)) and raise the discounted expected monopoly profits at Foreign. As a result, Foreign starts engaging in more R&D and gains back its competitiveness in both R&D and production. During the transition dynamics (i.e., as the equilibrium moves from point A’ to point B in Figure 3), the range of goods Home exports, \( \tilde{\theta} \), decreases, and the relative wage decreases to adjust the equilibrium in Home and Foreign labor markets. At point B in Figure 2, all industries at Home have adopted the new GPT.

\(^{24}\) There also exists a degenerate equilibrium where the adoption of the new GPT is not completed. Suppose that when a new GPT arrives, every potential producer expects that no one will produce more goods. As a result, it does not pay to increase production, because the new GPT will never be fully adopted. In this event, the pessimistic expectations are self-fulfilling, and no new GPTs are fully adopted. I do not discuss these types of equilibria in what follows.
Figure 3: Stability of the Balanced-Growth Equilibrium
5. Conclusions

The previous literature on “quality ladders” framework that analyzed Ricardian models of trade exhibits the scale effects property. In this paper, I have developed a model of trade based on “quality-ladders” growth without scale effects to analyze how GPTs affect the pattern of trade and the relative wage are determined in steady-state equilibrium. The absence of scale effects generates novel and interesting results.

Given the relatively simplicity of the model, this dynamic formulation provides a useful framework to examine other issues. For example, the introduction of trade instruments and their effect on the pattern of trade between countries can be examined under the two alternative models. Alternatively, a North-South model of trade might yield interesting implications.

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