Skill Bias, Trade, and Wage Dispersion

Monte, Ferdinando

The University of Chicago - Department of Economics

16 April 2009

Skill Bias, Trade, and Wage Dispersion

Ferdinando Monte*

April 16, 2009

Abstract

Skill-biased technical change and trade integration have both been indicated to be the cause of the wide increase in wage inequality in U.S. in the last 50 years. This paper shows in a simple unified framework why both mechanisms can reproduce the observed pattern of wage dispersion. Intra-firm rent distribution can be used to disentangle these causes.

1 Introduction

A number of studies have shown that wage inequality in the United States has been increasing throughout the last 40 or 50 years. The ratio of wages at the 50th vs. 10th percentile, and 90th vs. 50th percentile in the distribution, grew each approximately 10% from 1973 to 1987. After that, the lower tail flattened, while the upper tail continued to grow 10% more through 2004 (Autor, Katz and Kearney (2005), (2006)). At extreme high values in the wage distribution, inequality started increasing in the sixties, and grew even more markedly (Piketty and Saez (2003), (2004))[1].

While there seems to be a consensus that an increase in demand biased toward skilled workers is at the origin of these facts and that that much of the increase in inequality has been observed *within* sectors (see for example Juhn, Murphy and Pierce (1993), Berman, Bound and Griliches (1994)), the causes of this increase are the subject of an intense controversy.

*Department of Economics, Ph.D. program, The University of Chicago. Contact: fmonte@uchicago.edu. The first version of this work was discussed in February 2004 as undergraduate dissertation at Bocconi University, Italy, under the title "Commercio Internazionale, Eterogeneita' e Diseguaglianze nella Distribuzione del Reddito". I am deeply indebted toward Paolo Epifani and Fabrizio Onida for their guidance on that work. A longer version of this paper has circulated at various times in the past years under the title "Two Sided Heterogeneity, Technology and Trade". Thomas Chaney, James Heckman, Samuel Kortum and Ralph Ossa provided patient guidance, insightful critiques and careful comments and suggestions on this project. This paper has greatly benefited from discussions with Pierre Andre' Chiappori, Jonathan Eaton, Lance Lochner, Lars Nesheim, Jaromir Nosal, Nancy Stokey and participants at the working groups on Capital Theory and International Trade and the University of Chicago. All errors are my own.

[1] For example, from 1960 to 2004 the ratio between the wage at the 99th and the 95th percentile changed from 1.6 to 2; in the same period, the ratio 99.9-99 increased from 1.7 to 2.2, and the ratio 99.99-99.9 increased from 2.1 to 4.4. Looking at the overall wage distribution, Juhn, Murphy and Pierce (1993) document a 72% increase in the variance of log-weekly wages for males between 1963 and 1989, showing how the change in log real wages is essentially a linear function of the percentile in the wage distribution.

Skill-biased technical change and trade integration are the two most prominent candidates for an explanation[2]. In particular, under the assumption that trade shifts the derived demand of skilled workers *across* sectors, Berman, Bound and Griliches (1994) and Autor, Katz and Krueger (1998) argue that this pattern is only consistent with skill-biased technical change. This position is also supported by the fact that the increase in relative demand of skilled workers has been larger where computing technology has spread faster (Autor, Katz and Kruger (1998), Autor, Levy and Murnane (2003))[3]. On the other hand, Feenstra and Hanson (1996, 1999) find a significant role for international trade when they consider the import content of firms' intermediate inputs: a shift in demand away from unskilled and toward skilled workers within sectors occurs when firms respond to competition from low-wage countries by outsourcing abroad tasks intensive in unskilled labor.

The common denominator of this debate is that trade flows are mainly driven by differences in the price of skilled vs. unskilled workers across countries (i.e., on Hecksher-Ohlin motives). Although this channel has gained relevance in recent years with the rise of China and other developing countries (see the discussion in Krugman (2008)), intra-industry trade has been very important since long before (Baldwin and Martin (1999)). It is therefore surprising that a relative evaluation of the consequences of skill-biased technical change vs. intra-industry trade have not received much attention in the literature. This paper wants to contribute to fill this gap by asking two questions: 1) Can skill-biased technical change and trade integration produce the same observed pattern for inequality within a unified framework? And if so, 2) Can firm-level data help to disentangle the two causes? The answer to these questions are yes, and yes: by examining the equilibrium, micro-level responses of the wage function in a heterogeneous population of workers and firms, I show that the evolution of wage ratios described above can be rationalized independently by both the forces considered, and show what evidence needs to be provided to tell apart the two mechanisms. The neatness of the results is obtained at the price of strong reliance on functional form assumptions, which is admittedly in no way general, although often standard. However, it will be a virtue of the model the ability to clearly answer the questions posed.

To frame my argument, I consider two identical economies with a potentially unbounded mass of goods, where varieties are characterized by heterogeneous efficiencies in their technology, in the spirit of Eaton and Kortum (2002) and Melitz (2003). I extend this framework by assuming that workers are heterogeneous in their ability to run any firm (if they choose so), while they are identical as production workers at the firms' production lines: a firm is then made up by an idea, a manager, and production workers. Complementarities between technology and ability[4] imply positive assortative matching between managers and technological efficiency[5], producing a "superstars" effect as in Rosen

[2] These are by no means the only two causes put forth. Autor, Katz and Kearney (2005) consider the consequences of shifts in labor force compositions in terms of education and experience. Card and Di Nardo (2002) provide a general critique of the skill-biased technical change hypothesis.

[3] In particular, computers are argued to be substitute for unskilled and complementary to skilled workers: hence, rates of computer adoption should be correlated with rates of skill-upgrading across industries. Empirically, rates of skill-upgrading within industries explain most of the increase in demand of college workers.

[4] Log-supermodularity, in the terms of Costinot (2009))

[5] Sattinger (1979) is the first to propose this framework. This paper generalizes his contribution, introducing a fully-

(1981). The occupational choice implies that the wage of the manager if she was a production worker plays the role of the fixed selling cost in the domestic market, giving rise to increasing returns to scale at the firm level (as in Krugman (1979)). A fixed cost of exporting produces the endogenous selection of most productive firms in the foreign market.

To answer the first question of this paper, I build the theoretical counterpart of wage ratios at two (arbitrary) levels of the skill spectrum and examine how this ratio moves following exogenous changes in the two relevant forces in the economy. Predictions on the response of wage ratios can always be reduced to predictions of *local changes in inequality*. The local change in inequality describes how the wage ratio moves for two abilities similar enough, s' and $s'' > s'$. The wage of s'' is simply the wage of s' plus the marginal price of skills: hence, the direction of the change in this wage ratio can be determined by comparing two elasticities, the elasticity of the total price of skills (i.e., the wage of s') and the elasticity of the marginal price of skills (i.e., the slope of the wage at s'). Suppose now that the latter is more responsive than the former: then, an exogenous shock that raises both the total and the marginal price of skills increases the ratio between the two wages and increases local inequality.

I model skill-biased technical change as an increase in the contribution of ideas to firm-level productivity. While all firms tend to gain from this increase, complementarities in production ensure that better managers gain more than proportionately relative to average managers; on the other hand, the improvement in the overall efficiency of the economy exerts a stronger competitive pressure uniformly on all firms. With power law distributions for ideas and abilities, the marginal price of skills is always more responsive than the total price of skills. For low levels of abilities, they both decrease, but the former falls more, and local inequality decreases; for high levels of abilities, they are both larger, but the former raises more, and local inequality increases.

Trade integration in the form of reduction of iceberg transport costs also benefits firms asymmetrically: while the domestic sellers must bear stiffer competition from abroad, exporters face lower costs to sell their products in the foreign market. This argument is standard in all trade models with heterogeneous firms. I show that the marginal price of skills is again more sensitive, and the same behavior of local changes in inequality is produced. In both cases, the level of wage is less sensitive than the marginal wage because part of the adjustment on the level is borne by profits.

I can then answer positively to the first question because the wage ratio between two arbitrary levels of ability can always be expressed as the sum of all the local changes between them. To the best of my knowledge, this is the first paper which can reproduce the basic facts identified in the firm-level heterogeneity and trade literature, and provide a link between the empirically observed evolution of wage ratios and a general equilibrium theory of the consequences of skill-biased technical change and intra-industry trade.

If skill-biased technical change and trade integration can both rationalize the pattern for inequality in the U.S. economy that the literature has documented, what can help tell the two causes apart? I focus

fledged general equilibrium model where the outside options are endogenously determined. Sattinger (1993) gives a review and a motivation for using assingment models to study wage distributions.

on the intra-firm rent distribution, where I call "rent" the sum of profits and the manager's wage, less the opportunity cost of ideas and managers in the alternative occupation (zero and the production worker wage, respectively). This is the first paper to make a connection between intra-firm rent distribution, and the consequences of intra-industry trade and skill-biased technical change. I show that the intra-firm rent distribution is not modified by trade integration, which only affects competitiveness and productivities *across* firms. In equilibrium, in fact, the slope of the wage function (a marginal cost for the firm) must reflect the marginal benefit of a better manager. In comparing two similar firms, then, the fraction of the marginal rent captured by the better manager must be proportional to her contribution to the creation of that rent. Since the trade costs influence the marginal contribution of managers and ideas in the same way, their *relative* contribution cannot be a function of trade integration. Hence, changes in inequality not accompanied by change in the intra-firm rent distribution must be attributed to trade. Vice-versa, changes in the intra-firm rent distribution must imply changes in local inequality caused by skill-biased technical change.

This paper is consistent with the patterns identified in the empirical literature on trade and firm-level heterogeneity: exporting firms are larger, more productive, and earn higher profits; moreover, I add to the current literature a channel through which larger firms also pay higher wages (Oi and Idson (1999)), exporting firms employ higher quality of workers and pay higher wages than firms only selling on the domestic market (see for example Bernard and Jensen, (1995), (1997), (1999)).

The hypotheses of the model allow the arguments in this paper to dodge the most common rejection of trade-based explanations for the evolution of inequality. I analyze a progressive reduction of variable trasportation costs between two perfectly identical economies, because contemporary trade flows are mainly based on intra-industry trade between similar countries and on similar products[6]. This focus shuts down by construction motives for international trade based on relative differences in factor endowments (as in the classic Heckscher-Ohlin framework) and the associated distributional effects based on differences in relative factor intensities across goods (Stolper-Samuelson effects)[7]. Since trade flows are not generated by comparative advantage, rejections grounded on the reallocation of employment shares across sectors (Berman, Bound and Griliches (1994), Autor, Katz and Krueger (1998)) do not apply to this framework.

The model is consistent with the mechanisms used by Gabaix and Landier (2008) to describe the surge in CEO compensation in the last decades: this framework endogenizes the response of the size distribution of firms to different exogenous shocks. Also, it shows that a divergent behavior in the upper and lower tails of the wage distribution can be produced even when changes in the labor force composition (Autor, Katz and Kearney (2005)) are absent. The elasticity of total and marginal prices

[6] For example, Baldwin and Martin (1999) document that two-thirds of contemporary world trade occurs among rich countries with similar factor endowments, and three-fourths of this share is two-way trade within narrowly defined industries. See also Helpman (1999), for a discussion. The rise in the share of trade in manufactured goods with developing countries is not to be ignored, but is too recent to account for the evolution of inequality starting the sixties. Moreover, an Hecksher-Ohlin-based explanation would predict a *reduction* in the skill premium in China, which is counterfactual.

[7] A similar approach is undertaken in Epifani and Gancia (2008), who consider the case of two goods with different skill intensity, homogeneous firms and no choice between skilled and unskilled occupations.

of skills are all that matter, and when one considers only data on the wage distribution, these effects are difficult to disentangle.

This paper belongs to a series of recent studies which capitalize on firm-level heterogeneity models (Manasse and Turrini (2001), Eaton and Kortum (2002), Melitz (2003), Helpman, Melitz and Yeaple (2005), Yeaple (2005), Chaney (2008)) and extend them enriching the details of the human capital aspects.

The closest paper to this study is Costinot and Vogel (2009), who build a model of complementarities in production capable of addressing the consequences of technological change, North-North and North-South trade on the wage function of heterogeneous workers. Their analysis has less restrictive hypotheses, and allows for skill- and task-upgrading and downgrading, i.e., changes in the assignment function between workers and tasks: in the present paper, this possibility is shut down by the power law assumptions on the ideas and abilities' distribution. However, their model is still an Hecksher-Ohlin model, and there is no motive for trade between two identical countries. Moreover, the mass of goods (in their model, tasks) is fixed and there is no notion of selection into the home market. This result is driven - compared to the present study - by the absence of an occupational choice of workers between production and managerial activities: as a consequence, skill-biased technical change cannot then produce a divergent pattern for wage ratios in different regions of the ability spectrum. Finally, their model does not capture the well-documented selection of subsets of firms into the export market, and only compares autarky and free-trade equilibria when North-North trade is driven by differences in factor diversity[8] across countries.

This paper is also related to Helpman, Itskhoki and Redding (2008). Their paper focuses on the relation between trade, search and screening costs, and unemployment. Contrary to this model, it features firms with many heterogeneous workers. However, the consequences of skill-biased technical change are not addressed. Moreover, heterogeneity in wages arises because labor market frictions give workers a bargaining power within the firm, while in this paper it is simply a consequence of assortative matching and of demand and supply at each skill level.

Also related to this study is Verhoogen (2008), who proposes a partial equilibrium framework where wage inequality among heterogeneous workers in developing countries increases following trade integration: the reason is that an increase in the demand for product quality raises the demand of more skilled workers because of complementarities between input quality and output quality. This mechanism limits its usefulness in discussing trade among identical countries.

I see the present study as complementary to this literature in the endevour to better understand how international trade generates interdependence across labor market outcomes of different countries.

In the rest of the paper, I will describe the model in closed economy (section 2) and provide a motivation for the theoretical framework used in analyzing wage ratios, applying this to skill-biased technical change (section 3). In section 4, I extend the model to an open economy framework, while in section 5 I show how wage ratios respond to skill-biased techical change and trade integration. Section 6

[8] Factor diversity captures the relative abundance of agents with extreme abilities, be them very low or very high.

argues why the intra-firm rent distribution can help in disentangling the two causes, skill-biased technical change and trade integration. Section 7 provides some conclusive remarks.

2 The Closed Economy

2.1 Consumers, Managers, and Ideas

The representative consumer maximizes a standard CES utility function where, from an infinite mass of varieties potentially available, a subset J of them is produced and aggregated as

$$Y = \left[\int_{j \in J} y(j)^{(\sigma-1)/\sigma} \, dj \right]^{\sigma/(\sigma-1)}$$

with $\sigma \geq 1$. Standard optimization implies that each consumer will spend

$$x(j) = \left(\frac{p(j)}{P} \right)^{1-\sigma} X \tag{2.1}$$

on each variety produced, where

$$P = \left[\int_{j \in J} p(j)^{1-\sigma} \, dj \right]^{1/(1-\sigma)}$$

is the ideal price index of good Y and X is total consumers' expenditure on it.

To fix ideas, we think of different j as different varieties; however, I will interchangeably use the term "firm", implicitly assuming one product per firm.

Three inputs are necessary for a production line to exists: an idea, a manager, and production workers in proportion to output.

Varieties differ according to the status of the technology available for their production: denoting with $z \in (0, \infty)$ the quality of an idea, I assume that there is a measure $G(z) = Tz^{-\xi_z}$ (with $\xi_z \geq 1$) of ideas at least as good as z. This specification insures that there is a sufficient number of ideas, however bad, to accomodate any number of managers in equilibrium. Ideas are owned by a mutual fund that maximizes profits and redistributes them equally across agents[9].

The economy is populated by a mass L of agents, which, as in Lucas (1978), can be either production workers or managers. Agents are heterogeneous in their mangerial ability, while they all have a unit efficiency as production workers. The ability s is also distributed according to a power law: for $s \geq 1$, there is a measure $L(s) = Ls^{-\xi_s}$ ($\xi_s \geq 1$) of potential managers with ability of at least s. While in Lucas (1978) potential managers differ by their ability to run larger firms producing a homogeneous final product, here I assume that there are complementarities between managerial ability and idea efficiency. In particular, if production with an idea of efficiency z is run by a manager s, the total firm's productivity

[9] The assumption of equal redistribution is immaterial to the rest of the paper, since I am only interested in wage (rather than income) distribution. Juhn, Murphy and Pierce (1993) discuss relative merits of these two alternatives.

is

$$\varphi(z,s) = z^{\kappa} s^{\alpha}$$

with $\kappa > 0$, $\alpha > 0$.[10] Note that by suitably redefining units and parameters in the productivity, we can always write the distribution of ability as having a shape parameter of 1, and still recover the same distribution for s^{α}, which is what matters for the productivity of the firm; the same is true for the parameter ξ_z on the distribution of ideas' efficiency. The parameters ξ_s and ξ_z will then be set to 1 without loss of generality[11]. The parameter α uniquely measures the influence of managers' ability: while $\alpha = 0$ reduces this model to a simple one-sided heterogeneity framework, increasing α lets a firm gain from a better manager. Moreover, there is a simple mapping between abilities and percentiles in the skill and wage distributions: the ability s is always collocated at the $100(1-s^{-1})^{th}$ percentile.

Agents who choose to be production workers earn a wage $\bar{w}$, which is then also the opportunity cost of being a manager. This wage is the numeraire and will be normalized to 1; I leave it here explicitly for clarity. When y units of good are to be produced, y/φ efficiency units of work from production workers are used at a cost of $\bar{w}y/\varphi$ in total. Denote as:

$$v(p;\varphi) = x(p) - \frac{\bar{w}x(p)}{p\varphi}$$

the surplus of the firm with overall productivity φ, i.e. the excess of revenues over costs for production workers, when the price p is chosen. For any manager s, a firm with idea quality z sets a price which solves

$$\pi(z;s) \equiv \max_p v(p;\varphi(z,s))$$

implying

$$p(\varphi) = \frac{\sigma}{\sigma-1}\frac{\bar{w}}{\varphi} \Rightarrow p(z,s) = \frac{\sigma}{\sigma-1}\frac{\bar{w}}{z^{\kappa}s^{\alpha}} \tag{2.2}$$

For any given quality z, the optimal price is then a function of the quality of the manager s the owner of z chooses. The way in which the market balances incentives across firms is the subject of the next section.

[10] This assumption satisfies log-supermodularity as in Costinot (2007).

[11] More precisely, suppose that $G(z) = Tz^{-\xi_z}$; in this case, the distribution of z^{κ} satisfies $T\Pr\{z^{\kappa} > a\} = Ta^{-\xi_z/\kappa}$. We want to show that there are a $\tilde{z}$ and $\tilde{\kappa}$ such that the measure of ideas $\tilde{z}$ better than any value a is Ta^{-1} and that still assigns to $\tilde{z}^{\tilde{\kappa}}$ the same distribution that z^{κ} has. Let $\tilde{z} \equiv z^{\xi_z}$, and $\tilde{\kappa} = \kappa/\xi_z$. Hence, the measure of ideas $\tilde{z}$ better than a is $T\Pr\{\tilde{z} > a\} = T\Pr\left\{z^{\xi_z} > a\right\} = Ta^{-1}$: $\tilde{z}$ has a distribution with shape parameter 1. Moreover, $T\Pr\left\{\tilde{z}^{\tilde{\kappa}} > a\right\} = T\Pr\left\{\tilde{z} > a^{\xi_z/\kappa}\right\} = T\left(a^{\xi_z/\kappa}\right)^{-1}$ which is then equal to the distribution of z^{κ}. An analogous argument, with $\tilde{s} \equiv s^{\xi_s}$ and $\tilde{\alpha} = \alpha/\xi_s$, establishes the equivalence for the population of managers. Finally, since z^{κ} and s^{α} have the same distribution as $\tilde{z}^{\tilde{\kappa}}$ and $\tilde{s}^{\tilde{\alpha}}$, it must also be true that the product of these two variables, $\tilde{z}^{\tilde{\kappa}}\tilde{s}^{\tilde{\alpha}}$, has the same distribution as $z^{\kappa}s^{\alpha}$.

2.2 Assignment

Substituting the revenue function (2.1) and the optimal price (2.2) in the expression for $v(p;\varphi)$, the surplus for a firm (z,s) is rewritten as

$$\begin{aligned} v(z,s) &= M\left(\frac{z^{\kappa}s^{\alpha}}{\bar{w}}\right)^{\sigma-1} \\ M &\equiv \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} XP^{\sigma-1} \end{aligned} \tag{2.3}$$

The term M is a measure of the economically relevant size of the market. A larger expenditure level X, or weaker competition through higher average price P all make the market bigger and tend to raise the surplus for any firm.

This surplus must cover payments to the manager of ability s, residually determining profits for the idea z. The complementarity between managers and ideas - which drives the incentive to positive assortative matching - is manifest in the cross derivative of the surplus $v_{1,2}(z,s)$ being positive: a marginal increase in the quality of ideas always enhances the total net value of production, but this enhancement is larger when the manager running the firm is better. This complementarity creates an incentive for better firms to hire better managers, or equivalently, for more able managers to choose better firms. The assignment between managers and firms is *non-random*. The ideas' owner problem is then

$$\pi(z) = \max_{s\in[1,\infty)} \{v(z,s) - w(s)\}$$

Following Sattinger (1979), I assume that a firm is unable to affect the prevailing market conditions, so that the wage function is taken as given: the optimal ability s will be chosen to balance the marginal benefit of a better manager (higher productivity and larger surplus available for distribution) with the marginal costs of it (higher wage demanded). In an optimum,

$$v_2(z,s)\big|_{z=z(s)} = w'(s) \tag{2.4}$$

gives a condition that can be used to trace out the wage function when the left hand side is evaluated at the idea quality z which chooses s optimally, i.e., at $z = z(s)$.

To build the equilibrium wage function, I will then proceed under the tentative assumption of positive assortative matching, $z'(s) > 0$, and prove that this must be true in equilibrium because of complementarities in production. If the best managers work running firms with the best ideas, the assignment problem will imply (matching the measures at the right tail of the distributions) that $Tz^{-1} = Ls^{-1}$, or

$$z = ts \Leftrightarrow s = z/t, \text{ with } t \equiv T/L \tag{2.5}$$

under our assumptions. The parameter t, which we take as exogenous, is a measure of relative size of technology available in the country. A larger population size increases the availability of managers at all levels of ability, so that any idea z is matched with a better s; any potential manager gets hurt by a larger L, though, since this increases the mass of people better than her[12].

Differentiating then the surplus (2.3) with respect to s, plugging $z(s)$ from (2.5) in it, and substituting the resulting $v_2(z(s), s)$ in (2.4) we obtain

$$v_2(z,s)|_{z=z(s)} \equiv (\sigma-1) M \left(\frac{t^\kappa}{\bar{w}}\right)^{\sigma-1} \alpha s^{(\alpha+\kappa)(\sigma-1)-1} = w'(s)$$

This equation gives the equilibrium marginal rent for managers of different ability: its elasticity to different stimuli will play a crucial role in determining how inequality responds. Integrating over s, and using the fact that the marginal manager - denote her s_c - must be indifferent between occupations, we get:

$$\begin{aligned} w(s) &= \int_{s_c}^{s} v_2(z,s)|_{z=z(s)}\, ds = \\ &= \frac{\alpha}{\kappa+\alpha}\left(\frac{t^\kappa}{\bar{w}}\right)^{\sigma-1} M\left(s^{(\alpha+\kappa)(\sigma-1)} - s_c^{(\alpha+\kappa)(\sigma-1)}\right) + \bar{w} \end{aligned} \tag{2.6}$$

and with $w(s) = \bar{w}$ below s_c.

The profit function $\pi(z)$ must then be the difference between the surplus and the wage, and leaves the marginal idea z_c indifferent to the alternative of not being used. To find the equilibrium profit value, I use the optimal assignment function (2.5) and (2.6), and the fact that marginal idea z_c must have zero profits, to get

$$\begin{aligned} \pi(z) &= v(z, s(z)) - w(s(z)) = \\ &= \frac{\kappa}{\kappa+\alpha}\left(\frac{t^{-\alpha}}{\bar{w}}\right)^{\sigma-1} M\left(z^{(\kappa+\alpha)(\sigma-1)} - z_c^{(\kappa+\alpha)(\sigma-1)}\right) \end{aligned} \tag{2.7}$$

with $\pi(z) = 0$ below z_c[13].

The sufficient condition for an optimum will require, when looking at the choice of manager z, that $v_{22}(z,s) - w''(s) < 0$ when $z = z(s)$ (i.e., along the optimal assignment), which can be easily shown to be true by differentiating (2.4) again with respect to s, and then plugging in the assignment function: the complementarity assumption $v_{12}(z, s(z)) > 0$ insures that positive assortative matching emerges as an equilibrium outcome[14].

[12] To parallel the terminology of Costinot and Vogel (2008), this would be *skill-upgrading* from the standpoint of the firm, and *firm-downgrading* (which they call *task-downgrading*) from the point of view of a manager.

[13] This is not the only way to characterize the earning functions. Since the problem is symmetric in managers and ideas, we could have started with the managers taking as given the profit function $\pi(z)$ and choosing ideas. Alternatively, we could have had each side choosing the other (as Sattinger (1979)), particularizing each constant of integration with the relevant outside option.

[14] The second order condition for the optimality of s in the firm problem requires $v_{22}(z,s) - w''(s) < 0$. Differentiating

The equilibrium assignment of managers to firms provides a simple microfoundation for the rent sharing between managers and ideas within a firm, based on local scarcity of talents vs. ideas' and their contributions to the total productivity of the firm. The rent to be shared is $v(s, z(s)) - \bar{w}$, the excess of surplus over the sum of managers and ideas' opportunity cost ($\bar{w}$ and 0 respectively). The share of this rent going to managers' wages is

$$\theta \equiv \frac{\alpha}{\alpha + \kappa}$$

If managers do not influence the firm's efficiency ($\alpha = 0$), the rent for talent is zero, the equilibrium wage function reduces to the outside option, and we are back to the standard one-sided heterogeneity case similar to Melitz (2003), where workers' contributions are homogeneous and the wage per efficiency unit is flat across ability levels. On the other hand, if $\kappa = 0$ we recover a model similar to Lucas (1978) and Manasse and Turrini (2001), where heterogeneous workers are operating using homogeneous ideas: profits then are zero, and only a non-trivial wage function remains.

If complementarities exist in production, the distribution of each factor determines both the rent-sharing *and* the final productivity distribution of firms; in other words, making assumptions on both the productivity distribution of firms and rent sharing is equivalent to making a statement on the degree of heterogeneity of each factor. Observationally, this framework provides a link between firm size distribution and the share of rent which goes to each factor type.

For some purposes, one could go further, simply assuming a firm-level heterogeneity in φ and an ad-hoc rent splitting in constant shares. Such an approach would sidestep the description of the market mechanism underlying the assignment problem, at the cost of obscuring its economic content. For the purpose of this study, however, we cannot simply assume an exogenous firm-level productivity distribution and arbitrary rent-sharing proportions. As I will argue below, a proper way to think about skill-biased technical change is to keep fixed the distribution of managers' ability, while changing the effective distribution of idea quality through increases in κ. When this happens, not only does the share of rents to managers decrease, but the overall firm productivity distribution improves: studying the effect of a change in the share parameter we would miss the second part, while only studying an increase in the firm-level heterogeneity parameter we would ignore the former.

2.3 Equilibrium

To characterize the equilibrium in a closed economy, it is sufficient to determine the cutoff s_c for managers' ability and the expenditure level X in the economy. Note first that using the price index definition,

(2.4) again with respect to s we get $w''(s) = v_{22}(z, s) + v_{12}(z, s) z'(s)$, which implies $w''(z) - v_{22}(z, s) = v_{12}(z, s) z'(s)$. Using (2.3), we have

$$v_{12}(z, s) = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} XP^{\sigma-1}\kappa(\sigma - 1)\alpha(\sigma - 1) z^{\kappa(\sigma-1)-1} s^{\alpha(\sigma-1)-1} > 0$$

so that, $v_{22}(z, s) - w''(s) < 0 \Leftrightarrow v_{12}(z, s) z'(s) > 0 \Leftrightarrow s'(z) > 0$.

the individual firm price (2.2) and the assignment function (2.5), and assuming $(\kappa+\alpha)(\sigma-1)<1$, the price index has the form

$$P=\frac{\sigma}{\sigma-1}\bar{w}\left(\frac{\psi}{L}\right)^{1/(\sigma-1)}t^{-\kappa}s_c^{\psi/(\sigma-1)} \tag{2.8}$$

with

$$\psi\equiv 1-(\sigma-1)(\kappa+\alpha)\in(0,1) \tag{2.9}$$

where the assumption $(\kappa+\alpha)(\sigma-1)<1$ is needed to insure that the *joint knowledge* embodied in ideas and people never implies the existence of a few firms efficient enough to bring down the price index P to zero. Note that a larger relative measure of technology t reduces the price of the final good aggregate Y.

The expenditure on production workers for each firm is $x(\varphi)-v(\varphi)$; using the expression for revenues (2.1), surplus (2.3), the assignment function (2.5) and the price index (2.8), and integrating over all active firms, we get that the overall expenditure on production workers is $\frac{\sigma-1}{\sigma}X$.

On the supply side, when s_c is the managers' cutoff, $L\left(1-s_c^{-1}\right)$ efficiency units are provided for production. Equating total wages of production workers to total expenditure over them by firms, we get

$$X=\frac{\sigma}{\sigma-1}L\bar{w}\left(1-s_c^{-1}\right) \tag{2.10}$$

This curve describes an equilibrium relation in the labor market. When $s_c\to 1$, total earnings of production workers are zero, and so must be the expenditure X, while as $s_c\to\infty$, all agents are employed as production workers, and total expenditure on them approaches a finite constant; the monotonic increase in s_c is inherited by the properties of the CDF.

In addition to production labor market clearing, we need to make sure that the firm $(s_c, z(s_c))$ is made up by factors indifferent between production of varieties in Y and their alternative employment. This condition requires the surplus function (2.3) to be equal to the sum of the outside options for the indifferent pair of agents; using the assignment relation (2.5) and the price index (2.8),

$$X=\frac{\sigma}{\psi}L\bar{w}s_c^{-1} \tag{2.11}$$

This equation is a "zero cutoff earnings" condition. As $s_c\to 1$ the right-hand side becomes a strictly positive and finite number, while as s_c grows toward infinity, this curve goes to zero. Hence, as shown in Figure 2.1, the equilibrium (s_c, X) is always uniquely determined.

The simple functional form assumed allows to solve explicitly for s_c in terms of parameters: equating (2.10) and (2.11), and using the definition (2.9) for ψ, we have

$$s_c = 1+\frac{(\sigma-1)}{1-(\sigma-1)(\kappa+\alpha)} \tag{2.12}$$

$$X = \frac{\sigma}{\psi+\sigma-1}L \tag{2.13}$$

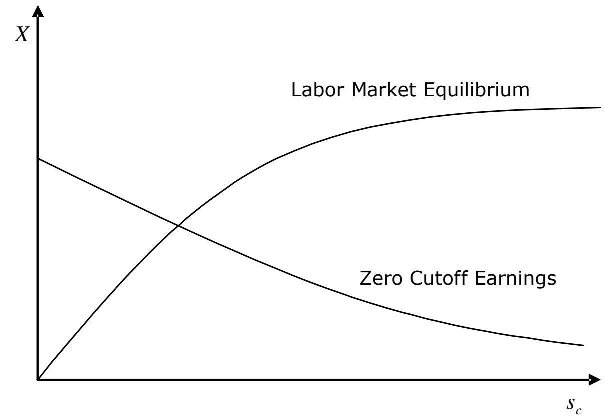

Figure 2.1: This figure shows the equilibrium determination of the cutoff s_c and the expenditure level X in closed economy. The labor market equilibrium represents the locus of pairs (s_c, X) where expenditure over and income of production workers are equalized; the zero cutoff earnings is the locus of points where the surplus of the marginal firm $(s_c, z(s_c))$ exactly covers the sum of outside options, so that there is no incentive for entry or exit in the differentiated sector.

where we note that the equilibrium cutoff is only a function of the *sum* of contributions of ideas and managers.

Having established the equilibrium conditions, we can also rewrite the earning functions in more explicit terms. Using the assignment function (2.5) to express the surplus (2.3) in terms of s, and imposing equality to $\bar{w}$ for the marginal firm, we have that

$$M = \bar{w}^{\sigma} t^{-\kappa(\sigma-1)} s_c^{-(\alpha+\kappa)(\sigma-1)} = \bar{w}^{\sigma} t^{\alpha(\sigma-1)} z_c^{-(\alpha+\kappa)(\sigma-1)}$$

Substituting the LHS out in the expressions for wages (2.6) and profits (2.7), we can rewrite them as

$$w(s) = \theta \left[\left(\frac{s}{s_c} \right)^{(\alpha+\kappa)(\sigma-1)} - 1 \right] \bar{w} + \bar{w} \tag{2.14}$$

$$\pi(z) = (1-\theta) \left[\left(\frac{z}{z_c} \right)^{(\alpha+\kappa)(\sigma-1)} - 1 \right] \bar{w} \tag{2.15}$$

Using the expression for s_c in (2.12) in the price index (2.8), we can express the profit and wage function above the cutoff in real terms only as a function of parameters, as $\pi(z)/P$ and $w(s)/P$ above the cutoffs, and 0 and P^{-1} below, respectively.

The equilibrium wage function is then determined jointly by the distribution of abilities *and* technology through a market mechanism which prices the relative scarcity of each type of agent.

The structure of the real earning functions has some characteristic elements.

The inverse of the price index gives a measure of the opportunity cost of keeping agents employed as production workers: in fact, their real wage is exactly P^{-1}, after normalizing $\bar{w}$ to 1.

The parameter θ represents a talent-specific component: total real rents in the firm $\left[(s/s_c)^{1-\psi} - 1\right]/P$ are split giving a share θ to managers and a share $1-\theta$ to ideas. For example, if $\alpha = 0$, managers do not contribute to the overall productivity distribution, and the share of rents going to managers is zero, and the wage reduces to $\bar{w}$ for all abilities.

A microeconomic component, s/s_c and z/z_c, determines then income differences between different levels of ability within managers, and within ideas.

In the next section, I use this framework to evaluate the consequences of skill-biased technical change on the wage ratio at different percentiles in the wage distribution.

3 Skill-Biased Technical Change

This section analyzes the effect of skill-biased technical change on an arbitrary wage ratio in a closed economy. Before focusing on the substantive side of the issue, I provide a general motivation for the theoretical framework used in analyzing the effect of skill-biased technical change on wage ratios. This discussion will also apply to the analysis of the open economy.

I will model skill-biased technical change as an exogenous increase in κ. For a given ability s, the elasticity of the productivity of the firm $t^{\kappa}s^{(\alpha+\kappa)}$ to κ is simply $\kappa \ln ts$, which is increasing in s: the percent change in productivity is biased toward firms which employ better managers[15]. This argument makes clear why we need a market mechanism and two distributions of heterogeneous agents, rather than an exogenous rent splitting between them: we think of skill-biased technical change as affecting a *fixed* ability distribution, which implies both an improvement in the assignment and a change in the overall distribution of productivities.

Empirically, the skill-biased technical change hypothesis is rooted in a positive correlation between adoption of computer-based technologies, skill-upgrading, and rising wage premia for education within and across firms and industries (see Autor, Katz and Krueger (1998) for a discussion and further references, and Card and DiNardo (2002) for a critique). This hypothesis has also been rationalized arguing that it impacts asymmetrically routine and non-routine tasks, thereby providing testable implications on the composition and shifts of job tasks over time (Autor, Levy and Murnane (2003)). My formulation captures these arguments in a simple model.

Since the distribution of abilities is fixed, there is a one-to-one correspondence between any ability level s and a percentile in the wage distribution, $100(1 - s^{-1})$. The analysis can then focus on the elasticity to κ of $w(s'')/w(s')$, with $s'' > s'$. Consider first two agents with very similar levels of ability. The difference in their wage is essentially the marginal price of skills at s', so that the wage ratio is just $1 + w_s(s')/w(s')$. When the marginal price of skills is more elastic to κ than is the wage, skill-biased technical change increases the premium for the ability difference $w_s(s')$ proportionally more than the

[15] For example, at $t = \kappa = 1$ a 1% increase in κ raises the productivity of a firm employing a top 10% manager by 1.61 percentage points *more* than the median firm (in fact, $\ln(s''/s')^{\kappa} = \ln(10/2)^{\kappa} = 1.61$).

price of ability $w(s')$ (i.e., the wage at s'). The wage ratio $w(s'')/w(s')$, which in this case is a measure of *local* inequality, increases.

More generally, the change in the wage ratio between two ability levels s' and s'' can always be thought of as the "sum" of all the local changes intervening between them. The formal argument (relegated to Appendix A.1) shows that the local change in inequality at s when κ changes is

$$\eta^{(\kappa)}(s) \equiv \frac{w_{\kappa s}(s)\,\kappa}{w_s(s)} - \frac{w_\kappa(s)\,\kappa}{w(s)} \tag{3.1}$$

where $w_\kappa(s) = \partial w(s)/\partial\kappa$ and $w_{\kappa s}(s) = \partial^2 w(s)/(\partial\kappa\partial s)$. The function $\eta^{(\kappa)}(s)$ is the difference between the elasticity of the marginal price of skills and the elasticity of the total price of skills. For two ability levels s' and s'', with $s'' > s'$, the elasticity of the wage ratio $w(s'')/w(s')$ to κ is simply

$$\int_{s'}^{s''} \frac{w_s(s)}{w(s)}\eta^{(\kappa)}(s)\,ds$$

The total change in inequality between s' and s'' is the integral of the local changes in inequality, weighted by $w_s(s)/w(s)$, a positive and unitless measure of the importance of ability differences. When for some s, $\eta^{(\kappa)}(s) > 0$, the local contribution of s is to increase *all* the wage ratios that contain it, and vice-versa.

Any argument about the behavior of wage dispersion is essentially a specification of eq. (3.1); the framework here proposed simply provides one. I now examine in detail how skill-biased technical change affects $\eta^{(\kappa)}(s)$.

3.1 Skill-Biased Technical Change and Wage Ratios

At the economy-wide level, skill-biased technical change raises the contribution of ideas to the productivity of all firms. However, any firm, holding constant its productivity, faces now a stiffer competition for its product. If s_c is the indifferent manager at expenditure level X_c, eq. (2.11) shows that a larger expenditure is necessary for her to stay in the managerial occupation: the Zero Cutoff Earnings curve in Figure 2.1 shifts up. Since the market for production workers is not directly affected by this change, selection is stricter among managers: the worst white collar managers become blue collar.

Having described how κ affects exit from the differentiated sector, I can characterize the behavior of the local change in inequality.

The elasticity of the wage to κ (the second term in (3.1)) is determined by the interaction of three effects. Eq. (2.14) shows that an increase in κ affects the wage through *(1)* the change in the share of rents θ going to managers, *(2)* the effect of selection, and *(3)* the effect of assignment[16]. The first term captures a negative "share" effect: for a fixed rent level in the firm, the share of it going to managers

[16] The effect of κ on the opportunity cost, which goes through real wages, is canceled because we are considering wage ratios, so that $[w(s'')/P]/[w(s')/P] = w(s'')/w(s')$.

decreases, because technology now contributes more overall to differences in firm-level productivities. The second term captures a negative "selection" effect: since the rent level is just the integral of the marginal rents from the worst firm upwards, when the worst agent select out of the market (we already saw that $\partial s_c / \partial \kappa > 0$) the total rent decreases. The third term represents an "assignment" effect, and is always positive: any manager gains from a larger contribution of z to the productivity of the firm; since the change is biased towards better agents, the assignment will grow in importance as s grows.

The elasticity to κ of the marginal price of skill $w_s(s)$ (the first term in (3.1)), on the other hand, is not directly influenced by the share of rents going to managers, since θ is affecting in the same proportion both $w_s(s)$ and its change when κ increases: the share effect is absent. However, skill-biased technical change tends to reduce the elasticity to κ of the marginal price of skills through selection, and to increase it through assignment. Simple calculations show that $\eta^{(\kappa)}(s)$ is positive if and only if

$$\frac{w_{\kappa s}(s)\,\kappa}{w_s(s)} > \frac{w_\kappa(s)\,\kappa}{w(s)} \Leftrightarrow$$

$$\underbrace{-\frac{(1-\psi)\,\kappa}{s_c}\frac{\partial s_c}{\partial \kappa}}_{\text{Selection}} + \underbrace{(\sigma-1)\,\kappa \ln \frac{s}{s_c}}_{\text{Assignment}} > -g_1(s)\underbrace{(1-\theta)}_{\text{Share}} - g_2(s)\underbrace{\frac{(1-\psi)\,\kappa}{s_c}\frac{\partial s_c}{\partial \kappa}}_{\text{Selection}} + g_2(s)\underbrace{(\sigma-1)\,\kappa \ln \frac{s}{s_c}}_{\text{Assignment}}$$

where

$$g_1(s) \equiv \frac{w(s)-1}{w(s)} \in (0,1)\,,\; g_2(s) \equiv \frac{w(s)-(1-\theta)}{w(s)} \in (\theta,1) \tag{3.2}$$

and $g_i'(s) > 0$ for $i = 1, 2$.

For managers bad enough, the assignment and share effects are negligible, and the negative selection effect dominates. Its impact is greater (more negative) on the marginal price of skills: the selection effect determines the wage elasticity only to the extent managers participate to the creation of surplus (in fact, $g_2(s) \to 0$ as $\theta \to 0$). Overall, the marginal price of skills falls proportionately more than the wage, so that the wage ratio between two close managers becomes smaller: the change in local inequality is negative.

For good enough managers, the assignment effects dominates the other two, but again, only a fraction $g_2(s)$ of it impacts the elasticity of the wage. Hence, for abilities high enough, skill-biased technical change *increases* the marginal price of skills proportionately more than the wage: for two close managers the ratio of wages will then increase, and the contribution of local inequality will be positive.

Note also that if the selection effect is close to zero, the region with negative local change in inequality tends to vanish, which means that selection is necessary for the inequality to decrease among bad managers.

Proposition 1 (proven in Appendix A.2) formally states this result:

Proposition 1. *There exists a unique skill level $s^{(\kappa)} > s_c$ such that the local change in inequality from skill-biased technical change is positive for high abilities and negative for low abilities, i.e., $\eta^{(\kappa)}(s) \geq 0 \Leftrightarrow s \geq s^{(\kappa)}$.*

With these results in hand, it is possible to rationalize a divergent pattern in the percentile ratio for high vs. low ability agents (as in Autor, Katz and Kearney 2005) and an increasing dispersion in the tails (as in Piketty and Saez 2004) only in terms of skill-biased technical change. The latter is already evident: local inequality increases for all $s > s^{(\kappa)}$, and the percentile ratio will increase at *any* pair of points in the ability distribution above the threshold, and in particular, at very high percentiles in the tail. To rationalize the former, it is sufficient to note that we can pick $s' < s'' < s_c$, and $s''' > s^{(\kappa)}$ to produce a constant ratio in the lower part of the income distribution $(w(s'')/w(s'))$ and an increasing ratio in the upper part $(w(s''')/w(s''))$; or, $s_c < s' < s'' < s^{(\kappa)}$ in the lower part and $s''' > s^{(\kappa)}$ to obtain one decreasing and one increasing wage ratio at different percentiles in the distribution.

4 The Open Economy

I now introduce the framework and show the equilibrium determination when 2 identical countries are allowed to trade with each other. Assuming identical countries is appropriate since we want to think of intra-industry trade as a source of inequality, and to do so it is necessary to neutralize differences in factor endowments or technologies.

In what follows, I lay out the main modification to the framework in an open economy; then I show how the equilibrium is determined.

4.1 Framework

I assume that a firm needs to produce τ units of a good for 1 unit to reach the foreign destination, and that f units of production workers are needed to sell in the export market. If the price of firm φ is $p(\varphi)$ in the domestic market, it will be $\tau p(\varphi)$ abroad. The surplus from sales on the domestic and export markets are given respectively by:

$$v_d(z,s) = M\left(\frac{z^{\kappa}s^{\alpha}}{\bar{w}}\right)^{\sigma-1} \tag{4.1}$$

$$v_x(z,s) = \tau^{1-\sigma}M\left(\frac{z^{\kappa}s^{\alpha}}{\bar{w}}\right)^{\sigma-1} - f\bar{w} \tag{4.2}$$

where v_i, with $i = d, x$, indicates the surplus reaped by a firm selling in the domestic and in the export market, respectively, and $M \equiv \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} XP^{\sigma-1}$.

The earning functions corresponding to equations (2.6) and (2.7) are built following steps analogous to the closed economy. The only difference is that the optimality condition (2.4) will depend on the export status of the firm. Since this is not known in advance, I postulate the existence of the two cutoffs s_d and s_x, and then build separately two sets of first order conditions, for firms selling only to the domestic market (which earn $v_d(z,s)$) and for firms selling to both markets (which earn $v_d(z,s) + v_x(z,s)$). Having obtained two expressions for $w_d(s)$ and $w_x(s)$, I impose two separate indifference conditions,

$w_d(s_d) = \bar{w}$ and $w_x(s_x) = f\bar{w}$, to actually pin down s_d and s_x.

Following these steps, simple calculations deliver the earning functions in each country:

$$\pi(z) = \begin{cases} (1-\theta)\left(\frac{t^{-\alpha}}{\bar{w}}\right)^{\sigma-1} M\left(z^{1-\psi} - z_d^{1-\psi}\right) & z \in [z_d, z_x) \\ (1-\theta)\left(\frac{t^{-\alpha}}{\bar{w}}\right)^{\sigma-1} M\left[\left(z^{1-\psi} - z_d^{1-\psi}\right) + \tau^{1-\sigma}\left(z^{1-\psi} - z_x^{1-\psi}\right)\right] & z \geq z_x \end{cases} \quad (4.3)$$

$$w(s) = \begin{cases} \theta\left(\frac{t^{\kappa}}{\bar{w}}\right)^{\sigma-1} M\left(s^{1-\psi} - s_d^{1-\psi}\right) + \bar{w} & s \in [s_d, s_x) \\ \theta\left(\frac{t^{\kappa}}{\bar{w}}\right)^{\sigma-1} M\left[\left(s^{1-\psi} - s_d^{1-\psi}\right) + \tau^{1-\sigma}\left(s^{1-\psi} - s_x^{1-\psi}\right)\right] + \bar{w} & s \geq s_x \end{cases} \quad (4.4)$$

To connect the selection of domestic and foreign sellers, I set the surplus in (4.1) and (4.2) to $\bar{w}$ and 0, respectively, to characterize s_d and s_x; substituting the assignment function (2.5), solving both expressions for M and equating them,

$$s_x = \left(\tau^{\sigma-1} f\right)^{1/(1-\psi)} s_d \quad (4.5)$$

where I assume that $\left(\tau^{\sigma-1} f\right)^{1/(1-\psi)} > 1$ in order to generate the empirically relevant pattern of partitioning in the export behavior of firms, i.e., $s_x > s_d$.

This equation allows us to write the price index as a simple function of the domestic cutoff. In fact, since

$$P = \frac{\sigma}{\sigma-1}\psi^{1/(\sigma-1)}\left[Lt^{\kappa(\sigma-1)}\bar{w}^{1-\sigma}s_d^{-\psi} + \tau^{1-\sigma}Lt^{\kappa(\sigma-1)}\bar{w}^{1-\sigma}s_x^{-\psi}\right]^{1/(1-\sigma)}$$

we can use (4.5) to eliminate $s_x^{-\psi}$ and get:

$$P = \frac{\sigma}{\sigma-1}\bar{w}\left(\frac{\psi}{L}\right)^{1/(\sigma-1)} t^{-\kappa}\left(1+\frac{1}{\delta}\right)^{-1/(\sigma-1)} s_d^{\psi/(\sigma-1)} \quad (4.6)$$

$$\delta \equiv \tau^{1/(\alpha+\kappa)} f^{\psi/(1-\psi)} \quad (4.7)$$

where δ is an index of distance between the two economies. While the general structure of the price index reflects its shape in closed economy (eq. (2.8)), the additional term $(1+1/\delta)^{-1/(\sigma-1)}$ shows how competition from abroad lowers the price index at home. In particular, note that heterogeneity in both skill and technology contribute to effectively reduce the distance between the two countries (as $\kappa+\alpha$ grows, δ becomes smaller). The relative size of ideas vs. population t lowers the price index only in proportion to the importance of the assignment: as $\kappa \to 0$, ideas play no role in the productivity, sorting is immaterial, and t no longer affects P.

4.2 Equilibrium

In an open economy, equilibrium will require for each country: (i) indifference for the marginal agent s_d between alternative occupation, (ii) equilibrium in the market for production workers, and (iii) trade balance.

The indifference of a firm to sell on the domestic market or shut down (condition (i)) simply requires the surplus in the domestic market given in (4.1) to be equal to the sum of the outside options $\bar{w}$ and 0 when evaluated at s_d and $z_d \equiv ts_d$. Substituting in such equality the expression for the price index (4.6), using (4.5) and rearranging, we get

$$X = \frac{\sigma}{\psi} L\bar{w}\left(1 + \frac{1}{\delta}\right) s_d^{-1} \tag{4.8}$$

This equation is the open economy equivalent of (2.11), the Zero Cutoff Earnings: it shows how competition from abroad affects occupational choices. Stronger trade integration (lower δ) makes competition stiffer, lowering the price index and increasing the real wage for production workers: at any expenditure level X, the cutoff agent s_d must be better to compete in her own market.

Equating total income of production workers to total expenditure of firms on them (condition (ii)) we obtain

$$L\bar{w}\left(1 - s_d^{-1}\right) = \frac{\sigma - 1}{\sigma} X + f\bar{w}Ls_x^{-1} \tag{4.9}$$

The right-hand side of this expression is found integrating separately labor demand for domestic and export sales, and including the fixed requirement to sell abroad, $f\bar{w}$, in proportion to the mass of exporters Ls_x^{-1}. Substituting (4.5) into (4.9) and rearranging,

$$X = \frac{\sigma}{\sigma - 1} L\bar{w}\left[1 - \left(1 + \frac{1}{\delta}\right) s_d^{-1}\right] \tag{4.10}$$

Equation (4.10) is the parallel in an open economy of eq. (2.10), the Labor Market Clearing condition: it shows how the possibility to sell abroad affects domestic demand of production workers. As economies become more integrated, more workers are demanded to pay the fixed costs of export (δ decreases), and a lower level of overall expenditure X is needed to equilibrate demand and supply of production workers.

To close the model, we need to make sure that these conditions are compatible in the world economy: if trade balance has to be satisfied (condition (iii)), this entails a relation between the relative wage of production workers in the two economies. When countries are identical, this ratio is simply 1.

Equations (4.8) and (4.10) pin down the 2 endogenous variables of this model, the national income X and the domestic cutoffs s_d. The exporter cutoff s_x can then be found using (4.5), and the price indicex using (4.6). Equating (4.8) and (4.10) and solving for s_d, we obtain

$$s_d = \left(1 + \frac{\sigma - 1}{1 - (\sigma - 1)(\alpha + \kappa)}\right)\left(1 + \frac{1}{\delta}\right) \tag{4.11}$$

Substituting this value back in (4.8), I obtain

$$X = \frac{\sigma}{\psi + \sigma - 1} L\bar{w} \tag{4.12}$$

Figure 4.1 shows graphically how the equilibrium is determined.

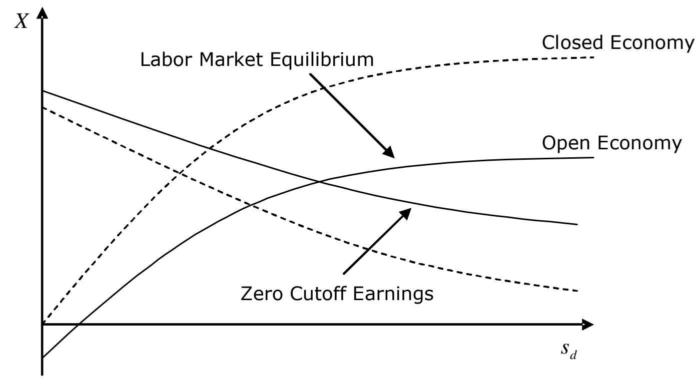

Figure 4.1: This figure shows the equilibrium determination of the cutoff s_d and the expenditure level X in an open economy. The possibility to sell abroad implies that a lower level of expenditure (for any supply of production workers) is sufficient for equilibrium, so that the Labor Market Equilibrium curve shifts down and to the right. On the other hand, competition from abroad implies that the marginal firm must employ a better manager (at any level of domestic expenditure) to stay indifferent when trade is allowed, and the Zero Cutoff Earnings curve shifts up and to the right. As a result, the cutoff for domestic producers is larger in open economy.

In equilibrium, the market size M can be written, using the assignment function (2.5) in the surplus (4.1) and imposing equality to $\bar{w}$, as

$$M = \bar{w}^{\sigma} t^{-\kappa(\sigma-1)} s_d^{-(\alpha+\kappa)(\sigma-1)} = \bar{w}^{\sigma} t^{\alpha(\sigma-1)} z_d^{-(\alpha+\kappa)(\sigma-1)}$$

We can use this expression and the equation (4.5) to substitute out M and the exporters' cutoffs in the profit and wage functions (4.3) and (4.4), to obtain

$$\pi(z) = \begin{cases} (1-\theta)\left[\left(\frac{z}{z_d}\right)^{1-\psi} - 1\right] & z \in [z_d, z_x) \\ (1-\theta)\left[\left(1+\tau^{1-\sigma}\right)\left(\frac{z}{z_d}\right)^{1-\psi} - (1+f)\right] & z \geq z_x \end{cases} \tag{4.13}$$

and

$$w(s) = \begin{cases} \theta\left[\left(\frac{s}{s_d}\right)^{1-\psi} - 1\right] + 1 & s \in [s_d, s_x) \\ \theta\left[\left(1+\tau^{1-\sigma}\right)\left(\frac{s}{s_d}\right)^{1-\psi} - (1+f)\right] + 1 & s \geq s_d \end{cases} \tag{4.14}$$

The real earnings can easily be obtained dividing by the price index (4.6). Below the cutoffs, we still have $\pi(z)/P = 0$ and $w(s)/P = P^{-1}$.

All the components identified in the closed economy case are present, suitably modified, in the open economy. The macro-economic component, P^{-1}, and the type-specific component θ enter in a similar way. The existence of an export market now raises the marginal price of skills for managers good enough to access it.

In the next section, I use this model to compare the consequences of skill-biased technical change and trade integration on wages ratios in different regions of the income distribution.

5 Implications for Wage Dispersion

As discussed in the Introduction, the recent evolution of the wage distribution can be characterized by a divergent pattern for wage ratios in the lower vs. the upper tail, and an increase in the dispersion on the right tail of the wage distribution. I now ask if, for given contribution of managers α to the productivity of the firm, trade integration and skill-biased technical change can both rationalize this pattern.

5.1 Skill-Biased Technical Change and Inequality

The basic components determining the direction of the local change in inequality (see eq. (3.1)) are unaltered when countries are allowed to trade: we still have to compare the elasticity of the marginal price of skill to the elasticity of the wage with respect to a change in κ. The difference lies in the latter term, which varies according to the export status. In particular, denoting $g_2^i(s)$ the weight on selection and assignment effects for domestic sellers ($i = d$) and exporters ($i = x$), the local change in inequality is positive if and only if

$$\underbrace{-\frac{(1-\psi)\,\kappa}{s_d}\frac{\partial s_d}{\partial \kappa}}_{\text{Selection}} + \underbrace{(\sigma-1)\,\kappa \ln\frac{s}{s_d}}_{\text{Assignment}} \;>\; -\,g_1(s)\underbrace{(1-\theta)}_{\text{Share}} - g_2^i(s)\underbrace{\frac{(1-\psi)\,\kappa}{s_d}\frac{\partial s_d}{\partial \kappa}}_{\text{Selection}} + g_2^i(s)\underbrace{(\sigma-1)\,\kappa \ln\frac{s}{s_d}}_{\text{Assignment}}$$

where

$$g_2^d(s) \;\equiv\; \frac{w(s)-(1-\theta)}{w(s)}, \text{ for } s \in (s_d, s_x) \tag{5.1}$$

$$g_2^x(s) \;\equiv\; \frac{w(s)-[1-\theta(1+f)]}{w(s)}, \text{ for } s \geq s_x \tag{5.2}$$

and $g_1(s) \equiv (w(s)-1)/w(s)$, as in (3.2). The analysis of each of these terms exactly mirrors the discussion in the closed economy section.

The elasticity of the marginal price of skills (left-hand side) is unaffected by the export status. Conditional on exporting ($i = x$), the elasticity of the wage to κ (right-hand side) now incorporates the additional fixed cost necessary to sell abroad. A larger fixed cost tends to reduce the level of skill rent (see eq. (4.14)) and hence to increase the percentage impact of any given change in κ on $w(s)$. For this reason, the selection and assignment effects receive a larger weight for exporters than for domestic sellers, i.e., $g_2^x(s) > g_2^d(s_x)\ \forall s \geq s_x$. The share effect, operating on a *given* rent, is not changed.

Note that while $g_2^d(s) \in (\theta, 1)$ always, fixed costs can be large enough (i.e., $1-\theta(1+f) < 0$, which is true if and only if $f > \kappa/\alpha$) to imply $g_2^x(s) > 1$. In this case, the selection and the assignment effect are stronger on the elasticity of the wage to κ (right-hand side) than on the elasticity of the marginal price of skills (left-hand side). In order to answer the question of this paper, whether or not the observed behavior on wage ratios can be rationalized both by skill-biased technical change and trade integration, it is sufficient to focus on the case in which $g_2^x(s) < 1$. I will maintain this restriction throughout the rest of the paper, although I will briefly discuss what happens when it is not satisfied[17].

Assumption 1. *Fixed costs of exporting are such that $f < \kappa/\alpha$.*

Proposition 2. *There exists a unique skill level $s^{(\kappa)} > s_d$ such that the local change in inequality from skill-biased technical change is positive for high abilities and negative for low abilities, i.e., $\eta^{(\kappa)}(s) \geq 0 \Leftrightarrow s \geq s^{(\kappa)}$.*

This proposition (proven in Appendix A.3) generalizes Proposition 1 to an open economy. Whether the threshold $s^{(\kappa)}$ is larger or smaller than the threshold for exporters s_x depends on the values of the parameters of the model. In the Proof, I show that along a path of trade integration (from $\tau \to \infty$ to $\tau \to 1$) it is possible to have a positive local change in inequality 1) for all exporters and the best domestic sellers ($s^{(\kappa)} \in (s_d, s_x)$), 2) only for all exporters ($s^{(\kappa)} = s_x$)[18], and finally 3) only for the best exporters ($s^{(\kappa)} > s_x$).

When Assumption 1 is not satisfied ($f < \kappa/\alpha$), the assignment and selection effects are proportionally more important on the level of the wage than on the marginal wage. In this case, it is still true that for the worst domestic sellers local inequality decreases, while it increases for the best exporters. However, it is possible to show that the threshold is no longer necessarily unique: we can have two ability levels $s^{(\kappa,d)}$ and $s^{(\kappa,x)}$, for domestic sellers and exporters respectively, above which local inequality increases, and below which it decreases. This is a case where *(1)* the best domestic sellers are good enough for the assignment effect to overcome the other two: as κ increases, the the total price of skills is less sensitive than the marginal price, and the local change in inequality is positive; and *(2)* at the same time, for some of the worst exporters fixed costs are high enough to make the total price of skills more sensitive to κ than the marginal price: for them the local change in inequality is negative[19].

5.2 Trade Integration and Inequality

In this section I adapt the conceptual framework introduced above to evaluate the effect of trade integration on the evolution of wage ratios. In this experiment, I will model trade integration as reduction

[17] At aggregate level, Assumption 1 is placing a lower bound on the ratio between rents to ideas and rent to managers. Total rents for managers (wages less opportunity costs) can be written as $W = \Psi \cdot L\theta s_d^{-1}$, and profits (i.e., ideas' rents) as $\Pi = \Psi \cdot L(1-\theta)s_d^{-1}$, where $\Psi \equiv \Psi_h + \Psi_f$ are two parameter aggregates coming from home and foreing market rents; in particular, $\Psi_h = [1/\psi - 1]$, and $\Psi_f = \left[\left(1+\tau^{1-\sigma}\right)/\psi - (1+f)\,\tau^{1-\sigma}/f\right]\left(\tau^{\sigma-1}f\right)^{-\psi/(1-\psi)}$. Hence, $\Pi/W = \kappa/\alpha$.

[18] In particular, there is a non-trivial interval of values for τ where this state occurs: in other words, $s^{(\kappa)} = s_x$ is not a knife-edge case (see Proof of Proposition 2).

[19] Since a complete taxonomy of the cases is not the objective of this paper, and divergent patterns of percentile ratios can be obtained even if $f < \kappa/\alpha$, I will focus on cases where this restriction holds. Also, note that even if $f > \kappa/\alpha$, we think of falling trade costs and skill-biased technical change as important facts in the major industrialized countries in recent decades. Hence, this inequality would tend to be reversed by declines in f and increases in κ.

in the proportional transportation cost τ. Following steps analogous to what we did before to analyze episodes of skill-biased technical change (as in Appendix A.1), we can then write the total change in inequality following trade integration between ability s' and $s'' > s'$ as

$$\eta^{(\tau)}(s) = \frac{w_{\tau s}(s)\,\tau}{w_s(s)} - \frac{w_\tau(s)\,\tau}{w(s)}$$

where $\eta^{(\tau)}(s)$ measures the local change in inequality at ability level s, $w_\tau(s) = \partial w(s)/\partial\tau$ and $w_{\tau s}(s) = \partial^2 w(s)/(\partial\tau\partial s)$. The local change in inequality is then proportional to the difference between the elasticity to τ of the marginal price of skills and the total price of skills. The change in the wage ratio between s' and s'', which can be written as

$$\int_{s'}^{s''} \frac{w_s(s)}{w(s)} \eta^{(\tau)}(s)\,ds$$

is positive when trade costs fall if $\frac{\partial}{\partial\tau}[w(s'')/w(s')] < 0$, so that now $\eta^{(\tau)}(s) < 0$ goes in the direction of increasing local inequality.

Trade integration is affecting wages through two channels, *(1)* the reduced marginal cost that exporters face in order to sell abroad, and *(2)* higher competition at home, which tends to reduce revenues on the domestic market and select some managers out of the differentiated sector.

For non-exporters, the selection effect is the only channel active when trade integration occurs. Using (4.14), simple calculations show that

$$\left.\frac{w_{\tau s}(s)\,\tau}{w_s(s)}\right|_{s<s_x} > \left.\frac{w_\tau(s)\,\tau}{w(s)}\right|_{s<s_x} \Leftrightarrow$$
$$\underbrace{-(1-\psi)\frac{\tau}{s_d}\frac{\partial s_d}{\partial\tau}}_{\text{Selection}} > -g_2^d(s)\underbrace{(1-\psi)\frac{\tau}{s_d}\frac{\partial s_d}{\partial\tau}}_{\text{Selection}} \Leftrightarrow \eta^{(\tau)}(s) > 0$$

with g_2^d given in (5.1). As τ falls, both the wage and the marginal price of skill decrease. However, the marginal price is more responsive (it drops more), since the elasticity of the wage level is reduced by the fact that managers are not getting all the surplus (and in fact, $g_2^d(s) < 1$): part of the adjustment will occur through profits. Hence, as trade integration occurs, $\eta^{(\tau)}(s)$ is positive, and local inequality decreases for all non-exporting managers.

For exporters, differentiation of (4.14) delivers

$$\left.\frac{w_{\tau s}(s)\,\tau}{w_s(s)}\right|_{s\geq s_x} < \left.\frac{w_\tau(s)\,\tau}{w(s)}\right|_{s\geq s_x} \Leftrightarrow$$
$$-\underbrace{\frac{(\sigma-1)}{\tau^{\sigma-1}+1}}_{\text{Price}} - \underbrace{(1-\psi)\frac{\tau}{s_d}\frac{\partial s_d}{\partial\tau}}_{\text{Selection}} < -\frac{\theta(s/s_d)^{1-\psi}}{w(s)}\underbrace{\frac{(\sigma-1)}{\tau^{\sigma-1}}}_{\text{Price}} - g_2^x(s)\underbrace{(1-\psi)\frac{\tau}{s_d}\frac{\partial s_d}{\partial\tau}}_{\text{Selection}} \Leftrightarrow \eta^{(\tau)}(s) < 0$$

where g_2^x is given in (5.2). The price effect increases the marginal price of skills (on the left hand side) and the component of the wage coming from the export market (on the right hand side). The former always receives a higher weight, since part of the adjustment on the rent level goes through profits. This force points towards an increase in local inequality.

Under Assumption 1, the selection effect is always pushing $\frac{w_{\tau s}(s)\tau}{w_s(s)}$ upward, in positive territory (recall that $\partial s_d/\partial\tau < 0$), and more than $\frac{w_\tau(s)\tau}{w(s)}$: as it happens for domestic sellers, trade integration lowers the price index, puts an upward pressure on real wages of production workers and makes the selection in the manager occupation stricter. This is a force towards the reduction of local inequality.

Overall, the price effect always prevails, and trade integration increases local inequality for exporters.

We can then state the following proposition (proven in Appendix A.4), which states that the unique threshold ability beyond which trade increases local inequality coincides with the exporters' cutoff ability, s_x.

Proposition 3. *There exists a unique skill level $s^{(\tau)} = s_x$ such that the local change in inequality is positive for high abilities and negative for low abilities, i.e., $\eta^{(\tau)}(s) \leq 0 \Leftrightarrow s \geq s^{(\tau)}$.*

Under Assumption 1, trade integration is producing increasing dispersion of the wages in the high part of the wage distribution and a compression of wages in the lower part. Evaluating the evolution of wage dispersion in different regions of the wage distribution is then not sufficient to disentangle the source of the pattern.

If $f > \kappa/\alpha$, Assumption 1 is not satisfied, local inequality decreases for both non-exporters and exporters (see proof of Proposition 3), and that the qualitative consequences of skill-biased technical change and trade integration no longer coincide. In this case, the selection effect on $\frac{w_\tau(s)\tau}{w(s)}$ is receiving a weight so high that the total price of skills is always more sensitive than the marginal price of skills, and local inequality decreases at all levels with trade integration.

Assumption 1 is then crucial for trade to imply the same qualitative behavior of skill-biased technical change and its validity is ultimately an empirical question. Note that along a path of skill-biased technical change (increase in κ), this assumption tends to be more and more restricitive. In absence of any information on its validity, we cannot exclude that trade has a role in causing the observed pattern of wage dispersion only on the basis of the wage distribution in the overall economy. I argue in the discussion which follows that intra-firm rent distribution can provide an alternative source of information which does not rely on Assumption 1.

6 Intra-Firm Rent Distribution

A larger wage dispersion at the top and smaller wage dispersion at the bottom of the distribution can be caused both by skill-biased technical change and by trade integration, so that information only on the wage distribution is not enough. The following question naturally arises: are there firm-level outcomes that have a different response to these two sources of changes in inequality? And are these responses independent on the validity of Assumption 1? In this simple framework, the answer is yes to both questions.

The rent created in a firm by a manager and an idea is given by the sum of profits and manager's wage (i.e., the surplus) less their opportunity cost in the alternative occupation. Noting that the assignment (2.5) allows us to write $z(s)/z(s_d) = s/s_d$, we can use the earning functions (4.13) and (4.14) to express the rent for an exporting and a non-exporting firm as

$$\pi(z(s)) + w(s) - \bar{w} \equiv \begin{cases} \left(\frac{s}{s_d}\right)^{1-\psi} - 1 & s \in [s_d, s_x) \\ \left(1+\tau^{1-\sigma}\right)\left(\frac{s}{s_d}\right)^{1-\psi} - (1+f) & s \geq s_x \end{cases}$$

The share of this rent that goes to managers is then

$$\lambda(\theta) \equiv \frac{w(s) - \bar{w}}{\pi(z(s)) + w(s) - \bar{w}} = \theta$$

where $\theta \equiv \alpha/(\alpha+\kappa)$; a share $1-\lambda(\theta) = 1-\theta$ is then left for profits. This fact is true for any firm, independently of its export status. Moreover, Assumption 1 may or may not hold, without compromising the validity of this result.

The intra-firm rent distribution is only a function of the relative contribution of types to the overall productivity of the firm. In particular, it is not a function of the level of trade integration, not even in exporting firms. The economic force that drives this result is the positive assortative matching between managers and ideas: in equilibrium, the wage function equates the marginal benefits and costs of a better manager *in all markets* where the firm chooses to sell, so that a fraction θ of the additional rent that a larger ability generates *in each market* is given to the manager. Hence, while trade integration affects the level of the rents reaped by a firm, it does not affect the way in which this rent is shared.

This suggests that a promising avenue for disentangling the two effects is to look at the intra-firm rent distribution. Firm-level data on employers and employees, properly interpreted, can give us a handle on the evolution of θ. Changes in inequality not accompanied by changes in the intra-firm rent distribution must be attributed to trade. Vice-versa, changes in the intra-firm rent distribution must imply changes in local inequality and wage ratios caused by skill-biased technical change.

7 Conclusion

I have shown how changes in local inequality determine the behavior of wage ratios across the ability spectrum. Although through partially different channels, local inequality responds in similar ways to both skill-biased technical change and trade integration: both shocks have asymmetric effects across firms, raising the competitive pressure on low productivity firms and favoring firms at the high end of the productivity range, which are also the firms which employ higher skilled managers. Hence, skill-biased technical change and trade integration can - under appropriate parameters restriction - both reproduce divergent patterns of wage ratios in the lower and the upper tail of the wage distribution, and increasing ratios at all levels in the upper tail, thus being consistent with the evidence on wage inequality in the last 50 years in the United States.

This result suggests the need to go beyond analyses based on wage distributions that do not consider also the firms that contribute to determine the level of wage as an equilibrium outcome. I argue that deeper, firm-level considerations of the mechanisms of operation of each force may help in the quantification of the magnitude of each channel. Since skill-biased technical change operates by increasing the importance of heterogeneity in technology in the determination of the relative fortune of different firms, managers get in *each* firm a smaller share θ of the rent. On the other hand, international trade influences the market size and the marginal cost of *each* firm, while it does not change the intra-firm rent distribution. Estimates of the evolution of intra-firm rent distribution, possibly based on the use of employer-employee matched data sets, can put (at least) a bound on the degree of skill-biased technical change, and help disentangle the size of each channel.

The model is still too highly stilyzed to attempt any reasonable quantification of the magnitude of the forces that have shaped the evolution of wage ratios in U.S. in the last 50 years. I leave this for future research.

A Proof of Results

A.1 The Local Change in Inequality

For two ability levels $s'' > s'$, we want to study the direction of the change in $w(s'')/w(s')$ as κ increases. Denote with $w_\kappa(s)$ the derivative of the wage function with respect to κ, and with $\varepsilon(s) \equiv \kappa w_\kappa(s)/w(s)$ the point elasticity of the wage. Then, the elasticity of the wage ratio with respect to κ, call it $\varepsilon(s', s'')$, is simply $\varepsilon(s'') - \varepsilon(s')$. Since the choice of the percentiles (and then the abilities) is arbitrary, it is convenient to this elasticity as

$$\varepsilon(s', s'') = \int_{s'}^{s''} \frac{\partial \varepsilon(s)}{\partial s} ds$$

The elasticity of the wage with respect to κ generally varies with the ability level: the function $\partial\varepsilon(s)/\partial s$ describes this dependence. Moreover, its sign will determine if the local contribution of the ability level s is to increase or decrease *all* the wage ratios that contain it. Calculating $\frac{\partial \varepsilon(s)}{\partial s}$ explicitly,

$$\begin{aligned} \frac{\partial \varepsilon(s)}{\partial s} &= \frac{w_s(s)}{w(s)} \eta^{(\kappa)}(s) \\ \eta^{(\kappa)}(s) &\equiv \frac{w_{\kappa s}(s)\kappa}{w_s(s)} - \frac{w_\kappa(s)\kappa}{w(s)} \end{aligned}$$

where $w_s(s)$ is the marginal wage at s and $w_{\kappa s}(s)$ is the cross-partial derivative of the wage function with respect to κ and s. The sign of $\eta^{(\kappa)}(s)$ is what matters to determine the direction of the local change in inequality.

A.2 Skill-Biased Technical Change in Closed Economy

Recall that eq. (3.1) defines $\eta^{(\kappa)}(s) \equiv \frac{w_{\kappa s}(s)\kappa}{w_s(s)} - \frac{w_\kappa(s)\kappa}{w(s)}$.

Differentiating (2.14) with respect to κ, multiplying by $\kappa/w(s)$, and normalizing $\bar{w}$ to 1, we have

$$\frac{w_\kappa(s)\kappa}{w(s)} = \frac{\theta(\sigma-1)\kappa}{\theta(j(s)-1)+1}\left[-\frac{j(s)-1}{1-\psi} + h(s)j(s)\right] \tag{A.1}$$

$$\frac{w_{\kappa s}(s)\kappa}{w_s(s)} = (\sigma-1)\kappa h(s) \tag{A.2}$$

with

$$\begin{aligned} j(s) &\equiv \left(\frac{s}{s_c}\right)^{1-\psi} \\ h(s) &\equiv \ln\frac{s}{s_c} - (\alpha+\kappa)\frac{1}{s_c}\frac{\partial s_c}{\partial \kappa} \end{aligned}$$

and where we recall that $\theta \equiv \alpha/(\alpha+\kappa)$ and $\psi \equiv 1-(\sigma-1)(\alpha+\kappa)$. The function $j(s)$ is always greater than or equal to 1, $j'(s) > 0$, and is such that $\lim_{s\to s_c} j(s) = 1$, $\lim_{s\to\infty} j(s) = +\infty$. The function $h(s)$ is always increasing in s and has the properties $\lim_{s\to s_c} h(s) = -(\alpha+\kappa)\frac{1}{s_c}\frac{\partial s_c}{\partial \kappa} < 0$, since the cutoff s_c is increasing in κ (this is immediate from eq. (2.12)), and $\lim_{s\to\infty} h(s) = +\infty$; hence, $h(s)$ crosses zero only once.

Proposition 1 *There exists a unique skill level $s^{(\kappa)} > s_c$ such that the local change in inequality from skill-biased technical change is positive for high abilities and negative for low abilities, i.e., $\eta^{(\kappa)}(s) \geq 0 \Leftrightarrow s \geq s^{(\kappa)}$.*

Proof. For $\eta^{(\kappa)}(s) > 0$ is necessary and sufficient that $\frac{w_{\kappa s}(s)\kappa}{w_s(s)} - \frac{w_\kappa(s)\kappa}{w(s)}$ is positive. Using (A.1) and (A.2) in this difference and rearranging, we have

$$\begin{aligned}
\eta^{(\kappa)}(s) &\geq 0 \Leftrightarrow \frac{w_{\kappa s}(s)\kappa}{w_s(s)} - \frac{w_\kappa(s)\kappa}{w(s)} > 0 \Leftrightarrow \\
(1-\theta)\,h(s) &> -\frac{1}{1-\psi}\theta\,(j(s)-1)
\end{aligned}$$

The left-hand side starts at $(1-\theta)\,h(s_c) < 0$ and always increases with s, crossing zero only once, while the right-hand side starts in zero and always decreases with s. Hence, there is one and only one $s^{(\kappa)}$ such that $\eta^{(\kappa)}(s) \geq 0 \Leftrightarrow s \geq s^{(\kappa)}$. This $s^{(\kappa)}$ is the unique solution of $(1-\theta)\,h\left(s^{(\kappa)}\right) = -\frac{1}{1-\psi}\theta\left(j\left(s^{(\kappa)}\right)-1\right)$. Note that if $h(s_c)$ were zero, the left- and right-hand side would touch for $s = s_c$ and then diverge from each other, so that we would have $s^{(\kappa)} = s_c$ and no region with a negative local change in inequality. ■

A.3 Skill-Biased Technical Change in Open Economy

The elasticity of wage to skill is now a piecewise function of the form

$$\frac{\kappa}{w(s)}\frac{\partial w(s)}{\partial \kappa} = \begin{cases} -(1-\theta)\,g_1^d(s) + g_2^d(s)\,(\sigma-1)\,\kappa h(s) & s \in (s_d, s_x) \\ -(1-\theta)\,g_1^d(s) + g_2^x(s)\,(\sigma-1)\,\kappa h(s) & s > s_x \end{cases}$$

with

$$\begin{aligned}
h(s) &\equiv \left[\ln\frac{s}{s_d} - (\alpha+\kappa)\frac{1}{s_d}\frac{\partial s_d}{\partial \kappa}\right] \\
g_1^d(s) &\equiv \frac{w(s)-1}{w(s)}, \\
g_2^d(s) &\equiv \frac{w(s)-(1-\theta)}{w(s)},\ g_2^x(s) \equiv \frac{w(s)-[1-\theta(1+f)]}{w(s)}
\end{aligned}$$

However, the elasticity of the marginal price of skills to κ is always

$$\frac{\kappa}{w_s(s)}w_{\kappa s}(s) = (\sigma-1)\,\kappa h(s)$$

Proposition 2 *There exists a unique skill level $s^{(\kappa)} > s_d$ such that the local change in inequality from skill-biased technical change is positive for high abilities and negative for low abilities, i.e., $\eta^{(\kappa)}(s) \geq 0 \Leftrightarrow s \geq s^{(\kappa)}$.*

Proof. For a domestic seller, $s \in (s_d, s_x)$, local inequality increases if and only if

$$\begin{aligned}\frac{\kappa}{w_s(s)} w_{\kappa s}(s) &> \frac{\kappa}{w_x(s)} \frac{\partial w_x(s)}{\partial \kappa} \Leftrightarrow \\ h(s) &> -\frac{\theta}{1-\theta} \frac{1}{1-\psi} (j(s) - 1) \qquad \text{(A.3)}\end{aligned}$$

In particular, as $s \to s_d$, the inequality is never satisfied: for the worst managers, local inequality always decreases. For an exporter, $s \geq s_x$, local inequality increases if and only if

$$\begin{aligned}&\frac{\kappa}{w_s(s)} w_{\kappa s}(s) > \frac{\kappa}{w_x(s)} \frac{\partial w_x(s)}{\partial \kappa} \Leftrightarrow \\ &h(s) > -\frac{\theta}{1-\theta(1+f)} \frac{1}{1-\psi} \left[\left(1+\tau^{1-\sigma}\right) j(s) - (1+f)\right] \qquad \text{(A.4)}\end{aligned}$$

Note that in both (A.3) and (A.4), *(i)* the left-hand side is always increasing in s, while *(ii)* the right-hand side is always non-positive and decreasing (in (A.4) it is decreasing because, by assumption, $f < \kappa/\alpha \Leftrightarrow [1-\theta(1+f)] > 0$): for the values of s where each relation applies, if the inequality is satisfied for an s', it is also satisfied for all $s > s'$, and vice-versa, if it is not satisfied for an s', it is also not true for all $s < s'$. To prove the existence of this threshold and identify the region it falls in, I will check the value of each inequality at $s = s_x$. Then, let:

$$\begin{aligned}lhs(\tau) &\equiv h(s)|_{s=s_x} = \frac{\ln f^{1/(1-\psi)} \tau^{1/(\alpha+\kappa)}}{1 + f^{-\psi/(1-\psi)} \tau^{-1/(\alpha+\kappa)}} - \frac{(\alpha+\kappa)(\sigma-1)^2/\psi^2}{\left(1+\frac{\sigma-1}{\psi}\right)} \\ rhs_d(\tau) &\equiv -\frac{\theta}{1-\theta}\frac{1}{1-\psi}(j(s_x)-1) = -\frac{\theta}{1-\theta}\frac{1}{1-\psi}\left(f\tau^{\sigma-1}-1\right) \\ rhs_x(\tau) &\equiv -\frac{\theta}{1-\theta(1+f)}\frac{1}{1-\psi}\left[\left(1+\tau^{1-\sigma}\right)j(s)-(1+f)\right] = -\frac{\theta}{1-\theta(1+f)}\frac{1}{1-\psi}\left(f\tau^{\sigma-1}-1\right)\end{aligned}$$

These functions describe the left-hand side and the right-hand side of (A.3) and (A.4) when $s = s_x$.

Note that $rhs_d(\tau) > rhs_x(\tau)\ \forall \tau$. The function $lhs(\tau)$ can be in three positions with respect to this inequality.

(i) Suppose that $rhs_d(\tau) > rhs_x(\tau) > lhs(\tau)$: at $s = s_x$ inequality (A.4) is not satisfied, and the local inequality for the worst exporters is decreasing; then, local inequality is also decreasing among the best domestic sellers, since $rhs_d(\tau) > lhs(\tau)$. The relation (A.3) is never satisfied for $s \in [s_d, s_x)$, local inequality is decreasing among all domestic sellers, and there must exists a threshold $s^{(\kappa)} > s_x$ such that local inequality decreases below it and increases above it.

(ii) Suppose that $lhs(\tau) > rhs_d(\tau) > rhs_x(\tau)$: at $s = s_x$ inequality (A.3) is satisfied, and local inequality for the best domestic sellers is increasing; then, local inequality is also increasing among all exporters, i.e., $lhs(\tau) > rhs_x(\tau)$. The relation (A.4) is always satisfied for $s \geq s_x$, local inequality is then increasing among all exporters, while there must exists a threshold $s^{(\kappa)} \in (s_d, s_x)$ such that local inequality increases above it and decreases below.

(iii) Suppose that $rhs_d(\tau) > lhs(\tau) > rhs_x(\tau)$: at $s = s_x$, (A.3) is not satisfied while (A.4) is, and so local inequality increases for all exporters and decreases for all non-exporters. In this cased dd , $s^{(\kappa)} = s_x$. Also, since $rhs_d(\tau) > rhs_x(\tau)$ holds strictly and $lhs(\tau)$ changes continuously with τ, this case is not a knife-edge possibility: it will happen for $\tau \in (\tau_d, \tau_x)$, with $\tau_d : rhs_d(\tau_d) = lhs(\tau_d)$ and $\tau_x : rhs_x(\tau_x) = lhs(\tau_x)$.

To show that each of these three cases can actually occur, suppose $f = 1$. Then, $\lim_{\tau\to 1} lhs(\tau) < 0$, $\lim_{\tau\to 1} rhs_d(\tau) = \lim_{\tau\to 1} rhs_x(\tau) = 0$. Since, as $\tau \to \infty$, $lhs(\tau)$ goes monotonically to $+\infty$ while both rhs functions go monotonically to $-\infty$, $lhs(\tau)$ will cross once $rhs_x(\tau)$ and then $rhs_d(\tau)$ from below. In this construction, moving τ from autarky $(+\infty)$ to perfect integration $(\tau = 1)$ will let the economies visit case *(ii)*, *(iii)* and finally *(i)*. ■

A.4 Trade Integration

Recall that, differentiating (4.14) for domestic sellers, and using $j(s) \equiv (s/s_d)^{(\alpha+\kappa)(\sigma-1)}$,

$$\begin{aligned} w_\tau(s) &= \theta(\psi-1)\, j(s) \frac{1}{s_d}\frac{\partial s_d}{\partial \tau} \\ w_{\tau s}(s) &= \theta(\psi-1)(1-\psi)\, s^{-1} j(s) \frac{1}{s_d}\frac{\partial s_d}{\partial \tau} \end{aligned}$$

so that the elasticity of wage to τ is

$$w_\tau(s)\frac{\tau}{w(s)} = -\frac{(1-\psi)\,\theta j(s)}{\theta(j(s)-1)+1}\frac{\tau}{s_d}\frac{\partial s_d}{\partial \tau} \tag{A.5}$$

The elasticity of the marginal price of skills to τ is

$$w_{\tau s}(s)\frac{\tau}{w_s(s)} = \frac{\theta(\psi-1)(1-\psi)\, s^{-1} j(s) \frac{\tau}{s_d}\frac{\partial s_d}{\partial \tau}}{(1-\psi)\,\theta s^{-1} j(s)} = -(1-\psi)\frac{\tau}{s_d}\frac{\partial s_d}{\partial \tau} \tag{A.6}$$

For exporters, the relevant functions are:

$$\begin{aligned} w_\tau(s) &= -\theta j(s)(\sigma-1)\left[(\alpha+\kappa)\frac{1}{s_d}\frac{\partial s_d}{\partial \tau}\left(1+\tau^{1-\sigma}\right)+\tau^{-\sigma}\right] \\ w_{\tau s}(s) &= -\theta(1-\psi)\, s^{-1} j(s)(\sigma-1)\left[(\alpha+\kappa)\frac{1}{s_d}\frac{\partial s_d}{\partial \tau}\left(1+\tau^{1-\sigma}\right)+\tau^{-\sigma}\right] \end{aligned}$$

so that the elasticity of wage to τ is

$$w_{\tau}(s)\frac{\tau}{w(s)}=\frac{-\theta j(s)(\sigma-1)\left[(\alpha+\kappa)\frac{\tau}{s_d}\frac{\partial s_d}{\partial\tau}\left(1+\tau^{1-\sigma}\right)+\tau^{1-\sigma}\right]}{\theta\left[j(s)\left(1+\tau^{1-\sigma}\right)-(1+f)\right]+1} \tag{A.7}$$

and the elasticity of the marginal price of skills to τ is

$$w_{\tau s}(s)\frac{\tau}{w_s(s)}=\frac{-(\sigma-1)\left[(\alpha+\kappa)\frac{\tau}{s_d}\frac{\partial s_d}{\partial\tau}\left(1+\tau^{1-\sigma}\right)+\tau^{1-\sigma}\right]}{\left(1+\tau^{1-\sigma}\right)} \tag{A.8}$$

Also, note that the square bracket is positive, since, differentiating (4.11) and using $\delta\equiv\tau^{1/(\alpha+\kappa)}f^{\psi/(1-\psi)}$

$$\begin{aligned}&(\alpha+\kappa)\frac{\tau}{s_d}\frac{\partial s_d}{\partial\tau}\left(1+\tau^{1-\sigma}\right)+\tau^{1-\sigma}=-\frac{1}{1+\delta}\left(1+\tau^{1-\sigma}\right)+\tau^{1-\sigma}>0\Leftrightarrow\\&(1+\delta)\tau^{1-\sigma}>1+\tau^{1-\sigma}\Leftrightarrow\left(\tau^{\sigma-1}f\right)^{\psi/(1-\psi)}>1\end{aligned}$$

which is always true.

Proposition 3 *There exists a unique skill level $s^{(\tau)}=s_x$ such that the local change in inequality is positive for high abilities and negative for low abilities, i.e., $\eta^{(\tau)}(s)\leq 0\Leftrightarrow s\geq s^{(\tau)}$.*

Proof. Local inequality increases with trade integration if and only if $\eta^{(\tau)}(s)<0$.

For domestic sellers, using (A.5) and (A.6), this will happen if and only if

$$\begin{aligned}w_{\tau s}(s)\frac{\tau}{w_s(s)}&<w_{\tau}(s)\frac{\tau}{w(s)}\Leftrightarrow-(1-\psi)\frac{\tau}{s_d}\frac{\partial s_d}{\partial\tau}<\frac{\theta(\psi-1)j(s)}{\theta(j(s)-1)+1}\frac{\tau}{s_d}\frac{\partial s_d}{\partial\tau}\Leftrightarrow\\1&<\frac{\theta j(s)}{\theta(j(s)-1)+1}\Leftrightarrow 1-\theta<0\end{aligned}$$

which is never true. Hence, local inequality always decreases for domestic sellers.

For exporters, using (A.7) and (A.8) local inequality increases if

$$\begin{aligned}w_{\tau s}(s)\frac{\tau}{w_s(s)}&<w_{\tau}(s)\frac{\tau}{w(s)}\Leftrightarrow\\\frac{-(\sigma-1)\left[(\alpha+\kappa)\frac{1}{s_d}\frac{\partial s_d}{\partial\tau}\left(1+\tau^{1-\sigma}\right)+\tau^{-\sigma}\right]}{\left(1+\tau^{1-\sigma}\right)}\tau&<\frac{-\theta j(s)(\sigma-1)\left[(\alpha+\kappa)\frac{1}{s_d}\frac{\partial s_d}{\partial\tau}\left(1+\tau^{1-\sigma}\right)+\tau^{-\sigma}\right]\tau}{\theta\left[j(s)\left(1+\tau^{1-\sigma}\right)-(1+f)\right]+1}\end{aligned}$$

Since the term in the square bracket is always positive, we can simplify further to obtain

$$\begin{aligned}&w_{\tau s}(s)\frac{\tau}{w_s(s)}<w_{\tau}(s)\frac{\tau}{w(s)}\Leftrightarrow\frac{1}{\left(1+\tau^{1-\sigma}\right)}>\frac{\theta j(s)}{\theta\left[j(s)\left(1+\tau^{1-\sigma}\right)-(1+f)\right]+1}\\\Leftrightarrow\ &1-\theta(1+f)>0\Leftrightarrow f<\frac{\kappa}{\alpha}\end{aligned}$$

Hence, local inequality increases for exporters as trade barriers fall if and only if $f<\kappa/\alpha$. ■

References

[1] **Autor, David H., Lawrence F. Katz and Alan B. Krueger. 1998.** "Computing Inequality: Have Computers Changed the Labor Market?", *The Quarterly Journal of Economics*, Vol. 113(4): 1169-1214.

[2] **Autor, David H., Lawrence F. Katz and Melissa S. Kearney. 2005.** "Rising Wage Inequality: The Role of Composition and Prices", *National Bureau of Economic Research*, Working Paper 11628, September 2005.

[3] **Autor, David H., Lawrence F. Katz and Melissa S. Kearney. 2006.** "The Polarization of the U.S. Labor Market", *American Economic Review*, Vol. 96(2): 189-194.

[4] **Autor, David H., Frank Levy and Richard J. Murnane. 2003.** "The Skill Content of Recent Technological Change: an Empirical Investigation", *The Quarterly Journal of Economics*, Vol. 118(4): 1279-1233.

[5] **Baldwin, Richard and Philippe Martin. 1999**. "Two Waves of Globalisation: Superficial Similarities, Fundamental Differences", in *Globalisation and Labour*, ed. H.Siebert, Chapter 1, pp 3-59. Tubingen: J.C.B. Mohr for Kiel Institute of World Economics.

[6] **Berman, Eli, John Bound and Zvi Griliches. 1994.** "Changes in the Demand for Skilled Labor within U.S. Manufacturing: Evidence from the Annual Survey of Manufactures", *The Quarterly Journal of Economics*, Vol. 109(2): 237-397.

[7] **Bernard, Andrew B., and Bradford J. Jensen. 1995.** "Exporters, Jobs and Wages in US Manufacturing: 1976-1987", *Brookings Papers on Economic Activity: Microeconomics*, Vol. 1995:67-119.

[8] **Bernard, Andrew B., and Bradford J. Jensen. 1997.** "Exporters, skill upgrading, and the wage gap", *Journal of International Economics*, Vol. 42(1-2): 3-31.

[9] **Bernard, Andrew B., and Bradford J. Jensen. 1999.** "Exceptional Exporter Performance: Cause, Effect or Both?", *Journal of International Economics*, Vol. 47(1): pp. 1-25.

[10] **Card, David and John E. DiNardo. 2002.** "Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles", *Journal of Labor Economics*, Vol. 20(4): 733-783.

[11] **Chaney, Thomas. 2008.** "Distorted Gravity: the Intensive and Extensive Margins of International Trade", *American Economic Review*, Vol. 98(4): 1707–1721.

[12] **Costinot, Arnaud. Forthcoming.** "An Elementary Theory of Comparative Advantage", *Econometrica*.

[13] **Costinot, Arnaud and Jonathan Vogel. 2009.** "Matching and Inequality in the World Economy", *National Bureau of Economic Research*, Working Paper 14672.

[14] **Eaton, Jonathan and Samuel Kortum. 2002.** "Technology, Geography, and Trade", *Econometrica*, Vol. 70(5): 1741-1779.

[15] **Epifani, Paolo and Gino Gancia. 2008.** "The Skill Bias of World Trade", *The Economic Journal*, Vol. 118: 927–960.

[16] **Feenstra, Robert C. and Gordon H. Hanson. 1996.** "Globalization, Outsourcing, and Wage Inequality", *The American Economic Review*, Vol. 86(2): 240-245.

[17] **Feenstra, Robert C. and Gordon H. Hanson. 1999.** "The Impact of Outsourcing and High-Technology Capital on Wages: Estimates For The United States, 1979–1990", *The Quarterly Journal of Economics*, Vol. 114(3): 907-940.

[18] **Juhn, Chinhui, Kevin M. Murphy and Brooks Pierce. 1993.** "Wage Inequality and the Rise in Returns to Skill", *Journal of Political Economy*, Vol. 101(3): 410-442.

[19] **Krugman, Paul. 1979**: "Increasing returns, monopolistic competition, and international trade", *Journal of International Economics*, Vol. 9(4): 469-479.

[20] **Krugman, Paul. 2008**: "Trade and Wages, Reconsidered", *Brooking Papers on Economic Activity*, Conference Draft.

[21] **Helpman, Elhanan. 1999**: "The Structure of Foreign Trade", *Journal of Economic Perspectives*, Vol. 13(2): 121-144.

[22] **Helpman, Elhanan, Oleg Itskhoki and Stephen Redding. 2008.** "Inequality and Unemployment in a Global Economy", http://www.economics.harvard.edu/faculty/helpman/files/Inequality_and_Unemployment.pdf

[23] **Helpman, Elhanan, Marc J. Melitz and Stephen R. Yeaple. 2004.** "Export versus FDI with Heterogeneous Firms", *The American Economic Review*, Vol. 94(1): 300-316.

[24] **Lucas, Robert E. 1978**: "On the Size Distribution of Business Firms.", *Bell Journal of Economics,* Vol. 9(2): 508-523.

[25] **Manasse, Paolo and Alessandro Turrini. 2001.** "Trade, wages and 'Superstars'", *Journal of International Economics*, Vol. 54(1): 97-117.

[26] **Melitz, Marc J. 2003.** "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity", *Econometrica*, Vol. 71(6): 1965-1726.

[27] **Oi, Walter Y. and Todd I. Idson. 1999.** "Firm Size and Wages", *Handbook of Labor Economics*, Edited by O. Ashenfeher and D. Card, Vol. 3, Elsevier Science B. V.

[28] **Piketty, Thomas and Emmanuel Saez. 2003.** "Income Inequality in the United States, 1913-1998", *The Quarterly Journal of Economics*, Vol. 118(1): 1-39.

[29] **Piketty, Thomas and Emmanuel Saez. 2004**: "Income Inequality in the United States, 1913-2002", Working Paper, November 2004.

[30] **Rosen, Sherwin. 1981.** "The Economics of Superstars", *The American Economic Review*, Vol. 71(5): 845-858.

[31] **Sattinger, Michael J. 1979.** "Differential Rents and the Distribution of Earnings", *Oxford Economic Papers*, New Series, Vol. 31(1): 60-71.

[32] **Sattinger, Michael J. 1993.** "Assignment Models of the Distribution of Earnings", *Journal of Economic Literature*, Vol. 31(2): 831-880.

[33] **Verhoogen, Eric. 2008.** "Trade, Quality Upgrading and Wage Inequality in the Mexican Manufacturing Sector." *Quarterly Journal of Economics*, Vol. 123(2): 489-530.

[34] **Yeaple, Stephen R. 2005.** "A Simple Model of Firm Heterogeneity, Trade and Wages", *Journal of International Economics*, Vol. 65(1): 1-20.