Effects of Patent Policy on Income and Consumption Inequality in an RD-Growth Model

Chu, Angus C.

Institute of Economics, Academia Sinica

February 2009

Online at https://mpra.ub.uni-muenchen.de/14472/
MPRA Paper No. 14472, posted 04 Apr 2009 18:11 UTC
Effects of Patent Policy on Income and Consumption Inequality in an R&D-Growth Model

Angus C. Chu*

Institute of Economics, Academia Sinica

February 2009

Abstract

To analyze the effects of patent policy on growth and inequality, this paper develops a quality-ladder model with wealth heterogeneity and elastic labor supply. The model predicts that strengthening patent protection increases (a) growth by stimulating R&D and (b) income inequality by raising the return on assets. Elastic labor supply creates an additional effect on income inequality. As for consumption inequality, the effect is ambiguous and depends on the elasticity of intertemporal substitution. Calibrating the model to US data shows that strengthening patent protection increases income inequality by more than consumption inequality, and this pattern is consistent with the data.

Keywords: endogenous growth, heterogeneity, income inequality, patent policy

JEL classification: D31, O34, O41

* Institute of Economics, Academia Sinica, Taipei, Taiwan. E-mail: anguscc@econ.sinica.edu.tw. I would like to thank Been-Lon Chen, Dmitriy Stolyarov, Cheng-Chen Yang and the referees for helpful comments and suggestions. The usual disclaimer applies.
1. Introduction

Since the seminal work of Simon Kuznets (1955), the tradeoff between growth and inequality has been an important issue in economics. Given that economic growth is driven by technological progress which in turn is influenced by innovation policies, this paper analyzes the effects of patent policy on economic growth and income inequality within an R&D-based endogenous-growth model. In the model, the effect of patent policy on income inequality is driven by the rate of return on assets. Therefore, even if patents do not represent a significant fraction of assets in reality, the effect of patent policy on income inequality can still be significant in the presence of other capital incomes that depend on the real interest rate. Although the prevailing wisdom is that the rising income inequality in the US is largely driven by an increase in the relative wage between skilled and unskilled workers, some studies, such as Atkinson (2000, 2003), suggest that inequality in capital income is also playing an increasingly important role. For example, Reed and Cancian (2001) show that capital income contributes to one quarter of the increase in income inequality in the 90’s while it accounts for less than one-tenth of the increase in the 70’s. The current study relates to this literature by providing a model that highlights the effects of capital income on the rising inequality in the US.

The growth-theoretical framework is a canonical quality-ladder model with the additions of heterogeneity in households’ wealth, variable patent breadth and elastic labor supply. The model predicts that strengthening patent protection increases (a) economic growth by stimulating R&D investment and (b) income inequality by raising the return on assets. However, whether it also increases consumption inequality depends on the elasticity of intertemporal substitution. If this elasticity is less (greater) than unity, strengthening patent protection would increase (decrease) consumption inequality. Calibrating the model to aggregate data of the US economy shows that strengthening patent protection leads to a larger increase in income inequality than consumption inequality. This divergence between income and consumption inequality is consistent with the empirical pattern in the US.

\footnote{According to Nakamura (2003), the market value of intangible assets in the US is at least $5 trillion in 2000 (i.e. about 50% of US GDP). Although intangible assets include patents and copyrights that are innovation-related, they also include trademarks and goodwill that may be unrelated to innovation.}
Krueger and Perri (2006) and Blundell et al. (2008) provide empirical evidence to show that the sharp increase in income inequality in the US since the 80’s was accompanied by a much smaller increase in consumption inequality. For example, based on the Consumer Expenditure Survey, Krueger and Perri (2006) find that the variance of log of income (consumption) increases by over 20% (about 5%) from 1980 to 2004. During the same period, R&D investment as a share of GDP has increased (see Figures 1) while patent protection in the U.S. has strengthened. Table 1 presents an index for the strength of patent protection in the US from Park (2008).

|-------|------|------|------|------|------|------|------|------|------|------|

Given this empirical pattern, I calibrate the R&D-growth model to see whether it can replicate a similar divergence in income and consumption inequality as in the data. The model predicts that the coefficient of variation of income over the coefficient of variation of consumption increases from 1.55 in 1980 to 1.69 in 2004. This finding suggests that patent policy may provide a partial explanation for the recent trend of income and consumption inequality in the US.

The intuition of the results is as follows. Strengthening patent protection increases R&D as well as the equilibrium growth rate that drives up the rate of return on assets. This higher return on assets increases the income of asset-wealthy households relative to asset-poor households. Furthermore, the allowance of elastic labor supply creates an additional effect on income inequality through labor income, and this effect will be discussed in details in the main text. As for the ambiguous effect on consumption inequality, the higher growth rate also increases the fraction of assets for saving. Therefore, whether the relative consumption between asset-wealthy households and asset-poor households increases or decreases

---

2 See Jaffe (2000), Gallini (2002) and Jaffe and Lerner (2004) for a discussion on the changes in patent policy. According to the Patent and Trademark Office, the number of patent applications and patents granted in the US was quite stable between 1963 and 1979. Then, the number of patent applications increases from 112,379 in 1980 to 484,955 in 2007 while the number of patents granted increases from 66,170 in 1980 to 182,901 in 2007.

3 The index is on a scale of 0 to 5, and a larger number indicates stronger patent protection. See Ginarte and Park (1997) and Park (2008) for details.
depends on the relative increase in the equilibrium growth rate and the real interest rate, which in turn is determined by the elasticity of intertemporal substitution.

**Literature Review**

Since the seminal Kuznets curve that hypothesizes an inverted U-shape effect of economic development on income inequality, economists have become interested in the empirical relationship between economic growth and income inequality. Early empirical studies, such as Alesina and Rodrik (1994), Persson and Tabellini (1994) and Perotti (1996), find a negative relationship while the more recent studies, such as Li and Zou (1998) and Forbes (2000), find a positive relationship. Forbes (2000) argues that the different results are due to omitted-variable bias and measurement error in previous studies and suggests the use of panel estimation and improved data on inequality to overcome these problems. Barro (2000) considers a larger sample of countries than Forbes (2000) and finds a positive (negative) relationship between growth and inequality in developed (developing) countries. Assuming that economic growth in developed countries is driven by innovation, the theoretical result that stronger patent protection increases income inequality is consistent with a positive relationship between growth and inequality in developed countries.

Early theoretical studies, such as Galor and Zeira (1993), Alesina and Rodrik (1994) and Persson and Tabellini (1994), tend to derive a negative relationship between growth and inequality. Garcia-Penalosa and Turnovsky (2006) argue that the theoretical relationship between growth and inequality should be ambiguous and depends on the underlying structural and policy changes. To explore this theoretical relationship, they incorporate heterogeneity in households’ wealth into a canonical AK growth model with capital externality and elastic labor supply. The comparative static results show that a positive growth-inequality relationship is more likely to emerge. They also derive a law of motion for the distribution of assets and show that the distribution is stationary in the model.\(^4\) The current study adopts a similar approach to show that the distribution of assets is also stationary in a canonical quality-ladder

\(^4\) See, also, Bertola (1993) for an early study on income distribution in the AK model and Caselli and Ventura (2000) for an interesting study that considers multiple dimensions of heterogeneity.
growth model. An interesting difference between the two models is that the AK model relies on elastic labor supply to generate an endogenous income distribution while the quality-ladder model does not. Nonetheless, the consideration of elastic labor supply is still interesting because it creates an additional channel through which growth affects income inequality through labor income.

Although the capital-accumulation-driven growth models are useful frameworks for analyzing many macroeconomic issues, they are not suitable for evaluating innovation policies. Therefore, the current study incorporates wealth heterogeneity and elastic labor supply into an R&D-driven endogenous-growth model to analyze the effects of patent policy on growth and inequality. Chou and Talmain (1996), Li (1998), Zweimuller (2000) and Foellmi and Zweimuller (2006) also consider wealth heterogeneity in R&D-growth models. However, they focus on the effects of wealth inequality on growth. The current paper differs from these studies by considering the effects of patent policy on income and consumption inequality given wealth inequality that is independent of growth in the model.

Bertola et al. (chapter 10, 2006) also consider an R&D-growth model in which wealth inequality is independent of growth due to homothetic preferences. Bertola et al. analyze the effect of firms’ market power determined by the elasticity of substitution between products on the distribution of income between workers and entrepreneurs, and they derive an inverse relationship between the labor share of income and the market power of firms. The current study explores a different issue. It firstly derives a closed-form expression for the coefficient of variation of income/consumption and then shows that stronger patent protection has a positive effect on income inequality but an ambiguous effect on consumption inequality. The current study also differs from Bertola et al. in the following ways. Firstly, they consider a variety-expanding model with inelastic labor supply while the current study considers a quality-ladder model with elastic labor supply that creates an additional effect on income inequality. Secondly, the current study analyzes consumption inequality in addition to income inequality and shows that these two measures of inequality could in theory go in opposite directions. Finally, the current study also provides a quantitative analysis to illustrate the effects of patent breadth on income and consumption inequality.
This paper also relates to the issue on the underinvestment in R&D. There is an important empirical literature that finds the social return to R&D to be much higher than the private return.\(^5\) Jones and Williams (1998, 2000) develop an R&D-growth model and use these empirical estimates to quantify that the socially optimal level of R&D is at least two to four times higher than the market level. Given this underinvestment in R&D, patent policy is a relevant instrument that can be used to correct for this market failure and increase growth. In the R&D-growth literature, Li (2001) and O’Donoghue and Zweimuller (2004) analyze the growth effects of patent breadth in a quality-ladder model that has a representative household. Given that patent policy may affect inequality, the current paper contributes to this literature by providing a theoretical framework that highlights the distributional consequences of patent policy. As for empirical studies, Goel and Ram (1994) and Goel et al. (2008) examine the effect of R&D on growth. Using cross-country data, Goel and Ram (1994) find a positive effect that is also significant for a subset of empirical specifications. Using time-series data in the US, Goel et al. (2008) also find a positive relationship between non-federal R&D and growth although the effect is even stronger for federal R&D and especially defense R&D.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 defines the equilibrium. Section 4 analyzes the effects of patent breadth and calibrates the model. The final section concludes with policy implications.

2. **A Quality-Ladder Model with Heterogeneous Households**

This section develops a quality-ladder model similar to Aghion and Howitt (1992) and Grossman and Helpman (1991) by adding mainly three features (a) heterogeneity in households’ wealth, (b) variable patent breadth as in Li (2001), and (c) elastic labor supply. Given that quality-ladder models have been well-studied, the model’s familiar components will be briefly described below while the new features will be described in more details.

\(^5\) See, for example, Griliches (1992) for a review of this literature.
2.1. Households

There is a unit continuum of identical households (except for the initial distribution of wealth) indexed by

$h \in [0,1]$. Each household $h$ has a standard iso-elastic utility function given by

$$U(h) = \int_0^\infty e^{-\rho t} \left[ \left( \frac{C_t(h)^{1-\gamma}}{1-\gamma} \right) - 1 \right] dt .$$

$\gamma \in (0,\infty)$ is the inverse of the intertemporal substitution elasticity $\varepsilon \equiv 1/\gamma$. $\gamma = \varepsilon = 1$ corresponds to the case of log utility. $C_t(h)$ is the consumption of final goods. Each household is endowed with one unit of time to allocate between leisure $l_t(h)$ and work $L_t(h)$. $\phi \geq 0$ is a preference parameter on leisure, and setting $\phi$ to zero corresponds to the case of inelastic labor supply. $\rho$ is the exogenous discount rate. To ensure that lifetime utility is bounded, the following parameter restriction is imposed on the discount rate.

**Condition D (Discount Rate):**

$$\rho > (1-\gamma)g ,$$

where $g$ denotes the balanced-growth rate of consumption.

Each household maximizes utility subject to a sequence of budget constraints given by

$$\dot{V}_t(h) = R_t V_t(h) + W_t L_t(h) - P_t C_t(h) .$$

$V_t(h)$ is the nominal value of assets owned by household $h$ at time $t$. The share of assets owned by household $h$ at time 0 is exogenously given by $s_{x,0}(h) \equiv V_0(h)/V_0$ that has a general distribution function with a mean of one and a standard deviation of $\sigma_v$. $R_t$ is the nominal rate of return on assets. Household $h$ endogenously supplies $L_t(h)$ to earn the nominal wage rate $W_t$. $P_t$ is the price of final goods. From the household’s intratemporal optimization, household $h$’s labor supply is determined by

$$1 - L_t(h) = l_t(h) = \phi P_t C_t(h) ,$$

where $W_t$ is normalized to one. From the household’s intertemporal optimization, the familiar Euler equation is given by
Lemma 1 shows that the consumption growth rate is the same across households. To ensure that the Euler equation has the usual properties, the following parameter restriction is imposed.

**Condition E (Euler Equation):** \( \gamma - \phi(1 - \gamma) > 0 \).

**Lemma 1:** Aggregate consumption and the consumption of household \( h \) evolve according to

\[
\frac{\dot{C}_t(h)}{C_t(h)} = \frac{1}{\gamma} \left( R_t - \frac{P_t}{P_t} - \rho \right) + \phi \left( \frac{1 - \gamma}{\gamma} \right) l_t(h).
\]

for all \( h \). Also, aggregate labor supply is determined by \( L_t = 1 - \phi P_t C_t \).

**Proof:** Differentiate (3) with time and substitute it into (4). As for \( L_t \), integrate (3) with \( h \).

Final goods are produced by a standard Cobb-Douglas aggregator over a continuum of differentiated intermediates goods \( i \in [0, 1] \) given by

\[
C_t = \exp \left( \int_0^1 \ln X_t(i) \, di \right).
\]

I define a stationary variable \( E_t \equiv P_t C_t \) that denotes the aggregate nominal expenditure, which will be used to analyze the stability of the balanced-growth path.

**2.2. Intermediate Goods**

There is a continuum of industries indexed by \( i \in [0, 1] \) producing the differentiated intermediate goods.

In each industry \( i \), there is a monopolistic leader, who holds a patent on the latest invention and dominates the market until the next invention occurs. The production function for the leader in industry \( i \) is

\[
X_t(i) = z^{\nu, (i)} L_{t, i}(i).
\]
$L_{s,t}(i)$ is the number of workers in industry $i$. $z > 1$ is the exogenous productivity improvement from each invention, and $n_t(i)$ is the number of inventions that have occurred as of time $t$. Given $z^{n_t(i)}$, 

(8) \[ MC_t(i) = W_t / z^{n_t(i)} = 1 / z^{n_t(i)} \]

is the nominal marginal cost of production for the leader in industry $i$.

As commonly assumed in the literature, the current and former industry leaders engage in Bertrand competition, and the profit-maximizing price for the current leader is a constant markup over the marginal cost given by

(9) \[ P_t(i) = \mu(z,b)MC_t(i), \]

where $\mu(z,b) = z^b$ for $b \in (0,1]$ that captures the level of patent breadth. In Aghion and Howitt (1992) and Grossman and Helpman (1991), there is complete patent protection against imitation such that $b = 1$. Li (2001) generalizes the policy environment to capture incomplete patent protection against imitation such that $b \in (0,1)$. Because of incomplete patent protection, the current leader’s invention enables the former leader to increase her productivity by a factor of $z^{1-b}$ without infringing the current leader’s patent. Therefore, the limit-pricing markup for the current leader is given by $z^b$. An increase in the level of patent breadth $b$ enables the current leader to charge a higher markup $\mu$, and the resulting increase in the amount of monopolistic profit improves the incentives for R&D and stimulates growth.

---

6 The degree of patent breadth $b$ can also be interpreted as the fraction of an invention $z - 1$ that is protected by its patent. To see this, $b(z - 1) = b \ln z \approx z^b - 1$ that is the markup. In reality, when an inventor applies for a patent to protect her invention, she makes a number of claims about this invention to be reviewed by a patent examiner. If some of these claims are too specific, then imitators may be able to imitate around them to avoid patent infringement.

7 O’Donoghue and Zweimuller (2004) refer to this form of patent protection as lagging breadth, and they formalize another form of patent protection known as leading breadth (i.e. patent protection against subsequent innovations). For the purpose of the current study, the consideration of lagging patent breadth is sufficient.
2.3. R&D and Aggregate Technology

Denote the nominal value of an invention in industry \( i \) as \( \tilde{V}_i(i) \). Due to the Cobb-Douglas specification in (6), the amount of monopolistic profit is the same across industries (i.e. \( \pi_i(i) = \pi_i \) for \( i \in [0,1] \)). As a result, \( \tilde{V}_i(i) = \tilde{V}_i \) for \( i \in [0,1] \). Because patents are the only assets in the economy, their market value equals the value of assets owned by households (i.e. \( \tilde{V}_i = V_i \)). The familiar no-arbitrage condition is

\[
RV_i = \pi_i + \tilde{V}_i - \lambda_i V_i.
\]

The left-hand side of (10) is the nominal return on this asset. The right-hand side of (10) is the sum of (a) the monopolistic profit \( \pi_i \) generated by this asset, (b) the potential capital gain, and (c) the expected capital loss due to creative destruction, in which \( \lambda_i \) is the Poisson arrival rate of inventions.

There is a continuum of R&D entrepreneurs indexed by \( j \in [0,1] \), and they hire workers to create inventions. The expected profit for entrepreneur \( j \) is

\[
\pi_{r,j}(j) = V_i \lambda_i(j) - W_i L_{r,j}(j).
\]

The Poisson arrival rate of inventions for entrepreneur \( j \) is \( \lambda_i(j) = \phi L_{r,j}(j) \), where \( \phi \) captures the productivity of R&D workers. The zero-profit condition from the R&D sector is given by

\[
V_i \phi = W_i = 1.
\]

This condition determines the allocation of labor between production and R&D.

The Cobb-Douglas specification in (6) implies that each industry \( i \) employs an equal number of workers. Substituting (7) into (6) yields \( C_i = Z_i L_{r,i} \), where the level of aggregate technology is defined as

\[
Z_i \equiv \exp \left( \int_0^1 n_i(i) di \ln z \right) = \exp \left( \int_0^1 \lambda_i d \tau \ln z \right)
\]

in which the last equality uses the law of large numbers.

Differentiating \( \ln Z_i \) with respect to time yields the growth rate of aggregate technology given by

\[
g_i \equiv \dot{Z}_i / Z_i = \lambda_i \ln z, \quad \text{where} \quad \lambda_i = \phi L_{r,i} \text{ is the economy-wide arrival rate of inventions. In other words,}
\]
although the invention process of each R&D entrepreneur is stochastic, the idiosyncratic uncertainty washes out at the aggregate level. As a result, aggregate technology increases at a constant rate along the balanced-growth path.

3. Decentralized Equilibrium

This section defines the equilibrium and shows that the aggregate economy is always on a unique and stable balanced-growth path. Given the balanced-growth behavior of the economy and a distribution of initial wealth, Section 3.1 shows that the distribution of assets in subsequent periods is stationary.

The equilibrium is a sequence of prices \( \{R, W, P_t(i), V_t\}_{t=0}^{\infty} \) and a sequence of allocations \( \{X_t(i), L_{x,t}(i), L_{r,t}(j), L_t(h), C_t(h)\}_{t=0}^{\infty} \) such that in each period,

a. household \( h \in [0,1] \) chooses \( \{C_t(h), L_t(h)\} \) to maximize utility taking \( \{R, W, P_t\} \) as given;

b. the monopolistic leader in industry \( i \in [0,1] \) chooses \( \{P_t(i), L_{r,t}(i)\} \) to maximize profit according to the Bertrand competition and taking \( \{W_t\} \) as given;

c. R&D entrepreneur \( j \in [0,1] \) chooses \( \{L_{r,t}(j)\} \) to maximize profit taking \( \{W_t, V_t\} \) as given;

d. the market for final goods clears such that \( \int_0^1 C_t(h)dh = C_t = \exp\left(\int_0^1 \ln X_t(i)di\right) \); 

e. the labor market clears such that \( \int_0^1 L_t(h)dh = L_t = \int_0^1 L_{x,t}(i)di + \int_0^1 L_{r,t}(j) dj \).

To prove that the aggregate economy is always on a unique and stable balanced-growth path, I derive the law of motion for \( E_t \) in the appendix and show that it must jump to its steady-state value. The following parameter restriction is a necessary condition for the saddle-point stability.\(^8\)

\[ \text{Condition S (Saddle-Point Stability): } \gamma > 1 - \frac{\rho + \varphi}{\varphi \ln \xi} \equiv \gamma \in (-\infty, 1) . \]

\(^8\) The sufficient condition will be derived in Appendix A.
Lemma 2: The aggregate economy is always on a unique and stable balanced-growth path, and the balanced-growth equilibrium is characterized by

\begin{align*}
L_{r,t} &= \frac{(1 + \phi)\mu/(1 + \phi\mu) - (1 + \rho/\varphi)}{(1 + \phi)\mu/(1 + \phi\mu) + (\gamma - 1)\ln z}, \\
g_t &= g = (\varphi\ln z)L_r, \\
r_t &= r = \rho + \gamma g, \\
C_t &= [(r - g)v_t + w_t]/(1 + \phi).
\end{align*}

Proof: See Appendix A. ■

\( r_t \equiv R_t - P_t/P_t \) denotes the real interest rate. \( w_t \equiv W_t/P_t \) and \( v_t \equiv V_t/P_t \) denote respectively the real wage rate and the real value of assets that are both increasing at rate \( g \) along the balanced-growth path. In (16), \( C_t (1 + \phi) > w_t \) because \( r - g = \rho + (\gamma - 1)g > 0 \) from Condition D. The effect of increasing patent breadth on the equilibrium is as follows. A larger \( \mu \) increases the incentives for R&D; as a result, R&D labor increases. The increase in \( L_r \) increases the equilibrium growth rate \( g \) and the real interest rate \( r \).

To ensure that \( L_r > 0 \), I impose the following lower bound on the productivity of R&D labor.

\textit{Condition R (R&D Productivity):} \( \varphi > \rho(1 + \phi\mu)/(\mu - 1). \)

3.1. Distribution of Assets

I adopt a similar approach as in Garcia-Penalosa and Turnovsky (2006) to show that the distribution of assets is stationary. To do this, it is more convenient to rewrite (2) in terms of real variables such that

\begin{align*}
\dot{v}_t(h) &= r_tv_t(h) + w_tL_t(h) - C_t(h).
\end{align*}

The real aggregate value of assets evolves according to

\begin{align*}
\dot{v}_t &= r_tv_t + w_tL_t - C_t.
\end{align*}
Combining (17) and (18) yields the law of motion for \( s_{v,t}(h) \equiv v_t(h)/v_t \) given by

\[
\dot{s}_{v,t}(h) = \frac{\dot{v}_t(h)}{v_t} - \frac{\dot{v}_t}{v_t} = \frac{C_t - w_tL_t}{v_t} - \frac{C_t(h) - w_tL_t(h)}{v_t(h)}.
\]

Using \( w_t[1 - L_t(h)] = \phi C_t(h) \) on (19), \( s_{v,t}(h) \) evolves according to a simple linear differential equation

\[
\dot{s}_{v,t}(h) = \frac{C_t(1 + \phi) - w_t}{v_t} s_{v,t}(h) - \frac{s_{v,t}(h)C_t(1 + \phi) - w_t}{v_t}.
\]

(20) describes the potential evolution of \( s_{v,t}(h) \) given an initial value of \( s_{v,0}(h) \). \( s_t(h) \equiv C_t(h)/C_t \) is a stationary variable from Lemma 1. Because \( C_t \), \( w_t \) and \( v_t \) all increase at rate \( g \), the coefficient on \( s_{v,t}(h) \) and the last term in (20) are constant. Also, given that the coefficient on \( s_{v,t}(h) \) is positive (recall that \( C_t(1 + \phi) > w_t \)), the only solution consistent with long-run stability is \( \dot{s}_{v,t}(h) = 0 \) for all \( t \).

Furthermore, from (20), \( \dot{s}_{v,t}(h) = 0 \) for all \( t \) implies that

\[
C_t(h) = \left[ (r - g)v_t(h) + w_t \right]/(1 + \phi).
\]

**Lemma 3**: For every household, \( s_{v,t}(h) = s_{v,0}(h) \) for all \( t \).

**Proof**: Proven in the text.

4. **Effects of Patent Policy on Income and Consumption Inequality**

Given that the economy is always on a unique and stable balanced-growth path and the distribution of assets is stationary, this section analyzes the effects of increasing patent breadth on income and consumption inequality. The amount of real income earned by household \( h \) is

\[
y_t(h) = rv_t(h) + w_tL_t(h) = [(r + \phi g)v_t(h) + w_t]/(1 + \phi),
\]

where the last equality uses \( w_tL_t(h) = \phi C_t(h) \) and (21). From (12) and Lemma 3, the share of income earned by household \( h \) simplifies to
Proposition 1: An increase in the level of patent breadth increases income inequality.

Proof: An increase in \( b \) (i.e. an increase in \( \mu \)) raises \( r \) and \( g \), which in turn increases \( \sigma_y \).

Intuitively, a larger patent breadth increases R&D and hence the equilibrium growth rate. This higher growth rate drives up the real interest rate, and the resulting higher return on assets increases the share of income \( s_y(h) \) earned by asset-wealthy households (i.e. \( s_y(h) > 1 \)) while it decreases that of asset-poor households (i.e. \( s_y(h) < 1 \)). In addition to this interest-rate effect, (24) shows that patent breadth also has a labor-income effect (captured by \( \phi g \)) on income inequality in the case of elastic labor supply. (21) shows that holding \( r \) constant, a higher growth rate reduces a household’s consumption by increasing its saving \( g v_i(h) \). This lower level of consumption reduces the household’s leisure and hence increases its labor income. Furthermore, this effect is stronger for a household that has a larger amount of assets as shown in (22). Therefore, when the supply of labor is elastic, there is an additional labor-income channel through which patent breadth affects income inequality.

The consumption of final goods for household \( h \) is given by (21). Using (12), (16) and Lemma 3 yields household \( h \)’s share of consumption given by
(25) \[ s_{c,t}(h) \equiv \frac{C_t(h)}{C_t} = \frac{(r-g)s_{c,t}(h) + \varphi}{r-g+\varphi} \]

for all \( t \). (25) implies that the standard deviation of consumption share \( \sigma_c \equiv \sqrt{\int_0^1 [s_{c,t}(h) - 1]^2 dh} \) is

\[ \sigma_c = \left( \frac{r-g}{r-g+\varphi} \right) \sigma_v. \]

Proposition 2 summarizes the effect of patent policy on consumption inequality measured by the coefficient of variation of consumption.

**Proposition 2:** An increase in the level of patent breadth increases (decreases) consumption inequality if and only if the elasticity of intertemporal substitution is less (greater) than unity.

**Proof:** An increase in \( b \) (i.e. an increase in \( \mu \)) raises \( r \) and \( g \). (15) shows that the resulting increases in \( r \) and \( g \) lead to a higher (lower) \( \sigma_c \) if and only if \( \varepsilon = 1/\gamma \) is less (greater) than one.\( \blacksquare \)

Intuitively, strengthening patent protection increases growth, and this higher growth rate increases each household’s saving \( g \nu_t(h) \). At the same time, the higher growth rate also increases each household’s asset income \( r

\nu_t(h) \). Given that the fraction of assets for consumption is given by \( r-g \), whether or not the increase in asset income is sufficient to compensate for the increase in saving depends on the value of \( \varepsilon \). For \( \varepsilon \) less (greater) than one, \( r-g \) increases (decreases). A larger \( r-g \) increases the share of consumption of asset-wealthy households (i.e. \( s_c(h) > 1 \)) and decreases that of asset-poor households (i.e. \( s_c(h) < 1 \)). The opposite occurs when \( r-g \) decreases. For the case of log utility, \( r-g = \rho \) and hence consumption inequality is simply given by \( \sigma_c = \sigma_v\rho/(\rho+\varphi) \).
Finally, Proposition 3 ranks the different measures of inequality according to their value, and the theoretical ranking is consistent with the empirical pattern in the US as documented by Budria-Rodriguez et al. (2002), Krueger and Perri (2006) and Blundell et al. (2008).

**Proposition 3**: *Wealth inequality > income inequality > consumption inequality.*

**Proof**: Comparing (24) and (26) shows that \( \sigma_y > \sigma_y > \sigma_c \). ■

### 4.1. Numerical Analysis

This section calibrates the model to aggregate data of the US economy in order to numerically evaluate the effects of patent breadth on income and consumption inequality. From the model, I express each of the following moments as a function of structural parameters and then use the values of these moments in the data to infer the parameter values. I use standard values for the fraction of time devoted to leisure \( l = 0.7 \), the real rate of return on assets \( r = 0.07 \), and total factor productivity growth \( g = 0.01 \). For the arrival rate of inventions, I set \( \lambda \) to 0.33 such that the average time between arrivals of inventions is 3 years as in Acemoglu and Akcigit (2008). R&D spending as a share of GDP is given by \( wL_r/(\pi + wL) \) in the model. Assuming that the increase in R&D spending since the 80’s has been driven by patent protection, the hypothetical exercise is to firstly use the time trend of R&D from 1980 to 2004 to infer a time path for patent breadth \( b \) and then examine how the increase in \( b \) affects the relative level of income and consumption inequality. Figure 1 plots R&D as a share of GDP and its trend.

For a given value of \( \gamma \), the five moment conditions determine respectively the values of \( \{\phi, \rho, z, \varphi, b\} \). As for \( \gamma \), I use a conservative value of 3 implying an intertemporal substitution elasticity of 0.33 that is within the usual range in the business-cycle literature.\(^9\) The calibrated parameter values are \( \{\gamma = 3, \phi = 2.33, \rho = 0.04, z = 1.03, \varphi = 71.4, b_{1980} = 0.62\} \). The values of the standard parameters are

\(^9\) At a lower value of \( \gamma \) (i.e. a larger \( \varepsilon \)), strengthening patent protection would increase income inequality relative to consumption inequality by even more.
reasonable, and the large value of $\varphi$ implies that asset income from patents $r v_t$ is very small compared to labor income $w_t L_t$, where $w_t = \varphi v_t$ from (12). This implication also seems reasonable given that labor income and industrial R&D are on average about 70% and less than 2% of GDP respectively.

The calibrated value of $b$ gradually increases from 0.62 in 1980 to 0.86 in 2004 implying a substantial increase in the level of patent breadth. As a result of the increase in $b$, the model predicts that the relative coefficient of variation between income and consumption (i.e. $\sigma_y / \sigma_c$) increases from 1.55 in 1980 to 1.69 in 2004. This illustrative exercise suggests that for a given degree of wealth inequality, increasing patent breadth leads to a larger increase in income inequality than consumption inequality such that $\sigma_y / \sigma_c$ increases over time, which is consistent with the empirical pattern in the US.

5. Conclusion

This paper analyzes the effects of patent policy on growth and inequality. In summary, strengthening patent protection increases growth but worsens income inequality. However, the effect on consumption inequality is ambiguous and depends on the elasticity of intertemporal substitution. Given these results, the policy implications are as follows. Firstly, when policymakers use patent policy as a policy instrument to increase R&D and economic growth, they need to take into account the distributional consequences which may have negative impacts on the society. Secondly, the possible divergence between income and consumption inequality suggests that policymakers need to choose the relevant and appropriate measure of inequality when evaluating the policy outcome.

The theoretical framework is a canonical quality-ladder model with heterogeneity in households’ wealth. In this model, the aggregate economy is always on a unique and stable balanced-growth path. Given the balanced-growth behavior of the aggregate economy and an exogenous distribution of initial wealth, the endogenous distribution of assets in subsequent periods is stationary and equal to its initial distribution. Therefore, one potential weakness of the model is that it takes the distribution of wealth as given. For example, if this model needs to capture the rise in wealth inequality in the US from 1983 to
1998 as documented by Wolff (1994, 1998), then it would have to be formulated as an exogenous shock to the model. Furthermore, the model does not feature capital accumulation for analytical tractability. Therefore, a useful direction for future research is to develop an R&D-growth model that endogenizes the wealth distribution and features capital accumulation in order to provide a more accurate assessment on the quantitative importance of patent policy on the distributions of wealth, income and consumption.

Finally, the canonical quality-ladder model belongs to the first generation of R&D-growth models that exhibit scale effects, in which a larger economy experiences faster growth and an economy that has a growing population experiences an increasing growth rate rather than a balanced-growth path. In this study, scale effects are eliminated by normalizing the population size to unity. Another useful direction for future research is to analyze the distributional consequences of patent policy in the later vintages of R&D-growth models.

References


---

10 See, for example, Jones (1999) for a discussion on scale effects in R&D-growth models.


Li, C.-W., 1998. Inequality and growth: a Schumpeterian perspective. manuscript.


Appendix A

**Proof of Lemma 2:** To show the stability and uniqueness of the balanced-growth path, I derive the law of motion for \( E_t = P_t C_t \) and analyze its dynamics. The labor-market clearing condition is

\[
L_t = L_{s,t} + L_{r,t}.
\]

From aggregate labor supply, \( L_t = 1 - \phi E_t \). From the labor share of aggregate expenditure, \( L_{s,t} = E_t / \mu \).

From the R&D production function, \( L_{r,t} = \lambda_t / \phi \). Substituting these conditions into (A1) yields

\[
\lambda_t = \phi \left( 1 - \frac{1 + \mu \phi}{\mu} E_t \right).
\]

From (5), the law of motion for \( E_t \) is

\[
\frac{\dot{E}_t}{E_t} = \frac{\dot{P}_t}{P_t} + \frac{\dot{C}_t}{C_t} = \left( \frac{\gamma - 1}{\gamma - \phi(1 - \gamma)} \right) \frac{\dot{P}_t}{P_t} + \frac{R_t - \rho}{\gamma - \phi(1 - \gamma)}.
\]

The price index is \( P_t = \exp \left( \int_0^t \ln P_i(i) \, di \right) = \mu / Z_t \). Therefore, \( \dot{P}_t / P_t = -\dot{Z}_t / Z_t = -\lambda_t \ln z \). As for \( R_t \), using (10) and (12) yields

\[
R_t = \frac{\pi_t + \dot{V}_t - \lambda_t V_t}{V_t} = \frac{\pi_t - \lambda_t / \phi}{1 / \phi}.
\]

Using the profit share \( \pi_t = E_t (\mu - 1) / \mu \) and substituting (A4) into (A3) yield

\[
\frac{\dot{E}_t}{E_t} = \frac{\phi}{\gamma - \phi(1 - \gamma)} \left( \frac{1}{\mu} \right) \left( \frac{\mu - 1}{\mu} \right) E_t - \left( \frac{\gamma - 1}{\gamma - \phi(1 - \gamma)} \right) \lambda_t - \frac{\rho}{\gamma - \phi(1 - \gamma)}.
\]

Substituting (A2) into (A5) yields

\[
\frac{\dot{E}_t}{E_t} = \frac{\phi}{\mu} \left( \frac{\mu(1 + \phi) + (1 + \mu \phi)(\gamma - 1) \ln z}{\gamma - \phi(1 - \gamma)} \right) E_t - \left( \frac{\rho + \phi(\gamma - 1) \ln z}{\gamma - \phi(1 - \gamma)} \right).
\]
Condition S and Condition R imply that \( \gamma > 1 - \frac{(1 + \phi)\mu}{(1 + \mu\phi)\ln z} \). This condition together with Condition E imply that the coefficient on \( E_i \) is positive in (A6), so that the dynamic system is characterized by global instability. Therefore, \( E_i \) must jump to its non-zero steady-state value given by

\[
(A7) \quad E = \frac{\mu}{\phi} \left( \frac{\rho + \phi + \phi(\gamma - 1)\ln z}{\mu(1 + \phi) + (1 + \mu\phi)(\gamma - 1)\ln z} \right) > 0.
\]

Substituting (A7) into (A2) and using the R&D production function \( \lambda = \phi L_i \) yield

\[
(A8) \quad L_i = \frac{\lambda}{\phi} = \frac{(1 + \phi)\mu/(1 + \phi\mu) - (1 + \rho/\phi)}{(1 + \phi\mu)/(1 + \phi\mu) + (\gamma - 1)\ln z} > 0.
\]

The aggregate production function \( C_i = Z_i L_i \) implies \( \dot{C}_i / C_i = \dot{Z}_i / Z_i \) while the price index \( P_i = \mu / Z_i \) implies \( \dot{P}_i / P_i = -\dot{Z}_i / Z_i \). Substituting these conditions into (5) yields

\[
(A9) \quad r = \rho + \gamma g.
\]

Finally, combining (18) and \( \phi C_i = w_i (1 - L_i) \) yields

\[
(A10) \quad C_i = (r - g)v_i + w_i L_i = (r - g)v_i + w_i - \phi C_i,
\]

because \( v_i = V_i / P_i \) grows at rate \( g \) along the balanced-growth path. ■
Data sources: (a) National Science Foundation: Division of Science Resources Statistics; and (b) Bureau of Economic Analysis: National Income and Product Accounts.

Footnote: R&D is net of federal spending, and non-farm business-sector output is calculated as GDP net of government spending and farm-sector output. The trend from the data is extracted using a standard HP-filter with a smoothing parameter of 100 for the annual frequency.