Why tenure?

Cater, Bruce and Lew, Byron and Pivato, Marcus

Department of Economics, Trent University, Department of Mathematics, Trent University

17 January 2009

Why tenure?

Bruce Cater, Byron Lew and Marcus Pivato*

January 17, 2009

Abstract

Academic research is a public good whose production is supported by the tuition-paying students that a faculty's research accomplishments attract. A professor's spot contribution to the university's revenues thus depends not on her spot research production, but rather on her cumulative research record. We show that a profit-maximizing university will apply a 'high' minimum retention standard to the production of a junior professor who has no record of past research, but a 'zero' retention standard to the spot production of a more senior professor whose background includes accomplishments sufficient to have cleared the 'high' probationary hurdle.

I Introduction

Under a tenure-track contract, a professor who fails to meet some positive standard of research production during a finite probationary period is dismissed at that period's end. Yet, a professor who meets that initial standard is granted *tenure* and retained *regardless* of her research output thereafter.[1]

The literature offers a number of possible economic rationales for the university's puzzling contractual choice. Freeman [8] suggests that risk averse professors are granted the security of tenure to compensate for the risk inherent in their research. [2] Yet, this explanation is unsatisfactory, for nonacademic employers manage to contract with workers who

*Cater and Lew: Department of Economics, Trent University, Ontario, Canada; Pivato: Department of Mathematics, Trent University, Ontario, Canada.

[1]Siow [25] notes that, in the 1989 *Survey Among College and University Faculty* sponsored by the Carnegie Foundation Survey, 4.7 percent and 36.4 percent of tenured faculty in doctoral-granting and non-doctoral-granting schools, respetively, reported no publications in the previous two years and no current research. Yet, in reviews of U.S. case law, legal scholars, including Hendrickson [10] and Morris [21], do not cite a single case in which a tenured professor was dismissed primarily on the grounds of low research productivity.

[2]Kahn and Huberman [14] and Waldman [26] offer explanations of the use of 'up-or-out' contracts, but do not address the issue of post-probationary minimum production standards. McKenzie [19] and McPherson and Shapiro [20] attempt to academic tenure on internal political, rather than economic, grounds.

are risk averse and whose productivity is uncertain without having to offer them anything akin to tenure.

Carmichael [4] suggests that a university is unique in that, because the state of academic knowledge is vast and expanding, it is the incumbent occupants of its scarce faculty slots who are best positioned to judge the research potential of candidates. With the aim of maximizing its research production, the university then provides those incumbents with the security of tenure to ensure that they are willing to identify and hire candidates superior to themselves. Yet, senior faculty from other institutions are just as qualified to rank candidates. There is, therefore, no reason why a university could not enlist externals to assess its applicants in much the same way as it utilizes them to evaluate its tenure and promotion cases.[3] The cost of doing so would surely pale in comparison to the foregone research output that results when scarce faculty slots are occupied, perhaps for decades, by unproductive scholars. Moreover, in focusing on the issue of incentives in hiring, Carmichael's [4] approach assumes that a professor's research productivity is governed only by ability, and not effort, and that ability is constant over a professor's lifetime. In doing so, it abstracts from a question at the heart of the tenure debate: why is academic research output observed to decline, on average, with age?[4] Critics, including Alchian [1], have long suggested that this pattern reflects some disincentive effect of the tolerance tenure extends. But if that were so, it seems unlikely that a university would choose to grant tenure. Could declining research production instead be understood to be optimal in some way?

Siow [25] assumes that production declines because research productivity falls with age, arguing that, as this occurs, it becomes socially efficient for a professor to spend less time on research and more time on teaching. Tenure is the means by which the university then induces its older professors to do less research. Yet, while research *production* declines with age, there is no empirical evidence to support an assumption that academic research *productivity* declines as well. How, then, should falling research production be understood? Moreover, it is not clear that Siow's theory can be fully reconciled with the observed facts. If the goal of tenure were to *eliminate* research effort among older faculty, we would not observe universities providing even tenured professors with considerable research incentives. On the other hand, if a university's goal were to merely *reduce* research effort among its older faculty, it might tolerate *reduced* production, but not utter failure.

The purpose of this paper is to develop a model of the unique way in which a university translates its research production into profits, and to show that such a model can explain why a university would retain only those professors who are initially successful in research, regardless of their research output thereafter, and continue to induce research effort, albeit at a rate that declines with the professor's age.

We suggest that the key to the tenure puzzle may lie in a number of observations regarding the nature of academic productivity and tenure itself.

A primary role of a university is to encourage research that is important, but would not be elsewhere undertaken. As Carmichael [4] notes, this includes the production of

[3] This point was first made by Ceci, Williams and Mueller-Johnson [6].

[4] Empirical evidence of declining research production is presented in Diamond [7] and Levin and Stephan [16].

knowledge that either cannot be appropriated or is of no value to private sector firms, such as that generated by research in the fields of philosophy, literature, public policy and pure mathematics. While a professor is hired to both research and instruct, it is, therefore, only her instruction that is sold.[5]

How is it, then, that a university can afford to reward research accomplishments? One might argue that a university is not subject to the same economic pressures that constrain other employers; that a university does not care about profits. Yet, while Hendrickson [10] and Morris [21] do not cite a single case in which a tenured professor was dismissed primarily on the grounds of low research productivity, as noted above, they, along with Lovain [17], do cite cases in which tenured professors have been dismissed for failing to perform their teaching duties. This clearly suggests that a university *is* concerned with the realization of revenues and with its own economic viability. More plausibly, the production of academic research may be viable because, as observers, including Bok [3], James [13] and Hearn [9], suggest, it benefits a university by attracting tuition-paying students. Siow [24] presents supporting evidence that students interpret observed research output as a signal that a university's faculty have the knowledge they seek.[6]

Combining these observations, we model a professor's contribution to the university's revenues, at any point, as depending not on her spot research production, but rather on the strength of her cumulative research record. A representative university operates in discrete time and aims to maximize its expected profits per period. Its problem, in general terms, involves choosing both the conditions under which it will retain an incumbent into the next period and the extent to which it will reward research accomplishments so as to induce a professor's unobservable research effort.

We show, for a range of distributions of research productivity, that the particular tolerance of tenure can be understood not as the solution to some hiring problem or as a way of inducing less research, but as the means by which the university retains those professors whose current research production may be poor or nonexistent, but whose past research accomplishments continue to make them profitable. We also show that declining research production over the life cycle need not result from some disincentive effect of tenure or from declining productivity. Falling production can be understood to result from the university optimally inducing less research effort as a professor approaches retirement and the opportunities to realize tuition revenues from any resulting research successes diminish.

This paper is related to Cater, Lew and Smith [5], which examines a simpler model in which research productivity is governed only by ability, not effort, and in which that ability is assumed to decline with age. The university's problem in that paper thus involves choosing only the conditions under which an incumbent will be retained into the next period. The main contribution of this paper is our consideration of a much more general model that allows for the simultaneous analysis of the university's choices of optimal research

[5] Note that Carmichael [4] assumes that a professor is hired only to research.

[6] Research may be a reliable proxy for knowledge either because knowledge is accumulated through research or because knowledge makes it easier to conduct research. Of course, students need not observe and process the research directly for the signal to be effective. Scholarly accomplishments of a university's faculty may filter down to students through media sources that rank universities, in part, on the basis of those accomplishments and their correlates.

standards and research effort inducement. The analysis presented here enables us to fully resolve the contractual puzzle described above.

II The model

A representative university

We conceive of a government or private donor providing a one-time capital endowment to create a representative university under the terms of a charter that directs it, in perpetuity, to produce and impart academic knowledge. The endowed capital is sufficient to support a fixed number of faculty 'slots'. Without loss of generality, we let that number be one.

Once endowed, the university is expected to be financially independent. It is risk neutral and has a zero rate of discount. Operating in discrete time, it expects to live for infinitely many periods, remaining viable by maximizing its expected profits per period.[7]

The university's only revenues are 'tuitions', defined here to include any revenues tied to student enrollment.[8] The university's only costs are the wages of its faculty.

Hiring and retention decisions can be made only at the beginning of a period. The university can condition a professor's wages on her observable research output. Its employment contracts are enforceable before the courts.

A representative professor

Professors are drawn from overlapping generations, each with a working lifetime of three periods. Those in their first, second and third periods will be referred to, respectively, as being 'junior', 'middle-aged' and 'senior'. All professors are identical *ex ante*.

In each period of her working life, a representative professor will occupy either a nonacademic or an academic job. The option of nonacademic employment always exists; as in Carmichael [4], its per period maximized utility is a constant C_o. If a professor chooses the nonacademic option at the beginning of any period, we, like Carmichael [4], assume that her academic abilities decay so that, in any subsequent period, nonacademic employment will be her only option.[9]

At the beginning of her first working period, our representative professor receives one offer of academic employment. If she accepts that offer, then, at the beginning of the second period, the university that employed her as a 'junior' may wish to retain her. Outside universities may also attempt to hire her, and a bidding war for her services may

[7] Rothschild and White [23] and Siow [25] also assume that a university seeks to maximize profits.

[8] Our model thus applies both to privately-endowed schools where tuitions are typically paid entirely by the students and to publicly-endowed universities where tuitions may be subsidized, in part or in full, by a government.

Note also that while research production may lead to subsequent grant income, we will abstract from this possibility to focus on the question of why tenure is granted both by universities that frequently realize such income and by universities that rarely, if ever, do.

[9] This is a simplifying assumption that rules out the possibilities of delayed or discontinuous academic employment.

occur. A similar process then occurs at the beginning of the third period if she remains in academic employment through her second working period.[10]

If employed by a university during the t^{th} period of her working life ($t = 1, 2, 3$), our representative professor will, at the beginning of that period, choose an unobservable level of research effort, e_t (≥ 0), the quadratic utility cost of which is e_t^2. At the end of the period she will then realize research output described by a single index that, as in Carmichael [4], measures quantity and quality with the correct weights. The value of that index, r_t, is drawn randomly from the probability distribution ρ_{e_t} on $[0, \infty)$ that is assumed to come from either the uniform, exponential or power-law family of distributions.[11] As in Carmichael [4], any knowledge accumulated through, or otherwise associated with, academic research is of no value in nonacademic employment. During any period of academic employment, our representative professor will also provide instruction, the disutility of which is a constant D. We normalize the professor's utility scale so that $C_o + D = 0$.

Our most critical assumption is that a professor's period t research output serves as a signal of knowledge that increases her contribution to the tuition revenues of *any* university that employs her in *any* subsequent period. Because all 'junior' professors in our model begin with no research record, they all contribute the same revenues during the first period of their working lives. We normalize those revenues to 0. The translation of observed research output into subsequent revenues is assumed to be linear: a 'middle-aged' and a 'senior' professor contribute kr_1 and $k(r_1 + r_2)$, respectively (where $k > 0$).[12]

Our representative professor is risk neutral and has a zero rate of discount. Her constant marginal utility of money is normalized to 1. In choosing between alternative employment offers, she will, therefore, attempt to maximize her expected lifetime income, less any research effort disutility.

At the beginning of the first period of her working life, our representative professor is assumed to accept the academic offer, provided that it matches or betters the expected lifetime utility of 0 she would obtain from a lifetime of nonacademic employment. Because of infinitesimally small but positive job change costs, an academic job which offers an expected future utility of 0 is similarly sufficient to deter a 'middle-aged' or 'senior' professor from quitting to pursue nonacademic employment. Those job change costs also mean that, for one university to successfully raid another university for a 'middle-aged' or 'senior' professor, the recruiting university must slightly better the (expected) wage she would receive by remaining with her current employer.

[10] Note that, in our model, because a professor's research record is publicly observable, there is no meaningful distinction between an outside university raiding for a professor and a professor seeking employment with an outside university. It is, therefore, sufficient to consider only the implications of raiding.

[11] These three probability distributions, each intuitively plausible and analytically tractable, are chosen to demonstrate that our results are robust across a range of models of intellectual creativity.

[12] There may be rare cases where a university continues to realize revenues from its association with a particularly accomplished professor even after her retirement. We abstract from this possibility, however, on the grounds that the use of tenure-track contracts seems to transcend such cases.

III Analysis

Academic contracts

Any equilibrium in our model necessarily involves at least some universities hiring 'junior' professors at least some of the time. The terms of employment offered to a 'junior' will not only determine whether she accepts the initial academic offer, but will also play a role in determining the relative value of her nonacademic and potential academic options in subsequent periods. It is, therefore, necessary for us to first describe the terms of employment that a 'junior' professor will be offered.

When attempting to hire a 'junior', a university must choose two inter-related features of its employment contract: (1) the conditions, if any, under which it wishes to retain the professor into subsequent periods of her working life and (2) the wage structure necessary to recruit her initially, to induce her 'optimal' effort, and to ensure that she chooses to remain with the university when her retention is sought.

We make a number of assumptions about the contractual form. The university sets minimum research standards that a professor's most recent research realization must equal or exceed for her to be given the option of remaining. This structure admits the tenure-track sequence of 'spot' standards as a possible (partial) solution to the university's problem, but in no way restricts the values of those standards. The university also adopts a simple variant of the linear incentive model of Holmstrom and Milgron [12], whereby it pays a base wage as well as a bonus that is linear in research output.[13] Our general payment structure places no restrictions on the timing of research bonuses, allowing them to be paid, if at all, immediately upon the research realization and/or in any subsequent period of retention.

The *academic contract* offered to a 'junior' professor is thus a structure, $\mathbf{C} := (w_1, w_2, w_3; b_1, b_2; b_{21}, b_{31}, b_{32}; s_1, s_2)$, comprised of base wages (w_1, w_2, w_3), bonus multipliers $(b_1, b_2; b_{21}, b_{31}, b_{32})$, and retention standards (s_1, s_2).

A professor who accepts $\mathbf{C}$ will receive a salary of

$$S_1(r_1) \quad := \quad w_1 + b_1 r_1 \tag{1}$$

at the end of her first period of employment. In the event that her first research draw $r_1 \geq s_1$, she then has the option of remaining with the university through her second period. If she chooses to remain, she receives a salary of

$$S_2(r_1, r_2) \quad := \quad w_2 + b_2 r_2 + b_{21} r_1 \tag{2}$$

at the end of that period. Similarly, if her $r_2 \geq s_2$, she is given the option of remaining with the university through the third and final period of her working life. If she takes that

[13] In Macleod and Malcomson [18], Pearce and Stacchetti [22], and Hogan [11], a similar base-plus-bonus payment scheme is considered. In those models, only the base wage is part of the explicit contract; the bonus for unobservable effort and is promised only 'implicitly', but, in repeated interaction, it is in the best interest of the firm to honor even the implicit component. Here, where the bonus is tied to observable research output, both the base wage and bonus components are explicit.

option, she receives a salary of

$$S_3(r_1, r_2) \quad := \quad w_3 + b_{31}r_1 + b_{32}r_2. \tag{3}$$

at that period's end. Note that the contract contains no bonus for r_3. Because the professor's working life ends immediately after any r_3 draw, that draw results in no additional revenues for the university, making it obvious that payment of a bonus for that draw will never be profitable. Note also that our contract's general payment structure places no restrictions on the timing of research bonuses, allowing them to be paid, if at all, immediately upon the research realization and/or in any subsequent period of retention.

A number of definitions are useful. For any level of research effort, e (≥ 0), let $\overline{r}(e) := \int_0^\infty r \; d\rho_e[r]$ be the expected value of r. For any $s \geq 0$, let

$$P(e, s) \;:=\; \int_s^\infty d\rho_e[r] \quad \text{and} \quad \overline{R}(e, s) \;:=\; \frac{1}{P(e, s)} \int_s^\infty r \; d\rho_e[r] \tag{4}$$

be, respectively, the probability that $r \geq s$, and the expected value of r, given that $r \geq s$.

We define the ***net benefit*** for the professor during period t to be the net benefit of remaining employed by *this* university under contract $\mathbf{C}$, rather than quitting to nonacademic employment. (We will deal with the possibility of quitting to another university later.) The expected net benefit she extracts from $\mathbf{C}$ depends upon the effort she exerts.

A professor's incentives

To maximize her lifetime expected net benefit under $\mathbf{C}$, the professor must choose optimal effort levels for each period; to do this, she must solve a dynamic programming problem, starting with period 3 and working backward.

Period 3. – Suppose the professor has been retained $\mathbf{C}$ through the first two periods of her career, and that her $r_2 \geq s_2$. If she were to remain with the university, then, in the absence of any bonus for third period research production, her optimal e_3 would be 0, so the net benefit of remaining with this university for the third period of her career would be

$$\mathrm{NB}_3(r_1, r_2) \;\underset{(3)}{=}\; w_3 + b_{31}r_1 + b_{32}r_2. \tag{5}$$

To ensure that the professor would not instead choose to pursue nonacademic employment, we require:

$$\mathrm{NB}_3(r_1, r_2) \quad \geq \quad 0, \quad \forall\; r_1, r_2 \geq 0. \tag{6}$$

Period 2. – Now suppose that a professor has completed the first period of $\mathbf{C}$, that her $r_1 \geq s_1$, and that (6) is satisfied. The expected net benefit of choosing to remain with the university for (at least) the second period of her working life would be

$$\mathrm{NB}_{2,3}(r_1, e_2) \quad = \quad \mathrm{NB}_2(r_1, e_2) + P(e_2, s_2)\,\mathrm{NB}_3(r_1, e_2). \tag{7}$$

Here,

$$\mathrm{NB}_2(r_1, e_2) \quad := \quad w_2 + b_2\overline{r}(e_2) + b_{21}r_1 - e_2^2 \tag{8}$$

is the net benefit of period 2 employment alone, while

$$\mathrm{NB}_3(r_1, e_2) \quad := \quad w_3 + b_{31} r_1 + b_{32} \overline{R}(e_2, s_2).$$

is the expected value of (5), given period-2 effort e_2. If the professor were to choose to remain with the university, she would then choose her optimal level of period-2 research effort, e_2^*, so as to maximize (7). For the university to ensure that a professor will not pursue her nonacademic option at this stage, the contract must satisfy

$$\mathrm{NB}_{2,3}(r_1, e_2^*) \quad \geq \quad 0, \quad \forall\, r_1 \geq 0. \tag{9}$$

Period 1. – Suppose (6) and (9) are satisfied. For a 'junior' professor, aware that her period-2 research effort will be e_2^*, the expected net benefit of the academic contract, given period-1 effort e_1, is

$$\mathrm{NB}_{1,2,3}(e_1, e_2^*) \quad = \quad \mathrm{NB}_1(e_1) + P(e_1, s_1)\, \mathrm{NB}_{2,3}(e_1, e_2^*). \tag{10}$$

Here,

$$\mathrm{NB}_1(e_1) \quad := \quad w_1 + b_1 \overline{r}(e_1) - e_1^2 \tag{11}$$

is the net benefit of period 1 alone, while

$$\mathrm{NB}_{2,3}(e_1, e_2^*) \;:=\; w_2 + b_2 \overline{r}(e_2^*) + b_{21} \overline{R}(e_1, s_1) - (e_2^*)^2 + P(e_2^*, s_2) \left(w_3 + b_{31} \overline{R}(e_1, s_1) + b_{32} \overline{R}(e_2^*, s_2) \right)$$

is the expected value of (7), given period-1 effort e_1 and anticipating optimal period-2 effort e_2^*. If the 'junior' professor were to accept the academic contract, she would choose her optimal level of period-1 research effort, e_1^*, so as to maximize (10). It will be rational for the potential 'junior' professor to accept the academic offer if and only if $\mathrm{NB}_{1,2,3}(e_1^*, e_2^*) \;\geq\; 0$.

To the university, w_1 represents a cost that has no influence on the professor's choice of effort profile, (e_1^*, e_2^*). To minimize its costs, the university will set $w_1 := -b_1 \overline{r}(e_1^*) + (e_1^*)^2 - P(e_1^*, s_1)\, \mathrm{NB}_{2,3}(e_1^*, e_2^*)$, so that the contract satisfies the *minimal recruitment* condition:

$$\mathrm{NB}_{1,2,3}(e_1^*, e_2^*) \quad = \quad 0. \tag{12}$$

We will say that $\mathbf{C}$ is *admissible* if it satisfies (6), (9) and (12). Period-specific expected profits, as of the beginning of each of the contract's three periods, are then given by:

$$\overline{\Pi}_1 \quad = \quad -w_1 - b_1 \overline{r}(e_1^*), \tag{13}$$

$$\overline{\Pi}_2(r_1) \quad = \quad -w_2 - b_2 \overline{r}(e_2^*) + (k - b_{21}) r_1 \qquad \text{and} \tag{14}$$

$$\overline{\Pi}_3(r_1, r_2) \quad = \quad -w_3 + (k - b_{31}) r_1 + (k - b_{32}) r_2. \tag{15}$$

Academic raiding

In addition to the issue of admissibility, a university that hires a 'junior' professor need also consider the possibility that it could be raided for its 'middle-aged' and/or 'senior' professors by another university.

Lazear [15], Bernhardt and Scoones [2] and Waldman [26] each describe a situation where one firm initially employs a worker and where that worker may be raided by an outside firm. Each of those papers establish that raiding will occur only if the worker is a better match with the outside firm; where match-quality is equal across the firms, the initial employer creates a contract to pre-empt raiding.

In our model, there is no match-quality heterogeneity. A professor has no preference for one university over another, and, conditional on her research record, she would generate the same revenues for any university that employs her. If an admissible **C** were then to, say, induce research effort by rewarding research output immediately and strictly upon the realization of that output, its profits would be 'back-loaded', and a strategy of raiding 'middle-aged' and 'senior' professors by offering slightly higher wages than those set out under **C** would earn the outside university higher profits per period than the initial employer. Even if the initial employer were to thwart the outside university by matching its offer, the initial employer's profits would be reduced. But if **C** were instead characterized by 'back-loaded' wages, profits would be 'front-loaded' and a strategy of raiding 'middle-aged' and 'senior' professors by offering slightly higher wages than defined under **C** would earn the outside university lower profits per period than those earned by the initial employer.

We can thus proceed under the assumption that all universities will operate in a *raid-proof equilibrium* where each adopts a strategy of recruiting 'junior' professors using a back-loaded or 'raid-proof' wage structure that acts to pre-empt raiding.

We say that the contract **C** is *strongly raid-proof* if, for all $r_1, r_2 \geq 0$, we have $\overline{\Pi}_2(r_1) \leq \overline{\Pi}_1$ and $\overline{\Pi}_3(r_1, r_2) \leq \overline{\Pi}_1$, where these quantities are as defined in equations (13-15). (Note that 'strong' raid-proofing is sufficient, but not necessary, to make **C** raid-proof.)

The optimal contract

We say that a contract is *tenure-track* if $s_1 > 0$ and $s_2 = 0$ (or, equivalently, $0 < P(e_1, s_1) < 1$ and $P(e_2, s_2) = 1$ for any $e_1, e_2 \geq 0$). We say that the contract induces a *declining effort profile* if $e_1^* > e_2^*$.

Our principle result can now be stated as follows.

Theorem 1 *Assume a raid-proof environment. Let $\{\rho_e\}_{e \in \mathbb{R}_{\not+}}$ be a family of probability distributions on $[0, \infty)$, and let **C** be an admissible, raid-proof contract that maximizes expected profits per period.*

- **(a)** *For all $e \geq 0$, suppose ρ_e is the uniform probability distribution on* $[0, e]$. (That is, $d\rho_e(r) = 1/e$ if $r \in [0, e]$ and $d\rho_e(r) = 0$ if $r > e$.) *Then **C** is tenure-track, with a declining effort profile.*
- **(b)** *For all $e \geq 0$, suppose ρ_e is the exponential probability distribution $d\rho_e(r) = \frac{1}{e}\exp(-r/e)$. Then **C** is tenure-track, with a declining effort profile.*
- **(c)** *For any $\alpha > 1$ and $e \geq 0$, let ρ_e^α be the power law distribution $d\rho_e^\alpha(r) = \frac{e^\alpha\,\alpha}{(e+x)^{\alpha+1}}$. There exist $\underline{\alpha}, \overline{\alpha} \in (1, \infty)$ such that, if $\alpha \in (1, \underline{\alpha})$ or $\alpha \in (\overline{\alpha}, \infty)$, then **C** is tenure-track, with a declining effort profile. In particular, this holds if $\alpha = 2$.*

In the above, we see that the maximization of profits leads the university both to adopt a tenure-track contract and to induce declining effort that results in research production declining, on average, over the life cycle.

The intuition is straightforward. In each of the first and second periods of the contract, the university will apply the same rule: induce a professor's research effort up to the point where the resulting marginal revenue product (MRP) is equal to the (increasing) marginal cost. But because more revenues can be realized from the first research draw than from the second, the optimal level of induced research effort declines from the first to the second period. Accordingly, research output will, on average, decline with age.

In choosing the retention standard to apply at the beginning of each of the second and third periods, the university faces a trade-off: increasing the minimum standard applied to the most recent research draw raises the conditional mean payoff associated with the remaining periods of the contract, but it involves foregoing any benefit from the past research accomplishments of those who fail to meet the current standard. For a professor entering the second period of her working life, whose first research draw will influence both second and third period profitability and who has no record of past accomplishments, a relatively high minimum standard is optimal. But for a professor entering her third period, whose second research draw will influence only third period profitability and whose background includes the accomplishments necessary to have cleared the high first standard, the university optimally tolerates little or even no research output on the professor's part.[14]

IV Outline of Theorem 1 proof

The proof of Theorem 1 is long and appears in the appendix. This section, however, describes the basis for that proof and outlines the major steps involved. (Detailed proofs of all statements appear in the Appendix.)

Recall that our representative university operates in a raid-proof equilibrium. In any period, the university will find itself in one of three 'states': its single faculty 'slot' will be occupied by a 'junior' professor (state 1), a 'middle-aged' professor (state 2), or a 'senior' professor (state 3). Whenever a 'junior' ('middle-aged') incumbent is retained into the following period, the university will transition from state 1 (2) to state 2 (3). If the university cannot 'raid' from other universities, then it can only hire junior professors; thus, whenever any incumbent is not retained into the following period, the university returns to state 1. If other universities will not 'raid' from our representative university, then the probability of retaining a professor is exactly the probability that her research exceeds the minimum standards s_1 and s_2 specified by the contract. Thus, the retention probabilities are $p_1 := P(e_1^*, s_1)$ and $p_2 := P(e_2^*, s_2)$. This data defines a 3-state Markov

[14] Because, in our model, inducing research contributes to revenues only *ex post*, and only on the condition that the worker is retained, it can be thought of as being analogous to a nonacademic employer's investment in training a worker. Indeed, our story can be thought of as being akin to one where the optimal rate of the training investment diminishes as the worker approaches retirement, and where the employer will not tolerate a worker who fails in his initial training, but *will* tolerate a worker who initially reaches an acceptable level of productivity, even if his productivity then ceases to increase any further.

process with transition probability matrix

$$\begin{bmatrix} 1-p_1 & p_1 & 0 \\ 1-p_2 & 0 & p_2 \\ 1 & 0 & 0 \end{bmatrix}. \tag{16}$$

This process has stationary probability distribution (π_1, π_2, π_3) given by

$$\pi_1 \;=\; \frac{1}{1+p_1+p_1p_2}, \quad \pi_2 \;=\; \frac{p_1}{1+p_1+p_1p_2}, \quad \text{and} \quad \pi_3 \;=\; \frac{p_1p_2}{1+p_1+p_1p_2}. \tag{17}$$

Recall that equation (14) gave the expected value of $\overline{\Pi}_2$ at the start of period 2 —i.e. once the realization of r_1 is already *known.* Likewise, (15) gave the expected value of $\overline{\Pi}_3$ at the start of period 3, when the realizations of r_1 and r_2 are both known. However, at the start of period 1, the future values of r_1 and r_2 are both unknown; at this moment, the expected profits which $\mathbf{C}$ will generate in each of three periods of a professor's career are

$$\begin{aligned} \overline{\Pi}_1 &\underset{(13)}{=\!=} -w_1 - b_1\overline{r}(e_1^*); \\ \overline{\Pi}_2 &\underset{(14)}{=\!=} -w_2 - b_2\overline{r}(e_2^*) + (k-b_{21})\overline{R}(e_1^*, s_1); \\ \text{and } \overline{\Pi}_3 &\underset{(15)}{=\!=} -w_3 + (k-b_{31})\overline{R}(e_1^*, s_1) + (k-b_{32})\overline{R}(e_2^*, s_2). \end{aligned} \tag{18}$$

Combining (18) and (17), the expected profit per period of the university is given by

$$\overline{\Pi}(\,\mathbf{C}) \quad := \quad \pi_1\overline{\Pi}_1 + \pi_2\overline{\Pi}_2 + \pi_3\overline{\Pi}_3. \tag{19}$$

The university must find the (raid-proof) contract which maximizes the value of $\overline{\Pi}$. The proof of Theorem 1 now proceeds in three steps:

1. We relax the need to optimize over raid-proof contracts, by showing that a non-raidproof contract can be 'retroactively raidproofed' without affecting its optimality.

2. We show that it suffices to solve the optimization problem over a particularly nice class of contracts we call MNQ ('marginal no-quitting').

3. We establish Theorem 1 for the class of MNQ contracts.

Steps 1 and 2 both use the concept of *contract equivalence.* Let $\mathbf{C}$ and $\widetilde{\mathbf{C}}$ be two academic contracts. We say that $\mathbf{C}$ and $\widetilde{\mathbf{C}}$ are *equivalent* if:

(Eq1) In both contracts, the professor's optimal effort profile (e_1^*, e_2^*) is the same.
(Eq2) Both contracts have the same research standards (s_1, s_2).
(Eq3) Both contracts yield the same expected lifetime net benefit $\mathrm{NB}_{1,2,3}$ for the professor.

In particular, (Eq2) implies that $\mathbf{C}$ is tenure-track if and only if $\widetilde{\mathbf{C}}$ is also tenure-track. (Eq3) implies that $\mathbf{C}$ satisfies minimal recruitment condition (12) if and only if $\widetilde{\mathbf{C}}$ does.

Lemma 2 *If contracts* $\mathbf{C}$ *and* $\widetilde{\mathbf{C}}$ *are equivalent, then both contracts yield the same value of* $\overline{\Pi}$ *in equation* (19). (Thus, $\mathbf{C}$ is $\overline{\Pi}$-maximizing if and only if $\widetilde{\mathbf{C}}$ is.) □

The next proposition accomplishes Step 1 in our proof strategy. Recall that $\overline{r}(e) := \int_0^\infty r\ d\rho_e[r]$.

Proposition 3 *Assume $\overline{r}'(e) \neq 0$ for all $e \geq 0$. Let $\mathbf{C}$ be any admissible, tenure-track contract which is not raid-proof. There exists an admissible, raid-proof contract $\widetilde{\mathbf{C}}$ which is equivalent to $\mathbf{C}$* (and hence, is also tenure-track). □

Proposition 3 says that, to demonstrate that the raid-proof $\overline{\Pi}$-maximizing contract is tenure-track, it suffices to first find a *non*-raid-proof contract which maximizes $\overline{\Pi}$ by being tenure-track, because we can always 'retroactively raidproof' it later.

We will focus on a class of contracts which are especially easy to optimize. We say that $\mathbf{C}$ is a *minimal no-quitting* (MNQ) contract if the conditions (6) and (9) are satisfied with *equalities* —that is,

$$\mathrm{NB}_{2,3}(r_1, e_2^*) = 0, \quad \text{and} \quad \mathrm{NB}_3(r_1, r_2) = 0, \quad \forall\ r_1, r_2 \geq 0. \tag{MNQ}$$

If $\mathbf{C}$ satisfies (MNQ), then $\mathrm{NB}_{2,3} = \mathrm{NB}_2$ and $\mathrm{NB}_{1,2,3} = \mathrm{NB}_1$; this will make it much easier to characterize (and control) the professor's utility-maximizing effort profile (e_1^*, e_2^*). Define $\beta : (0,\infty) \longrightarrow (0,\infty)$ by $\beta(e) := 2e/\overline{r}'(e)$ for all $e > 0$. We will require the family of distributions $\{\rho_e\}_{e\in\mathbb{R}_{\neq}}$ to satisfy the following assumption:

$$\beta \textit{ is a bijection from } (0,\infty) \textit{ to } (0,\infty). \tag{B}$$

One way to satisfy (B) is for β to be strictly increasing, with $\lim\limits_{e\searrow 0} \beta(e) = 0$, and $\lim\limits_{e\nearrow\infty} \beta(e) = \infty$. This just means that there are *not* strongly increasing returns to effort —a very weak assumption. It is easy to check that all the distribution families in Theorem 1 satisfy (B). The next proposition accomplishes Step 2 in our strategy.

Proposition 4 *Suppose $\{\rho_e\}_{e\in\mathbb{R}_{\neq}}$ satisfies* (B).

(a) *Let $\mathbf{C}$ be any contract satisfying minimal recruitment condition* (12). *There is a MNQ contract $\widetilde{\mathbf{C}}$ equivalent to $\mathbf{C}$.*

(b) *Let $\mathbf{C}$ be a profit-maximizing contract in the space of all admissible contracts. Let $\widetilde{\mathbf{C}}$ be a profit-maximizing contract in the space of all admissible MNQ contracts. Then $\widetilde{\mathbf{C}}$ provides the same expected profit per period as $\mathbf{C}$.* □

If hypothesis (B) holds, then Proposition 4(b) implies that, to find the $\overline{\Pi}$-maximizing contract, it suffices to maximize $\overline{\Pi}$ over the set of admissible MNQ contracts. For any MNQ contract, it can be shown that $b_{12} = b_{13} = b_{23} = w_3 = 0$, while the values of w_1 and w_2 are entirely determined by b_1 and b_2 (see Lemma A in the Appendix). Thus, an MNQ contract has only four free parameters: b_1, b_2, s_1, and s_2. Furthermore, we can achieve any desired effort profile (e_1, e_2) and retention probabilities (p_1, p_2) with a suitable choice of parameters $(b_1, b_2; s_1, s_2)$ (see Lemma B in the Appendix). Thus, the space of MNQ contracts can be parameterized by the set of all 4-tuples $(e_1, e_2; p_1, p_2)$. When an MNQ contract is expressed in this form, $\overline{\Pi}$ can be expressed as a function $\overline{\Pi}(e_1, e_2; p_1, p_2)$. With a mild technical assumption, we can then define functions $e_1^* : [0,1]^2 \longrightarrow \mathbb{R}_{\not-}$ and $e_2^* : [0,1]^2 \longrightarrow \mathbb{R}_{\not-}$ such that, for any fixed (p_1, p_2), the values of the parameters (e_1, e_2) which maximize $\overline{\Pi}(e_1, e_2; p_1, p_2)$ are $e_1^*(p_1, p_2)$ and $e_2^*(p_2)$ (see Lemma C). At this point, the $\overline{\Pi}$-maximization problem is reduced to finding the values of p_1^* and p_2^* in $[0,1]$ which maximize the function $\hat{\Pi}(p_1, p_2) := \overline{\Pi}[e_1^*(p_1, p_2), e_2^*(p_1, p_2); p_1, p_2]$. If the family of probability distributions $\{\rho_e\}_{e \in \mathbb{R}_{\not-}}$ and the derivative $\partial_2 \hat{\Pi}$ satisfy certain technical conditions, then the $\hat{\Pi}$-maximizing value of p_2 is $p_2^* = 1$ —in other words, the $\overline{\Pi}$-maximizing MNQ contract is tenure track (see Lemma E(a)). Furthermore, if p_1^* and p_2^* then satisfy certain conditions (in particular, if $p_1^* > 1/2$) then the $\overline{\Pi}$-maximizing MNQ contract induces a declining effort profile (see Lemma E(b)).

In particular, the uniform, exponential, and power-law families of distributions all satisfy the technical conditions required by Lemma E; thus, for all three families of distributions, the $\overline{\Pi}$-maximizing element in the space of MNQ contract is tenure-track, and induces a declining profile of effort (see Lemmas F, G, and H). In other words, the conclusions of Theorem 1 hold for the space of MNQ contracts. Then Proposition 4(b) implies that the conclusions of Theorem 1 hold for the space of *all* contracts. Finally, Proposition 3 implies that the conclusions of Theorem 1 hold for the restricted space of raid-proof contracts; this establishes Theorem 1.

V Concluding Remarks

This paper provides an explanation of the use of tenure-track contracts in academia that arises out of the unique nature of academic productivity and optimizing behavior on the part of the university. The theory, briefly, is that, because a university's mission involves encouraging its faculty to engage in research that is important but yields no saleable results, a professor's marginal revenue product does not depend simply on her current research production. Rather, because her research accomplishments act as a signal of knowledge that serves to attract tuition-paying students, a professor's contribution to the university's revenues, at any point in her career, will depend on the strength of her cumulative research record. The university then profits by dismissing a professor who fails to establish a strong research record initially, but by retaining a professor who establishes a strong record regardless of her research output thereafter.

The theory further provides an explanation for the observation that academic research production declines, on average, with age. The intuition is simple: because the university's opportunities to realize tuition revenues from a professor's spot research accomplishments diminish as she approaches the end of her career, the optimal level of induced research effort, and therefore the expected level of research output, diminishes with age.

Tenure, of course, does not amount to absolute job security. As noted in the Introduction, while tenured professors are not dismissed for poor research productivity, they will be dismissed for failing to perform their teaching duties. Our theory provides a simple explanation: the past research accomplishments of a tenured professor can be translated into the tuition revenues necessary to make her profitable only if she continues to teach.

The theory serves to correct some common misperceptions. In particular, our analysis shows that research production does not fall with age because tenure undermines incentives, and tenure itself is not a measure of security that a university concedes in lieu of compensation or with reluctance to a powerful faculty union.

The most important implication of our theory is that the tolerance for research failure that characterizes tenure is consistent with a university's interest in advancing knowledge through the research production. Although it might seem that a university could produce more research by replacing *any* unproductive scholar, or by providing older professors with greater research incentives, our analysis suggests that the gain in research output would be short-lived. By deviating from its profit-maximizing rule, the university's long-term viability would be undermined.

Similarly, if, under the pressure of system-wide funding constraints, universities as a group were to abolish tenure or adopt post-tenure reviews, our analysis suggests that an efficiency loss would result as the full value of past research accomplishments would go unrealized.

One final comment should be made regarding our model's prediction that a university will adopt tenure-track contracts exclusively. After all, in practice, a university typically hires some faculty on tenure-track while hiring others on a limited-term basis. How can this observed contractual mix be reconciled with our result? Our assumption regarding the translation of a professor's current research output into future tuition revenues is itself based on the implicit assumption that there will exist future demand for the university's instruction. Under these conditions, the tenure-track contract is optimal. But if the university was uncertain as to whether some portion of its current enrollment level would continue into the future, a limited-term hire could be used to meet that portion of current demand without any long-term commitment.

Appendix: Proofs

Proof of Lemma 2. Let $\overline{\Pi}$ be the expected profit per period under $\mathbf{C}$, as defined in eqn.(19). Let $\widetilde{\Pi}$ be the expected profit per period under $\widetilde{\mathbf{C}}$. Then clearly

$$\overline{\Pi} \;=\; \overline{R} - \overline{C} \quad \text{and} \quad \widetilde{\Pi} \;=\; \widetilde{R} - \widetilde{C}, \tag{2.1}$$

where $\overline{R}$ and $\widetilde{R}$ represent the university's expected *revenue* per period under the two contracts, while $\overline{C}$ and $\widetilde{C}$ represent the university's expected *costs* per period.

(Eq1) implies that the professor will exhibit the same probability distribution of research outputs; in particular she will have the same expected values $R_1^* := \overline{R}(e_1^*, s_1)$ and $R_2^* := \overline{R}(e_2^*, s_2)$. Then (Eq2) implies she will have the same retention probabilities (p_1, p_2) in both contracts. Thus equation (17) says both contracts have the same stationary probability distribution (π_1, π_2, π_3) over the three periods. Thus, both contracts generate the same expected revenue per period, namely

$$\widetilde{R} \quad = \quad \pi_1 \cdot 0 + \pi_2 \cdot k\, R_1^* + \pi_3 \cdot k\,(R_1^* + R_2^*) \quad = \quad \overline{R}. \tag{2.2}$$

Let $\overline{S}_1, \overline{S}_2, \overline{S}_3$ denote the professor's expected salaries in the three periods, under $\mathbf{C}$.

Then $\overline{C}$ is simply the professor's expected salary per period, namely:

$$\overline{C} \quad = \quad \pi_1\overline{S}_1 + \pi_2\overline{S}_2 + \pi_3\overline{S}_3 \;\underset{(17)}{=\!=}\; \frac{\overline{S}_1 + p_1(\overline{S}_2 + p_2\overline{S}_3)}{1+p_1+p_1p_2} \quad = \quad \frac{\overline{S}}{1+p_1+p_1p_2},$$

where $\overline{S} := \overline{S}_1 + p_1(\overline{S}_2 + p_2\overline{S}_3)$ is the professor's expected lifetime salary in $\mathbf{C}$. Likewise, $\widetilde{C} := \widetilde{S}/(1+p_1+p_1p_2)$, where $\widetilde{S}$ is the professor's lifetime salary in $\widetilde{\mathbf{C}}$. The professor's expected lifetime net benefit under the two contracts can be expressed by

$$\mathrm{NB}_{1,2,3} \quad = \quad \overline{S} - (e_1^*)^2 - p_1\cdot(e_2^*)^2 \quad \text{and} \quad \widetilde{\mathrm{NB}}_{1,2,3} \quad = \quad \widetilde{S} - (e_1^*)^2 - p_1\cdot(e_2^*)^2.$$

But (Eq3) says $\widetilde{\mathrm{NB}}_{1,2,3} = \mathrm{NB}_{1,2,3}$; hence $\widetilde{S} = \overline{S}$; hence $\widetilde{C} = \overline{C}$. Combining this with equations (2.1) and (2.2), we get $\widetilde{\Pi} = \overline{\Pi}$. □

Proof of Proposition 3. Let (e_1^*, e_2^*) be the utility-maximizing effort profile for $\mathbf{C}$. Let $r_1^* := \overline{r}(e_1^*)$ and $r_2^* := \overline{r}(e_2^*)$. If $\widetilde{\mathbf{C}}$ is equivalent to $\mathbf{C}$, then (e_1^*, e_2^*) will also be the utility-maximizing effort profile for $\widetilde{\mathbf{C}}$ (we will ensure this later). In that case, the expected profit of $\widetilde{\mathbf{C}}$ *before* each period will be given by:

$$\begin{aligned} \overline{\Pi}_1 \;&\underset{(13)}{=\!=}\; -w_1 - b_1 r_1^*; \\ \overline{\Pi}_2(r_1) \;&\underset{(14)}{=\!=}\; -w_2 - b_2 r_2^* + (k-b_{21})r_1; \\ \text{and} \quad \overline{\Pi}_3(r_1,r_2) \;&\underset{(15)}{=\!=}\; -w_3 + (k-b_{31})r_1 + (k-b_{32})r_2. \end{aligned}$$

To make $\widetilde{\mathbf{C}}$ raid-proof, it suffices to ensure that $\overline{\Pi}_3(r_1,r_2) = \overline{\Pi}_2(r_1) = \overline{\Pi}_1$ for all $r_1, r_2 \geq 0$. To do this, we must set

$$b_{21} \;:=\; b_{31} \;:=\; b_{32} \;:=\; k; \tag{3.1}$$

$$w_3 \;:=\; w_1 + b_1 r_1^*; \quad \text{and} \tag{3.2}$$

$$w_2 \;:=\; w_1 + b_1 r_1^* - b_2 r_2^*. \tag{3.3}$$

The net benefit of contract $\widetilde{\mathbf{C}}$ for the professor during period 3 is then

$$\widetilde{\mathrm{NB}}_3(r_1,r_2) \quad = \quad w_3 + kr_1 + kr_2, \qquad \text{by (5) and (3.1).} \tag{3.4}$$

At the beginning of period 2, the value of r_1 is known, and the expected future value of $\widetilde{\mathrm{NB}}_3$, as a function of e_2, is given:

$$\widetilde{\mathrm{NB}}_3(r_1,e_2) \;\underset{(3.4)}{=\!=}\; w_3 + kr_1 + k\overline{r}(e_2). \tag{3.5}$$

Let $\widetilde{\mathrm{NB}}_{2,3}$ be the net benefit of $\widetilde{\mathbf{C}}$ at the start of period 2 (including the anticipated future benefit of period 3). By hypothesis, $\mathbf{C}$ is tenure-track (i.e. $p_2 = 1$); hence, to be equivalent, $\widetilde{\mathbf{C}}$ must also be tenure-track. In this case, the expected value of $\widetilde{\mathrm{NB}}_{2,3}$ at the beginning of period 2, as a function of e_2, is given:

$$\begin{aligned}
\widetilde{\mathrm{NB}}_{2,3}(r_1, e_2) &\underset{(7)}{=} \widetilde{\mathrm{NB}}_2(r_1, e_2) + \widetilde{\mathrm{NB}}_3(r_1, e_2) \\
&\underset{(8,3.1)}{=\!=} w_2 + b_2\overline{r}(e_2) + kr_1 + \widetilde{\mathrm{NB}}_3(r_1, e_2) - e_2^2 \\
&\underset{(3.5)}{=} (w_2 + w_3) + 2k\, r_1 + (k + b_2)\overline{r}(e_2) - e_2^2 \\
&\underset{(3.2,3.3)}{=\!=} 2w_1 + 2b_1 r_1^* - b_2 r_2^* + 2k\, r_1 + (k + b_2)\overline{r}(e_2) - e_2^2. \qquad (3.6)
\end{aligned}$$

Let s_1 be the period 1 standard of $\mathbf{C}$ (and hence, of $\widetilde{\mathbf{C}}$). If the professor exerts effort e_1 during period 1, and is retained during period 2, then the conditionally expected value of r_1, given this information, is $\overline{R}(e_1) := \overline{R}(e, s_1)$ [see eqn.(4)]. Thus, the expected future value of $\widetilde{\mathrm{NB}}_{2,3}$ at the beginning of period 1, as a function of e_1 and e_2, is given:

$$\widetilde{\mathrm{NB}}_{2,3}(e_1, e_2) \underset{(3.6)}{=} 2w_1 + 2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1) + (k + b_2)\overline{r}(e_2) - e_2^2. \qquad (3.7)$$

Let $\widetilde{\mathrm{NB}}_{1,2,3}$ be the lifetime net benefit of $\widetilde{\mathbf{C}}$ at the start of period 1 (including the anticipated potential future benefits in periods 2 and 3). For any $e \geq 0$, let $P(e) := p(e, s_1)$ [see eqn.(4)]. Thus, the expected value of $\widetilde{\mathrm{NB}}_{1,2,3}$, as a function of e_1 and e_2, is

$$\begin{aligned}
\widetilde{\mathrm{NB}}_{1,2,3}(e_1, e_2) &\underset{(10,11)}{=\!=} w_1 + b_1\overline{r}(e_1) + P(e_1)\cdot\widetilde{\mathrm{NB}}_{2,3}(e_1, e_2) - e_1^2 \\
&\underset{(3.7)}{=} w_1 + b_1\overline{r}(e_1) + P(e_1)\Big(2w_1 + 2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1) + (k + b_2)\overline{r}(e_2) - e_2^2\Big) - e_1^2 \\
&= \Big(1 + 2P(e_1)\Big)\, w_1 + b_1\overline{r}(e_1) - e_1^2 \\
&\qquad + P(e_1)\Big(2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1) + (k + b_2)\overline{r}(e_2) - e_2^2\Big). \qquad (3.8)
\end{aligned}$$

Let $p_1 := P(e_1^*, s_1)$ and let $\overline{R}_1^* := \overline{R}(e_1^*)$. If the professor exerted effort profile (e_1^*, e_2^*), then the expected lifetime net benefit of $\widetilde{\mathbf{C}}$ would be

$$\begin{aligned}
&\widetilde{\mathrm{NB}}_{1,2,3}(e_1^*, e_2^*) \\
&\quad\underset{(3.8)}{=} \Big(1 + 2P(e_1^*)\Big)\, w_1 + b_1\overline{r}(e_1^*) - (e_1^*)^2 \\
&\qquad\qquad + P(e_1^*)\Big(2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1^*) + (k + b_2)\overline{r}(e_2^*) - (e_2^*)^2\Big) \\
&\quad= (1 + 2p_1)w_1 + b_1 r_1^* - (e_1^*)^2 + p_1\Big(2b_1 r_1^* + 2k\,\overline{R}_1^* + k r_2^* - (e_2^*)^2\Big). \qquad (3.9)
\end{aligned}$$

The expected lifetime net benefit offered by contract $\mathbf{C}$ is $\mathrm{NB}_{1,2,3} = 0$, because $\mathbf{C}$ is admissible by hypothesis. We must also make $\widetilde{\mathrm{NB}}_{1,2,3} = 0$. For any values of b_1 and b_2, we can achieve this by setting

$$w_1 = w_1(b_1) := \frac{-b_1 r_1^* - p_1\Big(2b_1 r_1^* + 2k\,\overline{R}_1^* + k r_2^* - (e_2^*)^2\Big) + (e_1^*)^2}{1 + 2p_1}. \qquad (3.10)$$

At this point, $\widetilde{\mathbf{C}}$ has only two free parameters: b_1 and b_2. Substituting eqn.(3.10) into (3.7) and (3.8), we define, for all $b_1, b_2 \in \mathbb{R}$, the functions

$$\begin{aligned}
&\widetilde{\mathrm{NB}}_{2,3}(b_1, b_2; e_1, e_2) \\
&\quad := \; 2w_1(b_1) + 2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1) + (k + b_2)\overline{r}(e_2) - e_2^2, \quad \text{and} \qquad (3.11) \\
&\widetilde{\mathrm{NB}}_{1,2,3}(b_1, b_2; e_1, e_2) \; := \; \Big(1 + 2P_1(e_1)\Big)\, w_1(b_1) + b_1 \overline{r}(e_1) - e_1^2 \\
&\quad + P(e_1)\left(2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1) + (k + b_2)\overline{r}(e_2) - e_2^2\right). \qquad (3.12)
\end{aligned}$$

Now we must choose b_1, b_2 so that the effort profile (e_1^*, e_2^*) is still optimal for the professor under contract $\widetilde{\mathbf{C}}$. That is, we must ensure that

$$\partial_{e_2}\, \widetilde{\mathrm{NB}}_{2,3}(b_1, b_2; e_1^*, e_2^*) \;=\; 0 \quad \text{and} \quad \partial_{e_1}\, \widetilde{\mathrm{NB}}_{1,2,3}(b_1, b_2; e_1^*, e_2^*) \;=\; 0; \qquad (3.13)$$

Differentiating eqn.(3.11) we get $\partial_{e_2}\, \widetilde{\mathrm{NB}}_{2,3}(b_1, b_2; e_1^*, e_2^*) = (k + b_2)\overline{r}'(e_2^*) - 2e_2^*$. Thus, we have $\partial_{e_2}\, \widetilde{\mathrm{NB}}_{2,3}(b_1, b_2; e_1^*, e_2^*) = 0$ if and only if

$$b_2 = \frac{2e_2^*}{\overline{r}'(e_2^*)} - k. \qquad (3.14)$$

Differentating eqn.(3.12), we get a (complicated) expression for $\partial_{e_1}\, \widetilde{\mathrm{NB}}_{2,3}(b_1, b_2; e_1^*, e_2^*)$. Solving for b_1 to satisfy eqn.(3.13), we get

$$b_1 \;=\; \frac{B}{\overline{r}'(e_1^*)\,(2\,p_1 + 1)}, \qquad (3.15)$$

where $B := 4\,P'(e_1^*)p_1\,k\overline{R}_1^* + 2\,P'(e_1^*)p_1\,kr_2^* - 2\,P'(e_1^*)p_1\,(e_2^*)^2 - 2\,P'(e_1^*)(e_1^*)^2 + 2\,e_1^* + 4\,e_1^* p_1 + P'(e_1^*)\;b_2\,r_2^* + 2\,P'(e_1^*)\;b_2\,r_2^*\,p_1 - 2\,P'(e_1^*)k\overline{R}(e_1^*) - 4\,P'(e_1^*)k\overline{R}(e_1^*)p_1 - P'(e_1^*)r(e_2^*)k - 2\,P'(e_1^*)r(e_2^*)kp_1 - P'(e_1^*)r(e_2^*)\;b_2 - 2\,P'(e_1^*)r(e_2^*)\;b_2\,p_1 + P'(e_1^*)(e_2^*)^2 + 2\,P'(e_1^*)(e_2^*)^2 p_1 - 2\,P(e_1^*)k\overline{R}'(e_1^*) - 4\,P(e_1^*)k\overline{R}'(e_1^*)p_1$.

Proof of contract equivalence. The expressions (3.14) and (3.15) are well-defined because $\overline{r}'(e_2^*) \neq 0$ and $\overline{r}'(e_1^*) \neq 0$ by hypothesis. If we define b_1 and b_2 as in (3.14) and (3.15), then the equations (3.13) hold, so the professor's optimal effort profile is (e_1^*, e_2^*), as desired. Thus, condition (Eq1) is satisfied. If we then substitute the value of $w_1(b_1)$ from eqn.(3.10) into expression (3.9), we will get $\widetilde{\mathrm{NB}}_{1,2,3} = 0 = \mathrm{NB}_{1,2,3}$; thus, condition (Eq3) is satisfied. Condition (Eq2) is satisfied automatically because we have assumed that both $\mathbf{C}$ and $\widetilde{\mathbf{C}}$ have the same value for s_1, and set $s_2 = 0$.

Proof that $\widetilde{\mathbf{C}}$ *is admissible.* $\widetilde{\mathbf{C}}$ satisfies (12) because $\mathbf{C}$ does, by condition (Eq3). Now, $\mathbf{C}$ also satisfies the 'no quitting' constraints (6) and (9), so $\mathrm{NB}_{2,3} \geq 0$ and $\mathrm{NB}_3 \geq 0$; thus, it suffices to show that $\widetilde{\mathrm{NB}}_{2,3} \geq \mathrm{NB}_{2,3}$ and $\widetilde{\mathrm{NB}}_3 \geq \mathrm{NB}_3$. To do this, first note that (5) implies

$$\widetilde{\mathrm{NB}}_3 - \mathrm{NB}_3 \;=\; \widetilde{S}_3 - \overline{S}_3. \qquad (3.16)$$

Also, $\widetilde{\mathbf{C}}$ and $\mathbf{C}$ induce the same effort profile (e_1^*, e_2^*); thus, the professor experiences the same disutility of effort $(e_2^*)^2$ in period 2 of both contracts; thus, equation (8) implies that $\widetilde{\mathrm{NB}}_2 - \mathrm{NB}_2 = \widetilde{S}_2 - \overline{S}_2$. Furthermore, $p_2 = 1$ in both contracts; thus, equation (7) implies that

$$\widetilde{\mathrm{NB}}_{2,3} - \mathrm{NB}_{2,3} \;=\; (\widetilde{\mathrm{NB}}_2 - \mathrm{NB}_2) + (\widetilde{\mathrm{NB}}_3 - \mathrm{NB}_3) \underset{(3.16)}{=\!=} (\widetilde{S}_2 - \overline{S}_2) + (\widetilde{S}_3 - \overline{S}_3). \quad (3.17)$$

Lemma 2 says $\overline{\Pi} = \widetilde{\Pi}$. But $\widetilde{\mathbf{C}}$ is raid-proof, while $\mathbf{C}$ was not. This means we must have $\widetilde{\Pi}_1 \geq \overline{\Pi}_1$, while $\widetilde{\Pi}_2 \leq \overline{\Pi}_2$ and $\widetilde{\Pi}_3 \leq \overline{\Pi}_3$. Since both contracts yield the same expected revenue (2.2) in each period, this can only mean that $\widetilde{S}_2 \geq \overline{S}_2$ and $\widetilde{S}_3 \geq \overline{S}_3$. Substituting this into equations (3.16) and (3.17) yields $\widetilde{\mathrm{NB}}_3 - \mathrm{NB}_3 \geq 0$ and $\widetilde{\mathrm{NB}}_{2,3} - \mathrm{NB}_{2,3} \geq 0$; hence $\widetilde{\mathbf{C}}$ satisfies (6) and (9). □

To prove Proposition 4, we need the following lemma.

Lemma A *Suppose contract* $\mathbf{C}$ *satisfies minimal recruitment condition* (12) *and constraint* (MNQ)*, and suppose* $\{\rho_e\}_{e\in\mathbb{R}_{\neq}}$ *satisfies* (B)*. Define* $\epsilon := \beta^{-1} : (0,\infty) \longrightarrow (0,\infty)$.

(a) *The professor's optimal effort profile is given by* $e_1^* = \epsilon(b_1)$ *and* $e_2^* = \epsilon(b_2)$.

(b) *Let* $\omega(b) := \epsilon(b)^2 - b\,\overline{r}[\epsilon(b)]$*. Then* $\mathbf{C}$ *must have* $b_{12} = b_{13} = b_{23} = w_3 = 0$, $w_2 = \omega(b_2)$*, and* $w_1 = \omega(b_1)$.

Proof: Hypothesis (B) implies β is invertible. Examining eqn.(5) reveals that, to make $\mathrm{NB}_3 = 0$ for all $r_1, r_2 \geq 0$, we must set $b_{13} := b_{23} := w_3 := 0$. We then have

$$\mathrm{NB}_{2,3}(r_1, e_2) \underset{(7)}{=} \mathrm{NB}_2(r_1, e_2) \underset{(8)}{=} w_2 + b_2\overline{r}(e_2) + b_{21}r_1 - e_2^2.$$

Thus, the optimal effort e_2^* is the solution to the equation $b_2\overline{r}'(e_2) \;=\; 2e_2$. It is easy to check that $e_2^* := \epsilon(b_2)$ is the unique solution to this equation. To ensure that $\mathrm{NB}_2 = 0$ for all $r_1 \geq 0$, we must then set $b_{21} := 0$ and set $w_2 = \omega(b_2)$. We then have

$$\mathrm{NB}_{1,2,3}(e_1, e_2^*) \underset{(10)}{=\!=} \mathrm{NB}_1(e_1) \underset{(11)}{=\!=} w_1 + b_1\overline{r}(e_1) - e_1^2.$$

Thus, e_1^* is the solution to the equation $b_1\overline{r}'(e_1) \;=\; 2e_1$; again, the unique solution is $e_1^* := \epsilon(b_1)$. If we finally set $w_1 = \omega(b_1)$, then we satisfy (12). □

Proof of Proposition 4. (a) Suppose $\mathbf{C}$ has optimal effort profile (e_1^*, e_2^*) and standards (s_1, s_2). Let $\widetilde{\mathbf{C}}$ have the same standards (s_1, s_2) (so that (Eq2) is satisfied), and set $b_1 := \beta(e_1^*)$, $b_2 := \beta(e_2^*)$, $b_{12} = b_{13} = b_{23} = w_3 = 0$, $w_2 = \omega(b_2)$, and $w_1 = \omega(b_1)$. Lemma A says that $\widetilde{\mathbf{C}}$ is a MNQ contract which also has optimal effort profile (e_1^*, e_2^*). Thus, (Eq1) is satisfied. Lemma A also says that $\widetilde{\mathbf{C}}$ satisfies (12); thus (Eq3) is satisfied.

(b) If $\mathbf{C}$ is the globally $\overline{\Pi}$-maximizing contract, then part (a) yields an MNQ contract $\widetilde{\mathbf{C}}$ which is equivalent to $\mathbf{C}$, hence yields the same value of $\overline{\Pi}$ (by Lemma 2), hence is also $\overline{\Pi}$-maximixing. If $\mathbf{C}$ satisfies (12), then so does $\widetilde{\mathbf{C}}$, by (Eq3). Finally, any MNQ contract automatically satisfies (6) and (9); thus, $\widetilde{\mathbf{C}}$ is admissible. □

For any $e \geq 0$, define $P_e(s) := P(e,s)$. Then $P_e : [0,\infty)\longrightarrow(0,1]$ is a strictly decreasing bijection; hence invertible. Define $\varsigma(e,p) := P_e^{-1}(p)$. It is easy to prove the next result.

Lemma B *For any $e_1, e_2 \geq 0$ and $p_1, p_2 \in [0,1]$, we can achieve the effort profile (e_1, e_2) and retention probabilities (p_1, p_2) with the MNQ contract $(b_1, b_2; s_1, s_2)$ defined by $b_k = \beta(e_k)$ and $s_k = \varsigma(e_k, p_k)$.* □

If $b_{12} = b_{13} = b_{23} = w_3 = 0$, with $w_2 = \omega(b_2)$, and $w_1 = \omega(b_1)$ as specified in Lemma A, and b_1, b_2, s_1 and s_2 are as specified in Lemma B, then equations (18) become:

$$\begin{aligned} \overline{\Pi}_1(e_1,e_2;p_1,p_2) &= -e_1^2; \\ \overline{\Pi}_2(e_1,e_2;p_1,p_2) &= k\,\widetilde{R}(e_1,p_1) - e_2^2; \\ \text{and}\quad \overline{\Pi}_3(e_1,e_2;p_1,p_2) &= k\,\widetilde{R}(e_1,p_1) + k\,\widetilde{R}(e_2,p_2), \end{aligned} \tag{B.1}$$

where $\widetilde{R}(e,p) := \overline{R}[e,\varsigma(e,p)]$. Substituting (B.1) and (17) into (19), the expected profit for the University is given by

$$\overline{\Pi}(e_1,e_2;p_1,p_2) \;=\; \frac{-e_1^2 + p_1\left(-e_2^2 + k\,\widetilde{R}(e_1,p_1)\right) + p_1p_2k\,\left(\widetilde{R}(e_1,p_1) + \widetilde{R}(e_2,,p_2)\right)}{1+p_1+p_1p_2}. \tag{B.2}$$

Lemma C *Assume hypothesis* (B). *For any $p \in [0,1]$ and $e > 0$, define $\gamma_p(e) := e/\partial_1\,\widetilde{R}(e,p)$. Suppose that, for all $p \in [0,1]$, the function $\gamma_p : \mathbb{R}_{\not-}\longrightarrow\mathbb{R}_{\not-}$ is bijective.*

(a) *For any fixed (p_1,p_2), the values of (e_1,e_2) which maximize the value of $\overline{\Pi}(e_1,e_2;p_1,p_2)$ are given by*

$$e_1^*(p_1,p_2) \;:=\; \gamma_{p_1}^{-1}\left(\frac{k\,p_1(1+p_2)}{2}\right) \qquad \textit{and} \qquad e_2^*(p_2) \;:=\; \gamma_{p_2}^{-1}\left(\frac{k\,p_2}{2}\right). \tag{C.1}$$

(b) *In particular, suppose $\widetilde{R}(e,p) = e\,L(p)$ for some function $L : [0,1]\longrightarrow\mathbb{R}_+$. Then $e_1^*(p_1,p_2) \;=\; L(p_1)\,k\,p_1(1+p_2)/2$ and $e_2^*(p_2) \;=\; L(p_2)\,k\,p_2/2$.*

Proof: (a) Differentiating (B.2) we get

$$\begin{aligned} \partial_{e_1}\,\overline{\Pi}(e_1,e_2;p_1,p_2) &= \frac{-2e_1 + k\,(p_1+p_1p_2)\,\partial_1\,\widetilde{R}(e_1,p_1)}{1+p_1+p_1p_2} \\ \text{and}\qquad \partial_{e_2}\,\overline{\Pi}(e_1,e_2;p_1,p_2) &= \frac{-2p_1e_2 + k\,p_1p_2\,\partial_1\,\widetilde{R}(e_2,p_2)}{1+p_1+p_1p_2}. \end{aligned}$$

To make the numerators of these expressions zero, we need

$$\frac{e_1}{\partial_1\,\widetilde{R}(e_1,p_1)} \;=\; \frac{k\,(p_1+p_1p_2)}{2} \qquad\text{and}\qquad \frac{e_2}{\partial_1\,\widetilde{R}(e_2,p_2)} \;=\; \frac{k\,p_2}{2},$$

which is achieved by eqn.(C.1).

(b) If $\widetilde{R}(e,p) = e\,L(p)$, then $\partial_1\,\widetilde{R}(e,p) = L(p)$, so $\gamma_p(e) = e/L(p)$, so $\gamma_p^{-1}(x) = L(p)\,x$. Now apply part (a). □

If the hypotheses of Lemma C are satisfied, then the $\overline{\Pi}$-maximization problem is reduced to finding the $(p_1, p_2) \in [0,1]^2$ which maximize the function

$$\hat{\Pi}(p_1, p_2) \quad := \quad \overline{\Pi}[e_1^*(p_1, p_2), e_2^*(p_1, p_2); p_1, p_2]. \tag{C.2}$$

The family of distributions $\{\rho_e\}_{e \in \mathbb{R}_{\neq}}$ is ***tenable*** if it satisfies two conditions:

(T1) $\overline{R}(e, s) = c_1 e + c_2 s$ for some constants $c_1, c_2 \in \mathbb{R}_+$.

(T2) $\varsigma(e, p) = e\, S(p)$ for some function $S : [0,1] \longrightarrow \mathbb{R}_+$, with $S(1) = 0$.

'Tenability' is a technical condition with no obvious economic interpretation. However, we will later see that all three distribution families in Theorem 1 are tenable.

Lemma D *Suppose* $\{\rho_e\}_{e \in \mathbb{R}_{\neq}}$ *is tenable. Then hypothesis* (B) *holds. Define* $L(p) := c_1 + c_2\, S(p)$*. Then* $\widetilde{R}(e, p) = e\, L(p)$*, so Lemma* C(b) *applies. Furthermore,*

$$\hat{\Pi}(p_1, p_2) \quad = \quad \frac{k^2\, p_1}{4} \left(\frac{p_1 L(p_1)^2 (1 + p_2)^2 + L(p_2)^2 p_2^2}{1 + p_1 + p_1 p_2} \right). \tag{D.1}$$

Thus,

$$\partial_2 \hat{\Pi}(p_1, p_2) \quad = \quad \frac{k^2\, p_1\, \Xi(p_1, p_2)}{4(1 + p_1 + p_1 p_2)^2}, \qquad \textit{where} \tag{D.2}$$

$$\begin{aligned} \Xi(p_1, p_2) \quad := \quad & 2(1 + p_1 + p_1 p_2) \left(p_1\, L(p_1)^2 (1 + p_2) + L(p_2) L'(p_2) p_2^2 + L(p_2)^2 p_2 \right) \\ & - p_1 \left(p_1 L(p_1)^2 (1 + p_2)^2 + L(p_2)^2 p_2^2 \right). \end{aligned} \tag{D.3}$$

Proof: For any $e \geq 0$, we have $\overline{r}(e) = \overline{R}(e, 0) \underset{(\mathrm{T1})}{=\!=} c_1 e$; thus, $\overline{r}'(e) = c_1 > 0$ is constant, so $\beta(e) := 2e/\overline{r}'(e) = 2e/c_1$ satisfies condition (B). Now,

$$\begin{aligned} \widetilde{R}(e, p) \quad &= \quad \overline{R}[e, \varsigma(e, p)] \underset{(\mathrm{T1})}{=\!=} c_1 e + c_2 \varsigma(e, p) \underset{(\mathrm{T2})}{=\!=} c_1 e + c_2 e\, S(p) \\ &= \quad e(c_1 + c_2 S(p)) = e\, L(p). \end{aligned} \tag{D.4}$$

Equation (D.4) means that Lemma C(b) is applicable, so the functions $e_1^*(p_1, p_2)$ and $e_2^*(p_2)$ are well-defined. We define

$$\widehat{R}_1(p_1, p_2) \quad := \quad \widetilde{R}[e_1^*(p_1, p_2), p_1] \underset{(\mathrm{D.4})}{=\!=} e_1^*(p_1, p_2)\, L(p_1), \tag{D.5}$$

$$\text{and} \quad \widehat{R}_2(p_2) \quad := \quad \widetilde{R}[e_2^*(p_2), p_2] \underset{(\mathrm{D.4})}{=\!=} e_2^*(p_2)\, L(p_2). \tag{D.6}$$

Substitute (B.2), (D.5) and (D.6) into (C.2) to obtain

$$\begin{aligned} \hat{\Pi}(p_1, p_2) \quad &= \quad \frac{-e_1^*(p_1, p_2)^2 + p_1 \left(-e_2^*(p_2)^2 + k\, \widehat{R}_1(p_1, p_2) \right) + k\, p_1 p_2 \left(\widehat{R}_1(p_1, p_2) + \widehat{R}_2(p_2) \right)}{1 + p_1 + p_1 p_2} \\ &= \quad \frac{k p_1 (1 + p_2) \widehat{R}_1(p_1, p_2) - e_1^*(p_1, p_2)^2}{1 + p_1 + p_1 p_2} \; + \; p_1 \left(\frac{k p_2 \widehat{R}_2(p_2) - e_2^*(p_2)^2}{1 + p_1 + p_1 p_2} \right). \end{aligned} \tag{D.7}$$

Now,

$$\begin{aligned}
kp_1(1+p_2)\widehat{R}_1(p_1,p_2) - e_1^*(p_1,p_2)^2 &\underset{\text{(D.5)}}{=\!=} kp_1(1+p_2)\, e_1^*(p_1,p_2)\, L(p_1) - e_1^*(p_1,p_2)^2 \\
&\underset{(*)}{=\!=} k^2p_1^2(1+p_2)^2L(p_1)^2/2 - k^2p_1^2(1+p_2)^2L(p_1)^2/4 \\
&= k^2p_1^2(1+p_2)^2L(p_1)^2/4. \qquad \text{(D.8)}\\
\text{and} \quad kp_2\widehat{R}_2(p_2) - e_2^*(p_2)^2 &\underset{\text{(D.6)}}{=\!=} kp_2\, e_2^*(p_2)\, L(p_2) - e_2^*(p_2)^2 \\
&\underset{(*)}{=\!=} L(p_2)^2\, k^2\, p_2^2/2 - L(p_2)^2\, k^2\, p_2^2/4 \\
&= L(p_2)^2\, k^2\, p_2^2/4. \qquad \text{(D.9)}
\end{aligned}$$

where $(*)$ is Lemma C(b). Substituting (D.8) and (D.9) into (D.7) we get

$$\hat{\Pi}(p_1,p_2) \quad = \quad \frac{k^2p_1^2(1+p_2)^2L(p_1)^2 \ + \ p_1L(p_2)^2\, k^2\, p_2^2}{4(1+p_1+p_1p_2)},$$

which we factor to obtain (D.1). Differentiating (D.1) yields (D.2). □

Lemma E *Suppose* $\{\rho_e\}_{e\in\mathbb{R}_{\neq}}$ *is tenable, and let* Ξ *be as in equation* (D.3).

(a) *Suppose* $\Xi(p_1,p_2) \geq 0$ *for all* $(p_1,p_2) \in [0,1]^2$. *Then the* $\overline{\Pi}$*-maximizing MNQ contract is tenure-track* (i.e. $p_2^* = 1$).

(b) *In this case* $e_1^* = k(c_1 + c_2S(p_1^*))p_1^*$ *and* $e_2^* = kc_1/2$. *Thus, if* $S(p_1) > c_1(1-2p_1)/2c_2p_1$ *then the* $\overline{\Pi}$*-maximizing MNQ contract induces a declining effort profile* (i.e. $e_1^* > e_2^*$). *In particular, if* $p_1^* > 1/2$, *then* $e_1^* > e_2^*$.

Proof: Part (a) follows immediately from eqn.(D.2). Part (b) follows by substituting $p_2^* = 1$ into Lemma C(b); note that $L(1) = c_1 + c_2S(1) = c_1$, because $S(1) = 0$. □

We are now in a position to prove the equivalent of Theorem 1 in the restricted setting onf MNQ contracts. This is the content of the next three lemmas.

Lemma F *Suppose* $\{\rho_e\}_{e\in\mathbb{R}_{\neq}}$ *is the family of uniform distributions from* Theorem 1(a). *Then the* $\overline{\Pi}$*-maximizing MNQ contract is tenure-track, with a declining effort profile.*

Proof: For all $0 \leq s \leq e$ we have $P(e,s) \ = \ (e-s)/e$; hence $\varsigma(e,p) = e(1-p)$. Also, $\overline{R}(e,s) \ = \ (e+s)/2$. Thus, setting $c_1 = c_2 = \frac{1}{2}$ and $S(p) = 1-p$, we see that $\{\rho_e\}$ is tenable, so we can apply Corollary E. We have $L(p) = (2-p)/2$ in Lemma D. Substitute this expression for $L(p)$ into eqn.(D.3) to get $\Xi(p_1,p_2) \ = \ f(p_1,p_2)/4$, where

$$\begin{aligned}
f(p_1,p_2) \ := \ & 16\,p_1p_2 + 8\,p_1 + 8\,p_2 - 12\,p_2^2 - 6\,p_1^3p_2 - 4\,p_1^2 - 2\,p_1^3 + 4\,p_2^3 - 8\,p_1p_2^2 \\
& + 4\,p_1^2p_2^2 - 4\,p_1^3p_2^2 + 2\,p_1^4p_2 + p_1^4p_2^2 + p_1^4 - 4\,p_1p_2^3 + 3\,p_1p_2^4.
\end{aligned}$$

Claim 1: $f(p_1,p_2) \geq 0$ *for all* $(p_1,p_2) \in [0,1]^2$.

Proof: Let

$$\begin{aligned} g(p_1,p_2) \quad := \quad & 16\,p_1p_2 + 8\,p_1 + 8\,p_2 - 12\,p_2^2 - 6\,\underline{p_1^1}p_2 - 4\,\underline{p_1^1} - 2\,\underline{p_1^1} + 4\,p_2^3 - 8\,p_1p_2^2 \\ & \quad + 4\,p_1^2p_2^2 - 4\,\underline{p_1^2}p_2^2 + 2\,p_1^4p_2 + p_1^4p_2^2 + p_1^4 - 4\,p_1\underline{p_2^2} + 3\,p_1p_2^4 \\ = \quad & 10\,p_1p_2 + 2\,p_1 + 8\,p_2 - 12\,p_2^2 + 4\,p_2^3 - 12\,p_1p_2^2 + 2\,p_1^4p_2 + p_1^4p_2^2 + p_1^4 + 3\,p_1p_2^4. \end{aligned}$$

Claim 1.1: $g(p_1,p_2) \leq f(p_1,p_2)$, *for all* $(p_1,p_2) \in [0,1]^2$.

Proof: Suppose $0 < n < m$. If $0 \leq x \leq 1$ then $x^n \geq x^m$; hence $-x^n \leq -x^m$.
We obtained $g(p_1,p_2)$ by taking the expression for $f(p_1,p_2)$ and decreasing the exponents on the underlined negative terms. Each of these terms is made smaller by this change (by previous paragraph); thus, $g(p_1,p_2) \leq f(p_1,p_2)$. ∇ `Claim 1.1`

Claim 1.2: $\partial_1\, g(p_1,p_2) \geq 0$ *for all* $(p_1,p_2) \in [0,1]^2$.

Proof: $\partial_1\, g(p_1,p_2) \;=\; (8\,p_2 + 4\,p_2^2 + 4)\,p_1^3 + 10\,p_2 + 2 - 12\,p_2^2 + 3\,p_2^4$. Thus, $\partial_1\, g(p_1,p_2) < 0$ if and only if $-p_1^3 \;>\; h(p_2)$, where

$$h(p_2) \quad := \quad \frac{2 + 10\,p_2 - 12\,p_2^2 + 3\,p_2^4}{8\,p_2 + 4\,p_2^2 + 4}.$$

The denominator of $h(p_2)$ is clearly positive for $p_2 \in [0,1]$. The numerator of $h(p_2)$ is $H(p_2) := 2 + 10\,p_2 - 12\,p_2^2 + 3\,p_2^4$. It suffices to show that $H(p_2) \geq 0$ for $p_2 \in [0,1]$. But $H'(p_2) \;=\; 10 - 24\,p_2 + 12\,p_2^3$ has only one root in $[0,1]$, which corresponds to a (positive) maximum of H. Thus, H has no interior minima in $[0,1]$. Now, $H(0) = 2 > 0$ and $H(1) = 3 > 0$; thus, $H(p_2) > 0$ for all $p_2 \in [0,1]$. Thus, $h(p_2) > 0$ for all $p_2 \in [0,1]$, so it is impossible for $-p_1^3 \;>\; h(p_2)$ (because $p_1 > 0$). Thus, $\partial_1\, g(p_1,p_2) \geq 0$. ∇ `Claim 1.2`

Claim 1.3: $g(p_1,p_2) \geq 0$ *for all* $(p_1,p_2) \in [0,1]^2$.

Proof: Claim 1.2 implies that $g(p_1,p_2)$ is increasing in p_1; thus, it suffices to check that $g(0,p_2) \geq 0$ for all $p_2 \in [0,1]$. But $g(0,p_2) = G(p_2) := 8\,p_2 - 12\,p_2^2 + 4\,p_2^3$. Now, $G'(p_2) = 8 - 24\,p_2 + 12\,p_2^2$ has roots $1 \pm \sqrt{3}/3$. Only one of these roots is in $[0,1]$, and it corresponds to a maximum of G. Also, $G(0) = 0 = G(1)$. Thus, $G(p_2) \geq 0$ for all $p_2 \in [0,1]$. Thus, $g(p_1,p_2) \geq 0$ for all $(p_1,p_2) \in [0,1]^2$. ∇ `Claim 1.3`

Claims 1.1 and 1.3 together imply that $f(p_1,p_2) \geq 0$ for all $(p_1,p_2) \in [0,1]^2$. $\diamond$ `Claim 1`

Claim 1 and Corollary E(a) imply that the $\overline{\Pi}$-maximizing contract is tenure-track. It remains to demonstrate the declining effort profile. The maximum of $\hat{\Pi}$ occurs along the boundary $p_2 = 1$. Thus, to identify p_1^*, it suffices to maximize

$$\Upsilon(p_1) \quad := \quad \frac{\hat{\Pi}(p_1,1)}{k^2} \quad \underset{\text{(D.1)}}{=\!=} \quad \frac{p\,(16\,p - 16\,p^2 + 4\,p^3 + 1)}{16(1 + 2\,p)}$$

The zeros of

$$\Upsilon'(p_1) \quad = \quad \frac{32\,p_1 + 1 + 24\,p_1^4 - 48\,p_1^3 - 16\,p_1^2}{16\,(1 + 2\,p_1)^2}$$

are the zeros of the numerator $32\,p_1 + 1 + 24\,p_1^4 - 48\,p_1^3 - 16\,p_1^2$. Only one of these zeros is in the interval $[0,1]$; it is located at $p_1^* \approx 0.8422568359$, and corresponds to a maximum of Υ. Since $p_1^* > 1/2$, Corollary E(b) implies that $e_1^* > e_2^*$. $\Box$

Lemma G *Suppose* $\{\rho_e\}_{e\in\mathbb{R}_{\not<}}$ *is the family of exponential distributions from* Theorem 1(b). *Then the* $\overline{\Pi}$*-maximizing MNQ contract is tenure-track, with a declining effort profile.*

Proof: We have $P(e,s) = \exp(-s/e)$, so $\varsigma(e,p) = -e\,\ln(p)$. Also, $\overline{R}(e,s) = e + s$. Setting $S(p) = -\ln(p)$ and $c_1 = c_2 = 1$, we see that $\{\rho_e\}_{e\in\mathbb{R}_{\not<}}$ is tenable; thus, we can apply Corollary E. In Lemma D, we have $L(p) = (1-\ln(p))$. Substitute into (D.3) to get

$$\begin{aligned} \Xi(p_1,p_2) &= \lambda(p_1,p_2) + p_1 g(p_1,p_2), \qquad\qquad (G.1)\\ \text{where}\quad g(p_1,p_2) &:= 2 - p_2^2 + p_1 + 2\,p_2 + 2\,p_1p_2 + p_1p_2^2, \end{aligned}$$

and

$$\begin{aligned}\lambda(p_1,p_2) \;:=\; & \left(2\,p_1 + p_1^2 + p_1^2p_2^2 + 2\,p_1p_2 + 2\,p_1^2p_2\right)\ln(p_1)^2 \;+\; \left(p_1p_2^2 + 2\,p_1p_2 + 2\,p_2\right)\ln(p_2)^2 \\ & -\;\left(4\,p_1 + 4\,p_1p_2 + 4\,p_1^2p_2 + 2\,p_1^2 + 2\,p_1^2p_2^2\right)\ln(p_1) \;-\; \left(2\,p_1p_2 + 2\,p_2\right)\ln(p_2)\,.\end{aligned}$$

Claim 1: $\Xi(p_1,p_2) > 0$ *for all* $(p_1,p_2) \in [0,1]^2$.

Proof: $\lambda(p_1,p_2) \geq 0$ for all $(p_1,p_2) \in [0,1]^2$, because $\ln(x)^2 \geq 0$ for all $x > 0$, and $-\ln(x) \geq 0$ for all $x \in (0,1]$. Thus, it suffices to show $g(p_1,p_2) > 0$. Let $h(p_2) := -p_2^2 + 2p_2 + 2$.

Claim 1.1: $g(p_1,p_2) > h(p_2)$ *for all* $p_1, p_2 > 0$.

Proof: Write $g(p_1,p_2)$ as polynomial in p_2 to get: $g(p_1,p_2) = (-1+p_1)\,p_2^2 + (2+2\,p_1)\,p_2 + 2 + p_1$. If $p_1 > 0$, then $-1+p_1 > -1$, $2+2p_1 > 2$ and $2+p_1 > 2$. Thus, each p_2-coefficient of $g(p_1,p_2)$ is strictly larger than the corresponding coefficient of $h(p_2)$, for any $p_1 > 0$. Thus, $g(p_1,p_2) > h(p_2)$ for all $p_1,p_2 > 0$. $\bigtriangledown$ `Claim 1.1`

Now, $h(0) = 2 > 0$, $h(1) = 3 > 0$, and h has no extremal points in $[0,1]$; thus $h(p_2) > 0$ for all $p_2 \in [0,1]$. Thus, Claim 2.1 implies that $g(p_1,p_2) > 0$ for all $(p_1,p_2) \in [0,1]^2$. Thus, eqn.(G.1) implies that $\Xi(p_1,p_2) \geq 0$ for all $(p_1,p_2) \in [0,1]^2$, as desired. $\diamond$ `Claim 1`

Claim 2 and Corollary E(a) imply that the $\overline{\Pi}$-maximizing contract is tenure-track. It remains to demonstrate the declining effort profile. The maximum of $\hat{\Pi}$ occurs along the boundary $p_2 = 1$. Thus, to identify p_1^*, it suffices to maximize

$$\Upsilon(p_1) \quad := \quad \frac{\hat{\Pi}(p_1,1)}{k^2} \;\underset{(D.1)}{=\!=}\; \frac{p_1\left(4\,p_1 - 8\,p_1\ln(p_1) + 4\,p_1\ln(p_1)^2 + 1\right)}{4(1+2\,p_1)}.$$

The zeros of

$$\Upsilon'(p_1) \quad = \quad \frac{-8\,p_1\ln(p_1) + 8\,p_1\ln(p_1)^2 + 8\,p_1^2\ln(p_1)^2 + 1 - 8\,p_1^2}{4\,(1+2\,p_1)^2}$$

are the zeros of the numerator $-8\,p_1 \ln(p_1) + 8\,p_1 \ln(p_1)^2 + 8\,p_1^2 \ln(p_1)^2 + 1 - 8\,p_1^2$. This is a transcendental function, and it is not possible to find closed-form expressions for its zeros. However, numerically, the numerator has only one zero, located at $p_1^* \approx 0.7121849555$; this corresponds to the unique maximum of $\Upsilon(p_1)$. Since $p_1^* > 1/2$, Corollary E(b) implies that $e_1^* > e_2^*$. $\square$

Lemma H *For any $\alpha > 1$, let $\{\rho_e^\alpha\}_{e\in\mathbb{R}_{\neq}}$ be the family of power law distributions from* Theorem 1(c). *There exist $\underline{\alpha}, \overline{\alpha} \in (1,\infty)$ such that, if $\alpha \in (1,\underline{\alpha})$ or $\alpha \in (\overline{\alpha},\infty)$, then the $\overline{\Pi}$-maximizing MNQ contract is tenure-track, with a declining effort profile. In particular, this holds if $\alpha = 2$.*

Proof: For any $\alpha > 1$, we have $P_\alpha(e,s) = \left(\frac{e}{e+s}\right)^\alpha$; thus $\varsigma_\alpha(e,p) = e(p^{-1/\alpha} - 1)$. Also, $\overline{R}_\alpha(e,s) = \frac{e+\alpha s}{\alpha-1}$. Thus, setting $S_\alpha(p) := (p^{-1/\alpha} - 1)$, $c_1 = 1/(\alpha-1)$ and $c_2 = \alpha/(\alpha-1)$, we see that $\{\rho_e^\alpha\}_{e\in\mathbb{R}_{\neq}}$ is tenable. In Lemma D, we have

$$L_\alpha(p) \quad = \quad \frac{\alpha p^{-1/\alpha}}{(\alpha-1)} - 1 \quad \text{thus} \quad L'_\alpha(p) \quad = \quad \frac{-1}{(\alpha-1)\, p^{\frac{\alpha+1}{\alpha}}}.$$

Substituting into eqn.(D.3) we get $\Xi_\alpha(p_1,p_2) = \xi(p_1,p_2)/(\alpha-1)^2$, where $\xi(p_1,p_2) :=$ $2\,p_1 + 2\,p_2 - 4\,pa - 4\,p_2 a - 2\,p_1 p_2^{\frac{\alpha-2}{\alpha}}\alpha + p_1^2 + 4\,p_1^{\frac{2\,\alpha-1}{\alpha}} a p_2 + 2\,p_1^{\frac{2\,\alpha-1}{\alpha}} p_2^2\alpha + 4\,p_1 p_2 + p_1 p_2^{2\frac{\alpha-1}{\alpha}}\alpha^2 - 2\,p_2^{\frac{\alpha-2}{\alpha}}\alpha - 4\,p_2^{\frac{\alpha-1}{\alpha}}\alpha^2 + p_1 p_2^2 + 2\,p_1 p_2^{\frac{\alpha-2}{\alpha}}\alpha^2 + 6\,p_1 p_2^{\frac{\alpha-1}{\alpha}}\alpha + 2\,p_1^{\frac{\alpha-2}{\alpha}}\alpha^2 + 2\,p_1^{\frac{\alpha-2}{\alpha}}\alpha^2 p_2 + p_1^2 p_2^2 + 4\,p_1^{\frac{\alpha-1}{\alpha}}\alpha - 2\,p_1^{\frac{2\,\alpha-1}{\alpha}}\alpha^2 + 2\,\alpha^2 p_2 + p_1^{2\frac{\alpha-1}{\alpha}} p_2^2\alpha^2 + 4\,p_1\alpha^2 p_2 - 8\,p_1 p_2 a + 2\,p_1\alpha^2 - 2\,p_1 p_2^{\frac{\alpha-1}{\alpha}} - 4\,p_1^{\frac{\alpha-1}{\alpha}}\alpha^2 p_2 - 4\,p_1^{\frac{2\,\alpha-1}{\alpha}}\alpha^2 p_2 + p_1^{2\frac{\alpha-1}{\alpha}}\alpha^2 + 2\,p_1^{\frac{2\,\alpha-1}{\alpha}}\alpha - 2\,p_1 p_2^{\frac{2\,\alpha-1}{\alpha}} - 2\,p_2^{\frac{\alpha-1}{\alpha}} + 2\,p_2^{\frac{\alpha-2}{\alpha}}\alpha^2 + 6\,p_2^{\frac{\alpha-1}{\alpha}}\alpha + 4\,p_1 p_2^{\frac{2\,\alpha-1}{\alpha}}\alpha + 2\,p_1^2 p_2 + p_1\alpha^2 p_2^2 - 2\,pap_2^2 - 2\,p_1^{\frac{2\,\alpha-1}{\alpha}} p_2^2\alpha^2 + 2\,p_1^{2\frac{\alpha-1}{\alpha}}\alpha^2 p_2 - 2\,p_1 p_2^{\frac{2\,\alpha-1}{\alpha}}\alpha^2 - 2\,p_1 p_2^{2\frac{\alpha-1}{\alpha}}\alpha - 2\,p_1^2\alpha + 2\,p_1^2\alpha^2 p_2 - 4\,p_1^2 p_2 a + p_1^2 p_2^2\alpha^2 - 2\,p_1^2 p_2^2\alpha + p_1^2\alpha^2 - 4\,p_1 p_2^{\frac{\alpha-1}{\alpha}}\alpha^2 + 4\,p_1^{\frac{\alpha-1}{\alpha}} a p_2 - 4\,p_1^{\frac{\alpha-1}{\alpha}}\alpha^2$. Thus, it suffices to show that $\xi_\alpha(p_1,p_2) > 0$ for all $(p_1,p_2) \in [0,1]^2$. As promised in Theorem 1(b), we consider only the asymptotics as $\alpha \searrow 1$ or $\alpha\to\infty$.

Asymptotics as $\alpha \searrow 1$. We have

$$\lim_{\alpha\searrow 1} \xi_\alpha(p_1,p_2) \quad = \quad \xi_1(p_1,p_2) \quad := \quad (1-p_1) + 2\,p_2 + p_2^2 + 2\,\frac{p_2}{p_1} + \frac{2}{p_1}. \qquad \text{(H.1)}$$

Now, $\xi_1(p_1,p_2)$ is positive for all $(p_1,p_2) \in [0,1]^2$, because $(1-p_1) \geq 0$ if $p_1 \leq 1$, and all the other terms in expression (H.1) are nonegative. Thus, if α is small enough, then $\xi_\alpha(p_1,p_2) > 0$ for all $(p_1,p_2) \in [0,1]^2$; hence Corollary E(a) implies that the $\overline{\Pi}$-maximizing contract is tenure-track.

Indeed, if $\alpha = 2$, we have $\xi_2(p_1,p_2) = 12 - 6\sqrt{p_2} + 10\,p_2 - 8\,p_1^{3/2} p_2 - 2\,p_1\,p_2^{3/2} - 8\sqrt{p_1}\,p_2 - 4\,p_1^{3/2} p_2^2 - 4\,p_1^{3/2} - 8\sqrt{p_1} - 6\,p_1\sqrt{p_2} + 12\,p_1\,p_2 + 5\,p_1\,p_2^2 + p_1^2 p_2^2 + p_1^2 + 2\,p_1^2 p_2 + 10\,p_1$. A numerical plot reveals that $6 < \xi_2(p_1,p_2) < 16$ for all $(p_1,p_2) \in [0,1]^2$. Thus, the $\overline{\Pi}$-maximizing contract is tenure-track when $\alpha = 2$.

It remains to demonstrate the declining effort profile. For all $p \in [0,1]$, we have $\lim_{\alpha \searrow 1} S_\alpha(p) - \frac{c_1(1-2p)}{2c_2 p} = \frac{1}{2p} > 0$. Thus, if α is small enough, then Corollary E(b) implies that $e_1^* > e_2^*$, as desired.

Asymptotics as $\alpha \to \infty$. A computation reveals that $\lim_{\alpha\to\infty} \xi_\alpha(p_1,p_2) = \Xi(p_1,p_2)$, where $\Xi_\alpha(p_1,p_2)$ is exactly as in eqn.(G.1) from the exponential case. Thus, Claim 2 implies that $\lim_{\alpha\to\infty} \xi_\alpha(p_1,p_2) > 0$ for all $(p_1,p_2) \in [0,1]^2$. Thus, if α is sufficiently large, then Corollary E(a) implies that the $\overline{\Pi}$-maximizing contract is tenure-track.

It remains to demonstrate the declining effort profile. Substituting $L_\alpha(p) = \frac{\alpha p^{-1/\alpha}}{(\alpha-1)} - 1$ into eqn.(D.1) and differentiating yields

$$\partial_1 \hat{\Pi}_\alpha(p_1,1) \quad = \quad k^2 \, \frac{8\alpha^2 f_\alpha(p_1) \ + \ \alpha\, g_\alpha(p_1) \ + \ h_\alpha(p_1)}{4(\alpha-1)^2(1+2\,p_1)^2}, \tag{H.2}$$

where $f_\alpha(p) := \left(p^2 + p^{2-\frac{2}{a}} - 2p^{2-\frac{1}{a}}\right) + \left(p + p^{1-\frac{2}{a}} - 2p^{1-\frac{1}{a}}\right)$, $g_\alpha(p) := -16\,p^{2\,\frac{a-1}{a}} - 8\,p^{\frac{a-2}{a}} + 32\,p^{\frac{2\,a-1}{a}} + 24\,p^{\frac{a-1}{a}} - 16\,p^2 - 16\,p$, and $h_\alpha(p) := 1 + 8\,p - 16\,p^{\frac{2\,a-1}{a}} - 8\,p^{\frac{a-1}{a}} + 8\,p^2$.

Claim 1: *If $p \in (0,1)$, then $f_\alpha(p) > 0$.*

Proof: If $p \in (0,1)$ then the function $x \mapsto p^x$ is convex. Thus, $p^x + p^y > 2p^{(x+y)/2}$. Setting $x = 2$ and $y = 2 - \frac{2}{\alpha}$, we get $p^2 + p^{2-\frac{2}{a}} > 2p^{2-\frac{1}{a}}$. Setting $x = 1$ and $y = 1 - \frac{2}{\alpha}$, we get $p^1 + p^{1-\frac{2}{a}} > 2p^{1-\frac{1}{a}}$. Thus, each of the two bracketed terms in $f_\alpha(p)$ is strictly positive; thus $f_\alpha(p) > 0$, as desired. ◇ Claim 1

In the limit as $\alpha \to \infty$, the term $(\alpha-1)^2$ in the denominator of expression (H.2) annihilates all terms in the numerator except $f_\alpha(p)$. Thus, if α is extremely large, then the sign of $\partial_1 \hat{\Pi}_\alpha(p_1,1)$ is the same as the sign of $f_\alpha(p_1)$, and $f_\alpha(p_1) > 0$ by Claim 3. Thus, $\partial_1 \hat{\Pi}_\alpha(p_1,1) > 0$ for all $p \in (0,1)$; thus, the optimal value of p_1 is $p_1^* = 1$.

This means that, if α is large enough, then $p_1^* > 1/2$; thus, Corollary E(b) implies that $e_1^* > e_2^*$, as desired. □

Proof of Theorem 1. Lemmas F, G, and H state that, under any of the hypotheses (a), (b) or (c), the $\overline{\Pi}$-maximizing element in the space of MNQ contract is tenure-track, and induces a declining profile of effort. Thus, by Proposition 4, the same statement is true for the $\overline{\Pi}$-maximizing element in the space of *all* contracts. Thus, by Proposition 3, the same statement is true for the $\overline{\Pi}$-maximizing element in the space of *raid-proof* contracts. This proves Theorem 1. □

References

[1] Alchian, Armen. "Private Property and the Relative Cost of Tenure." In *The Public Stake in Union Power*, edited by P.D. Bradley (1959). University of Virginia Press, Charlottesville.

[2] Bernhardt, D. and Scoones, D. "Promotion, Turnover and Preemptive Wage Offers." *American Economic Review* 83 (1993): 771-91.

[3] Bok, D. "Toward Education of Quality." *Harvard Magazine* (May/June 1986): 49-64.

[4] Carmichael, H. Lorne. "Incentives in Academics: Why Is There Tenure?" *Journal of Political Economy* 96(3) (June 1988): 453-72.

[5] Cater, Bruce; Lew, Byron and Smith, Barry. "A Theory of Tenure-Track Contracts." *Education Economics* 16(2) (June 2008): 203-18.

[6] Ceci, Stephen; Williams, Wendy and Mueller-Johnson, Katrin. "Tenure and Academic Freedom: Prospects and Constraints." *Behavioral and Brain Sciences* 29 (2006): 586-92.

[7] Diamond, Arthur. "The Life Cycle Research Productivity of Mathematicians and Scientists." *Journal of Gerentology* 41 (1986): 520-25.

[8] Freeman, Smith. "Wage Trends as Performance Displays Productive Potential: A Model and Application to Academic Early Retirement." *Bell Journal of Economics* 8(2) (Autumn 1977): 419-43.

[9] Hearn, J.C. "The Teaching Role of Contemporary American Higher Education: Popular Imagery and Organizational Reality." In *The Economics of American Higher Education*, edited by W.E. Becker and D.R. Lewis (1992). Kluwer Academic, Boston.

[10] Hendrickson, R.M. "Removing Tenured Faculty for Cause." *Education Law Reporter* 44 (1988): 483-94.

[11] Hogan, C. "Enforcement of Implicit Employment Contracts Through Unionization." *Journal of Labor Economics* 18(1) (2002): 171-95.

[12] Holmstrom, Bengt, and Milgrom, Paul. "Aggregation and Linearity in the Provision of Intertemporal Incentives." *Econometrica* 55(2) (1987): 308-28.

[13] James, E. "Decision Processes and Priorities in Higher Education." In *The Economics of American Universities*, edited by S.A. Hoenack and E.L. Collins (1990). State University of New York Press, Albany.

[14] Kahn, Charles, and Huberman, Gur. "Two-Sided Uncertainty and 'Up-or-Out' Contracts." *Journal of Labor Economics* 6 (October 1988): 423-44.

[15] Lazear, Edward P. "Raids and Offer Matching." In *Research in Labor Economics* 8 Part A, edited by Ronald G. Ehrenberg (1986). JAI Press, Greenwich, CT.

[16] Levin, Sharon, and Stephan, Paula. "Research Productivity over the Life Cycle: Evidence for Academic Scientists." *American Economic Review* 81(1) (1991): 114-32.

[17] Lovain, Timothy. "Grounds for Dismissing Tenured Post-Secondary Faculty for Cause." *Journal of College and University Law* 10 (1983/84): 419-33.

[18] MacLeod, W.B., and Malcomson, J. "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment." *Econometrica* 57 (1989): 447-80.

[19] McKenzie, Richard B. "In Defense of Academic Tenure." *Journal of Institutional and Theoretical Economics* 152(2) (June 1996): 325-41.

[20] McPherson, Michael, and Schapiro, Morton Owen. "Tenure Issues in Higher Education." *Journal of Economic Perspectives* 13(1) (Winter 1999): 85-98.

[21] Morris, A. A. *Dismissal of Tenured Higher Education Faculty: Legal Implications of the Elimination of Mandatory Retirement.* Topeka KS.: National Organization on Legal Problems in Education (1992).

[22] Pearce, D.G. and Stacchetti, E. "The Interaction of Implicit and Explicit Contracts in Repeated Agency." *Games and Economic Behavior* 23 (1998): 75-96.

[23] Rothschild, Michael, and White, Lawrence. "The Analytics of the Pricing of Higher Education and Other Services in which the Customers Are Inputs." *Journal of Political Economy* 103 (1995): 573-86.

[24] Siow. A. "Some Evidence on the Signalling Role of Research in Academia." *Economic Letters* 54 (1997): 271-6.

[25] Siow, A. "Tenure and other Unusual Personnel Practices in Academia." *Journal of Law, Economics and Organization* 14 (1998): 152-73.

[26] Waldman, Michael. "Up-or-Out Contracts: A Signaling Perspective." *Journal of Labor Economics* 8 (April 1990): 230-50.