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The Neolithic Revolution from a
price-theoretic perspective

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Abstract

The adoption of agriculture during the Neolithic triggered the first demographic explosion in
history. When fertility returned to its original level, early farmers found themselves more poorly
nourished than hunter-gatherers and working longer hours to make ends meet. We develop a
dynamic, price-theoretic model that rationalizes these events: in the short-run, fertility and
utility increase; in the long-run, consumption, leisure, and utility fall below their initial levels.
This, we argue, can be attributed to the rise in child labor productivity that followed the
adoption of agriculture. Counter-intuitively, an increase in the productivity of children may
lead to a permanent reduction in utility.

Keywords: Neolithic Revolution; hunter-gatherers; child labor; Thomas Malthus.

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1 Introduction

The shift from hunting and gathering to agriculture, a transition known as the Neolithic Revolution, was followed by a sharp increase in fertility (Bocquet-Appel, 2002). Global population increased a hundred times faster during the Neolithic Revolution than during the preceding hunting-gathering period (Hassan, 1973). In the course of a few centuries, typical communities grew from about 30 individuals to 300 or more, and population densities increased from less than one hunter-gatherer per square mile to 20 or more farmers (Johnson and Earle, 2000, pp. 43, 125, 246).

This demographic explosion has been attributed to two main causes. First, food was available to early farmers in unprecedented quantities (Price and Gebauer, 1995). Second, having children was cheaper for early farmers than for hunter-gatherers, since caring for children interfered more with hunting and gathering than with farming (Locay, 1983, pp. 60–98). More importantly, the children of farmers contributed substantially more to food production than the children of hunter-gatherers (Kramer and Boone, 2002).

Although early farmers produced food in larger quantities, they were more poorly nourished and suffered from poorer health than did hunter-gatherers (Armelagos et al., 1991; Cohen and Armelagos, 1984). The archeological record of the Eastern Mediterranean region reveals that male hunter-gatherers who reached adulthood had a life expectancy of 33 years, while females who reached adulthood had a life expectancy of 29 years. After the Neolithic Revolution, the life expectancy of adult males dropped to 32 years, and the life expectancy of adult females dropped to 25 years. Average statures also diminished, from 177 to 165 centimeters among males, and from 165 to 152 centimeters among females (Angel, 1975).

To make matters worse, the average daily working time increased upon the arrival of agriculture. Ethnographic studies suggest that hunter-gatherers worked less than six hours per day, whereas primitive horticulturists worked seven hours on average, and intensive agriculturalists worked nine (Sackett, 1996, pp. 338–42).

We develop a dynamic, price-theoretic model that makes sense of these perplexing events, provided that a set of plausible conditions hold. Of these, the key conditions are: (1) \textit{ceteris paribus},
fertility increases with wage, and (2) wage decreases with population size. Together, conditions 1 and 2 constitute the classical Malthusian-Ricardian assumptions.

The story begins with a tribe of hunter-gatherers, whose members derive utility from consumption, leisure, and childbearing. The tribe is in demographic equilibrium: given his “wage” (the amount of food he can produce per unit of time) and the prices he faces, each tribesman chooses to reproduce at the replacement rate. At some point, the tribe discovers agriculture and adopts it because doing so increases the wage and the productivity of children. As a result, the demographic equilibrium breaks down: the tribesmen increase their fertility above the replacement rate, and the population begins to grow. Wages, which are negatively correlated with population size, progressively fall; and, as the wages fall, fertility falls accordingly. Eventually, fertility returns to the replacement rate and the population restabilizes, only now at higher numbers. The wage, on the other hand, settles below its original level. The later generations of farmers reproduce at the same rate, consume less food, and work longer hours than the hunter-gatherers. Hence, while adopting agriculture increases the utility of the first farmers, the utility of the later generations of farmers turns out to be lower than the utility of their hunter-gatherer ancestors.

The increase in the productivity of children, which amounts to a fall in their price, drives the results of our model. Counter-intuitively, an increase in the productivity of children may cause a permanent reduction in utility. This result distinguishes our work from previous attempts to explain the Neolithic Revolution. In other work, enhancements in food production technology play the main role. In our model, these have only one effect in the long-run, namely population growth.

Our model sheds some light on a central puzzle in the literature: did agriculture emerge out of opportunity? Or were our ancestors forced into farming? Traditional scholarship regards agriculture as highly desirable: once humans discovered agriculture and recognized the potential productivity gains, beginning to farm was an obvious decision (Trigger, 1989). In recent times, this idea has been called into question, partly because taking up farming reduced the standard of living of our ancestors (Harlan, 1992; Sahlins, 1974). The reduction in the standard of living suggests that our ancestors did not become farmers willingly. Instead, they may have been forced into agriculture by
some external pressure [e.g., overpopulation (Binford, 1968; Flannery, 1969) or the overkill of the mammoths and other game (Smith, 1975)].

We propose a third, hybrid answer to the puzzle—one that is purely economic. Since wages decrease as population increases, a generation’s fertility imposes a negative externality to subsequent generations. Increased child productivity is equivalent to a subsidy on childbearing that accentuates the intergenerational externality. The first farmers were lured into agriculture; they benefited from the higher wages and from this subsidy. But the demographic explosion that ensued impoverished later generations. Still, members of later generations were forced to remain farmers for two reasons. First, reverting to hunting and gathering would have reduced their own wages even further. Second, renouncing the subsidy on childbearing would have made them even worse off. More productive children were an asset to the early adopters of agriculture. In the long-run, however, the increase in the productivity of children pushed primitive societies deeper into the Malthusian trap.

1.1 Related literature

A variety of theories have been advanced to explain the emergence of agriculture. These range from excessive hunting (Smith, 1975), to warfare (Rowthorn and Seabright, 2008), to climatic variation (Dow et al., 2009). Theories on the adoption of agriculture have been extensively surveyed by Weisdorf (2005) and Sharp and Weisdorf (2009). We will thus limit this review to previous explanations for the loss of welfare that followed the Neolithic Revolution.

Locay (1989) argues that substitution effects can explain the events of the Neolithic. According to Locay, the fall in the price of children motivated early farmers to substitute fertility for consumption and leisure. He maintains that these changes were welfare enhancing, since the fall in the price of children expanded the farmers’ feasible set. According to Weisdorf (2004), early farmers gave up their leisure time in exchange for goods produced by an emerging class of non-food-producing specialists (e.g., craftsmen and bureaucrats). Marceau and Myers (2006) model the fall in consumption and leisure as a tragedy of the commons. Locay (1989), Weisdorf (2004), and Marceau and Myers (2006) assume that population remains constant during the transition to agriculture. Demographics plays no role in their models.
Two papers, one by Weisdorf (2008) and the other by Robson (2008), explore the influence of demographics on the loss of welfare experienced by early farmers.

Weisdorf (2008) incorporates Malthusian-Ricardian population principles into a model with two sectors: hunting and agriculture. In his model, the higher productivity of agriculture motivates its adoption, but the subsequent population growth overrides the productivity gain. Weisdorf implicitly assumes that people’s only desire in life is to reproduce and that they will work as many hours as it takes to maximize their fertility. He also assumes that food consumption is an increasing function of the energy spent at work, which explains why farmers have to work more than hunter-gatherers.

Robson (2008) develops a model with two goods: children and children’s health. In Robson’s model, infectious diseases become more prevalent as population increases. This makes the health of children more expensive for farmers than for the (less populous) hunter-gatherers. Farmers respond by having more children and investing less in their health than did their predecessors.

2 A model of agriculture adoption

2.1 Model setup

A tribe has \( N > 0 \) identical adult members or tribesmen. A tribesman chooses food consumption \( c > 0 \), leisure \( r > 0 \), and the number of his children \( n > 0 \) in order to maximize a certain utility function \( u(c, r, n) \), which is increasing in all its arguments.\(^1\)

The tribesman is subject to the following budget constraint:

\[
w(A, N) \cdot (T - r + \alpha n) \geq c + \kappa n.
\]

Function \( w(A, N) > 0 \) is the tribesman’s “wage,” measured in units of food, \( T > 0 \) is his disposable time, and \( T - r > 0 \) is his labor supply. The wage is increasing in a technology parameter \( A \) and decreasing in population \( N \); these are the classical Ricardian assumptions. Using subscripts

\(^1\)There is ample evidence that pre-modern peoples deliberately chose the number of their children. The methods they used included abstinence, celibacy, prolonged breast-feeding, abortion, and infanticide (Douglas, 1966; Cashdan, 1985).
to denote derivatives, these assumptions translate into \( w_A > 0 \) and \( w_N < 0 \). Parameter \( \alpha \geq 0 \) represents child productivity measured in man-hours. The children of hunter-gatherers contribute nothing to production, which we capture by setting \( \alpha = 0 \). When agriculture is adopted, \( \alpha \) rises to a positive amount. Parameter \( \kappa > 0 \) corresponds to the food requirements of a child.

The budget constraint can be rewritten as follows:

\[
I \geq c + p^r r + p^n n,
\]

where \( I = wT \) is total income, \( p^r = w \) is the price of leisure, and \( p^n = \kappa - \alpha w > 0 \) is the price of children.

Finally, the following equation governs the population dynamics:

\[
N_{\text{next}} = nN,
\]

where \( N_{\text{next}} \) is the adult population of the next period.

In the short-run, population \( N \) is fixed, and fertility \( n \) responds to income and prices. In the long-run, \( N \) adjusts so that the wage \( w \) converges to the value at which the tribesman’s optimal decision is to bear children at the replacement rate, \( n = 1 \). These assumptions link our model to the family of endogenous fertility models with Malthusian-Ricardian checks on population.\(^2\)

Without loss of generality, let the tribesman’s choices be summarized by the Marshallian demands that stem from his utility function:

\[
\begin{align*}
c & = c^m(p^r, p^n, I), \\
r & = r^m(p^r, p^n, I), \\
n & = n^m(p^r, p^n, I).
\end{align*}
\]

2.2 Effects of the adoption of agriculture

We model the adoption of agriculture as two technological improvements: a positive shock to child productivity, and a one-time increase in total factor productivity (TFP). We will explore the effects of each improvement separately.

First, we will show that a rise in the productivity of children will produce the stylized facts of the Neolithic Revolution, provided that a set of plausible conditions hold. Second, we will show that a one-time increase in TFP will produce a short-run increase in fertility and in utility but will have no long-lasting effects other than an increase in population.

The effects of an increase in child productivity

Let $A$ be constant. By totally differentiating $w(A, N)$ and the Marshallian demands with respect to $\alpha$, we obtain a linear system for the effects of an increase in child productivity:

\begin{align*}
    w_\alpha &= w_N N_\alpha, \quad (1) \\
    c_\alpha &= r_p^{m} w_\alpha + c_p \cdot (-w - \alpha w_\alpha) + c_l w_\alpha T, \quad (2) \\
    r_\alpha &= r_p^{m} w_\alpha + r_p^{n} \cdot (-w - \alpha w_\alpha) + r_l^{m} w_\alpha T, \quad (3) \\
    n_\alpha &= n_p^{m} w_\alpha + n_p^{n} \cdot (-w - \alpha w_\alpha) + n_l^{m} w_\alpha T, \quad (4)
\end{align*}

where $w_\alpha$, $c_\alpha$, $r_\alpha$, $n_\alpha$, and $N_\alpha$ are the unknown total derivatives. The system is closed with $N_{\alpha}^{SR} = 0$ for the short-run and with $n_{\alpha}^{LR} = 0$ for the long-run, where superscripts $SR$ and $LR$ distinguish short-run and long-run solutions. In other words, the population is fixed in the short-run (i.e., $N_{\alpha}^{SR} = 0$), but may change in the long-run as a consequence of today’s fertility decisions. In the long-run, the population must adjust so that the wage is just enough to persuade the tribesman to reproduce at the replacement rate (i.e., $n_{\alpha}^{LR} = 1$, which entails $n_{\alpha}^{LR} = 0$).
Initially, the tribesman hunts and gathers, so \( \alpha = 0 \). Replacing \( \alpha = 0 \) in equations (2)–(4), we obtain:

\[
\begin{align*}
    c_\alpha &= (c_{cp} + c_{IT}^m)w_\alpha - wc_{pn}^m, \\
    r_\alpha &= (r_{cp} + r_{IT}^m)w_\alpha - wr_{pn}, \\
    n_\alpha &= (n_{cp} + n_{IT}^m)w_\alpha - wn_{pn}.
\end{align*}
\]

From equations (1) and (7), together with condition \( N^\alpha_{sr} = 0 \), we obtain:

\[
\begin{align*}
    w_\alpha^{sr} &= 0, \\
    n_\alpha^{sr} &= -wn_{pn}.
\end{align*}
\]

The wage remains constant in the short-run \( (w_\alpha^{sr} = 0) \), since population is fixed in the short-run. Equation (8) implies that fertility \( n \) will increase in the short-run if \( \alpha \) increases and the demand for children has a negative slope: \( \eta_{pn}^m < 0 \).

Will the tribesman adopt agriculture? Yes, and this is why. The tribesman’s wage remains constant in the short-run, which implies that total income and the price of leisure remain constant as well:

\[
\begin{align*}
    \frac{dI}{d\alpha}^{sr} &= Tw_\alpha^{sr} = 0, \\
    \frac{dp^r}{d\alpha}^{sr} &= w_\alpha^{sr} = 0.
\end{align*}
\]

Adopting agriculture, however, increases \( \alpha \), which brings the price of children down:

\[
\frac{dp^m}{d\alpha}^{sr} = -w < 0.
\]

The fall in \( p^m \), while \( I \) and \( p^r \) remain unchanged, pushes the tribesman’s budget constraint outwards, unambiguously increasing his utility: \( u_\alpha^{sr} > 0 \).
From equations (1) and (5)–(7), together with condition $n_{lr}^{\text{fr}} = 0$, we obtain the long-run comparative statics:

\[
\begin{align*}
W_{\alpha}^{LR} &= \frac{wn_{p_{\alpha}}^{m}}{n_{p_{\alpha}}^{m} + n_{I_{\alpha}}^{m}T}, \\
C_{\alpha}^{LR} &= \frac{c_{p_{\alpha}}^{m} + c_{I_{\alpha}}^{m}T}{n_{p_{\alpha}}^{m} + n_{I_{\alpha}}^{m}T}wn_{p_{\alpha}}^{m} - wc_{p_{\alpha}}^{m}, \\
R_{\alpha}^{LR} &= \frac{r_{p_{\alpha}}^{m} + r_{I_{\alpha}}^{m}T}{n_{p_{\alpha}}^{m} + n_{I_{\alpha}}^{m}T}wn_{p_{\alpha}}^{m} - wr_{p_{\alpha}}^{m}, \quad (9) \\
N_{\alpha}^{LR} &= \frac{wn_{p_{\alpha}}^{m}}{w_{N}\cdot (n_{p_{\alpha}}^{m} + n_{I_{\alpha}}^{m}T)}. \quad (11)
\end{align*}
\]

Equations (8)–(11) yield a set of conditions that suffice to reproduce the stylized facts of the Neolithic Revolution—that is, a short-run rise in fertility ($n_{\alpha}^{sr} > 0$), a long-run increase in population ($N_{\alpha}^{lr} > 0$), and long-term drops in food consumption and leisure ($c_{\alpha}^{lr} < 0$ and $r_{\alpha}^{lr} < 0$). The conditions are:

1. Wages are decreasing in population: $w_{N} < 0$ (the Ricardian assumption).

2. Children are ordinary goods: $n_{p_{\alpha}}^{m} < 0$.

3. Fertility is increasing in wages: $n_{w} = n_{p_{\alpha}}^{m} + Tn_{I_{\alpha}}^{m} > 0$ (the Malthusian assumption).

4. Consumption is a normal good and a gross substitute for leisure and children: $c_{I_{\alpha}}^{m}, c_{p_{\alpha}}^{m}, c_{p_{\alpha}}^{m} > 0$.

5. Leisure is a gross substitute for children: $r_{p_{\alpha}}^{m} > 0$.

6. Leisure is decreasing in wages: $r_{w} = r_{p_{\alpha}}^{m} + r_{I_{\alpha}}^{m}T < 0$.

7. Leisure is more responsive to changes in wage than to changes in the price of children: $|r_{p_{\alpha}}^{m} + r_{I_{\alpha}}^{m}T| > r_{p_{\alpha}}^{m}$.

8. Fertility is more responsive to changes in the price of children than to changes in wage: $n_{p_{\alpha}}^{m} + n_{I_{\alpha}}^{m}T < |n_{p_{\alpha}}^{m}|$. 

9
The long-run fall in consumption and leisure, while fertility returns to its original level, implies that the later generations of farmers will obtain a lower utility than did their hunter-gather predecessors — that is, $u_{\alpha}^{LR} < 0$. Still, the later generations of farmers will not revert to hunting and gathering, as that would increase the price of children while total income and the price of leisure remain constant. The utility of farmers would thus fall in the short-run. Given their numbers, hunting and gathering is not an option for the farmers: they are trapped in an undesirable equilibrium with plenty of people, low incomes, scarce food, and too much work.

What lies behind these puzzling results?

An increase in the productivity of children promotes a present and future rise in fertility, since more productive children are cheaper:

$$\frac{dp_n^{SR}}{d\alpha} = \frac{dp_n^{LR}}{d\alpha} = -w < 0.$$  

However, equilibrium requires later generations to reproduce at the replacement rate ($n = 1$). Since children will remain cheap in the long-run, the wage must fall below its original level ($w_{\alpha}^{LR} < 0$) to induce future generations to limit their fertility to one child per capita. The fall in the wage is achieved through the increase in population ($N_{\alpha}^{LR} > 0$) that results from a temporary increase in fertility ($n_{\alpha}^{SR} > 0$).

In addition, the long-run reduction in the wage implies that the price of leisure will fall:

$$\frac{dp_r^{LR}}{d\alpha} = w_{\alpha}^{LR} < 0.$$  

Since food is a normal good and a gross substitute for leisure and for children, the long-run reductions in $w_I$, $p_r$ and $p_n$ will all push food consumption downwards ($c_{\alpha}^{LR} < 0$).

Two opposing forces act upon leisure in the long-run. On the one hand, leisure will be cheaper, so the tribesmen will tend to rest more and work less. On the other hand, total income will be lower and children will be cheaper; since leisure is a normal good and a gross substitute for children, the tribesmen will want to reduce leisure and increase their working hours. Conditions 7

10
and 8 guarantee that the second effect will dominate the first, so leisure decreases in the long-run \( (r^\text{lr}_\alpha < 0) \).

The effects of improvements in food production technology

Let \( \alpha \) be constant. By totally differentiating \( w(A, N) \) and the Marshallian demands with respect to \( A \), we obtain a linear system for the effects of improved food production technology:

\[
\begin{align*}
    w_A &= w_A + w_N N_A, \\
    c_A &= c^m_p w_A + c^m_n \cdot (-\alpha w_A) + c^m_I w_A T, \\
    r_A &= r^m_p w_A + r^m_n \cdot (-\alpha w_A) + r^m_I w_A T, \\
    n_A &= n^m_p w_A + n^m_n \cdot (-\alpha w_A) + n^m_I w_A T, \\
        &\quad \left(\frac{dp^r}{dA} \right) \left(\frac{dp^n}{dA} \right) \left(\frac{dI}{dA} \right) 
\end{align*}
\]

where \( w_A, c_A, r_A, n_A, \) and \( N_A \) are the unknown total derivatives. The system is closed with \( N_A^\text{sr} = 0 \) for the short-run and with \( n^\text{lr}_A = 0 \) for the long-run.

Using \( \alpha = 0 \), the system reaches:

\[
\begin{align*}
    w_A &= w_A + w_N N_A, \\
    c_A &= c^m_p w_A + c^m_I w_A T, \\
    r_A &= r^m_p w_A + r^m_I w_A T, \\
    n_A &= n^m_p w_A + n^m_I w_A T.
\end{align*}
\]
And, under the conditions stated in the previous section, we obtain:

\[
\begin{align*}
    w_{A}^{SR} &= w_A > 0, \\
    n_{A}^{SR} &= n_{m}^{m}w_A + n_{m}^{m}w_AT > 0, \\
    w_{A}^{LR} &= c_{A}^{LR} = r_{A}^{LR} = n_{A}^{LR} = 0, \\
    N_{A}^{LR} &= -\frac{w_{A}}{w_{N}} > 0.
\end{align*}
\]

It follows that wages increase in the short-run and so does fertility. The only long-run effect of a higher TFP is an increase in population. Consumption and leisure do not change in the long-run, since neither total income nor prices change:

\[
\begin{align*}
    dI^{LR}_{A} &= T_{w_{A}^{LR}} = 0, \\
    dp^{LR}_{A} &= w_{A}^{LR} = 0, \\
    dp^{n}_{A} &= 0.
\end{align*}
\]

This is the typical Malthusian result.

For any given population level, the TFP of agriculture is higher than the TFP of hunting and gathering. Therefore, a generation of farmers who reverted to hunting and gathering would see their wages fall. This contributes to the stickiness of agriculture.

### 3 An example: inelastic supply of labor

The purpose of this simple example is to clarify the inner mechanics of the model. As the effects of a rise in TFP are intuitive, we will only discuss here the short-run and long-run effects of an increase in child productivity. We will assume that the tribesman supplies a fixed amount of labor. These assumptions will allow us to concentrate on the behavior of consumption and on changes in
utility. We will also assume that consumption and children are ordinary goods, normal goods, and gross substitutes of each other.

Initially, the tribe hunts and gathers, and the wage is just enough to keep the population constant; given his income and the prices he faces, each tribesman chooses to bear one child. The tribesman’s choice is represented by bundle $B_0$ in Figure 1. When the tribe adopts agriculture, children become more productive. This is equivalent to a reduction in the price of children, so the budget constraint rotates outwards from $BC_0$ to $BC_{sr}$. Since children are ordinary, the tribesman switches to bundle $B_{sr}$, which contains more children than bundle $B_0$. And, since consumption and children are gross substitutes, bundle $B_{sr}$ contains less consumption than bundle $B_0$. Because $BC_{sr}$ is less stringent than $BC_0$, the change is welfare enhancing: utility rises from $u_0$ to $u_{sr}$. [This point was previously made by Locay (1989, pp. 746–47).]

In the short-run, fertility rises above the replacement rate, so the population begins to grow. As a result, the descendants of the original farmers earn ever lower wages, so the budget constraint moves inwards. The typical tribesman consumes progressively less, and his fertility falls. Both the fall in consumption and the fall in fertility are a consequence of normality. When the budget constraint reaches $BC_{lr}$, the tribesman chooses bundle $B_{lr}$, which again contains just one child. Population restabilizes, only at a higher level. The wage, consumption, and utility are lower than they were before the price shock, since $B_{lr}$ is to the left of $B_0$. [Locay (1989), p. 748, speculates that consumption and utility may fall in the long-run, but he does not prove or explain why they must fall in the long-run when both goods are normal.]

Why should $B_{lr}$ contain less consumption than $B_0$? We argue by contradiction. Suppose that $BC_{lr}$ has settled to the right of bundle $B_0$ — Figure 2 represents this situation. In equilibrium, the tribesman must choose bundle $B_{lr}$; otherwise, the population would grow or shrink. This bundle contains more consumption than bundle $B_0$. Now imagine that the wage falls for some reason (e.g., a negative productivity shock), pushing the budget constraint inwards to $BC_1$, which contains bundle $B_0$. Before the price shock, the tribesman revealed that he preferred bundle $B_0$ to all bundles that lay on the thin segment of $BC_1$. Therefore, he will now choose a bundle that lies on the thick segment of $BC_1$. He will not choose $B_0$, because that would imply that indifference curves cross at
Therefore, he must choose a bundle that lies on the thick segment of $BC_1$ and that is strictly above the dashed line. But the wage is lower in $BC_1$ than it is in $BC_{lr}$. And, since children are normal goods, the tribesman must choose a bundle on $BC_1$ that contains fewer children than $B_{lr}$—that is, a bundle that lies on the thin segment of $BC$ and that is strictly below the dashed line. We have reached a contradiction. Therefore, if children are normal, the wage and consumption will fall in the long-run whenever the price of children decreases, as occurs when child productivity rises.
Figure 1: Short-run and long-run effects of a fall in the price of children when the supply of labor is inelastic, consumption and children are normal goods, and both goods are substitutes of each other.

Figure 2: If children are normal, the wages and consumption will fall in the long run whenever the price of children decreases.
References


