On the J-test for nonnested hypotheses and Bayesian extension

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Abstract

Davidson and MacKinnon's J-test was developed to test non-nested model specification. In empirical applications, however, when the alternate specifications fit the data well the *J* test may fail to distinguish between the true and false models: the *J* test will either reject, or fail to reject both specifications. In such cases we show that it is possible to use the information generated in the process of applying the J-test to implement a Bayesian approach that provides an unequivocal and acceptable solution. Jeffreys' Bayes factors offer ways of obtaining the posterior probabilities of the competing models and relative ranking of the competing hypotheses. We further show that by using approximations of Schwarz Information Criterion and Bayesian Information Criterion we can use the classical estimates of the log of the maximum likelihood which are available from the estimation procedures used to implement the *J* test to obtain Bayesian posterior odds and posterior probabilities of the competing nested and non- nested specifications without having to specify prior distributions and going through the rigorous Bayesian computations.

I. INTRODUCTION

One of the most widely used tests for comparing non-nested hypotheses is the *J* test proposed by Davidson and MacKinnon (1981). The non-nested tests of hypotheses arise in situations when the alternate hypothesis cannot be derived as a special case of the null hypothesis. This may arise either due to completely different sets of regressors in competing model specifications or different distributions of the stochastic terms. This test appears in standard econometrics' textbooks [e.g. Greene, 2003, and Davidson and MacKinnon, 2004], the Handbook of Econometrics [Vol. 4, 1994], is included in the literature of standard econometrics programs [e.g. EViews 5, and Shazam], and is the most commonly used non-nested test procedure (McAleer's (1995)).

When each of the competing hypotheses is successful in explaining the variations in the data, the *J*-test may not be able to discriminate between alternative specifications. Some of the situations in which the *J* test does not discriminate between the competing specifications have already been noted. Godfrey and Pesaran (1982) state the following one or more conditions where the *J* test is likely to over reject the true hypothesis: (i) a poor fit of the true model; (ii) low or moderate correlation between the regressors of the two models; and (iii) the false model includes more regressors than the correct specification. Davidson and MacKinnon (2004) agree that the *J* test will over-reject, "often quite severely" in finite samples when the sample size is small or where conditions (i) or (iii) above are obtained. Gourieroux and Monfort (1994) conclude that the test is very sensitive to the relative number of regressors in the two hypotheses; in particular, the power of the *J* test is poor when the number of regressors in the null hypothesis is smaller than the number of regressors in the alternative one.

It is possible, however, to find examples in the literature where none of the above noted conditions are violated[1] and where the *J* test rejects all models.[2]

[1] That is to say that each of the alternative hypotheses fit the data extremely well, where the regressors of the alternative hypotheses are correlated, where the alternatives have the same numbers of regressors, J-test is inconclusive.

[2] McAleer's (1995) survey of the use of non-nested tests in applied econometric work reports that out of 120 applications all models were rejected in 43 applications. However, he did not break down the rejections by the type of test used.

Here, we give three examples of empirical work on the consumption functions that illustrates this situation.

In the econometrics software EViews 5, the *J* test is used to compare two hypotheses regarding the determinants of consumption. The first hypothesis is that consumption is a function of GDP and lagged GDP. The alternative expresses consumption as a function of GDP and lagged consumption. The data used are quarterly observations, 1947:2 – 1994:4. The conclusion reads: “we reject both specifications, against the alternatives, suggesting that another model for the data is needed.” [p. 581]. This conclusion is surprising, for the coefficient of determination reported for each of the models was .999833, a value that would have lead most researchers to accept either of the models as providing full explanation for the quarterly variability of consumption over almost half a century.

Greene [2003] reported the results of comparing the same two consumption function hypotheses using quarterly data for the period 1950:2 – 2000:4. The results of the test lead him to a similar conclusion: “Thus, H_o should be rejected in favor of H_1. But reversing the roles of H_o and H_1... H_1 is rejected as well.” Although Greene did not report on the goodness of fit, it is very likely, as in the EViews 5 data, that each of the models had explained almost all of the variation in consumption.

The third example is found in Davidson and MacKinnon [1981] where they report on the results of applying the *J* test to the five alternative consumption function models examined by Pesaran and Deaton [1978]. In spite of the fact that the coefficients of determination for all the models are quite high, ranging from .997933 to .998756 [Pesaran and Deaton, 1978, 689-91], each of the models is rejected against one or more of the alternatives.

In this paper we show that when we wish to test alternative non-nested specifications that are successful in explaining the observed variations, the *J* test is likely to be inconclusive. While advances and improvements on the *J* test such as the Fast Double Bootstrap procedure [Davidson and MacKinnon, 2002] have been made and are reported to increase the power of the test, it appears that in doing empirical work researchers still use the standard *J* test [see for examples:

Faff and Gray (2006) or Singh (2004)]. In the discussion below we use the original test as this allows for clarity of exposition.

In section II we point out the theoretical reasons why the test may lack power in testing model specifications that fit a given set of data well. We do this by expressing the test statistic in terms of the correlation between the variables in the alternative specification.

In section III we illustrate the problems encountered in using the *J* test in empirical work by applying the test to two alternative specifications designed to explain monthly output behavior in 24 industries.

In section IV, we present a testing paradigm for non-nested hypothesis that can be implemented to supplement the *J* test when the *J* test proves inconclusive. We use log-likelihood values which are obtained in the process of applying the *J* test to approximate Bayesian information criteria and Bayes factors. This allows us to circumvent the complexities of the Bayesian approach: specifying the prior distributions and computations of marginal likelihoods. This specification testing method yields results that do not depend on the choice of the null or the maintained hypothesis. We illustrate the use of the Bayes factors in specification testing by applying it to the same data on the 24 industries studied in section III.

II. THE *J* TEST

An "artificial regression" approach for testing non-nested models was proposed by Davidson and MacKinnon [1981, 1993]. Consider two non-nested hypotheses that are offered as alternative explanations for Y:

(2.1) H_0: $Y = X\beta + \varepsilon_1$, and

(2.2) H_1: $Y = Z\gamma + \varepsilon_2$,

Both disturbances satisfy the classical normal model assumptions, X has k_1 and Z has k_2 independent non-stochastic regressors.

We write the artificial compound model as:

(2.3) $$Y = (1-\alpha)X\beta + \alpha Z\gamma + \varepsilon$$

If this model is estimated, we test the non-nested model by testing one parameter: when $\alpha = 0$, the compound model collapses to equation (2.1) and when $\alpha = 1$, the compound model collapses to equation (2.2).

Because the parameters of this model are not identifiable, Davidson and MacKinnon suggest replacing the compound model (2.3) by one "in which the unknown parameters of the model *not* being tested are replaced by estimates of those parameters that would be consistent if the DGP [data generating process] actually belonged to the model they are defined." (Davidson and MacKinnon, 1993, p. 382). Thus, to test equation (2.1), we replace γ in (2.3) by its estimate $\hat{\gamma}$ obtained by regressing Y on Z. If we write $\hat{Y}_z = Z\hat{\gamma}$, the equation to be estimated to test whether $\alpha = 0$ is:

(2.3') $$Y = (1-\alpha)X_1\beta + \alpha\hat{Y}_z + \varepsilon.$$

Similarly, to test equation (2.2) we estimate $\hat{\beta}$ by fitting equation (2.1) to the data and replace $X\beta$ in (2.3) by $X\hat{\beta}$, or $\hat{Y}_x$. The equation to be estimated to test (2.2) is then,

(2.3") $$Y = (1-\alpha)\hat{Y}_x + \alpha Z\gamma + \varepsilon.$$

The Davidson and MacKinnon *J*-test applies the *t*-test for the estimated coefficients on $\hat{Y}_z$ in equation (2.3') and $\hat{Y}_x$ in equation (2.3''). A statistically significant *t*-statistic on the coefficient of $\hat{Y}_z$ rejects H_0 as the appropriate model and a significant t-statistic on the $\hat{Y}_x$ coefficient results in the rejection of H_1. For instance, in the consumption functions described in the introduction, both *t*-statistics result in the rejection of each model. As some of the regressors in (2.3') and (2.3") are stochastic, the *t*-test is not strictly valid. Davidson and MacKinnon (1993, pp. 384-5) show as to why the *J* and *P* tests (which in [linear models] are identical) are asymptotically valid.[3]

In this section we show that the *t*-test statistic for the significance of $\widehat{\alpha}$ in (2.3'), thus the decision we make regarding the hypothesis (2.1), depends on the goodness of fit of the regression of Y on Z, the goodness of fit of the regression (2.3') as well as the correlation between the two sets of regressors in (2.3'). We show this using the *F* ratio for testing $\alpha = 0$, which is identical to the square of the *t*-value since we are interested in the contribution of only one regressor $\hat{Y}_z$. A similar statement applies to the test of the significance of $(1 - \alpha)$ in (2.3").

Consider the OLS estimator of the coefficient α of the model (2.3'). Using a theorem due to Lovell (1963, p. 1001)[4], the OLS estimate of α and the estimated residuals will be the same as those obtained from regressing the residuals of the regression of Y on X, $M_x Y$, on the residuals of regressing $\hat{Y}_z$ on X, , $M_x \hat{Y}_z$::

$$(2.4) \quad M_x Y = \alpha M_x \hat{Y}_z + M_x \varepsilon$$

Where, $M_x = [I - X(X'X)^{-1}X']$, and $\hat{Y}_z = Z\hat{\gamma}$ and $\hat{\gamma}$ is the estimated regression coefficients of Y on Z:

Writing, $P_z = Z(Z'Z)^{-1}Z'$, we write (2.4) as:

$$M_x Y = \alpha M_x P_z Y + M_x \varepsilon .$$

The OLS estimator of α is then:

$$(2.5) \quad \hat{\alpha} = [Y'P_z M_x P_z Y]^{-1} Y'P_z M_x Y$$

[3] They also add, "also indicates why they (*J* and *P* tests) may not be well behaved in finite samples. When the sample size is small or *Z* contains many regressors that are not in *S*(*X*)…" We do not consider these situtions in what follows.

[4] Lovell's theorem 4.1 generalizes (to deal with seasonal adjustment) a theorem that was developed by R. Frisch and F. Waugh for dealing with detrending data. Green (2003) extends the application to any partitioned set of regressors.

The residuals of OLS estimation of (2.4) are:

$$M_x\hat{\varepsilon} = [M_x Y - \hat{\alpha} M_x P_z Y]$$

Since (2.4) has only one regressor, under the null hypothesis that $\alpha = 0$ the *F*-statistic is the square of the *t*-statistic.

The sum of squares due to regression of equation (2.4), Q, is given by:

(2.6) $Q = \hat{\alpha}' D \hat{\alpha}$, where $D = \hat{Y}_z' M_x \hat{Y}_z$

Consider regressing Y on X only and denote the residuals of that regression by $\hat{u} = M_x Y$ and regressing $\hat{Y}_z$ on X and denote the residuals of that regression by: $\hat{v} = M_x \hat{Y}_z$. We can then write:

(2.5') $\hat{\alpha} = (\hat{v}'\hat{v})^{-1}\hat{v}'\hat{u}$, and

(2.6') $Q = Y' M_x \hat{Y}_z [\hat{Y}_z' M_x \hat{Y}_z]^{-1} \hat{Y}_z' M_x Y = \hat{u}'\hat{v}(\hat{v}'\hat{v})^{-1}\hat{u}'\hat{v} = (\Sigma \hat{u}\hat{v})^2 \big/ \Sigma \hat{v}^2$

The residuals from OLS estimation of (2.4) can be written as:

$$M_x\hat{\varepsilon} = [M_x Y - \hat{\alpha} M_x P_z Y] = \hat{u} - \hat{\alpha}\hat{v} = \hat{u} - (\hat{v}'\hat{v})^{-1}(\hat{v}'\hat{u})\hat{v}$$

The sum of the squared residuals from estimating (2.4) is:

(2.7) $\hat{\varepsilon}' M_x \hat{\varepsilon} = \sum \hat{u}^2 - [(\sum \hat{u}\hat{v})^2 / \sum \hat{v}^2]$

This sum of squares has $(T–k_1–1)$ degrees of freedom, where T is the number of observations and k_1 is the number of variables in X.

Thus, under the hypothesis that $\alpha = 0$, the *F*-statistic is[5]:

$$(2.9) \quad F(1, T-k_1-1) = Q/[\ \hat{\varepsilon}' M_x \hat{\varepsilon} / (T\text{-}k_1\text{-}1)] = \left[\frac{(\sum \hat{u}\hat{v})^2}{\sum \hat{u}^2 \sum \hat{v}^2 - (\sum \hat{u}\hat{v})^2} \right] (T-k_1-1)$$

This test statistic can be expressed in terms of correlations between the variables. We show in the Appendix that:

$$(2.10) \quad {}^2F(1, T-k_1-1) = \frac{(T-k_1-1)[R_{yz} - R_{yx} R_{\hat{y}_x \hat{y}_z}]^2}{(1-R^2_{yx})(1-R^2_{\hat{y}x}) - [R_{yz} - R_{yx} R_{\hat{y}_x \hat{y}_z}]^2}$$

Where we placed the superscript 2 to denote that it is a test for the second model, equation (2.2), under the assumption that the first model is true, and where:

R^2_{yx} is the coefficient of determination of the regression of Y on X only,

R^2_{yz} is the coefficient of determination of the regression of Y on Z only,

$R^2_{\hat{y}x}$ is the coefficient of determination of the regression of $\hat{Y}_z$ on X,

$R_{\hat{y}_x \hat{y}_z}$ is the correlation coefficient of $\hat{Y}_x$ and $\hat{Y}_z$, and since these are linear combinations of X and Z respectively, $R_{\hat{y}_x \hat{y}_z}$ is the canonical correlation of the alternative regressors X and Z.[6]

Because the *J* test is symmetric, the second part of the *J* test, maintaining (2.2) and testing for the significance of (1 - α) in (2.3"), the test statistic, denoted as ${}^1F$ is:

$$(2.11) \quad {}^1F(1, T-k_2-1) = \frac{(T-k_2-1)[R_{yx} - R_{yz} R_{\hat{y}_x \hat{y}_z}]^2}{(1-R^2_{yz})(1-R^2_{\hat{y}z}) - [R_{yx} - R_{yz} R_{\hat{y}_x \hat{y}_z}]^2}$$

[5] See equation (22) of Godfrey and Peseran (1983).

[6] Where there is only one regressor in each of *X* and *Z*, the coefficient $R_{\hat{y}_x \hat{y}_z}$ is the correlation between the two regressors and the test statistic simplifies to: ${}^2F(1, T-k_1-1) = \dfrac{(T-k_1-1)[R_{yz} - R_{yx} R_{xz}]^2}{1 - R^2_{xz} - R^2_{yx} - R^2_{yz} + 2R_{yz} R_{yx} R_{xz}}$.

From these two test statistics we note the following:

a) When sample size is small, the difference between the numbers of regressors in the competing model will affect the sizes of the test statistics. If the data were generated by the model of (2.1), the test statistic ${}^{1}\mathrm{F}$ will get smaller as the number of regressors in the alternative model, k_2, increases. This may lead to the rejection of (2.1) in favor of the alternative model (2.2), particularly if the number of regressors k_1 is small. This is consistent with Godfrey and Pesaran's (1982) simulation-based findings as well as with Gourieroux and Monfort (1994) who conclude that the test "is very sensitive to the relative number of regressors in the two hypotheses; in particular the power of the *J* test is poor when the number of regressors in the null hypothesis is smaller than the number of regressors in the alternative one." However, the influence of the differentials in the number of regressors will become negligible as sample size increases.

b) When a model is successful in explaining the variations in Y the *J* test is likely to reject it. To see this clearly, assume that the alternative regressors are orthogonal so that $\mathrm{R}_{\hat{y}_x \hat{y}_z} = 0$. If model (2.1) is successful, the high coefficient of determination R^2_{yx} will increase the numerator of (2.11) while reducing the denominator, thus increasing the value of the test statistic ${}^{1}\mathrm{F}$ which leads to rejection of the model (2.1). Similarly, if the model (2.2) is successful in explaining the variations in Y, the high value of R^2_{yz} will increase the value of the test statistic ${}^{2}\mathrm{F}$ which leads to the rejection of model (2.2). When both models are successful in explaining the variations in Y, the combined effect of high R^2_{yx} and R^2_{yz} leads us to reject both models. Such was the situation in Davidson and MacKinnon [1981] report on the five alternative consumption function where the coefficients of determination for all the models ranged from .997933 to .998756, yet all the models were rejected. This would be at variance with the conclusion reached by Godfrey and Pesaran (1983) "when sample sizes are small the application of the (unadjusted) Cox test or the J-test to non-nested linear regression models is most likely to result in over rejection of the null hypothesis, even when it happens to be true, if …the true model fits poorly", unless the fit of the false model also fits poorly.

c) $R_{\hat{y}_x \hat{y}_z}$ is the canonical correlation coefficient of the sets of regressors X and Z. Higher values of this correlation would reduce the numerator and increase the denominator of both (2.10) and (2.11), lowering the values of the *F* statistics. The effect would reduce the likelihood of rejecting either of the competing hypotheses. The reverse, as stated in Godfrey and Pesaran (1983) is also true: "when the correlation among the regressors of the two models is weak" the *J* test "is most likely to result in over rejection of the null hypothesis, even when it happens to be true,"

The effect of the coefficients of determination of the alternative model specifications on the *F* statistic is shown in Figure 1.[7] In this figure, the light grey areas represent combinations of R^2_{yx} and R^2_{yz} that result in rejecting the X model (2.1) and failing to reject the Z model (2.2). This is appropriate since for those combinations, the model using the set of explanatory variables Z is clearly superior to that which uses the set X. The dark grey areas represent combinations that result in rejecting the model that uses the set of explanatory variables Z and failing to reject the model which uses X. Again, this is clearly appropriate. The interior white areas represent combinations of the coefficients of determination for which the *J* test fails to reject both models. Within those areas comparisons of the coefficients of determination for the two alternative models, particularly for large samples would have led to the conclusion that neither model is

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[7] The coefficients of determination and the canonical correlations are subject to restrictions. Since the quadratic form $\hat{\varepsilon}' M_x \hat{\varepsilon} = \sum \hat{u}^2 - [(\sum \hat{u}\hat{v})^2 / \sum \hat{v}^2]$ is positive semi-definite, $\sum \hat{u}^2 \sum \hat{v}^2 \geq (\sum \hat{u}\hat{v})^2$, that is: $(1 - R^2_{yx})(1 - R^2_{\hat{y}x}) \geq [R_{yz} - R_{yx}.R_{\hat{y}_x \hat{y}_z}]^2$. The restrictions imply that when the canonical correlation between X and Z is zero (the two sets of alternative explanatory variables are orthogonal), $R^2_{yx} + R^2_{yz} \leq 1$. Thus, in figures (1.a), (1.b) and (1.c) where the canonical correlation is set at zero, the only feasible region is the triangle below the line connecting the points $R^2_{yx} = 1$ and $R^2_{yz} = 1$. When the canonical correlation is different from zero, the restriction on the relationship between the coefficients of determination result in the elliptical shape of the feasible region shown in the second and third columns of Figure 1. Combinations of the coefficients of determinations outside of the ellipse violate the restriction.

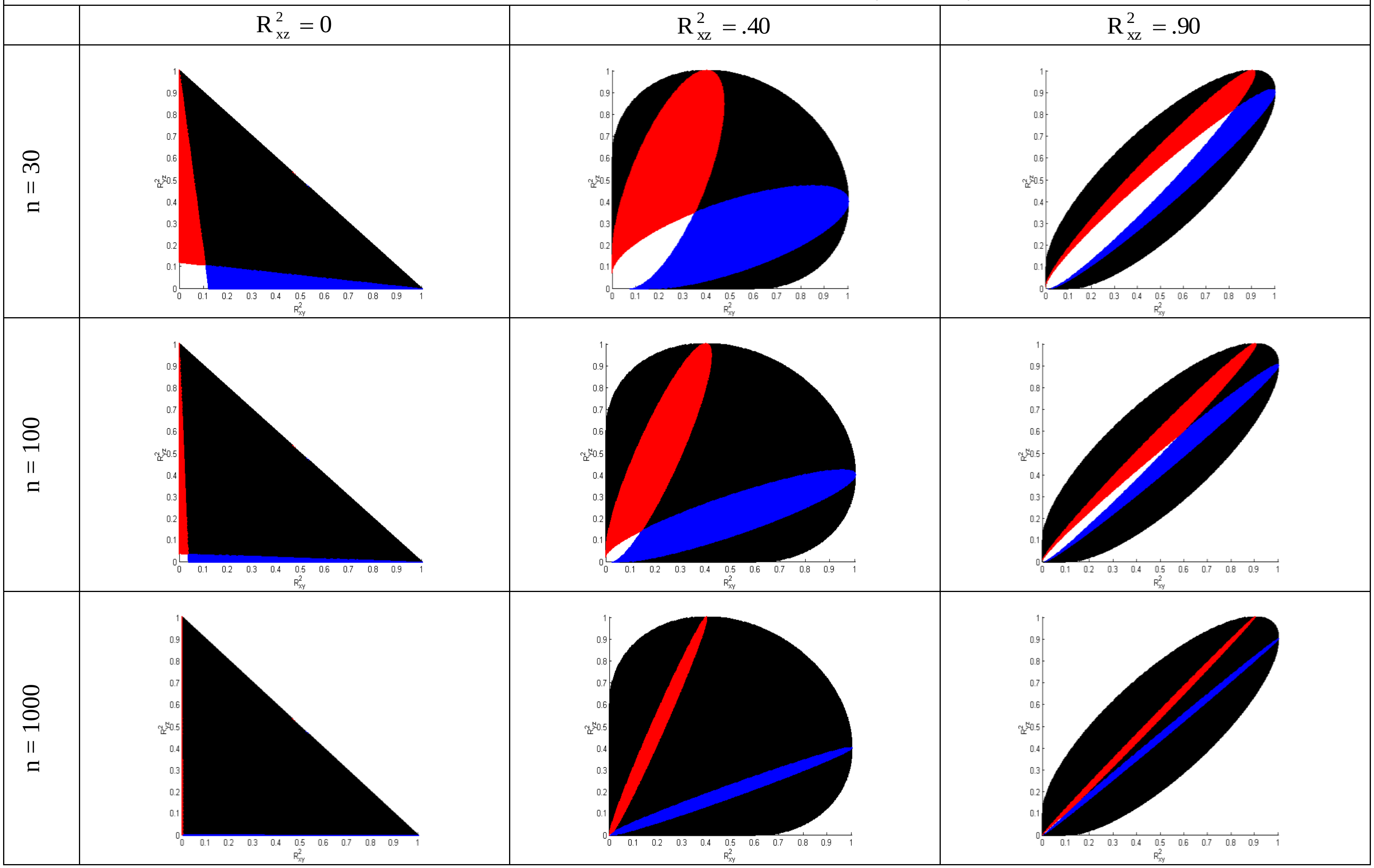
particularly useful in explaining Y. The black areas represent combinations of R^2_{yx} and R^2_{yz} that result in rejecting both hypotheses. What is remarkable is the size of these areas compared to the other areas and the fact that the black area encompasses combinations of R^2_{yx} and R^2_{yz} that, because of their large difference, would reasonably preclude a researcher from employing the *J* test.[8] For instance, consider two competing models in the middle panel (the case of n = 100 and $R^2_{xz} = .4$). If one model had $R^2_{yx} = .9$ and the other $R^2_{yz} = .6$, the *J* test would reject both models despite the fact that the X model would be traditionally viewed as the superior model based solely on the comparisons of the coefficients of determination.

The canonical correlation of the competing model's independent variables impacts the permissible values of the *J* test. The first panel demonstrates a canonical correlation between regressors set at zero, in the second panel it is set at .40 and in the third it is set at .90. It is worth noting that when the canonical correlation is greater than zero, the size of the permissible region decreases as the correlation increases. In the extreme case where the canonical correlation approaches 1, so that each of the variables X and Z are a linear combination of the other, the permissible combinations of the coefficients of determination, R^2_{xy} and R^2_{yz} will lie on the 45 degree diagonal emanating from the origin.

The effect of sample size on both the permissible region, we present the figures for sample sizes 30, 100 and 1000 in each of the three panels. The size of the permissible region depends only on the value of the canonical correlation, and is independent of sample size as would be expected. It is clear from these figures that as sample size increases, the area in which the *J* test would lead to the rejection of both hypotheses expands and thus covers increasingly larger areas of the permissible region.

[8] The white spaces outside of the shaded areas are regions where the combinations of the coefficients of determination that are not permissible- they result in violating the requirement that $\hat{\varepsilon}' M_x \hat{\varepsilon} = \sum \hat{u}^2 - [(\sum \hat{u}\hat{v})^2 / \sum \hat{v}^2$] is positive semi-definite.

Figure 1: J-Test Results for Various N, R^2_{xz}, R^2_{xy}, and R^2_{yz}



Notes: Black area represents reject both region, red (light grey) area represents reject the X model and fail to reject the Z model, blue (dark grey) represents reject the Z and fail to reject the X model, interior white areas represents fail to reject both models. All graphs were produced at a 5% level of significance.

III. DETERMINANTS OF MONTHLY VARIATIONS IN INDUSTRY OUTPUT

3.1 Alternative Model Specifications for Production Behavior

We now apply the *J* test to compare two model specifications that have been used to explain monthly variations in industry output (Ghali, 2004). In both specifications monthly output is determined by sales. In one specification the stock of inventories also influences production, while in the other specification inventory stock does not play a role. The two specifications also differ in the way in which the sales variable enter into the specification.

Minimizing the discounted cost over an infinite horizon for the traditional cost function used by many researchers results in the Euler equation reported by Ramey and West (1999, p. 885). Solving for current period output, Q_t and assuming the cost shocks to be random,[9] we get the following equations:

$$(3.1) \quad Q_i = \beta_0 + \beta_1[\Delta Q_i - 2b\Delta Q_{i+1} + b^2\Delta Q_{i+2}] + \beta_2 Q_{i+1} + \beta_3 S_{i+1} + \beta_4 H_i + u_i$$

where Q_i is output in month *"i"*, S_i is sales and H_i is the inventory stock at the end of the month.

The minimization of the cost using an alternative cost function (Ghali, 1987) and solving the resulting Euler equation for output we get:

$$(3.2) \quad Q_{it} = \gamma_0 + \gamma_1 S_{it} + \gamma_2 \overline{S}_t + u_{it}\ ,$$

where S_{it} represents sales in month "*i*" of production planning horizon "*t*" and $\overline{S}_t$ is the average sales over the production planning horizon.

We apply the *J*-test to the two specifications M1 in equation (3.1) and M2 in equations (3.2). as specifications are non-nested hypotheses explaining the monthly variability of production.

[9] The empirical justification for this assumption is that the estimates reported in the literature for the effect of factor price variations on cost are not strongly supportive of the assumption that the cost shocks are observable. Ramey and West (1999) tabulated the results of seven studies regarding the significance of the estimated coefficients for input prices. They reported that wages had a significant coefficient in only one study and material prices in one study. (Ramey and West, 1999, 907). More detailed discussion is given in Ghali (2004).

3.2 The Data

The data we use are those used by Krane and Braun (1991).[10] These data are in physical quantities, thus obviating the need to convert value data to quantity data and eliminating the numerous sources of error involved in such calculations[11]. The data are monthly, eliminating the potential biases that may result from temporal aggregation.[12] They are at the four-digit SIC level or higher, reducing the potential biases that may result from the aggregation of heterogeneous industries into the two-digit SIC level.[13] The data are not seasonally adjusted, thus obviating the need for re-introducing seasonals in an adjusted series.[14] A description of the data and their sources is available in Table 1 of Krane and Braun (1991, 564-565). We use the data on the 24 industries studied by Ghali (2005).

3.3 Empirical Results[15]

In Table 1 the results of applying the *J* test to compare the two specifications are reported. In the first set of columns we report the results of testing M1, equation (3.1) assuming that M2, equation (3.2) is maintained. This is done by estimating the parameters of equation (3.2) using OLS as suggested by Davidson and MacKinnon,[16] The coefficient of determination, R^2, of those regressions are reported in the first column, We then used the predicted values of Q_i from that regression as an added regressor in the estimation of equation (1'). The coefficient of the added regressor, $\hat{Q}_i$, is reported in the second column and its *t* value in the third column. If the coefficient of the added regressor is significantly different from zero, the model specification (3.1) is rejected in favor of the model specification (3.2). As the fourth column shows, this was the case for all of the industries.

The process is reversed in the second set of columns of Table 1. We now maintain the model specification of equation (3.1) and test that of equation (3.2). The last column of this set of

[10] We are very grateful to Spencer D. Krane who made this data available.
[11] For discussion of the potential measurement errors in converting value to quantity data for inventory stocks,see Krane and Braun (1991, 560 –562).
[12] See Ghali (1987) and Lai (1991).
[13] For discussion of the potential biases see Krane and Braun (1990, 7).
[14] For example see Ramey (1991). She had to re-introduce seasonality as the data she was using was seasonally adjusted.
[15] The results reported in Table (1) are from Ghali 2007 reproduced with permission from publisher.
[16] All equations were estimated under the assumption of an AR(1) process for the error term.

columns shows that the model of equation (3.2) is rejected in favor of the model specification of equation (3.1).

As can be seen from Table 1, for all industries studied both competing specifications are rejected by the *J*-test. "When both models are rejected, we must conclude that neither model is satisfactory, a result that may not be welcome but that will perhaps spur us to develop better models."(Davidson and MacKinnon, 1993, p. 383). However, it should be noted that each of the model specifications explains very high proportion of the *monthly* variation of output that as seen by the high coefficients of determination reported for each. It may be that because each of the specifications is so successful in explaining the behavior of output, the *J* test is not able to distinguish between them. In other words, if the maintained specification is successful in explaining the dependant variable, the correlation between the predicted value and the dependant value will be significant, and so will be the coefficient of the predicted value when added as a regressor in the artificial compound model.

TABLE 1: DAVIDSON- MACKINNON *J* TEST

INDUSTRY	SAMPLE PERIOD	H_2Maintained, H_1Tested				H_1Maintained H_2 Tested			
		R_2^2	α	t	H_1	R_1^2	α	t	H_2
Asphalt1	1977:01-1988:12	.955	.767	16.76	R	.980	.959	14.98	R
Bituminous Coal	1977:01-1988:09	.606	.371	8.432	R.	.944	.958	34.276	R
Cotton Fabric	1975:01-1986:12	.936	.507	13.201	R.	.944	.578	14.242	R
Distillate Fuel	1977:01-1988:12	.686	.330	7.000	R	.943	1.060	28.310	R
Gasoline	1977:01-1988:12	.733	.246	3.750	R	.904	1.058	19.085	R
Glass Containers	1977:01-1989:03	.670	.231	4.301	R	.902	.969	20.822	R
Iron and Steel Scrap	1956:01-1988:12	.969	.596	15.868	R	.966	.642	18.185	R
Iron Ore	1961:01-1988:12	.873	.376	12.489	R	.966	1.087	37.225	R
Jet Fuel	1977:01-1988:12	.899	.182	3.932	R	.983	.961	28.234	R
Kerosene	1977:01-1989:03	.823	.318	5.551	R	.949	1.155	26.824	R
Liquefied Gas	1977:01-1988:12	.376	.441	7.015	R	.917	1.045	31.742	R
Lubricants	1977:01-1988:12	.496	.198	2.940	R	.912	1.046	25.697	R
Man-made Fabric	1975:01-1986:12	.904	.587	11.894	R	.903	.553	10.606	R
Newsprint Canada	1961:01 1988:12	.838	.243	8.769	R	.966	.965	44.514	R
Newsprint US	1961:01-1988:12	.991	.402	12.963	R	.996	.773	30.423	R
Petroleum Coke	1977:01 1989:03	.933	.216	5.377	R	.983	.847	20.469	R
Pig Iron	1961:01-1988:12	.997	.924	39.214	R	.985	.242	8.294	R
Pneumatic Casings	1966:01-1988:12	.802	.317	12.310	R	.956	.983	40.722	R
Residual Fuel	1977:01-1988:12	.965	.313	6.950	R	.991	.997	29.659	R
Slab Zinc	1977:01-1988:03	.888	.254	5.283	R	.976	1.065	28.974	R
Sulfur	1961:01-1988:12	.889	.281	6.348	R	.988	1.024	72.875	R
Super Phosphates	1981:01-1988:12	.941	.325	6.743	R	.981	.936	21.897	R
Synthetic Rubber	1961:01 1984:12	.833	.198	5.567	R	.965	1.026	34.948	R
Waste Paper	1977:01 1988:02	.874	.440	6.253	R	.962	.896	19.073	R

IV. A BAYESIAN SOLUTION

In many non-standard testing of hypotheses situations when the classical procedures lead to inconsistent results as in the case of *J*-test, the Bayesian approach provides an alternative that is consistent (see, for example; Zellner (1971, 1994), Berger and Pericchi (2001)). The Bayesian paradigm is generally more involved as it necessitates the specification of prior distribution for the parameters as well as the hypotheses, obtaining marginal likelihoods, Bayesian posterior odds and Bayes factors for the competing hypotheses. Therefore, it is not surprising that we find a rather limited number of applications of the Bayesian approach even though it is intuitively more appealing and provides consistent and meaningful results.

Schwarz (1978) suggested approximations to Schwarz Information criterion (SIC) and Bayesian Information criterion (BIC) using the log of the likelihood values. Later, Kass and Raftrey (1995) provided extensions and applications for computing Bayes factors. By combining these approaches we can use the maximum likelihood values obtained from the estimation needed for the *J* test to approximate the Bayesian posterior odds and the Bayes factor.

We give below a brief overview of the Bayesian approach and then describe how the log of the maximum likelihood can be used to asymptotically approximate the Bayes factor and to provide consistent results for non nested model selection.

4.1 An overview of the Bayesian Hypothesis Testing for Nested and Non-nested hypotheses

The theory of Bayesian testing of hypotheses is built around the concept of posterior probabilities of hypotheses and the Bayes factor, which were first introduced by Jeffreys (1935, 1961). Bayesian model comparison concepts and the issues that arise in empirical applications have been discussed by Zellner (1971), Kass and Raftrey (1995), Berger and Pericchi (2001) and Koop (2003), amongst many others. Schwarz (1978) paved the way for interplay between the Information Criteria and the Bayes factor for Bayesian specification test. We use Schwarz' approximation of Bayesian information criteria and the log likelihood values to calculate Bayes factors for the competing models.

If M1, M2 are two different model specifications[17] for a given data D, the posterior odds ratio K_{12} is given by

(4.1) $K_{12} = [P\,(\text{Data}/H_1)/P\,(\text{Data}/H_2)]^* \, [P\,(H_1)\,/P\,(H_2)]$

Or:

(4.2) Posterior Odds = Bayes factor X Prior odds

In the absence of any definitive information or if we have little information we treat the two hypotheses *a priori* equally likely implying $P(H_1) = P(H_2) = ½$, and the prior odds ratio $[P\,(H_1) / P\,(H_2)]$ is equal to 1. If prior odds equal one, from (4.2), the Posterior odds ratio is same as the Bayes factor.

The Bayes factor is the ratio of the posterior probability of observing the data if H_i, i=1, 2 were true. Bayes factor K_{12} measures the extent to which data supports Hypothesis l over Hypothesis 2 and the evidence against Hypothesis 2.

$P(D/H_i$, i=1,2…k) , the marginal likelihood and is also known as the weighted likelihood or the predictive likelihood and is given by

(4.4) $P\,(D/H_i) = \int P(D/\theta_i, H_i)\ \pi\,(\theta_i/ H_i)\ d\theta_i$ i=1.2….,k

Where θ_i, is the parameter under H_i and $\pi\,(\theta_i/ H_i)\ d\theta_i$ is its prior probability density and $\int P(D/\theta_i, H_i)$ is the probability density of D given the value of θ_i under the hypothesis H_i or the likelihood function of θ.

In the traditional Bayesian approach, we must specify the prior distribution $\pi(\theta_i/ H_i)$ for the parameter(s) θ_i. The use of prior distribution is the double edged sword for the Bayesian approach. This is what provides that extra information in applications and the advantage over

[17] The two model specifications must be exahustive if we need to obtain Posterior probabilities of hypothesis from the posterior odds. The results can be easily extended for k model specifications.

the classical approach. On the other hand, the specification of the prior is one of the most controversial aspects of the Bayesian approach.

The quantity $P(D/H_i)$, is the predictive probability of the data; that is the probability of seeing this data which is calculated before the data is observed. Bayes factor which is the ratio of these marginal probabilities of the data shows the evidence in favor of or against the hypothesis. In case of two hypotheses, i=1,2:

(4.5) $K_{12} = P(D/H1) / P(D/H2)$

(4.6) $K_{12} = \int P(D/\theta_1, H_1)\, \pi(\theta_1/ H_1)\, d\theta_1 \,/ \int P(D/\theta_2, H_2)\, \pi(\theta_2/ H_2)\, d\theta_2$

If K_{12} is greater than 1, the data favors Hypothesis 1 (Model M1) over Hypothesis 2(Model M2) and if K_{12} is less than 1, the data favors Hypothesis 2 (model M2).

4.2 Bayes factor, BIC and the Likelihood values

Although Bayes factors are fairly versatile and universally applicable for specification testing, calculation of marginal likelihoods is extremely demanding and sometimes these may not even exist (Leamer 1978). There has been great interest in finding alternate methods and approximations to Bayes factors. Various information criterions have been developed to this effect, which are not very rigorous, yet they are approximations to quantities that are either Bayesian or have a Bayesian justification. Akaike, Schwarz and Bayesian information criterion are frequently used.

From Schwarz (1978) we note that the log of the Marginal likelihood can be approximated by the log of the Likelihood minus a correction term. This asymptotic approximation to the marginal likelihoods can be used to compute Bayes factor and can be applied to obtain the Posterior odds and the posterior probabilities of the two competing models from the likelihood values for each model (Koop 2003 and Kass and Raftrey 1995). This is how it works in practice:

(4.7) $-2\ \text{SIC} \cong \text{BIC}$

(4.8) $\text{SIC} = (\log[\ pr(\ D\ \theta_1/,,M1] - \log[\ pr(\ D/\ \widetilde{\theta 2}\ , M2]\ ) - ½\ (p_{1\ -}\ p_2)\ \log(n),$

Where $\widetilde{\theta i}$ i-1,2 are the MLE under Model Mi, p1, and p2 are the number of parameters in models 1 and 2 respectively and n is the sample size.

(4.9) $\text{BIC}\ (M_1) = 2 \log pr\ (D/\ \theta\hat{1}\ /, M1] - p_1 \ln(n)$,

(4.10) $\text{BIC}\ (M_2) = 2\log pr\ [\ p(\ D/\ \theta 2\ \ M2] - p_2 \ln(n)$, and

(4.11) $2 \log K_{12} = \text{BIC}\ (M2) - \text{BIC}\ (M1)$

Since BICs can be calculated from likelihood values, we can calculate twice the Bayes factor from (4.11) without specifying the prior distribution. Once we know the 2 log K_{12} and since Models M1 and M2 are exhaustive in this case we can obtain posterior probabilities Π_1 and Π_2 for Models M1 and M2 by using the relationship:

(4.12) $\Pi_1 = \dfrac{\mathbf{K12}}{\mathbf{1 + K12}}$; and $\Pi_2 = \dfrac{1}{1 + K12}$

Although in all empirical applications we use only the posterior odds and the Bayes factors, we can also use the posterior probabilities of individual specifications and hypothesis for ranking and comparing different model specifications in case of larger number of alternate model specifications. A decision to accept or reject a particular model generally requires choosing a model that minimizes an appropriate loss function.

Jeffreys in (1961, appendix B) also proposed some rules of thumb for interpreting Bayes factor. If we consider $0 < 2\ \text{Log}(K_{21}) < 2$; the evidence against M1 is not worth more than a bare mention, if $2 < 2 \text{ Log}(B_{21}) < 6$, the evidence against M1 is positive and if $6 < 2 \text{ Log}(B_{21}) < 10$, the evidence against is strong and if $2 \text{ Log}(B_{21}) > 10$, the evidence against M1 is very strong. We shall use these guiding rules to make decisions for choosing between two cost functions for all twenty five industries in our data set.

4.3 Bayes Factors and the Model Specification:

Let us consider that the model specification M_1 in equation 3.1 is the Null Hypothesis H_0 and the maintained hypothesis is model specification M2 in equation 3.2. Bayes factor K_{12} will measure the evidence for Model 1 against model 2 and K_{21} will measure the evidence against M1. These results are given in the Table 2 below. The results are quite consistent and unequivocal that specification M2 in equation 3.2 is strongly supported by the data for 23 of the 25 industries (except, iron scrap and pig iron) irrespective of the choice of the Null and the maintained hypotheses.

Table 2: Bayes factors and Posterior Probabilities of Models M1 and M2

	2 Log K21	K21	Evidence against M1	2 log (K12)	K12	Evidence Against M2	Π1	Π2
Asphalt	101.99	1.4E+22	very Strong	-101.99	7.12E-23	Not worth mention	7.12E-23	1
Beer	18.56	10695.36	very Strong	-18.56	9.35E-05	Not worth mention	9.35E-05	0.999907
Bituminous Coal	322.56	1.1E+70	very Strong	-322.56	9.06E-71	Not worth mention	9.06E-71	1
Cotton Fabric	25.09	280624.6	very Strong	-25.09	3.56E-06	Not worth mention	3.56E-06	0.999996
Distillate Fuel	249.23	1.32E+54	very Strong	-249.23	7.6E-55	Not worth mention	7.6E-55	1
Gasoline	154.64	3.79E+33	very Strong	-154.64	2.64E-34	Not worth mention	2.64E-34	1
Glass Containers	178.87	6.94E+38	very Strong	-178.87	1.44E-39	Not worth mention	1.44E-39	1
Iron Scrap	-18.37	0.000103	not worth mention	18.37	9739.505	Very strong	0.999897	0.000103
Iron Ore	353.21	4.99E+76	very Strong	-353.21	2E-77	Not worth mention	2E-77	1
Jet Fuel	257.77	9.43E+55	very Strong	-257.77	1.06E-56	Not worth mention	1.06E-56	1
Kerosene	175.58	1.34E+38	very Strong	-175.58	7.47E-39	Not worth mention	7.47E-39	1
Liquified Gas	277.70	2E+60	very Strong	-277.70	5E-61	Not worth mention	5E-61	1
Lubricants	232.60	3.22E+50	very Strong	-232.60	3.11E-51	Not worth mention	3.11E-51	1
Man-made Fabric	7.61	44.98217	Strong	-7.61	0.022231	Not worth mention	0.021748	0.978252
Newsprint Canada	520.39	1E+113	very Strong	-520.39	1E-113	Not worth mention	1E-113	1
Newsprint US ARD	513.09	2.6E+111	very Strong	-513.09	3.8E-112	Not worth mention	3.8E-112	1
Newsprint US	276.53	1.12E+60	very Strong	-276.53	8.97E-61	Not worth mention	8.97E-61	1
Petroleum Coke	125.52	1.81E+27	very Strong	-125.52	5.53E-28	Not worth mention	5.53E-28	1
Pig Iron	-508.78	3.3E-111	not worth mentioning	508.78	3E+110	Very strong	1	3.3E-111
Pneumatic casings	512.75	2.2E+111	very Strong	-512.75	4.5E-112	Not worth mention	4.5E-112	1
Residual Fuel	197.12	6.36E+42	very Strong	-197.12	1.57E-43	Not worth mention	1.57E-43	1
Slab Zinc	216.21	8.88E+46	very Strong	-216.21	1.13E-47	Not worth mention	1.13E-47	1
Sulphur	722.68	8.5E+156	very Strong	-722.68	1.2E-157	Not worth mention	1.2E-157	1
Super Phosphate	148.91	2.17E+32	very Strong	-148.91	4.62E-33	Not worth mention	4.62E-33	1
Synthetic Rubber	460.03	7.8E+99	very Strong	-460.03	1.3E-100	Not worth mention	1.3E-100	1
Waste Paper	180.21	1.36E+39	very Strong	-180.21	7.37E-40	Not worth mention	7.37E-40	1

IV. CONCLUSION

In earlier research the original *J* test has been shown to over reject when the true model fits the data poorly, when the regressors in the models being compared are highly correlated, or when the false model contains more regressors than the true model. We presented examples where the alternative specifications fit the data well but the *J* test did not distinguish between them: the *J* test either rejects, or fails to reject both specifications.

To supplement the J test when such situations arise we proposed a Bayesian approach that uses the estimated maximum likelihood values obtained in the process of conducting the test. Bayesian posterior odds allow us to overcome the problems associated with the *J*-test. Jeffreys' Bayes factors offer ways of obtaining the posterior probabilities of the competing model specifications and relative ranking of the competing specifications. We showed that by using approximations of Schwarz Information Criterion and Bayesian Information Criterion we can use the classical estimates of the log of the maximum likelihood to obtain Bayesian posterior odds and posterior probabilities of the competing nested and non- nested models.

Bayesian testing for nested and non-nested specifications is currently an active area of research. Bayes intrinsic factors and Bayes fractional intrinsic factors and default Bayes factors of Berger and Pericchi (2001), the approximations by Gelfand and Dey (1994) and Chib (1995) are all very promising for applications in complex economic models and in case of panel data models. The method we proposed in this paper has an advantage as it gives us all the benefits of the Bayesian paradigm and the Bayes factors without having to specify prior probabilities and going through the extensive Bayesian computations.

APPENDIX

Expressing the *F* statistic in terms of correlations between the variables

$$(2.10)\quad F(1, T\text{-}k_1\text{-}1) = Q/[\ \hat{\varepsilon}'M_x\hat{\varepsilon}/(T\text{-}k_1\text{-}1)] = \left[\frac{(\sum \hat{u}\hat{v})^2}{\sum \hat{u}^2 \sum \hat{v}^2 - (\sum \hat{u}\hat{v})^2}\right](T-k_1-1)$$

This test statistic can be expressed in terms of correlations between the variables. For simplicity, we assume that all variables are deviations from means. Now:

$$\sum \hat{u}^2 = \sum Y^2 - \sum \hat{Y}_x^2 = \sum Y^2[1-R_{yx}^2] = (T-1)s_y^2[1-R_{yx}^2]$$

$$\sum \hat{v}^2 = \sum \hat{Y}_z{}^2 - \sum \hat{\hat{Y}}_{zx}^2 = \sum \hat{Y}_z{}^2[1-R_{\hat{y}x}^2] = (T-1)s_{\hat{y}_z}^2[1-R_{\hat{y}x}^2]$$

$$\hat{u}'\hat{v} = Y'M_xP_zY = Y'P_zY - Y'P_xP_zY = \sum \hat{Y}_z^2 - \sum \hat{Y}_x\hat{Y}_z$$

$$(\hat{u}'\hat{v})^2 = [\sum \hat{Y}_z^2 - \sum \hat{Y}_x\hat{Y}_z]^2 = (\sum \hat{Y}_z^2)^2 + (\sum \hat{Y}_x\hat{Y}_z)^2 - 2\sum \hat{Y}_z^2 \sum \hat{Y}_x\hat{Y}_z$$

$$= (\sum Y^2)^2[(\sum \hat{Y}_z^2)^2/(\sum Y^2)^2 + (\sum \hat{Y}_x\hat{Y}_z)^2/(\sum Y^2)^2 - \{2(\sum \hat{Y}_z^2)/(\sum Y^2)\}\{(\sum \hat{Y}_x\hat{Y}_z)/(\sum Y^2)\}]$$

$$= (T-1)^2 s_y^4 R_{y_z}^2 [R_{y_z}^2 + R_{y_x}^2 .R_{\hat{y}_x\hat{y}_z}^2 - 2(\sum \hat{Y}_x\hat{Y}_z)/(\sum Y^2)]$$

$$= (T-1)^2 s_y^4 R_{y_z}^2 [R_{y_z}^2 + R_{y_x}^2 .R_{\hat{y}_x\hat{y}_z}^2 - 2(\mathrm{Cov}(\hat{Y}_x\hat{Y}_z)/s_y^2)]$$

$$= (T-1)^2 s_y^4 R_{y_z}^2 [R_{y_z}^2 + R_{y_x}^2 .R_{\hat{y}_x\hat{y}_z}^2 - 2(R_{\hat{y}_x\hat{y}_z} s_{\hat{y}x} s_{\hat{y}z}/s_y^2)]$$

$$F = \frac{(\sum \hat{u}\hat{v})^2}{\sum \hat{u}^2 \sum \hat{v}^2 - (\sum \hat{u}\hat{v})^2}(T-k_1-1)$$

$$= \frac{(T-k_1-1)(T-1)^2 s_y^4 R_{y_z}^2 [R_{y_z}^2 + R_{y_x}^2 .R_{\hat{y}_x\hat{y}_z}^2 - 2(R_{\hat{y}_x\hat{y}_z} s_{\hat{y}x} s_{\hat{y}z}/s_y^2)]}{(T-1)^2 s_y^2 s_{\hat{y}_z}^2 (1-R_{yx}^2)(1-R_{\hat{y}x}^2) - \left[(T-1)^2 s_y^4 R_{y_z}^2 [R_{y_z}^2 + R_{y_x}^2 .R_{\hat{y}_x\hat{y}_z}^2 - 2(R_{\hat{y}_x\hat{y}_z} s_{\hat{y}x} s_{\hat{y}z}/s_y^2)]\right]}$$

$$= \frac{(T-k_1-1)s_y^2 R_{y_z}^2 [R_{y_z}^2 + R_{y_x}^2 .R_{\hat{y}_x\hat{y}_z}^2 - 2(R_{\hat{y}_x\hat{y}_z} s_{\hat{y}x} s_{\hat{y}z}/s_y^2)]}{s_{\hat{y}_z}^2 (1-R_{yx}^2)(1-R_{\hat{y}x}^2) - \left[s_y^2 R_{y_z}^2 [R_{y_z}^2 + R_{y_x}^2 .R_{\hat{y}_x\hat{y}_z}^2 - 2(R_{\hat{y}_x\hat{y}_z} s_{\hat{y}x} s_{\hat{y}z}/s_y^2)]\right]}$$

But $s_{\hat{y}_z}^2 = s_y^2 R_{y_z}^2$, so that the expression can be written as:

$$F = \frac{(T-k_1-1)[R_{y_z}^2 + R_{y_x}^2 .R_{\hat{y}_x\hat{y}_z}^2 - 2(R_{\hat{y}_x\hat{y}_z} s_{\hat{y}x} s_{\hat{y}z}/s_y^2)]}{(1-R_{yx}^2)(1-R_{\hat{y}x}^2) - [R_{y_z}^2 + R_{y_x}^2 .R_{\hat{y}_x\hat{y}_z}^2 - 2(R_{\hat{y}_x\hat{y}_z} s_{\hat{y}x} s_{\hat{y}z}/s_y^2)]}$$

The F statistic for the model (2.2) with model (2.1) as maintained hypothesis, which we denote by ${}^2F$ is given by:

$${}^2F(1, T-k_1-1) = \frac{(T-k_1-1)[R_{y_z} - R_{y_x} .R_{\hat{y}_x\hat{y}_z}]^2}{(1-R_{yx}^2)(1-R_{\hat{y}x}^2) - [R_{y_z} - R_{y_x} .R_{\hat{y}_x\hat{y}_z}]^2}$$

The second part of the *J* test consists of maintaining (2.2) and testing for the significance of $(1-\alpha)$ in (2.3”). This can be similarly derived with the roles of X and Z reversed. If the number of regressors in *Z* is k_2, the test statistic which we denote by ${}^{a}F$ is:

$$ {}^{1}F(1, T-k_2-1) = \frac{(T-k_2-1)[R_{y_x} - R_{y_z}.R_{\hat{y}_x\hat{y}_z}]^2}{(1-R^2_{yz})(1-R^2_{\hat{y}z}) - [R_{y_x} - R_{y_z}.R_{\hat{y}_x\hat{y}_z}]^2} $$

$R^2_{\hat{y}x}$ is the coefficient of determination of the regression of $\hat{Y}_z$ on X. Since $\hat{Y}_z$ is a linear transformation of Z, $\hat{Y}_z = Z\hat{\gamma}$, the coefficient $R^2_{\hat{y}x} = R^2_{xz}$.
$R_{\hat{y}_x\hat{y}_z}$ is the correlation coefficient of $\hat{Y}_x$ and $\hat{Y}_z$, and since these are linear transformations of X and Z respectively, $R_{\hat{y}_x\hat{y}_z}$ is the canonical correlation of the alternative regressors X and Z.

If Z has only one variable and *X* has only one variable, $R_{\hat{y}_x\hat{y}_z} = R_{xz}$, and the *F* statistic for the model (2.2) with model (2.1) as maintained hypothesis, which we denote by ${}^{2}F$ is given by:

$$ {}^{2}F(1, T-2) = \frac{(T-2)[R_{\hat{y}_z} - R^2_{xz}]^2}{(1-R^2_{yx})(1-R^2_{xz}) - [R_{\hat{y}_z} - R^2_{zx}]^2} $$

This can be written as:

$$ {}^{2}F(1, T-2) = \frac{(T-2)[R_{y_z} - R_{y_x}.R_{xz}]^2}{1 - R^2_{xz} - R^2_{yx} - R^2_{y_z} + 2R_{y_z}R_{y_x}.R_{xz}} $$

The second part of the *J* test consists of maintaining (2.2) and testing for the significance of (1-α) in (2.3”). This can be similarly derived with the roles of X and Z reversed. If the number of regressors in Z is k_2, the test statistic which we denote by ${}^{1}F$ is:

$$ {}^{1}F(1, T-k_1-1) = \frac{(T-k_1-1)[R_{\hat{y}_x} - R^2_{xz}]^2}{(1-R^2_{yz})(1-R^2_{xz}) - [R_{\hat{y}_x} - R^2_{zx}]^2}. $$

REFERENCES

Balakrishnan, P, K. Surekha and B.P. Vani, 1994, The Determinants of Inflation in India, Bayesian and Classical Analyses, *Journal of Quantitative Economics*, vol. 10, no.3, 325-336.

Berger , J.O. and L. Pericchi, 2001, Objective Bayesian Methods for Model Selection: Introduction and Comaprison, in P. Lahiri ed. Institute of Mathematical Statistics Lecture Notes -- Monograph Series volume 38, Beachwood Ohio, 135--207.

Chib, S, (1995), Marginal Likelihood from the Gibbs Sampler, Journal of the American Statistical Association, 90, 1331-1350

Davidson, R and MacKinnon, J. G.. 1981, "Several Tests for Model Specification in the Presence of Alternative Hypotheses" *Econometrica*, vol. 49, pp. 781-793.

Davidson, R and MacKinnon, J. G., 1982, "Some Non-Nested Hypothesis Tests and the Relations Among Them," *The Review of Economic Studies*, XLIX, 1982, pp. 551-565.

Davidson, Russell and MacKinnon, James G., 1993, *Estimation and Inference in Econometrics*, Oxford University Press.

Davidson, Russell and MacKinnon, James G., 2002, "Fast Double Bootstrap Tests of Non-nested Linear Regression Models," *Econometric Reviews*, vol. 21, No. 4, pp. 419-429.

Davidson, Russell and MacKinnon, James G., 2004, *Econometric Theory and Methods*, Oxford University Press.

Eviews 5 User Guide, 2004, Quantitative Micro Software LLC, Irvine, California.

Faff, Robert and Gray, Philip, 2006, "On the Estimation and Comparison of Short-Rate Models Using the Generalized Method of Moments" *Journal of Banking and Finance*, Vol. 30, No. 11, pp. 3131-3146.

Gaver,K.M. and Geisel, M.S., 1974, "Discriminating among Alternative Models: Bayesian and Non-Bayesian Methods," in *Frontiers in Econometrics*, P. Zarembka, ed., Academic Press, New York.

Gelfand, A., and Dey, D.K., Bayesian Model Choice: Asymptotics and Exact calculations, Journal of the Royal Statistical Society Series B, 56, 501-514.

Ghali, M., 1987, "Seasonality, Aggregation and the Testing of the Production Smoothing Hypothesis," *The American Economic Review*, vol. 77, no. 3, pp. 464-469.

Ghali, M., 2005, "Measuring the Convexity of the Cost Function", *International Journal of Production Economics*, vol. 93-94, Elsevier, Amsterdam.

Ghali, M., 2007, "Comparison of Two Empirical Cost Functions," *International Journal of Production Economics*, 108, pp. 15-21

Godfrey, L.G., and Pesaran, M.H., 1983, " Tests of Non-Nested Regression Models: Small Sample Adjustments and Monte Carlo Evidence," *Journal of Econometrics*, vol. 21, pp. 133-154.

Goldberger, A.S., 1968, *Topics in Regression Analysis*, Macmillan, London.

Gourieroux, C. and Monfort, A, 1994, "Testing Non-Nested Hypotheses," Chapter 44 in *Handbook of Econometrics*, R.F. Engle and D.L. McFadden eds., Elsevier Science.

Greene, William, 2003, *Econometric Analysis*, 5th edition, Prentice Hall, New Jersey.

Jeffreys, H. (1935), Some Tests Of Significance, Treated by the Theory of Probability, Proceedings of the Cambrdige Philosophical Society, 31 , 203-222.

Jeffreys, H., 1961, " *Theory of Probability*", *3rd edition*, Oxford University Press, Oxford, UK.

Kass R. and A.E. Raftery, (1995)," Bayes Factors", Journal *of the American Statistocal Association*, Vol 90, no 430, 773-795

Krane, Spencer, and Steven Braun, (1991), "Production Smoothing Evidence from Physical-Product Data," *Journal of Political Economy*, vol. 99,no 3, pp. 558-581.

Koop, Gary, 2003, *Bayesian Econometrics*, John Wiley and Sons, England.

Lai, Kon S., 1991, "Aggregation and Testing of the Production Smoothing Hypothesis," *International Economic Review*, vol.32, no. 2, pp.391- 403.

Lovell, Michael C., 1963, "Seasonal Adjustment of Economic Time Series and Multiple Regression Analysis," *Journal of the American Statistical Association*, vol. 58, pp. 993-1010.

Leamer, E.E., 1978, *Specification Searches, Ad Hoc Inference with Non –experimental data*, John Wiley, New York.

Malinvaud, E., 1966, *Statistical Methods of Econometrics*, Rand McNally & Co., Chicago.

McAleer, Michael, 1995, "The significance of testing empirical non-nested models," *Journal of Econometrics*, vol. 67, pp. 149-171.

Pesaran, M.H, and Weeks, M., 2001, "Non-nested Hypotheses Testing: An Overview," chapter 13 in *Theoretical Econometrics*, Baltagi, B.H. ed., Blackwell Publishers, Oxford.

Pesaran, M.H., 1983, "Comment," *Econometric Reviews*, vol. 2, no. 1, pp. 145-9. Marcel Dekker, Inc.

Pesaran, M.H., and Deaton, A.S, 1978, "Testing Non-nested Nonlinear Regression Models," *Econometrica*, Vol. 46, N0. 3, pp. 677-94.

Ramey, Valerie A., and Kenneth D. West, 1999 "Inventories," in Handbook of Macroeconomics, Volume 1B, John B. Taylor and Michael Woodford eds., Elsevier Science, pp. 863-923.

Singh, Tarlok, 2004, "Optimising and Non-optimising Balance of Trade Models: A Comparative Evidence from India," International Review of Applied Economics, Vol. 18, No. 3, pp. 349-368.

Surekha, K. and W.E. Griffiths, 1990, "Comparison of Some Bayesian Heteroscedastic Error Models", *Statistica,* anno l, n. 1 pp. 109 -117

Surekha K. and M. Ghali, 2001, "Speed of Adjustment and Production Smoothing: Bayesian Estimation", *International Journal of Production Economics, vol. 71, no. 3, pp.55-65*

Schwarz, G, 1978, "Estimating the Dimensions of a Model, *The Annals of Statistics*, Vol. 6, no 2, pp. 461-464.

West, Kenneth, 1986 "A Variance Bounds Test of the Linear Quadratic Inventory Model," *Journal of Political Economy*, vol.94, no.2, pp. 347-401.

Zellner, A. 1971, *An Introduction to Bayesian Inference in Econometrics,* John Wiley, New York

Zellner, A. 1984, *Basic Issues in Econometrics,* University of Chicago Press, Chicago.