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Jiang Janet Hua and Dong Mei

University of Manitoba, Simon Fraser University

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One or Two Monies?*

Mei Dong

Janet Hua Jiang

Simon Fraser University

University of Manitoba

mdonga@sfu.ca

janet_jiang@umanitoba.ca

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Abstract

We investigate whether money constitutes a perfect substitute for the missing record-keeping technology in a quasi-linear environment, where private information and limited commitment are present. We adopt the mechanism design approach and solve a social planner's problem subject to the resource constraint, the incentive constraints imposed by the existing frictions, and the available memory technologies. The result is that when money is divisible, concealable and in variable supply, a single money may or may not be sufficient to replace the record-keeping technology. We further show that two monies serve as a perfect substitute for the record-keeping technology so that there is no need for a third money.

JEL Categories: E40, F30, D82

Keywords: Record-keeping, Money, Private Information, Limited Commitment, Mechanism Design

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1 Introduction

Micro-founded monetary theory explains how an intrinsically useless object can be valued in exchange. Recent advances in the literature seem to have reached a consensus that the role of money is to make up for the missing memory or the record-keeping technology, i.e., see Kocherlakota (1998*a, b*). A natural question to follow is whether money constitutes a perfect substitute for the record-keeping technology. Most micro-founded monetary models feature one single money and there is no welfare-enhancing role for a second money. In this paper, we show that when money is divisible, concealable and in variable supply, a single money may or may not be sufficient to replace the record-keeping technology. We then show that in the latter case, introducing a second money improves welfare, and that two monies act as a perfect substitute for the record-keeping technology.

We construct a heterogeneous agent model in a quasilinear environment as introduced by Lagos and Wright (2005). There are two types of agents and two locations indexed by a and b . In every period, a location/preference shock randomly assigns agents to one of the two locations and determines their marginal utilities from consumption. Type a (b) agents have high marginal utilities at location a (b) and low marginal utilities at location b (a). Since agents at the same location are endowed with the same amount of goods, the first-best allocation requires that agents with low marginal utilities transfer some of their endowment to agents with high marginal utilities.

There are two frictions in the economy: limited commitment and private information about types. In the presence of these frictions, an implementable allocation must be incentive compatible to ensure participation and truthful revelation of types. Throughout the paper, we adopt a mechanism design approach and solve the conditions under which the first-best allocation satisfies the relevant incentive constraints.

We first analyze mechanisms with a perfect record-keeping technology. The mechanism can directly record nonparticipation and impose perpetual autarky as the punishment.¹ With regard to private information, the mechanism asks agents to report their types, records the information and uses it later on to infer agents' marginal utilities. Due to the symmetry structure of the preferences, agents have the incentive to truthfully report their types *ex ante* (we call this 'early-sorting' because the incentive is aligned before the realization of the location shock) at the first-best allocation. The first-best allocation can be achieved as long as the participation constraint is satisfied, or agents are patient enough.

Next, we assume that the record-keeping technology is not available, but the society has access to one fiat money. By rewarding participants with more money and requiring an ever increasing amount of money for future participation, one-money mechanisms can deal with limited commitment as effectively as mechanisms with the record-keeping technology. However, one-money mechanisms are not as powerful in dealing with private information. With a single concealable money, encoding and passing information on *ex ante* type reporting becomes problematic. Different reports can only be encoded into different money balances, which however, can be hidden to prevent credible information communication. In this case, the only effective way to deal with private information is to induce agents to reveal their types/marginal utilities after the period location shock (we call this 'late-sorting').

Due to quasilinearity, the late-sorting mechanism involves no *ex ante* welfare cost; it, however, imposes an extra constraint on the patience parameter. As a result, the restriction on the patience parameter to implement the first-best allocation is more stringent than with the record-keeping technology. There exists a positive measure of the patience parameter such that the first-best allocation can be achieved with the record-keeping technology, but not with one money. Hence, one money is not a

¹We assume that the most severe societal penalty is ostracism.

perfect substitute for the record-keeping technology in the quasilinear environment that we consider.

We then investigate mechanisms when a second money is introduced. We find that having two monies allows the mechanism to use monetary portfolios and total money balances to record *ex ante* reporting about types. Moreover, the information can be credibly passed into the future. It follows that two monies act as a perfect substitute for the record-keeping technology. We also extend the above results to an environment with more than two types of agents. We argue in general that two monies are a perfect substitute for the record-keeping technology so that there is no need for a third money.

Our work is most closely related to Kocherlakota and Krueger (1999), and Kocherlakota (2002) which also study the essentiality of multiple monies.²

Kocherlakota and Krueger (1999) share with us a common feature that a second money improves welfare in that a second money serves as a signalling device to deal with private information. Their model, however, builds on Trejos and Wright (1995) with indivisible money. The result that there is no need for a third money cannot be extended to multiple-type-agent models. Moreover, the quasilinear preferences in our model introduce an additional way (the late-sorting mechanism) to align incentives. It follows that a second money is inessential if agents are patient enough (because late-sorting is powerful enough to deal with both private information and limited commitment).

Kocherlakota (2002) correctly points out that when money is concealable, it is necessary to establish a monotonic relationship between ‘proper’ behavior and money

²There is another strand of literature investigating whether multiple currencies can coexist or circulate at the same time. Examples are Trejos and Wright (2001), Camera and Winkler (2003), Camera, Craig and Waller (2004), and Craig and Waller (2004). Our paper’s goal is to study the welfare enhancing role of multiple currencies, or whether multiple currencies are essential.

balances. In Kocherlakota (2002), limited commitment makes it impossible to establish such a relationship and renders the need for a second money. This conclusion, however, hinges critically on the assumption of a fixed money supply. When money supply is fixed, the only way to record whether an agent behaves properly is to transfer some money to him from somebody else (who might also behave properly). Agents' money holdings will differ in general. The first-best allocation, however, requires that future allocation should not discriminate those with less money balances. The mechanism that we propose circumvents the problem by increasing money supply to reward all properly behaved participants. Limited commitment thus does not justify a role for a second money if money supply is allowed to change.

The rest of the paper proceeds as follows. Section 2 lays out the physical environment and characterizes the first-best allocation. Section 3 introduces private information and limited commitment. We solve for the condition to achieve the first-best allocation when the society has access to a record-keeping technology. Section 4 studies the optimal monetary mechanisms when the record-keeping technology is absent, and establishes the condition under which a second money is essential. Section 5 extends the results to a multi-type-agent model. Section 6 argues in general that two monies constitute a perfect substitute for the record-keeping technology. We conclude and suggest directions for future research in section 7.

2 The Physical Environment

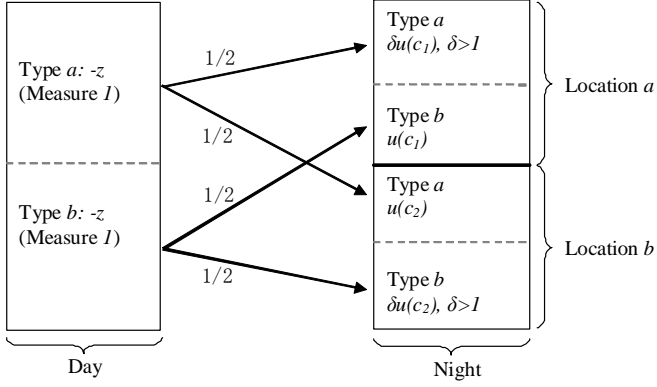
The framework that we adopt is the quasi-linear environment suggested by Lagos and Wright (2005) without the search friction. Time is discrete and runs from 0 to ∞ . Each period consists of two stages: day and night. There are two locations a and b . Inter-location interaction is allowed during the day but prohibited at night. There are

three non-storable goods, one day good and two location-specific night goods indexed by 1 and 2. Good 1 is local to location a and good 2 is local to location b . There are two types of agents – each is of measure 1.

During the day, all agents can produce or consume the day good. They have the same linear preference over the good. Let z be the amount of production (consumption if z is negative). The disutility of production (utility of consumption if z is negative) is $-z$.

At night, agents consume one of the two night goods. The two types of agents are distinguished by their preferences over the two night goods. Type a value good 1 more than good 2, and type b value good 2 more than good 1. It might be helpful to think of type a as local consumers of good 1 and foreign consumers of good 2; similarly, think of type b as local consumers of good 2 and foreign consumers of good 1. The utility of a local consumer is $\delta u(c)$ and the utility of a foreigner is $u(c)$, where $\delta > 1$, $u(0) = 0$, $u'' < 0 < u'$ and $u'(0) = +\infty$. Which night good an agent consumes is determined by a preference shock realized upon entering the night stage. With probability $1/2$, an agent becomes a local consumer and has a high valuation of the night good. With probability $1/2$, the agent becomes a foreign consumer and has a low valuation of the night good. We assume that each agent is endowed with y units of the location-specific good after the realization of the preference shock. Note that at the night stage, each location is inhabited by two types of agents who value the night good differently. Refer to Figure 1 for a graphical illustration of the environment.

Figure 1: Environment



The life-time expected utility of a type a agent $i \in (0, 1)$ is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -z_t^a(i) + \frac{1}{2} [\delta u(c_{1,t}^a(i)) + u(c_{2,t}^a(i))] \right\}.$$

where $0 < \beta < 1$ is the discount factor, $z_t^a(i)$ is the production (consumption if negative) of the day good, and $c_{1,t}^a(i)$ and $c_{2,t}^a(i)$ are the consumption of (night) good 1 and 2 respectively.

Similarly, the life-time expected utility of a type b agent $j \in (0, 1)$ is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -z_t^b(j) + \frac{1}{2} [\delta u(c_{2,t}^b(j)) + u(c_{1,t}^b(j))] \right\}.$$

The resource constraints are given by:

$$\int_0^1 z_t^a(i) di + \int_0^1 z_t^b(j) dj = 0$$

at the day stage, and

$$\begin{aligned}\int_0^1 c_{1,t}^a(i) I_t^a(i) di + \int_0^1 c_{1,t}^b(j) I_t^a(j) dj &= 2y \text{ at location } a; \\ \int_0^1 c_{2,t}^a(i) I_t^b(i) di + \int_0^1 c_{2,t}^b(j) I_t^b(j) dj &= 2y \text{ at location } b;\end{aligned}$$

at the night stage for all $t \geq 0$. $I_t^k(\bullet)$ is an indicator function and is equal to 1 if the agent is at location $k \in \{a, b\}$ at date t .

We will focus on symmetric stationary allocations where for all i and $j \in (0, 1)$ and $k \in \{a, b\}$,

- $c_{1,t}^a(i) = c_{2,t}^b(j) = c_h$, $c_{2,t}^a(j) = c_{1,t}^b(i) = c_\ell$ with $c_h + c_\ell = 2y$ for $t \geq 0$;
- $z_0^a(i) = z_0^b(j) = 0$;
- $z_t^k(\cdot) = \begin{cases} z_h, & \text{if the agent consumed } c_h \text{ at the night stage of time } t-1; \\ z_\ell, & \text{if the agent consumed } c_\ell \text{ at the night stage of time } t-1; \end{cases}$
with $z_h + z_\ell = 0$ for $t \geq 1$.

The social planner's problem is to choose $(c_h, c_\ell, z_h, z_\ell)$ to maximize the *ex ante* utility:

$$W(c_h, c_\ell, y) = \frac{1}{2} \frac{1}{1-\beta} [\delta u(c_h) + u(c_\ell)]; \quad (1)$$

subject to

$$c_h + c_\ell = 2y.$$

The solution is characterized by:

$$\begin{aligned}
\delta u'(c_h^*) &= u'(c_\ell^*); \\
c_h^* + c_\ell^* &= 2y; \\
W^* &= \frac{1}{2} \frac{1}{1-\beta} [\delta u(c_h^*) + u(c_\ell^*)].
\end{aligned}$$

Note that since $\delta > 1$, $c_h^* > y > c_\ell^*$. The planner can instruct the night stage low-valuation agents to ‘lend’ $\tau^* \equiv y - c_\ell^*$ units of his endowment to high-valuation agents. For the day stage allocation, note that since z_t enters linearly in preferences, any $z_t^a(i)$ and $z_t^b(j)$ that satisfy $E_0 z_t^a(i) = E_0 z_t^b(j) = 0$ would satisfy the day stage resource constraint and entail no *ex ante* welfare loss. In the current context, one such allocation is $z_t^a(i) = z_t^b(j) = 0$ for all i and j and $t \geq 0$.

The first-best allocation can be achieved if agents’ types are public information, and agents are able to commit to sticking with the allocation.

3 Limited Commitment and Private Information

Assume that agents cannot commit, and agents’ types and thus their valuations of the night goods are private information. In this case, a record-keeping technology becomes essential to overcome the frictions caused by limited commitment and private information (see Kocherlakota, 1998*a, b*).

In the presence of limited commitment and private information, an implementable allocation must satisfy individual rationality or participation constraints (so that individuals have the incentive to stick with the mechanism), and the incentive constraints (so that individuals have the incentive to truthfully reveal their private information).³

³We assume that group deviation and side trades can be prevented to avoid extra constraints incurred by the market structure.

When agents cannot commit, the allocation (c_h, c_ℓ, z) must respect *ex post* rationality. Assuming that the punishment for nonparticipation is autarky, the welfare is

$$W_0 = \frac{1}{2} \frac{1}{1 - \beta} [\delta u(y) + u(y)]. \quad (2)$$

It is straightforward that $W^* > W_0$. At the night stage, there are two individual rationality conditions: one for high-valuation agents and one for low-valuation agents,

$$\delta u(c_h) + \beta(-z_h + W) \geq \delta u(y) + \beta W_0; \quad (3)$$

$$u(c_\ell) + \beta(-z_\ell + W) \geq u(y) + \beta W_0; \quad (4)$$

where W is as defined in (1). At the day stage, there are also two individual rationality conditions:

$$-z_h + W \geq W_0; \quad (5)$$

$$-z_\ell + W \geq W_0. \quad (6)$$

Note that if $c_h > c_\ell$, for night stage high-valuation agents, the day stage individual rationality condition (5) implies the night stage individual rationality condition (3). For night stage low-valuation agents, the night stage individual rationality condition (4) implies the day stage individual rationality condition (6). An implementable allocation with $c_h > c_\ell$ must satisfy (5) and (4), which we rewrite and label as (*IRH*) and (*IRL*) respectively. To simplify notation, let $z = z_h = -z_\ell$.

$$z \leq W - W_0; \tag{IRH}$$

$$z \geq W_0 - W + \frac{u(y) - u(c_\ell)}{\beta}; \tag{IRL}$$

If a mechanism prescribes higher night consumption for high-valuation agents (which is the case at the first-best allocation), low-valuation agents will have the incentive to claim to be high-valuation agents. Private information about types implies that agents can potentially lie about their valuations of the night goods.

Due to the structure of the preference shocks, there are two ways to deal with the incentive problem caused by private information. Since the two types of agents always value the same night good differently, the planner can induce agents to truthfully reveal their types at the day stage and use the information to infer an agent's valuation of the night good. For example, if an agent reports to be a type a agent and shows up at location b , the planner can infer that the agent is a low-valuation agent. Note that since recorded information can be passed into the infinite future, the mechanism only needs to ask agents to report their types once at the day stage of periods 0. The information will then be used in all the following periods.⁴ We call this mechanism the early-sorting mechanism because information used to identify agents' valuations is revealed before the realization of the preference shocks. To use the early-sorting mechanism, the following constraint needs to be satisfied:

$$\frac{1}{1-\beta} \left\{ \frac{\delta u(c_h) - \beta z_h}{2} + \frac{u(c_\ell) - \beta z_\ell}{2} \right\} \geq \frac{1}{1-\beta} \left\{ \frac{\delta u(c_\ell) - \beta z_\ell}{2} + \frac{u(c_h) - \beta z_h}{2} \right\}; \tag{ICT}$$

which holds if $c_h > c_\ell$.⁵

⁴This explains why we use life-time utilities in the *ICT*.

⁵If $c_h > c_\ell$, using *ICT* does not impose extra constraints on the day stage allocation z other than the resource constraint.

Alternatively, the planner can skip type reporting and try to induce the agents to truthfully report their valuations of the night goods by resorting to variations in production/consumption at the following day stage. We call this the late-sorting mechanism, which is effective if and only if the following two conditions are satisfied:⁶

$$\delta u(c_h) + \beta(-z_h + W) \geq \delta u(c_\ell) + \beta(-z_\ell + W);$$

$$u(c_\ell) + \beta(-z_\ell + W) \geq u(c_h) + \beta(-z_h + W).$$

We can rearrange the two incentive constraints as (again, let $z = z_h = -z_\ell$ to simplify notation):

$$z \leq \frac{\delta[u(c_h) - u(c_\ell)]}{2\beta}, \tag{ICH}$$

$$z \geq \frac{u(c_h) - u(c_\ell)}{2\beta}. \tag{ICL}$$

The first constraint ensures that high-valuation agents do not want to imitate low-valuation agents (note that this means that type a agents do not want to imitate type b agents at location a , and that type b agents do not want to imitate type a agents at location b). The second constraint ensures that low-valuation agents do not want to imitate high-valuation agents.⁷

Proposition 1 states the condition under which the first-best allocation can be achieved when the planner has access to a record-keeping technology,

Proposition 1 *When agents lack commitment and hold private information about their types, a record-keeping technology can achieve the first-best allocation if and*

⁶We call this ‘late sorting’ because information used to identify agents’ valuations is revealed after the realization of the preference shocks.

⁷Strictly speaking, there is a third way to align agents’ incentives by allowing each agent to consume his endowment, which is obviously not optimal.

only if $\beta \geq \beta_0$ where β_0 is defined as:

$$\beta_0 = \frac{u(y) - u(c_\ell^*)}{\delta[u(c_h^*) - u(y)]}.$$

Proof. With a record-keeping technology, the planner can use *ICT* to deal with private information. Since $c_h^* > c_\ell^*$, the first-best allocation meets *ICT* automatically. The first-best can be achieved if and only if there exists a z such that *IRH* and *IRL* are satisfied, or

$$W_0 - W^* + \frac{u(y) - u(c_\ell)}{\beta} \leq W^* - W_0;$$

which, with some manipulation, can be rewritten as:

$$\beta \geq \frac{1}{\delta} \frac{u(y) - u(c_\ell^*)}{u(c_h^*) - u(y)} \equiv \beta_0. \blacksquare$$

Proposition 1 states that the first-best allocation is implementable when agents are patient enough. Andolfatto (2008) has a similar result. The key friction that generates this result is limited commitment. Private information can be overcome since the first-best allocation entails $c_h^* > c_\ell^*$. As long as a record-keeping technology is available, *ICT* is automatically satisfied. There is no need to use *ICH* and *ICL*.

4 Monetary Mechanisms

Now suppose that the society has no access to the record-keeping technology. Then it is impossible to directly pass information across time. In this case, the planner uses tokens – which we call money – as a substitute for the missing record-keeping technology to communicate information across stages.⁸ We assume that money is

⁸The society, though, has access to a contemporaneous memory technology which can remember agents' actions *within* a stage.

perfectly divisible, concealable and in variable supply.

4.1 One-Money Mechanisms

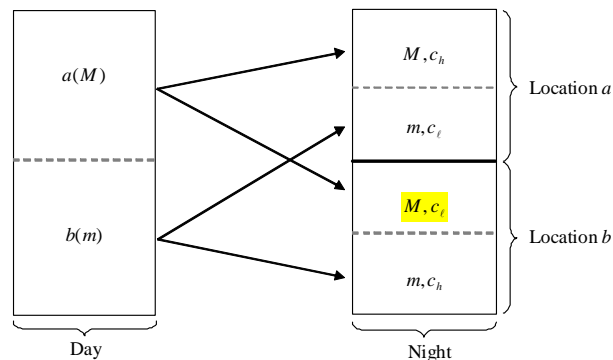
We first assume that there is a single money available and study if one money is a perfect substitute for the record-keeping technology. One-money mechanisms can deal with limited commitment as follows. By rewarding participants with money and increasing the amount of money required for future participation, a one-money mechanism can effectively catch nonparticipants and cast them into perpetual autarky. The individual rationality constraints remain the same as in the case with a record-keeping technology. Note that the concealability of money balances does not pose a problem here since the proposed mechanism establishes a monotonically increasing relationship between participation and money balances so that people do not have the incentive to hide money.

Now we show how one-money mechanisms deal with private information. The question we ask is whether the planner can encode type reports into money holdings and use them later on to identify agents' valuations of the night goods. The answer is no. It means that early-sorting cannot be used in one-money mechanisms.

With one money, the only way to encode type reports is to associate different types with different money balances. For example, the planner can give those who report to be type a more money. To use early-sorting, the planner is supposed to give high consumption to those with more money (or those reported to be type a) at location a , and those with less money (or those reported to be type b) at location b . The problem is that at location b , those with more money have the incentive and ability to mimic those with less money to demand for higher consumption. A quick examination of figure 2 shows that holding more money is strictly preferred to holding less money, so all agents will report to be type a agents at the day stage of period 0.

All agents will hold the same amount of money. The planner will not be able to infer agents' valuations of the night goods based on their money holdings.

Figure 2: Early sorting ineffective with a single concealable money
($M > m$)



To induce agents to truthfully reveal their valuations of the night goods, the planner must rely on the late-sorting mechanism. The planner can give all agents the same amount of money at the day stage of period 0. High-valuation agents can choose to consume more at the night stage, but they need to work more in the future. They will leave the night stage with less money and work more in the following day stage to accumulate more money. In this case, we need to replace *ICT* by *ICH* and *ICL*.

Proposition 2 states the condition under which one-money mechanisms can achieve the first-best allocation.

Proposition 2 *When agents lack commitment and hold private information about their types, one-money mechanisms can achieve the first-best allocation if and only if $\beta \geq \beta_1$, with β_1 given by:*

$$\beta_1 = \frac{u(c_h^*) - u(c_\ell^*)}{(\delta + 1)[u(c_h^*) - u(y)]} > \beta_0.$$

Proof. Consider the following mechanism.

Let $0 < \rho_h < \rho_\ell < 1$.

At date 0 day stage, the mechanism endows each agent with one unit of money (\$).

At date 0 night stage, after preference shocks are realized, the mechanism offers agents the following choices:

$$\text{Show } 1 \text{ \$, } \begin{cases} \text{use } \tau^* \text{ good (1 or 2) to exchange for } \rho_\ell \text{ \$; or} \\ \text{receive } \tau^* \text{ good (1 or 2) and } \rho_h \text{ \$} . \end{cases}$$

With this mechanism, non-participants leave with 1 \$ and participants leave with more than 1 \$. Participants receiving transfers consume more and leave with lower money balances; those giving up endowment consume less and leave with higher balances.⁹

At date 1 day stage, the mechanism offers agents the following options:

Show $(1 + \rho_h)$ \$, use z day good to exchange for $(1 - \rho_h)$ \$;

Show $(1 + \rho_\ell)$ \$, receive z day good and $(1 - \rho_\ell)$ \$;

With this mechanism, all participants leave the stage with 2 units of money and non-participating agents leave with less than 2 units of money. Participants entering with lower balances work to earn extra money.

At date 1 night stage, the choices are:

$$\text{Show } 2 \text{ \$, } \begin{cases} \text{use } \tau^* \text{ good (1 or 2) to exchange for } \rho_\ell \text{ \$; or} \\ \text{receive } \tau^* \text{ good (1 or 2) and } \rho_h \text{ \$} . \end{cases}$$

At date $t \geq 2$ day stage, the choices are:

Show $(t + \rho_h)$ \$, use z day good to exchange for $(1 - \rho_h)$ \$;

Show $(t + \rho_\ell)$ \$, receive z day good and $(1 - \rho_\ell)$ \$;

At date $t \geq 2$ night stage, the choices are:

⁹Since there is a contemporaneous memory technology, the mechanism can prevent agents from participating more than once.

Show $(t + 1)$ \$, $\left\{ \begin{array}{l} \text{use } \tau^* \text{ good (1 or 2) to exchange for } \rho_h \text{ \$; or} \\ \text{receive } \tau^* \text{ good (1 or 2) and } \rho_\ell \text{ \$;} \end{array} \right.$

Note that under this mechanism, if an agent skips a stage, his money balance will fall short of the required balances to participate in all of the following stages. The mechanism effectively catches non-participants and casts them into perpetual autarky. The individual rationality conditions thus remain the same as in the case with the record-keeping technology.

The first-best allocation can be achieved if and only if there exists a z such that at (c_h^*, c_ℓ^*, z) , ICH , ICL , IRH and IRL are satisfied, or

$$\frac{u(c_h^*) - u(c_\ell^*)}{2\beta} \leq z \leq \frac{\delta[u(c_h^*) - u(c_\ell^*)]}{2\beta}; \quad (\text{IC})$$

$$\frac{u(y) - u(c_\ell^*)}{\beta} + W_0 - W^* \leq z \leq W^* - W_0. \quad (\text{IR})$$

z is non-empty if and only if

$$\frac{u(c_h^*) - u(c_\ell^*)}{2\beta} \leq W^* - W_0;$$

which can be rearranged as:

$$\beta \geq \frac{u(c_h^*) - u(c_\ell^*)}{(\delta + 1)[u(c_h^*) - u(y)]} \equiv \beta_1.$$

It follows from $\delta u(c_h^*) + u(c_\ell^*) > (1 + \delta)u(y)$ that $\beta_1 > \beta_0$. ■

The one-money mechanism outlined above deals with frictions caused by limited commitment and private information as follows. By rewarding participants with newly issued money and increasing the money balances required for future participation, the mechanism effectively catches non-participants and casts them into per-

petual autarky. By requiring previous high-valuation agents to work for previous low-valuation agents at the day stage, the mechanism induces agents to truthfully reveal private information and signal preferences by choosing different money balances at the night stage. The first-best allocation can be implemented if and only if $\beta \geq \beta_1$. When $\beta_0 < \beta < \beta_1$, the one-money mechanism cannot implement the first-best allocation, while the mechanism with a record-keeping technology can. In this sense, the one-money mechanism is less powerful in dealing with private information about types.

4.2 Two-Money Mechanisms

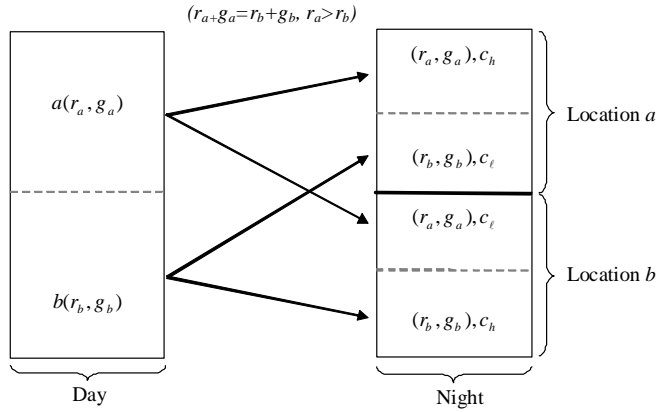
Given that one-money mechanisms cannot fully replicate the allocations that are implementable with a record-keeping technology, we introduce a second money in this subsection and show that two monies constitute a perfect substitute for the record-keeping technology.

Label the two monies as "red" and "green". Similar to the one-money mechanism, the two-money mechanism can reward participants with more money balances and effectively exclude nonparticipants from the mechanism forever. The individual rationality conditions thus stay the same as in the case with the record-keeping technology.

What is different from the one-money mechanism is that two-money mechanisms make *ICT* feasible again. The planner can encode type reports into monetary portfolios with the same total balances but different compositions of the two monies, and request agents to show the same total balances at the night stage. For example, suppose that the planner gives those reporting as type a more red money and those reporting as type b more green money. At the following night stage, the planner requires more red money for high consumption at location a and more green money

for high consumption at location b . By requesting all agents to show the same total money balances, agents will not be able to juggle their portfolios to renege on their earlier reports. The early-sorting mechanism is thus reinstated (see Figure 3). As in the case with record-keeping technology, the first-best allocation can be achieved if and only if $\beta \geq \beta_0$.

Figure 3: Early sorting effective with two monies



Proposition 3 *When agents lack commitment and hold private information about their types, two monies act as a perfect substitute for the record-keeping technology and can achieve the first-best allocation if and only if $\beta \geq \beta_0$.*

Proof. Call the two monies red (R) and green (G). Consider the following mechanism.

At date 0 day stage, the mechanism asks agents to choose from two monetary portfolios: 1 R or 1 G .

At date 0 night stage, after the shocks are realized, the mechanism offers agents the following choices:

At location a ,

Show R , receive τ^* good 1 and ρR , where $0 < \rho < 1$;

Show G , use τ^* good 1 to exchange for ρR ;

At location b ,

Show G , receive τ^* good 2 and ρG ;

Show R , use τ^* good 2 to exchange for ρG ;

With this mechanism, non-participants leave the night stage with 1 unit of money and participants leave with more than 1 unit of money. Participants with higher consumption leave the night stage with a single type of money (i.e., $1 + \rho$ units of R at location a); participants with lower consumption leave with two types of money (for example, 1 unit of G and ρ units of R at location a). All participants exit the night stage with the same total money balances $1 + \rho$.

At date 1 day stage,

Show $(1 + \rho) R$, use z day output to exchange for $(1 - \rho) R$;

Show $(1 + \rho) G$, use z day output to exchange for $(1 - \rho) G$;

Show $R + \rho G$, use ρG to exchange for z day output and 1 R ;

Show $G + \rho R$, use ρR to exchange for z day output and 1 G ;

At date 1 night stage,

At location a ,

Show 2 R , receive τ^* good 1 and ρR ;

Show 2 G , use τ^* good 1 to exchange for ρR ;

At location b ,

Show 2 G , receive τ^* good 2 and ρG ;

Show 2 R , use τ^* good 2 to exchange for ρG ;

At date $t \geq 2$ day stage,

Show $(t + \rho) R$, use z day output to exchange for $(1 - \rho) R$;

Show $(t + \rho) G$, use z day output to exchange for $(1 - \rho) G$;

Show $t R + \rho G$, use ρG to exchange for z day output and 1 R ;

Show $t G + \rho R$, use ρR to exchange for z day output and 1 G ;

At date $t \geq 2$ night stage,

At location a :

Show $(t + 1) R$, receive τ^* good 1 and ρR ;

Show $(t + 1) G$, use τ^* good 1 to exchange for ρR ;

At location b :

Show $(t + 1) G$, receive τ^* good 2 and ρG ;

Show $(t + 1) R$, use τ^* good 2 to exchange for ρG .

The two-money mechanism described here rewards participants with more money balances, effectively catches non-participants and bars them from participating in the mechanism forever. The individual rationality conditions thus stay the same as in the case with the record-keeping technology.

The two-money mechanism can induce the two types of agents to hold different monetary portfolios with the same total balances. At the night stages, low-valuation agents will not be able to falsely claim to be high-valuation agents. For example, suppose that type a choose to hold red money and type b choose to hold green money at date 0. At the following night stage, a type a agent at location b is a low-valuation agent and cannot claim to be a high-valuation agent since he does not have the green money required for higher consumption. We verify in the following that the two types of agents indeed have the incentive to differentiate themselves from each other by choosing different monetary portfolios at date 0. Take type a agents as an example.

The expected life-time utility from holding the red money is:¹⁰

$$W_r^a = \frac{1}{2} \frac{1}{1-\beta} [\delta u(c_h^*) + u(c_\ell^*)];$$

and the expected utility from holding the green money is:

$$W_g^a = \frac{1}{2} \frac{1}{1-\beta} [\delta u(c_\ell^*) + u(c_h^*)].$$

It is straightforward that $W_r^a > W_g^a$ so that type a agents prefer holding the red money.

The mechanism outlined above can achieve the first-best allocation if the allocation (c_h, c_ℓ, z) satisfies *ICT*, *IRH* and *IRL* at (c_h^*, c_ℓ^*) . As in the case with a record-keeping technology, the first-best allocation can be achieved if and only if $\beta \geq \beta_0$. ■

The two-money mechanism induces the two types of agents to hold different monetary portfolios. Since the two portfolios feature the same total balances, it is impossible to juggle one's portfolio to renege on earlier type reports. The early-sorting mechanism (*ICT*) is thus reinstated and two monies provide a perfect substitute for the record-keeping technology. The introduction of a second money improves welfare when $\beta < \beta_1$.¹¹

¹⁰Note that under the proposed mechanism, agents hold the same color of money while entering all night stages; they basically make only one type reporting choice when they decide what portfolio to hold at the day stage of period 0. This is why we compare the *life-time* utilities from holding different monetary portfolios.

¹¹Note that when $\beta < \beta_0$, the first-best allocation cannot be achieved even with a record-keeping technology. It can be shown that two monies are still a perfect substitute for the record-keeping technology and two-money mechanisms strictly improve welfare over one-money mechanisms.

5 Extension to Multi-type-agent Models

In this section, we show that as in Townsend (1987), two monies consist of a perfect substitute for the record-keeping technology even when there are more than two types of agents.¹² The optimal two-money mechanism is to let different types hold different combinations of the red and green monies, with all combinations giving the same total money balances.

There are $N < +\infty$ symmetric locations and N location specific night goods. There are N types of agents distinguished by their preferences over the night goods. A type $m \in \{1, 2, \dots, N\}$ agent derives utility $\delta_{mn}u(c_{mn})$ from consuming c_{mn} units of night good $n \in \{1, 2, \dots, N\}$, where $\delta_{mn} = \delta_{m-n+I(m<n)N}$, $\delta_0 > \delta_1 > \delta_2 > \dots > \delta_{N-1} > 0$, and $I(m < n) = 1$ if $m > n$ and 0 otherwise. For example, type 1 agents derive utility $\delta_0(c)$ from goods at location 1, $\delta_{N-1}u(c)$ from goods at location 2, ..., and $\delta_1u(c)$ from goods at location N ; type N agents derive utility $\delta_0u(c)$ from goods at location N , $\delta_{N-1}u(c)$ from goods at location 1, ..., and $\delta_1u(c)$ from goods at location $N - 1$. See table 1 for an illustration of the structure of the preferences.

Table 1: Preference shocks

Good Agent type	1	2	...	n	...	$N-1$	N
1	δ_0	δ_{N-1}	...	δ_{1-n+N}	...	δ_2	δ_1
2	δ_1	δ_0	...	δ_{2-n+N}	...	δ_3	δ_2
.
.
.
m	δ_{m-1}	δ_{m-2}	...	$\delta_{m-n+I(m<n)N}$...	δ_{m+1}	δ_m
.
.
.
$N-1$	δ_{N-2}	δ_{N-3}	...	δ_{N-1-n}	...	δ_0	δ_{N-1}
N	δ_{N-1}	δ_{N-2}	...	δ_{N-n}	...	δ_1	δ_0

¹²Kocherlakota and Krueger (1999) also mention that two monies are sufficient in their model which has only two types of agents. With indivisible monies, however, more monies will be needed if there are more than two types of agents.

During the day, all agents can produce and consume the day good. At night, each agent is subject to a preference shock and goes to each of the N locations with the same probability $1/N$. After agents are relocated, at location $n \in \{1, 2, \dots, N\}$, all agents are endowed with y units of good n , but they differ in their valuations of the good. Agents cannot commit and agents' types and thus their valuations of the night goods are private information.

We focus on symmetric stationary allocations where all agents with the same valuations of night goods consume the same amount, or for any $m, n \in \{1, 2, \dots, N\}$,

- $c_{mnt} = c_{mn} = c_{m-n+I(m<n)N}$ for $t \geq 0$ where c_{mnt} is the consumption of type m agents at location n in period t night stage;
- $z_{m0} = 0$;
- $z_{mnt} = z_{m-n+I(m<n)N}$ if the agent consumed c_{mn} at the night stage of time $t-1$ for $t \geq 1$;

The first-best night stage consumption is characterized by

$$\begin{aligned} \delta_q u'(c_q^*) &= \delta_{q'} u'(c_{q'}^*) \text{ for all } q \neq q' \in \{0, 1, \dots, N-1\}; \\ \sum_{q=0}^{N-1} c_q^* &= Ny. \end{aligned}$$

Any day stage allocation $(z_0, z_1, \dots, z_{N-1})$ satisfying $\sum_{q=0}^{N-1} z_q = 0$ satisfies the resource constraint and is consistent with the first-best allocation. The first-best life-time welfare of a representative agent is:

$$W^{N*} = \frac{1}{N} \frac{1}{1-\beta} \left[\sum_{q=0}^{N-1} \delta_q u(c_q^*) \right].$$

We first characterize the condition under which the first-best can be achieved

when the planner has access to a record-keeping technology. Suppose that $c_0^* > c_1^* > \dots > c_{B-1}^* > y > c_B^* > \dots > c_{N-1}^*$ so that at the first-best allocation, B types of agents are borrowers who consume more than their endowment, and $N - B$ types of agents are lenders who consume less than their endowment.

To deal with limited commitment, the following $2N$ individual rationality conditions must be satisfied: for all $q, q' \in \{0, 1, \dots, N - 1\}$ and $q \neq q'$,

$$\begin{aligned} \delta_q u(c_q) + \beta(-z_q + W^N) &\geq \delta_q u(y) + \beta W_0^N; \\ -z_q + W^N &\geq W_0^N; \end{aligned}$$

where

$$\begin{aligned} W^N &= \frac{1}{N} \frac{1}{1 - \beta} \left[\sum_{q=0}^{N-1} \delta_q u(c_q) \right] \text{ and} \\ W_0^N &= \frac{1}{N} \frac{1}{1 - \beta} u(y) \sum_{q=0}^{N-1} \delta_q. \end{aligned}$$

There are two ways to deal with the friction caused by private information. If early-sorting is used, the following $(N^2 - N)$ constraints must be satisfied:

$$\frac{1}{N} \frac{1}{1 - \beta} \left[\sum_{n=1}^N \delta_{mn} u(c_{mn}) \right] \geq \frac{1}{N} \frac{1}{1 - \beta} \left[\sum_{n=1}^N \delta_{m'n} u(c_{m'n}) \right];$$

for all $m, m' \in \{1, 2, \dots, N\}$ and $m' \neq m$. If late-sorting is used, the following $(N^2 - N)$ constraints must be satisfied:

$$\delta_q u(c_q) + \beta(-z_q + W^N) \geq \delta_{q'} u(c_{q'}) + \beta(-z_{q'} + W^N);$$

for all $q \neq q' \in \{0, 1, \dots, N - 1\}$.

Following the steps in section 3, it can be shown that the *ICT*s hold at the first-best allocation. When there is a record-keeping technology, the first-best allocation can be achieved if and only if

$$\beta \geq \beta_0^N = \frac{\sum_{q=B}^{N-1} \delta_q [u(y) - u(c_q^*)]}{\sum_{q=0}^{B-1} \delta_q [u(c_q^*) - u(y)]}.$$

In the absence of a record-keeping technology, one-money mechanisms must resort to late-sorting to align incentives, and the first-best allocation can be achieved if and only if

$$\beta \geq \beta_1^N = \frac{\sum_{q=1}^{N-1} (N-q) \delta_q [u(c_{q-1}^*) - u(c_q^*)]}{\sum_{q=1}^{N-1} (N-q) \delta_q [u(c_{q-1}^*) - u(c_q^*)] + \sum_{q=0}^{N-1} \delta_q [u(c_q^*) - u(y)]} \geq \beta_0^N.$$

The following proposed mechanism with two monies (R and G) shows that two monies act as a perfect substitute for the missing record-keeping technology. Two-money mechanisms improve welfare over one-money mechanisms when $\beta < \beta_1^N$.

At the day stage of date 0, the mechanism asks agents to choose from N monetary portfolios:

$$r_m R + (1 - r_m) G;$$

with $0 < r_m < 1$ for all $m \in \{1, 2, \dots, N\}$ and $r_m \neq r_{m'}$ for all $m \neq m'$.

At the night stage of date 0, after the shocks are realized, the planner offers each agent the following choices:

At location $n \in \{1, 2, \dots, N\}$,

Show $r_m R + (1 - r_m) G$, get $c_{mn} = c_{m-n+I(m < n)N}$ good n and $\varepsilon[r_n R + (1 - r_n) G]$ where $0 < \varepsilon < \min_{m \neq m' \in \{1, 2, \dots, N\}} \{|r_m - r_{m'}|\}$;

The mechanism requires agents to show 1 unit of money to participate in the stage, and proposes consumption contingent on the composition of monetary portfolios held

by agents. The mechanism rewards participating agents with ε units of money, the composition of which differs across locations. We restrict ε to ensure that agents of different types and consuming at different locations exit the night stage with different monetary portfolios.

At the day stage of the $t \geq 1$ period,

Show $t[r_m R + (1 - r_m) G] + \varepsilon[r_n R + (1 - r_n) G]$, use $z_{mn} = z_{m-n+I(m < n)N}$ day output and $\varepsilon[r_n R + (1 - r_n) G]$ to exchange for $[r_m R + (1 - r_m) G]$;

At the night stage of the $t \geq 1$ period,

At location n ,

Show $(t+1)[r_m R + (1 - r_m) G]$, get $c_{mn} = c_{m-n+I(m < n)N}$ good n and $\varepsilon[r_n R + (1 - r_n) G]$.

The two-money mechanism outlined here deals with limited commitment and private information exactly the same way as the two-money mechanism with two types of agents. As long as N is finite, we can see that two monies are always a perfect substitute for the record-keeping technology. If money is indivisible as in Kocherlakota and Krueger (1999), we will need at least N monies to replace the record-keeping technology when there are N types of agents.

6 Two Monies as A Perfect Substitute for the Record-Keeping Technology

In our model's environment, two monies are sufficient to replace the record-keeping technology. Townsend (1987) and Kocherlakota (2002) have similar results in different environments. Here we develop an intuitive argument to show that two monies are always sufficient as a substitute for the record-keeping technology so there is no need

for a third money.

If money balances are not concealable, there is a one-to-one match between records and money balances, money balances will thus carry the relevant information into future periods. When money balances are concealable, however, the one-to-one match will be destroyed since individuals can change balances by hiding money. Or, the concealability of money balances makes it possible for individuals to change records to their own benefits. The introduction of a second money solves the problem by encoding records into different monetary portfolios with the same total balances and different compositions of the two monies. Agents will not be able to juggle their monetary portfolios to mimic other portfolios by concealing money. Note that when money is divisible, it is possible to encode any finite number of records into different monetary portfolios so a third money will not be needed.

7 Conclusion

In this paper, we show in a quasi-linear environment that in the presence of private information and limited commitment, a second money can potentially improve welfare by providing an efficient way to pass information across time.

In the absence of a record-keeping technology, monetary mechanisms (with either a single money or two monies) can effectively deal with limited commitment by rewarding participants with more money balances and requiring ever increasing money for future participation. The individual rationality conditions stay the same as in the case with the record-keeping technology where defectors are directly caught and forced into perpetual autarky.

There are two options to deal with private information about preferences. The first is to induce agents to truthfully report their types before the realization of the

preference shocks and use the information later on to infer agents' valuations of the night goods. We call this early-sorting. The second option is to induce agents to report their valuations after the realization of the preference shocks, and use the day stage consumption/production to align the incentives. We call this late-sorting.

Mechanisms with a single concealable money rule out early-sorting. When agents are patient enough, the late-sorting mechanism effectively aligns the incentive by inducing agents to leave the night stage with different money balances and produce/consume different amounts in the following day stage. When agents are not patient enough, the late-sorting mechanism is not powerful enough to align incentives. The introduction of a second money permits the early-sorting mechanism and allows agents to signal their preferences by holding different monetary portfolios (with the same total money balances).

We intend to extend the paper in the following way. In the paper, we take the mechanism design approach and there is no market in the mechanisms proposed in the paper. We would like to follow Waller (2007) to see if the allocations can be decentralized with market mechanisms (and with the help of monetary and fiscal policies).

References

Andolfatto, David (2008), "The Simple Analytics of Money and Credit in a Quasi-linear Environment," Manuscript, Simon Fraser University.

Camera, Gabriele, Ben Craig and Christopher J. Waller, (2004), "Currency Competition in a Fundamental Model of Money," *Journal of International Economics*, 64, 521-544.

Camera, Gabriele and Johannes Winkler, (2003), "International Monetary Trade and the Law of One Price," *Journal of Monetary Economics*, 50, 1531-1553.

Craig, Ben and Christopher J. Waller, (2004), "Dollarization and Currency Exchange," *Journal of Monetary Economics*, 51, 671-689.

Kocherlakota, Narayana (1998a), "The Technological Role of Fiat Money," *Federal Reserve Bank of Minneapolis Quarterly Review*, 22 (3), 2–10.

Kocherlakota, Narayana (1998b), "Money is Memory," *Journal of Economic Theory*, 81(2), 232–251.

Kocherlakota, Narayana and Thomas Krueger (1999), "A Signaling Model of Multiple Currencies", *Review of Economic Dynamics*, 2, 231-244.

Kocherlakota, Narayana (2002). "The Two-money Theorem," *International Economic Review*, 43(2), 333-346.

Lagos, Ricardo and Randall Wright (2005), "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113(3), 463-484.

Townsend, Robert M. (1987), "Economic Organization with Limited Communication", *The American Economic Review*, 77(5), 954–971.

Trejos, Alberto and Randall Wright (1995), "Search, Money, Bargaining, and

Prices," *Journal of Political Economy*, 103, 118-141.

Trejos, Alberto and Randall Wright (2001), "International Currency," *Advances in Macroeconomics*, Vol 1, Issue 1, Article 3.

Waller, Christopher (2007), "Dynamic Taxation, Private Information and Money," Manuscript, University of Notre Dame.