Residual income and value creation: An investigation into the lost-capital paradigm

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Abstract

This paper presents a new way of measuring residual income, originally introduced by Magni (2000a,b,c, 2001a,b, 2003). Contrary to the standard residual income, the capital charge is equal to the *capital lost* by investors multiplied by the cost of capital. The lost capital may be viewed as (a) the foregone capital, (b) the capital implicitly infused into the business, (c) the outstanding capital of a *shadow project*, (d) the claimholders' credit. Relations of the lost capital with book values and market values are studied, as well as relations of the lost-capital residual income with the classical standard paradigm; many appealing properties are derived, among which an aggregation property. Different concepts and results, provided by different authors in such different fields as economic theory, accounting and corporate finance, are considered: O'Hanlon and Peasnell's (2002) unrecovered capital and Excess Value Created; Ohlson's (2005) Abnormal Earnings Growth; O'Byrne's (1997) EVA improvement; Miller and Modigliani's (1961) investment opportunities approach to valuation; Young and O'Byrne's (2001) Adjusted EVA; Keynes's (1936) user cost; Drukarczyk and Schueler's (2000) Net Economic Income; Fernández's (2002) Created Shareholder Value; Anthony's (1975) profit. They are all conveniently reinterpreted within the theoretical domain of the lost-capital paradigm and conjoined in a unified view. The results found make this new theoretical approach a good candidate for firm valuation, capital budgeting decision-making, managerial incentives and control.

Keywords. Accounting, corporate finance, residual income, value creation, management, incentive compensation, lost capital, Net Present Value, book value, market value.

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Introduction

Residual income is income in excess of an income that could be obtained if investors invested their funds at the opportunity cost of capital. Introduced in the first half of the past century (e.g. Preinreich, 1936, 1938) the term "residual income" has been first used by Solomons (1965). The terms "abnormal earnings" and "excess profit" are also used in management accounting and business economics to mean earnings (profit) in excess of normal earnings (profit). While it was a minor area of research in the 1950s and 1960s, the massive literature developed on project and firm valuation in the last forty years have induced a renewed interest on residual income, both as a valuation tool and as a basis for management compensation. Important works such as Peasnell's (1981, 1982) and Ohlson's (1989, 1995) in accounting finance, Rogerson's (1997) and Reichelstein's (1997) in management accounting and the proposal of Economic Value Added in applied corporate finance (Stewart, 1991) have triggered a considerable amount of contributions in various fields. In particular, the notion of residual income is used in several kinds of optimization problems. For example, a lively debate concerns the so-called principal-agent problem, where a firm's owner (principal) delegates investment to a better informed manager (agent), who is to be induced to optimal investment through an appropriate rewarding contract. In this research area, Rogerson (1997) consider the situation where the manager exerts an unobservable level of effort each period that increases the firm's cash flow; the principal is willing to maximize the NPV of the firm whereas the manager is willing to maximize his own utility. The principal knows the relative productivity of the asset, but not the absolute productivity which is known only to the manager. The author considers the use of an allocation rule (depreciation schedule+opportunity cost) and a contract according to which the manager's wage is weakly increasing with respect to residual income. In this context, the author shows that there is a unique allocation rule that maximizes both the firm's NPV and the manager's utility: such a rule is the so-called *Relative Marginal Benefit* rule and is given by the ratio of periodic relative productivity to the discounted sum of all periodic relative productivities: $\varrho_t/\big(\sum_{k=1}^{T} \frac{\varrho_k}{(1+i)^k}\big)$ where ϱ_t is the relative productivity and i is the opportunity cost of capital (Rogerson, 1997, eq. 25). In case productivity is constant, this allocation rule boils down to $1/\big(\sum_{k=1}^{T} \frac{1}{(1+i)^k}\big)$ which corresponds to the instalment of a T-year annuity whose present value is equal to one. In this operations management context, a significant contribution is Baldenius and Reichelstein (2005), who examine efficient inventory management from an incentive and control perspective: the firm delegates decisions on production to a manager who has superior information and affects sales revenues with his productive efforts. They propose to value inventory with a compounded historical cost valuation rule that capitalizes production costs and periodic holding costs and, in addition, treats inventory as an interest-accruing asset (the value of each unit remaining in ending inventory in a given period increases at the cost of capital i). The authors assume: (i) the manager's objective is to maximize the (expected) NPV of bonus payments, which are proportional to residual income, (ii) the optimal sales exceed the available production capacity in each period of the inventory cycle, (iii) the LIFO (last-in-first-out) inventory flow valuation rule is employed. This implies that value of inventory is $\big[\mathfrak{c}(1+i)^{t-t^*}+\sum_{k=0}^{t-t^*}(1+i)^k\big]x_t$, with $\mathfrak{c}$=unit production cost, x_t=ending inventory, t^*= beginning of inventory buildup. The authors show that the optimal production and sales plan that maximizes the firm's NPV is also the one that maximizes the NPV of manager's bonus payments; in the case where the manager receives updated information about future revenues after the initial production decision, the residual income based on the lower-of-cost-or-market rule becomes the optimal incentive mechanism (see also Dutta and Zhang, 2002, on production incentives, and Pfeiffer and Schneider, 2007). Dutta and Reichelstein (2005) analyze several different transactions: multi-year construction contracts, long-term leases, asset disposals, research and development. Stoughton and Zechner (2007) consider optimal capital allocation based on residual income in financial institutions assuming frictions in the market and the presence of an institution constituted by a risky division and a riskless division. Baldenius, Dutta, and Reichelstein (2006) deal with the case of optimal project selection in presence of several divisional managers. Mohnen and Bareket (2007) provide a special residual income that is capable of inducing the agent to optimally select a portfolio of projects, whereby the NPV is maximized. Grinyer and Walker (1990) and Stark (2000) focus on real-option frameworks and find that a residual income-type performance measure can be designed which supports optimal investment and disinvestment decisions (see also Friedl, 2005). Pfeiffer (2000) copes with performance measures based on residual income and adjustment of hurdle rates (in another context, Antle and Eppen, 1995, discuss the design of hurdle rates in a contracting setting including agency costs and asymmetric information). Anctil (1996) and Anctil et al. (1998) find appropriate assumptions such that even if the manager myopically maximizes residual income ignoring both future residual incomes and future cash flows, the resulting policy will lead, asymptotically, to NPV maximization. The importance of this area of research for management science is testified by Balachandran's findings (2006). In portfolio optimizazion, Claus and Thomas (2001) use the expected returns implied by the residual-income approach for forecasting the equity premium. The authors argue that the use of residual income is superior compared to the dividend growth model and estimate the equity premium for six countries, whose robustness is corroborated by sensitivity analyses. Hagemeister and Kempf (2006) use expected returns implied by the residual-income model for Markowitz-optimization: they optimally combine the residual-income-based estimator with the time series estimator using the Bayesian approach and find that such a combination generates a better performance when compared to traditional estimation and investment strategies. Hagemeister and Kempf (2007) use the same approach to test different versions of the Capital Asset Pricing Model. Desroisiers, Lemaire, and L'Her (2007) use residual income to deduce the implicit expected rates of return of nineteen countries: They consider zero-investment portfolios and implement a ranking strategy and a mean-variance optimization strategy, finding that the strategies posted positive performances.

A major element in residual income is played by the opportunity cost (capital charge), which represents the income that could be obtained by investing funds at the cost of capital. While the counterfactual feature of the opportunity cost as a foregone income is well known (Coase, 1968; Buchanan, 1969), no debate has ever taken place in the literature about possible alternative ways of computing such a counterfactual income. The traditional accepted formalization of opportunity cost rests on the assumption of investment of the *actual* capital at the cost of capital. In recent years, a new definition of residual income, called *Systemic Value Added*, has been proposed in Magni (2000a,b,c, 2001a,b, 2003), derived from the comparison between two alternative dynamic systems: The first one describes the net worth's evolution in case of project acceptance, the second one refers to project rejection. Rather than a particular metric, the Systemic Value Added is a paradigm, on the basis of which one can construct infinite possible metrics. The paradigm has been thoroughly studied by the author from several points of view: Conceptual, formal, theoretical, cognitive, empirical, historical (see Magni, 2004, 2005, 2006, 2009a,b; Ghiselli Ricci and Magni, 2006).

This paper revisits the Systemic-Value-Added paradigm, which is here renamed *lost-capital* paradigm. The purpose is to show that this new paradigm may be useful for both valuation and management compensation, and that it is capable of encompassing seemingly disparate perspectives conjoining them in one single theoretical domain. To this end, the lost-capital paradigm is thoroughly investigated in two senses: (i) Formal results are provided aimed at clarifying both the link between performance and value creation and the link between residual income and compensation plan; in addition, the formal and conceptual relations that the two paradigms bear one another are studied; (ii) several notions, models and results in the literature are considered, spanning from the 1930s up to most recent years, ranging from microeconomics to management accounting and corporate finance. Different as they are in aims and scope, they are here unified in the comprehensive theoretical domain of the lost-capital paradigm. In particular, after a brief introduction of the standard paradigm (section 1) the lost-capital paradigm is presented in section 2; in section 3 the two paradigms are connected via a cumulation procedure and an aggregation property is shown for the lost-capital paradigm. Section 4 focuses on Ohlson's (2005) Abnormal Earnings Growth and O'Byrne's (1997) Economic-Value-Added improvement; section 5 shows that a project's NPV is equal to the difference between its market value and the lost capital, and relates NPV, Market Value Added and lost-capital residual income. In section 6 lost capital is shown to coincide with the notion of O'Hanlon and Peasnell's (2002) *unrecovered capital* and the NPV (=Excess Value Created) is split in past and prospective lost-capital residual incomes; in section 7 it is shown that the lost-capital residual incomes is enfolded in Keynes's notion of user cost; the latter originates a goal congruent subclass of lost-capital residual incomes, here named *Keynesian Excess Profit*. Within this subclass, Drukarczyk and Schueler's (2000) *Net Economic Income* and the lost-capital companion of Fernández's (2002) *Created Shareholder Value* are briefly investigated (section 8). A final unification of the two paradigms is shown to be implied by Anthony's (1975) notion of profit: The use of his argument leads to a subclass of residual income models that belong to both paradigms (section 9). Some concluding remarks end the paper and a brief numerical example is illustrated in the Appendix.

Some caveats are worth underlining: (a) the analysis is meant to be valid for projects, firms, divisions, businesses. We will interchangeably use the terms 'project', 'firm', 'business'; (b) the terms 'outstanding balance' and 'outstanding capital' are used as synonyms and refer to the *actual* capital employed (which will be distinguished from the capital *infused* into the business); (c) we will be concerned with a project (firm) described by the cash-flow stream $\vec{d} = (-d_0, d_1, \ldots d_n)$. With no loss of generality, we will assume that the final cash flow d_n is inclusive of the project's terminal value (a finite-time horizon is assumed); (d) for the sake of notational convenience, cost of capital is constant (generalization to variable costs of capital is just a matter of symbology); (e) main notational conventions are collected in Table 0 at the end of the paper.

1 The standard paradigm

Consider a project (firm) $\vec{d}$=$(-d_0, d_1, d_2, \ldots, d_n)$ and a sequence of n uniperiodic subprojects $\vec{d_t}$ such that $\vec{d_t} = -y_{t-1} \cdot \vec{I_t} + (y_t + d_t) \cdot \vec{I}_{t+1}$, where $\vec{I_t}$:=$(0, 0, \ldots, 0, 1, 0, 0, \ldots, 0)$$\in \mathbb{R}^{n+1}$ is the vector with all zeros except the t-th entry which is equal to 1, $t = 1, 2, \ldots, n$. Consider the vector $\vec{y}$=$(y_0, y_1, y_2, \ldots, y_n)$ such that $y_0 := d_0$ and y_t is arbitrary in $\mathbb{R}$ for $t = 1, 2, \ldots n-1$. Let r_t be such that

$$y_{t-1} = \frac{y_t + d_t}{1 + r_t} \tag{1}$$

for t≥ 1. We may interpret y_{t-1} as the capital employed in $\vec{d_t}$ and r_t as the period rate of return. From eq. (1), one finds the recurrence equation linking successive capitals:

$$y_t(\vec{r}) = y_{t-1}(\vec{r})(1 + r_t) - d_t \tag{2}$$

(Soper, 1959; Teichroew et al. 1965a, 1965b. See also Peasnell, 1982, p. 366), where the functional dependence of the capital on the return rates is highlighted, being $y_t(\vec{r})$:=$y_t(r_1, r_2, \ldots, r_t)$ for $t \geq 1$. While $y_t(\vec{r})$ may be any number, from a financial point of view it is possible to interpret it as the actual capital employed in $\vec{d}$ at the beginning of the $(t+1)$-th period and define income as the product of capital invested $y_{t-1}(\vec{r})$ and rate of return r_t. The final $y_n(\vec{r})$ is determined by picking $t = n$ in eq. (2). If $y_n(\vec{r})$=0, then $\vec{d} = \sum_{t=1}^{n} \vec{d}_t$. The initial condition $y_0(\vec{r}) := d_0$ says that the initial outstanding capital employed to undertake the project coincides with the capital infused by the investors (it is a negative dividend). The Net Present Value (NPV) of subproject $\vec{d}_t$ is

$$-\frac{y_{t-1}(\vec{r})}{(1+i)^{t-1}} + \frac{y_t(\vec{r}) + d_t}{(1+i)^t},$$

which becomes, owing to eq. (2),

$$\frac{y_{t-1}(\vec{r})(r_t - i)}{(1+i)^t} \qquad t = 1, \ldots, n. \tag{3}$$

It is widely known that the sum of these uniperiodic NPVs is just the project's NPV:

$$\text{NPV} = \sum_{t=1}^{n} \frac{y_{t-1}(\vec{r})(r_t - i)}{(1+i)^t}$$

(see Peasnell, 1981, 1982; Peccati, 1987, 1989; Martin and Petty, 2000; Lundholm and O'Keefe, 2001; Martin et al. 2003; Vélez-Pareja and Tham, 2003; Ohlson, 2005).

Remark 1. It is worth noting that solving eq. (2) one finds

$$d_0(1+r)^{0,n} - \sum_{t=1}^{n} d_t(1+r)^{t,n} = y_n(\vec{r}) \tag{4}$$

where $(1+r)^{\tau,h}$:=$(1+r_{\tau+1})\ldots(1+r_h)$. The vector $\vec{r} = (r_1, r_2, \ldots r_n)$ is a discount function that generalizes the notion of internal rate of return. It is therefore an *internal discount function* (IDF) (see also Peasnell, 1981, p. 367). We stress that eq. (4) holds for any choice of $\vec{r}$ satisfying eq. (1) and for any choice of the outstanding balances $y_t(\vec{r})$ as well (r_t and $y_t(\vec{r})$ are in a biunivocal correspondence). This means that any such discount function $\vec{r}$ is an IDF for the cash-flow stream $(-d_0, d_1, \ldots, d_n + y_n(\vec{r}))$ (see also Ohlson, 2005). Let $\vec{r}^* = (r_1^*, r_2^*, \ldots, r_n^*)$ be such that $y_n(\vec{r}^*) = 0$. Then, $\vec{r}^*$ is an IDF for the cash-flow stream $(-d_0, d_1, \ldots, d_n)$.

Remark 2. If $y_t(\vec{r})$ is the equity book value B_t^e, then r_t is the Return On Equity (ROE), which determines an IDF for firm $\vec{d}$. Therefore, the ROE is an index with a genuine economic meaning (for relations between ROEs and internal rate of return, see also Peasnell, 1982; Brief and Lawson, 1990). The amount $r_t B_{t-1}^e$ is obviously the shareholders' net profit.

Remark 3. The recurrence equation (2) is a familiar relation in finance, used in the construction of amortization plans, and is consistent with the clean surplus relation often advocated in management accounting (Peasnell, 1982; Ohlson, 1989, 1995):

$$\text{cash flow} = \text{income} + \text{capital's depreciation}.$$

This sets a link between accounting and loan theory: The time-t outstanding balance is, in an amortization plan, the residual principal debt at time t; the IDF represents the contractual rate(s), the variation of the outstanding balance is the principal repayment, the cash flows are the instalments, and the product $r_t y_{t-1}(\vec{r})$ is the interest charge (see also Kellison, 1991; Promislow, 2006). The idea of income as interest is unambiguous and already recognized in the relevant literature (see Forker and Powell, 2000, p. 237). This analogy is perfectly fulfilled in Anthony's perspective, where equity is seen as a shareholders' credit (see section 9. See also Table 1).

Let $\vec{r}^* = (r_1^*, r_2^*, \dots, r_n^*)$ be an IDF for project $\vec{d}$, so that $y_n(\vec{r}^*) = 0$. We give the following definition:

Definition 1. The classical paradigm of residual income is formally represented by the set $\{x_t^a\}$ such that

$$x_t^a = y_{t-1}(\vec{r}^*)(r_t^* - i). \tag{5}$$

In the standard definition of residual income a capital charge $iy_{t-1}(\vec{r}^*)$, representing counterfactual income, is deducted from the actual income $r_t^* y_{t-1}(\vec{r}^*)$. The set $\{x_t^a\}$ of the standard paradigm consists of many infinite residual income (RI) models, depending on the choice of $\vec{r}^*$ and the choice of the cost of capital i. The former automatically determines the choice of $\vec{y}(\vec{r}^*)$, the latter depends on the perspective taken: Cost of equity if equity cash flows are considered, weighted average cost of capital if free cash flows are used, pre-tax weighted average cost of capital if capital cash flows are employed (see Ruback, 2002, and Fernández, 2002, for the notion of capital cash flow). Among others, the following ones belong to the set of the standard RI models:

Entity approach. Stewart's *Economic Value Added* (EVA) (Stewart, 1991) is found by selecting i=$wacc$, r^*=ROA, and d_t=free cash flow (consequently, $y(\vec{r}^*)$ is the book value of total liabilities). Madden's (1999) *Cash Flow Return On Investment* ($cfroi$) is an (inflation-adjusted) internal rate of return of the business, obtained by equating to zero the sum of the discounted free cash flows. The $cfroi$-based residual income is found by picking d_t=free cash flows, i=$wacc$, and r_t^*=IRR (the outstanding balance $y(\vec{r}^*)$ is automatically determined by eq. (2)).

Equity approach. The *Edwards-Bell-Ohlson* (EBO) model (Edwards and Bell, 1961; Ohlson, 1989, 1995) is obtained by choosing i=k_e, d_t=equity cash flow, and r^*=ROE (therefore $y(\vec{r}^*)$ is the book value of equity. See also Arnold, 2005). Fernández's (2002) *Created Shareholder Value* (CSV) is found by picking $y_t(\vec{r}^*)$=V_t^e ($\vec{r}^*$ is automatically determined by eq. (2)) and i=k_e. An equity version of the $cfroi$ is what we here name *Cash Flow Return On Equity* ($cfroe$), which is the internal rate of return obtained by equating to zero the sum of the discounted equity cash flows, i.e. d_t=equity cash flow ($y(\vec{r}^*)$ is automatically determined by the usual recursive equation). The resulting RI model is found by selecting i=k_e.

2 The lost-capital paradigm

In this section we revisit the *Systemic Value Added* model, relabelling it the "lost-capital paradigm". In Magni (2000a,b, 2001a,b, 2003) attention is drawn on shareholders' wealth. It is assumed that, in case of acceptance of the project, shareholders reinvest the equity cash flows at the cost of capital i (this is the standard assumption of the NPV rule). Therefore, in each period shareholders' wealth is a portfolio of the project and the proceeds of the reinvestments. The all-comprehensive profit (inclusive of income from the project and earned interest from the reinvestments) is $r^* y_{t-1}(\vec{r}^*) + iC_{t-1}$, where C_{t-1} is the value, at time $t-1$, of the reinvestment proceeds, which evolves according to the dynamic system $C_t = C_{t-1}(1+i) + d_t$. Suppose, instead, that the project is not undertaken and the amount d_0 is invested at the cost of capital: Letting C^t be its compounded value at time t (C^0=C_0+d_0 is the initial investor's wealth), wealth evolves according to the dynamic system $C^t = C^{t-1}(1+i)$, so that the income is iC^{t-1}. The residual income is given by the difference of the two alternative incomes, and is called *Systemic Value Added* because it is deduced from the two dynamic systems:

$$\text{Systemic Value Added} = \big(r^* y_{t-1}(\vec{r}^*) + iC_{t-1}\big) - iC^{t-1}. \tag{6}$$

This residual income consists of three parts: $r^* y_{t-1}(\vec{r}^*)$ represents income from investment in the business, iC_{t-1} represents earned interest from reinvestment proceeds, iC^{t-1} is the income that shareholder forgo if project is undertaken. Note that, in Magni's model, C_{t-1} is part of the investor's actual wealth, whereas C^{t-1} is a foregone capital. We here revisit this paradigm by adopting an arbitrage-type perspective, which enables us to dispense with the reinvestment assumption of interim cash flows. To this end, one can construct a twin asset that replicates the project's payoff. This is accomplished by assuming that d_0 is invested at the cost of capital and that, at the end of each period, cash flow d_t is withdrawn from the asset's balance. So doing, the cash-flow stream of the project is replicated and, at the end of the n-th period, the residual capital $y_n(i)$ is obtained as an arbitrage gain (or loss). In other terms, the two alternatives are represented by a double application of eq. (2) with two different IDFs: The first one is an arbitrary vector $\vec{r} = (r_1^*, r_2^*, \ldots, r_n^*)$ such that $y_n(\vec{r}^*) = 0$, the second one is the vector of the costs of capital (which, we remind, are here assumed to be constant: $\vec{\imath} = (i, i, \ldots, i)$):

$$y_t(\vec{r}^*) = y_{t-1}(\vec{r}^*)(1 + r_t^*) - d_t \tag{7}$$

$$y_t(i) = y_{t-1}(i)(1 + i) - d_t; \tag{8}$$

the first dynamic system represents the evolution of the actual outstanding balance, the second one represents the path the balance would follow if investors invested their funds at the cost of capital while withdrawing, at each period, the cash flow d_t from the balance. Under this interpretation, $y_t(\vec{r}^*)$ is the actual capital employed by investors, whereas $y_t(i)$ is the capital that would be (or have been) employed if, at time 0, investors decided (or had decided) to invest funds at the cost of capital. The amount $y_t(i)$ is therefore the capital sacrificed by investors: The lost capital. Thus, $r_t^* y_{t-1}(\vec{r}^*)$ represents the actual income in the t-th period, whereas $iy_{t-1}(i)$ represents the lost income. The difference between actual income and lost income gives the lost-capital (LC) residual income.

Definition 2. The lost-capital paradigm is formally represented by the set $\{\xi_t^a\}$ such that

$$\xi_t^a = r_t^* \, y_{t-1}(\vec{r}^*) - i \, y_{t-1}(i) \tag{9}$$

Remark 4. Eq. (9) is just eq. (6) disguised in a different shape, given that $C^t - C_t = y_t(i)$ for every t (see Magni, 2000a, 2003, 2005): The lost capital may therefore be decomposed into an actual capital C_t and a foregone capital C^t. In his papers Magni shows that the lost capital is just the outstanding capital of a *shadow* project whose standard residual income coincides with the lost-capital residual income of project $\vec{d}$.[1]

Remark 5. Eq. (9) may be conveniently derived from an accounting perspective. Consider two mutually exclusive courses of action: Investing funds at the corporate rate of return, as opposed to investing funds at the corporate cost of capital. The two alternative courses of action give rise to two alternative clean-surplus type relations:

$$d_t = r_t^* \, y_{t-1}(\vec{r}^*) - \Delta \, y_t(\vec{r}^*)$$
$$d_t = i \, y_{t-1}(i) - \Delta \, y_t(i).$$

Subtracting the latter from the former, we have $\Delta \, y_t(\vec{r}^*) - \Delta \, y_t(i) = \xi_t^a$. Given that depreciation is capital's variation changed in sign, the latter equality informs that periodic performance is positive if and only if the depreciation of the firm's capital is higher upon investing funds at the cost of capital rather than at the corporate actual rate of return.

Remark 6. The LC residual income is linked to depreciation in two different senses:

[1] In this paper, we do not focus on this interpretation for reasons of space. See Magni (2000a, 2004, 2005, 2006).

- *depreciation through time*: eq. (2) and eq. (9) imply

$$\xi_t^a = \underbrace{[y_{t-1}(i) - y_t(i)]}_{\text{rejection}} - \underbrace{[y_{t-1}(\vec{r}^*) - y_t(\vec{r}^*)]}_{\text{acceptance}} \tag{10}$$

where each depreciation charge refers to time, in the two alternative cases of project rejection and acceptance, respectively

- *depreciation through use*: eq. (10) may be rewritten as

$$\xi_t^a = \underbrace{[y_{t-1}(i) - y_{t-1}(\vec{r}^*)]}_{\text{time } t-1} - \underbrace{[y_t(i) - y_t(\vec{r}^*)]}_{\text{time } t} \tag{11}$$

where each depreciation charge refers to different uses of the funds, at time $t-1$ and time t respectively. Scott (1953) observes that "economists cannot afford to lump together, as "depreciation", changes in present value caused by the passage of time, and by use" (p. 371). In fact, the LC paradigm does enable one to lump together *depreciation through time* and *depreciation through use*.

Whenever a metric in the classical paradigm is constructed, a corresponding metric in the LC paradigm is univocally determined. Let $\mathcal{L}$ the mathematical operator that transforms standard metrics in LC metrics: $\mathcal{L}: x_t^a \to \xi_t^a$. If ξ_t^a is the image of x_t^a via $\mathcal{L}$, i.e. $\xi_t^a = \mathcal{L}(x_t^a)$, then ξ_t^a is said to be the LC-companion of x_t^a. For example, the LC companions of EVA, EBO, and CSV are, respectively,

$$\mathcal{L}(\text{EVA}_t) = \text{ROA}_t \cdot B_t^l - wacc \cdot y_t(wacc)$$

$$\mathcal{L}(\text{EBO}_t) = \text{ROE}_t \cdot B_t^e - k_e \cdot y_t(k_e)$$

$$\mathcal{L}(\text{CSV}_t) = \begin{cases} r_t^* d_0 - k_e\, d_0 & t = 1 \\ k_e\, V_t^e - k_e\, y_t(k_e) & t > 1 \end{cases}$$

where $r_1^* = (V_1^e + d_1 - d_0)/d_0$ (see also Table 2 and the numerical example in the Appendix).

3 Valuation and aggregation property

This section shows that a cumulation of past residual incomes is intrinsically incorporated in the definition of LC residual income, which enables one to show an important aggregation property of LC residual income.

Proposition 1. *The lost-capital RI is equal to the sum of the standard RI plus accumulated interest on past standard RIs:*

$$\xi_1^a = x_1^a \qquad \text{and} \qquad \xi_t^a = x_t^a + i\sum_{k=1}^{t-1} x_k^a u^{t-1-k} \qquad \text{for } t > 1 \tag{12}$$

where $u := 1 + i$.

Proof. The first equation is obvious, given that $y_0(\vec{r}^*) = y_0(i)$. Using the usual recursive (clean surplus) relation $d_k = y_{k-1}(\vec{r}^*)(1 + r_k^*) - y_k(\vec{r}^*)$ one finds

$$y_{t-1}(i) = y_0(i)u^{t-1} - \sum_{k=1}^{t-1} d_k u^{t-1-k} = y_0(i)u^{t-1} - \sum_{k=1}^{t-1}\big(y_{k-1}(\vec{r}^*)(1 + r_k^*) - y_k(\vec{r}^*)\big)u^{t-1-k}.$$

Upon algebraic manipulations, $y_{t-1}(i) = y_{t-1}(\vec{r}^*) - x_1^a u^{t-2} - x_2^a u^{t-3} - \ldots - x_{t-1}^a$. Therefore, $\xi_t^a = r_t^* y_{t-1}(\vec{r}^*) - i y_{t-1}(i)$, which is equal to $r_t^* y_{t-1}(\vec{r}^*) - i(y_{t-1}(\vec{r}^*) - x_1^a u^{t-2} - x_2^a u^{t-3} + \ldots - x_{t-1}^a)$. □

Remark 7. Using induction on eq. (12) it is easily proved that

$$\sum_{k=1}^{t} \xi_k^a = \sum_{k=1}^{t} x_t^a u^{t-k} \qquad \text{for every } t \geq 1 \tag{13}$$

(see Magni, 2005, Lemma 2.4 and Theorem 2.2, for a generalization of eqs. (12) and (13)). Applying both eqs. (12) and (13) one finds

$$\xi_t^a = x_t^a + i \sum_{k=1}^{t-1} \xi_k^a \qquad \text{for every } t > 1 \tag{14}$$

which expresses the LC residual income in terms of cumulations of past LC residual incomes.

Eq. (13) implies that projects and firms can be appraised through the LC paradigm by reversing the role of summing and discounting: The standard-type residual income model is tied to the Net Present Value via a *discount-and-sum* procedure, whereas the LC paradigm employs a *sum-and-discount* procedure. Letting $v{:=}u^{-1}{=}(1+i)^{-1}$ and reminding that $\sum_{k=1}^{n} x_k^a v^k{=}\text{NPV}$, if one picks $t{=}n$ in eq. (13) one obtains

$$v^n \sum_{k=1}^{n} \xi_k^a = v^n \sum_{k=1}^{n} x_k^a u^{n-k} = \sum_{k=1}^{n} v^k x_k^a = \text{NPV}. \tag{15}$$

Residual incomes are first summed, and then discounted: The reverse of the classical procedure. In terms of Net Final Value one gets, at time n,

$$N_n = \text{NPV}(1+i)^n = \sum_{k=1}^{n} \xi_k^a. \tag{16}$$

The Net Final Value is given by the *uncompounded* sum of all residual incomes ξ_t^a. This means that the LC residual income is additively coherent.[2] Note also that, replacing r with i in eq. (4), the terminal lost capital is just the project's Net Final Value (changed in sign): $y_n(i) = -N_n$. Thus, the terminal lost capital may be found by summing the past residual incomes: $y_n(i) = -\sum_{t=1}^{n} \xi_t^a$. The additive coherence, far from being a mere elegant formal property, unfolds the powerful property of income aggregation, as opposed to discounting. That is, equations (15) and (16) show that capital budgeting problems may be solved by dispensing with forecasting each and every cash flow and, in addition, by dispensing with forecasting each and every residual income. If the lost-capital paradigm is used, only the grand total residual income that a firm (project) releases within the fixed horizon is relevant. One does not have to worry about timing. This additive coherence reflects the aggregation property of accounting. Given that NPV$=V_0{-}d_0$ with $d_0{=}y_0(\vec{r}^*)$, one can express the firm's market value as a function of the outstanding capital and the grand total residual income:

$$V_0 = y_0(\vec{r}^*) + v^n \sum_{k=1}^{n} \xi_k^a. \tag{17}$$

Picking $y_k(\vec{r}^*){=}B_k^e$ and $i{=}k_e$ one may write

$$V_0^e = B_0^e + \frac{1}{(1+k_e)^n} \sum_{k=1}^{n} \text{abnormal earnings}. \tag{18}$$

Lost-capital abnormal earnings aggregate in a value sense and prediction in each of the following years is not needed. Value is derived from knowledge about total abnormal earnings in a span of n years, no matter how

[2] See Magni (2009a) for the property of antisymmetry of the LC residual income and its implications.

they distribute across periods. One may estimate an average abnormal earning for a future span of years and multiply by the number of years to obtain the Net Final Value. By discounting back and adding the equity book value one gets the equity market value. Section 6 provides a generalization of eq. (18) when the analysis starts at time $t>0$.

Remark 8. The Net Final Value N_n may be reexpressed in a further fashion, where no capitalization process is involved for the standard RIs, while the lost-capital RIs are only linearly compounded. Expanding eq. (14),

$$\begin{aligned}
\xi_1^a &= x_1^a \\
\xi_2^a &= x_2^a + i\xi_1^a \\
\xi_3^a &= x_3^a + i(\xi_1^a + \xi_2^a) \\
\ldots &= \ldots \\
\xi_n^a &= x_n^a + i(\xi_1^a + \xi_2^a + \ldots \xi_{n-1}^a)
\end{aligned} \tag{19}$$

and, summing by column,

$$\sum_{t=1}^{n} \xi_t^a = \sum_{t=1}^{n} x_t^a + \sum_{t=1}^{n} i(n-t)\xi_t^a. \tag{20}$$

which implies $N_n = \sum_{t=1}^n x_t^a + \sum_{t=1}^n i(n-t)\xi_t^a$, owing to $\sum_{t=1}^n \xi_t^a = N_n$. The project's Net Final Value may therefore be viewed as a double sum of residual incomes: A sum of uncompounded conventional RIs plus a sum of linearly compounded LC residual incomes.

Remark 9. Eq. (12) is significant for management compensation purposes. It tells us that any LC measure may be reinterpreted in standard terms. An LC performance metric may be interpreted as an index ξ_t^a constructed from the standard paradigm on the basis of (not only current but also) past (standard) performances: $\xi_t^a = \xi_t^a(x_1^a, x_2^a, \ldots, x_t^a)$. In a compensation plan based on the function ξ_t^a (where compensation is increasing with respect to ξ_t^a) performances are amplified with respect to the usual standard metrics: past positive performances play an insurance role against current negative performance: if $x_t^a < 0$ and $x_k^a > 0$ for $1 \leq k \leq t-1$, one finds $\xi_t^a > x_t^a$ and, possibly, $\xi_t^a > 0 > x_t^a$; past negative performance play a penalty role for current positive performances: if $x_t^a > 0$ and $x_k^a < 0$ for $1 \leq k \leq t-1$, one finds $\xi_t^a < x_t^a$, and, possibly, $\xi_t^a < 0 < x_t^a$ (in case the sequence $\{x_k^a\}$ has nonhomogenous sign, the net effect depends on the relative weight of positive past performances on negative past performances). The behaviour of an LC metric should therefore attract value-based management scholars, who often recommend a functional dependence of compensation on past performances in the form of cumulation of residual incomes (e.g. Stewart, 1991; Ehrbar, 1998; Young and O'Byrne, 2001): a compensation plan should somehow consider capitalization of previous residual incomes. Equation (12) incorporates it in the definition of residual income itself. An LC-based compensation plan should in principle imply a less myopic management's behaviour, for (i) managers are rewarded on the basis of two benchmarks: the rate of return they should guarantee (profitability) and the capital they should reach (capital accumulation) in a given period, (ii) managers are aware that their current performance reverberate on the future: past performances will not be "forgotten", (iii) dependance of compensation on past performances is nontrivial, so that managers will have problems in gaming the measure. It is also worth underlining that any LC metric may be viewed as a standard metric with an adjusted cost of capital: from eqs. (5) and (9) one finds $\xi_t^a = y_t(r_t^* - j_t)$ with $j_t = i + i \cdot [y_{t-1}(i) - y_{t-1}(\vec{r^*}))/y_{t-1}(\vec{r^*})]$: the benchmark for a positive management compensation is given by the return rate i plus the return on the relative excess capital (which may be positive or negative). If, for example,

i=0.1, $y_{t-1}(\vec{r^*})$=80, $y_{t-1}(i)$=90, the capital invested in case of rejection of $\vec{a}$ would have been greater than the capital actually invested by a 12.5%=(90-80)/80. This means that a 10% on that 12.5% could have been earned, so that an additional 1.25% would have accrued. Therefore, for a positive performance to occur, the period internal rate must be greater than 10%; in particular, the threshold level is i=11.25%=10% +1.25%.

4 Ohlson's Abnormal Earnings Growth, O'Byrne's EVA improvement, and LC residual income

The notion of Abnormal Earnings Growth (AEG), recently proposed by Ohlson (2005) as a method of firm valuation, is arousing interest among management accounting scholars (see Ohlson and Juettner-Nauroth, 2005; Penman, 2005; Brief, 2007). AEG is the difference between two (standard) consecutive residual earnings (equity perspective). This very concept has been previously used and studied for value-based management purposes by O'Byrne (1996, 1997) and Young and O'Byrne (2001). The so-called EVA improvement is just the AEG in an entity perspective. In their 2001 book, Young and O'Byrne illustrate a numerical example (p. 29) where the future value of EVA improvement is calculated period by period.[3] They explain the way they compute the future value of EVA improvement as follows: "We do this by multiplying the prior-year future value by 1.10 (1+the WACC of 10 percent) and then adding current-year excess EVA improvement" (p. 40). Formalizing their algorithm and denoting with F_t the future value of EVA improvement,

$$F_t = F_{t-1}(1+i) + \text{EVA}_t - \text{EVA}_{t-1}. \tag{21}$$

Let us generalize the above equation by replacing EVA with the generic residual income x_t^a so as to redefine AEG to include both equity and entity perspective:

$$z_{t-1} = x_t^a - x_{t-1}^a \qquad t = 1, 2, \ldots, n \tag{22}$$

with z_0:=x_1^a. The future value of cumulated AEGs may be formalized as

$$F_t = F_{t-1}(1+i) + z_{t-1}. \tag{23}$$

We may interpret the above equation as representing the growth in the "AEG account". As the account starts from zero (at the beginning of the project, no residual income has been generated), it is natural to take the boundary condition F_0:=0. The account grows by a normal return iF_{t-1} plus an abnormal return z_{t-1}.[4]. Using

[3]Rigorously speaking, the authors compute the future value of the *Excess* EVA improvement but, given their assumptions of no excess future growth value, excess EVA improvement equals EVA improvement (see O'Byrne, 1997, for relations among excess EVA improvement, future growth value, and excess return).

[4]The notion of future value of cumulated AEGs is quite natural, given that AEG measures the growth of abnormal earnings (for this reason GAE might be a better acronym. See Brief, 2007, p. 433)

eq. (23), one finds

$$\begin{aligned}
F_1 &= 0(1+i) + z_0 \\
F_2 &= z_0(1+i) + z_1 \\
F_3 &= z_0(1+i)^2 + z_1(1+i) + z_2 \\
\vdots &= \quad \vdots \\
F_t &= z_0(1+i)^{t-1} + z_1(1+i)^{t-2} + z_2(1+i)^{t-3} + \ldots + z_{t-1}.
\end{aligned} \tag{24}$$

We may then prove the following

Proposition 2. *The future value of cumulated AEGs is equal to the lost-capital residual income*

$$\xi_t^a = F_t = \sum_{k=1}^{t} z_{k-1} u^{t-k} \tag{25}$$

Proof. Reminding that $z_0 = x_1^a$ and using eqs. (22) and (24), simple manipulations lead to

$$\begin{aligned}
F_t &= x_1^a u^{t-1} + (x_2^a - x_1^a) u^{t-2} + \ldots + (x_t^a - x_{t-1}^a) \\
F_t &= i x_1^a u^{t-2} + i x_2^a u^{t-3} + \ldots + i x_{t-1}^a + x_t^a \\
F_t &= x_t^a + i \sum_{k=1}^{t-1} x_k^a u^{t-1-k}
\end{aligned}$$

From eq. (12), $x_t^a + i \sum_{k=1}^{t-1} x_k^a u^{t-1-k} = \xi_t^a$, so that $F_t = \xi_t^a$. □

Remark 10. Young and O'Byrne (2001, p. 42) illustrate a numerical example where the notions of Adjusted Invested Capital and Adjusted EVA are introduced. In the example, they assume earnings=dividends. It is easy to show that the two notions correspond to the notions of lost capital and LC residual income. The recurrence equations for the two notions, inferred from the authors' explanations at p. 42 and the numbers in the Table, are as follows:

$$\begin{aligned}
\mathrm{AIC}_t &= \mathrm{AIC}_{t-1} - \mathrm{AE}_t \\
\mathrm{AE}_t &= \mathrm{Earnings}_t - wacc \cdot \mathrm{AIC}_{t-1}.
\end{aligned} \tag{26}$$

where *wacc* coincides with the cost of equity, given their assumption of zero debt. The two equations yield

$$\begin{aligned}
\mathrm{AIC}_t &= \mathrm{AIC}_{t-1} - \mathrm{Earnings}_t + wacc \cdot \mathrm{AIC}_{t-1} \\
&= \mathrm{AIC}_{t-1} \cdot (1 + wacc) - \mathrm{Earnings}_t
\end{aligned} \tag{27}$$

If one assumes $\mathrm{Earnings}_t$=dividends, eq. (27) corresponds to the recurrence equation for $y_t(wacc)$ (see eq.(8)), so that $\mathrm{AIC}_t = y_t(wacc)$. As a result, AE_t in eq. (26) is equal to the lost-capital EVA as well as to the future value of cumulated AEGs: $\mathrm{AE}_t = \mathcal{L}(\mathrm{EVA}_t) = F_t$.

Remark 11. Reminding that $y_0(\vec{r}^*) = d_0 = B_0$, eqs. (17) and (25) imply

$$\begin{aligned}
V_0 = B_0 + \mathrm{NPV} = B_0 + v^n \sum_{t=1}^{n} \xi_t^a &= B_0 + v^n \sum_{t=1}^{n} F_t \\
&= B_0 + v^n \sum_{t=1}^{n} \sum_{k=1}^{t} z_{k-1} u^{t-k} = B_0 + \sum_{t=1}^{n} \sum_{k=1}^{t} z_{k-1} v^{n-t+k}
\end{aligned} \tag{28}$$

Disentangling the double sum in eq. (28), one finds

$$\begin{array}{lllllll}
\sum_{t=1}^n \sum_{k=1}^t v^{n-t+k} z_{k-1} & = z_0 v^n & & & & & \\
& + z_0 v^{n-1} & + z_1 v^n & & & & \\
& + z_0 v^{n-2} & + z_1 v^{n-1} & + z_2 v^n & & & \\
& \vdots & \vdots & \vdots & & & \\
& + z_0 v & + z_1 v^2 & + z_2 v^3 & + z_3 v^4 \quad \dots & + z_{n-2} v^{n-1} & + z_{n-1} v^n
\end{array}$$

The t-th column of the above sum may be written as $\sum_{k=t}^n z_{t-1} v^k$. Summing the n columns, $\sum_{t=1}^n \sum_{k=t}^n z_{t-1} v^k = \sum_{t=1}^n \sum_{k=1}^t z_{k-1} v^{n-t+k}$. Hence,

$$V_0 = B_0 + \sum_{t=1}^{n} \sum_{k=t}^{n} z_{t-1} v^k. \tag{29}$$

Therefore, the lost-capital paradigm gives us the opportunity of viewing AEG with the book value as the anchoring value.[5] The generalization for infinite-lived firms is straightforward:

$$\begin{aligned}
V_0 &= B_0 + \lim_{n\to\infty} \sum_{t=1}^{n} \sum_{k=t}^{n} z_{t-1} v^k \\
&= B_0 + \sum_{t=1}^{\infty} \sum_{k=t}^{\infty} z_{t-1} v^k = B_0 + \sum_{t=1}^{\infty} z_{t-1} \frac{v^t}{1-v} \\
&= B_0 + \frac{1}{i} \sum_{t=1}^{\infty} z_{t-1} v^{t-1} = B_0 + \frac{z_0}{i} + \frac{1}{i} \sum_{t=1}^{\infty} z_t v^t.
\end{aligned} \tag{31}$$

The latter is just the fundamental EVA equation. O'Byrne (1996, p. 117) introduces this equation by making use of Miller and Modigliani's (1961) investment opportunities approach to valuation; Miller and Modigliani's approach is substantiated in their equation (12), where they include the excess profit generated by the increase in physical assets. Such an excess profit, in the language of EVA, is just the EVA improvement.[6]

5 Tying lost capital to value creation

The Net Present Value of an asset is commonly defined as the difference between the market value of the asset and the capital infused into it at a certain time. This implies that the capital infused may defined as follows:

[5] If one is willing to highlight the first-period earnings as anchoring value (as is done in Ohlson, 2005), one finds

$$\sum_{t=1}^{n} v^t z_{t-1} = \sum_{t=1}^{n} v^t x_t^a - v(\sum_{t=1}^{n} v^t x_t^a) + v^{n+1} x_n^a = N_0 - vN_0 + v^{n+1} x_n^a = ivN_0 + v^{n+1} x_n^a$$

where N_0:=NPV. Reminding that x_{n+1}^a=0 (the project ends at time n), so that z_n=$-x_n^a$, one finds

$$N_0 = \frac{(1+i)}{i} \left(\sum_{t=1}^{n} v^t z_{t-1} + v^{n+1} z_n \right) = \frac{1}{i} \left(\sum_{t=1}^{n} v^{t-1} z_{t-1} + v^n z_n \right) = \frac{1}{i} \left(\sum_{t=0}^{n} v^t z_t \right).$$

Using the fact that z_0=x_1^a=$(r_1^* - i) y_0(\vec{r}^*)$ with $r_1^* y_0(\vec{r}^*)$ being the first-period income, one gets

$$V_0 = N_0 + y_0(\vec{r}^*) = \frac{r_1^*}{i} y_0(\vec{r}^*) + \left(\sum_{t=1}^{n} v^t z_t \right) = \frac{\text{Income}_1}{i} + \frac{1}{i} \left(\sum_{t=1}^{n} v^t z_t \right). \tag{30}$$

[6] An equivalent formulation of Miller and Modigliani's equation (12) is anticipated in Bodenhorn (1959) and in Walter (1956).

Definition 3. *At each time t, the capital infused by an investor into an asset is given by the difference between the market value of the asset and its Net Present Value.*

Armed with the above definition, we show the following

Proposition 3. *For every t, the lost capital $y_t(i)$ is the capital infused into the project at time t:*

$$y_t(i) = V_t - N_t. \tag{32}$$

Proof. Reminding that $y_0(\vec{r}){:=}d_0$ for any return rate r_t, using eq. (8) one finds

$$y_t(i) = d_0 u^t - \sum_{k=1}^{t} d_k u^{t-k}; \tag{33}$$

however, $V_t = \sum_{k=t+1}^{n} d_k u^{t-k}$ and $N_t{=}u^t{\cdot}\text{NPV}{=}\sum_{k=1}^{n} d_k u^{t-k} - d_0 u^t$, whence

$$V_t - N_t = d_0 u^t - \sum_{k=1}^{t} d_k u^{t-k}. \tag{34}$$

Eqs. (33) and (34) coincide. □

While the notion of lost capital has been previously introduced as a foregone capital, Proposition 3 allows us to reinterpret it as the capital infused by investors into the firm at the beginning of each period: The time-t Net Present Value N_t just measures by how much the (market) value of the firm exceeds (if positive) the capital infused into the business. Such a capital is not $y_t(\vec{r}^*)$, as could erroneously be expected: It is just the lost capital. If one deducts $y_t(\vec{r}^*)$ from V_t, one obtains what may be called the *generalized* Market Value Added (gMVA). If book values are selected for $\vec{y}$, the gMVA boils down to the well-known Market Value Added (MVA).

$$N_t = V_t - y_t(i) \tag{35}$$

$$\text{gMVA}_t = V_t - y_t(\vec{r}^*) \tag{36}$$

Proposition 4. *For every $t{\geq}1$, the difference between the Net Present Value and the Market Value Added is given by the (uncompounded) past lost-capital residual incomes:*

$$N_t - \text{MVA}_t = \sum_{k=1}^{t} \xi_k^a \tag{37}$$

Proof. From eq. (10) we have

$$\begin{aligned}\sum_{k=1}^{t} \xi_k^a &= \sum_{k=1}^{t} [y_{k-1}(i) - y_k(i)] - [y_{k-1}(\vec{r}^*) - y_k(\vec{r}^*)] \\ &= y_t(\vec{r}^*) - y_t(i).\end{aligned} \tag{38}$$

Picking $y_t(\vec{r}^*) = B_t^e$, eq. (36) becomes $\text{MVA}_t = V_t - B_t^e$. Deducting the latter from eq. (35) and using eq. (38) one gets eq. (37). □

Proposition 4 says that if one uses the Market Value Added to measure value creation, one forgets the past residual incomes. In other words, value creation is obtained by adding to the firm's Market Value Added the LC residual incomes generated in the past. This very Proposition highlights the major role played by the LC residual income as a measure of excess variation of Net Present Value upon Market Value Added.

Corollary 1. *The LC residual income is the difference between NPV's variation and MVA's variation:*

$$\xi_t^a = \Delta N_t - \Delta MVA_t. \tag{39}$$

Proof. From eq. (37) we have $N_{t-1} - \text{MVA}_{t-1} = \sum_{k=1}^{t-1} \xi_t^a$. Subtracting the latter from eq. (37) one gets eq. (39). □

Proposition 5. *The firm's outstanding balance is given by the sum of the capital infused and the (uncompounded) past lost-capital residual incomes:*

$$y_t(\vec{r}^*) = y_t(i) + \sum_{k=1}^{t} \xi_k^a. \tag{40}$$

Proof. Straightforward from eq. (38) □

The above Proposition provides the relation among the outstanding balance, the lost capital and past residual incomes. The relation holds for any $y_t(\vec{r}^*)$, in particular for $y_t(\vec{r}^*) = B_t$, so one is given the link connecting book value, lost capital and past residual incomes.

Propositions 3-5 show that the investors' commitment to the business is the lost capital, not the actual outstanding capital, and, in particular, not the book value. The relation between $y_t(\vec{r}^*)$ and $y_t(i)$ unveils the relation between the MVA and the NPV. At each date, the Net Present Value N_t is an overall measure taking account of the entire life of the project. Therefore, it comprises both a forward-looking and a backward-looking perspective. In contrast, the Market Value Added erases the past and limits its perspective to prospective cash flows: In its view the firm incorporates (the project begins) at time t. Net Present Value and Market Value Added may be seen as different ways of splitting the market value of equity: From eqs. (35) and (36),

$$V_t = N_t + y_t(i) \tag{41}$$

$$V_t = g\text{MVA}_t + y_t(\vec{r}^*). \tag{42}$$

Eq. (41) determines an unambiguous partition of V_t, given a cash-flow $\vec{d}$ and a cost of capital i. Eq. (42) originates a set of infinite partitions, one for any choice of $\vec{r}^*$.[7]

6 O'Hanlon and Peasnell's approach and the lost capital

This section shows that the approach of O'Hanlon and Peasnell (2002) is consistent with the LC paradigm and proposes lost-capital splitting identities. In their paper, O'Hanlon and Peasnell (OP) introduce the notion of *Excess Value Created* (EVC), which is based on the notion of "unrecovered capital". They define EVC as the difference

$$EVC_t = V_t^e - U_t^0 \tag{43}$$

where U_t^0 is the unrecovered capital: $U_t^0 = d_0(1+k_e)^t - \sum_{k=1}^{t} d_k(1+k_e)^{t-k}$. Owing to eq. (33), the unrecovered capital is just the capital lost by shareholders: $U_t^0 = y_t(k_e)$. The EVC, which OP acknowledge as analogous to

[7] To be rigorous, one should write $g\text{MVA}_t(\vec{r}^*)$ rather than $g\text{MVA}_t$, because the generalized MVA changes as $\vec{r}^*$ changes.

Young and O'Byrne's (2001) excess return, actually coincides with the time-t Net Present Value N_t, and eq. (43) is the equity version of our eq. (35): $U_t^0=y_t(k_e)$ and $N_t=\text{EVC}_t$. In their Proposition 1 (p. 233), OP show that the book value of equity may be written as the sum of the unrecovered capital and the compounded past residual incomes, and in their Proposition 2 (pp. 233-234) they show that the EVC equals the sum of compounded residual incomes and the Market Value Added. Using our symbols, OP show that

$$B_t^e = y_t(k_e) + \sum_{k=1}^{t} x_t^a(1+k_e)^{t-k} \tag{44}$$

$$N_t = \sum_{k=1}^{t} x_k^a(1+k_e)^{t-k} + \sum_{k=t+1}^{n} x_k^a(1+k_e)^{t-k} \tag{45}$$

It is worth noting that our Propositions 5 and 4 are, respectively, the LC-companions of OP (2002)'s Propositions 1 and 2. In particular, to pass from eq. (40) to eq. (44) and from eq. (37) to eq. (45) one just has to use eq. (13) with $i=k_e$ and $r^*=\text{ROE}$. However, the following Propositions directly tie the LC paradigm to value creation, dispensing with the notion of market value added (and, therefore, dispensing with the standard RI models).

Proposition 6. *For every $t \geq 1$, the time-t Net Present Value is given by the sum of all LC residual incomes, discounted at time t:*

$$N_t = v^{n-t} \sum_{k=1}^{n} \xi_k^a$$

Proof. We have $N_t=v^{n-t}\sum_{k=1}^{n} x_k^a u^{n-k}$. Using eq. (13) with $t=n$ the thesis follows. □

Consider now the project generated by the truncation of $\vec{d}$ from time 0 to time $t-1$, or, which is the same, generated by the sum of subprojects $\vec{d}_{t+1}$, $\vec{d}_{t+2}$, … $\vec{d}_n$ (with $\vec{r}=\vec{r}^*$). Denote this project by $\vec{d}_{t,n}$. Then,

$$\vec{d}_{t,n} = \sum_{k=t+1}^{n} \vec{d}_k = (0, 0, \dots, 0, -y_t(\vec{r}^*), d_{t+1}, d_{t+2}, \dots, d_n) \in \mathbb{R}^{n+1}.$$

In other words, $\vec{d}_{t,n}$ is the future part of project $\vec{d}$. Letting

$$\vec{d}_{0,t} = \sum_{k=1}^{t} \vec{d}_k = (-d_0, d_1, d_2, \dots, d_t + y_t(\vec{r}^*), 0, 0, \dots, 0) \in \mathbb{R}^{n+1}$$

be the first part of project $\vec{d}$, then project $\vec{d}$ is the sum of the two parts: $\vec{d} = \vec{d}_{0,t} + \vec{d}_{t,n}$.

The following Proposition holds.

Proposition 7. *The Net Present Value of project $\vec{d}$ is decomposed into two shares: (i) the sum of the LC residual incomes of project $\vec{d}$'s first part, and (ii) the discounted sum of the LC residual incomes of project $\vec{d}$'s future part:*

$$N_t = \sum_{k=1}^{t} \xi_k^a + v^{n-t} \sum_{k=t+1}^{n} \xi_{k,(\vec{d}_{t,n})}^a \tag{46}$$

where $\xi_{k,(\vec{d}_{t,n})}^a$ is the LC residual income from $\vec{d}_{t,n}$.

Proof. Project $\vec{d}_{t,n}$ begins at time t with initial outstanding capital equal to $y_t(\vec{r}^*)$. The initial boundary condition is $y_t(\vec{r}^*)=y_t^\circ(i)$, where $y_t^\circ(i)$ denotes the initial lost capital of project $\vec{d}_{t,n}$; its evolution is given by

$y^\circ_k(i) = y^\circ_{k-1}(i)(1+i) - d_k$ for $k > t$. Therefore, any result previously found for project $\vec{d}$ holds for project $\vec{d}_{t,n}$ as well. In particular, eq. (13) applied to project $\vec{d}_{t,n}$ becomes

$$\sum_{k=t+1}^{\tau} \xi^a_{k,(\vec{d}_{t,n})} = \sum_{k=t+1}^{\tau} x^a_{k,(\vec{d}_{t,n})} u^{\tau-k} \qquad \text{for every } \tau > t$$

where $x^a_{k,(\vec{d}_{t,n})}$ is the standard RI for project $\vec{d}_{t,n}$. However, the right-hand side holds for both $\vec{d}$ and $\vec{d}_{t,n}$, because cash flows, outstanding capitals, rates of return of the two projects coincide ($\vec{d}_{t,n}$ is the future part of $\vec{d}$). Therefore, $x^a_{k,(\vec{d}_{t,n})} = x^a_k$. This implies $\sum_{k=t+1}^{\tau} \xi^a_{k,(\vec{d}_{t,n})} = \sum_{k=t+1}^{\tau} x^a_k u^{\tau-k}$ for every $\tau > t$. Picking $\tau{=}n$, and using the fact that $v^{n-t} \sum_{k=t+1}^{n} x^a_k u^{n-k} = g\text{MVA}_t$, one gets $v^{n-t} \sum_{k=t+1}^{n} \xi^a_{k,(\vec{d}_{t,n})} = g\text{MVA}_t$. Eq. (46) is finally derived by using eq. (37) with $g\text{MVA}_t$ replacing MVA_t.[8] □

Proposition 7 says that the Net Present Value (the Excess Value Created, in OP's words) is reached by summing the lost-capital RIs of the first part of $\vec{d}$ and by discounting the aggregated lost-capital RIs of the future part of $\vec{d}$. Picking $i{=}k_e$ and $r^*{=}\text{ROE}$ in eq. (46) one finds the equivalent of OP's eq. (45) expressed in genuine LC terms. The same Proposition induces a generalization of eq. (17). Using the equality $g\text{MVA}_t{=}V_t - y_t(\vec{r}^*)$ and the fact that $g\text{MVA}_t{=}v^{n-t} \sum_{k=t+1}^{n} \xi^a_{k,(\vec{d}_{t,n})}$ (see proof of Proposition 7), one finds

$$V_t = y_t(\vec{r}^*) + v^{n-t} \sum_{k=t+1}^{n} \xi^a_{k,(\vec{d}_{t,n})}. \tag{47}$$

Choosing the equity perspective and selecting book values as outstanding capitals, the above equality becomes

$$V^e_t = B^e_t + \frac{1}{(1+k_e)^{n-t}} \sum_{k=t+1}^{n} \xi^a_{k,(\vec{d}_{t,n})} \qquad \text{for every } t. \tag{48}$$

Setting $t{=}0$ one finds back eq. (17), given that $\vec{d}_{0,n} = \vec{d}$, which implies $\xi^a_{k,(\vec{d}_{0,n})}{=}\xi^a_k$ for all k. Eq. (48) says that to get the equity market value one does not need to forecast dividends nor residual incomes: Only the total amount of prospective residual incomes is relevant.

7 User cost, lost-capital residual income and Keynesian Excess Profit

In 1936, Keynes introduces the notion of *user cost* in *The General Theory of Employment, Interest and Money*. Referring to an entrepreneur, user cost is defined as the difference between "the value of his capital equipment at the end of the period ... and ... the value it might have had at the end of the period if he had refrained from using it" (Keynes, 1967, p. 66). The same concept is investigated by Coase (1968), who relabels it "depreciation through use". User cost is equal to $G' - G$ where "G' is the value of the entrepreneurial stock and equipment had they not been used and G is their value after use" (Scott, 1953, p. 370). User cost therefore compares two different choices: "The choice between ... using a machine for a purpose and using it for another" (Coase, 1968, p. 123) and the result represents a depreciation in the value of the asset. Such a depreciation represents the "opportunity cost of putting goods and resources to a certain use" (Scott, 1953, p. 369), and is therefore an economic measure of "the opportunity *lost* when another decision is carried through" (Scott, 1953, p. 375, italics added). In this section we apply this concept to the situation where the entrepreneur may either put his resources in asset $\vec{d}$ or

[8] Obviously, eq. (37) does hold if MVA_t is replaced by $g\text{MVA}_t$.

invest them in an asset yielding return at the market rate i. To compute G and G', one must calculate "the present value of the net receipts ... by discounting them at a rate of interest" (Coase, 1968, p. 123). This "rate of discount coincides with that in the market" (Scott, 1953, p. 378). Using the arbitrage-type description given in section 2, if project is undertaken the cash-flow stream is $(-d_0, d_1, d_2, \ldots, d_n)$; if the entrepreneur abstains from investing in the project, his cash-flow stream is $(-d_0, d_1, d_2, \ldots, d_n + y_n(i))$. In the former case, the value of the entrepreneurial stock at time t is $G = \sum_{k=t+1}^{n} d_k v^{k-t}$. In the latter case, it is $G' = \sum_{k=t+1}^{n} d_k v^{k-t} + y_n(i)v^{n-t}$. User cost is therefore

$$G' - G = \sum_{k=t+1}^{n} d_k v^{k-t} + y_n(i)v^{n-t} - \sum_{k=t+1}^{n} d_k v^{k-t} = y_n(i)v^{n-t} \tag{49}$$

which, as Keynes acknowledges, represents "the discounted value of the additional prospective yield which would be obtained at some later date" (Keynes, 1967, p. 70). In other terms, reminding that the Net Final Value is the final lost capital changed in sign ($N_n = -y_n(i)$), user cost is the time-t NPV (changed in sign): $G' - G = -N_t$. It is worth noting that $G{=}V_t$ by definition of market value. Also, $G'{=}y_t(i)$. To prove the latter, just note that, using eq. (4) with $r{=}i$, one gets $d_0u^n - \sum_{k=1}^{t} d_k u^{n-k}{=}\sum_{k=t+1}^{n} d_k u^{n-k} + y_n(i)$. Dividing by u^{n-t},

$$d_0u^t - \sum_{k=1}^{t} d_k u^{t-k} = \sum_{k=t+1}^{n} d_k v^{k-t} + y_n(i)v^{n-t}. \tag{50}$$

The left-hand side is $y_t(i)$, the right-hand side is G'. Hence, user cost is equal to $G' - G{=}y_t(i) - V_t$. Consider now the depreciation through time of user cost, which is $(y_{t-1}(i) - V_{t-1}) - (y_t(i) - V_t)$: The latter is just a particular case of eq. (11) where V_t is chosen as the outstanding capital $y_t(\vec{r})$. The notion of lost-capital residual income is then just a generalization of the keynesian excess profit which is implicitly originated by user cost. We have then the following:

Definition 4. The Keynesian Excess Profit (KEP) is the subclass of lost-capital RIs generated by the choice of market values as outstanding capitals, so that $y_t(\vec{r}^*){=}V_t$, for $t = 1, \ldots, n-1$.

User cost is the change in the value of the asset due to a different use of it, and, in turn, the KEP is the change in the value of user cost due to time. The notion of user cost enables us to present residual income in terms of periodic variation of user cost.

The following Proposition shows that the KEP enjoys the important property of *goal congruence*: every residual income has the same sign as the NPV (see Martin et al., 2003)

Proposition 8. *The KEP has the same sign as the net value N_t.*

Proof. Proposition 3 implies

$$\xi_t^a = r_t^* \, y_{t-1}(\vec{r}^*) - i\,(V_{t-1} - N_{t-1}). \tag{51}$$

Definition 4 implies $\vec{r} = (\frac{V_1+d_1-d_0}{d_0}, i, i, \ldots i)$, so that eq. (51) becomes

$$\text{KEP}_t = \begin{cases} i\big(V_{t-1} - d_0\big) + \big(V_{t-1} - d_0\big) & \text{if } t = 1 \\ i\big(V_{t-1} - y_{t-1}(i)\big) & \text{if } t > 1 \end{cases} \tag{52}$$

where we have used the equalities $V_0{-}N_0{=}d_0$ and $V_1 + d_1{=}V_0(1+i)$. Therefore, given that $N_t{=}V_t{-}y_t(i)$ for all t, we have, for $t{>}\ 1$, $\text{KEP}_t{>}0$ if and only if $N_{t-1}{>}0$; as for $t{=}1$, we have $\text{KEP}_1{>}0$ if and only if $N_1{>}0$, given that $N_1{=}N_0(1+i){=}(V_0 - d_0)(1+i)$. □

The above Proposition compellingly proposes a subclass of RI models that always signal a positive residual income if and only if Net Present Value is positive, i.e if and only if value exceeds capital infused into the business. Note that the uncompounded sum of all the KEPs is equal to the project's Net Final Value N_n[9] and that the goal congruence property holds irrespective of the sign of the cash flows.[10]

Remark 12. The significance of the LC paradigm is also appreciated in terms of evolutions of NPV and gMVA. From eq. (32) and eq. (52) we find

$$N_t = \begin{cases} \mathrm{KEP}_t & \text{if } t = 1 \\ N_{t-1} + \mathrm{KEP}_t & \text{if } t > 1. \end{cases} \tag{53}$$

The KEP is the periodic addition to the Net Present Value or, equivalently, the KEP is just the RI model generated by the NPV. Using induction upon eq. (53),

$$N_t = \sum_{k=1}^{t} \mathrm{KEP}_k. \tag{54}$$

The above equation and eq. (37) imply MVA_t=0. This is obvious, given that in the KEP model the outstanding balance $y_t(\vec{r}^*)$ equals the market value for all $t \geq 1$. Eq. (54) gives some insights on the Net Present Value. Having previously found that the NPV may be written as sum of *future* LC residual incomes and *past* LC residual incomes (eq. (46)), we have now rewritten the NPV by using only *past* LC residual incomes. This result shows that NPV and LC paradigm are strictly connected, and that the use of the KEP-class enables one us to dismiss the future LC residual incomes.

8 Created Shareholder Value and Net Economic Income

The LC perspective gives us the opportunity of conjoining two seemingly disparate metrics in a unified view, introduced in a value-based management book and in a corporate finance book, respectively. The former is the *Net Economic Income* (NEI) and its use is suggested by Drukarczyk and Schueler (2000) for managerial purposes. The latter is the *Created Shareholder Value* (CSV) and is fostered by Fernández's (2002) for measuring value creation. It is easy to see that NEI and the LC-companion of CSV belong to the class of KEP metrics. As for NEI, the authors define current invested capital IC_t as $IC_t = IC_\tau(1 + wacc)^{t-\tau} - \sum_{k=\tau+1}^{t} NCF_k(1 + wacc)^{t-\tau}$, where $\tau < t$ is the time of the initial investment and NCF_k is the free cash flow. Evidently, setting τ=0, IC_t is just $y_t(i)$, and i=$wacc$ (which also means that their notion of invested capital coincides with the entity version of O'Hanlon and Peasnell's unrecovered capital). Net Economic Income is defined as

$$\mathrm{NEI}_t = NCF_t + (MV_t - MV_{t-1}) - wacc \cdot IC_{t-1} \tag{55}$$

with MV_t being the market value of the firm. It is evident that this perspective is consistent with the LC paradigm and that NEI is just an instantiation of the KEP class in an entity approach:

Proposition 9. *Net Economic Income is an entity-approach version of KEP.*

[9] This result just derives from eq. (16).

[10] Grinyer (1985, 1987, 1995) and Rogerson (1997) present a goal congruent measure under the assumption that all cash flows have the same sign. However, "this latter constraint is very unrealistic as many positive NPV projects have a mixture of positive and negative cash flows throughout the project's life." (Martin et al., 2003, pp. 20–21).

Proof. Pick i=$wacc$ and V=V^l in eq. (52), so that

$$\text{KEP}_t = \begin{cases} \big(V^l_{t-1} - d_0\big) + wacc \cdot \big(V^l_{t-1} - d_0\big) & \text{if } t = 1 \\ wacc \cdot \big(V^l_{t-1} - y_{t-1}(wacc)\big) & \text{if } t > 1. \end{cases}$$

Therefore eqs. (55) and (52) coincide, given that $wacc \cdot V^l_{t-1}$=NCF_{t-1}+$(MV_t - MV_{t-1})$. □

Net Economic Income is therefore the KEP from the point of view of all capital providers. As for Fernández's metric, it lies within the boundaries of the conventional paradigm, as seen in section 1. The author suggests the choice $y_t(\vec{r}^*)$=V^e_t, $t = 1, 2, \ldots, n-1$, so that

$$\text{CSV}_t = \begin{cases} d_0(r^*_t - k_e) & \text{if} \quad t = 1 \\ V^e_{t-1}(r^*_t - k_e) & \text{if} \quad t > 1. \end{cases} \tag{56}$$

The author's choice of equity market values as outstanding capitals implies that, in the first period, the internal rate of return is r^*_1=$(V^e_1 + d_1)/d_0$−1 (see Fernández, 2002, p. 281) and r^*_t=k_e otherwise. This in turn implies that the CSV model imputes value creation to the first period only (assuming expectations are met): CSV_1=$d_0(\frac{V^e_1+d_1-d_0}{d_0} - k_e)$ and CSV_t=0 for $t > 1$.[11] This metric is not aligned to N_t, because (if expectations are met) residual incomes after $t > 1$ are all zero, regardless of the sign of the Net Present Value. However, the LC-companion of CSV is aligned with N_t, because it is just the KEP in an equity approach.

Proposition 10. *The LC-companion of Fernández's CSV is the equity-approach version of the KEP.*

Proof. For t =1, one gets

$$\mathcal{L}(\text{CSV}_1) = \text{CSV}_1 = d_0(\frac{V^e_1 + d_1 - d_0}{d_0} - k_e) = (V^e_0 - d_0) + k_e(V^e_0 - d_0) = \text{KEP}_1.$$

As for $t > 1$, to pass from CSV_t to $\mathcal{L}(\text{CSV}_t)$ we replace $k_e\, V^e_{t-1}$ with $k_e\, y_{t-1}(k_e)$ in eq. (56). One finds $\mathcal{L}(\text{CSV})_t = r^*_t V^e_{t-1} - k_e\, y_{t-1}(k_e) = k_e\,(V^e_{t-1} - y_{t-1}(k_e)) = \text{KEP}_t$. □

Propositions 9 and 10 show that seemingly dissimilar metrics (NEI and CSV) share common conceptual and formal analogies if they are connected via a LC perspective: Both are goal congruent measures (the NEI directly, the CSV after transforming it into its LC-companion).

9 Anthony's argument and the unification of the two paradigms

In his *Accounting for the Cost of Interest*, Anthony (1975) advocates the use of a charge on equity capital in accounting statements: The interest on the use of equity capital should be accounted for as an item of cost. Evidently, to record equity interest as a cost boils down to redefine the notion of profit: In this view, profit is earnings in excess of the equity interest. Anthony's profit is therefore just *residual* income, as he himself recognizes (Anthony, 1975, p. 3). The idea of recording equity interest as a cost for accounting purposes implies that, for certain assets, the amounts recorded is higher, and shareholders' equity is correspondingly higher. Anthony describes an enlightening example that is worth quoting extensively:

[11] We also have CSV_1=N_1=NPV$(1 + k_e)$.

> Consider, for example, a corporation that is formed to invest in land. It buys a parcel for $1,000,000, holds it for five years, sells the land for $2,000,000 at the end of the fifth year, and liquidates ... In the proposed system, interest cost would be added to the cost of the land each year, and there would be a corresponding credit to shareholder's equity. At the end of the fifth year, there would be an additional entry to shareholders' equity, representing the net income realized from the sale; that is, the difference between the sales revenue and the accumulated cost of the land. Thus, the statements would show an increase in shareholders' equity in each of the five years. During the first four years, the company would report neither income nor loss; instead, the costs incurred in holding the land, here assumed to be only equity interest, would be added to the original cost of the land. In the fifth year, when the sale took place, net income would be reported as the difference between the selling price and the costs accumulated in inventory up to that time. (Anthony, 1975, p. 30)

As shown below, paraphrasing in a formal way Anthony's suggestion, an interesting residual income model is generated. Under Anthony's proposal, the book value of the land increases periodically by the cost of equity capital. This means that the depreciation charge for the land is negative (i.e. it is an increase in shareholders' equity) and is equal to the interest on equity. In other words, the periodic rate of return in the first four years is set equal to the ROE, and the ROE is set equal to the cost of equity: Formally, the project is $\vec{d}$=$(-1, 0, 0, 0, 0, 2)$ (in millions), and Anthony is choosing r_t^*=ROE=k_e and therefore $y_t(\vec{r}^*) = B_t^e = y_t(k_e)$ for $t = 1, 2, 3, 4$. The lost capital coincides with the equity book value and the latter evolves according to $y_t(k_e)$=$y_{t-1}(k_e) \cdot (1 + k_e)$ for all $1 \leq t < 5$, which is just eq. (8) with i=k_e and d_t=0 for $t < 5$. Thus, the equity book values are $y_0(\vec{r}^*) = 1$, $y_1(\vec{r}^*) = (1 + k_e)$, $y_2(\vec{r}^*) = (1 + k_e)^2$, $y_3(\vec{r}^*) = (1 + k_e)^3$, $y_4(\vec{r}^*) = (1 + k_e)^4$. During the first four years, residual income (Anthony's profit) is neither positive nor negative, because net income is equal to the increase in shareholders' equity,[12] which is just equal to the capital charge $iy_{t-1}(i)$=$k_e y_{t-1}(k_e)$, so that RI=Net Income − equity capital charge=$k_e\, y_{t-1}(k_e) - k_e\, y_{t-1}(k_e)$=0. At time $t = 5$, the accumulated cost is $y_4(k_e) \cdot (1+k_e)$=$(1+k_e)^5$ and the net income is given by the sum of the negative depreciation (=appreciation) charge $k_e y_4(k_e)$ and the surplus generated by the sale of the land: $2 - y_4(k_e) \cdot (1 + k_e)$. Therefore,

$$\begin{aligned}\text{Residual Income} &= \text{Net Income} - \text{equity capital charge}\\ &= \big[k_e y_4(k_e) + 2 - y_4(k_e)(1 + k_e)\big] - k_e y_4(k_e)\\ &= 2 - y_4(k_e)(1 + k_e) = 2 - (1 + k_e)^5\end{aligned}$$

As Anthony acknowledges, last year's (residual) profit is just the difference between the selling price and the costs accumulated up to that time. This residual income may be written as $y_4(k_e)(r_5^* - k_e)$, with r_5^*=$\big[(k_e y_4(k_e) + 2 - y_4(k_e)(1 + k_e)\big]/y_4(k_e)$=$\big[k_e(1 + k_e)^4 + 2 - (1 + k_e)^5\big]/(1 + k_e)^4$. It is worth noting that this model provides zero residual incomes for all years except the last one, when residual income is equal to the project's Net Final Value, amounting to $2 - (1 + k_e)^5 = N_5$=NPV$(1 + k_e)^5$.

Applying Anthony's argument to a generic project, the project's outstanding capital is set equal to the lost capital: $y_t(\vec{r}^*)$=$y_t(i)$ so that eqs. (7) and (8) coincide. Also, taking an equity approach, r^* is set equal to ROE and i is set equal to k_e for $t = 1, 2, \ldots, n - 1$; a new RI model is thus generated, here named Anthony's Residual Income (ARI):

$$\text{ARI}_t = r_t^* y_{t-1}(k_e) - k_e y_{t-1}(k_e) \tag{57}$$

with r_t^*=k_e if $t < n$, and r_n^*=$\frac{k_e y_{n-1}(k_e) + d_n - y_{n-1}(k_e)(1+k_e)}{y_{n-1}(k_e)}$.

[12] This is because revenues are zero and the depreciation charge is negative (equity appreciates).

By suggesting that the lost capital be directly recorded in accounting statements, because it represents a real cost, Anthony implicitly maintains that the appropriate book value of assets should be given by the value assets would have had if the initial sum d_0 had been invested at the cost of equity. This is a conceptual shift: In his view the book value equals the lost capital, i.e. the capital shareholders renounce to when investing in the project (firm). However, this lost-opportunity interpretation is not given by Anthony, who, instead, considers the lost capital not lost at all: It is just the shareholders' credit. Therefore, he uses a metaphor from loan theory (see Table 1), and to him the clean surplus relation is derived by interpreting equity as a shareholders' credit.

This conceptual shift brings about some interesting consequences: [a] The lost capital may be interpreted as the capital which is "borrowed" from claimholders; [b] Anthony's residual income is a mirror-image of Fernández's CSV: According to the latter value is created in the first period, according to the former value is created in the last period. Therefore, the latter is, so to say, finance-derived, whereas the former is accounting-derived; [c] Anthony's RI model realizes a unification of the two paradigms. His argument is the only one that is consistent with both paradigms. As for claim [a], it gives us a fourth interpretation of the lost capital, besides the three previously found: The lost capital is the capital which is lost by investors (section 2), is the outstanding capital of a shadow project whose standard RI coincides with the lost-capital RI (Magni, 2000a, 2005, 2006), is the capital infused into the business (section 5), and is the capital "borrowed" from shareholders, whose interest rate is the equity cost of capital. These four interpretations, while seemingly discordant, are coherently harmonized under the formal lens of the LC paradigm.Claim [b] is evident from Table 3, which uses the definition of CSV and ARI given in the previous and current section respectively: In Anthony's view, value is recorded only in the last period, whereas the previous RIs are zero. This is consistent with accounting principles: "In accordance with the realization concept, income would be reported only in the fifth year, when the land was sold" (Anthony, 1975, p. 31). In Fernández's view, value is created in the first period, when the project is undertaken, whereas the subsequent RIs are zero. This is consistent with a financial perspective, according to which market immediately recognizes value creation (see also Robichek and Myers, 1965, pp. 11-12). Referring to dates instead of periods: Fernández recognizes value creation at time 0 as a windfall gain (value creation=Net Present Value), Anthony recognizes value at time n (value creation=Net Final Value). As for claim [c], looking at eqs. (5) and (9), the two sets of models intersect if and only if $y_{t-1}(r_t^* - i){=}r_t^* y_{t-1} - iy_{t-1}(i)$ for every $t = 1, 2, \ldots, n$. The above equality implies $y_{t-1}(\vec{r}^*) = y_{t-1}(i)$ for every $t = 1, 2, \ldots, n$, which is just Anthony's suggestion. Thus, Anthony's argument gives rise to a theoretically significant subclass of RI models: They are the only models that belong to both paradigms. Putting it in equivalent terms, the notion of residual income is univocal if Anthony's argument is used, because the project's outstanding capital is made to coincide with the lost capital.[13,14]

10 Concluding remarks

This paper presents an investigation into an alternative non-standard notion of residual income (RI), originally introduced in Magni (2000a,b,c, 2001a,b) with the name *Systemic Value Added*. The paradigm is here renamed "lost-capital" (LC) paradigm, owing to the central role played by the capital that investors *lose* by undertaking

[13] Strictly speaking, Anthony selects r^*=ROE=i=k_e, but obviously his argument also implies the possible choice of r^*=ROA=i=$wacc$, which means that an entity perspective is adopted.

[14] Anthony's example may be interpreted as a particular case of either EBO or $\mathcal{L}$(EBO), where ROE is set equal to k_e in all periods but the last one.

the project. The LC paradigm is a theoretical domain which enables one to embrace varied notions, results, and models which have been developed in different fields with disparate scopes and aims. In particular, this paper has shown that the LC paradigm is easily interpretable in an arbitrage theory setting, and that Keynes's notion of user cost is a basic ingredient of it. The lost-capital residual income enables one to lump together two types of depreciation: Depreciation through time and depreciation through use (user cost). Gathering the two one obtains the Keynesian Excess Profit, which is a (market-based) subclass of LC residual incomes, which have the nice property of being aligned in sign with the project's NPV (goal congruence). Among such measures, we analyze Drukarczyk and Schueler's (2000) Net Economic Income and the LC-companion of Fernández's Created Shareholder Value. While different from the standard paradigm, strict relations of the two paradigms are underlined and it is shown that the LC approach, contrary to the standard one, enjoys a nice aggregation property which enables one to aggregate residual incomes for forecasting purposes without worrying about forecasts of each and every residual income. The lost capital itself is also given a fourfold interpretation: capital foregone, capital infused into the business, capital borrowed from shareholders, and capital of a shadow project (for the latter, see Magni, 2005). Relations are thoroughly investigated among NPV, book values and market values, shedding lights on O'Hanlon and Peasnell's (2002) results and providing counterparts in the lost-capital approach. Furthermore, the LC residual income is interpreted as the future value of cumulated Ohlson's (2005) Abnormal Earnings Growth, a notion which is equivalent to O'Byrne's (1997) EVA improvement, based on Miller and Modigliani's investment opportunities approach to valuation. Furthermore, Young and O'Byrne's (2001) notion of Adjusted EVA coincides with the LC residual income if earnings equal dividends. Finally, Anthony's (1975) notion of profit is shown to give rise to a class of metrics which just represents the intersection of the sets of standard residual incomes and LC residual incomes.

Future researches may be devoted to deepen the theoretical network originated by the LC paradigm, which seems to be susceptible of embracing several different notions and models and providing a fruitful integration among concepts in various fields such as economics, accounting, finance. An enrichment of this conceptual environment will possibly address in a more thorough way the issue of practical usefulness of this paradigm for value creation, capital budgeting decisions, managerial incentives and control. The results found seem to be auspicious.

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Table 0. Notational Conventions

Symbol	means	is equal to
AE	Adjusted EVA	
AEG	Abnormal Earnings Growth	
AIC	Adjusted Invested Capital	
ARI_t	Anthony's Residual Income	
B_t	book value	
B_t^e	book value of equity	
B_t^l	book value of total liabilities	
$cfroe$	Cash Flow Return On Equity	
$cfroi$	Cash Flow Return On Investment	
C_t	value of the reinvestment proceeds at time $t-1$	
C^t	amount of wealth at time t if project is not undertaken	
CSV	Created Shareholder Value	
$\vec{d}$	project, firm, business	
d_t	cash flow from $\vec{d}$ available for distribution	equity cash flow/free cash flow
$\vec{d}_t$	uniperiodic project	
$\vec{d}_{0,t}$	first part of project $\vec{d}$	
$\vec{d}_{t,n}$	second part of project $\vec{d}$	
ΔMVA_t	Market Value Added's variation	$\text{MVA}_t - \text{MVA}_{t-1}$
ΔN_t	Net Present Value's variation	$N_t - N_{t-1}$
$\Delta y_t(\cdot)$	capital's variation	$y_t(\cdot) - y_{t-1}(\cdot)$
EBO	Edwards-Bell-Ohlson	
ECF	Equity Cash Flow	
EVA	Economic Value Added	
EVC_t	Excess Value Created	N_t
FCF	Free Cash Flow	
F_t	future value of cumulated AEGs	
G	value of assets if they are used	V_t
G'	value of assets if they had not been used	$y_t(i)$
$G' - G$	user cost	$y_t(i) - V_t$
gMVA	generalized Market Value Added	$V_t - y_t(\vec{r}^*)$
i	(opportunity) cost of capital	
IC_t	current invested capital	$y_t(i)$
k_D	cost of debt (required return on debt)	
k_e	cost of equity (required return on equity)	
k_U	cost of assets (required return on assets)	
KEP_t	Keynesian Excess Profit	
LC	lost capital	

(The Table is continued on the next page)

continued - Table 0. Notational Conventions

Symbol	means	is equal to
$\mathcal{L}$(CSV)	LC companion of CSV	
$\mathcal{L}$(EBO)	LC companion of EBO	
$\mathcal{L}$(EVA)	LC companion of EVA	
$\mathcal{L}(x_t^a)$	LC companion of x_t^a	
$\mathcal{L}^{-1}$(NEI)	standard companion of NEI	
MV_t	market value	V_t
MVA	Market Value Added	$V_t^e - B_t^e$
N_t	time-t Net Present Value	$\text{NPV}(1+i)^t$
N_n	Net Final Value	$\text{NPV}(1+i)^n$
NCF_k	Net Cash Flow	Free Cash Flow
NEI_t	Net Economic Income	
NPV	Net Present Value	$\sum_{k=1}^{n} \frac{d_k}{(1+i)^k} - d_0$
r_t	period rate of return	
r_t^*	period internal rate of return	
$\vec{r}^*$	internal discount function for project $\vec{d}$	
RI	residual income	
ROA	Return On Assets	
ROE	Return On Equity	
u	compounding factor	$1+i$
U_t^0	unrecovered capital	$y_t(i)$
v	discount factor	$(1+i)^{-1}$
V_t	market value	$\sum_{k=t+1}^{n} \frac{d_k}{(1+i)^{k-t}}$
V_t^e	market value of equity	$\sum_{k=t+1}^{n} \frac{\text{ECF}_k}{(1+k_e)^{k-t}}$
V_t^l	market value of total liabilities/assets	$\sum_{k=t+1}^{n} \frac{\text{FCF}_k}{(1+wacc)^k}$
V^U	market value of the unlevered firm	$\sum_{k=t+1}^{n} \frac{\text{FCF}_k}{(1+k_U)^{k-t}}$
$wacc$	weighted average cost of capital	
x_t^a	standard RI	
$x^a_{k,(\vec{d}_{t,n})}$	standard RI of project $\vec{d}$'s second part	
ξ_t^a	lost-capital RI	
$\xi^a_{k,(\vec{d}_{t,n})}$	lost-capital RI of project $\vec{d}$'s second part	
y_{t-1}	capital invested	
$y_{t-1}(\vec{r}^*)$	outstanding capital of project $\vec{d}$	
$y_{t-1}(\vec{r})$	outstanding capital growing at rate r_t	
$y_{t-1}(i)$	lost capital of project $\vec{d}$	
$y_t^\circ(i)$	initial lost capital of project $\vec{d}_{t,n}$	$y_t(\vec{r}^*)$
z_t	Abnormal Earnings Growth	$\text{Residual earnings}_{t+1} - \text{Residual Earnings}_t$

Table 1. The firm and the loan

FIRM		LOAN	
cash flow	$\longrightarrow$	instalment	d_t
capital employed	$\longrightarrow$	residual debt (outstanding balance)	y_{t-1}
capital's depreciaton	$\longrightarrow$	principal repayment	$y_{t-1} - y_t$
periodic rate of return	$\longrightarrow$	contractual rate	r_t
income	$\longrightarrow$	interest	$r_t y_{t-1}$

Table 2. Constructing residual incomes in the two paradigms

	IDF	$y_t(\vec{r}^*)$	i	$y_t(i)$	d_t
Entity approach					
EVA	ROA	B_t^l	$wacc$	$y_t(wacc)$	FCF
RI^{cfroi}	$cfroi$	$y_t(cfroi)$	$wacc$	$y_t(wacc)$	FCF
NEI	$\begin{cases} r_1^* = \frac{V_1^l + d_1 - d_0}{d_0} \\ r_t^* = wacc \end{cases}$	V_t^l	$wacc$	$y_t(wacc)$	FCF
Equity approach					
EBO	ROE	B_t^e	k_e	$y_t(k_e)$	ECF
RI^{cfroe}	$cfroe$	$y_t(cfroe)$	k_e	$y_t(k_e)$	ECF
CSV	$\begin{cases} r_1^* = \frac{V_1^e + d_1 - d_0}{d_0} \\ r_t^* = k_e \end{cases}$	V_t^e	k_e	$y_t(k_e)$	ECF

Table 3. Anthony's and Fernández's symmetric residual incomes

	period					
	1	2	3	...	$n-1$	n
Anthony's Residual Income	0	0	0	...	0	$\text{NPV}(1+k_e)^n$
Created Shareholder Value	$\text{NPV}(1+k_e)$	0	0	...	0	0

Appendix. A firm is incorporated to undertake a 5-year project. Tables 4 and 5 collect the input data (in boldface), the firm's accounting statements, the resulting expected cash flows and rates of return, while Table 6 computes market values. To this end, it is assumed that the required return to total assets is k_U=11.9%, and that the nominal debt is equal to the market value of debt, i.e. the interest rate is equal to the required return on debt (here assumed to equal 8%). The market value of equity turns out to be 30 457 and the Net Present Value is 456.9. Table 7 focusses on the 3+3 groups of metrics, and the residual incomes are calculated for each period. Logically, each of them represents a decomposition of 456.9, and is obtained by a "Discount&Sum" procedure for the standard metrics and by a "Sum&Discount" method for the lost-capital metrics. Inspecting Table 7, the reader may appreciate the considerable differences across paradigms and across metrics. It is worth noting that the KEP metrics are perfectly aligned in sign with the NPV (goal congruence) and they are the only metrics that enjoy this property. It is also worth noting that discrepancies among standard metrics and LC metrics concern not only magnitude, but sign as well: for example in the third and fourth periods RI^{cfroi} signals value destruction in the standard paradigm and value creation in the LC paradigm; in the second period RI^{cfroe} signals value creation in the standard paradigm and value destruction in the LC companion. As for Anthony's Residual Income, from Table 3 one finds ARI_t=0 for $t < 5$ and ARI_5=861 (value creation is signalled only in the final period).

Table 4. Input data, Balance Sheet, Income Statement

Time	0	1	2	3	4	5
BALANCE SHEET						
Net fixed assets	**45 000**	**36 000**	**27 000**	**18 000**	**9 000**	**0**
Working capital	**15 000**	**15 000**	**15 000**	**15 000**	**15 000**	**0**
TOTAL ASSETS	60 000	51 000	42 000	33 000	24 000	0
Debt	**30 000**	**20 000**	**10 000**	**5 000**	**2 000**	**0**
Equity (book value)	30 000	31 000	32 000	28 000	22 000	0
TOTAL LIABILITIES	60 000	51 000	42 000	33 000	24 000	0
INCOME STATEMENT						
Sales		**31 500**	**32 100**	**31 500**	**27 500**	**28 000**
−Cost of sales and other expenses		**14 000**	**14 000**	**14 000**	**14 000**	**14 000**
−Depreciation		9 000	9 000	9 000	9 000	9 000
Earnings before interest and taxes		8 500	9 100	8 500	4 500	5 000
Interest rate		**8%**	**8%**	**8%**	**8%**	**8%**
−Interest		2 400	1 600	800	400	160
Profit before taxes		6 100	7 500	7 700	4 100	4 840
Tax rate		**33%**	**33%**	**33%**	**33%**	**33%**
−Taxes		2 013	2 475	2 541	1 353	1 597
Profit after taxes		4 087	5 025	5 159	2 747	3 243

Table 5. Cash flow and rates						
Time	0	1	2	3	4	5
ECF†	−30 000	3 087	4 025	9 159	8 747	25 243
FCF‡	−60 000	14 695	15 097	14 695	12 015	27 350
Cash flow to debt (instalment)	−30 000	12 400	11 600	5 800	3 400	2 160
roi		9.49%	11.95%	13.56%	9.14%	13.96%
roe		13.62%	16.21%	16.12%	9.81%	14.74%
cfroi		11.23%	11.23%	11.23%	11.23%	11.23%
cfroe		14.28%	14.28%	14.28%	14.28%	14.28%

†ECF=Profit after taxes+Depreciation + Δ Debt − Δ Working Capital; ‡FCF=ECF+Cash Flow to Debt − Tax rate· Interest

Table 6. Valuation						
Time	0	1	2	3	4	5
$\boldsymbol{k}_U$		**11.9%**	**11.9%**	**11.9%**	**11.9%**	**11.9%**
$\boldsymbol{k}_D$		**8%**	**8%**	**8%**	**8%**	**8%**
V^U	58 928	51 246	42 247	32 580	24 441	0
Value of Tax Shield†	1529	859	400	167	49	0
V^l	60 457	52 105	42 647	32 747	24 490	0
$V^e = V^l -$Debt	30 457	32 105	32 647	27 747	22 490	0
k_e		15.55%	14.23%	13.05%	12.58%	12.24%
$V^e = \sum_{j=t+1}^{n} \frac{\mathrm{ECF}_j}{\prod_{\tau=t+1}^{j}(1+k_{e_\tau})}$	30 457	32 105	32 647	27 747	22 490	0
$wacc$		10.49%	10.82%	11.24%	11.48%	11.68%
$V^e = \sum_{j=t+1}^{n} \frac{\mathrm{FCF}_j}{\prod_{\tau=t+1}^{j}(1+wacc_\tau)} -$Debt	30 457	32 105	32 647	27 747	22 490	0
N_t	457	528	603	682	767	861

†Three consistent methods are here used to find the equity value of the firm: the Adjusted Present Value, the ECF method, the FCF method. The value of tax shield is computed by discounting the tax shields at the required return on debt (8%). While not relevant to the subject of this paper, it is worth noting that there is a lively debate in the literature on the correct discount rate for computing the tax shield (see Myers, 1974; Harris and Pringle, 1985; Tham and Vélez-Pareja, 2001; Arzac and Glosten, 2005; Fernández, 2005; Cooper and Nyborg, 2006).

Table 7. Performance metrics and its companions†

Period	1	2	3	4	5		1	2	3	4	5
EVA	−600	578	972	−772	548	$\mathcal{L}$(EVA)	−600	513	963	−672	571
EBO	−577	615	984	−775	550	$\mathcal{L}$(EBO)	−577	533	978	−658	584
RI^{cfroi}	445	214	−5	−80	−109	$\mathcal{L}(\text{RI}^{cfroi})$	445	262	75	10	−17
RI^{cfroe}	−380	17	390	459	451	$\mathcal{L}(\text{RI}^{cfroe})$	−380	−37	335	448	495
$\mathcal{L}^{-1}$(NEI)	505	0	0	0	0	NEI‡	505	55	63	71	81
CSV	528	0	0	0	0	$\mathcal{L}$(CSV)‡	528	75	79	86	94

†Numbers are rounded off; ‡Keynesian Excess Profit