Mobile telephony and internet growth: impacts on consumer welfare

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Abstract

Innovation in digital technology has allowed rapid growth in mobile telephone and Internet adoption among consumers. The implication underlying the high rates of subscription growth is that consumers generally place a high valuation on telecommunication services. Moreover, since mobile telephone and Internet are predominantly telecommunication services, it is reasonable to presume that the network effect may be largely responsible for this growth. The implication of the network effect, where the consumer’s valuation of service increases with the size of the network is that subscription growth is endogenous. However, to date there have been few attempts to measure the change in consumer welfare as networks increase. Following Hausman (1981), this paper measures the change in consumer surplus based on the compensating variations approach. The result is an annual measure of the change in consumer surplus for the representative consumer for the OECD region. In addition, the approach reveals whether marginal consumer surplus is a decreasing or increasing function of network size. Measurement of the change in consumer welfare thus provides an additional tool for public policy analysis.

Keywords: consumer welfare, network effect, compensating variation

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I. Introduction

Since the early-1990s, mobile telephony and Internet network subscription sustained rapid growth in Europe (Welfens and Jungmittag, 2003). Pricing, technical innovation and regulatory framework change are seen as important to continued network growth. In March 2002, the European Union adopted a package of Directives that significantly revised the 1998 regulatory framework for electronic communications networks and services in Europe. In particular, the new regulatory approach seeks to be responsive to technological and market developments by being more neutral in its treatment of similar services provided via alternative technical means (such as narrowband, broadband and mobile), and by allowing regulation to be withdrawn as effective competition develops (Cawley 2004). However, as mobile telephony and the Internet are network delivered services, positive demand externality effects may also be important in explaining such network growth. That is, when network externalities are important, consumers’ valuation of network subscription is increased with network size—or equivalently—subscription can expand independently of any change to market conditions. Should network effects be shown to be empirically important in explaining network evolution, then they should also be considered in future regulatory framework changes.

This study develops a procedure to determine the importance of network effects. Model estimates provide an annual measure of consumer welfare change for the representative OECD region subscriber. Following Hausman (1981), our model is based on the compensating variation (CV) approach, which consists in assessing welfare improvement due to a price fall as the extra income the consumer would be willing to accept in place of the price fall. Further, the study indicates the welfare gain, as measured by CV, decreases with network size.

The chapter is organised as follows. Section II states the methodology and shows how Hausman’s CV formula may be adapted to the context of a dynamic demand model incorporating network effects, as specified by Madden et al. (2004). The demand system is such that current network size depends on the past size of the network, and expectations for future size. This specification reflects the dynamic optimising behaviour by a representative consumer whose subscription choice is influenced by a
telecommunications service network effect. Section III reports model parameter estimates based on annual OECD data for 30 Member States for the period 1996 through 2000. CV values are constructed from the parameter estimates and discussed. The services considered are fixed-line and mobile telephony, and the Internet. Section IV concludes.

II. Methodology

A feature of electronic communication networks is that consumers receive more utility the larger is the subscriber base. That is, a consumer’s welfare increases monotonically with the network size (Squire 1973; Rohlfs 1974). Accordingly, the presence of a network effect impacts on both current subscriber and marginal non-subscriber welfare. That is, for subscribers, network subscription growth provides greater consumer welfare. Further, marginal non-subscribers are more likely to subscribe, at current prices, the larger is network subscription. To identify the increase in consumer welfare due to a price fall then requires the separation of any consumer welfare gain due to a movement along the subscription demand curve (direct price effect), from any indirect welfare gain originating from network expansion (network externality effect).

Hausman (1981) uses the CV method to measure consumer welfare rise in response to a price fall for a non-network good. The change in consumer welfare induced by a network effect is obtained by adapting Hausman’s (1997) measure of the change in consumer welfare caused by the introduction of a new good. The consumer welfare change is obtained by treating the prevailing subscription price, before the price change, as the reservation price of a new subscriber, i.e., the price at which she will decide to subscribe. As such, the network’s growth leads to a change in the current reservation price. With the reservation price change identified, the variation to consumer welfare is then calculated.

Compensating Variation and Hausman’s approach

A price fall impacts on the demand for any given good in two different ways. First, the price fall has an effect which is equivalent to an income increase, stimulating demand for the good considered and for all other goods (income effect). Second, it makes the consumer demand less for other goods and
more for the good considered, the relative price of which is lowered (substitution effect). In order to separate the income effect from the substitution effect, consider a two-step process. In the first step (substitution effect), price is lowered from its initial to its final level, while the income effect is controlled by lowering income simultaneously with price, so as to hold utility constant. In the second step (income effect), price remains unchanged whereas the initial level of income is restored. By definition, the income removed in the first step, and then restored in the second step, is the compensating variation, CV. The CV is interpreted as the amount that the consumer should be compensated if he were not to benefit from the price fall, i.e., CV is the exact measure of the change in consumer welfare caused by a price fall.

Defining the compensated demand function as the hypothetical demand that would prevail under compensating income variation (during the first step), CV measure is simply the area A in Fig. 1, viz., the area lying to the left of the compensated demand curve, between the initial and the final price lines, $P_0$ and $P_1$. The difficulty is that compensated demand functions are not readily obtainable from market data. Only uncompensated demand functions are observed, indicating equilibrium demand at given prices and income. Now, using the uncompensated demand curve instead of the compensated demand curve leads to an inexact measure of welfare change. In particular, this measure includes the
income effect, and based on consumer surplus variation $SV$, results in the area $A+B$ in Fig. 1, rather than on the compensating variation $CV$. A sufficient condition for $CS$ and $SV$ to be equivalent is that the marginal utility of income is constant, which is not generally consistent with observed behaviour\(^1\). The measurement of $CV$, not $SV$, is thus required in order to obtain a proper assessment of welfare change.

Hausman (1981) employed the microeconomic theory of consumer behaviour to derive the unobserved compensated demand curve from the observed uncompensated demand curve. From the observed demand function, and Roy’s identity (see Appendix), the expenditure function\(^2\) is first derived, i.e., the variable income the consumer must be allocated when price varies in order to keep her utility constant. Then, the compensated demand function is obtained as the derivative of the expenditure function with respect to price. Finally, $CV$, i.e., the exact measure of welfare variation due to a price change, is calculated.

The case of network goods

Next, Hausman’s approach must be adapted to the context of network goods. Following Madden et al. (2004), an uncompensated network demand equation system is first specified as:

$$
N_{ij,t} = \theta_{i0} + \theta_{i1} (N_{i,t-1} + \beta N_{i,t-1}) + \sum_{i,j} \theta_{ij} N_{ij,t} + \sum_{j} \theta_{ji} N_{j,t-1} + \theta_{iP} P_{i,t} + \gamma Y_t,
$$

where $N_{ij,t}$ is the demand for service $i$ at time $t$ (defined in terms of the size of network $i$), $P_{i,t}$ is the price of subscription to network $i$ and $Y_t$ is the real per capita income. The response of demand to a price fall is indicated by parameter $\theta_{iP}$ (for a normal good $\theta_{iP} < 0$). Besides the price effect,

\(^1\) This condition requires that indifference curves are collinear. Decreasing marginal utility of income produces a compensated demand curve steeper than the corresponding uncompensated demand curve, and results in $SV$ being an inexact measure of welfare.

\(^2\) The prime approach to the analysis of consumer behaviour involves the maximisation of a strictly quasi-concave utility function, subject to some budget constraint. The dual approach considers the minimisation of the expenditure function, subject to utility being set at or greater than some prescribed level. When the indirect utility function is monotonically increasing in income, and the expenditure function is monotonically increasing in utility, either function can be inverted to derive the other corresponding function.
increasing network size at time \( t-1 \) leads to another impact on demand at time \( t \), i.e., a network effect indicated by parameter \( \theta_\beta \) \((0<\theta_\beta<1)\). Also, as the current size \( N_{i,t} \) of network \( i \) is a function of network size \( N_{i,t+1} \) in the next period, an anticipated fall in future price \( P_{i,t+1} \) for network \( i \) yields an increase in current subscription if \( \beta > 0 \). Moreover, \( \theta_\beta > 0 \) implies the anticipated fall in the price of network \( j \) induces a current period increase in subscription for network \( i \). A permanent price fall implies a larger increase in current subscription than for a temporary price fall, since the permanent price fall combines a fall in current and all future prices.

Following Hausman’s approach (see Appendix), the exact measure, \( CV_{i,t} \), of change in consumer welfare due to a price change from \( P_{i,t} \) to \( P_{i,t+1} \), is then calculated as:

\[
CV_{i,t} = \frac{1}{\gamma} \left( N_{i,t+1} + \frac{\theta_\gamma}{\gamma} \right) - \frac{1}{\gamma} \left( N_{i,t} + \frac{\theta_\gamma}{\gamma} \right) e^{\gamma(P_{i,t+1} - P_{i,t})}
\]  

(2)

The compensating variation \( CV_{i,t} \) reflects a movement along the compensated demand curve and reflects the direct price effect on welfare.

In order to assess the network effect from (2), one has to determine the change \( P_{i,t+1} - P_{i,t} \) in reservation subscription price which produces a one unit increase in network size, \( N_{i,t+1} - N_{i,t} = 1 \). For new subscribers, the size of the network is just large enough to induce them to join at the prevailing subscription price. Assuming network size is perfectly observable, and subscribers are able to join at any time, the prevailing network subscription price is equal to new subscriber’s reservation price \( P_i^* \). Then, calculating the change in the reservation price \( P_i^* \) with respect to a network subscription \( N_j \) increase gives \( \frac{\partial P_i^*}{\partial N_j} = \frac{(1-\theta_\gamma)}{\theta_\gamma} \), which being negative reveals the reservation

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3 Within a representative consumer framework, for a new subscriber over time, demand is zero until the market price falls to a level below the subscriber’s reservation price. To measure the change in consumer welfare for a marginal increase in network size then appears quite similar to measuring a change in consumer welfare with the introduction of a new service, as Hausman (1997) asserts the correct price for a new good in the pre-introduction period is the reservation price, i.e. the virtual price which sets demand equal to zero.
price decreases in network subscription. The change in consumer welfare induced by a unit increase in network size (from \( N_{i,t} = N_i \) to \( N_{i,t+1} = N_i + 1 \)) and a fall in reservation price \( \frac{P_{i,t+1} - P_{i,t}}{P_{i,t}} = \frac{\partial P_{i}^t}{\partial N_i} (1 - \theta_p) / \theta_p \), (2) becomes

\[
CV_{i,t}^{\text{NE}} = \frac{1}{\gamma} \left( N_{i,t} + 1 + \frac{\theta_p}{\gamma} \right) - \frac{1}{\gamma} \left( N_{i,t} + \frac{\theta_p}{\gamma} \right) e^{\gamma (1 - \theta_p) / \theta_p}.
\]

Equation (3) measures the change in welfare at time \( t \) due to the price adjustment caused by a unitary network increase.

**III. OECD Consumer Surplus Change**

Biannual rental price data are required to estimate (2) and (3). These data are collected for 30 OECD Member Country markets for 1996, 1998 and 2000 from the OECD *Communications Outlook* (1997, 1999, 2001, 2003). Annual quantity (network size) and income data for 1996 to 2000 are obtained from the International Telecommunication Union *World Telecommunications Indicators Database* (2003).

Fixed-line price data are the fixed component of the OECD’s basket of residential telephony charges. Mobile telephony price is the fixed component of the OECD’s basket of consumer mobile telephony charges. Internet price is the OECD’s Internet access basket for 20 hours using discounted PSTN rates. Income (GDP per capita) and price data are denominated in United States dollars (US$) according to OECD purchasing power parity. Both income and prices are deflated by the US CPI to allow comparison through time. Fixed-line telephony quantity is the number of main telephone lines. Mobile telephony quantity is mobile telephone subscribers, while Internet quantity is the number of Internet users. Quantity variables are per 100 persons. The resulting index data is comprised of 79 observations.

The demand function specification (1) is from Madden et al. (2004), a perfect foresight model that holds the marginal utility of wealth constant for each individual, but allows variation across

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4 An implicit assumption in using these data, following Becker et al. (1994), is that per capita telecommunications consumption reflects the behaviour of a representative consumer.
individuals. Thus, in the present context, the intercepts \( \theta_{\alpha_i} \) in the cross-country model capture, in part, country-specific variation of the marginal utility of wealth. The specification is relaxed further by allowing time-specific effects to capture unanticipated growth in wealth. Deviations in country- and time-specific means are captured by adding an argument for per capita income to the demand function, which is associated with changes in marginal wealth across countries and through time. The resulting augmentation is a two-way (country and time) effects model. In addition, given there is a possibility for simultaneity between network effects, the regression model is specified as a standard form vector autoregressive model. Estimates of the network, price and income coefficients for fixed-line and mobile telephony, and Internet service are from Madden \textit{et al.}^5

\begin{table}[h]
\centering
\begin{tabular}{lccc}
\hline
 & Fixed-line & Mobile & Internet \\
\hline
\( \theta_{\alpha_i} \) & 0.96240 & 0.86260 & 0.96510 \\
\( \theta_{p} \) & -0.00256 & -0.00497 & -0.02889 \\
\( \gamma \) & 0.00015 & 0.00045 & 0.00025 \\
\hline
\end{tabular}
\caption{Network, Price and Income Coefficients Estimates}
\end{table}


Table I presents estimates of network, price and income coefficients for fixed-line and mobile telephony and Internet services, respectively. All coefficient estimates are correctly signed and significant. Table II reports estimates of the change in welfare resulting from a fall in price. As shown, a price fall has an immediate impact on CV of 0.1%-0.2% of income. From Table II, a fall in mobile telephone subscription price provides the most direct benefit to consumers after 2000, followed by that for fixed-line service. Table III shows the indirect (or network externality) effect on welfare in fixed-line and mobile telephony and Internet service, respectively. CV estimates indicate that the benefit derived from a small increase in network size is large relative to that for the direct price effect.

\footnote{The presence of unobserved components means that two-stage instrumental variables estimation is required. Past network size is an endogenous variable because of the dependence of network size on the unobserved components. However, caution is required when implementing instrumental variables since, as Nelson and Startz (1990) warn, the use of lagged values as instruments when estimating stochastic Euler equations can lead to bias. Thus, instruments for network size are restricted to future access and use prices. Particular care is taken to ensure instruments are good predictors of network size. The resulting equations are estimated in Limdep 8.0 using the unbalanced panel data set described above.}
Table II. Direct Welfare Gain by Telecommunications Service

<table>
<thead>
<tr>
<th>Service</th>
<th>Year</th>
<th>Subscription per 100 persons</th>
<th>CV (US$)</th>
<th>CV / Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-line</td>
<td>1996</td>
<td>50</td>
<td>28</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>1998</td>
<td>52</td>
<td>31</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>54</td>
<td>33</td>
<td>0.001</td>
</tr>
<tr>
<td>Mobile</td>
<td>1996</td>
<td>13</td>
<td>13</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>1998</td>
<td>26</td>
<td>26</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>56</td>
<td>56</td>
<td>0.002</td>
</tr>
<tr>
<td>Internet</td>
<td>1996</td>
<td>6</td>
<td>6</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>1998</td>
<td>15</td>
<td>15</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>29</td>
<td>28</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Note: Benchmark is actual annual demand.

Table III. Indirect Welfare Gain by Telecommunications Service

<table>
<thead>
<tr>
<th>Service</th>
<th>Year</th>
<th>Subscription per 100 persons</th>
<th>CV (US$)</th>
<th>CV / Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-line</td>
<td>1996</td>
<td>50</td>
<td>58</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>1998</td>
<td>52</td>
<td>58</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>54</td>
<td>57</td>
<td>0.002</td>
</tr>
<tr>
<td>Mobile</td>
<td>1996</td>
<td>13</td>
<td>22</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>1998</td>
<td>26</td>
<td>18</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>56</td>
<td>10</td>
<td>0.000</td>
</tr>
<tr>
<td>Internet</td>
<td>1996</td>
<td>6</td>
<td>41</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>1998</td>
<td>15</td>
<td>41</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>29</td>
<td>41</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Note: Benchmark is actual annual demand.

Table III reveals the relative welfare benefit (relative increase in CV) at 2000 from a network effect for fixed-line telephony subscribers is almost 6 times larger than that for mobile telephony subscribers. This finding is in part explained by the relatively large fall in reservation price \((1 - \theta_p) / \theta_p\), for mobile telephony (-27.7) compared to that for fixed-line telephony (-14.7). Adding to the difference in magnitude is the negative scale factor \(\theta_p / \gamma\), which is -17.1, -11.0 and -115.6 for fixed-line and mobile telephony demand, and Internet demand, respectively. The ratio indicates the relative importance of price-to-income effects on network growth. The greater is the magnitude of the ratio the greater the impact on CV. While differences in scale explain some of the difference in CV for 1996, the scale effect has a negligible impact by 2000. Thus, the difference in valuation is due mainly to relative access prices between fixed-line and mobile telephone service. Table III also reveals
that the welfare gain, as measured by annual CV, decreases with network size. Namely, an 8% growth in the fixed-line network accords with an inelastic fall in annual CV of 2% for the period. The substantial growth in mobile and Internet networks (330% and 380%, respectively) also corresponds with a decline in annual CV for the mobile and Internet networks of 54% and less than 1%, respectively.

Further, Table IV provides an overview of changes to the mobile and fixed-line service price ratio. Mean values for 1996 and 1998 indicate annual mobile telephone subscription price is almost double that for fixed-line subscription. Sample standard deviations indicate substantial cross-country variation. For example, 1996, highest price mobile subscription (France) is higher than fixed-line telephony by a factor of 10. At 2000, the subscription price of mobile telephony in Korea (the highest in the sample) is almost 7 times higher than that for fixed-line telephony. Thus, the increase in consumer welfare induced due to an expansion in mobile telephony subscription is constrained by the relatively high subscription price.\(^6\)

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>2.64</td>
<td>1.89</td>
<td>0.64</td>
<td>9.95</td>
</tr>
<tr>
<td>1998</td>
<td>2.43</td>
<td>1.74</td>
<td>0.25</td>
<td>6.92</td>
</tr>
<tr>
<td>2000</td>
<td>1.29</td>
<td>1.23</td>
<td>0.01</td>
<td>6.60</td>
</tr>
</tbody>
</table>

Finally, Internet service expenditure is a relatively small proportion of communications expenditure compared to that for mobile telephony. Accordingly, the network effect induced by increased network size is less constrained than that for mobile telephony. However, since the subscription price includes the fixed-line subscription charge, it too is relatively constrained when compared to fixed-line access. These findings suggest that concentrating solely on the welfare impact of price falls yields an underestimate of consumer benefit. That is, while the direct effect from a price fall in

\(^6\) The economic constraint is reflected in the size of the price and income coefficients across services.
telecommunications networks matters, indirect benefit from subsequent network expansion is also important. This finding has important consequences for the conduct of universal service policy.

**IV. Conclusion**

This study uses Hausman’s (1981) CV approach to measure the direct consumer welfare change from a price fall and an indirect network externality effect. The study demonstrates a method to obtain exact estimates of welfare change. Once econometric price and income parameter estimates are obtained the corresponding change in consumer welfare is calculated. Additionally, this study demonstrates the importance of controlling for the network effect to obtain an accurate assessment of the total impact on consumer welfare. Estimates indicate a direct increase in welfare from a subscription price fall is at most 0.2% of income. However, larger indirect welfare increases occur *via* a network effect. Surprisingly, mobile telephony welfare increases appear to provide the smallest improvement. However, this result is explained by a high mobile subscription price when compared to fixed-line telephony and Internet service prices. That is, the relatively high mobile subscription price constrains the network effect. Finally, study findings support competition policy designed to place downward pressure on subscription prices. The study provides indirect justification for the continuation of universal service policy.

While the empirical estimates of CV for telephony and Internet networks contained in this study are revealing, it is important to note that the underlying source of welfare gains differ by network. That is, the network effects linked to (fixed-line and mobile) point-to-point communication services differ from that due primarily to information services. Conversely, information service growth is related to the installed base of terminals, in particular connected PCs. Accordingly, fixed and mobile telephones mostly generate communications network effects and, so far, few information service effects (which could change with the emergence of the 3rd generation mobile telephony and growth in mobile data markets). Additionally, the Internet generates mostly information service network effects, typically through e-mail, the dominant but not exclusive use. This latter network externality is very important
because the quantity and quality of data available on the Web is highly correlated with the size of network subscriber base. Further, it is reasonable to expect that this Internet communications externality will become increasingly important. Finally, as the model cannot distinguish these effects, the results are likely to underestimate the true impact of Internet network growth on CV.

Appendix: Hausman’s approach and proof of equation (2)

Consider the dynamic and deterministic demand system specified by Madden et al. (2004),

\[ N_{i,t} = \theta_{i0} + \theta_{iP} P_{i,t} + \gamma Y_t + F_{i,t} + G_{i,t} \]

\[ F_{i,t} = \theta_{iN} (N_{i,t-1} + \beta N_{i,t+1}) \] (A1)

\[ G_{i,t} = \sum_{j \neq i} \theta_{ij} N_{j,t-1} + \sum_{j \neq i} \theta_{ji} N_{j,t-1} + \sum_{j \neq i} \theta_{ij} N_{j,t+1} \]

where \( N_{i,j} \) is consumer demand for network \( i \) at time \( t \), under price level \( P_{i,j} \), and \( Y_t \) is consumer’s income at time \( t \). Terms \( F_{i,t} \) and \( G_{i,t} \) reflect the own-network and cross-network externalities, respectively.

Allowing price \( P_i \) and income \( Y \) to vary at time \( t \) from their observed levels \( P_{i,t} \) and \( Y_t \), the uncompensated demand function \( N_{i,t}(P_i,Y) \) is defined:

\[ N_{i,t}(P_i,Y) = \theta_{i0} + \theta_{iP} P_i + \gamma Y + F_{i,t} + G_{i,t} \] (A2)

The linear uncompensated demand function, \( N_{i,t}(P_i,Y) \), derives from its generating quadratic indirect utility function, \( U_i(P_i,Y) \), through Roy’s identity:

\[ N_{i,t}(P_i,Y) = -\frac{\partial U_i(P_i,Y)/\partial P_i}{\partial U_i(P_i,Y)/\partial Y} = \left( \frac{\partial Y}{\partial P_i} \right)_{U_i} \] (A3)

Following Hausman (1981), to enable a valid welfare comparison to be made, before and after a price change, requires that the consumer remain at constant utility \( U_i \), i.e., on a same indifference curve. If price of service \( i \) departs from \( P_{i,t} \) at time \( t \), to remain on the same indifference curve requires that,
simultaneously, income departs from $Y_t$. Denoting $Y_t(P_t)$ the expenditure function, i.e. the current income associated with current price $P_t$ along a constant utility path, Roy’s identity implies:

$$
\frac{dY_t(P_t)}{dP_t} = N_{t,i,t}(P_t, Y_t(P_t)) = \gamma Y_t(P_t) + \theta_{\theta}P_t + \theta_{\theta} + F_{t,i,t} + G_{-i,t} .
$$

(A4)

The solution to differential equation (A4) is:

$$
Y_t(P_t) = \frac{1}{\gamma} \left( \theta_{\theta}(P_t + 1/\gamma) + \theta_{\theta} + F_{t,i,t} + G_{-i,t} \right) + U_{i,t}e^{\theta_{\theta}P_t}
$$

(A5)

in which the constant of integration $U_{i,t}$ (reflecting the invariance of utility) derives from the initial condition $Y_t(P_{i,t}) = Y_t$:

$$
U_{i,t} = \frac{1}{\gamma} \left( N_{i,j} + \theta_{\theta}/\gamma \right) e^{-\theta_{\theta}P_t} .
$$

(A6)

Hence, finally:

$$
Y_t(P_t) = \frac{1}{\gamma} \left( N_{i,j} + \theta_{\theta}/\gamma \right)(1 - e^{\gamma(P_t - P_{i,t})}) - \frac{\theta_{\theta}}{\gamma}(P_t - P_{i,t}) .
$$

(A7)

From the expenditure function $Y_t(P_t)$, the compensated demand function $N^C_t(P_t)$ is derived as:

$$
N^C_t(P_t) = N_t[P_t, Y_t(P_t)] = \frac{dY_t(P_t)}{dP_t} = \left( \frac{\theta_{\theta}}{\gamma} \right) e^{\gamma(P_t - P_{i,t})} - \frac{\theta_{\theta}}{\gamma}
$$

(A8)

Now consider a virtual transition from time $t$ to time $t+1$, in which the price falls from $P_{i,t}$ to $P_{i,t+1} < P_{i,t}$ and income is simultaneously lowered from $Y_t$ to $Y_t(P_{i,t+1}) < Y_t$, in order to make the consumer insensitive to the price fall. The difference, or ‘compensating variation’, $CV_{i,t} = Y_t - Y_t(P_{i,t+1})$, is the income reduction which offsets the price fall benefit to the consumer. Conversely, $CV_{i,t}$ is a measure of the consumer’s welfare gain due to the price reduction from $P_{i,t}$ to $P_{i,t+1}$. 

\[ \frac{1}{3} \]
The explicit expression of $CV_{i,t}$, as in equation (2), then derives from (A7) and (A8), i.e.:

$$CV_{i,t} = Y_t - Y_t(P_{i,t+1}) = \int_{P_{i,t}}^{P_{i,t+1}} N_i^C(P_t) dP_t$$

$$= \frac{1}{\gamma} \left( N_{i,t} + \frac{\theta P_t}{\gamma} \right) - \frac{1}{\gamma} \left( N_{i,t} + \frac{\theta P_t}{\gamma} \right) e^{\gamma(P_{i,t+1}-P_{i,t})}$$

(A9)

References


