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Abstract

I study the cross sectional variation of stock returns and technological progress using a dynamic equilibrium model with production. In the model, technological progress is endogenously driven by R&D investment and is composed of two parts. One part is product innovation devoted to creating new products; the other part is dedicated to increasing the productivity of physical investment and is embodied in new tangible capital (e.g., structures and equipment). The model breaks the symmetry assumed in standard models between intangible capital and tangible capital, in which the accumulation processes of tangible capital stock and intangible capital stock do not affect each other. The model explains qualitatively and in many cases quantitatively well-documented empirical regularities: (i) the positive relation between R&D investment and the average stock returns; (ii) the negative relation between physical investment and the average stock returns; and (iii) the positive relation between book-to-market ratio and the average stock returns.

JEL Classification: E23, E44, G12

Keywords: Technological Progress, R&D Investment, Physical Investment, Stock Return

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1 Introduction

This paper investigates intangible capital, tangible capital and the cross section of stock returns using a dynamic equilibrium model. The primary type of intangible capital the paper focuses on is the accumulation of firms' research and development (R&D) efforts.[1] The central insight of the paper is that physical capital embodied-technological progress is essential to simultaneously explaining the well-documented puzzling facts regarding R&D investment and physical investment: i) high R&D-intensive firms earn *higher* average stock returns than low R&D-intensive firms [e.g., Chan, Lakonishok and Sougiannis 2001; Li 2006][2]; and ii) high physical investment-intensive firms earn *lower* average stock returns than low physical investment-intensive firms [e.g., Titman, Wei and Xie 2004; Xing 2008][3].[4] Moreover this paper directly links technological innovation to the differences between the value and the growth firms. Hence it provides a fresh explanation for the value premium, which is different from the existing literature.

Indeed, the positive covariation between R&D investment and expected stock returns is puzzling for the neoclassical Q-theory of investment. As shown by Cochrane (1991), under constant returns to scale stock returns equal investment returns. Since investment negatively forecasts expected investment returns, it must also be negatively correlated with expected stock returns. However, this prediction is inconsistent with R&D's positive forecasting of expected stock returns.

Standard models cannot simultaneously explain the different covariations between R&D investment, physical investment and expected stock returns. For example, Hansen, Heaton and Li (2004) and McGrattan and Prescott (2005) treat tangible capital and intangible capital symmetrically. More specifically, in their respective models, the accumulation processes of tangible capital stock and intangible capital stock do not affect each other. However, these models predict that R&D investment and physical investment forecast future stock returns in the same direction,

[1] Tangible capital consists primarily of equipment, machines, and plants, which is usually labelled as physical capital. Throughout the paper I use tangible and physical interchangeably, and intangible and R&D interchangeably as well.

[2] At aggregate level, Hsu (2006) finds that aggregate cumulative R&D growth rate positively forecasts future stock market returns.

[3] Cochrane (1991) and Lamont (2000) find that aggregate physical investment also negatively forecasts future stock market returns.

[4] These findings regarding R&D investment and physical investment still hold after controlling for size and book-to-market ratio.

which is counterfactual.

Three basic assumptions underpin the model. The first assumption is that technological progress is endogenously driven by R&D investment. This assumption is familiar from Romer (1990) who argues that technological progress largely arises from firms' R&D investment decisions. In the model, I assume technological progress is a result of firms' explicit R&D decisions and is represented by intangible capital. Here, intangible capital primarily refers to successful innovations in advances in manufacturing technologies and processes, new designs and formulas that generates new products, etc.

The second assumption is that part of the firms' technological progress is devoted to new products. This assumption comes from R&D literature. Cohen and Klepper (1996) and Lin and Saggi (2001) document that a large proportion of firms' R&D expenditures are used in innovations to generate new products. For example, in pharmaceuticals, software companies, etc., more than half of the total R&D expenditures are dedicated to new product innovations. Typically, product innovation increases firms' cash flows through the introduction of new product features that increases the price buyers are willing to pay for firms' products, or allows firms to reach new buyers[5]. In the model, product innovations combined with physical capital produce products.

The third assumption, which is the key assumption in the paper, is that the other part of technological progress is innovation devoted to increasing the productivity of physical investment in producing new physical capital. Hence, in the model, the advances of new physical capital embody current technological progress. This assumption is crucial to simultaneously generate a positive covariation between R&D investment and future stock returns, and a negative covariation between physical investment and future stock returns. The assumption of embodiment captures the fact that successful innovations increase the productivity of equipment and machines and reduce the costs of production process (Levin and Reiss 1988, Cohen and Klepper 1996). For instance, in petroleum refining, biochemical industry, etc., more than two thirds of the total R&D expenditure is dedicated to innovations in reducing production costs. Likewise, a number of other industries, including petrochemicals, food and beverage manufacturing, semiconductor plants,

[5]Firms with new product usually can raise prices through some degree of transient monopoly power.

invest R&D in manufacturing technology for designing, analyzing and controlling manufacturing through timely measurements (during processing) of critical quality and performance attributes of raw and in-process materials and processes, with the goal of ensuring final product quality.

The main economic implications of the model are as follows. First, firms' expected returns on physical investment are *increasing* in R&D investment but *decreasing* in physical investment. Intuitively, expected physical investment return is the ratio of the expected marginal benefit of physical investment to the marginal cost of physical investment. All else being equal, on one hand, R&D investment increases the expected marginal benefit of physical investment; on the other hand, R&D investment (physical investment) decreases (increases) the marginal cost of physical investment. These two effects reinforce each other and imply that R&D investment (physical investment) increases (decreases) expected returns on physical investment.

The second economic implication is that high R&D-intensive firms earn higher expected stock returns than low R&D-intensive firms, while high physical investment-intensive firms earn lower expected stock returns than low physical investment-intensive firms. Intuitively, in the model, the stock price is the sum of the market value of physical capital and R&D capital, and the stock return is the weighted average of physical investment return and R&D investment return. Since physical capital embodies current technological progress (R&D capital) and its share in output production dominates that of R&D capital, the market value of physical capital is higher than the market value of R&D capital. This relation implies that the weight on physical investment return is greater than the weight on R&D investment return.[6]Therefore, firms' stock returns covary with R&D investment and physical investment in the same way as physical investment returns do. The implication is that stock returns are increasing in R&D investment but decreasing in physical investment.

The third economic implication is that value firms earn higher expected stock returns than do growth firms[7]. Intuitively, with high book-to-market ratio, value firms have low physical investment, which implies that they must earn high expected physical investment returns. Growth

[6]In the model, the weight on physical (R&D) investment return is the ratio of the market value of physical (R&D) capital to the stock price.

[7]An incomplete list of studies exploring why book-to-market ratio positively forecasts average stock returns includes Carlson, Fisher and Giammarino (2004), Zhang (2005), Cooper (2006), etc.

firms have low expected physical investment returns because they have high physical investment with low book-to-market ratio. Hence, value firms earn high expected stock returns while growth firms earn low expected stock returns because the weight on physical investment return is larger than the weight on R&D investment return. More specifically, in the model, the productivity of the existing physical capital of value firms is lower than that of growth firms, because value firms invest less in R&D. In recessions, value firms are burdened with excessive physical capital and do not have as much technological progress in upgrading the efficiency of the existing physical capital as do growth firms, so they are more risky given that the market price of risk is high in bad times. The value premium in my model hinges on the interactions between technological progress and physical investment, which differs from Zhang (2005) who work through physical capital adjustment costs in generating the value premium. Given that most of the studies on book-to-market ratio and stock returns focus on physical investment only, this paper sheds light on the relation between technological progress and the value premium.

Cochrane (1991, 1996) are the first to study asset prices from firms' perspective using the Q-theory of investment. Different from Cochrane who focuses on aggregate physical investment and expected stock returns, this paper explores the relations between firms' technological progress, physical investment and the cross-section of returns.

Li (2006) has a paper close to mine. In it Li constructs a dynamic real options model in which R&D investment and stock returns change in predictable ways when R&D firms are financially constrained. The key distinction between Li and my model is that the real option model of Li features exogenous cash flows, systematic risk and financing constraints; while my model employs a neoclassical framework in which technological progress is *endogenously* determined. Hence, in my model the key economic fundamental variables, i.e., R&D investment, physical investment and stock returns, are determined endogenously in competitive equilibrium. My model can therefore shed light on the fundamental determinants of technological progress, and the covariations between R&D investment, physical investment and future stock returns without resorting to financing frictions.

Notably, Lustig, Syverson and Van Nieuwerburgh (2008) also investigate technology change at firm level, but with a different focus than this paper. Lustig, Syverson and Van Nieuwerburgh explores the implications of IT adoption on corporate payout, organizational capital and changes in labor market reallocation, while this paper examines the implications of firms' technological change on asset prices and returns.

2 A Two-Period Example

I use a simple two-period example to provide intuition for the link between expected returns and firm characteristics.

2.1 The Setup

2.1.1 Technology

Firms use physical capital, intangible capital and a vector of costlessly adjustable inputs to produce output. Firms choose the levels of these inputs each period to maximize their operating profits, defined as revenues minus the expenditures on these inputs. Taking operating profits as given, firms then choose optimal physical investment and R&D investment to maximize their market value.

There are only two periods, t and $t+1$. Firm j starts with physical capital stock $k^m_{j,t}$ and intangible capital stock $k^u_{j,t}$, invests in period t, and produces in both t and $t+1$. Physical capital, including structures, equipment and machines, can be measured. So I denote it with m. Intangible capital, including innovations in designs and formulas, new technologies in manufacturing, etc., can be unmeasured. So I denote it with u. The firm exits at the end of period $t+1$ with a liquidation value of $(1-\delta_m)k^m_{j,t+1} + (1-\delta_u)k^u_{j,t+1}$, in which δ_m and δ_u are rates of depreciation for physical capital and intangible capital, respectively. Operating profits, $\pi(k^m_{j,t}, \theta k^u_{j,t}, \Theta_{j,t})$, depend upon physical capital, $k^m_{j,t}$, intangible capital stock, $\theta k^u_{j,t}$, where θ is the proportion of intangible capital devoted to producing new products with $0 \leq \theta < 1$, and a vector of exogenous aggregate and firm-specific productivity shocks, denoted as $\Theta_{j,t}$. Operating profits exhibit constant returns to scale in $\left(k^m_{j,t}, k^u_{j,t}\right)$, that is, $\pi(k^m_{j,t}, \theta k^u_{j,t}, \Theta_{j,t}) = \pi_m(k^m_{j,t}, \theta k^u_{j,t}, \Theta_{j,t})k^m_{j,t} + \pi_u(k^m_{j,t}, \theta k^u_{j,t}, \Theta_{j,t})k^u_{j,t}$, in which the subscripts m and u denote partial derivatives w.r.t. $k_{j,t}^m$ and $k_{j,t}^u$. The expression $\pi_m(k_{j,t}^m, \theta k_{j,t}^u, \Theta_{j,t})$ is therefore the marginal product of physical capital, and $\pi_u(k_{j,t}^m, \theta k_{j,t}^u, \Theta_{j,t})$ is the marginal product of intangible capital.

In the rest of the paper, I drop the firm index j when no confusion results.

2.1.2 Intangible Capital and Tangible Capital Production

As is standard in the literature, intangible capital production follows the standard capital accumulation process given by

$$k_{t+1}^u = (1-\delta_u)k_t^u + i_t^u, \tag{1}$$

where i_t^u is R&D investment. Standard models also commonly assume that physical capital follows a symmetric process, $k_{t+1}^m = (1-\delta_m)k_t^m + i_t^m$, where i_t^m is physical investment.[8]

However, specifying physical capital and intangible capital symmetrically produces a model that predicts that both R&D investment and physical investment forecast expected stock returns in the same direction, which is counterfactual. I therefore abandon the symmetry of standard models and specify the following accumulation process for physical capital

$$k_{t+1}^m = (1-\delta_m)k_t^m + \Phi\left[i_t^m, (1-\theta)\, k_{t+1}^u\right], \tag{2}$$

where

$$\Phi\left[i_t^m, (1-\theta)\, k_{t+1}^u\right] \equiv A\left\{a(i_t^m)^\rho + (1-a)\left[(1-\theta)k_{t+1}^u\right]^\rho\right\}^{\frac{1}{\rho}}. \tag{3}$$

is a constant-elasticity-of-substitution (CES) technology for physical capital production.

Here, $\{a, \rho, A\}$ are constants with the constraints $0 < a \leq 1$, $\rho \leq 1$, $\rho \neq 0$, and $A > 0$. Note that $(1-\theta)\, k_{t+1}^u$ is the proportion of intangible capital dedicated to producing new physical capital. The CES function $\Phi\left[i_t^m, (1-\theta)\, k_{t+1}^u\right]$ in equation (2) generalizes the standard accumulation process

[8] See Hansen, Heaton and Li (2004) and McGrattan and Prescott (2005).

as a special case when $A = a = \rho = 1$.[9] It satisfies

$$\begin{aligned} \Phi_1\left[i_t^m, (1-\theta)\, k_{t+1}^u\right] &> 0, \Phi_2\left[i_t^m, (1-\theta)\, k_{t+1}^u\right] > 0, \\ \Phi_{11}\left[i_t^m, (1-\theta)\, k_{t+1}^u\right] &< 0, \Phi_{12}\left[i_t^m, (1-\theta)\, k_{t+1}^u\right] > 0, \text{ and } \Phi_{22}\left[i_t^m, (1-\theta)\, k_{t+1}^u\right] < 0, \end{aligned}$$

where numerical subscripts denote partial derivatives. That is, the total product of physical capital increases in the level of physical investment and intangible capital; moreover, the marginal product of physical investment decreases in physical investment but increases in intangible capital, and the marginal product of R&D capital decreases in R&D capital but increases in physical investment[10]. The elasticity of substitution between k_{t+1}^u and i_t^m is $\frac{1}{1-\rho}$.

The most important aspect of equation (2) is the inclusion of the intangible capital $(1-\theta)\, k_{t+1}^u$ that represents the current state of technological progress for producing new physical capital. A high realization of $(1-\theta)\, k_{t+1}^u$ increases the productivity of physical investment and directly upgrades the efficiency of physical capital from the current vintage to the next. The increases in $(1-\theta)\, k_{t+1}^u$ formalizes the notion of embodied technological progress.

The motivations for equation (2) come from the macro literature on embodied technological change[11]. Theoretically, as is shown in Greenwood et al (1997, 2000), technological progress, such as faster and more efficient means of telecommunications and transportation, new and more powerful computers, robotization of assembly lines, the advances of manufacturing technologies, etc., have made production of new physical capital more efficient and less expensive. More specifically, Greenwood et al. assume that the physical capital accumulation process follows $k_{t+1}^m = (1-\delta_m)k_t^m + q_t i_t^m$, where q_t is an exogenous technological progress different than the aggregate productivity shock. The technological progress q_t determines the productivity of physical investment. In particular, it makes the new physical capital production more efficient by reducing the marginal cost of physical investment, which equals $\frac{1}{q_t}$ in equilibrium. Fisher (2006) estimates

[9] The CES production function in equation (3) contains several well-known production functions as special cases, depending on the value of parameter ρ. For instance, when $\rho = 1$, $\Phi\left[i_t^m, (1-\theta)k_{t+1}^u\right]$ is a linear production function; when $\rho \to 0$, $\Phi\left[i_t^m, (1-\theta)k_{t+1}^u\right]$ is the Cobb-Douglas technology; when $\rho \to -\infty$, $\Phi\left[i_t^m, (1-\theta)k_{t+1}^u\right]$ reduces to the Leontif technology.

[10] Note that $\Phi_{21}\left[i_t^m, (1-\theta)\, k_{t+1}^u\right] = \Phi_{12}\left[i_t^m, (1-\theta)\, k_{t+1}^u\right] > 0$.

[11] A different label for capital embodied technological change is investment-specific technological change. See Greenwood et al (1997) for interpretations.

q_t using the real equipment price and finds q_t is important to account for economic growth both in the short run and the long run in addition to the aggregate productivity shock. Huffman (2007) assumes embodied technological progress is driven by R&D investment and reduces adjustment costs of physical capital. Economic growth takes place directly through aggregate R&D spending in his model.

I endogenize q_t in Greenwood et al. (1997) by assuming technological progress occurs at the level of firms and is a result of firms' R&D decisions in equation (2). Therefore equation (2) provides a direct microfoundation for the embodied technological change in the macro literature, and offers rich interactions between the current technological progress k_{t+1}^u and physical investment i_t^m.

Note that equation (2) can be rewritten as

$$k_{t+1}^m = (1-\delta_m)k_t^m + i_t^m \Phi_1\left[i_t^m, (1-\theta)\, k_{t+1}^u\right] + (1-\theta)\, k_{t+1}^u \Phi_2\left[i_t^m, (1-\theta)\, k_{t+1}^u\right], \tag{4}$$

where the equality follows from the fact that $\Phi\left[i_t^m, (1-\theta)\, k_{t+1}^u\right]$ is constant returns to scale in $\left(i_t^m, (1-\theta)\, k_{t+1}^u\right)$. So the role of intangible capital $(1-\theta)\, k_{t+1}^u$ in equation (2) can be interpreted in two ways. First, $\frac{1}{\Phi_1\left[i_t^m,(1-\theta)k_{t+1}^u\right]}$ can be considered as representing the cost of producing a new unit of physical capital in terms of final output using physical investment only. This cost decreases in k_{t+1}^u. In other words, one can imagine that in each period a new vintage of physical capital is produced by physical investment. The productivity of a new unit of physical investment is given by $\Phi_1\left[i_t^m, (1-\theta)\, k_{t+1}^u\right]$, which is increasing in $(1-\theta)\, k_{t+1}^u$. Second, $\Phi_2\left[i_t^m, (1-\theta)\, k_{t+1}^u\right]$ can be considered as representing the productivity of a new unit of intangible capital $(1-\theta)\, k_{t+1}^u$ in producing new physical capital k_{t+1}^m. This productivity $\Phi_2\left[i_t^m, (1-\theta)\, k_{t+1}^u\right]$ increases in i_t^m. In sum, technological progress makes new physical capital either less expensive or better than old physical capital, allowing for increased output.

Let $M_{t,t+1}$ be the stochastic discount factor from time t to $t+1$. It is correlated with the aggregate component of $\Theta_{j,t}$. Firm j chooses (i_t^m, i_t^u) to maximize the market value of equity and the constraints are equations (1) and (2):

$$\underbrace{\max_{(i_t^m, i_t^u)} \left\{ \overbrace{\pi(k_t^m, \theta k_t^u, \Theta_{j,t}) - i_t^m - i_t^u}^{\text{Cash flow at period t}} + \mathbb{E}_t \left[M_{t,t+1} \left[\overbrace{\pi(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1}) + (1-\delta_m) k_{t+1}^m + (1-\delta_u) k_{t+1}^u}^{\text{Cash flow at period t+1}} \right] \right] \right\}}_{\text{Cum dividend market value of equity at period t}} \tag{5}$$

The first part of this expression, denoted by $\pi(k_t^m, \theta k_t^u, \Theta_{j,t}) - i_t^m - i_t^u$, is net cash flow during period t. Firms use operating profits $\pi(k_t^m, \theta k_t^u, \Theta_{j,t})$ to invest in physical investment and R&D investment, (i_t^m, i_t^u). The price of investment is normalized to one[12]. If net cash flow is positive, firms distribute it to shareholders, and if net cash flow is negative firms collect external equity financing from shareholders. The second part of equation (5) contains the expected discounted value of cash flow during period $t+1$, which is equal to the sum of operating profits and the liquidation value of the physical capital stock and intangible capital stock at the end of period $t+1$.

Taking the partial derivative of equation (5) with respect to (i_t^m, i_t^u) yields the first-order conditions:

$$\overbrace{\frac{(i_t^m)^{1-\rho}}{aA[a(i_t^m)^\rho + (1-a)(1-\theta)^\rho (k_{t+1}^u)^\rho]^{\frac{1}{\rho}-1}}}^{\text{Marginal cost of physical investment at } t} = \mathbb{E}_t \left\{ M_{t,t+1} \left[\overbrace{\pi_m(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1}) + (1-\delta_m)}^{\text{Marginal benefit of physical investment at } t+1} \right] \right\} \tag{6}$$

$$\underbrace{1 - \frac{(1-a)(1-\theta)^\rho}{a} \left(\frac{i_t^m}{k_{t+1}^u}\right)^{1-\rho}}_{\text{Marginal cost of R\&D investment at } t} = \mathbb{E}_t \left\{ M_{t,t+1} \left[\underbrace{\pi_u(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1}) + (1-\delta_u)}_{\text{Marginal benefit of R\&D investment at } t+1} \right] \right\} \tag{7}$$

The left hand sides of the equations (6) and (7) are the marginal cost of physical investment and the marginal cost of R&D investment, respectively; and the right sides of the equations (6) and (7) are the marginal benefit of physical investment and R&D investment, respectively. To generate one additional unit of physical capital and intangible capital at the beginning of next period, $\left(k_{t+1}^m, k_{t+1}^u\right)$, a firm must pay the price of physical capital and intangible capital (equal to the marginal cost of physical investment and R&D investment at the optimum) ,

[12] Physical investment and R&D investment are assumed to be homogenous goods.

$\left(\frac{(i_t^m)^{1-\rho}}{aA[a(i_t^m)^\rho+(1-a)(1-\theta)^\rho(k_{t+1}^u)^\rho]^{\frac{1}{\rho}-1}}, 1-\frac{(1-a)(1-\theta)^\rho}{a}(\frac{i_t^m}{k_{t+1}^u})^{1-\rho}\right)$. The next-period marginal benefit of this additional unit of physical capital and intangible capital includes the marginal product of capital, $\left(\pi_m(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1}), \pi_u(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})\right)$, and the liquidation values of physical capital and intangible capital net of depreciation, $(1-\delta_m, 1-\delta_u)$, respectively.

To derive asset pricing implications from this two-period model, I first define the physical investment return as the ratio of the marginal benefit of physical investment at period $t+1$ to the marginal cost of physical investment at period t:

$$\underbrace{r_{t+1}^m}_{\text{physical investment return from period } t \text{ to } t+1} \equiv \frac{\overbrace{\pi_m(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1}) + (1-\delta_m)}^{\text{Marginal benefit of physical investment at period } t+1}}{\underbrace{\left[\frac{(i_t^m)^{1-\rho}}{aA[a(i_t^m)^\rho + (1-a)(1-\theta)^\rho (k_{t+1}^u)^\rho]^{\frac{1}{\rho}-1}}\right]}_{\text{Marginal cost of physical investment at period } t}}. \tag{8}$$

Similarly, I define the R&D investment return as the ratio of the marginal benefit of R&D investment at period $t+1$ to the marginal cost of R&D investment at period t:

$$\underbrace{r_{t+1}^u}_{\text{R\&D investment return from period } t \text{ to } t+1} \equiv \frac{\overbrace{\pi_u(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1}) + 1 - \delta_u}^{\text{Marginal benefit of R\&D investment at period } t+1}}{\underbrace{1 - \frac{(1-a)(1-\theta)^\rho}{a}(\frac{i_t^m}{k_{t+1}^u})^{1-\rho}}_{\text{Marginal cost of R\&D investment at period } t}}. \tag{9}$$

Using the definitions of physical investment and R&D investment returns in equations (8) and (9), together with equations (6) and (7) , I get the standard asset pricing equations for physical investment return and R&D investment return.

$$\begin{aligned} \mathbb{E}_t\left[M_{t,t+1} r_{t+1}^m\right] &= 1 \\ \mathbb{E}_t\left[M_{t,t+1} r_{t+1}^u\right] &= 1. \end{aligned}$$

To simplify notations, I define q_t^m and q_t^u as the marginal costs of physical investment and R&D

investment, respectively:

$$q_t^m \equiv \frac{(i_t^m)^{1-\rho}}{aA[a(i_t^m)^\rho + (1-a)(1-\theta)^\rho (k_{t+1}^u)^\rho]^{\frac{1}{\rho}-1}} \tag{10}$$

$$q_t^u \equiv 1 - \frac{(1-a)(1-\theta)^\rho}{a}\left(\frac{i_t^m}{k_{t+1}^u}\right)^{1-\rho}. \tag{11}$$

I now show that under constant returns to scale of operating profits $\pi(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})$, stock returns equal the weighted average of physical investment returns and R&D investment returns. From equation (5) I define the *ex*-dividend equity value at period t, denoted p_t^s, as:

$$\underbrace{p_t^s}_{\text{Ex-dividend equity value at } t} = \mathbb{E}_t\left\{M_{t,t+1}\left[\overbrace{\pi(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1}) + (1-\delta_m)k_{t+1}^m + (1-\delta_u)k_{t+1}^u}^{\text{Cash flow at period } t+1}\right]\right\} \tag{12}$$

$$= \underbrace{q_t^m k_{t+1}^m}_{\text{Market value of physical capital}} + \underbrace{q_t^u k_{t+1}^u}_{\text{Market value of intangible capital}}. \tag{13}$$

Equation (12) says that the ex-dividend equity value, p_t^s, equals the *cum*-dividend equity value minus the net cash flow over period t. Equation (13) states that at the optimum the *ex*-dividend equity value, p_t^s, is the sum of the market values of physical capital and intangible capital[13].

We can define the stock return, r_{t+1}^s, as

$$\begin{aligned}\underbrace{r_{t+1}^s}_{\text{Stock return from period } t \text{ to } t+1} &= \frac{\overbrace{\pi(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1}) + (1-\delta_m)k_{t+1}^m + (1-\delta_u)k_{t+1}^u}^{\text{Cash flow at period } t+1}}{\underbrace{p_t^s}_{\text{Ex-dividend equity value at period } t}} \\ &= \frac{\pi_m(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})k_{t+1}^m + (1-\delta_m)k_{t+1}^m}{p_t^s} \\ &\quad + \frac{\pi_u(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})k_{t+1}^u + (1-\delta_u)k_{t+1}^u}{p_t^s},\end{aligned} \tag{14}$$

[13] The details about the derivation from equation (12) to equation (13) is the following:
$p_t^s = \mathbb{E}_t\left\{M_{t,t+1}\left[\pi(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1}) + (1-\delta_m)k_{t+1}^m + (1-\delta_u)k_{t+1}^u\right]\right\}$
$= \mathbb{E}_t\left\{M_{t,t+1}\left[\pi_m(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})k_{t+1}^m + (1-\delta_m)\right]k_{t+1}^m\right\} + \mathbb{E}_t\left\{M_{t,t+1}\left[\pi_u(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})k_{t+1}^u + (1-\delta_u)k_{t+1}^u\right]\right\}$
$= \left[\frac{(i_t^m)^{1-\rho}}{aA[a(i_t^m)^\rho + (1-a)(1-\theta)^\rho (k_{t+1}^u)^\rho]^{\frac{1}{\rho}-1}}\right] k_{t+1}^m + \left[1 - \frac{(1-a)(1-\theta)^\rho}{a}\left(\frac{i_t^m}{k_{t+1}^u}\right)^{1-\rho}\right] k_{t+1}^u$
$= q_t^m k_{t+1}^m + q_t^u k_{t+1}^u$.
The second equality follows that operating profits $\pi(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})$ is constant returns to scale in (k_{t+1}^m, k_{t+1}^u). The third equality follows from equations (6) and (7). The last equality follows from equations (10) and (11).

The *ex*-dividend market equity in the numerator is zero in this two-period setting.

Dividing both the numerator and the denominator of the first term in equation (14) by $q_t^m k_{t+1}^m$, and dividing both the numerator and the denominator of the second term in equation (14) by $q_t^u k_{t+1}^u$, and invoking the constant returns to scale assumption for $\pi(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})$ yields

$$r_{t+1}^s = \frac{q_t^m k_{t+1}^m}{p_t^s} r_{t+1}^m + \frac{q_t^u k_{t+1}^u}{p_t^s} r_{t+1}^u.$$

The equality follows from the definitions of physical investment return r_{t+1}^m and R&D investment return r_{t+1}^u in equations (8) and (9).

2.2 Intuition

I use the equivalence of stock returns and the weighted average of physical investment returns and R&D investment returns to provide the driving forces behind expected returns:

$$\begin{aligned}\underbrace{\mathbb{E}_t\left[r_{t+1}^s\right]}_{\text{Expected stock return}} \quad = \quad & \frac{\overbrace{q_t^m k_{t+1}^m}^{\text{Weight on physical investment return}}}{p_t^s} \quad \underbrace{\mathbb{E}_t\left[r_{t+1}^m\right]}_{\text{Expected physical investment return}} \\ + \quad & \frac{\overbrace{q_t^u k_{t+1}^u}^{\text{Weight on R\&D investment return}}}{p_t^s} \quad \underbrace{\mathbb{E}_t\left[r_{t+1}^u\right]}_{\text{Expected R\&D investment return}}. \end{aligned} \tag{15}$$

Justification for this approach is in Cochrane (1997) and Liu, Whited and Zhang (2008), who show that average equity returns are well within the range of plausible parameters for average investment returns[14].

Equation (15) is useful for interpreting the empirical facts relating to R&D investment, physical investment, market-to-book ratio and expected stock returns because it ties expected returns directly to firm characteristics. The equation implies that there are four variables affecting expected stock returns: the expected physical investment returns, the expected R&D investment returns and their respective weights. I discuss them in detail below.

[14] Cochrane (1997) considers aggregate equity returns, while Liu, Whited and Zhang (2007) investigate the cross section of equity returns.

Physical Investment Returns, R&D Investment Returns and Stock Returns

From the definition of the physical investment return in equation (8), the expected physical investment returns, $\mathbb{E}_t\left[r^m_{t+1}\right]$, is given by

$$\underbrace{\mathbb{E}_t\left[r^m_{t+1}\right]}_{\text{Expected physical investment return from period } t \text{ to } t+1} \equiv \frac{\overbrace{\mathbb{E}_t\left[\pi_m(k^m_{t+1}, \theta k^u_{t+1}, \Theta_{j,t+1})\right]}^{\text{Expected marginal product of physical capital at period } t+1} + (1-\delta_m)}{\underbrace{q^m_t}_{\text{Marginal cost of physical investment at period } t}}.$$

The first implication is that $\mathbb{E}_t\left[r^m_{t+1}\right]$ is *increasing* in R&D investment but *decreasing* in physical investment. There are two effects determining the physical investment returns: (i) the productivity effect, the expected marginal product of physical capital $\mathbb{E}_t\left[\pi_m(k^m_t, \theta k^u_t, \Theta_{j,t})\right]$ in the numerator[15]; and (ii) the investment effect, the marginal cost of physical investment q^m_t in the denominator. All else equal, R&D investment, appearing in the numerator, increases the expected marginal product of physical capital because R&D capital creates new products which increase cash flows[16]; R&D (physical) investment, appearing in the denominator, decreases (increases) the marginal cost of physical investment.[17] The productivity effect and the investment effect reinforce each other and imply that R&D (physical) investment increases (decreases) the expected physical investment return.

In contrast, the expected R&D investment return, $\mathbb{E}_t\left[r^u_{t+1}\right]$, is *decreasing* in R&D investment but *increasing* in physical investment. From the definition of R&D investment return in equation (9), expected R&D investment return is given by

$$\underbrace{\mathbb{E}_t\left[r^u_{t+1}\right]}_{\text{Expected R\&D investment return from period } t \text{ to } t+1} \equiv \frac{\overbrace{\mathbb{E}_t\left[\pi_u(k^m_{t+1}, \theta k^u_{t+1}, \Theta_{j,t+1})\right]}^{\text{Expected marginal product of R\&D capital at period } t+1} + 1-\delta_u}{\underbrace{q^u_t}_{\text{Marginal cost of R\&D investment at period } t}}.$$

[15] The term $1-\delta_m$ is constant in the numerator, so the expected marginal benefit of physical investment is effectively the marginal product of physical capital, $\pi_m(k^m_t, \theta k^u_t, \Theta_{j,t})$.

[16] More precisely, $\frac{\partial \pi_m(k^m_{t+1}, \theta k^u_{t+1}, \Theta_{j,t+1})}{\partial i^u_t} = \frac{\partial \pi_m(k^m_{t+1}, \theta k^u_{t+1}, \Theta_{j,t+1})}{\partial k^u_{t+1}} \frac{\partial k^u_{t+1}}{\partial i^u_t} > 0$, since $\pi_m(k^m_{t+1}, \theta k^u_{t+1}, \Theta_{j,t+1})$ is strictly concave in k^u_{t+1}.

[17] Taking the partial derivative of marginal cost of physical investment w.r.t. physical investment and R&D investment, respectively, we have $\frac{\partial [q^m_t]}{\partial i^m_t} > 0$, and by chain rule, $\frac{\partial [q^m_t]}{\partial i^u_t} = \frac{\partial [q^m_t]}{\partial k^u_{t+1}} \frac{\partial k^u_{t+1}}{\partial i^u_t} < 0$.

All else equal, physical investment, which appears in the numerator, increases the marginal product of R&D capital $\mathbb{E}_t\left[\pi_u(k_t^m, \theta k_t^u, \Theta_{j,t})\right]$[18]; and, R&D (physical) investment, which appears in the denominator, increases (decreases) the marginal cost of R&D investment q_t^u.[19] These two effects imply that R&D (physical) investment decreases (increases) the expected R&D investment return.

Given that expected physical investment return and expected R&D investment return covary with R&D investment and physical investment *oppositely*, I need to investigate the weights on investment returns to determine whether physical investment return or R&D investment return dominates in stock return. Since new physical capital embodies (part of) the intangible capital and the share of physical capital in output production dominates the share of intangible capital (see details in Section 4.1 for my calibration results.), market value of physical capital $q_t^m k_{t+1}^m$ is larger than the market value of intangible capital $q_t^u k_{t+1}^u$, which implies that the weight on physical investment return $\frac{q_t^m k_{t+1}^m}{p_t^s}$ is greater than the weight on R&D investment return $\frac{q_t^u k_{t+1}^u}{p_t^s}$. Therefore, physical investment return multiplied by its weight, $\frac{q_t^m k_{t+1}^m}{p_t^s} r_{t+1}^m$, dominates R&D investment returns multiplied by its weight, $\frac{q_t^u k_{t+1}^u}{p_t^s} r_{t+1}^u$. Thus firms' stock returns covary with R&D investment and physical investment in the same way as their physical investment returns. The implication is that stock returns are increasing in R&D investment but decreasing in physical investment.

The Value Anomaly

Value firms and growth firms have different expected stock returns because they have different levels of technological progress embodied in physical capital in the model. Book equity is identified as physical capital in the model, so from equation (13), market-to-book ratio is $q_t^m + q_t^u \frac{k_{t+1}^u}{k_{t+1}^m}$. The Market value of physical capital $q_t^m k_{t+1}^m$ is much larger than the market value of intangible capital $q_t^u k_{t+1}^u$, which implies $q_t^m \gg q_t^u \frac{k_{t+1}^u}{k_{t+1}^m}$. So there is an approximately monotonic mapping from market-to-book ratio to the marginal cost of physical investment q_t^m. Value firms with low market-to-book ratios have low q_t^m's, and therefore have high expected physical investment returns. Growth firms with high market-to-book ratios have high q_t^m's, so they earn low expected physical investment

[18] More exactly, $\frac{\partial \pi_u(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})}{\partial i_t^m} = \frac{\partial \pi_u(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})}{\partial k_{t+1}^m} \frac{\partial k_{t+1}^m}{\partial i_t^m} > 0$, because $\pi_m(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})$ is strictly concave in k_{t+1}^m which implies that $\frac{\partial \pi_u(k_{t+1}^m, \theta k_{t+1}^u, \Theta_{j,t+1})}{\partial k_{t+1}^m} > 0$ and $\frac{\partial k_{t+1}^m}{\partial i_t^m} = \frac{1}{q_t^m} > 0$.

[19] More exactly, by the chain rule, $\frac{\partial [q_t^u]}{\partial i_t^u} = \frac{\partial [q_t^u]}{\partial k_{t+1}^u} \frac{\partial k_{t+1}^u}{\partial i_t^u} > 0$, and $\frac{\partial [q_t^u]}{\partial i_t^m} < 0$.

returns. Because physical investment returns are dominant in stock returns, value firms earn high expected stock returns and growth firms earn low expected stock returns.

3 Dynamic Model

The equilibrium model I present is constructed with production, aggregate uncertainty and firm-specific uncertainty. Section 4.1 presents the benchmark model, and Section 4.2 presents the solutions.

3.1 The Economic Environment

The economy is comprised of a continuum of competitive firms that produce a homogeneous product. Firms behave competitively, taking the product price as given.

3.1.1 Technology

Production requires two inputs, physical capital, k^m, and R&D capital, k^u, and is subject to both an aggregate shock, x, and an idiosyncratic shock, z. The aggregate productivity shock has a stationary and monotone Markov transition function, denoted by $Q_x(x_{t+1}|x_t)$, as follows:

$$x_{t+1} = \bar{x}(1 - \rho_x) + \rho_x x_t + \sigma_x \varepsilon^x_{t+1}, \tag{16}$$

where ε^x_{t+1} is an IID standard normal shock.

The idiosyncratic productivity shocks, denoted by $z_{j,t}$, are uncorrelated across firms, indexed by j, and have a common stationary and monotone Markov transition function, denoted by $Q_z(z_{j,t+1}|z_{j,t})$, as follows:

$$z_{j,t+1} = \rho_z z_{j,t} + \sigma_z \varepsilon^z_{j,t+1}, \tag{17}$$

where $\varepsilon^z_{j,t+1}$ is an IID standard normal shock and $\varepsilon^z_{j,t+1}$ and $\varepsilon^z_{i,t+1}$ for any pair (i, j) with $i \neq j$. Moreover, ε^x_{t+1} is independent of $\varepsilon^z_{j,t+1}$ for all j.

In the model, the aggregate productivity shock is the driving force of economic fluctuations and systematic risk, and the idiosyncratic productivity shock is the driving force of the cross-sectional heterogeneity of firms.

The production function is constant returns to scale:

$$y_t = e^{x_t + z_{j,t}} \left(k_t^m\right)^{\alpha} \left(\theta k_t^u\right)^{1-\alpha}, \tag{18}$$

where y_t is output.

3.1.2 Stochastic Discount Factor

Following Berk, Green and Naik (1999) and Zhang (2005), I directly specify the pricing kernel without explicitly modeling the consumer's problem. The pricing kernel is given by

$$\log M_{t,t+1} = \log \beta + \gamma_t (x_t - x_{t+1}) \tag{19}$$

$$\gamma_t = \gamma_0 + \gamma_1 (x_t - \bar{x}) , \tag{20}$$

where $M_{t,t+1}$ denotes the stochastic discount factor from time t to $t+1$. The parameters $\{\beta, \gamma_0, \gamma_1\}$ are constants satisfying $1 > \beta > 0$, $\gamma_0 > 0$ and $\gamma_1 < 0$.

Equation (19) can be motivated as a reduced-form representation of the intertemporal marginal rate of substitution for a fictitious representative consumer. In particular, following Zhang (2005), I assume in equation (20) that γ_t is time varying and decreases in the demeaned aggregate productivity shock $x_t - \bar{x}$ to capture the countercyclical price of risk[20] with $\gamma_1 < 0$.

[20] The precise economic mechanism driving the countercyclical price of risk is, e.g., time-varying risk aversion as in Campbell and Cochrane (1999).

3.1.3 Dynamic Value Maximization

I assume that firms own their capital, and are financed purely by equity. As such, once investment has been made, the residual is distributed as a dividend[21], d_t, i.e.,

$$d_t = y_t - i_t^m - i_t^u, \tag{21}$$

Let $v(k_t^m, k_t^u, x_t, z_{j,t})$ denote the *cum*-dividend market value of the firm. I state the firm's dynamic value maximization problem as

$$v(k_t^m, k_t^u, x_t, z_{j,t}) = \max_{k_{t+1+n}^m, k_{t+1+n}^u, i_{t+n}^m, i_{t+n}^u} \mathbb{E}_t \sum_{n=0}^{\infty} M_{t,t+n} d_{t+n} \tag{22}$$

$$s.t. \quad (1) \textit{ and } (2) \textit{ with } k_t^m, k_t^u \textit{ given.}$$

3.2 Solutions

3.2.1 First-order Conditions

The first-order conditions can be written as:

$$i_t^m \;:\; q_t^m = \frac{(i_t^m)^{1-\rho}}{aA[a(i_t^m)^\rho + (1-a)(1-\theta)^\rho (k_{t+1}^u)^\rho]^{\frac{1}{\rho}-1}} \tag{23}$$

$$i_t^u \;:\; q_t^u = 1 \tag{24}$$

$$k_{t+1}^m \;:\; 1 = \mathbb{E}_t \left\{ M_{t,t+1} \left[\frac{\alpha e^{x_{t+1}+z_{j,t+1}} \left(k_{t+1}^m\right)^{\alpha-1} \left(\theta k_{t+1}^u\right)^{1-\alpha} + (1-\delta_m) q_{t+1}^m}{q_t^m} \right] \right\} \tag{25}$$

$$k_{t+1}^u \;:\; 1 = \mathbb{E}_t \left\{ M_{t,t+1} \left[\frac{(1-\alpha)\,\theta e^{x_{t+1}+z_{j,t+1}} \left(k_{t+1}^m\right)^{\alpha} \left(\theta k_{t+1}^u\right)^{-\alpha} + 1 - \delta_u}{1 - \frac{(1-a)(1-\theta)^\rho}{a} \left(\frac{i_t^m}{k_{t+1}^u}\right)^{1-\rho}} \right] \right\}, \tag{26}$$

where q_t^u and q_t^m are Lagrange multipliers associated with equations (1) and (2), respectively.

Equations (23) and (24) are the optimality conditions for physical investment and R&D investment that equate the marginal costs of investing in physical capital and intangible capital, $\left(\frac{(i_t^m)^{1-\rho}}{aA[a(i_t^m)^\rho + (1-a)(1-\theta)^\rho (k_{t+1}^u)^\rho]^{\frac{1}{\rho}-1}} \text{ and } 1 \right)$, with their marginal benefits, (q_t^m and q_t^u). Here, (q_t^m, q_t^u)

[21] Negative dividend is considered as equity issuance.

are the shadow values of physical capital and intangible capital. Equations (25) and (26) are the Euler equations that describe the optimality conditions for physical investment and R&D investment. Note that the direct marginal cost of R&D investment in equilibrium is 1, but with an indirect benefit of $q_t^m \Phi_u \left[i_t^m, (1-\theta)\, k_{t+1}^u\right]$, the effective marginal cost of R&D investment is $\tilde{q}_t^u = 1 - q_t^m \Phi_u \left[i_t^m, (1-\theta)\, k_{t+1}^u\right] = 1 - \frac{(1-a)(1-\theta)^\rho}{a}(\frac{i_t^m}{k_{t+1}^u})^{1-\rho}$, where Φ_u denotes the partial derivative of $\Phi\left[i_t^m, (1-\theta)\, k_{t+1}^u\right]$ w.r.t to k_{t+1}^u. The effective marginal cost of R&D investment $\tilde{q}_t^u$ is the same as the variable q_t^u in the two-period example.

3.2.2 Investment Returns and Stock Return

From equations (25) and (26), I define one period returns for physical investment and R&D investment as

$$r_{t+1}^m \equiv \frac{\alpha e^{x_{t+1}+z_{j,t+1}} \left(k_{t+1}^m\right)^{\alpha-1} \left(\theta k_{t+1}^u\right)^{1-\alpha} + (1-\delta_m) q_{t+1}^m}{q_t^m} \tag{27}$$

$$r_{t+1}^u \equiv \frac{(1-\alpha)\,\theta e^{x_{t+1}+z_{j,t+1}} \left(k_{t+1}^m\right)^{\alpha} \left(\theta k_{t+1}^u\right)^{-\alpha} + 1 - \delta_u}{\tilde{q}_t^u}. \tag{28}$$

Intuitively, the investment (both physical and R&D) return from time t to time $t+1$ is the ratio of the marginal benefit of investment at time $t+1$ divided by the marginal cost of investment at time t.

I also define one period stock return as

$$r_{t+1}^s \equiv \frac{p_{t+1}^s + d_{t+1}}{p_t^s}, \tag{29}$$

where p_t^s is the *ex*-dividend stock price.

Proposition 1 *The ex-dividend stock price, p_t^s, equals the sum of the market values of physical capital and intangible capital. The stock return is a weighted average of the physical investment* *and R&D investment returns:*

$$p_t^s = q_t^m k_{t+1}^m + \tilde{q}_t^u k_{t+1}^u \tag{30}$$

$$r_{t+1}^s = \frac{q_t^m k_{t+1}^m}{p_t^s} r_{t+1}^m + \frac{\tilde{q}_t^u k_{t+1}^u}{p_t^s} r_{t+1}^u. \tag{31}$$

Proof. See Appendix A. ■

Intuitively, the market value of the equity of a firm made up of the market values of two economic fundamentals, physical capital and intangible capital; accordingly, the return on equity consists of the returns on these two economic fundamentals.

3.2.3 Risk and Expected Stock Return

In the model, risk and expected stock returns are determined endogenously along with firms' value-maximization. Evaluating the value function in equation (22) at the optimum,

$$v(k_t^m, k_t^u, x_t, z_{j,t}) = d_t + \mathbb{E}_t \left[M_{t,t+1} v(k_{t+1}^m, k_{t+1}^u, x_{t+1}, z_{j,t+1}) \right] \tag{32}$$

$$\Rightarrow 1 = \mathbb{E}_t \left[M_{t,t+1} r_{t+1}^s \right] \tag{33}$$

where equation (32) is the Bellman equation for the value function and equation (33) follows from the standard formula for stock return $r_{t+1}^s = v(k_{t+1}^m, k_{t+1}^u, x_{t+1}, z_{j,t+1}) / \left[v(k_t^m, k_t^u, x_t, z_{j,t}) - d_t \right]$. Note that if I define $p_t^s \equiv v(k_t^m, k_t^u, x_t, z_{j,t}) - d_t$ as the *ex*-dividend market value of equity, r_{t+1}^s reduces to the usual definition in equation (29), $r_{t+1}^s \equiv \left(p_{t+1}^s + d_{t+1} \right) / p_t^s$.

Now I rewrite equation (33) as the beta-pricing form, following Cochrane (2001 p. 19):

$$\mathbb{E}_t \left[r_{t+1}^s \right] = r_{ft} + \beta_t \zeta_{mt} \tag{34}$$

where $r_{ft} \equiv \frac{1}{\mathbb{E}_t[M_{t,t+1}]}$ is the real interest rate, and β_t is the risk defined as:

$$\beta_t \equiv \frac{-Cov_t \left[r_{t+1}^s, M_{t,t+1} \right]}{Var_t \left[M_{t,t+1} \right]} \tag{35}$$

and ζ_{mt} is the price of risk defined as

$$\zeta_{mt} \equiv \frac{Var_t\left[M_{t,t+1}\right]}{E_t\left[M_{t,t+1}\right]}.$$

Equation (34) and (35) imply that risk and expected returns are endogenously determined along with optimal investment decisions. All the endogenous variables are functions of four state variables (the endogenous state variables, k_t^m and k_t^u, and two exogenous state variables, x_t and $z_{j,t}$, which can be solved numerically.

4 Main Findings

Section 5.1 presents the calibration of the model. Section 5.2 presents the properties of the model solutions. Section 5.3 presents the main quantitative results. Section 5.4 investigates the economic sources of the empirical predictions of the model, and lastly Section 5.5 discusses the crucial assumption of the model.

4.1 Calibration

I divide the parameters of the benchmark model into two groups and then calibrate their quarterly values. Panels A and B in Table 1 summarize these values. Table 2 reports the model-implied moments and the data (See Appendix B for data construction.).

The first group includes parameters that can be restricted by empirical research or quantitative studies: the share of physical capital is 0.65, estimated using NIPA data (See Appendix C.1 for estimation details.). The average proportion of R&D capital devoted to new product, θ, is set at 70% following the estimate in Cohen and Klepper (1996). The physical capital depreciation rate $\delta_m = 2.5\%$ is from Jermann (1998); the intangible capital depreciation rate δ_u is set at 5%;[22] persistence ρ_x and conditional volatility σ_x of aggregate productivity are from Cooley and

[22] There is no agreement on the depreciation rate for R&D capital. However, it is generally agreed that R&D capital depreciates faster than physical capital. I choose to use a quarterly rate of 5% implying an annual rate of 20%, consistent with Chan, Lakonishok and Sougiannis (2001). The calibration results are not sensitive to the depreciation rate of R&D capital.

Prescott (1995), $\rho_x = 0.95, \sigma_x = 0.007$. The long-run average level of aggregate productivity, $\bar{x}$, is a scaling variable. Following Zhang (2005), I set the average long-run R&D capital in the economy at one, which implies that the long-run average of aggregate productivity $\bar{x} = -2.08$. To calibrate persistence ρ_z and conditional volatility σ_z of firm-specific productivity, I follow Zhang (2005) and restrict these two parameters using their implications on the degree of dispersion in the cross-sectional distribution of firms' stock return volatilities. Thus $\rho_z = 0.91$, and $\sigma_z = 0.17$, which[23] implies an average annual volatility of individual stock returns of 24.4%, approximately the average of 25% reported by Campbell at al (2001) and 32% reported by Vuolteenaho (2001).

Following Zhang (2005), I pin down the three parameters governing the stochastic discount factor, β, γ_0, and γ_1 to match three aggregate return moments: the average real interest rate, the volatility of the real interest rate, and the average annual Sharpe ratio. This procedure yields $\beta = 0.995$, $\gamma_0 = 23$, and $\gamma_1 = -900$, which generate an average annual real interest rate of 1.65%, an annual volatility of real interest rate of 3.2%, an average annual stock market return of 10.82% and an annual volatility of the stock market return of 16.26%. Those values are close to the values obtained in the data. The calibrated Sharpe ratio of the model is 0.56, close to 0.54 for the last 30 years (1975-2005) of data.

Prior studies provide only limited guidance for the calibration of the second group of parameters. These parameters are: (i) a, the weight on physical investment in $\Phi\left[i_t^m, (1-\theta)\, k_{t+1}^u\right]$; (ii) $\frac{1}{1-\rho}$, the elasticity of substitution between physical investment and intangible capital in $\Phi\left[i_t^m, (1-\theta)\, k_{t+1}^u\right]$; and (iii) A, the constant term in $\Phi\left[i_t^m, (1-\theta)\, k_{t+1}^u\right]$. I pin down these three parameters to match three moments: the average annual rate of physical investment, the average annual market-to-book ratio, and the average annual R&D investment to physical investment ratio. This procedure yields $a = 0.79$, $A = 0.42$, and $\rho = 0.25$. The calibrated mean and volatility of physical investment rate in the model are 0.15 and 0.07, respectively, close to 0.15 and 0.06 reported in Hennessy and Whited (2005). The calibrated mean and volatility of market-to-book ratio are 1.94 and 0.27, respectively, close to 1.50 and 0.24 reported by Hennessy and Whited. The average ratio of R&D investment to physical investment is 0.57 in the model, close to the value of 0.51 in the data. In

[23] Persistence ρ_z and conditional volatility σ_z are set to match monthly values from Zhang (2005), $\rho_z = 0.97^3 = 0.91, \sigma_z = 0.10\sqrt{1+\rho_z+\rho_z^2} = 0.17$

sum, the calibrated parameter values seem reasonable representative of reality.

4.2 Properties of Model Solutions

In this section, I investigate the qualitative properties of the key variables in the model.

4.2.1 Marginal Cost of Investments

The formulation of the production function and the evolution of new physical capital have the following implications for the behavior of the marginal cost of physical investment and the effective marginal cost of R&D investment:

Marginal Cost of Physical Investment

The critical variable in the model is q_t^m, the equilibrium marginal cost of physical investment. Panels A and B in Figure 1 plot the numerical examples of q_t^m as functions of physical investment i_t^m and intangible capital k_{t+1}^u. In Panel A, I plot q_t^m against physical investment i_t^m in four curves, each of which corresponds to one value of intangible capital k_{t+1}^u, where the arrow indicates the direction along which k_{t+1}^u increases. In Panel B I plot q_t^m against intangible capital k_{t+1}^u in four curves, each of which corresponds to one value of physical investment i_t^m, where the arrow indicates the direction along which i_t^m increases. Marginal cost of physical investment q_t^m is increasing in physical investment i_t^m due to diminishing-marginal-returns of $\Phi\left[i_t^m, (1-\theta)\,k_{t+1}^u\right]$ in i_t^m,[24] and is decreasing in intangible capital k_{t+1}^u because current technological progress makes new capital production more efficient and less expensive.[25]

Effective Marginal Cost of R&D Investment

Panels C and D in Figure 1 plot the numerical examples of the effective marginal cost of R&D investment $\tilde{q}_t^u$ as functions of physical investment i_t^m and intangible capital k_{t+1}^u. In Panel C I plot $\tilde{q}_t^u$ against physical investment i_t^m in four curves, each of which corresponds to one value of intangible capital k_{t+1}^u, where the arrow indicates the direction along which k_{t+1}^u increases. In Panel D I plot $\tilde{q}_t^u$ against intangible capital k_{t+1}^u in four curves, each of which corresponds

[24] In the model, $q_t^m = \frac{1}{\Phi_1\left[i_t^m,(1-\theta)k_{t+1}^u\right]}$. Since $\Phi_{11}\left[i_t^m, (1-\theta)\,k_{t+1}^u\right] < 0$, $\frac{\partial q_t^m}{\partial i_t^m} > 0$.

[25] To see why this is the case, $q_t^m = \frac{1}{\Phi_1\left[i_t^m,(1-\theta)k_{t+1}^u\right]}$. Since $\Phi_{12}\left[i_t^m, (1-\theta)\,k_{t+1}^u\right] > 0$, $\frac{\partial q_t^m}{\partial k_{t+1}^u} < 0$.

to one value of physical investment i_t^m, where the arrow indicates the direction along which i_t^m increases. The effective marginal cost of R&D investment $\tilde{q}_t^u$ is decreasing in physical investment because the term of indirect benefit $q_t^m \Phi_u \left[i_t^m, (1-\theta) k_{t+1}^u\right]$ is increasing in physical investment. The effective marginal cost of R&D investment $\tilde{q}_t^u$ is increasing in R&D capital due to the concavity of $\Phi \left[i_t^m, (1-\theta) k_{t+1}^u\right]$ in k_{t+1}^u.

4.2.2 Value Functions and Policy Functions

Using the numerical solution to the benchmark model, I plot and discuss the value and policy functions as functions of the underlying state variables.

Because there are four state variables (physical capital stock k_t^m, intangible capital stock k_t^u, the aggregate productivity shock x_t, and idiosyncratic productivity shock z_t), and the focus of the paper is the cross-sectional variations, I fix the aggregate productivity shock at its long-run average, $x_t = \bar{x}$. Panels A and C in Figure 2 plot the variables against k_t^m and z_t, with k_t^u and x_t fixed at their long-run average levels $\bar{k}^u$ and $\bar{x}$. Panels B and D in Figure 2 plot the variables against k_t^u and z_t, with k_t^m and x_t fixed at their long-run average level $\bar{k}^m$ and $\bar{x}$. Each one of these panels has a set of curves corresponding to different values of z_t, and the arrow in each panel indicates the direction along which z_t increases.

In Panels A and B in Figure 2, the firms' *cum*-dividend market value of equity is increasing in the firm-specific productivity, the physical capital stock and the intangible capital stock. Because of constant returns to scale in the output production technology, firm value is linear in the physical capital stock and intangible capital stock. In Panels C and D in Figure 2, the optimal physical investment and R&D investment are increasing in the firm-specific productivity. This indicates that the more profitable firms with higher firm-specific productivity invest more than less profitable firms with lower firm-specific productivity. This finding is consistent with the evidence documented by Fama and French (1995). In Panels C and D of Figure 2, the optimal investment rates are decreasing in capital stocks. Small firms with less capital invest more and grow faster than big firms with more capital. That prediction is consistent with the evidence provided by Evans (1987) and Hall (1987).

4.2.3 Fundamental Determinants of Risk

I find that risk, measured as β_t from equation (35), is decreasing in the three firm-specific state variables: the physical capital stock, the intangible capital stock and the firm-specific productivity. Using the benchmark parametrization, Panels A and B of Figure 3 plot β_t against physical capital, k_t^m, and intangible capital, k_t^u, and firm-specific productivity, z_t, with the aggregate productivity fixed at its long-run averages, $x_t = \bar{x}$. Doing so allows me to focus on the cross-sectional variation of risk. Panels A and B plot β_t in four curves, each of which corresponds to one value of firm-specific productivity, z_t. The arrow in the panels indicates the direction along which z_t increases. Small firms with less physical capital are more risky than big firms with more capital. That is consistent with Li, Livdan and Zhang (2008). Consistent with Zhang (2005), less profitable firms are riskier than more profitable firms, .

4.3 Empirical Predictions

Here, the quantitative implications concerning the cross section of returns in the model are investigated. I show that a neoclassic model with endogenous technological progress driven by R&D investment is capable of simultaneously generating a positive relation between R&D investment and the subsequent average of stock returns and a negative relation between physical investment and the subsequent average of stock returns. The model also generates a positive relation between book-to-market ratio and the subsequent average of stock returns.

The design of the quantitative experiment follows Kydland and Prescott (1982), Berk, Green and Naik (1999) and Zhang (2005). I simulate 100 samples, each with 3000 firms. And each firm has 120 quarterly observations. The empirical procedure on each artificial sample is implemented and the cross-simulation results are reported. I then compare model moments with where possible those in the data.

4.3.1 R&D Investment and Stock Returns

I now investigate the empirical predictions of the model on the cross section of stock returns and R&D investment. I focus on the work of Chan, Lakonishok and Sougiannis (2001) and Li

(2006). They document a positive relation between R&D intensity[26] and the subsequent average of stock returns. Chan, Lakonishok and Sougiannis (2001) interpret their results as indicating that investors are overly pessimistic about R&D firms' prospects. Li (2006) attributes her results to the fact that R&D firms are more likely to be financially constrained. I show that a neoclassical model without investor irrationality or financing frictions can quantitatively replicate their evidence.

I follow Chan, Lakonishok and Sougiannis (2001) in constructing 5 equal-weighted R&D portfolios for each simulated panel (See Appendix C.2 for details about the empirical procedure.). The market value of equity in the model is defined as the *ex*-dividend stock price. I sort all firms into 5 portfolios based on firms' ratio of R&D investment to market value of equity, i_{t-1}^u/p_{t-1}^s, and the ratio of R&D investment to physical investment, i_{t-1}^u/i_{t-1}^m, in ascending order as of the beginning of year t. I then calculate the equal-weighted annual average stock returns and average excess returns for each R&D investment portfolio. Following Chan, Lakonishok and Sougiannis (2001), I measure excess returns relative to benchmarks constructed to have similar firm characteristics such as size and book-to-market (See Appendix C.1 for details about the empirical procedure.). I construct a R&D investment-spread portfolio long in the high R&D intensity $\left(i_{t-1}^u/p_{t-1}^s,\ i_{t-1}^u/i_{t-1}^m\right)$ portfolio and short in the low R&D intensity $\left(i_{t-1}^u/p_{t-1}^s,\ i_{t-1}^u/i_{t-1}^m\right)$ portfolio. I repeat the entire simulation 100 times and report the cross-simulation averages of the summary statistics in Table 3.

From Panel A and Panel B in Table 3, consistent with Chan, Lakonishok and Sougiannis (2001) and Li (2006), firms with high R&D intensity, i_{t-1}^u/p_{t-1}^s (i_{t-1}^u/i_{t-1}^m), earn higher average stock returns and higher excess returns than firms with low R&D intensity. The model generates a reliable R&D investment-spread in Panel B, which is 8.75% (10.03%) per annum for portfolios sorted on i_{t-1}^u/p_{t-1}^s and i_{t-1}^u/i_{t-1}^m, respectively, close to those in the data, 12.06% (11.67%).

4.3.2 Physical Investment and Stock Returns

I now investigate the empirical predictions of the model for the cross section of stock returns and physical investment. I focus on Xing (2006), who documents that physical investment contains

[26] Chan, Lakonishok and Sougiannis (2001) and Li (2006) use R&D investment scaled by market value of equity, and R&D investment to physical investment ratio as R&D intensity, respectively.

information similar to the book-to-market ratio in explaining the value effect and that firms with higher rate of physical investment earn lower average subsequent stock returns.

I follow Xing (2006) in constructing 10 (both value-weighted and equal-weighted) portfolios sorted on physical investment. I sort all firms into 10 portfolios based on firms' rate of physical investment, i^m_{t-1}/k^m_{t-1}, in ascending order as of the beginning of year t. I construct a physical-investment-spread portfolio long in the low i^m_{t-1}/k^m_{t-1} portfolio and short in the high i^m_{t-1}/k^m_{t-1} portfolio, for each simulated panel. Table 4 reports the average stock returns of 10 portfolios sorted on physical investment. Consistent with Xing (2006), firms with low i^m_{t-1}/k^m_{t-1} on average earn higher stock returns than firms with high i^m_{t-1}/k^m_{t-1}. The model-implied average value-weighted (equal-weighted) physical investment-spread is 14.21% (17.51%) per annum. This spread is higher than that in the data, 5.28% (5.64%).

4.3.3 Abnormal Physical Investment and Stock Returns

I now investigate the empirical predictions of the model for the cross section of stock returns and abnormal physical investment. I focus on Titman, Wei and Xie (2004), who document that firms with higher abnormal physical investment, defined as $CI^m_{t-1} = \frac{CE^m_{t-1}}{(CE^m_{t-2}+CE^m_{t-3}+CE^m_{t-4})/3} - 1$ in the portfolio formation year t, earn lower subsequent average stock returns after controlling size, book-to-market and momentum (prior year return), where CE^m_{t-1} is physical capital expenditure scaled by sales during year $t-1$. Titman, Wei and Xie (2004) attribute their findings to investors' underreacting to the overinvestment behavior of empire building managers. I show that a neoclassical model without investor irrationality can quantitatively replicate their evidence. It is worth noting that Li, Livdan and Zhang (2008) also generate similar quantitative results, but with a different model.

I measure CE^m_{t-1} in the model as the physical investment-to-output ratio, i^m_{t-1}/y_{t-1}. The last three-year moving-average physical capital expenditure in the denominator of CI^m_{t-1} is used to proxy for firms' benchmark physical investment. I sort all firms into quintiles based on CI^m_{t-1} in ascending order as of the beginning of year t. I construct a CI-spread portfolio long in the low CI portfolio and short in the high CI portfolio, for each simulated panel.

I calculate the value-weighted annual excess returns for each CI portfolio. Following Titman, Wei, and Xie (2004), I measure excess returns relative to benchmarks constructed to have similar firm characteristics such as size, book-to-market, and momentum. (See Appendix C.3 for details about the empirical procedure.) Table 6 reports the average excess stock returns of 5 portfolios sorted on abnormal physical investment, CI. Consistent with the findings of Titman, Wei and Xie (2004), firms with low CI earn higher average excess stock returns than firms with high CI. The model-implied average CI-spread is 2.14% per annum. This spread is close to that documented in the data, 2.03%.

In sum, the benchmark model can *simultaneously* generate a positive covariation between R&D investment and future average stock returns, and a negative covariation between physical investment and future average stock returns.

Notably, the stochastic discount factor with countercyclical market price of risk is necessary to generate spreads of R&D investment portfolios and physical investment portfolios that are consistent with the data. With a constant price of risk, i.e., $\gamma_1 = 0$, the spreads of portfolio returns are smaller than those with a countercyclical price of risk. The results with constant price of risk are available upon request.

4.3.4 The Value Premium

Here, I explore the relation between endogenous technological progress and the value premium.

First I investigate if the model can generate a positive relation between the book-to-market ratio and expected stock returns. I construct 10 value-weighted and equal-weighted book-to-market portfolios. The book value of a firm in the model is identified as its physical capital stock. I sort all firms into 10 portfolios based on firms' book-to-market ratio, k^m_{t-1}/p^s_{t-1}, in ascending order as of the beginning of year t. I construct a value-spread portfolio long in the high book-to-market portfolio and short in the low book-to-market portfolio for each simulated panel. Table 7 reports the average stock returns of 10 portfolios sorted by book-to-market ratio. Consistent with the findings of Fama-French (1992, 1993), firms with low book-to-market ratios earn lower stock returns on average than do firms with high book-to-market ratios. The model-implied average value-weighted (equal-weighted) value-spread is 13.45% (19.27%) per annum. This spread is close to that documented in the data, 8.72% (19.36%).

4.4 Causality

I now focus on causal relations why R&D investment positively forecasts average stock returns while physical investment negatively forecasts average stock returns in the model. I also investigate the relation between endogenous technological progress and the value premium.

4.4.1 Investment Returns and Investment

First I examine the covariations between investment returns (both R&D and physical) and investment. In Panel A of Table 8, I report simulated average physical investment returns and R&D investment returns of 5 portfolios sorted on R&D intensity and rate of physical investment. The expected return on physical investment is negatively related to physical investment but *positively* related to R&D investment. This is because R&D investment increases the marginal product of physical capital; and R&D (physical) investment decreases (increases) the marginal cost of physical investment, which is negatively related to the expected physical investment returns. The expected returns on R&D investment covaries *positively* with physical investment and covaries *negatively* with R&D investment. That is because the expected marginal product of R&D capital (the effective marginal cost of R&D investment) is decreasing (increasing) in R&D investment but increasing (decreasing) in physical investment. So investments (both R&D and physical) covary with the expected physical investment return and R&D investment return in opposite ways. That leads to two countervailing effects on the predictability of investments on future average stock returns. We need to examine the weights on R&D investment return and physical investment return to determine which effect dominates.

4.4.2 Weights on Investment Returns

Panel B in Table 8 reports simulated average weights on physical investment return and on R&D investment return for 5 portfolios sorted on R&D intensity and rate of physical investment. The weight on physical investment return $\frac{q_t^m k_{t+1}^m}{p_t^s}$ is much greater than the weight on R&D investment return $\frac{\tilde{q}_t^u k_{t+1}^u}{p_t^s}$. This is because physical capital production involves both intangible capital and physical investment, and the share of intangible capital in the output production is smaller than that of the tangible capital. The difference in weights between physical investment return and R&D investment return implies that physical investment return together with its weight $\frac{q_t^m k_{t+1}^m}{p_t^s} r_{t+1}^m$ dominates in stock returns. That is why R&D investment positively forecasts average future stock returns while physical investment negatively forecasts average future stock returns.

4.4.3 Endogenous Technological Progress and the Value Premium

In the model, value firms invest less in intangible capital than do growth firms, so value firms do not gain as much from technological progress in increasing the productivity of physical capital as growth firms do. When a recession comes, value firms are stuck with excessive physical capital and do not have much endogenous technological progress to upgrade the efficiency of physical capital. They are therefore more risky than growth firms, given that the price of risk is high in economic downturns. This interaction between endogenous technological progress and physical capital reinforces the mechanism emphasized in Zhang (2005) who demonstrates that costly reversibility of physical capital is one of key mechanisms driving the value premium.

4.5 Discussion

The crucial channel in the model in generating a positive relation between R&D investment and the average stock returns is productivity increasing innovation. This is because, on one hand, if all R&D investment is devoted to creating new products ($\theta = 1$), the model reduces to standard models which predict that both R&D investment and physical investment forecast the expected stock returns in the same way[27], which is counterfactual; on the other hand, if all R&D investment is dedicated to increasing productivity of physical investment ($\theta = 0$) and output production is linear in physical capital, the model can still simultaneously explain the relationships of R&D

[27] Then output production has to be decreasing returns to scale in physical capital and R&D capital to guarantee an interior solution.

investment and physical investment with the average stock returns[28].

5 Concluding Remarks

Following Cochrane (1991, 1996), I show that a neoclassical model with endogenous technological progress driven by R&D investment can explain a number of empirical regularities in the cross section of stock returns. Most notably, technological progress endogenously driven by R&D investment raises expected marginal benefit of physical capital and reduces the marginal cost of physical investment, causing expected returns in physical investment increasing in R&D investment. The expected physical investment return is decreasing in physical investment due to diminishing marginal returns of physical capital production. In the model the weight on physical investment return dominates the weight on R&D investment return, thus the model simultaneously explains why R&D investment-intensive firms earn high average stock returns while physical investment-intensive firms earn low average stock returns. The positive predictability of R&D investment on expected stock returns, interpreted by Chan et al (2001) as excessive pessimism, is in principle consistent with rational expectations. The model also explains why value firms are more risky than growth firms; value firms invest less in R&D capital, and thus do not have as much technological progress in upgrading the efficiency of the existing physical capital as growth firms, especially in bad times.

Future research can proceed in a few directions. Theoretically, a full-fledged general equilibrium model with Epstein-Zin preferences can link endogenous technological progress to long-run consumption risk. The neoclassical framework in the model can also be extended to link asset prices to other types of intangible capital, e.g., human capital and organizational capital. Empirically, the correlation between human capital, organizational capital and physical capital, and their relations with the cross section of stock returns is worth further investigating.

[28] The quantitative results of the model with only productivity increasing R&D is available upon request.

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Appendix A: Proof

Proof of Proposition 1. I first show $p_t^s = q_t^m k_{t+1}^m + \tilde{q}_t^u k_{t+1}^u$. Production function is constant return to scale:

$$y_t = e^{x_t + z_{j,t}} \left(k_t^m\right)^\alpha \left(\theta k_t^u\right)^{1-\alpha}. \tag{36}$$

Transversality conditions for k_{t+1+j}^m and k_{t+1+j}^u are

$$\lim_{n\to\infty} \mathbb{E}_t M_{t,t+n} q_{t+j}^m k_{t+1+n}^m = 0 \tag{37}$$

$$\lim_{n\to\infty} \mathbb{E}_t M_{t,t+n} k_{t+1+n}^u = 0. \tag{38}$$

Define firms' *cum*-dividend market value as

$$v(k_t^m, k_t^u, x_t, z_{j,t}) \equiv p_t^s + d_t. \tag{39}$$

Dividend is given by

$$\begin{aligned} d_t &= y_t - i_t^m - i_t^u \\ &= e^{x_t + z_{j,t}} \left(k_t^m\right)^\alpha \left(\theta k_t^u\right)^{1-\alpha} - i_t^m - i_t^u. \end{aligned} \tag{40}$$

Combining equation (39) and (40), I get

$$\upsilon(k_t^m, k_t^u, x_t, z_{j,t}) = p_t^s + e^{x_t+z_{j,t}} \left(k_t^m\right)^\alpha \left(\theta k_t^u\right)^{1-\alpha} - i_t^m - i_t^u \tag{41}$$

Physical capital accumulation process in equation (2) can be rewritten as

$$k_{t+1}^m = (1-\delta_m)k_t^m + i_t^m \Phi_m \left[i_t^m, (1-\theta)\, k_{t+1}^u\right] + k_{t+1}^u \Phi_u \left[i_t^m, (1-\theta)\, k_{t+1}^u\right], \tag{42}$$

where Φ_m and Φ_u are partial derivatives of $\Phi\left[i_t^m, (1-\theta)k_{t+1}^u\right]$ w.r.t. i_t^m and k_{t+1}^u. Expanding the value function in equation (22) and using equation (42), I get

$$\begin{aligned}
&\upsilon(k_t^m, k_t^u, x_t, z_{j,t}) \\
&= \mathbb{E}_t \sum_{n=0}^{\infty} M_{t,t+n}\{(e^{x_t+z_{j,t}} \left(k_t^m\right)^\alpha \left(\theta k_t^u\right)^{1-\alpha} - i_{t+n}^m - i_{t+n}^u) - q_{t+n}^m[k_{t+1+n}^m - (1-\delta_m)k_{t+n}^m - \Phi_m i_{t+n}^m - \Phi_u k_{t+1+n}^u] \\
&\quad - q_{t+n}^u[k_{t+1+n}^u - (1-\delta_u)k_{t+n}^u - i_{t+n}^u]\}.
\end{aligned}$$

Recursively substituting equation (1), (2) and (23)-(26), I find

$$\begin{aligned}
&\upsilon(k_t^m, k_t^u, x_t, z_{j,t}) \\
&= e^{x_t+z_{j,t}} \left(k_t^m\right)^\alpha \left(\theta k_t^u\right)^{1-\alpha} + q_t^m(1-\delta_m)k_t^m + (1-\delta_u)k_t^u - \lim_{j\to\infty} \mathbb{E}_t M_{t+j} q_{t+j}^m k_{t+1+j}^m - (1-\delta_u) \lim_{j\to\infty} \mathbb{E}_t M_{t+j} k_{t+1+j}^u \\
&= e^{x_t+z_{j,t}} \left(k_t^m\right)^\alpha \left(\theta k_t^u\right)^{1-\alpha} + q_t^m(1-\delta_m)k_t^m + (1-\delta_u)k_t^u.
\end{aligned}$$

Combining with equation (41), I get

$$p_t^s + e^{x_t+z_{j,t}} \left(k_t^m\right)^\alpha \left(\theta k_t^u\right)^{1-\alpha} - i_t^m - i_t^u = e^{x_t+z_{j,t}} \left(k_t^m\right)^\alpha \left(\theta k_t^u\right)^{1-\alpha} + q_t^m(1-\delta_m)k_t^m + (1-\delta_u)k_t^u.$$

Re-arranging and using equation (4) leads to

$$\begin{aligned}
p_t^s &= q_t^m k_{t+1}^m + \left[1 - q_t^m \Phi_u \left[i_t^m, (1-\theta)k_{t+1}^u\right]\right] k_{t+1}^u && (43)\\
&= q_t^m k_{t+1}^m + \tilde{q}_t^u k_{t+1}^u. && (44)
\end{aligned}$$

Q.E.D. ■

Now I show $r^s_{t+1} = \frac{q^m_t k^m_{t+1}}{p^s_t} r^m_{t+1} + \frac{\tilde{q}^u_t k^u_{t+1}}{p^s_t} r^u_{t+1}$.

Define stock return as

$$r^s_{t+1} \equiv \frac{p^s_{t+1} + d_{t+1}}{p^s_t}.$$

Using equation (40) implies

$$r^s_{t+1} = \frac{p^s_{t+1} + e^{x_{t+1}+z_{j,t+1}} \left(k^m_{t+1}\right)^{\alpha} \left(\theta k^u_{t+1}\right)^{1-\alpha} - i^m_{t+1} - i^u_{t+1}}{p^s_t}.$$

Using equation (43) I find

$$r^s_{t+1} = \frac{q^m_{t+1} k^m_{t+2} + (1 - q^m_t \Phi_u) k^u_{t+2} + e^{x_{t+1}+z_{j,t+1}} \left(k^m\right)^{\alpha} \left(\theta k^u\right)^{1-\alpha} - i^m_{t+1} - i^u_{t+1}}{p^s_t}.$$

Since $\Phi\left[i^m_{t+1}, (1-\theta) k^u_{t+2}\right]$ is constant returns to scale in (i^m_{t+1}, k^u_{t+2}), I get

$$\Phi\left[i^m_{t+1}, (1-\theta) k^u_{t+2}\right] = i^m_{t+1} \Phi_m \left[i^m_{t+1}, (1-\theta) k^u_{t+2}\right] + k^u_{t+2} \Phi_u \left[i^m_{t+1}, (1-\theta) k^u_{t+2}\right].$$

This implies

$$\begin{aligned}
r^s_{t+1} &= \frac{q^m_{t+1}(1-\delta_m) k^m_{t+1} + (1-\delta_u) k^u_{t+1} + e^{x_{t+1}+z_{j,t+1}} \left(k^m_{t+1}\right)^{\alpha} \left(\theta k^u_{t+1}\right)^{1-\alpha}}{p^s_t} \\
&= \frac{k^m_{t+1}\left[\alpha e^{x_{t+1}+z_{j,t+1}} \left(k^m_{t+1}\right)^{\alpha-1} \left(\theta k^u_{t+1}\right)^{1-\alpha} + (1-\delta_m) q^m_{t+1}\right]}{p^s_t} \\
&+ \frac{\left[(1-\alpha)\, e^{x_{t+1}+z_{j,t+1}} \left(k^m_{t+1}\right)^{\alpha} \left(\theta k^u_{t+1}\right)^{-\alpha} + (1-\delta_u)\right] k^u_{t+1}}{p^s_t} \\
&= \frac{q^m_t k^m_{t+1} [\frac{\alpha e^{x_{t+1}+z_{j,t+1}} \left(k^m_{t+1}\right)^{\alpha-1} \left(\theta k^u_{t+1}\right)^{1-\alpha} + (1-\delta_m) q^m_{t+1}}{q^m_t}]}{p^s_t} \\
&+ \frac{\tilde{q}^u_t k^u_{t+1} \left[\frac{(1-\alpha)\theta e^{x_{t+1}+z_{j,t+1}} \left(k^m_{t+1}\right)^{\alpha} \left(\theta k^u_{t+1}\right)^{-\alpha} + (1-\delta_u)}{\tilde{q}^u_t}\right]}{p^s_t} \\
&= \frac{q^m_t k^m_{t+1}}{p^s_t} r^m_{t+1} + \frac{\tilde{q}^u_t k^u_{t+1}}{p^s_t} r^u_{t+1}.
\end{aligned}$$

Q.E.D.

Appendix B: Data Construction

1. **Stock returns.** I use annual CRSP value-weighted returns (1975-2005) from Ken French website[29] as stock market returns. The annual risk-free rate is from Ken French's website. Monthly returns are from CRSP. The annual return of a stock is compounded from monthly returns, recorded from the beginning of June to the end of May. The market value of equity is taken from CRSP at the end of May. The size of a firm is its market capitalization at the end of May, taken from CRSP.

2. **Rate of inflation.** To get real returns, I use price index of personal consumption expenditures in National Income and Product Accounts (NIPA) Table 2.3.4 to calculate rate of inflation.

3. **Physical capital and physical investment.** COMPUSTAT data item 128 is used for physical investment, i_t^m, and the net book value of property, plant, and equipment (data item 8) is used for the net fixed assets, k_t^m

4. **R&D investment.** COMPUSTAT data item 46 is used for R&D investment, i_t^u.

Appendix C: Empirical Procedure

C.1 Estimating Output Production Function

Output production function is given by

$$y_t = e^{x_t + z_{j,t}} \left(k^m\right)^{\alpha} \left(\theta k^u\right)^{1-\alpha},$$

I estimate α using NIPA data. Output y is GDP from NIPA Table 1.1.5 , physical capital k^m is private nonresidential fixed assets from NIPA Table 4.1, and R&D capital k^u is net stock of private R&D assets from NIPA Table 3.4. Sample period is 1975-2002.

[29] I thank Eugene Fama and Kenneth French for making their datasets available.

C.2 Calculating Charateristic-Adjusted Excess Returns for the R&D Investment Portfolios à la Chan, Lakonishok and Sougiannis (2001)

To calculate the characteristic-adjusted excess returns of the R&D investment portfolios, I follow Chan, Lakonishok and Sougiannis (2001). Specifically, I form 25 benchmark portfolios that capture these characteristics. Starting in year t, the universe of common stocks is sorted into five portfolios based on firm size at the end of year t-1. And the breakpoints for size are obtained by sorting all firms into quintiles based on their size measures at the end of year t-1 in ascending order. The size of each firm in our sample is then compared with the breakpoints to decide which portfolio the firm belongs to. Firms in each size portfolio are further equally sorted into quintiles based on their book-to-market ratio at the end of year t-1. In all, I obtain 25 benchmark portfolios.

I calculate excess returns using these 25 characteristic-based benchmark portfolios. Each year, each stock is assigned to a benchmark portfolio according to its rank based on size and book-to-market. Excess annual returns of a stock are then calculated by subtracting the returns of the corresponding benchmark portfolio from the returns of this particular stock. The excess returns on individual stocks are then used to calculate the equal-weighted excess annual returns on the test portfolios that are formed based on R&D intensity.

C.3 Calculating Characteristic-Adjusted Excess Returns for Abnormal Physical Investment Portfolios à la Titman, Wei and Xie (2004)

To calculate the characteristic-adjusted excess returns of the physical investment portfolios, I follow Titman, Wei and Xie (2004). Specifically, I form 125 benchmark portfolios that capture these characteristics. Starting in year t, the universe of common stocks is sorted into five portfolios based on firm size at the end of year t-1. And the breakpoints for size are obtained by sorting all firms into quintiles based on their size measures at the end of year t-1 in ascending order. The size of each firm in our sample is then compared with the breakpoints to decide which portfolio the firm belongs to. Firms in each size portfolio are further equally sorted into quintiles based on their book-to-market ratio at the end of year t-1. Finally, the firms in each of the 25 size and book-to-market portfolios are equally sorted into quintiles based on their prior-year-return. In all, I obtain 125 benchmark portfolios.

I calculate excess returns using these 125 characteristic-based benchmark portfolios. Each year, each stock is assigned to a benchmark portfolio according to its rank based on size, book-to-market and prior year returns. Excess annual returns of a stock are then calculated by subtracting the returns of the corresponding benchmark portfolio from the returns of this particular stock. The excess returns on individual stocks are then used to calculate the value-weighted excess annual returns on the test portfolios that are formed based on abnormal physical investment.

Appendix D: Numerical Method

To solve the model numerically, I use the value function iteration procedure to solve the firm's maximization problem. The value function and the optimal decision rule are solved on a grid in a discrete state space. I specify a grid with 00 points each for the physical capital and intangible capital, respectively with upper bounds $\bar{k}_m$, $\bar{k}_u$ (large enough to be nonbinding at al times). The grids for physical capital and intangible capital stocks are constructed recursively, following McGrattan (1999), that is, $\mathrm{k}_i = k_{i-1} + c_{k1}\exp(c_{k2}(i-2))$, where i=1,...,100 is the index of grids points and c_{k1} and c_{k2} are two constants chosen to provide the desired number of grid points and two upper bounds $\bar{k}_m$, $\bar{k}_u$, given two pre-specified lower bounds $\underline{\mathrm{k}}_m$, $\underline{\mathrm{k}}_u$. The advantage of this recursive construction is that more grid points are assigned around $\underline{\mathrm{k}}_m$, $\underline{\mathrm{k}}_u$, where the value function has most of its curvature.

The state variable x is defined on continuous state space, which has to be transformed into discrete state space. I use the method described in Rouwenhorst (1995) that works well when persistence level is above 0.9. I use 9 grid points for x process and 15 grid points for z process. In all cases the results are robust to finer grids as well. Once the discrete state space is available, the conditional expectation can be carried out simply as a matrix multiplication. Linear interpolation is used extensively to obtain optimal investments which do not lie directly on the grid points. Finally, I use a simple discrete, global search routine in maximizing problems.

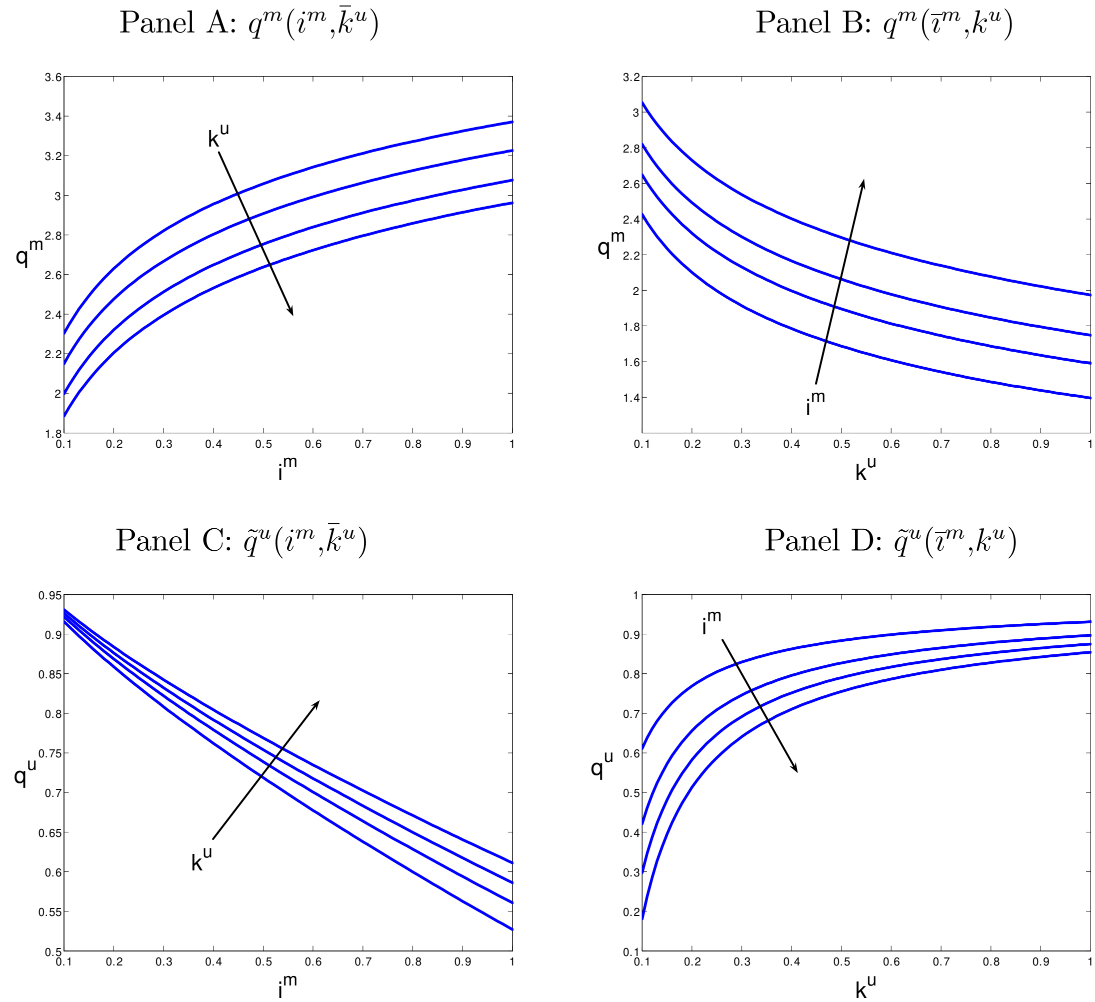

Figure 1 Marginal Cost of Physical Investment q^m and the Effective Marginal Cost of R&D Investment $\tilde{q}^u$. This figure plots the marginal cost of physical investment q^m and the effective marginal cost of R&D investment $\tilde{q}^u$ as a function of physical investment i^m and intangible capital k^u. In panel A and panel C, I plot q^m and $\tilde{q}^u$ against i^m. The arrow indicates the direction along which k^u increases. I then plot q^m and $\tilde{q}^u$ against k^u in panel B and panel D. The arrow indicates the direction along which i^m increases.

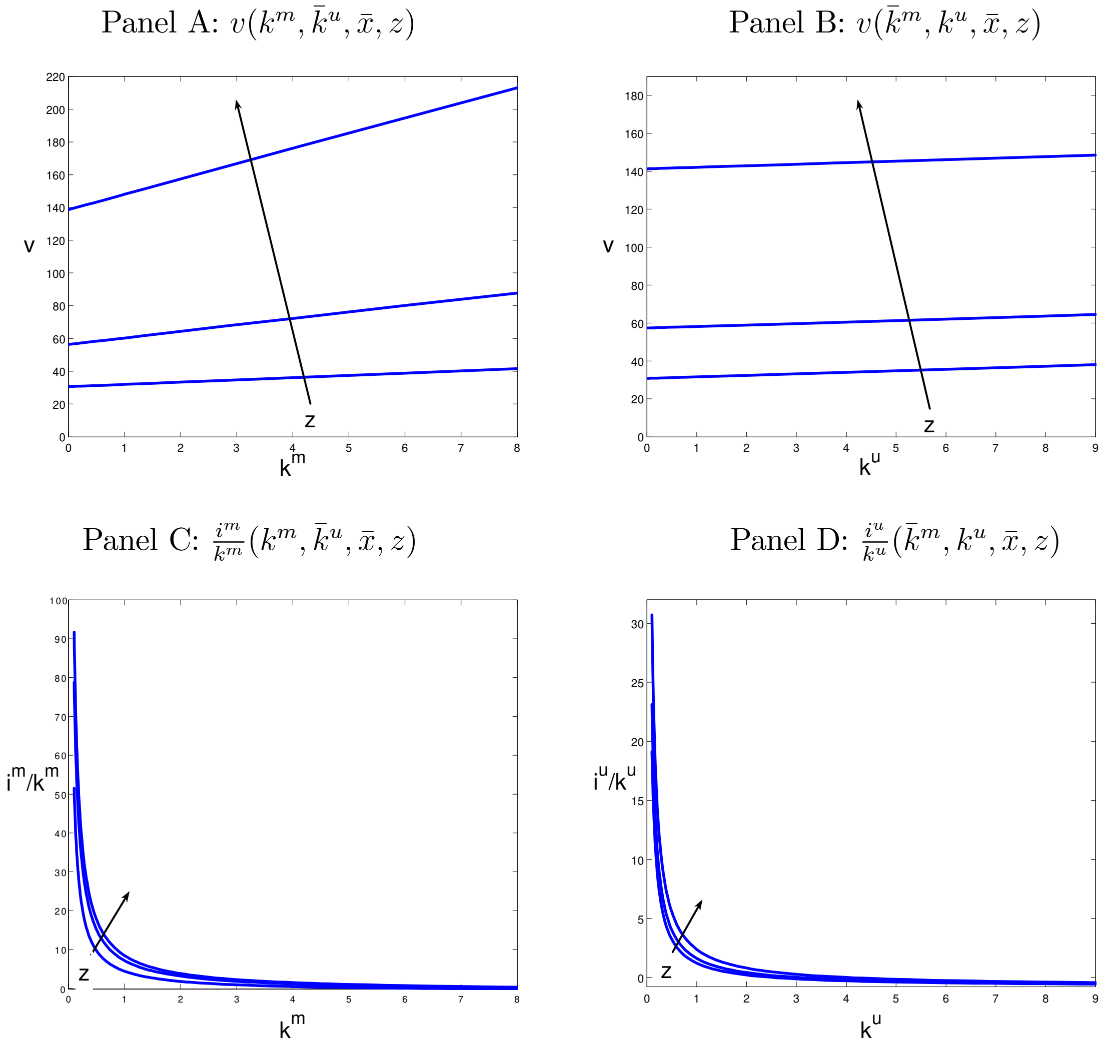

Figure 2 Value Functions and Policy Functions of the Model. This figure plots the value function $v(k^m, k^u, x, z)$ and the physical-investment-to-physical-capital ratio $\frac{i^m}{k^m}(k^m, k^u, x, z)$ and R&D investment-to-R&D-capital ratio $\frac{i^u}{k^u}(k^m, k^u, x, z)$ as functions of two endogenous state variable k^m and k^u, and two exogenous state variable x and z. Because there are four state variables, I fix $k^u = \bar{k}^u$ and $x = \bar{x}$, and plot the value and policy functions against k^m in Panels A and C, respectively, in which the arrows indicate the direction along which z increases. In panel B and D, I fix $k^m = \bar{k}^m$ and $x = \bar{x}$, and plot the value and and policy functions against k^u, respectively, in which the arrows indicate the direction along which z increases.

Panel A: $\beta(k^m, \bar{k}^u, \bar{x}, z)$ Panel B: $\beta(\bar{k}^m, k^u, \bar{x}, z)$

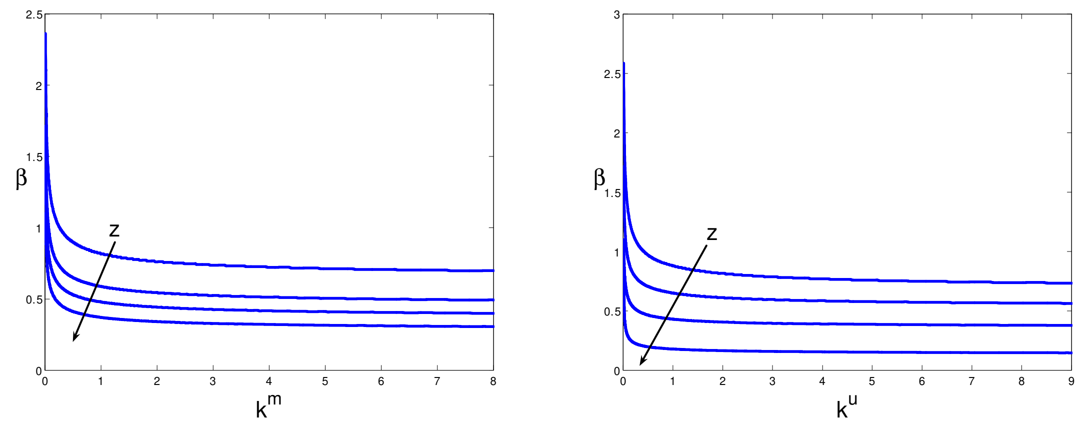

Figure 3 Determinants of Risk. This figure plots beta, $\beta(k^m, k^u, x, z)$ as functions of two endogenous state variable k^m and k^u, and two exogenous state variable x and z. Because there are four state variables, I fix $k^u = \bar{k}^u$ and $x = \bar{x}$, and plot $\beta(k^m, k^u, x, z)$ against k^m in Panels A, in which the arrows indicate the direction along which z increases. In panel B, I fix $k^m = \bar{k}^m$ and $x = \bar{x}$, and plot $\beta(k^m, k^u, x, z)$ against k^u, in which the arrows indicate the direction along which z increases.

Table 1 Parameter Values under Benchmark Parameterization

Notation	Values	Description
		Group I
α	0.65	Share of physical capital in output production
θ	0.70	Proportion of R&D capital devoted to new product
δ_m	0.025	Quarterly rate of physical capital depreciation
δ_u	0.05	Quarterly rate of intangible capital depreciation
ρ_x	0.95	Persistence coefficient of aggregate productivity
σ_x	0.007	Conditional volatility of aggregate productivity
$\bar{x}$	-2.08	Long-run average of aggregate productivity
ρ_z	0.91	Persistence coefficient of firm-specific productivity
σ_z	0.17	Conditional volatility of firm-specific productivity
β	0.995	Time-preference coefficient
γ_0	23	Constant price of risk
γ_1	-900	Time-varying price of risk
		Group II
a	0.79	Weight of physical investment in physical capital production
A	0.42	Constant term in physical capital production
ρ	0.25	Elasticity between physical investment and intangible capital

This table lists the benchmark parameter values used to solve and simulate the model.

Table 2 Key Moments

Moments	Data	Model
The annual average risk-free rate	2.00	1.65
The annual volatility of risk-free rate	2.12	3.20
The annual average of market return	10.31	10.82
The annual volatility of market returns	15.21	16.26
The average annual Sharpe ratio	0.54	0.54
The average annual investment-to-asset ratio	0.15	0.15
The volatility of investment-to-asset ratio	0.08	0.07
The average market-to-book ratio	1.50	1.94
The volatility of market-to-book ratio	0.24	0.27
The annual R&D-to-physical investment ratio	0.51	0.57

This table reports unconditional moments generated from the simulated data and the real data. I simulate 100 artificial panels, each of which has 3000 firms and each firm has 120 quarterly observations. I report the cross-simulation averaged annual moments. The data moments of annual average physical-investment-to-asset ratio (physical investment scaled by physical capital in simulated data) and average market-to-book ratio are from Hennessy and Whited (2005). The other data moments are estimated from a sample from 1975 to 2005.

Table 3 Stock Returns of R&D Investment Portfolios

	Data Low	Data 2	Data 3	Data 4	Data High	Data High-Low	Model Low	Model 2	Model 3	Model 4	Model High	Model High-Low
Panel A: Stock Returns												
$\frac{R\&D}{mkt}$	8.27	9.64	13.27	18.18	20.61	12.34	8.15	8.75	9.24	11.94	17.85	9.60
$\frac{R\&D}{physial}$	9.96	13.35	14.18	16.91	17.10	7.14	7.92	8.69	8.91	11.61	18.54	10.62
Panel B: Excess Returns After Controlling Size and Book-to-Market												
$\frac{R\&D}{mkt}$	-2.43	-1.62	1.66	5.52	9.63	12.06	-2.61	-2.43	-1.11	1.01	6.14	8.75
$\frac{R\&D}{physial}$	-2.78	0.86	2.44	7.71	8.89	11.67	-3.13	-2.13	-1.47	0.83	6.90	10.03

This table reports stock returns of 5 portfolios sorted on R&D intensity (in the first column), where the R&D intensity is measured as the ratio of R&D investment to market value of equity and R&D investment to physical investment ratio, respectively. In each June of year t from 1975 to 2005, I rank all firms based on the R&D intensity into 5 equal-numbered portfolios. I compute the subsequent annual equal-weighted returns from July of year t to June of year t + 1 and reform the portfolios in June of year t+1. The excess return is the difference between each individual stock's return and the return of its matching portfolio by its size and book-to-market ranks. To form matching portfolios, I sort all firms (in both the real data and the simulated panel) each year into one of 25 size and book-to-market portfolios. See Appendix C.2 for construction details of the benchmark. I simulate 100 artificial panels, each of which has 3000 firms and each firm has 120 quarterly observations. I perform the empirical analysis on each simulated panel and report the cross-simulation average results. All returns are simple, annualized real returns in percentages. In the first column, R&D refers to R&D investment, physical refers to physical investment, and mkt refers to market value of equity.

Table 4 Stock Returns of Physical Investment Portfolios

	VW		EW	
	Data	Model	Data	Model
low	11.75	17.38	17.75	22.42
2	11.51	16.11	15.11	17.51
3	11.27	13.55	14.75	14.68
4	11.40	11.83	15.47	12.56
5	11.75	9.81	15.59	10.33
6	10.43	7.66	14.99	8.27
7	9.71	6.57	14.51	6.99
8	9.35	5.65	14.15	6.08
9	8.75	4.16	13.55	5.50
high	6.47	3.17	12.11	4.91
low-high	5.28	14.21	5.64	17.51

This table reports the value-weighted (VW) and equal-weighted (EW) stock returns of physical investment portfolios. Each year in June, firms are sorted into 10 deciles by their previous fiscal year physical-investment-to-asset ratio (ratio of physical investment to physical capital in simulated data) and I compute value-weighted and equal-weighted returns on each decile portfolio. The Low-High variable is the return difference between the lowest physical investment decile and the highest physical investment decile. The sample period is from June 1975 to December 2005. I simulate 100 artificial panels, each of which has 3000 firms and each firm has 120 quarterly observations. I perform the empirical analysis on each simulated panel and report the cross-simulation average results. All returns are simple, annualized real returns in percentages.

Table 5 Equal-Weighted Stock Returns of Size and Physical Investment Portfolios

	Data						Model					
	physical investment quintile											
size	Low	2	3	4	High	Low-High	Low	2	3	4	High	Low-High
Small	16.68	15.36	11.52	13.08	6.96	9.72	25.14	14.57	10.73	6.39	5.66	19.48
2	15.01	15.72	16.08	14.76	13.20	1.81	21.42	14.14	9.71	5.35	4.40	17.02
3	15.48	15.84	13.32	13.68	11.88	3.60	18.68	13.83	8.10	4.88	3.83	14.85
4	12.48	12.84	13.56	13.08	11.28	1.20	17.67	13.23	7.81	4.25	3.46	14.21
Big	11.28	12.48	11.52	10.8	8.64	2.64	16.99	12.79	7.03	4.01	3.19	13.80

This table reports equal-weighted (EW) stock returns of 25 portfolios sorted on size and physical-investment-to-asset ratio (ratio of physical investment to physical capital in simulated data). Each year in June, the firms are first sorted into 5 quintiles based on NYSE market capitalization breakpoints at the end of May; within each quintile, firms are sorted on previous fiscal year physical investment-to-asset ratio. I compute the value-weighted and equal-weighted returns for each quintile portfolio. The Low-High variable is the return difference between low physical-investment-to-asset ratio quintile and high investment-to-asset ratio quintile within each size quintile. The sample period is from June 1975 to December 2005. I simulate 100 artificial panels, each of which has 3000 firms and each firm has 120 quarterly observations. I perform the empirical analysis on each simulated panel and report the cross-simulation average results. All returns are simple, annualized real returns in percentages.

Table 6 Excess Stock Returns of Abnormal Physical Investment Portfolios

	Low	2	3	4	High	Low-High	Low	2	3	4	High	Low-High
	Data						Model					
	0.50	1.01	0.66	-1.01	-1.52	2.03	1.43	0.62	-0.31	-0.48	-0.71	2.14

This table reports stock returns of 5 portfolio ranking by abnormal physical investment, $CI^m_{t-1} = CE^m_{t-1}/[(CE^m_{t-2} + CE^m_{t-3} + CE^m_{t-4})/3] - 1$ in the portfolio formation year t, where CE^m_{t-1} is physical capital expenditure scaled by sales during year t-1. Data is the annualized returns from Titman et al (2004). In each year t in simulated panel, I rank firms based on abnormal physical investment, CI, into 5 equal-numbered portfolios. I compute the subsequent annual value-weighted returns from year t to year t + 1 and reform the portfolios in year t+1. The excess return is the difference between each individual stock's return and the return of its matching portfolio by its size, book-to-market and prior-year-return ranks. To form matching portfolios, I sort all firms each year into one of 125 size, book-to-market and prior-year-return portfolios. See Appendix C.4 for construction details of the benchmark. The Low-High variable is the return difference between the lowest book-to-market decile and the highest book-to-market decile. I simulate 100 artificial panels, each of which has 3000 firms and each firm has 120 quarterly observations. I perform the empirical analysis on each simulated panel and report the cross-simulation average results. All returns are simple, annual real returns in percentages.

Table 7 Stock Returns of Book-to-Market Portfolios

	VW		EW	
	Data	Model	Data	Model
low	7.98	4.09	6.91	4.48
2	10.76	5.15	12.93	6.91
3	11.41	6.33	14.23	7.52
4	12.53	8.08	15.97	8.71
5	11.92	8.85	17.51	8.89
6	12.24	10.09	18.24	10.26
7	13.82	11.48	19.47	11.88
8	13.03	11.89	19.78	12.87
9	14.51	12.69	22.37	13.53
high	16.70	17.54	26.27	23.75
high-low	8.72	13.45	19.36	19.27

This table reports the value-weighted (VW) and equal-weighted (EW) stock returns of 10 portfolios sorted on book-to-market ratio. Data is from Ken French's website. The High-Low variable is the return difference between the highest book-to-market decile and the lowest book-to-market decile. The sample period is from 1975 to 2005. I simulate 100 artificial panels, each of which has 3000 firms and each firms has 120 quarterly observations. I perform the empirical analysis on each simulated panel and report the cross-simulation average results. All returns are simple, annualized real returns in percentages.

Table 8 Investment Returns and Weights on Investment Returns

	Low	2	3	4	High	Low	2	3	4	High
	Panel A: Physical Investment Returns					Panel B: R&D Investment Returns				
$\frac{R\&D}{mkt}$	8.56	9.71	10.02	14.26	22.87	6.73	6.40	5.37	4.44	4.17
$\frac{R\&D}{physial}$	8.67	9.33	9.51	13.91	23.98	6.79	6.36	5.34	4.45	4.16
$\frac{physical}{physical\ capital}$	20.48	16.57	10.71	7.12	5.70	1.94	2.30	3.31	4.13	5.17
	Panel C: Weights on Physical Investment Returns					Panel D: Weights on R&D Investment Returns				
$\frac{R\&D}{mkt}$	0.78	0.79	0.78	0.76	0.72	0.22	0.21	0.22	0.24	0.28
$\frac{R\&D}{physial}$	0.78	0.78	0.78	0.76	0.73	0.22	0.22	0.22	0.24	0.27
$\frac{physical}{physical\ capital}$	0.78	0.74	0.77	0.79	0.76	0.22	0.26	0.23	0.21	0.24

This table reports equal-weighted simulated R&D investment returns, physical investment returns, and their respective weights in stock returns of 5 portfolios sorted on R&D intensity (measured as R&D investment scaled by market value of equity, and R&D investment to physical investment ratio) and the rate of physical investment. I simulate 100 artificial panels, each of which has 3000 firms and each firm has 120 quarterly observations. All returns are simple, annualized real returns in percentages. In the first column, R&D refers to R&D investment, mkt refers to market value of equity, physical refers to physical investment. All weights are in percentages.