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Revenue Smoothing in an ARIMA Framework: Evidence from the United States*

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Abstract

This paper tests Mankiw's (1987) revenue-smoothing hypothesis, that the inflation rate moves one-for-one with the marginal tax rate in the long run, using the new average marginal tax rate series constructed by Stephenson (1998) and the long-horizon regression approach developed by Fisher and Seater (1993). It reports considerable evidence against revenue-smoothing.

Keywords: Optimal seigniorage; Integration; Long-run derivative.

JEL classification: C22, F31

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1 Introduction

A crucial implication of Mankiw's (1987) revenue-smoothing (or optimal seigniorage) hypothesis is that higher tax rates are associated with higher inflation rates (and nominal interest rates). There have been many attempts to test this hypothesis. For example, Mankiw (1987) and Poterba and Rotemberg (1990) using the OLS method find support of the hypothesis. However, more general tests (based on the cointegration and/or VAR methodology) by Trehan and Walsh (1990), Ghosh (1995), Evans and Amey (1996), and Serletis and Schorn (1999) generally reject revenue smoothing.

The present paper extends the literature by testing whether the inflation rate moves one-for-one with the marginal tax rate in the long run, using the new average marginal tax rate series constructed by Stephenson (1998) and the long-horizon regression approach developed by Fisher and Seater (1993). Long-horizon regressions have received a lot of attention in the recent economics and finance literature, because studies based on long-horizon variables seem to find significant results where short-horizon regressions commonly used in economics and finance have failed.

In what follows, we provide a brief summary of Mankiw's (1987) theory of optimal seigniorage and of the econometric approach developed by Fisher and Seater (1993). In section 4, we discuss the data, investigate the integration properties of the variables, and present the results. The paper closes with a brief summary and conclusion.

2 The Theory of Optimal Seigniorage

Following Mankiw (1987), let Y be the exogenous level of real output and τ the tax rate on output. The revenue raised by this tax is τY . It is assumed that the government finances expenditure in excess of taxes from seigniorage. Assuming that the demand for money is described by the quantity equation, $M/P = kY$, the real revenue from seigniorage is

$$\frac{\dot{M}}{P} = \frac{\dot{M}}{M} \frac{M}{P} = (\pi + g)kY$$

where π is the inflation rate and g is the growth rate of real output. The total real tax revenue, T , is therefore the sum of the receipts from direct taxation, τY , and seigniorage, $(\pi + g)kY$. That is, $T = \tau Y + (\pi + g)kY$.

The social costs of taxation and inflation are assumed homogenous in output and denoted by $f(\tau)Y$ and $h(\pi)Y$, respectively, where $f' > 0$, $h' > 0$ and $f'' > 0$, $h'' > 0$. The government's goal is to minimize, with respect to τ and π , the expected present value of the social losses

$$E_t \sum_{j=0}^{\infty} \beta^j [f(\tau_{t+j}) + h(\pi_{t+j})] Y$$

subject to the present value budget constraint

$$\sum_{j=0}^{\infty} \beta^j G_{t+j} + B_t = \sum_{j=0}^{\infty} \beta^j T_{t+j}$$

where G_t is real government expenditure at time t (taken to be exogenous), B_t is real government debt at time t , and β is the real discount factor, assumed constant over time.

The first-order conditions necessary for optimal intertemporal monetary and fiscal policy are [see Mankiw (1987)]

$$E_t [f'(\tau_{t+j})] = f'(\tau_t), \tag{1}$$

$$E_t [h'(\pi_{t+j})] = h'(\pi_t), \tag{2}$$

$$h'(\pi_t) = k f'(\tau_t). \tag{3}$$

The intertemporal first-order conditions (1) and (2) equate the marginal social cost of taxation and inflation, respectively, today and in the future. The static first-order condition (3), which relates the tax rate to the rate of inflation, equates the marginal social cost of raising revenue through taxation and the marginal social cost of raising revenue through seigniorage. This last condition expresses a crucial implication of the theory of optimal seigniorage. Increases in the government revenue requirement increase both taxation and inflation. Hence, over time higher tax rates are associated with higher inflation rates and higher nominal interest rates.

3 Econometric Methodology

As already noted, we test the theory of optimal seigniorage using the long-horizon regression approach developed by Fisher and Seater (1993). One important advantage to working with the long-horizon regression approach is that cointegration is neither necessary nor sufficient for tests on the long-run derivative. We start with the following bivariate autoregressive representation

$$\begin{aligned}\alpha_{\pi\pi}(L)\Delta^{\langle\pi\rangle}\pi_t &= \alpha_{\pi\tau}(L)\Delta^{\langle\tau\rangle}\tau_t + \varepsilon_t^\pi \\ \alpha_{\tau\tau}(L)\Delta^{\langle\tau\rangle}\tau_t &= \alpha_{\tau\pi}(L)\Delta^{\langle\pi\rangle}\pi_t + \varepsilon_t^\tau\end{aligned}$$

where $\alpha_{\pi\pi}^0 = \alpha_{\tau\tau}^0 = 1$, $\Delta = 1-L$, where L is the lag operator, π is the inflation rate, τ is marginal tax rate, and $\langle z \rangle$ represents the order of integration of z , so that if z is integrated of order γ , then $\langle z \rangle = \gamma$ and $\langle \Delta z \rangle = \langle z \rangle - 1$. The vector $(\varepsilon_t^\pi, \varepsilon_t^\tau)'$ is assumed to be independently and identically distributed normal with zero mean and covariance Σ_ε , the elements of which are $\text{var}(\varepsilon_t^\pi)$, $\text{var}(\varepsilon_t^\tau)$, $\text{cov}(\varepsilon_t^\pi, \varepsilon_t^\tau)$.

According to this approach, revenue smoothing can be tested in terms of the long-run derivative of π with respect to a permanent change in τ , which is defined as follows. If $\lim_{k \rightarrow \infty} \partial\tau_{t+k}/\partial\varepsilon_t^\tau \neq 0$, then

$$LRD_{\pi,\tau} = \lim_{k \rightarrow \infty} \frac{\partial\pi_{t+k}/\partial\varepsilon_t^\tau}{\partial\tau_{t+k}/\partial\varepsilon_t^\tau}$$

Thus, in the present context $LRD_{\pi,\tau}$ expresses the ultimate effect of an exogenous marginal tax rate disturbance on π , relative to that disturbance's ultimate effect on the marginal tax rate τ . When $\lim_{k \rightarrow \infty} \partial\tau_{t+k}/\partial\varepsilon_t^\tau = 0$, there are no permanent changes in τ and thus $LRD_{\pi,\tau}$ is undefined. In terms of this framework, revenue smoothing requires that $LRD_{\pi,\tau} = 1$.

The above bivariate autoregressive system can be inverted to yield the following vector moving average representation

$$\begin{aligned}\Delta^{\langle\pi\rangle}\pi_t &= \theta_{\pi\tau}(L)\varepsilon_t^\tau + \theta_{\pi\pi}(L)\varepsilon_t^\pi \\ \Delta^{\langle\tau\rangle}\tau_t &= \theta_{\tau\tau}(L)\varepsilon_t^\tau + \theta_{\tau\pi}(L)\varepsilon_t^\pi\end{aligned}$$

In terms of this moving average representation, Fisher and Seater (1993) show that $LRD_{\pi,\tau}$ depends on $\langle\tau\rangle - \langle\pi\rangle$, as follows

$$LRD_{\pi,\tau} = \frac{(1-L)^{\langle\tau\rangle-\langle\pi\rangle} \theta_{\pi\tau}(L)|_{L=1}}{\theta_{\tau\tau}(1)}$$

Hence, meaningful long-horizon regression tests of the revenue smoothing hypothesis can be conducted if both π_t and τ_t satisfy certain nonstationarity conditions. In particular, long-horizon regression tests require that both π_t and τ_t are at least I(1) and of the same order of integration. In fact, when $\langle\pi\rangle = \langle\tau\rangle = 1$, the long-run derivative becomes

$$LRD_{\pi,\tau} = \frac{\theta_{\pi\tau}(1)}{\theta_{\tau\tau}(1)}$$

where $\theta_{\pi\tau}(1) = \sum_{j=1}^{\infty} \theta_{\pi\tau}^j$ and $\theta_{\tau\tau}(1) = \sum_{j=1}^{\infty} \theta_{\tau\tau}^j$. Above, the coefficient $\theta_{\pi\tau}(1)/\theta_{\tau\tau}(1)$ is the long-run value of the impulse-response of π with respect to τ , suggesting that $LRD_{\pi,\tau}$ can be interpreted as the long-run elasticity of π with respect to τ .

Under the assumptions that $\text{cov}(\varepsilon_t^\pi, \varepsilon_t^\tau) = 0$ and that the marginal tax rate is exogenous in the long-run, the coefficient $\theta_{\pi\tau}(1)/\theta_{\tau\tau}(1)$ equals the zero-frequency regression coefficient in the regression of $\Delta^{\langle\pi\rangle}\pi$ on $\Delta^{\langle\tau\rangle}\tau$ — see Fisher and Seater (1993, note 11). This estimator is given by $\lim_{k \rightarrow \infty} b_k$, where b_k is the coefficient from the regression

$$\left[\sum_{j=0}^k \Delta^{\langle\pi\rangle} \pi_{t-j} \right] = a_k + b_k \left[\sum_{j=0}^k \Delta^{\langle\tau\rangle} \tau_{t-j} \right] + e_{kt}$$

In fact, when $\langle\pi\rangle = \langle\tau\rangle = 1$, consistent estimates of b_k can be derived by applying ordinary least squares to the regression

$$\pi_t - \pi_{t-k-1} = a_k + b_k [\tau_t - \tau_{t-k-1}] + e_{kt}, \quad k = 1, \dots, K. \quad (4)$$

The null of revenue smoothing is $b_k = 1$. If the null is not rejected across a range of k -forecast horizons, the data supports the theory of optimal seigniorage.

4 Empirical Results

4.1 The Data

We examine two variables in this paper — the inflation rate, π_t , and the average marginal tax rate, τ_t . As a measure of the average marginal tax rate we use Stephenson’s (1998) average marginal effective tax rate on personal income (*AMETR*). The data is annual from 1934 to 1994 (a total of 61 observations).

It is to be noted that Mankiw (1987) mostly uses federal government receipts as a percent of GNP (*TAX*), as a measure of the average tax rate, and his analysis is over the 1951 to 1985 period (that is, over 35 observations). He also uses the average marginal tax rate (*MAR*) on labor income (including social security) as estimated by Barro and Sahasakul (1983), and finds a positive relation to both the inflation rate and nominal interest rate. However, only the relation to the nominal interest rate is statistically significant, with the coefficient on *MAR* (in a regression of ΔINT on a constant and ΔMAR) being 0.50.

Of course, as Mankiw (1987, p. 339) puts it “[i]t is not clear *a priori* which of the two tax measures, *TAX* or *MAR*, is preferable. One might argue that the average marginal tax rate is the best measure of the marginal social cost of raising revenue. Yet consider what makes these two variables different. Changes in the mix of taxes, such as a shift between personal and corporate taxes, would change *MAR* without changing *TAX*. It is not obvious whether such a change in the tax mix should be associated with a change in the reliance on seigniorage as a source of revenue. Resolving this issue requires a model more extensive than that presented here.” With this in mind, in what follows we use Stephenson’s (1998) average marginal effective tax rate on personal income (*AMETR*) as a measure of the average tax rate.

4.2 Integration Tests

As it was argued in the introduction, meaningful long-run revenue-smoothing tests can only be conducted if both the π_t and τ_t variables satisfy certain nonstationarity conditions. In particular, revenue-smoothing tests require that both π_t and τ_t are at least integrated of order one and of the same order of integration. Hence, the first step in conducting revenue-smoothing tests is

to test for stochastic trends (unit roots) in the autoregressive representation of each individual time series. In doing so, in what follows we use four alternative testing procedures, to deal with anomalies that arise when the data are not very informative about whether or not there is a unit root.

In Table 1 we report p -values [based on the response surface estimates given by MacKinnon (1994)] for the augmented Weighted Symmetric (WS) unit root test [see Pantula et al. (1994)], the augmented Dickey-Fuller (ADF) test [see Dickey and Fuller (1981)], and the nonparametric, $Z(t_{\hat{\alpha}})$, test of Phillips (1987) and Phillips and Perron (1988). We also report the KPSS [see Kwiatkowski et al. (1992)] $\hat{\eta}_{\mu}$ and $\hat{\eta}_{\tau}$ t -statistics.¹ For the WS and ADF tests, the optimal lag length is taken to be the order selected by the Akaike Information Criterion (AIC) plus 2 - see Pantula et al. (1994) for details regarding the advantages of this rule for choosing the number of augmenting lags. The $Z(t_{\hat{\alpha}})$ test is done with the same Dickey-Fuller regression variables, using no augmenting lags.

Table 1. Unit Root Tests in the Levels

Variable	p -values			KPSS	
	WS	ADF	$Z(t_{\hat{\alpha}})$	$\hat{\eta}_{\mu}$	$\hat{\eta}_{\tau}$
π_t	.666	.741	.072	.154	.081
τ_t	.946	.128	.501	.671	.146

Notes: Numbers in the WS, ADF, and $Z(t_{\hat{\alpha}})$ columns are tail areas of tests.

Based on the p -values for the WS, ADF, and $Z(t_{\hat{\alpha}})$ unit root tests reported in Table 1, the null hypothesis of a unit root in levels cannot be rejected. Also, the t -statistic $\hat{\eta}_{\mu}$ that tests the null hypothesis of level stationarity is large

¹Kwiatkowski et al. (1992) argue that unit root tests have low power against relevant alternatives and they propose tests (known as the KPSS tests) of the hypothesis of stationarity against the alternative of a unit root. They argue that such tests should complement unit root tests and that by testing both the unit root hypothesis and the stationarity hypothesis, one can distinguish series that appear to be stationary, series that appear to be integrated, and series that are not very informative about whether or not they are stationary or have a unit root.

relative to the 5% critical value of .463 given in Kwiatkowski et al. (1992), for the τ_t series. However, the t -statistic $\hat{\eta}_\tau$ that tests the null hypothesis of trend stationarity does not exceed the 5% critical value of .146 [also given in Kwiatkowski et al. (1992)], for both series. Although the series do not appear to be very informative as to their integration properties, combining the results of our tests of the stationarity hypothesis with the results of our tests of the unit root hypothesis, we conclude that both series have at least one unit root.

To test the null hypothesis of a second unit root, in Table 2 we test the null hypothesis of a unit root [using the WS, ADF, and $Z(t_{\hat{\alpha}})$ tests] as well as the null hypotheses of level and trend stationarity in the first differences of the series. Clearly, the differenced series appear to be stationary, since the unit root null is rejected and the level and trend stationarity null hypotheses cannot be rejected.

Table 2. Unit Root Tests in the First Differences of Levels

Variable	p -values			KPSS	
	WS	ADF	$Z(t_{\hat{\alpha}})$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$
$\Delta\pi_t$.000	.000	.000	.136	.086
$\Delta\tau_t$.003	.000	.074	.296	.081

Notes: Numbers in the WS, ADF, and $Z(t_{\hat{\alpha}})$ columns are tail areas of tests.

4.3 Cointegration Tests

Although cointegration is neither necessary nor sufficient for tests on the long-run derivative, for information purposes we also test the null hypothesis of no cointegration (against the alternative of cointegration) between π_t and τ_t using the Engle and Granger (1987) two-step procedure. In particular, we regress one variable against the other (including a constant and a trend variable in the regression) to obtain the OLS regression residuals $\hat{\zeta}_t$. A test of the null hypothesis of no cointegration is then based on testing for a unit root in $\hat{\zeta}_t$, using the ADF test (with the number of augmenting lags being

chosen based on the AIC+2 rule mentioned earlier) and asymptotic p -values using the coefficients in MacKinnon (1994).

The cointegration tests are first done with π_t as the dependent variable in the cointegrating regression and then repeated with τ_t as the dependent variable.² When π_t is the dependent variable the p -value of the null hypothesis of no cointegration is .511 and when τ_t is the dependent variable the p -value is .143. Clearly, the null hypothesis of no cointegration between π_t and τ_t cannot be rejected (at the 5% level).

4.4 Long-Horizon Regression Tests

We start by estimating equation (4) for values of k ranging from 1 to 30, as in Fisher and Seater (1993), and present the estimates of b_k along with the 95% confidence bands [using the Newey and West (1987) procedure] in Figure 1. The evidence shows that the null hypothesis that $b_k = 1$ can be rejected for any $k \in [1, 30]$. Thus, we find strong evidence that revenue-smoothing does not hold.

To investigate the robustness of this result, we also examine the relationship between the three-month Treasury bill rate, R_t , and τ_t . In particular, we investigate the univariate time series properties of R_t and ΔR_t , we test for cointegration between R_t and τ_t , and estimate equation (4) with $R_t - R_{t-k-1}$ as the dependent variable. The integration tests in Table 3 indicate that the time series properties of R_t are very similar to those of τ_t , investigated in Tables 1 and 2. Also, when we test the null hypothesis of no cointegration between R_t and τ_t , we cannot reject the null (irrespective of which variable is treated as the dependent variable).

Finally, we present the estimates of b_k along with the 95% confidence bands in Figure 2. The evidence shows that again the null hypothesis that $b_k = 1$ can be rejected for most values of $k \in [1, 30]$.

²We should wary of a result indicating cointegration using one series as the dependent variable, but no cointegration when the other series is used as the dependent variable.

Table 3. Unit Root Tests in R_t and ΔR_t

Variable	p -values			KPSS	
	WS	ADF	$Z(t_{\hat{\alpha}})$	$\hat{\eta}_{\mu}$	$\hat{\eta}_{\tau}$
R_t	.575	.730	.396	.690	.101
ΔR_t	.000	.000	.001	.159	.115

Notes: Numbers in the WS, ADF, and $Z(t_{\hat{\alpha}})$ columns are tail areas of tests.

5 Conclusion

We have tested the revenue-smoothing hypothesis using annual data for the United States over the period from 1934 to 1994. In doing so, we have used the Fisher and Seater (1993) methodology, paying particular attention to the integration properties of the data, since meaningful long-horizon regression tests critically depend on these properties. Overall, although Mankiw (1987) and Poterba and Rotemberg (1990) found evidence supporting revenue-smoothing in the United States using contemporaneous ordinary least squares regressions, the evidence presented here, as well as in Trehan and Walsh (1990), Ghosh (1995), Evans and Amey (1996), and Serletis and Schorn (1999), does not support the theory of optimal seigniorage.

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Figure 1. The LRD for the Inflation Rate

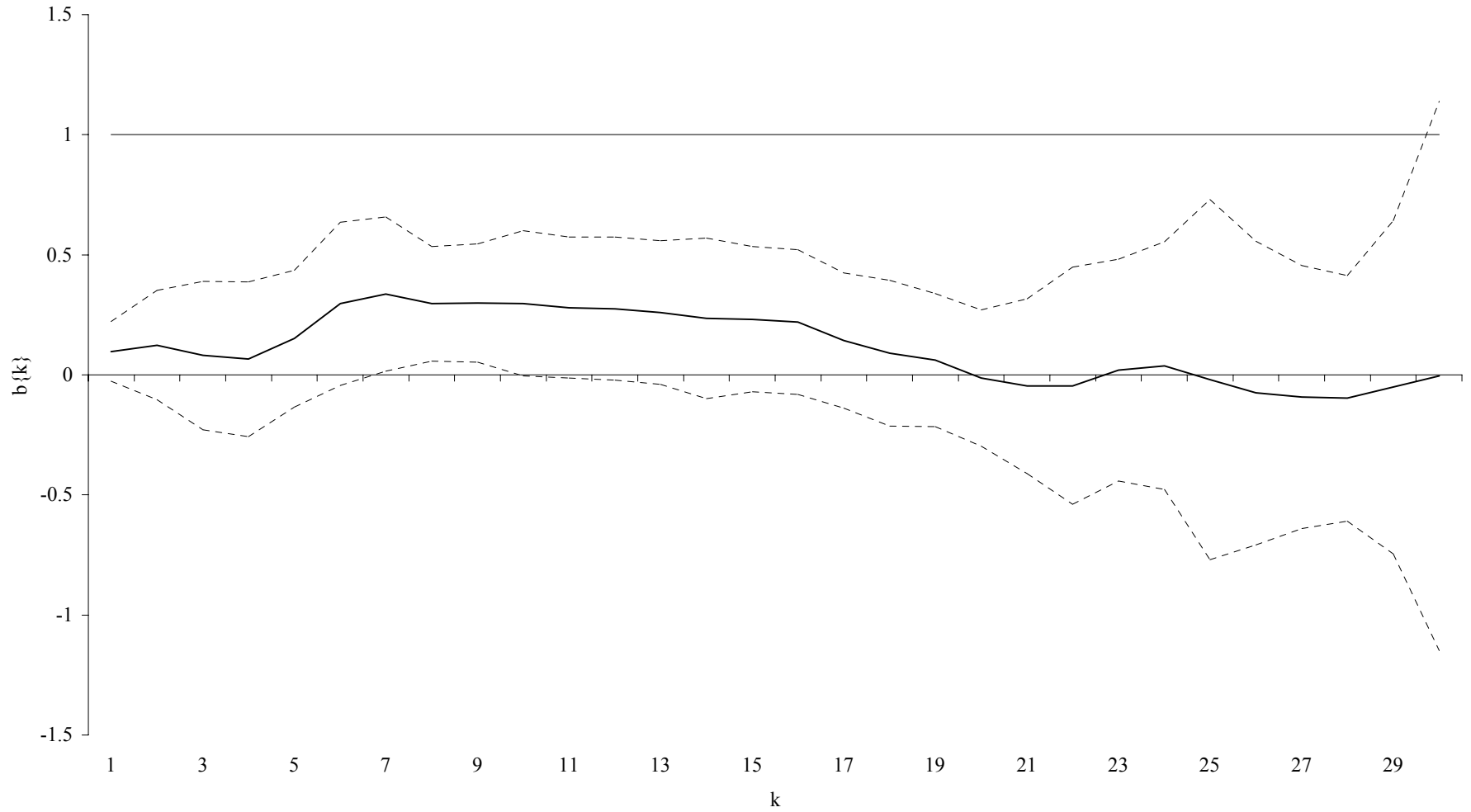


Figure 2. The LRD for the Interest Rate

