The peak of oil extraction and a modified maximin principle

Bazhanov, Andrei

Far Eastern National University

3 December 2006

The Peak of Oil Extraction and a Modified Maximin Principle*

Andrei V. Bazhanov†‡§

February 12, 2007

*This is revised version (numerical examples) of the paper prepared for the 21st-Century COE-GLOPE International Conference, Tokyo, December 22-23, 2006

†Institute of Mathematics and Computer Science, Far Eastern National University, Vladivostok, Russia or Department of Economics, Queen's University, Kingston, Canada, e-mail: bazhanov@econ.queensu.ca

‡Thanks to the Good Family Visiting Faculty Research Fellowship for financial support.

§The author is grateful to John M. Hartwick for very useful comments and advice.

Abstract The term "oil peak" usually is connected with the positive analysis problem, namely, with the problem of defining the year when the increase in the rate of oil extraction will be physically impossible. However, a normative approach to the problem of optimal extraction of a nonrenewable resource seems more important. We consider the economy which depends on the essential nonrenewable resource and the rate of the resource extraction increases over time. At some instant the government gradually switches to a sustainable (in sense of non-decreasing consumption over time) pattern of the resource extraction. Different criteria are considered for the construction some curves of switching to decreasing paths of the resource depletion. Consumption paths have diverse behavior patterns along these curves, including a path of unlimited growth. A new approach to the Rawlsian maximin criterion which allows for growth of consumption is offered.

Keywords Nonrenewable resource · Intergenerational justice · Generalized Rawlsian criterion

JEL Classification Numbers Q32 · Q38

1 Introduction

The anxiety about limiting world resources has persisted since the famous work of Thomas Malthus, published in 1798. Many observers are trying to estimate the time of the peak in world's production of the nonrenewable resources (see e.g., works of D. Meadows et al. and theories based on the Hubbert's peak of oil production). Economic decline is assumed to follow. The report of Cambridge Energy Research Associates (CERA)[1] (November, 2006) contains a very optimistic evaluation of the world oil reserves (3.74 trillion barrels) in comparison with the estimates of "peak oil" theorist (1.2 trillion: Oil & Gas J., 2005, 103, 47: p. 25). CERA's scenario of oil extraction is also very encouraging, since it promises that the rates of extraction will grow for at least another 24 years before entering the "undulating plateau" followed by decline. In contrast, the "peak oil" forecasts predict that the world oil production has already peaked or will have a peak in the next 5-10 years.

But when we worry about the peak and the associated scarcity of the resource we think mostly about the influence of this impending shortage on the output of our economy and on our consumption and that of our descendents. And it is not obvious that we must adjust our demand for the nonrenewable resource strictly in accord with the "physical" peak of extraction. In other words a "sustainable"[2] peak may not coincide with the physical one.

For the modeling various scenarios of the world oil extraction we will use the transition paths, developed in (Bazhanov, 2006) which have been constructed for an economy with the growing rates of extraction with a switch to a hypothetical sustainable path. We assume that a rapid decrease in oil extraction can be extremely costly in terms of consumption foregone and

[1] See http://www.cera.com/aspx/cda/public1/home/home.aspx

[2] We will consider the simplest sustainability criterion meaning nondecreasing consumption over time.

this leads us to consider various more gradual transitions. This in turn leads us to reflect on welfare criteria for the case of significant changes in consumption levels across generations.

We consider the Hartwick saving rule (Hartwick, 1977) for the Solow (1974) model which implies that the economy must involve investing current exhaustible resource returns in reproducible capital in order to maintain constant per capita consumption over time. We review this for the case of a Cobb-Douglas technology. For simplicity we consider the case with zero population growth and so all the paths of our economy such as output $q(t)$, consumption $c(t)$, capital $k(t)$ and so on are defined below in per capita units. For the case with no capital depreciation, no technological progress, and zero extraction cost, we have output $q = f(k, r) = k^{\alpha} r^{\beta}$ where k is produced capital, r - current resource use, $r = -\dot{s}$, s - per capita resource stock ($\dot{s} = ds/dt$), α, $\beta \in (0, 1)$ are constants. Prices of capital and the resource are $f_k = \alpha q/k$, $f_r = \beta q/r$ where $f_x = \partial f/\partial x$. Per capita consumption is $c = q - \dot{k}$. The Hartwick savings rule implies $c = q - r f_r$ or, substituting for f_r, $c = q(1 - \beta)$, which means that instead of $\dot{c} = 0$ we can check $\dot{q} = 0$.

From Hotelling rule $\dot{f}_r/f_r = f_k$ we have $\alpha\beta q/k + \dot{r}(\beta - 1)/r = f_k = \alpha q/k$ which yields

$$\dot{r}/r = -\alpha q/k. \tag{1}$$

Then

$$\dot{q}/q = \alpha\dot{k}/k + \beta\dot{r}/r = \beta(\alpha q/k + \dot{r}/r) = 0, \tag{2}$$

which means that we really have $\dot{q} = \dot{c} = 0$ or $q = const$. Then $r f_r = \beta q = const$ and we have $\dot{k} = \beta q = const$ for deriving $k(t)$ and (1) for deriving $r(t)$. We can find two constants of integration k_0 for $k(t) = k_0 + \beta q t$ and the constant of equation $\dot{r}/r = -1/\left(k_0/\alpha q + \beta t/\alpha\right)$ using initial conditions $r(0) = r_0$ and $s(0) = s_0$, where s_0 is the given resource stock which must be used for production over infinite time: $s_0 = \int_0^\infty r(t)dt$. Then we have

$$r(t) = r_0 \left[1 + r_0\beta t/s_0(\alpha - \beta)\right]^{-\alpha/\beta}, \qquad (3)$$

where $\alpha > \beta$ (Solow condition) and

$$\dot{r}(t) = -\ddot{s}(t) = -\alpha r_0^2/s_0(\alpha-\beta)\left[1 + r_0\beta t/s_0(\alpha - \beta)\right]^{-(\alpha+\beta)/\beta}. \qquad (4)$$

Since we assume that our economy depends on the resource essentially, we obtain path $r(t)$, asymptotically approaching zero and the path of extraction changes $\dot{r}(t)$ (or negative acceleration of stock $s(t)$ diminishing) also approaching zero, but starting from the negative value $\dot{r}_0 = -\alpha r_0^2/[s_0(\alpha-\beta)]$. Note, that path (3), asymptotically approaching zero, is necessary, but not sufficient condition of following Hartwick rule for Cobb-Douglas economy under the Hotelling rule assumption. By definition of $f(k,r)$ it can be seen, that if economy is extracting resource in accord with (3) and resource rent is consuming (total investments are less than resource rent), then $q(t)$ and $c(t)$ are asymptotically approaching zero, but from a greater starting value $c(0)$. Assuming that our economy has some "additional" savings, besides resource rent, it is possible to relax the assumption of zero population growth (as in (Stiglitz, 1974) and (Asheim et al., 2005), or zero capital depreciation. But in any case, if we assume, that

1) economy at every instant of time depends on resource (even if we gradually introduce substituting technologies and this dependence asymptotically approaches zero), and

2) we really want to maintain nondecreasing per capita consumption,

then rate of extraction $r(t)$ must tend to zero.

Capital - resource substitution is a fundamental topic in energy economics and there is an empirical evidence (Nordhaus, 1972, Pindyck, 1979) which can support the assumption that the elasticity of substitution between natural resources and capital exceeds unity. This implies that resource can be inessential. Other investigations (Fuss, 1977, Magnus, 1979, and partly in Halvorsen and Ford, 1979) show that energy and capital are rather strong complements than substitutes (elasticity is less than unity) and some researches find that this value is rather close to unity (Griffin and Gregory, 1976, Pindyck, 1979). In any case empirical evidence is not a proof and as Dasgupta and Heal (1979, p. 207) noted "Past evidence may not be a good guide for judging substitution possibilities for large values of k/r". And so, we can assume that for the world economy oil is essential, especially taking into account that no adequate immediate substitutes are available for transportation fuels, a main area of oil use (Heinberg, 2003, Nemoto, 2005). However, as we can see, e.g., from oil extraction data in December issues of Oil and Gas Journal, rates of extraction are in fact both growing on the world level (see Fig. 1 before the year 2005) and for the leading oil producers, not declining. Per capita world oil extraction (Fig. 2) is also not declining though after the period of growth it is following an undulating plateau since the oil crisis of 1979-1980.[3]

Assume that the government after a period of oil-rent consumption and growing rate of extraction decided to conform to the intergenerational justice principle and switch at t_0 to some sustainable path of saving, e.g., to the Hartwick rule.[4] An example with $\alpha = 0.3$ and $\beta = 0.05$ gives us $r(t)$ and $\dot{r}(t)$ for world oil extraction in Fig. 1 and Fig. 3 after the year 2005 (dotted lines).

An abrupt switch to the Hartwick rule means that people in oil-producing countries must instantly forget about this principal source of income and in a moment substantially re-structure

[3] We took the world population in 2006 equal to unity and as a source of information for the world population dynamics we used http://www.census.gov/ipc/www/worldpop.html .

[4] "Strong policy action is needed to move the world onto a more sustainable energy path." [The World Energy Outlook 2006 Maps Out a Cleaner, Cleverer and More Competitive Energy Future, IEA Press Release, http://www.iea.org/Textbase/press/pressdetail.asp?PRESS_REL_ID=187]

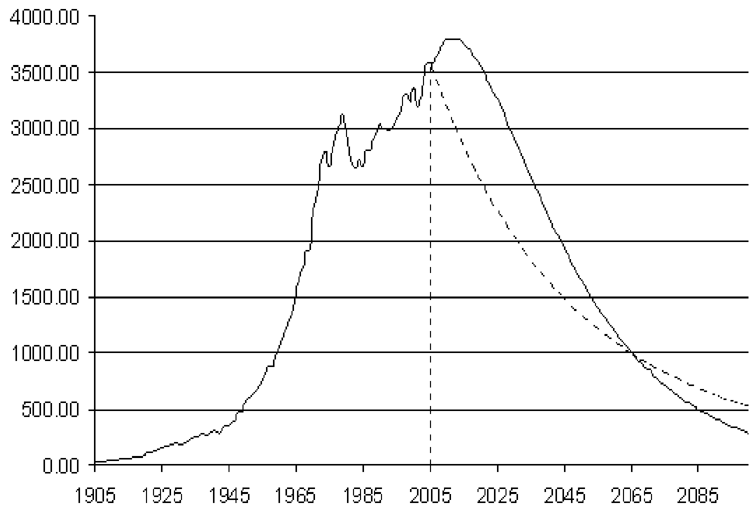

Figure 1: World oil extraction: historical data (before 2005); Hartwick curve (dotted); LA curve (solid)

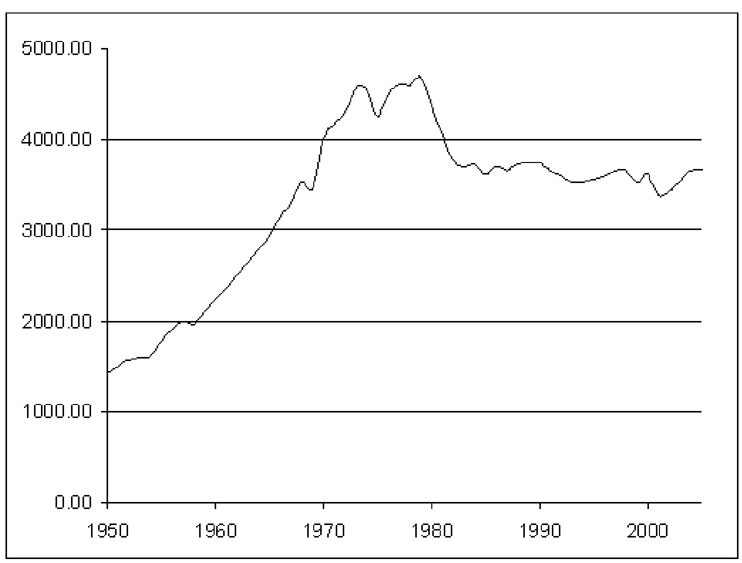

Figure 2: Per capita world oil extraction (historical data).

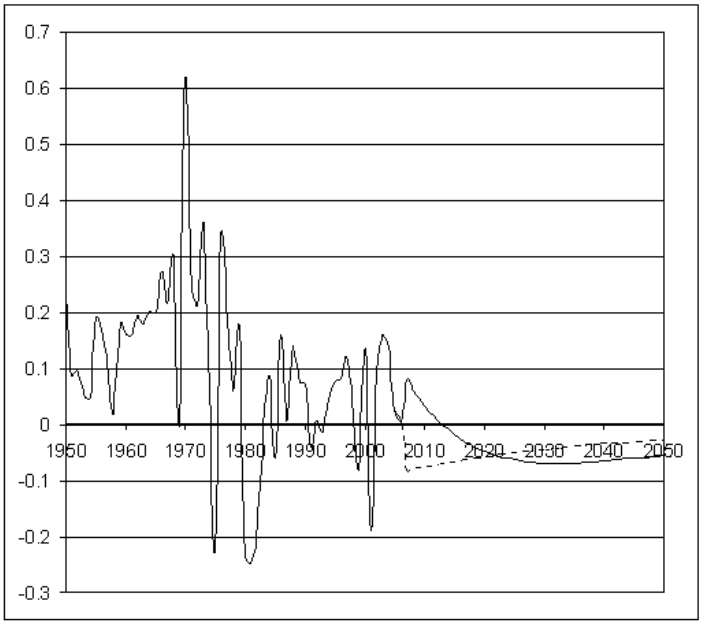

Figure 3: Per capita extraction accelerations: historical data (before 2005); Hartwick curve (dotted); LA curve (solid)

their living style. Moreover, countries must instantly reorganize their economies, because of the sharp decrease of consumption, which in turn leads to a decrease in production, a possible increase in unemployment, a further decrease of demand and so on. Thus, for an economy not following the Hartwick rule, the sudden invocation of intergenerational justice creates the dilemma of choosing between two awkward futures: diminishing consumption to zero in the future because of the inevitable shortage of essential exhaustible resources or diminishing consumption to a sustainable level right from the moment of switching to the Hartwick rule.

Solow's model implies that oil-rent is invested from the very beginning and that there is no time gap between the moment of oil extraction and correspondent increase of reproducible capital according to the Hartwick rule. We can consider it as an adequate model if we assume that reproducible capital is a fund of some high-return securities and oil profit can be instantly invested in some shares or bonds. But suppose that money bills are not able to substitute gasoline in engines of our cars when we have shortage of oil. And the shortage will be the inevitable result of growing demand because of economic growth and decreasing, according to (3), supply of oil. It means, that in order to sustain non-decreasing output with the same structure, we must invest at least part of oil profit into development of oil-substituting technologies. In other words, we must create an "anti-oil market" with the oil rent. And under this assumption the model of instant investment can not be really adequate because of the difficulties of a rapid re-structuring. Historical examples show that the development and the introduction of coal-based technologies took decades despite the obvious benefit of the new technologies for economy. The same can be said about the switch from a coal to an oil economy. Now we must consider the problem of switching to technologies, based on renewable resources not because they are economically more preferable but just because of anticipated shortage of profitable but exhaustible raw materials. And this process will occur over decades, not months.

The second dimension of the impossibility of an instant switch to the Hartwick rule is the awkward requirement of an abrupt and very substantial change of saving patterns for oil producing countries. As an illustration we can compare nonrenewable resource profit only from oil with the total amount of investments for a selection of countries. For example, oil gives Kuwait about 50% of GDP but gross fixed investments are only 6.6% of GDP. For Saudi Arabia these numbers are 45% and 16.3%, United Arab Emirates - 30% and 20.7%, Venezuela - 33% and 23.8%.[5] A very detailed analysis of the investment and consumption patterns in the world is in (Arrow et al., 2004). From leaders of oil producers only Norway can boast

[5] Source of information:
http://www.cia.gov/cia/publications/factbook/docs/profileguide.html (March 2006)

almost coinciding numbers (about 18.6%[6]), because of investing oil rent to Petroleum Fund, though there is no direct connection between this Fund and development of oil-substituting technologies.

However, the well-known empirical research of Kuznets (1946) tells us that consumer behavior is very persistent over time despite changes of governments and government policy. Subsequent analyses, for example, the work of Duesenberry (1949), tried to explain this phenomenon, and later papers examined why consumers do not react on "natural experiments" such as the Reagan cuts in taxes (Poterba, 1988). In any case, there is evidence that at least in the short run saving rate is very stable, and it is much more difficult to change it instantly, than to change a government policy toward maximin.

Hence, the problem of switching to sustainable path of essential resource extraction must take into account the next factors:

1) the path must have a period of a gradual slow-down in the rate of extraction;

2) there is a time lag between the moment of resource rent investment and correspondent increase in capital;

3) there is a non-zero period length for changing saving patterns from resource rent consumption to resource rent investment.

In this paper we suppose, for simplicity, that the third problem is already solved (as in Norway), and also we will temporarily neglect the influence of the second factor. So, we will concentrate on the question of the construction the trajectories for the transition period using various optimality criteria and examine consumption behavior along the paths.

[6]Source of information: http://www.ssb.no/en/indicators/ (March 2006)

2 Rational Transition Curves

The transition path can be found in the same class of rational functions as the Hartwick curve (3). The difference is in the numerator, which must depend on t with a negative coefficient to control "smooth breaking" in the neighborhood of $t = 0$. Namely, $A(t)$ must be in the form of

$$A(t, b, c, d) = \dot{r}(t) = (A_0 + bt)/(1 + ct)^d, \tag{5}$$

where $b < 0, c > 0, d > 1$ (for convergence $A(t) \to -0$ with $t \to \infty$). Corresponding to (5) $r(t)$ has a dependence on $b, c,$ and d in

$$r(t) = \{-\left[A_0 + b/[c(d-2)]\right]/[c(d-1)] + bt/[c(2-d)]\}/(1+ct)^{d-1}.$$

Note, that a constant of integration for $\dot{r}(t) = A(t)$ must be zero for the convergence of $\int_0^\infty r(t)dt$, and also for the convergence, d actually must be greater than 3. Then we have $r_0 = -\left[A_0 + b/[c(d-2)]\right]/[c(d-1)]$, which can be used to express $b:$

$$b = -c(d-2)\left[r_0 c(d-1) + A_0\right], \tag{6}$$

and then the transition curve has a dependence on c and d in

$$r(t) = r_0 \{1 + [c(d-1) + A_0/r_0]\, t\}/(1+ct)^{d-1}. \tag{7}$$

Coefficient c can be expressed from the condition that resource is finite $s_0 = \int_0^\infty r(t)dt:$

$$\begin{aligned} s_0/r_0 &= \int_0^\infty (1+ct)^{1-d}dt + [c(d-1) + A_0/r_0] \int_0^\infty t/(1+ct)^{d-1}dt \\ &= [1 + \{r_0 c(d-1) + A_0\}/\{r_0 c(d-3)\}]/[c(d-2)], \end{aligned}$$

which means that c is a solution of quadratic equation

$$c^2 s_0/r_0 - 2c/(d-3) - A_0/[r_0(d-3)(d-2)] = 0.$$

The only relevant root (because we are looking for $c > 0$) is

$$c(d) = \left[r_0/(d-3) + \left\{ r_0^2/(d-3)^2 + s_0 A_0/[(d-3)(d-2)] \right\}^{0.5} \right] /s_0.$$

Hence we have a single independent parameter d which defines the shape of the curve (including its peak) and we can use this parameter as a control variable in some optimization problem

$$F\left[r(t,d)\right] \to \max_d$$

which can be connected with the short- or long-run policy in output or consumption behavior.

A numerical example based on data for recent world oil extraction[7] (we set $A_0 = 0.08$) gives $d = 36.8837$ as a solution of the problem of minimization of the short-run negative shock on output because of the resource shortage (Bazhanov, 2006), namely,

$$\begin{aligned} F\left[r(t,d)\right] &= \min_t \dot{r}(t,d) \to \max_d, \\ s.t. \quad r(0) &= r_0, \\ \int_0^\infty r(t)dt &= s_0. \end{aligned} \tag{8}$$

We call such a path $r(t,d^*)$ the "Least Acceleration" (LA) curve. This value of $d = 36.8837$ implies c =0.001459 and b =-0.0136. Plots of $r(t)$ and $A(t)$ are on Fig. 1 and Fig. 3 after the year 2005 (solid lines). Maximum negative acceleration along the LA curve is A^*_{LA} =-0.06959 at t^* =25.136, which is less in absolute value than maximum negative acceleration along the Hartwick curve A^*_H =-0.08350 right from the very start at $t^* = 0$. Note, that $d \to \infty$ with $A_0 \to +0$ and we failed to find the numerical solution of (8) for $A_0 \leq 0.06$.

[7] Since we assumed that current world population is equal to unity, we used as per capita world oil reserves and extraction on January 1, 2006 (Oil & Gas J., 2005, 103, 47: p.25.): $r_0 = 71,793.8$ [1,000 bbl/day] $\times 365 = 26,204,737$ [1,000 bbl/year] (or 3.58969 bln t/year); $s_0 = 1,292,549,534$ [1,000 bbl] (or 177.06 bln t). We use coefficient 1 ton of crude oil = 7.3 barrel.

3 Consumption Along Transition Curves

We are going to examine, for simplicity, the case of saving pattern when all the resource rent is always invested in capital (zero net investments) and there are no time lags between the moments of investment and the corresponding capital increase. The only reason for government to change the pattern of extraction is that sustainable (in sense of constant consumption) path of the essential resource extraction must be decreasing and asymptotically approaching zero.

Note, that constant per capita consumption over time in this case is the result of

1) total investment of oil rent in capital (with no time lag) and

2) fulfillment of the Hotelling rule.

In this paper we are going to analyze the case when some reasons cause the deviation from an efficient path of extraction and we must find the optimal path across inefficient curves. We set down these assumptions below in the definitions 1 - 4, and the Propositions 1 and 2.

Definition 1 An *intertemporal program* $\langle f(t), c(t), k(t), r(t)\rangle_{t=0}^{\infty}$ is a set of paths $f(t)$, $c(t)$, $k(t)$, $r(t)$, $t \geq 0$ such that $f(t) = f[k(t), r(t)]$ and $c(t) = f(t) - \dot{k}(t)$.

Definition 2 For positive initial stock of capital and resource $(k_0, s_0) \gg 0$ the set of the programs $F = \{\langle f(t), c(t), k(t), r(t)\rangle_{t=0}^{\infty}\}$ is *a feasible sheaf* at $t = 0$ and each of the paths $f(t)$, $c(t)$, $k(t)$, $r(t)$ is *a feasible path* if any program $\langle f(t), c(t), k(t), r(t)\rangle_{t=0}^{\infty}$ from F for all $t \geq 0$ satisfies the conditions:

1) $(f(t), c(t), k(t), r(t)) \gg 0$;

2) $r(t), k(t), c(t)$ are continuously differentiable and $\sup_t |\dot{r}(t)| \leq \dot{r}_{\max} < \infty$;

3) $f(t)$ is twice continuously differentiable;

4) $\int_t^{\infty} r(t)dt \leq s(t)$;

5) $k(0) = k_0, c(0) = c_0, r(0) = r_0, \dot{r}(0) = A_0 \leq \dot{r}_{\max}$.

Definition 1 is based on the definition of the interior feasible path in (Asheim et al., 2005). The differences reflect our assumptions: a) population is constant; b) the speed of change of the extraction rate $\dot{r}$ is limited and continuous for all t including $t = 0$. Henceforth, a "program" and a "path" will refer to a feasible program and a feasible path.

Definition 3 (Dasgupta, 1979, p. 214) A feasible program $\langle f(t), c(t), k(t), r(t)\rangle_{t=0}^{\infty}$ from F is *intertemporally inefficient* if there exists a program $\langle \overline{f}(t), \overline{c}(t), \overline{k}(t), \overline{r}(t)\rangle_{t=0}^{\infty}$ from F such that $\overline{c}(t) \geq c(t)$ for all $t \geq 0$ and $\overline{c}(t) > c(t)$ for some t.

Definition 4 (Dasgupta, 1979, p. 214) A set of feasible programs $E = \{\langle f(t), c(t), k(t), r(t)\rangle_{t=0}^{\infty}\}$ is a *set of efficient programs* if all the programs $\langle f(t), c(t), k(t), r(t)\rangle_{t=0}^{\infty}$ from E are not inefficient.

Proposition 1 *If $\dot{f}_r(0)/f_r(0) \neq f_k(0)$ then $F \cap E = \emptyset$ or all the feasible paths are inefficient.*

Proof. Since $f(t)$ is twice continuously differentiable at $t = 0$, then there exists $\varepsilon > 0$ such that for any $t \in [0, \varepsilon)$ and for any feasible program $\langle f(t), c(t), k(t), r(t)\rangle_{t=0}^{\infty} \in F$ the Hotelling rule is not satisfied: $\dot{f}_r(t)/f_r(t) \neq f_k(t)$. Necessity of the Hotelling rule for the efficiency of a program (see, e.g., Asheim et al., Dasgupta, 1979) follows the assertion of the Proposition.

Now we will show that in our assumptions (zero extraction cost) all the growing paths of extraction are inefficient.

Proposition 2 *For an economy with technology* $q = k^\alpha r^\beta$ *where* $\alpha, \beta \in (0,1)$; $k(t), r(t) > 0$ *and* $\dot{k}(t) < q(t)$ *for all* t, *the path of extraction is inefficient if there is* $\bar{t} \geq 0$ *such that* $\dot{r}(\bar{t}) > 0$.

Proof. Since the Hotelling rule is a necessary condition for efficiency, it is enough to show that it does not hold for the growing rate of extraction. Indeed, we can write the Hotelling rule $\dot{f}_r(t)/f_r(t) = f_k(t)$ as $\dot{f}_r/f_r = r\beta \left[\alpha\dot{k}/kr + \beta q\dot{r}/r^2\right]/(\beta q) - \dot{r}/r = \alpha\dot{k}/k - (1-\beta)\dot{r}/r = \alpha q/k$ (since $f_k = \alpha q/k$). Then we have $\alpha\dot{k}/k + (\beta-1)\dot{r}/r = \alpha q/k$ or $(\beta-1)\dot{r}/r = \left(q - \dot{k}\right)\alpha/k$. The right hand side of the last equation is always positive and the left hand side can be positive only for $\dot{r} < 0$ for any $t \geq 0$ (since $(\beta - 1) < 0$ and $r > 0$).

So, the transition path (7) is not efficient (extraction grows in a neighborhood of $t = 0$) unlike the Hartwick curve (3) which is derived from the Hotelling rule and so satisfies it identically. Hence, to examine the consumption behavior in our case along some path we should check the fulfillment of the Hotelling rule along this curve. In common case $\dot{q} = f_k\dot{k} + f_r\dot{r}$. Then $\dot{f}_r = \beta d\left(q/r\right)/dt = \beta\left[f_k\dot{k}/r + f_r\dot{r}/r\right] - \beta\dot{r}q/r^2$. Dividing on $f_r = \beta q/r$ we have $\dot{f}_r/f_r = r\beta\left[\alpha\dot{k}/kr + \beta q\dot{r}/r^2\right]/(\beta q) - \dot{r}/r = \alpha\dot{k}/k - (1-\beta)\dot{r}/r$. Since $f_k = \alpha q/k$ we have $\dot{f}_r/f_r = f_k\left[\dot{k}/q - (1-\beta)k\dot{r}/(\alpha q r)\right]$ and substitution for $\dot{k}$ the saving rule $\dot{k} = \beta q$ gives us

$$\dot{f}_r/f_r = f_k\left[\beta - (1-\beta)k\dot{r}/(\alpha q r)\right]. \tag{9}$$

Just to check, we can see, that for the Hartwick curve $[\cdot] \equiv 1$, because the Hotelling rule implies $\dot{r}/r = -\alpha q/k$.

Hence, if $[\cdot] < 1$, then $\dot{q} > 0$, because $\dot{f}_r/f_r < f_k$, which follows $-\dot{r}/r < \alpha q/k$ or $\alpha q/k + \dot{r}/r > 0$. And the latter, using expression in the left hand side of (2), means $\dot{q} > 0$. In the same way, $[\cdot] > 1$ follows $\dot{q} < 0$ and, in general, $\operatorname{sgn}\dot{q} = \operatorname{sgn}\{1 - [\cdot]\}$. So, to examine long-run consumption $c = (1-\beta)q$ along the LA curve, we can check asymptotic behavior of $[\cdot]$.

Proposition 3 *If an economy with technology $q = k^\alpha r^\beta$ is such that $\alpha, \beta \in (0,1)$; $\beta < \alpha$ and*

1) resource rent is completely invested in capital;

2) there is no time lag between the moment of investment and correspondent increase in capital;

3) rate of extraction $r(t)$ is such that

$$\dot{r}(t) = (A_0 + bt)/(1+ct)^d,\ b < 0,\ c > 0,\ d > 3,$$

then the output q asymptotic behavior for different β is:

$$\lim_{t\to\infty} \operatorname{sgn} \dot{q}(t) = \begin{cases} -1, & \beta(d-2) \geq 1, \\ \operatorname{sgn} L(d,\alpha,\beta), & \beta(d-2) < 1, \end{cases} \tag{10}$$

where

$$L(d,\alpha,\beta) = \frac{[\alpha - \beta(d-2)]}{[\alpha - \alpha\beta(d-2)]}.$$

Proof of the Proposition is in (Bazhanov, 2006, Appendix).[8]

Corollary 1. Under the assumption of the Proposition 3 the consumption $c(t)$ is

1) asymptotically decreasing if $d > \alpha/\beta + 2$;

2) asymptotically constant if $d = \alpha/\beta + 2$;

3) asymptotically growing if $3 < d < \alpha/\beta + 2$.

Proof. Note that for $\beta(d-2) < 1$ or $d < 1/\beta + 2$ denominator of $L(d,\alpha,\beta)$ is positive. Then the sign of $L(d,\alpha,\beta)$ is defined by nominator. Since $c = (1-\beta)q$ and $\operatorname{sgn} \dot{c} = \operatorname{sgn} \dot{q}$ then substituting the expressions for d into $L(d,\alpha,\beta)$ in (10) we obtain the assertion of the Corollary. In the case when $d \geq 1/\beta + 2$ or $\beta(d-2) \geq 1$ we define the sign of $\dot{c}$ by the first line in (10) which is included in the first case of the Corollary.

[8] The simplified expression for $L(d,\alpha,\beta)$ was obtained by direct substitution of expressions for b, c and ρ.

4 Numerical Examples for the Oil & Gas Journal's Reserve Estimates

The Hartwick saving rule which we use in our economy implies that the consumption path is $c = q - \dot{k} = (1-\beta)q = (1-\beta)k^{\alpha}r^{\beta}$ where $r(t)$ is a known transition curve and $k(t)$ is an unknown path of capital. We can calculate $k(t)$ from the equation for the saving rule $\dot{k} = \beta k^{\alpha} r^{\beta}$ assuming that we have estimation of k_0. From (2) we have $\dot{q}/q = \beta(\alpha q/k + \dot{r}/r)$ which implies the expression for k_0, given $r_0, \dot{r}_0$, and our output percent change $(\dot{q}/q)_0$:

$$k_0 = \left\{ \left[\left(\frac{\dot{q}}{q} \right)_0 \frac{1}{\beta} - \frac{\dot{r}_0}{r_0} \right] / \left(\alpha r_0^{\beta} \right) \right\}^{\frac{1}{\alpha-1}} .$$

For the Hartwick's curve (3), derived from the Hotelling rule, we know the expression for $\dot{r}_0$: $\dot{r}_0 = -\alpha r_0^2/[s_0(\alpha-\beta)]$. We know also that $(\dot{q}/q)_0$ must be equal to zero for this curve and so we can estimate k_{0Hart} for these values. Then we can make a seemingly plausible assumption that regardless the path of extraction which we are going to follow, the initial capital in our economy at the same starting point must be the same. We used this assumption for the numerical examples in the previous version of the paper (Bazhanov, 2007). And indeed, the long-run consumption behavior was in all the examples consistent with the Proposition and the Corollary. However, if we apply this assumption backward, namely, calculate k_0 for the transition curve, given positive values of $\dot{r}_0$ and $(\dot{q}/q)_0$, and then use this k_0 for the numerical construction of the consumption path along the Hartwick's curve, we will obtain that consumption is constant only asymptotically despite the fulfillment of the Hotelling rule and the Hartwick investment rule. This means that the physical amount of initial capital, which is indeed always the same, must have different values in different examples. This is because our approach for the estimation of k_0 is based on it's expression via different values of other variables in our problem. Then in order to make a correct comparison of consumption behavior in different examples we must calculate different values of k_0 and use the scale factor for all the capital path. Otherwise, using the same value for k_0 in different examples, we will obtain distortions in the short-run behavior and can come to some wrong conclusions. The significance of these distortions can be estimated by the comparison of the results presented below and the results in the previous version of the paper (Bazhanov, 2007).

For the example with $\alpha = 0.3, \beta = 0.05$ we have $\alpha/\beta + 2 = 8$ and the Corollary implies that sustainable in the sense of nondecreasing consumption are the paths with $3 < d < 8$. Given $r_0, \dot{r}_{0=}A_0$ for the world oil extraction, we have the short-run optimal (in the sense of problem (8)) value of $d^* = 36.8837$ (or $\beta(d-2) = 1.74 > 1$). This means, that consumption and output decrease in the long run along the LA curve. Using $(\dot{q}/q)_0 = 0.04$ which implies k_0 =0.2809628328 and $c_0 = 0.6919442652$, we obtained the consumption path shown on Fig. 4 (the numerical solutions for $k(t)$ here and below were obtained in Maple by the procedure $rkf45$). For $\alpha = 0.2, \beta = 0.05$ (estimates from Nordhaus and Tobin, 1972) we also have decreasing to zero consumption in finite time.

We can fit the parameter d to obtain the "oil peak" around $t = 5$ (the forecast of "oil peak" theorists). Then we have $d = 11$ and according to the Corollary it follows the consumption asymptotically decreasing to zero (as on Fig. 4) after the maximum $c_{\max} = 1.915$ at $t = 504$. Note, that we obtained qualitatively the same result as in (Bazhanov, 2007), but now we have the growth of consumption much longer and $c_{\max}$ relatively higher.

We can see from the Corollary that there are sets of α and β for which, given d^* we have $L(d^*, \alpha, \beta) = 0$ or consumption tends to a constant along the transition curve. For example, $L(d^*, 0.697, 0.02) = L(d^*, 0.872, 0.025) = 0$.

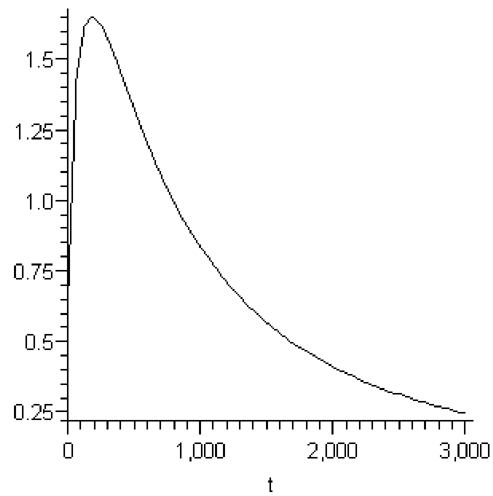

Figure 4: Consumption decrease along the LA curve

Selection of different values for (α, β) for which $\lim_{t\to\infty} \dot{c} = 0$ makes some sense if we wish to get a feeling of how far can some real extraction path be from the stable one, given that we don't know true values of α and β. In fact it is unrealistic to speak about short-term regulation of these magnitudes by the government's decisions. Our examples make the path of resource extraction look more controllable. We can try to fit the single free parameter d and recalculate $c(d)$ and $b(d)$ using some welfare criterion, e.g., constant consumption over time in the long run (asymptotically constant consumption) instead of the least negative output shock during the transition period. An example with $\alpha = 0.3$ and $\beta = 0.05$ gives us $d = 8.0$. In this case the maximum negative output shock takes place a little bit earlier ($t_{\max} = 19.6$) in comparison with $t_{\max} = 25.136$ for the LA curve; the value of the shock is larger ($A_{\max} = -0.0716$) in comparison with $A_{\max LA} = -0.06959$, but the shock is weaker than for the curve (3), for which

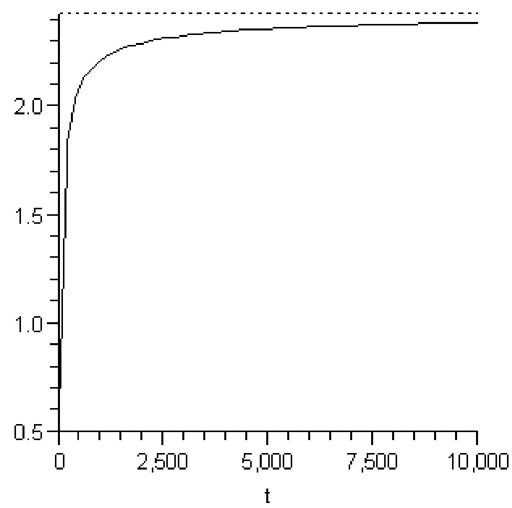

Figure 5: Consumption along the TCC curve

$A_{\max} = -0.08350$. The oil peak for this curve must be closer, namely, at $t = 4.27$.

To check that the level of consumption along this curve, which we will call the "Transition Constant Consumption" (TCC) curve, is far enough from zero, we can solve numerically for $k(t)$ and then plot $c(t)$ (Fig. 5). The value of constant consumption for the t, big enough, is around $c_{const} = 2.42801$.

Note that for this example we obtained the same long-run result as in (Bazhanov, 2007), namely, the convergence of consumption to almost the same constant. But now we have very important qualitative difference. Consumption is always growing while approaching the constant unlike the same case in (Bazhanov, 2007) where we had limited decline in consumption after a very short increase. And now this example already do not resemble the Asheim's counter example.[9]

[9] Asheim (1994) examines a theoretical example where the consumption decreases to a sustainable level

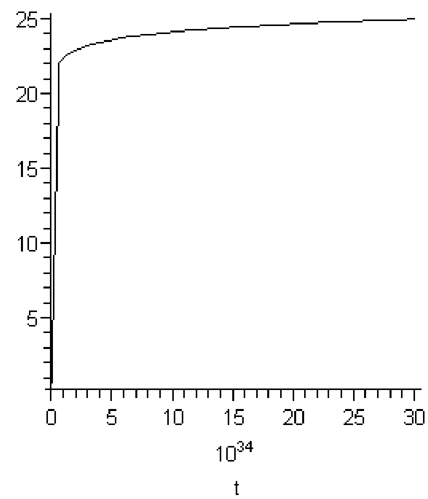

Figure 6: Unlimited growth.

Another interesting case is $L(d, \alpha, \beta) > 0$. What is the "cost" of infinitely growing consumption in this case? And can it be "optimal" in some sense or is it just a result of overinvestment? An example with $L(7.5671, 0.3, 0.05) = 0.1$ gives us unlimited growth of consumption (Fig. 6). The only "cost" of this growth is that the oil peak for this case must be even more closer, at $t = 4.13$. The important difference of this case from the same example in (Bazhanov, 2007) is that now we have no period of decline in consumption in the short run which in (Bazhanov, 2007) was just the result of incorrect choice of initial value for k_0.

$\overline{c} < c_0$ after a period of "over-consuming" with $c(t) > c_0$. He considers the consumption behavior as a result of changes in saving pattern with presumably efficient path of extraction. We, vice versa, examine the consumption behavior with the fixed saving rule but with the deviations of extractions from the efficient path.

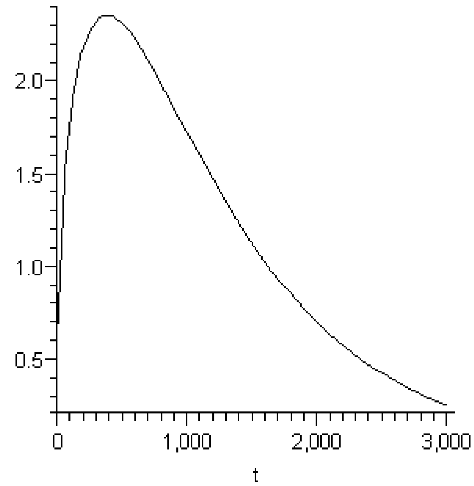

Figure 7: Consumption path for the CERA's scenario of oil extraction.

5 Numerical Examples for the CERA's Reserve Estimates

In order to construct the transition path of extraction with the peak at $t = 24$ as in CERA's scenario, we must take a rather large value of d. For $\alpha = 0.3$ and $\beta = 0.05$ we must take $d = 10^{10}$ which already means that this path (even without the undulating plateau) is unsustainable. Numerically it is expressed in the peak of consumption at $t = 375$ with $c_{\max} = 2.36$ (Fig. 7).

The next step of comparison involves constructing the path of extraction which is borderline between sustainable and unsustainable paths. As it was shown above, this path for $\alpha = 0.3$ and $\beta = 0.05$ has a value of $d = 8$. The only difference of this path (Fig. 8) from the one on Fig. 5 is that consumption approaches a higher asymptote with $\overline{c} = 3.6484$. The peak of oil extraction in this case must be at $t = 17.9$ ($r_{\max} = 4.187$ bln t/year) which is at least 6 years

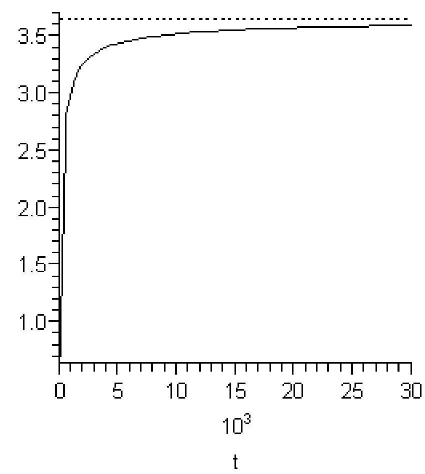

Figure 8: Consumption for the TCC curve of extraction for the CERA's reserves estimates.

earlier than in CERA's scenario.

For $\alpha = 0.2$ and $\beta = 0.05$ which are recommended by Nordhaus and Tobin, a "borderline curve" needs $d = 6$ which implies the oil peak at $t = 15.15$ ($r_{\max} = 4.093$ bln t/year). For these parameters we have $k_0 =$0.1983598129, $c_0 =$0.7327691725 and the asymptote $\bar{c} =$1.947.

We complete the comparison with the case when d is defined as a solution of the short-run problem (8). For $\alpha = 0.3$ and $\beta = 0.05$ the larger reserves give us $d = 7.52$ which is already a sustainable value in comparison with the result obtained for the Oil & Gas Journal reserve estimates (see Fig. 4). In this case we have rather slow but unlimited growth of consumption like on Fig. 6. Oil peak in this case must be at $t = 17.4$. Note that for the same d but $\alpha = 0.2$ and $\beta = 0.05$ we have already an unsustainable pattern of extraction (decreasing to zero consumption in the long run) since the "borderline value" of d for such an economy is 6.

6 The Generalized Rawlsian maximin principle

The examples are the illustrations of the answer to the question "what is worse": a small decrease of consumption in the present or the depriving of oneself and (or) one's descendants of any prospects for improving their lives in the future. According to Rawls's maximin principle, the patterns of sustainable growth of consumption are obviously the results of overinvestment. But actually Rawls (1971, p. 291) objected to applying his maximin principle to the questions of justice among generations because of unacceptable consequences. In (Bazhanov, 2006) we offer a generalized approach for the defining a "relevant position" in Rawls's theory which implies that we must take into account not only the values of some indicators of life quality in the present but rather such indicators combined with their time changes or differences in consumption from previous years. Then the utility in its simplest form is $u = u(c, \dot{c})$. Applying maximin principle, e.g., for u in additive form we have $u(c, \dot{c}) = wc(t) + (1-w)\dot{c}(t) = \gamma = const$ for any $t > 0,\ w \in [0, 1]$ which with $c_0 = c(0)$ follows

$$c(t) = \left[\gamma - \exp\{-wt/(1-w)\}(\gamma - c_0 w)\right]/w \tag{11}$$

or we have a case of limited growth (Fig. 9 compare with Fig. 8) for $\gamma > c_0 w$ and (11) is *desirable* in a sense "...that an extra bit of consumption at t is more valuable than the same *extra* bit at $t+1$, since individuals will, in any case, have more consumption at $t+1$" (Dasgupta and Heal, 1979, p. 284). Observe that (11) describes a limited decline for $\gamma < c_0 w$ and identically constant consumption (as in the Hartwick rule) for $\gamma = c_0 w$.

We do not claim that everybody favors this type of just path, particularly when it is apparent that rather small sacrifices in present can bring slow but *unlimited* growth in the long run (Fig. 6). For those, who prefer this form of intertemporal distribution, the more ap-

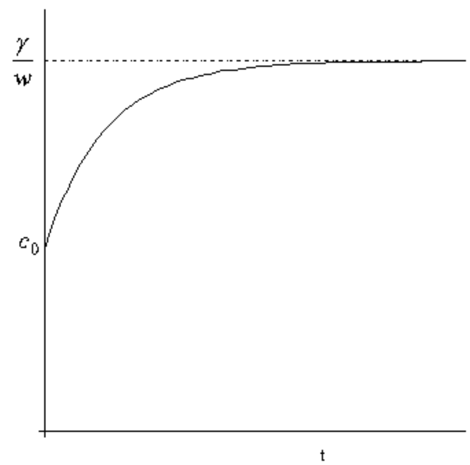

Figure 9: Generalized "fair Rawlsian growth" (11), $w = 0.5$

propriate consumption utility function would be the function with essential factors, e.g., the Cobb-Douglas case. Then the rule of intertemporal distribution is $c^w \dot{c}^{1-w} = \gamma = const$ which gives us $c(t) = c_0 (1 + \mu t)^{\varphi}$where $c_0 = c(0), \mu = (\gamma / c_o)^{1/\varphi} / \varphi, \varphi = 1 - w$ or a pattern of unlimited (quasi-arithmetic, Asheim et al., 2005, p. 5) growth which (for w close to 1) looks like the curve on Fig. 6.

In general, utility can be written as a CES function, or as a function with a variable elasticity where the elasticity parameter and w are to be chosen by the government. Then the specific just savings principle can be deduced for the specific utility function and the transition path of extraction can be adjusted to approach as close as possible (depending on constraints) the asymptotically optimal (in the long run) pattern of intertemporal distribution of consumption.

7 Concluding Remarks

Using our transition path analysis, we have shown for the Cobb-Douglas economy that the sustainable (in terms of nondecreasing consumption) or "normative" peak of oil extraction must be earlier than the "physical" peak when the growth of oil production is already technically impossible.

Analysis of long-run consumption along transition curves shows that even for the Oil & Gas Journal's estimates of the current world oil reserves which are about three times less than CERA's estimates, there is a path of extraction with asymptotically constant (separated from zero) consumption over time. Moreover, a "worsening " of the short-run situation (shortening the period of transition and introducing a stronger negative shock on output) yields the possibility of slow, but unlimited growth of the consumption in the long run.

The situation is brighter with the CERA's reserve estimates though the qualitative result is the same: the sustainable oil peak must be earlier than the "physical" one. The anxiety about possible violation the intergenerational justice criterion increases when we consider the examples with technological parameters α and β estimated by Nordhaus and Tobin. For the economy with these parameters the sustainable oil peak for the CERA's reserves estimates must be in the next 15 years.

For the cases of different patterns of consumption growth the transition curve (to be exact, the single free parameter - d) can be fitted to satisfy desirable qualitative behavior of consumption in accord with the various optimality criteria for the long run. And it again raises the long-standing question about the fairest ethical theory for the distribution of consumption across generations. If decreasing oil consumption is really necessary, which criterion must we follow? (A detailed analysis of ethical theories is, e.g., in (Konow, 2003)).

Aside from equivocation on the main welfare criterion there are some other questions and limitations of the model we have presented.

(1) We examined the transition curve as an interior solution neglecting restrictions imposed by technical possibilities and difficulties connected with a change in the saving rate. Constraints on the speed of changing savings behavior can restrict us from implementing even the path of extraction with asymptotically constant consumption, as well as paths with unlimited growth in the long run (questions of optimal path existence and uniqueness).

(2) There is an interesting question of the path stability with respect to errors in estimations of parameters α and β.

(3) Transition curves can be constructed in a different class of functions, e.g., as a solution of calculus of variation problem.

We also assumed that:

(4) The cost of extraction is zero and population is constant though it would be interesting to consider the problem of transition when extraction costs are present.

(5) There is no time lag between the moment of oil extraction and the corresponding increment of capital; this is not true if the oil rent is invested in alternative technologies.

(6) All oil rent is invested into reproducible capital. In general, this is not observed and we should consider some period of increasing investments along some smooth (maybe hysteresis-like) curves and examine the influence of this curve on the long-run consumption behavior.

(7) We can consider the problem of smooth switching to the efficient path of extraction after using the transition curve for entering the decreasing path.

We think that all these questions need special careful consideration in separate papers.

References

[1] Arrow, K., Dasgupta, P., Goulder L., Daily G., Ehrlich P., Heal G., Levin S., Maler K.-G., Schneider S., Starrett D. and B. Walker, 2004. Are we consuming too much? J. Econ. Perspect. 18: 147-172.

[2] Asheim, G.B., 1994. Net national product as an indicator of sustainability. Scand. J. Econ., 96: 257-265.

[3] Asheim, G.B., Buchholz, W., Hartwick, J.M., Mitra, T. and C. Withagen, 2005. Constant savings rates and quasi-arithmetic population growth under exhaustible resource constraints. CESifo Working Paper No 1573, October 2005.

[4] Bazhanov, A.V., 2006. Decreasing of Oil Extraction: Consumption behavior along transition paths. MPRA Paper No. 469, posted October 16, 2006. Online at http://mpra.ub.uni-muenchen.de/469/.

[5] Bazhanov, A.V., 2007. The peak of oil extraction and a modified maximin principle. MPRA Paper No. 1671, posted February 06, 2007. Online at http:// mpra.ub.uni-muenchen.de/ 1671/.

[6] Dasgupta, P. and G. Heal 1979. Economic Theory and Exhaustible Resources. Cambridge University Press, Cambridge, Eng., 501 pp.

[7] Duesenberry, J., 1949. Income, Saving and Theory of Consumer Behavior. Harvard University Press, Cambridge, MA, 128 pp.

[8] Fuss, M. A., 1977. The demand for energy in Canadian manufacturing. J. Econometrics., 5: 89-116.

[9] Griffin, J.M. and P.R. Gregory, 1976. An intercountry translog model of energy substitution responses. Amer. Econ. Rev., 66: 845-857.

[10] Halvorsen, R. and J. Ford, 1979. Substitution among energy, capital and labor inputs in American manufacturing. In: R. Pindyck (Editor), Advances in the Economics of Energy and Resources, 1: 51-75.

[11] Hartwick, J.M., 1977. Intergenerational equity and the investing of rents from exhaustible resources. Amer. Econ. Rev., 67: 972-974.

[12] Heinberg, R., 2003. The Party's Over. New Society Publishers, Gabriola Island, Canada, 274 pp.

[13] Konow, J., 2003. Which is the fairest one of all? A positive analysis of justice theories. J. Econ. Lit., 41: 1188-1239.

[14] Kuznets, S., 1946. National Product Since 1869. National Bureau of Economic Research, NY, 239 pp.

[15] Magnus, J.R., 1979. Substitution between energy and non-energy inputs in the Netherlands 1950-1976. Int. Econ. Rev., 20: 465-484.

[16] Nemoto, K., 2005. High oil prices dampening Asia-Pacific product demand. Oil & Gas J., 103: 58-62.

[17] Nordhaus, W.D. and J. Tobin, 1972. Is economic growth obsolete? In: Economic Growth, 5th Anniversary Colloquium, V, National Bureau of Economic Research, New York; 1-80.

[18] Pindyck, R.S., 1979. Interfuel Substitution and the Demand for Energy: An international comparison. Rev. Econ. Statist., 61: 169-179.

[19] Poterba, J., 1988. Are consumers forward looking? Evidence from fiscal experiments. Amer. Econ. Rev., 78: 413-418.

[20] Rawls, J., 1971. A Theory of Justice. Belknap Press of Harvard University Press, Cambridge, MA, 607 pp.

[21] Solow, R.M., 1974. Intergenerational equity and exhaustible resources. Rev. Econ. Stud., 41: 29-45.

[22] Stiglitz, J.E., 1974. Growth with exhaustible natural resources: efficient and optimal growth paths. Rev. Econ. Stud., 41: 123-137.

[23] United Nations (UN), 1993. Agenda 21, United Nations, New York.