Outsourcing induced by strategic competition

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Abstract

We show that intermediate goods can be sourced to firms on the "outside" (that do not compete in the final product market), even when there are no economies of scale or cost advantages for these firms. What drives the phenomenon is that "inside" firms, by accepting such orders, incur the disadvantage of becoming Stackelberg followers in the ensuing competition to sell the final product. Thus they have incentive to quote high provider prices to ward off future competitors, driving the latter to source outside.

Keywords: Intermediate goods, outsourcing, Cournot duopoly, Stackelberg duopoly

JEL Classification: D41, L11, L13

1   Introduction

One of the principal concerns of any firm is to configure the supply of intermediate goods essential to its production. Of late, with the liberalization of trade and the lowering of barriers to entry, supply chain configurations have assumed global proportions. Indeed, in several industries, it has become the trend for firms to cut across national boundaries and outsource their supplies "offshore", provided the economic lure is strong enough. Many diverse factors influence firms' decisions. First, of course, there is the immediate cost of procuring the goods which—other things being equal—firms invariably seek to minimize. Then there is the question of risk: a firm may be unwilling to commit itself to a single party and instead spread its orders among others, even if they happen to be costlier, in order to ensure a steady flow of inputs. Sometimes a firm may tie up with a broad spectrum of suppliers so as to increase its access to the latest technological innovation, which could be forthcoming from any one of them. There can arise situations when a firm is impelled to select suppliers that will be strategic allies in its endeavor to penetrate newly emerging markets. For the analysis of these and other factors, and how they impinge on firms' decisions, see, e.g., Jarillo (1993), Spiegel (1993), Vidal and Goetschalkx (1997), Domberger (1998), Aggarwal (2003), Shy and Stenbacka (2003) and Chen et al. (2004).

One intriguing possibility that has been alluded to, but not much explored, is that *strategic incentives* may arise in an oligopoly which outweigh other considerations and play the pivotal role in firms' selection of suppliers. Instances of this are presented by Jarillo and Domberger, of which we recount only two.

The first case comes from Germany. AEG[1] used to be a traditional supplier to both BMW[2] and Mercedes Benz. At some point, with a view to vertical integration, Mercedes Benz acquired AEG. This caused BMW to look for a different supplier, despite the inevitable extra costs of the switch (see p. 67, Jarillo, 1993).

The second case involves General Electric (GE) in the United States. In the early 1980's, GE investigated the possibility of outsourcing its lower brand microwave ovens from outside, since these had become too costly to manufacture at its factory in Maryland. Discussions were first held with, and even trial orders given to, Matsushita which happened to be a major rival of GE and also the world leader for this product in terms of both volume and technology. But ultimately GE turned to Samsung, then a small company with little experience in microwaves. The strategy entailed additional costs, such as sending American engineers to Korea, but it worked well for GE (see, pp. 84-86, Jarillo, 1993; and also Case Study 6.2, p. 108, Domberger, 1998).

Such case studies clearly point to the need for a game-theoretic analysis. In this paper we bring to light a scenario in which the outsourcing patterns emerge out of the strategic competition between firms. We find that it is typically *not* the case that a firm will outsource supplies to its rivals. There are two distinct reasons for this. The first is based on increasing returns to scale: if a firm places a sizeable order with its rival, it significantly lowers the rival's costs on account of the increasing returns, and this stands to its detriment in the ensuing competition on the final product. Thus the firm is led to outsource to others who may be costlier but, being out of the final product market, do not pose the threat of future

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[1]Allgemeine Deutsche Electricität sgesellschaft
[2]Bayerische Motoren Werke (or, Bavarian Motor Works)

competition. The second reason is more subtle and persists even in the case of constant returns to scale (i.e., linear costs)—indeed, it comes to the fore in this case. It is the main focus of this paper.

To be precise, suppose there are firms $\mathbb{N}$ competing in the market for a final product α. Intermediate goods η are critical inputs in the production of α, but only *some* of the firms $\mathbb{I} \subset \mathbb{N}$ have the competence to manufacture η at reasonable cost. The other firms $\mathbb{J} \equiv \mathbb{N}\backslash\mathbb{I}$ must obtain η from elsewhere. One possibility is to outsource η to their rivals in $\mathbb{I}$. But there is also a fringe of firms $\mathbb{O}$ on the "outside" which can manufacture η. What distinguishes $\mathbb{O}$ from $\mathbb{I}$ is that no firm in $\mathbb{O}$ can enter the market for the final product α. (This could be because it lacks the technology to convert η to α, or else faces high set-up costs—and, possibly, other barriers to entry—in the market α.[3]) To keep matters simple, we consider a purely linear model, i.e., in which the costs of production for both η and α are linear; as is the market demand for α.

Our main result is that, in this scenario, strategic considerations can come into play that will cause the firms in $\mathbb{J}$ to outsource η (outside) to $\mathbb{O}$ rather than (inside) to $\mathbb{I}$, *even if the costs of manufacturing η are higher in $\mathbb{O}$ than in $\mathbb{I}$*, so long as they are not much higher.

The intuition goes roughly as follows and is best seen when $\mathbb{I}$ and $\mathbb{J}$ consist of single firms and the outside fringe $\mathbb{O}$ is a competitive sector whose members simply quote their cost as the price at which they will provide the intermediate good. (Thus $\mathbb{O}$ has no strategic role here. However, we show that our results remain intact when $\mathbb{O}$ is taken to be strategic, indeed a monopolist,[4] though at the cost of a more complicated analysis.) Suppose (i) $\mathbb{I}$ and $\mathbb{J}$ are Cournot duopolists which compete in the market for the final product α; (ii) $\mathbb{I}$ and $\mathbb{O}$ can produce the intermediate good η, but $\mathbb{J}$ cannot; and (iii) $\mathbb{O}$ cannot enter the market for α. Thus $\mathbb{J}$ must decide how to allocate its order of η between $\mathbb{I}$ and $\mathbb{O}$, and then how much of it to use in the production of α, freely disposing of the unused portion of η. We show that the optimal course of action for $\mathbb{J}$ is to outsource *exclusively* to either $\mathbb{I}$ or $\mathbb{O}$, never to both, and to use the entire input to produce α. (Thus, in equilibrium, free disposal will not be availed of even though it is permitted.) Now if $\mathbb{J}$ outsources to $\mathbb{I}$, then $\mathbb{I}$ immediately knows the amount outsourced. This has the effect of establishing $\mathbb{J}$ as leader in the Stackelberg game that ensues in the market for α, in which $\mathbb{I}$ is forced to become the follower. In contrast, if $\mathbb{J}$ outsources to $\mathbb{O}$ then—thanks to the sanctity of the secrecy clause—$\mathbb{I}$ will only know that $\mathbb{J}$ has struck a deal with $\mathbb{O}$ but not the quantity that $\mathbb{J}$ has outsourced. Thus $\mathbb{I}$ and $\mathbb{J}$ will remain Cournot duopolists in the ensuing game on market α.

If costs for manufacturing η do not vary too much between $\mathbb{I}$ and $\mathbb{O}$, then $\mathbb{I}$ will earn less as a Stackelberg follower than as a Cournot duopolist. This will tempt $\mathbb{I}$ to push $\mathbb{J}$ towards $\mathbb{O}$ by quoting so high a price for the intermediate good η that, in spite of the premium that

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[3]In particular, think of the following set-up. The market for α is concentrated in the "developed world". The firms in $\mathbb{O}$, on the other hand, are located offshore in the "developing world" and can manufacture η but lack the (advanced) technology for converting η to α. Even if some of them were to make the technological breakthrough, they would face not just the standard set-up costs for penetrating the market α, but further barriers to entry that pertain to foreign firms. This international setting perhaps makes our hypothesis of an outside fringe $\mathbb{O}$ more viable. But we do not need it, and all we formally postulate is the existence of this fringe.

[4]The moment $\mathbb{O}$ has two or more (identical) firms, Bertrand competition will bring the price they quote down to the level of their cost and $\mathbb{O}$ will in effect be a competitive sector. (Note that we postulate linear costs and unbounded capacity). Thus, within the parameters of our model, $\mathbb{O}$ can be strategic only if it is a monopolist.

$\mathbb{J}$ is willing to pay for the privilege of being the leader, $\mathbb{J}$ prefers to go to $\mathbb{O}$. The temptation can only be resisted if it is feasible for $\mathbb{I}$ to provide η at such an exorbitant price that it can recoup as provider what it loses as follower. But such an exorbitant price is undercut by the competitive price prevailing at $\mathbb{O}$, as long as $\mathbb{O}$'s costs are not too much higher than $\mathbb{I}$'s. The upshot is that in any subgame-perfect pure strategy Nash equilibrium (SPNE) of the game, $\mathbb{J}$ will outsource to $\mathbb{O}$. *Throughout our analysis, we confine ourselves to pure strategies.*

Which subgame gets played between $\mathbb{I}$ and $\mathbb{J}$ on market α—Cournot or Stackelberg—is thus not apriori fixed, but endogenous to equilibrium. This is all the more striking since, in our overall game, the option is open for firm $\mathbb{J}$ to outsource to both $\mathbb{I}$ and $\mathbb{O}$ and to thus bring any "mixture" of the Stackelberg and Cournot games into play. The logic of the SPNE rules out mixing and shows that only one of the two pure games will occur along the equilibrium play.

Worthy of note is the fact that it is not $\mathbb{J}$ who has the "primary" strategic incentive to outsource to $\mathbb{O}$. This incentive resides with $\mathbb{I}$ who is anxious to ward off $\mathbb{J}$ and force $\mathbb{J}$ to turn to $\mathbb{O}$. The anxiety gets played out when $\mathbb{O}$ does not have a severe cost disadvantage compared to $\mathbb{J}$. Otherwise, $\mathbb{I}$ is happy to strike a deal with $\mathbb{J}$ since it can get high provider prices that compensate it for becoming a follower. That inside firms would want to provide inputs to their competitors at high prices is quite common and has been commented on (e.g., Hart and Tirole, 1990; Ordover et al., 1990; Rey and Tirole, 2006). What is surprising in our scenario is that high prices are quoted not to earn revenue at the expense of the competitor but instead to ward it off and compel it to seek its supply elsewhere.

The actual argument is more intricate and the exact result is presented in Section 4. As was said, there are no economies of scale or cost advantages for the outside firm $\mathbb{O}$. In fact, we suppose that $\mathbb{O}$ has a higher cost than $\mathbb{I}$ for manufacturing η. Our main result states that, if $\mathbb{O}$'s cost does not exceed a well-defined threshold, $\mathbb{J}$ will outsource to $\mathbb{O}$ in any SPNE.

Our formal model is as follows. The market for the intermediate good η meets first, followed by duopolistic competition between $\mathbb{I}$ and $\mathbb{J}$ on the final good α. Since the outside competitive fringe $\mathbb{O}$ stands ready to supply η at its cost price, firm $\mathbb{I}$ must counter this with a price quote of its own for η. Then both firms $\mathbb{I}$ and $\mathbb{J}$, seeing these prices, decide how much of η to outsource to $\mathbb{I}$ and to $\mathbb{O}$. The outsourcing orders are subject to a secrecy clause, which is tantamount to $\mathbb{I}$ and $\mathbb{J}$ placing their orders simultaneously. The only act that could destroy the simultaneity is a preliminary announcement by $\mathbb{I}$ of the quantity of η it intends to produce (outsource to itself). But, in the absence of an external enforcement agency, such an announcement would not constitute a credible commitment and would be like "cheap talk" which can be ignored. We discuss this issue in more detail in Section 6.1.

The secrecy clause is crucial for our analysis. It can be upheld on the simple ground that it is routinely seen in practice (see, e.g., Temponi and Lambert, 2001; Ravenhill, 2003; Hoecht and Trott, 2006) and it is often a legally binding provision (see, e.g., Khalfan, 2004; Vagadia, 2007). The evidence supporting the secrecy clause is discussed in Section 6.2. Moreover we give a plausibility argument that, in certain scenarios, it holds endogenously in equilibrium.

It must also be pointed out that our model is one-shot (corresponding to discounting the future very heavily if one were to think of a multi-period setting) and in effect all goods are perishable. With a long time horizon, durable goods and players who are patient, other kinds of SPNE would surely emerge. All these considerations—possibility of credible commitments, breakdown of the secrecy clause, long time horizon—are clearly important

issues but lie beyond the scope of this paper.

Our analysis indicates that firms which position themselves on the "outside", by *not* entering the market for the final product, are more likely to attract orders for intermediate goods. There is some evidence that this can happen in practice. By the mid-1980's (see Ravenhill, 2003), US companies in the electronics industry were looking "to diversify their sources of supply" in order to fare better against their Japanese competitors. Malaysia and Singapore made a strong bid to get the US business. A key feature of the government policies of both nations was that "they were not attempting to promote national champions in the electronic industry", but the objective was rather "to build a complementary supply base, not to create local rivals that might displace foreign producers". Their success in becoming major supply hubs for electronic components is well documented. Of course it is true that they had the advantage of low-cost skilled labor. But what we wish to underscore is their deliberate and well-publicized *abstention* from markets for the final products, which by itself gave them a competitive edge. To reiterate this implication of our analysis in more ambitious terms: *the current widespread trend of outsourcing to offshore locations may well persist for strategic reasons, even if offshore costs were to rise, so long as the offshore companies abstain from the final product markets of their clients.*

The paper is organized as follows. We discuss the related literature in Section 2. The model is presented in Section 3, and the main result is described in Section 4. It holds for both scenarios, the one where the outside firm is strategic and the other where it is part of a competitive fringe. An outline of its proof, followed by the formal proof itself (modulo some technical lemmas) are in Section 5. In Section 6 we discuss several extensions and variations of the model. Proofs of the technical lemmas are relegated to an Appendix.

2   Related Literature

Outsourcing of inputs is such an important and widespread phenomenon that it has been studied from various points of view. First, it can be driven by cost considerations or by differences in productivity across firms (see, e.g., Grossman and Helpman, 2002; Antràs and Helpman, 2004; and the references therein). A second argument is based on niche markets. A firm may be attracted to an exclusive outsourcing contract with a supplier who can provide highly specialized inputs, in order to market a differentiated final product that limits the intensity of price competition (Shaked and Sutton, 1982). There are explanations based on economies of scale. It is cost-effective to outsource inputs instead of undertaking redundant investment of one's own, all the more so if one's rivals have already gone to an outsider and created the scale there (Shy and Stenbacka, 2003). Even when the rival is doing in-house production, there is pressure to outsource because placing orders with the rival will only enhance its economies of scale and lower its costs, making it a more formidable competitor in the final goods market (Chen and Dubey, 2009). Another rationale for outsourcing is based on imperfect competition in the input markets. By outsourcing orders with a supplier to which its rival has already gone, a firm accomplishes two things: first, it raises the input price, which softens the competition in the final goods market and can generate a net gain (Buehler and Haucap, 2006); second, it prevents the rival from extracting monopolistic benefits from the supplier (Arya et al., 2008a). Another line of argument in favor of outsourcing runs as follows. A vertically integrated provider, which competes in the final product market, will

have incentive to "hold up" its supply once investments have been sunk in the procurement process (Heavner, 2004). Furthermore there is the considerable risk that the business plans of the input-seeking firm will get revealed to its rival via the orders placed. On both counts, it is safer for the firm to outsource inputs to an outsider.

Of course, the other side of the story has also been discussed. If a negotiated agreement can be reached to share the gains, it could become optimal to order inputs from a rival (Spiegel, 1993). Indeed trade of intermediate goods may enable rivals to collude, via contractual agreements based on that trade, and thereby sustain a high price in the downstream market for the final product (Chen 2001; Chen et al., 2004; Arya et al., 2008b).

Each of these explanations clearly has its own merit, but our purpose here is to bring a new strategic consideration to the fore which can sometimes be critical to firms' behavior. To this end, we present a stripped-down model in which none of the factors above are present. There are no cost benefits, comparative advantage, niche markets, economies of scale or possibility of collusion; nor is there reluctance in the firm to reveal its business plans to the provider. On the contrary, the strategic competition in our model creates incentive for the input-seeking firm to fully reveal its plans to the vertically integrated rival, with the intent of becoming Stackelberg leader in the final product market. It is the rival who, seeing through this ploy, refuses to play the role of the input provider and drives the firm to outsource elsewhere.

There is also considerable literature on endogenous Stackelberg leadership.[5] The paper most closely related to ours, and inviting immediate comparison, is Baake et al (1999). They consider a duopoly model to examine what they call "cross-supplies" within an industry—in our parlance, this is the phenomenon that a firm outsources to its rival. The "endogenous Stackelberg effect" is indeed pointed out by them: firm A, upon accepting the order outsourced by its rival B, automatically becomes a Stackelberg follower in the ensuing game on the final markets. But there are set-up costs of production in their model, and provided these costs are high enough, A can charge B a sufficiently high price so as to be compensated for being a follower. The upshot is that cross-supplies can be sustained in SPNE.

There are several points of difference between their model and ours. First, their argument relies crucially on the presence of sufficiently strong economies of scale (set-up costs). If these are absent or weak, there is no outsourcing in SPNE in their model. In contrast, in our model, outsourcing occurs *purely* on account of the endogenous Stackelberg effect (recall that we have constant returns to scale[6]). Second, outsourcing occurs only in *some* of their SPNE: there always coexist other SPNE where it does not occur. In our model, the outsourcing is invariant across *all* SPNE. In short, they show that outsourcing *can* occur, while we show that it *must*. Third, it is critical for their result that there be no outside suppliers.[7] Such suppliers would generate competition that would make it infeasible for A to charge a high price to B, invalidating their result. In our model, the situation is different. We allow for both kinds of suppliers: those that are inside as potential rivals and others that are outside. It turns out that increasing the number of either type leaves our result intact (see Section 6.4). Finally—and this, to our mind, is the most salient difference—the economic phenomena depicted in Baake et al. and here are different, indeed almost complementary. In Baake et

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[5]E.g., Hamilton and Slutsky (1990), Robson (1990), Mailath (1993), Pal (1996), van Damme and Hurkens (1999)—in all of which the timing of entry by firms is viewed as strategic.

[6]Though outsourcing is further boosted by economies of scale in our model. See Section 6.4.

[7]Recall that these are suppliers who are not present as rivals in the final product market.

al., the issue is to figure out when a firm will outsource to its rival. Here we consider precisely the opposite scenario and pinpoint conditions under which a firm will turn away from its rival and outsource instead to an outsider, even if the outsider happens to have a costlier technology. The fact that both models take cognizance of the endogenous Stackelberg effect is a technical—albeit interesting—point. What is significant is that this effect is embedded in disparate models and utilized to explain complementary economic phenomena.

3   The model

We shall first present Model 1 in which $\mathbb{O}$ is a strategic monopolist and then Model 2 where $\mathbb{O}$ is a competitive sector. The reason for this order is mathematical brevity: Model 2 can immediately be derived from Model 1 by the simple expedient of setting $\mathbb{O}$'s provider price equal to its cost.

Model 1: strategic outside firm

For ease of notation, we substitute 0, 1, 2 for $\mathbb{O}$, $\mathbb{I}$, $\mathbb{J}$. As was said, firms 1 and 2 are duopolists in the market for a final good α. An intermediate good η is required to produce α. Firm 1 can manufacture η, but 2 cannot. There is an "outside" firm 0 which can also manufacture η. What distinguishes 0 from 1 is that 0 cannot enter the market for the final good α. Firm 0's sole means of profit is the manufacture of good η for the "inside" firms 1 and 2.

Let x_1 and x_2 be the respective quantities of α produced by firms 1 and 2, and $P(.)$ be the price of α. The inverse market demand for good α is

$$P(x_1 + x_2) = [a - x_1 - x_2]_+ \text{ where } y_+ \equiv \max\{y, 0\} \text{ and } a \text{ is a positive constant.} \quad (1)$$

The constant marginal cost of production of good η is c_0 for 0 and c_1 for 1. Furthermore both 1 and 2 can convert one unit of good η into one unit of good α at the (for simplicity) same constant marginal cost, which w.l.o.g we normalize to zero. We assume

$$0 < c_1 < c_0 < (a + c_1)/2. \quad (2)$$

The condition $c_1 < c_0$ gives a cost disadvantage to the outside firm 0 and loads the dice against good η being sourced to it. The inequality $c_0 < (a + c_1)/2$ prevents 1 from automatically becoming a monopolist in the market for good α.

The extensive form game between the three firms is completely specified by the parameters c_1, c_0, a and so we shall denote it $\Gamma(c_0, c_1, a)$. For $i \in \{0, 1\}$ and $j \in \{1, 2\}$, let

$$q_j^i \equiv \text{ quantity of good } \eta \text{ outsourced by firm } j \text{ to firm } i \text{ and}$$

$$x_j \equiv \text{ quantity of good } \alpha \text{ supplied by firm } j.$$

The game $\Gamma(c_0, c_1, a)$ is played as follows.

Stage 1: Firms 0 and 1 simultaneously and publicly announce prices p_0 and p_1 at which they are ready to provide good η.

Stage 2: Firm 2 chooses q_2^1 (quantity of η to order from firm 1 at price p_1).

Stage 3: For every (p_0, p_1, q_2^1), 1 and 2 play the simultaneous-move game $G(p_0, p_1, q_2^1)$ where

1. Firm 1 chooses q_1^0 (quantity of η to order from firm 0 at price p_0), q_1^1 (quantity of η to produce by itself at cost c_1) and x_1 (quantity of α to put up in the final goods market) subject to:

   (i) $q_1^0 + q_1^1 \geq q_2^1$ (the total amount of η is at least q_2^1 so that it is able to honor its commitment to supply q_2^1 units of η to firm 2) and (ii) $x_1 \leq q_1^0 + q_1^1 - q_2^1$ (quantity of α must be producible from the amount of η left on hand after fulfilling the order of firm 2), so that the strategy set of firm 1 in $G(p_0, p_1, q_2^1)$ is

$$S_1(q_2^1) = \{(q_1^0, q_1^1, x_1) \in \mathbf{R}_+^3 | q_1^0 + q_1^1 \geq q_2^1 \text{ and } x_1 \leq q_1^0 + q_1^1 - q_2^1\}. \tag{3}$$

   Note that "$\leq$" is tantamount to allowing free disposal of the intermediate good (input).[8]

2. Firm 2 chooses q_2^0 (quantity of η to order from firm 0 at price p_0) and x_2 (quantity of α to put up in the final goods market) subject to $x_2 \leq q_2^0 + q_2^1$ (quantity of α must be producible from the total η it has ordered). Therefore the strategy set of firm 2 in $G(p_0, p_1, q_2^1)$ is

$$S_2(q_2^1) = \{(q_2^0, x_2) \in \mathbf{R}_+^2 | x_2 \leq q_2^0 + q_2^1\}. \tag{4}$$

It remains to describe the payoffs of the three firms at the terminal nodes of the game tree. Any such node is specified by $p \equiv (p_0, p_1)$, $q \equiv \{q_j^i\}_{j=1,2}^{i=0,1}$ and $x \equiv (x_1, x_2)$. For $i = 0, 1, 2$, the payoff $\Pi_i(p, q)$ is given by

$$\Pi_0(p, q) = \underbrace{p_0(q_1^0 + q_2^0)}_{\text{revenue from } \eta} - \underbrace{c_0(q_1^0 + q_2^0)}_{\text{cost of } \eta}, \tag{5}$$

$$\Pi_1(p, q, x) = \underbrace{P(x_1 + x_2)x_1}_{\text{revenue from } \alpha} + \underbrace{p_1 q_2^1}_{\text{revenue from } \eta} - \underbrace{(p_0 q_1^0 + c_1 q_1^1)}_{\text{cost of } \eta} \text{ and} \tag{6}$$

$$\Pi_2(p, q, x) = \underbrace{P(x_1 + x_2)x_2}_{\text{revenue from } \alpha} - \underbrace{(p_0 q_2^0 + p_1 q_2^1)}_{\text{cost of } \eta}. \tag{7}$$

This completes the description of the game $\Gamma(c_0, c_1, a)$.

Remark (On the Timing of Moves) We have supposed in our game that firm 2 chooses q_2^1 in stage 2 after finding out p_0, p_1 announced in stage 1; and that subsequently it chooses q_2^0, x_2 in stage 3 but without knowing anything further than p_0, p_1, q_2^1. This is entirely equivalent to choosing q_2^1, q_2^0, x_2 simultaneously in stage 2 after finding out p_0, p_1.

But for firm 1, the timing we have described is important. Firm 1 chooses q_1^0, q_1^1, x_1 *after* finding out the order q_2^1 placed by firm 2. This timing is logically necessary. Were firm 1 to announce q_1^0, q_1^1, x_1 before knowing q_2^1, it could always renege on its announcement and choose a new $\tilde{q}_1^0, \tilde{q}_1^1, \tilde{x}_1$ after knowing q_2^1. Due to the secrecy clause, firm 2 does not even know if and when firm 1 has gone to 0, leave aside the quantity q_1^0 it has ordered. As for q_1^1 and x_1, these are known to firm 1 alone. Thus firm 1 has the full power to revise q_1^0, q_1^1, x_1. In the absence of binding contracts, which could be written before a third party such as a courthouse empowered to enforce it, the prior announcement of q_1^0, q_1^1, x_1 by firm 1 does not constitute a credible commitment and has no effect (see Section 6.1).

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[8]We thank an anonymous referee for suggesting that we formally incorporate free disposal in our model.

Model 2: competitive outside fringe

For contrast, and to gain better perspective, we shall consider a variant[9] Model 2 in which $\mathbb{O}$ is a *competitive* fringe of many outside firms. In this scenario we may take the choice made by a representative firm 0 of $\mathbb{O}$ to be $p_0 \equiv c_0$ (so that $\Pi_0 \equiv 0$). Thus firm 0 is in effect a "strategic dummy" and we wind up with strategic competition only between firms 1 and 2. We denote this game $\widetilde{\Gamma}(c_0, c_1, a)$. The formal mathematical definition of $\widetilde{\Gamma}(c_0, c_1, a)$ is exactly the same as that of $\Gamma(c_0, c_1, a)$ except that we take $p_0 \equiv c_0$ to be an exogenously fixed constant. We seek to find subgame-perfect pure strategy Nash equilibrium (SPNE) of $\Gamma(c_0, c_1, a)$ and $\widetilde{\Gamma}(c_0, c_1, a)$. Henceforth we shall often denote the games by simply Γ and $\widetilde{\Gamma}$.

4   The main result

Our main result asserts that if the cost disadvantage of the outside firm 0 is not too significant (i.e. $c_0 - c_1$ is not too large), then there is outsourcing to 0 in any SPNE of both Γ and $\widetilde{\Gamma}$.

The Main Result (Strategic outside firm or competitive outside fringe) *There is a threshold $\widehat{\theta}(c_1) \in (c_1, (a + c_1)/2)$ such that if $c_0 \in (c_1, \widehat{\theta}(c_1))$, then in any SPNE of either $\Gamma(c_0, c_1, a)$ or $\widetilde{\Gamma}(c_0, c_1, a)$, firm 2 orders η exclusively from the outside firm 0.*

Observe that when $c_0 \in (c_1, \widehat{\theta})$, firm 0 has a cost disadvantage compared to firm 1, yet 2 outsources η to 0 rather than to 1. Strategic considerations dominate firms' behavior here. To keep these strategic incentives in the foreground, we have assumed in the paper that $c_0 > c_1$.[10] The main result is also summarized in Figure 1 below, in which c_0 is varied on the horizontal axis, holding a and c_1 fixed.
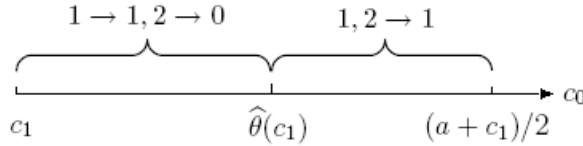

Figure 1: The Main Result

This figure confirms our claim that outsourcing to offshore locations will persist for strategic reasons, even if offshore costs rise moderately so long as the offshore companies abstain from the final product markets of their clients.

4.1   SPNE of Γ and $\widetilde{\Gamma}$: The detailed characterization

In this section we fully characterize SPNE of Γ (Theorem 1) and $\widetilde{\Gamma}$ (Theorem 2). The proofs are in Section 5. It will be useful to first state the following lemma.

Lemma 1 *In any SPNE of $\Gamma(c_0, c_1, a)$ or $\widetilde{\Gamma}(c_0, c_1, a)$, we must have* (i) $p_0 \geq c_1$ *and* (ii) $q_1^0 = 0$ *(firm 1 does not outsource to firm 0).*

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[9]We are grateful to an anonymous referee for suggesting this variant.

[10]Below this interval, when $c_0 \leq c_1$, 0 has a cost advantage over 1 and so 2 even more readily outsourced to 0; in fact, for small enough c_0, *both* 1 and 2 outsource to 0.

Proof See the Appendix. ∎

By Lemma 1, in any SPNE play of Γ or $\widetilde{\Gamma}$, firm 1's constant marginal cost is c_1. Fix the inverse demand as in (1) and 1's cost at c_1. For constant marginal cost $c_2 \geq 0$ of firm 2, let

- $\mathbb{C}(c_2)$ be the Cournot duopoly with firms 1 and 2, and

- $\mathbb{S}^{21}(c_2)$ be the Stackelberg duopoly with firm 2 as the leader and firm 1 as the follower.

Let $(x_1^{\mathbb{C}}(c_2), x_2^{\mathbb{C}}(c_2))$ be the quantities of firms 1 and 2 in the unique NE of $\mathbb{C}(c_2)$ and $(x_1^{\mathbb{S}}(c_2), x_2^{\mathbb{S}}(c_2))$ be the quantities in the unique SPNE of $\mathbb{S}^{21}(p_1)$. Also define

$$\widetilde{x}_1^{\mathbb{S}}(c_2) := \max\{x_1^{\mathbb{S}}(c_2), x_1^{\mathbb{C}}(0)\} \text{ and } \widetilde{x}_2^{\mathbb{S}}(c_2) := \min\{x_2^{\mathbb{S}}(c_2), x_2^{\mathbb{C}}(0)\}. \tag{8}$$

Let (p_0, p_1) be the pair of prices announced in Stage 1 in an SPNE of Γ ($p_0 \equiv c_0$ for $\widetilde{\Gamma}$). Denote by $G(p_0, p_1)$ the game played between firms 1 and 2 following the announcement (p_0, p_1). Let $x \equiv (x_1, x_2)$ and $q \equiv \{q_j^i\}_{j=1,2}^{i=0,1}$. By Lemma 1, $q_1^0 = 0$ in any SPNE of $G(p_0, p_1)$. The following definitions will be useful.

(a) We say that the *Cournot outcome* is played in $G(p_0, p_1)$ if $(x_1, x_2) = (x_1^{\mathbb{C}}(p_0), x_2^{\mathbb{C}}(p_0))$.

(b) We say that the *Stackelberg outcome* is played in $G(p_0, p_1)$ if $(x_1, x_2) = (\widetilde{x}_1^{\mathbb{S}}(p_1), \widetilde{x}_2^{\mathbb{S}}(p_1))$.

(c) Firm 2's order of η is called *exclusive* if $q_1^0 q_2^1 = 0$ (i.e., firm 2 does not order a positive amount of η from both 0 and 1).

(d) For $j = 1, 2$, (x_j, q_j^0, q_j^1) is said to satisfy the *no wastage* property for firm j if every unit of η ordered by j is completely utilized (either to supply α or to provide η to another firm). As $q_1^0 = 0$, this property holds for firm 1 if $q_1^1 = q_2^1 + x_1$. It holds for firm 2 if $q_1^0 + q_2^0 = x_2$.

For any interval $[u, v] \subseteq [c_1, (a + c_1)/2]$, define

$$(\text{Graph } \tau)[u, v] \equiv \{(p_0, \tau(p_0)) | p_0 \in [u, v]\} \text{ and abbreviate}$$

$$\text{Graph } \tau \equiv (\text{Graph } \tau)[c_1, (a + c_1)/2].$$

$(\text{Graph } \tau)[u, v]$ is portrayed in Figure 2.

Theorem 1 (Strategic outside firm) *There is a threshold $\widehat{\theta}(c_1) \in (c_1, (a + c_1)/2)$ such that, in the game $\Gamma(c_0, c_1, a)$ the following hold.*

(I) *In any SPNE, (a) firm 1 never outsources to firm 0 ($q_1^0 = 0$), (b) firm 2's order of η is exclusive ($q_1^0 q_2^0 = 0$) and (c) the no wastage property holds for both firms 1 and 2.*

(II) *If $c_0 \in (c_1, \widehat{\theta}(c_1))$, there is a continuum of SPNE, indexed by supplier prices $(p_0, p_1) \in (\text{Graph } \tau)[c_0, \widehat{\theta}(c_1)]$. For any such (p_0, p_1), $G(p_0, p_1)$ has a unique SPNE where firm 2 outsources η to the outside firm 0 and the Cournot outcome is played, i.e., $q_2^1 = 0$, $q_1^1 = x_1 = x_1^{\mathbb{C}}(p_0)$ and $q_2^0 = x_2 = x_2^{\mathbb{C}}(p_0)$.*
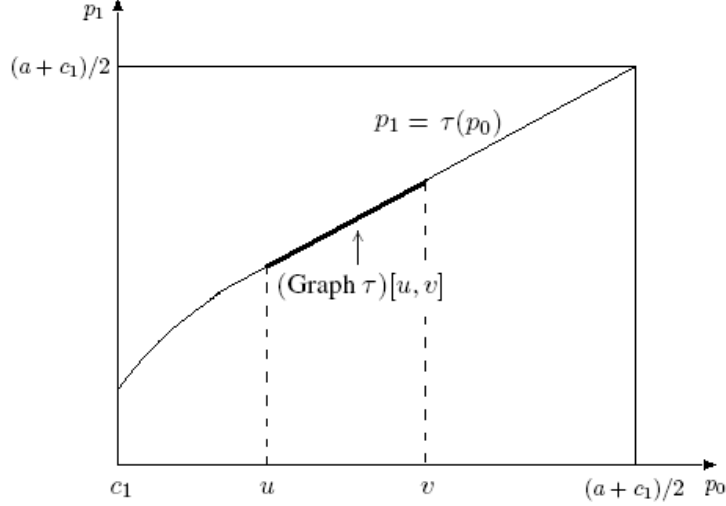
Figure 2: Graph $\tau[u, v]$

(III) *If $c_0 \in (\widehat{\theta}(c_1), (a + c_1)/2)$, there is a continuum of SPNE, indexed by supplier prices $(p_0, p_1) \in (Graph\ \tau)[c_1, c_0]$. For any such (p_0, p_1), $G(p_0, p_1)$ has a unique SPNE where firm 2 outsources η to firm 1 and the Stackelberg outcome is played, i.e., $q_2^0 = 0$, $q_1^1 = q_2^1 + x_1$, $q_2^1 = x_2 = x_2^{\mathbb{S}}(p_1)$ and $x_1 = x_1^{\mathbb{S}}(p_1)$.*

(IV) *Finally, if $c_0 = \widehat{\theta}(c_1)$, there are two SPNE with the same provider prices $(p_0, p_1) = (c_0, \tau(c_0))$. In the first SPNE firm 2 outsources η to 0 and the Cournot outcome is played in $G(p_0, p_1)$; in the the second SPNE firm 2 outsources η to 1 and the Stackelberg outcome is played.*

Proof See Section 5. ∎

Essentially the same qualitative result can be stated for Model 2 (the game $\widetilde{\Gamma}(c_1, c_0, a)$), with some obvious modifications spelled out below. First, a little terminology. We say that two SPNE are "*equivalent in real terms*" if they only differ in the provider prices quoted by the firm to which nothing is outsourced. In other words, the two SPNE must have the same quantities $q \equiv \{q_j^i\}_{j=1,2}^{i=0,1}$, and also the same prices except perhaps the provider price p_i ($i = 0, 1$) at which there is no outsourced order (i.e., $q_1^i + q_2^i = 0$).

Theorem 2 (Competitive outside fringe) *There is a threshold $\widehat{\theta}(c_1) \in (c_1, (a + c_1)/2)$ (same as in Theorem 1 with the strategic outside firm) such that, in the game $\widetilde{\Gamma}(c_0, c_1, a)$ the following hold.*

(I) *Same as Theorem 1.*

(II) *Same as Theorem 1 except that $p_1 \in [\tau(c_0), \infty)$ and the continuum of SPNE are equivalent in real terms.*

(III) *Same as Theorem 1 except that the continuum of SPNE collapses to a unique SPNE with $p_1 = \tau(c_0)$.*

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(IV) *Finally, if $c_0 = \widehat{\theta}(c_1)$, there is a continuum of SPNE. In one of these, $p_1 = \tau(c_0)$ and firm 2 outsources η to firm 1. The rest are indexed by $p_1 \in [\tau(c_0), \infty)$, are equivalent in real terms and have firm 2 outsourcing to firm 0.*

Proof See Section 5. ∎

5 Proof of the main results

First we give an intuitive outline of the proof in Section 5.1. The formal proof, modulo some technical lemmas, follows in Section 5.2. The proofs of the lemmas are in the Appendix.

5.1 Outline of the proof

We apply backward induction to determine SPNE of Γ and $\widetilde{\Gamma}$. We therefore begin from Stage 3 of these games. Here p_0, p_1, q_2^1 are given and firms 1, 2 play the simultaneous-move game $G(p_0, p_1, q_2^1)$. It can be viewed as a Cournot game in which firm 2 has built a "capacity" of q_2^1 prior to the game. We show that that if the capacity q_2^1 is too small, the game yields the standard Cournot outcome and if it is too large, part of the capacity remains unutilized (thus for any $p_1 > 0$, building a very large capacity cannot be optimal for firm 2). Intermediate capacities have a commitment value and effectively establishes firm 2 as the Stackelberg leader in the final good market α (Lemma S3).

Next we move back to Stage 2. Here p_0, p_1 are given and firm 2 has to choose its capacity q_2^1. We show that firm 2's optimal choice is either

(i) to order nothing from firm 1 (i.e. $q_2^1 = 0$) and instead order exclusively from firm 0 to obtain the Cournot profit with cost p_0

or

(ii) to order nothing from firm 0 and order exclusively the Stackelberg leader output from firm 1 (i.e. build a capacity q_2^1 exactly equal this output).

Whether firm 2 prefers to be a Stackelberg leader or a Cournot duopolist depends on the relative values of p_0 and p_1. For any p_0, we identify a function $\tau(p_0)$ that represents the *leadership premium*: if $p_1 < \tau(p_0)$, firm 2 prefers to be the Stackelberg leader and if $p_1 > \tau(p_0)$ it prefers to be a Cournot duopolist (Lemma S2).

Finally we arrive at Stage 1 where firms 0 and 1 simultaneously set prices p_0, p_1 for the intermediate good η (for $\widetilde{\Gamma}$, where firm 0 is part of a competitive fringe, it has no strategic role and p_0 automatically equals c_0). Any p_0, p_1 leads to the game $G(p_0, p_1)$ whose SPNE results in either the Cournot outcome or the Stackelberg outcome. In the Cournot outcome, 2 orders the good η exclusively from firm 0 and then the Cournot game ensues between firms 1 and 2 in the final good market α. In the Stackelberg outcome, 2 orders η exclusively from firm 1 and then the Stackelberg game ensues between firms 1 and 2 in the market α with 2 as the leader and 1 the follower. Firm 1's profit in the the market α is clearly lower when it is the Stackelberg follower rather than a Cournot duopolist. Therefore 1 prefers to be the follower only if it can obtain a sufficiently high supplier price of η from firm 2 so that it can recover its losses in the market α. However, if firm 0 is not too inefficient compared to 1,

it can undercut a high price of η set by 1. We identify a threshold $\widehat{\theta}$ such that when 0's unit cost c_0 is below $\widehat{\theta}$, any SPNE will entail firm 2 ordering η from firm 0, followed by the Cournot outcome in the market α. If $c_0 > \widehat{\theta}$, firm 2 orders η exclusively from firm 1, followed by the Stackelberg outcome in the market α with 2 as the leader and 1 the follower (Lemma S1, which immediately leads to Theorems 1 and 2).

5.2 The formal proof

Fix the inverse demand as in (1) and firm 1's constant marginal cost at c_1. Recall that $\mathbb{C}(c_2)$ is the Cournot duopoly with firms 1 and 2 where 2 has constant marginal cost c_2. For $i = 1, 2$, denote by $\kappa_i(c_2)$ the NE profit of firm i at $\mathbb{C}(c_2)$.

Lemma 2 *In any SPNE of Γ or $\widetilde{\Gamma}$, (i) firm 0 obtains at least zero and (ii) firm 1 obtains at least $\kappa_1(c_0)$.*

Proof (i) Follows by noting that firm 0 can always ensure zero payoff by setting a sufficiently high input price (e.g., $p_0 > a$) so that no firm places an order of η with it.

(ii) Observe that firm 1 always has the option of setting a sufficiently high input price (e.g., $p_1 > a$) to ensure that 2 does not order η from 1. For any such p_1, 2 orders η only from 0 and the game $\mathbb{C}(p_0)$ is played in the market α. If $x_2 = 0$ (i.e. 2 supplies nothing in the market α) in the NE of $\mathbb{C}(p_0)$, then firm 1 obtains the monopoly profit which is higher than $\kappa_1(c_0)$. If $x_2 > 0$, we must have $p_0 \geq c_0$ (otherwise firm 0 will obtain a negative payoff, contradicting (i)) and 1 obtains $\kappa_1(p_0) \geq \kappa_1(c_0)$. ∎

5.2.1 Stage 3 of Γ and $\widetilde{\Gamma}$

Let $p_0 \geq c_1$ and $p_1 \geq 0$. In Stage 3 of Γ, the simultaneous-move game $G(p_0, p_1, q_2^1)$ is played between firms 1 and 2 ($p_0 \equiv c_0$ for $\widetilde{\Gamma}$) where the strategy sets are given by (3) and (4) and the payoffs are given in (6)-(7). From Lemma 1(ii), $q_1^0 = 0$ (i.e. firm 1 produces η entirely by itself), so by (6), its cost of η is $c_1 q_1^1$. Optimality requires that

(i) For firm 1, $q_1^1 = x_1 + q_2^1$ (every unit of η produced by firm 1 is utilized completely either to supply α or to fulfill the order of η for firm 2).

(ii) For firm 2, $q_2^0 = \max\{x_2 - q_2^1, 0\}$. If $x_2 \leq q_2^1$ then $q_2^0 = 0$ (if 2's supply of α does not exceed the amount q_2^1 of η that it has ordered from 1, then it does not order η from 0) and if $x_2 > q_2^1$ then $q_2^0 = x_2 - q_2^1$ (if 2's supply of α exceeds q_2^1, its order of η from firm 0 equals exactly the additional amount it needs to meet its supply).

By (i) and (ii) above, $G(p_0, p_1, q_2^1)$ reduces to the game where firms 1 and 2 simultaneously choose $x_1, x_2 \geq 0$. By (i) and (6), the payoff function of firm 1 is

$$\Pi_1(x_1, x_2) = P(x_1 + x_2)x_1 - c_1 x_1 + (p_1 - c_1)q_2^1. \tag{9}$$

By (ii) and (7), the payoff function of firm 2 is

$$\Pi_2(x_1, x_2) = \begin{cases} P(x_1 + x_2)x_2 - p_1 q_2^1 & \text{if } x_2 \in [0, q_2^1], \\ P(x_1 + x_2)x_2 - p_0 x_2 - (p_1 - p_0)q_2^1 & \text{if } x_2 > q_2^1. \end{cases} \tag{10}$$

By (9) and (10), $G(p_0, p_1, q_2^1)$ is a Cournot duopoly between firms 1 and 2 where

(i) firm 1 has constant unit cost c_1,

(ii) firm 2's constant unit cost is 0 for $x_2 \in [0, q_2^1)$ and p_0 for $x_2 > q_2^1$.

In a Cournot duopoly under inverse demand (1), if firm i has constant marginal cost c_i, its unique best response to its rival firm j's quantity x_j is $b^{c_i}(x_j) = [(a - c_i - x_j)/2]_+$. Let $(x_1^{\mathbb{C}}(c_2), x_2^{\mathbb{C}}(c_2))$ be the quantities of firms 1 and 2 in the unique NE of $\mathbb{C}(c_2)$. We know that

$$(x_1^{\mathbb{C}}(p_0), x_2^{\mathbb{C}}(p_0)) = \begin{cases} ((a - 2c_1 + p_0)/3, (a + c_1 - 2p_0)/3) & \text{if } p_0 < (a + c_1)/2 \\ ((a - c_1)/2, 0) & \text{if } p_0 \geq (a + c_1)/2 \end{cases} \tag{11}$$

$$(x_1^{\mathbb{C}}(0), x_2^{\mathbb{C}}(0)) = \begin{cases} ((a - 2c_1)/3, (a + c_1)/3) & \text{if } c_1 < a/2 \\ (0, a/2) & \text{if } c_1 \geq a/2 \end{cases} \tag{12}$$

The following lemma follows from the fact that, under a Cournot duopoly, the best response of a firm is decreasing in its rival's quantity.

Lemma S3 (Stage 3) (i) $G(p_0, p_1, q_2^1)$ *has a unique NE where* $q_1^0 = 0$, $q_1^1 = x_1 + q_2^1$ *and which is given as follows:*

(a) *If* $q_2^1 \in [0, x_2^{\mathbb{C}}(p_0))$, *then* $x_1 = x_1^{\mathbb{C}}(p_0)$, $x_2 = x_2^{\mathbb{C}}(p_0)$ *and* $q_2^0 = x_2^{\mathbb{C}}(p_0) - q_2^1$;

(b) *If* $q_2^1 \in [x_2^{\mathbb{C}}(p_0), x_2^{\mathbb{C}}(0)]$, *then* $x_1 = b^{c_1}(q_2^1)$, $x_2 = q_2^1$ *and* $q_2^0 = 0$;

(c) *If* $q_2^1 \in (x_2^{\mathbb{C}}(0), \infty)$, *then* $x_1 = x_1^{\mathbb{C}}(0)$, $x_2 = x_2^{\mathbb{C}}(0)$ *and* $q_2^0 = 0$.

(ii) *Suppose* $p_0 \geq (a + c_1)/2$. *Then the NE of* $G(p_0, p_1, q_2^1)$ *is invariant of* p_0. *Hence w.l.o.g. we may restrict* $p_0 \leq (a + c_1)/2$.

Proof See the Appendix. ∎

5.2.2 Stage 2 of Γ and $\widetilde{\Gamma}$: The leadership premium

Any node in Stage 2 of Γ corresponds to a specific price pair $(p_0, p_1) \equiv p$ (for $\widetilde{\Gamma}$, $p_0 \equiv c_0$) where firm 2 chooses $q_2^1 \geq 0$ determining the ensuing game $G(p_0, p_1, q_2^1)$. This game can be viewed as a Cournot duopoly between firms 1, 2 in which 2 has built a capacity q_2^1 prior to the game. Lemma S3 above has shown that building a capacity that is too high $[q_2^1 > x_2^{\mathbb{C}}(0)]$ leads to some part of it being unutilized while a capacity that is too low $[q_2^1 < x_2^{\mathbb{C}}(p_0)]$ provides no strategic advantage to firm 2. By Lemma S3(i)(b), intermediate capacities $(q_2^1 \in [x_2^{\mathbb{C}}(p_0), x_2^{\mathbb{C}}(0)])$ are fully utilized, and moreover firm 2 does not make any additional order of η from firm 0 (i.e. its order is exclusively from firm 1). Thus in this range q_2^1 constitutes a credible commitment that establishes firm 2 as the Stackelberg leader in the (unique) NE of $G(p_0, p_1, q_2^1)$. By (10) and Lemma S3, the payoff of firm 2 at the unique NE of $G(p_0, p_1, q_2^1)$ is

$$\Pi_2^p(q_2^1) = \begin{cases} \kappa_2(p_0) + (p_0 - p_1)q_2^1 & \text{if } q_2^1 \in [0, x_2^{\mathbb{C}}(p_0)), \\ P\left(b_1^c(q_2^1) + q_2^1\right)q_2^1 - p_1 q_2^1 & \text{if } q_2^1 \in [x_2^{\mathbb{C}}(p_0), x_2^{\mathbb{C}}(0)], \\ \kappa_2(0) - p_1 q_2^1 & \text{if } q_2^1 > x_2^{\mathbb{C}}(0). \end{cases} \tag{13}$$

For $q_2^1 \leq x_2^{\mathbb{C}}(p_0)$, firm 2's choice is driven purely by cost considerations (whether $p_0 > p_1$ or not). On the other hand, for any $p_1 > 0$, choosing $q_2^1 \geq x_2^{\mathbb{C}}(0)$ is not optimal for firm 2.

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Recall that $\mathbb{S}^{21}(p_1)$ is the Stackelberg duopoly with firm 2 as the leader with cost p_1 and firm 1 the follower with cost c_1. For $q_2^1 \in [x_2^{\mathbb{C}}(p_0), x_2^{\mathbb{C}}(0)]$, 2 solves the *constrained* problem of the Stackelberg leader in $\mathbb{S}^{21}(p_1)$ that is restricted to choose its output in this interval. Denote by $(x_1^{\mathbb{S}}(p_1), x_2^{\mathbb{S}}(p_1))$ the unique SPNE of $\mathbb{S}^{21}(p_1)$ and let $\ell(p_1)$ and $f(p_1)$ be the SPNE profits of the leader (firm 2) and the follower (firm 1). It will be useful to define

$$\widetilde{x}_2^{\mathbb{S}}(p_1) := \min\{x_2^{\mathbb{S}}(p_1), x_2^{\mathbb{C}}(0)\} \text{ and } \widetilde{x}_1^{\mathbb{S}}(p_1) := b^{c_1}\left(\widetilde{x}_2^{\mathbb{S}}(p_1)\right) = \max\{x_1^{\mathbb{S}}(p_1), x_1^{\mathbb{C}}(0)\}.$$

It is clear that if $p_1 \le p_0$ (i.e. the unit cost of building the capacity q_2^1 is less than the unit cost of the standard Cournot game), then 2 would prefer to be the Stackelberg leader. Since $p_0 \ge c_1$ in any SPNE (Lemma 1), if firm 1 sets $p_1 \le c_1 \le p_0$, then it does not obtain positive profit as a supplier of η and moreover incurs the disadvantage of becoming a Stackelberg follower. The next lemma shows that this cannot be sustained in an SPNE.

Lemma 3 *In any SPNE of Γ or $\widetilde{\Gamma}$ (i) if $0 < p_1 \le p_0$, then firm 2 chooses $q_2^1 = \widetilde{x}_2^{\mathbb{S}}(p_1)$ in Stage 2 and consequently (ii) we must have $p_1 > c_1$.*

Proof See the Appendix. ∎

In the light of Lemma 3, consider $p_1 > c_1 > 0$. Then by (13), firm 2 will choose $q_2^1 \le x_2^{\mathbb{C}}(0)$. The SPNE of $\mathbb{S}^{21}(p_1)$ for $p_1 > c_1$ is given as

$$\left(x_1^{\mathbb{S}}(p_1), x_2^{\mathbb{S}}(p_1)\right) = \begin{cases} ((a - 3c_1 + 2p_1)/4, (a + c_1)/2 - p_1) \text{ if } c_1 < p_1 \le (a + c_1)/2 \\ ((a - c_1)/2, 0) \text{ if } p_1 \ge (a + c_1)/2. \end{cases} \quad (14)$$

By (14), firm 2's optimal choice of q_2^1 is invariant in p_1 for $p_1 \ge (a + c_1)/2$. Henceforth, without loss of generality, we shall consider $p_1 \le (a + c_1)/2$. Note that

$$\left(\widetilde{x}_1^{\mathbb{S}}(p_1), \widetilde{x}_2^{\mathbb{S}}(p_1)\right) = \begin{cases} \left(x_1^{\mathbb{C}}(0), x_2^{\mathbb{C}}(0)\right) \text{ if } c_1 < a/2 \text{ and } p_1 \le (a + c_1)/6 \\ \left(x_1^{\mathbb{S}}(p_1), x_2^{\mathbb{S}}(p_1)\right) \text{ otherwise} \end{cases} \quad (15)$$

If firm 2 chooses $q_2^1 = \widetilde{x}_2^{\mathbb{S}}(p_1)$ (then $q_2^0 = 0$ and firm 0 obtains zero payoff), the market α has the NE $\left(\widetilde{x}_1^{\mathbb{S}}(p_1), \widetilde{x}_2^{\mathbb{S}}(p_1)\right)$ where firm 2 obtains the Stackelberg leader (possibly constrained) profit

$$\widetilde{\ell}(p_1) := \begin{cases} \kappa_2(0) - p_1 x_2^{\mathbb{C}}(0) \text{ if } c_1 < a/2 \text{ and } p_1 \le (a + c_1)/6 \\ \ell(p_1) \text{ otherwise} \end{cases} \quad (16)$$

Firm 1's payoff has two components: (i) the Stackelberg follower's profit and (ii) its supplier revenue $(p_1 - c_1)\widetilde{x}_2^{\mathbb{S}}(p_1)$. Hence its payoff is

$$F(p_1) := \begin{cases} \kappa_1(0) + (p_1 - c_1)x_2^{\mathbb{C}}(0) \text{ if } c_1 < a/2 \text{ and } p_1 \le (a + c_1)/6 \\ f(p_1) + (p_1 - c_1)x_2^{\mathbb{S}}(p_1) \text{ otherwise} \end{cases} \quad (17)$$

We know by Lemma 3 that if $p_1 \le p_0$, it is optimal for firm 2 to choose $q_2^1 = \widetilde{x}_2^{\mathbb{S}}(p_1)$. Now consider $p_1 > p_0$. Then by (13), over $q_2^1 \le x_2^{\mathbb{C}}(p_0)$, it is optimal for firm 2 to choose $q_2^1 = 0$ that yields the Cournot outcome with profits $\kappa_1(p_0)$ and $\kappa_2(p_0)$. Firm 0 supplies $q_2^0 = x_2^{\mathbb{C}}(p_0)$ to firm 2 at price p_0, so it obtains

$$\Psi(p_0) = (p_0 - c_0)x_2^{\mathbb{C}}(p_0). \quad (18)$$

If 2 chooses $q_2^1 \in [x_2^{\mathbb{C}}(p_0), x_2^{\mathbb{C}}(p_0)]$ by paying $p_1 > p_0$ for the capacity q_2^1, it can acquire the leadership (possibly constrained) position in the market α. Firm 2 would prefer to be the

leader as long as p_1 is not too high compared to p_0. Lemma S2 below shows that there is a function $\tau(p_0) \in [p_0, (a + c_1)/2]$ (representing the *leadership premium*) such that 2 prefers to be the Stackelberg leader as long as $p_1 < \tau(p_0)$. The detailed description of this function is given in Appendix A1.

Let $p_0, p_1 \in [c_1, (a + c_1)/2]$ and $G(p_0, p_1)$ be the game that follows the announcement $p \equiv (p_0, p_1)$. If $(x, q) \equiv \langle (x_1, q_1^0, q_1^1), (x_2, q_2^0, q_2^1) \rangle$ is an SPNE of $G(p_0, p_1)$, then (i) q_2^1 is an optimal choice of firm 2 that maximizes $\Pi_2^p(q_2^1)$ given in (13) and (ii) (x_1, x_2) is the (unique) NE of $G(p_0, p_1, q_2^1)$ (Lemma 3). For $i = 0, 1, 2$ let $\Pi_i^p(x, q)$ be the payoff of firm i at (x, q).

Lemma S2 (Stage 2) (Leadership premium) *There is a continuous and strictly increasing function $\tau : [c_1, (a + c_1)/2] \to \mathbf{R}_+$ such that for $p_0, p_1 \in [c_1, (a + c_1)/2]$*

(i) *If (x, q) is an SPNE of $G(p_0, p_1)$, then $q_2^0 q_2^1 = 0$ (firm 2 orders η either exclusively from firm 0 or exclusively from firm 1).*

(ii) *If $p_1 < \tau(p_0)$, the Stackelberg outcome is played in the unique SPNE of $G(p_0, p_1)$ where $x_2 = q_2^1 = \widetilde{x}_2^{\mathbb{S}}(p_1)$, $x_1 = q_1^1 - q_2^1 = \widetilde{x}_1^{\mathbb{S}}(p_1)$ and $q_1^0 = q_2^0 = 0$. The payoffs are $\Pi_0^p(x, q) = 0$, $\Pi_1^p(x, q) = F(p_1)$ and $\Pi_2^p(x, q) = \widetilde{\ell}(p_1)$.*

(iii) *If $p_1 > \tau(p_0)$, the Cournot outcome is played in the unique SPNE of $G(p_0, p_1)$ where $x_2 = q_2^0 = x_2^{\mathbb{C}}(p_0)$, $x_1 = q_1^1 = x_1^{\mathbb{C}}(p_0)$ and $q_1^0 = q_2^1 = 0$. The payoffs are $\Pi_0^p(x, q) = \Psi(p_0)$, $\Pi_1^p(x, q) = \kappa_1(p_0)$ and $\Pi_2^p(x, q) = \kappa_2(p_0)$.*

(iv) *If $p_1 = \tau(p_0)$, $G(p_0, p_1)$ has two SPNE: the Stackelberg outcome is played in one and the Cournot outcome is played in the other.*

Proof See the Appendix. ∎

5.2.3 Stage 1 of Γ and $\widetilde{\Gamma}$

Now we go to the first stage of Γ and $\widetilde{\Gamma}$ where firms 0 and 1 simultaneously announce prices p_0, p_1 ($p_0 \equiv c_0$ for $\widetilde{\Gamma}$) that result in the game $G(p_0, p_1)$. If $p_1 > \tau(p_0)$ in any SPNE of Γ or $\widetilde{\Gamma}$, then the Cournot outcome is played in the induced SPNE of $G(p_0, p_1)$, firm 0 obtains $\Psi(p_0)$ and firm 1 obtains $\kappa_1(p_0)$. If $p_1 < \tau(p_0)$, the Stackelberg outcome is played, 0 obtains zero and 1 obtains $F(p_1)$. Lemma 4 summarizes some properties of the functions $\Psi(p_0)$ and $F(p_1)$.

Lemma 4 *There is $\theta_0(c_0, c_1) \in (c_0, (a + c_1)/2)$ and $\widehat{\theta}(c_1) \in (c_1, \theta_0(c_1, c_0))$ such that*

(i) *$\Psi(p_0)$ is strictly increasing for $p_0 \in [c_1, \theta_0)$, strictly decreasing for $p_0 \in (\theta_0, (a + c_1)/2]$ and its unique maximum is attained at $p_0 = \theta_0$.*

(ii) *$\Psi(c_0) = \Psi((a + c_1)/2) = 0$, $\Psi(p_0) < 0$ for $p_0 \in [c_1, c_0)$ and $\Psi(p_0) > 0$ for $p_0 \in (c_0, (a + c_1)/2)$.*

(iii) *$F(\theta_0) > \kappa_1(\theta_0)$.*

(iv) *$F(p_1)$ is strictly increasing for $p_1 \in [0, (a + c_1)/2]$.*

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(v) *For $p_0 \in [c_1, \theta_0]$, $F(\tau(p_0)) \gtreqqless \kappa_1(p_0) \Leftrightarrow p_0 \gtreqqless \widehat{\theta}$.*

Proof See the Appendix. ∎

Part (v) of Lemma 4 asserts that firm 1 prefers the Stackelberg outcome over the Cournot outcome for relatively large values of p_0. To see the intuition for this, observe that both $\kappa_1(p_0)$ and $F(\tau(p_0))$ are strictly increasing in p_0. While p_0 has a direct effect on $\kappa_1(p_0)$, its effect on $F(\tau(p_0))$ takes place through the function $\tau(p_0)$. The latter causes a stronger effect since $\tau(p_0)$ (the leadership premium) itself increases with p_0. As firm 2 is willing pay a higher premium for larger values of p_0, it leads to higher supplier revenue for firm 1. This in turn provides a better compensation to firm 1 for its follower position in the ensuing Stackelberg game. This is the reason why the Stackelberg outcome is preferred by firm 1 for relatively large values of p_0.

Lemma S1 (Stage 1) (i) *In any SPNE of $\Gamma(c_1, c_0, a)$:* (a) $p_1 > c_1$, (b) $p_1 = \tau(p_0)$ *and* (c) $p_0 \in [c_1, \theta_0]$.

(ii) *In any SPNE of $\widetilde{\Gamma}(c_1, c_0, a)$:* (a) $p_1 > c_1$ *and* (b) $p_1 \geq \tau(c_0)$.

(iii) *In any SPNE of $\Gamma(c_0, c_1, a)$*

(a) *the Cournot outcome is played if and only if $p_1 = \tau(p_0)$, $p_0 \in [c_0, \theta_0]$ and $p_0 \leq \widehat{\theta}$.*

(b) *the Stackelberg outcome is played if and only if $p_1 = \tau(p_0)$, $p_0 \in [c_1, c_0]$ and $p_0 \geq \widehat{\theta}$.*

(iv) *In any SPNE of $\widetilde{\Gamma}(c_0, c_1, a)$:*

(a) *the Cournot outcome is played if and only if $p_1 \geq \tau(c_0)$ and $c_0 \leq \widehat{\theta}$.*

(b) *the Stackelberg outcome is played if and only if $p_1 = \tau(c_0)$ and $c_0 \geq \widehat{\theta}$.*

Proof See the Appendix. ∎

5.2.4 Proofs of Theorems 1 and 2

Proof of Theorem 1 (I)(a) follows from Lemma 1. I(b) and (c) follow from Lemma S2.

(II) Observe from Lemma S1(iii) that if $c_0 < \widehat{\theta}$, there is no SPNE where the Stackelberg outcome is played and a continuum of SPNE (indexed by (p_0, p_1) where $p_0 \in [c_0, \widehat{\theta}]$ and $p_1 = \tau(p_0)$) where the Cournot outcome is played. The outsourcing pattern under the Cournot outcome follows from Lemma S2.

(III) Again from Lemma S1(iii), if $c_0 > \widehat{\theta}$, there is no SPNE where the Cournot outcome is played and a continuum of SPNE (indexed by (p_0, p_1) where $p_0 \in [c_1, c_0]$ and $p_1 = \tau(p_0)$) where the Stackelberg outcome is played. The outsourcing pattern follows from Lemma S2.

(IV) If $c_0 = \widehat{\theta}$, there are two SPNE, one where the Cournot outcome is played and one where the Stackelberg outcome is played, each having $p_0 = c_0 = \widehat{\theta}$ and $p_1 = \tau(c_0)$ (Lemma S1(iii)). The outsourcing pattern again follows from Lemma S2. ∎

Proof of Theorem 2 (I) follows from lemmas 1 and S2.

(II) Observe from Lemma S1(iv) that if $c_0 < \widehat{\theta}$, there is no SPNE where the Stackelberg outcome is played and a continuum of SPNE, indexed by $p_1 \in [\tau(c_0), \infty)$, where the Cournot

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outcome is played. The outsourcing pattern under the Cournot outcome follows from Lemma S2. As firm 2 does not order any input from firm 1 in any of these SPNE, they are equivalent in real terms.

(III) If $c_0 > \widehat{\theta}$, there is a unique SPNE, where $p_1 = \tau(c_0)$ and the Stackelberg outcome is played (Lemma S1(iv)). The outsourcing pattern under the Stackelberg outcome follows from Lemma S2.

(IV) If $c_0 = \widehat{\theta}$, then there is one SPNE where $p_1 = \tau(c_0)$ and the Stackelberg outcome is played and a continuum of SPNE, indexed by $p_1 \in [\tau(c_0), \infty)$ and equivalent in real terms, where the Cournot outcome is played (Lemma S1(iv)). The outsourcing pattern again follows from Lemma S2. ∎

6 Discussion and extensions

6.1 Credible commitment versus public announcement

When firm 2 orders q_2^1 units of the intermediate good η from firm 1, this constitutes a contract between the two parties, whereby 2 is able to *credibly* commit itself to the purchase of q_2^1 while 1 credibly commits to supply q_2^1 to firm 2. Were one party to renege on its purchase or sale, the other party would have recourse to the signed contract to take it to task.

The situation is quite different when firm 1 simply announces that it will produce q_1^1 units for *itself*. If 1 were to renege, would 1 take itself to task? In the absence of an external enforcement agency (like a courthouse), where 1 could go and write a binding contract to produce q_1^1 or else be liable for severe punishment, 1's announcement is simply that: just "cheap talk" and not a credible commitment.

We rule out the possibility of such binding contracts in our model. Thus if we envisage the game where, anxious to be a Stackelberg leader, firm 1 first announces q_1^1 and then firm 2 comes to it to order q_2^1, 1 will always be free to change its mind later regarding q_1^1 upon hearing q_2^1. This fact is common knowledge to all the players. Hence every subgame that follows an announcement by 1, is still $\Gamma(c_0, c_1, a)$. Since we are looking at Subgame Perfect Nash Equilibria, adding the initial announcements will have no effect and the same equilibria will occur in the subgames as before.

Were an external agency in place to enable 1 to make credible commitments,[11] our analysis would no longer hold and it may well be possible for 1 to emerge as a Stackelberg leader, with 2 outsourcing to 1. But our model rules out such a mechanism.

6.2 The secrecy clause

It is crucial to our analysis that the quantity outsourced by any firm $j = 1, 2$ to 0 cannot be observed by the rival firm. This can be justified on the ground that outsourcing contracts in practice do incorporate secrecy (non-disclosure) clauses (see, e.g., Temponi and Lambert, 2001; Ravenhill, 2003; Hoecht and Trott, 2006). In fact, in many cases it is legally binding to have such clauses. Offshore outsourcing contracts are likely to come under the general

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[11]Since we have linear costs and unbounded capacity, there is no autonomous costly action (such as building up excess capacity à la Dixit, 1980) that 1 can undertake to signal its commitment. An external agency is needed for this purpose.

purview of international trade laws that ensure protection of confidential information. For example, Article 39 of the agreement of TRIPS (Trade-Related Aspects of Intellectual Property Rights) of The World Trade Organization states:[12]

> "Natural and legal persons shall have the possibility of preventing information lawfully within their control from being disclosed to, acquired by, or used by others without their consent in a manner contrary to honest commercial practices...so long as such information:
>
> (a) is secret in the sense that it is not...generally known among or readily accessible to persons within the circles that normally deal with the kind of information in question;
>
> (b) has commercial value because it is secret; and
>
> (c) has been subject to reasonable steps under the circumstances, by the person lawfully in control of the information, to keep it secret."

Turning more specifically to outsourcing contracts, there is evidence of widespread use of the secrecy clause. For example, in his study of projects outsourced to Kuwait, Khalfan (2004: 61) states that:

> "Data confidentiality should be viewed as a critical element by the different parties, and therefore should be respected by the vendor throughout the contracting period and after termination. Taking into consideration that a particular vendor may work simultaneously with two competing organizations, extra caution must therefore be exercised to ensure that data confidentiality is not compromised..."

Discussing data protection laws of the European Union in the context of outsourcing, Vagadia (2007: 121) also points out:

> "Each party should recognize that under the agreement it may receive or become appraised of information belonging or relating to the other, including information concerning business and marketing plans...Each party should agree...not to divulge confidential information belonging to the other to any third party..."

The secrecy clause is indeed widely used in practice, which is why we took it be exogenously given in our model. However, it can often be *deduced* to hold endogenously in equilibrium (in appropriately "enlarged" games). Indeed suppose that the quantity q outsourced by 2 to 0 can be made "public" (and hence observable by 1) or else kept "secret" between 2 and 0. We argue that a public contract can never occur (be active) at an SPNE, as long as the game provides sufficient "strategic freedom" to its various players. For suppose it did occur: 1 knew that 2 buys q units of η from 0 at price p_0. Thus 1 is a Stackelberg follower in the final market α, regardless of whom 2 chooses to outsource η to. It would be better for 1 to quote a lower price $p_0 - \varepsilon$ for η. This would be certain to lure 2 to outsource to 1. But $p_0 \geq c_0$, since 0 could not be making losses at the presumed SPNE; hence $p_0 - \varepsilon > c_1$ for small enough ε (recall $c_0 > c_1$). By manoeuvering 2's order to itself, firm 1 thus earns a significant profit on the manufacture of η. It does lose a little on the market for α, because 2 has a lower cost $p_0 - \varepsilon$ of η (compared to the p_0 earlier), but the loss is of the order of ε. Thus 1 has made a profitable unilateral deviation, contradicting that we were at an SPNE.

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[12]Source: http://www.wto.org/english/tratop_e/trips_e/t_agm3_e.htm

Note that our argument relies on the fact that 1 has the strategic freedom to "counter" the public contract. If, furthermore, 0 also has the freedom to reject the public contract and counter it with a secret contract, then—foreseeing the above deviation by firm 1—firm 0 will only opt for secret contracts.

The most simple instance of such an enlarged game is obtained by inserting an initial binary move by 0 at the start of our game Γ. This represents a declaration by 0 as to whether its offer to 2 is by way of a public or a secret contract. The game Γ follows 0's declaration. It is easy to verify that any SPNE of the enlarged game must have 0 choosing "secret", followed by an SPNE of Γ. Of course, more complicated enlarged games can be thought of. For example, after the simultaneous announcement of p_0 and p_1 in our game Γ, suppose firm 2 has the option to choose "Public q" or "Secret q" in the event that it goes to 0, followed by "Accept" or "Reject" by 0. Clearly 1 finds out q only if "Public q" and "Accept" are chosen. On the other hand, if 0 chooses "Reject" we (still having to complete the definition of the enlarged game) could suppose that 2's order of η is automatically directed to 1. This game is more complex to analyze, but our argument above still applies and shows that a public contract will never be played out in any SPNE.

We thus see that the secrecy clause can often emerge endogenously from strategic considerations, even though—for simplicity—we postulated it in our model. It is apparent that the firm placing orders (firm 2 in our model) may demand secrecy in order to protect sensitive information from leaking out to its rivals and destroying its competitive advantage. Our analysis reveals that the firm *taking* the orders (i.e., firm 0) may also—for more subtle strategic reasons—have a vested interest in maintaining the secrecy clause.

6.3 Alternative pricing schemes

The pricing scheme we have considered in our model is unit-based, i.e., an input provider charges a constant price for each unit that it supplies. Although other schemes such as flat fees and profit-sharing arrangements are sometimes used in outsourcing, unit-based pricing is the most prevalent (see, e.g., Barthélemy, 2003; Robinson and Kalakota, 2004 and Vagadia, 2007).

Not surprisingly, many papers (for instance, most of those that we have cited) are based on unit pricing. However, it is important to go beyond this benchmark and investigate alternative pricing schemes and their influence on the pattern of outsourcing. This is an issue that we hope to take up in future work.

6.4 Variations of the model

Our model can be varied in many ways, but the essential theme remains intact: if $\mathbb{O}$'s costs are not much higher than $\mathbb{I}$'s, $\mathbb{J}$ will outsource to $\mathbb{O}$. Here we briefly indicate a few possible variations.

6.4.1 Economies of scale

When there are there are increasing, instead of constant, returns to scale in the manufacture of the intermediate good η, a new strategic consideration arises, though it does not affect the tenor of our results. The primary strategic incentive to outsource to the outside firm 0

can shift from firm 1 to firm 2. For now 2 must worry that if it outsources η to 1, then 1 will develop a cost advantage on account of economies of scale. In other words, 1 will be able to manufacture η for itself at an average cost that is significantly lower than what it charges to 2. This might outweigh any leadership advantage that 2 obtains by going to 1. So, foreseeing a competitor in 1 that is fierce in spite of being a follower, 2 would prefer to outsource to 0 as long as 0's price is not too much above 1's. This, in turn, will happen if 0's costs are not significantly higher than 1's. But then, if 2 is outsourcing to 0, economies of scale can drive 1 to outsource to 0 as well!

These two strategic considerations, the first impelling 1 to push 2 towards 0 and the second impelling 2 to turn away from 1 on its own and to seek out 0, are intermingled in the presence of economies of scale. It is hard to disentangle them and say precisely when one fades out, leaving spotlight on the other. But by eliminating economies of scale altogether, we were able to focus on just the first scenario, wherein the game turns essentially on the informational content of the strategies.

Economies of scale can easily be incorporated in our model. Suppose the average cost $c_i(q)$ of manufacturing q units of η falls (as q rises) for both $i = 0, 1$. For simplicity, suppose $c_i(q)$ falls linearly and that $c_0(q) = \lambda c_1(q)$ for some positive scalar[13] λ. It can then be shown that there exists a threshold $\lambda^* > 1$ such that if $\lambda < \lambda^*$:

(i) firm 2 outsources to firm 0 in any SPNE,

(ii) both firms 1 and 2 outsource to firm 0 in any SPNE when economies of scale are not too small.

This result is established in Chen and Dubey (2009).[14]

6.4.2 Discriminatory pricing

The outsourcing result for increasing returns hinges on the fact that 0 cannot quote *discriminatory* provider prices to 1 and 2. Otherwise firm 0 can benefit by setting different prices in sequence to 1 and 2 (see, e.g., Arya et al, 2008a). However, in our model with constant returns to scale, discriminatory prices are of no avail. This is so because firm 1's cost c_1 is by assumption less than 0's cost c_0. The only reason 1 would contemplate buying from 0, at a price necessarily as high as c_0, is to make public its order and become a Stackelberg leader vis-à-vis 2. But this is ruled out by the secrecy clause. So 1 will always produce the intermediate good by itself at a lower cost c_1 and ignore 0's offer.

6.4.3 Multiple firms of each type

Suppose there are n_1, n_2 replicas of firms 1 and 2. The timing of moves is assumed to be as before, with the understanding that all replicas of a firm move simultaneously wherever that firm had moved in the original game. Restricting attention to type-symmetric SPNE, Theorem 1 again remains intact with a lower threshold in terms of the value of $\widehat{\theta}(c_1)$.

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[13]Thus $c_1(q) = \max\{0, c - bq\}$ and $c_0(q) = \lambda \max\{0, c - bq\}$ for positive scalars b, c, λ.

[14]It is needed here that the economies of scale be not too pronounced, otherwise pure strategy SPNE may fail to exist. More precisely, for the average cost function $c_1(q) = \max\{0, c - bq\}$, it is assumed that $0 < b < c/2a$ to guarantee (i) the existence of pure strategy SPNE and (ii) in equilibrium, the quantity produced entails positive marginal cost.

6.4.4 Only Outside Suppliers

The strategic incentives that we have analyzed can arise in other contexts. Suppose, for instance, that 1 and 2 both need to outsource the supply of the intermediate good η to outsiders $\mathbb{O} = \{O_1, O_2, \ldots\}$. If 2 goes first to $\mathbb{O}$ and 1 knows which O_i has received 2's order, then 1 will have incentive to outsource to some O_j that is distinct from O_i, even if O_j's costs are higher than O_i's, so long as they are not much higher. For if 1 went to O_i, it might have to infer the size of 2's orders and thus be obliged to become a Stackelberg follower (e.g., because O_i has limited capacity and can attend to 1's order only after fully servicing the prior order of 2). Alternatively, even if 1 does not know who 2 has outsourced to, or indeed if 2 has outsourced at all, it may be safer for 1 to spread its order among several firms in $\mathbb{O}$ so that it minimizes the probability of becoming 2's follower. We leave the precise modeling and analysis of such situations for future research.

Appendix

Appendix A1

The function $\tau(p_0)$

It will be useful to begin by defining $\tau_1, \tau_2 : [c_1, (a + c_1)/2] \to \mathbf{R}_+$ as

$$\tau_1(p_0) := 4p_0(a + c_1 - p_0)/3(a + c_1) \text{ and } \tau_2(p_0) := (3 - 2\sqrt{2})(a + c_1)/6 + 2\sqrt{2}p_0/3. \quad (19)$$

Denote $g(p_0) := (2/3)p_0 + (1/3)(a + c_1)/2$. Observe that for $t = 1, 2$, $\tau_t(p_0)$ is strictly increasing, $\tau_t(p_0) \in (p_0, g(p_0))$ for $p_0 \in [c_1, (a+c_1)/2)$ and $\tau_2((a+c_1)/2) = (a+c_1)/2$. Denote

$$\bar{\theta}(c_1) := (\sqrt{2} - 1)(a + c_1)/2\sqrt{2}. \quad (20)$$

Observe that $\bar{\theta}(c_1) \in (0, (a + c_1)/6)$ and (i) $\tau_1(p_0) < \tau_2(p_0) < (a + c_1)/6$ if $p_0 \in [0, \bar{\theta}(c_1))$, (ii) $\tau_1(\bar{\theta}(c_1)) = \tau_2(\bar{\theta}(c_1)) = (a + c_1)/6$ and $(a + c_1)/6 < \tau_1(p_0) < \tau_2(p_0)$ if $p_0 \in (\bar{\theta}(c_1), (a + c_1)/2]$. The function $\tau(p_0)$ is the continuous and strictly increasing function given as

$$\tau(p_0) = \begin{cases} \tau_1(p_0) \text{ if } p_0 < \bar{\theta}(c_1) \\ \tau_2(p_0) \text{ if } p_0 \geq \bar{\theta}(c_1) \end{cases} \quad (21)$$

The functions $\theta_0(c_1, c_0)$ and $\widehat{\theta}(c_1)$

Define

$$\theta_0(c_1, c_0) := c_0/2 + (a + c_1)/4 \in (c_0, (a + c_1)/2) \text{ and} \quad (22)$$

$$\widehat{\theta}_1(c_1) := \frac{4c_1 + a}{5} - \frac{\sqrt{a^2 - 7ac_1 + c_1^2}}{5} \text{ and } \widehat{\theta}_2(c_1) := \frac{a}{14} + \frac{13c_1}{14}. \quad (23)$$

Observe that $\widehat{\theta}_2(c_1) \in (c_1, \theta_0(c_1, c_0))$ for $c_1 < a$ and

$$\widehat{\theta}_2(c_1) \gtreqqless \bar{\theta}(c_1) \Leftrightarrow c_1 \gtreqqless \underline{\rho}a \text{ where } \underline{\rho} \equiv 23/[121 + 84\sqrt{2}] \in (0, 1/2). \quad (24)$$

Also note that for $c_1 < \underline{\rho}a$, $\widehat{\theta}_1(c_1)$ is real and $c_1 < \widehat{\theta}_1(c_1) < \bar{\theta}(c_1) < \theta_0(c_1, c_0)$. Define

$$\widehat{\theta}(c_1) := \begin{cases} \widehat{\theta}_1(c_1) \text{ if } c_1 < \underline{\rho}a \\ \widehat{\theta}_2(c_1) \text{ if } c_1 \geq \underline{\rho}a \end{cases} \quad (25)$$

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Observe that $\widehat{\theta}(c_1)$ is continuous and

$$\widehat{\theta}(c_1) \lesseqgtr \overline{\theta}(c_1) \Leftrightarrow c_1 \lesseqgtr \underline{\rho}a. \tag{26}$$

Appendix A2

Proof of Lemma 1 (i) For $\widetilde{\Gamma}$, we have $p_0 \equiv c_0 > c_1$, so consider Γ and suppose $p_0 < c_1$. Since $c_1 < c_0$, firm 0 makes $(p_1 - c_0) < 0$ dollars per unit of the total outsourced order $q_1^0 + q_2^0$ that it receives. If it could be shown that $q_1^0 + q_2^0 > 0$, there would be an immediate contradiction, because firm 0 can in fact ensure zero payoff by deviating from p_0 to some sufficiently high p_0' (e.g., $p_0' > a$), at which price neither firm will outsource anything to it.

To complete the proof, we now show that $q_1^0 + q_2^0 > 0$.

Let $q_2^0 = 0$ (otherwise we are done). If $x_2 > 0$, we must have $q_2^1 > 0$. Then, since $p_0 < c_1$, 1 will pass on this order to 0, i.e., $q_1^0 > 0$.

If $x_2 = 0$, then, as is easily verified, $x_1 > 0$, i.e., $q_1^0 + q_1^1 > 0$. But the cost of producing $q_1^0 + q_1^1$ is $p_0 q_1^0 + c_1 q_1^1$. Since $p_0 < c_1$, optimality requires that $q_1^1 = 0$, so $q_1^0 > 0$.

(ii) It is clear by (6) that if $p_0 > c_1$, then firm 1 will choose $q_1^0 = 0$. If $p_0 = c_1 < c_0$ and $q_1^0 > 0$, then firm 0 obtains a negative payoff. As it can ensure a zero payoff by quoting a sufficiently high price, we must have $q_1^0 = 0$. ■

To prove Lemma S3, we first prove Lemma A1.

Lemma A1 *The following hold for $G(p_0, p_1, q_2^1)$.*

(i) *The unique best response of firm 1 to $x_2 \geq 0$ is $b^{c_1}(x_2)$. The unique best response of firm 2 to $x_1 \geq 0$ is (a) $b^{p_0}(x_1)$ if $q_2^1 < b^{p_0}(x_1)$, (b) $b^0(x_1)$ if $q_2^1 > b^0(x_1)$ and (c) q_2^1 if $b^{p_0}(x_1) \leq q_2^1 \leq b^0(x_1)$.*

(ii) *If (x_1, x_2) is an NE, then (a) $x_1 = b^{c_1}(x_2)$, (b) $x_2 = b^{p_0}(x_1)$ if $x_2 > q_2^1$, (c) $x_2 = b^0(x_1)$ if $x_2 < q_2^1$ and (d) $b^{p_0}(x_1) \leq x_2 \leq b^0(x_1)$ if $x_2 = q_2^1$.*

(iii) *(a) If $q_2^1 \leq x_2^{\mathbb{C}}(p_0)$, there is no NE where $x_2 < q_2^1$ and (b) if $q_2^1 \geq x_2^{\mathbb{C}}(0)$, there is no NE where $x_2 > q_2^1$.*

Proof (i) The first part is direct by (9). To determine firm 2's best response(s), denote

$$m(x_1) := \min\{b^0(x_1), q_2^1\} \quad \text{and} \quad M(x_1) := \max\{b^{p_0}(x_1), q_2^1\}. \tag{27}$$

By (10), for $x_1 \geq 0$, the unique optimal strategy of firm 2 over $x_2 \in [0, q_2^1]$ is $m(x_1)$ while over $x_2 \in [q_2^1, \infty)$, it is $M(x_1)$. As $b^{p_0}(x_1) \leq b^0(x_1)$ and $x_2 = q_2^1$ is feasible for both $[0, q_2^1]$ and $[q_2^1, \infty)$, (a)-(c) follow by (27).

(ii) Follows by (i).

(iii) (a) If (x_1, x_2) is an NE where $x_2 < q_2^1$, then by (ii)(a) and (c), $x_1 = b^{c_1}(x_2)$ and $x_2 = b^0(x_1)$. The unique solution to this system has $x_1 = x_1^{\mathbb{C}}(0)$ and $x_2 = x_2^{\mathbb{C}}(0) > x_2^{\mathbb{C}}(p_0) \geq q_2^1$, contradicting $x_2 < q_2^1$.

(iii) (b) If (x_1, x_2) is an NE where $x_2 > q_2^1$, then by (ii)(a) and (b), $x_1 = b^{c_1}(x_2)$ and $x_2 = b^{p_0}(x_1)$. The unique solution to this system is $x_1 = x_1^{\mathbb{C}}(p_0)$ and $x_2 = x_2^{\mathbb{C}}(p_0) < x_2^{\mathbb{C}}(0) \leq q_2^1$, contradicting $x_2 > q_2^1$. ■

Proof of Lemma S3 (i)(a) Let $0 \leq q_2^1 < x_2^{\mathbb{C}}(p_0)$. First we show that $(x_1^{\mathbb{C}}(p_0), x_2^{\mathbb{C}}(p_0))$ is an NE. Clearly $x_1^{\mathbb{C}}(p_0)$ is the unique best response of firm 1 to $x_2^{\mathbb{C}}(p_0)$. Since $b^{p_0}\left(x_1^{\mathbb{C}}(p_0)\right) = x_2^{\mathbb{C}}(p_0) > q_2^1$, $x_2^{\mathbb{C}}(p_0)$ is the unique best response of firm 2 to $x_1^{\mathbb{C}}(p_0)$.

To prove the uniqueness, note by Lemma A1 (iii)(a) that if (x_1, x_2) is an NE, we must have $x_2 \geq q_2^1$.

If (x_1, q_2^1) is an NE, then by Lemma A1(ii)(a) and (d), $x_1 = b^{c_1}(q_2^1)$ and $q_2^1 \geq b^{p_0}(x_1) = b^{p_0}(b^{c_1}(q_2^1))$. Since $x_2 \lesseqgtr x_2^{\mathbb{C}}(p_0) \Leftrightarrow x_2 \lesseqgtr b^{p_0}(b^{c_1}(x_2))$, we have $q_2^1 \geq x_2^{\mathbb{C}}(p_0)$, a contradiction.

Hence if (x_1, x_2) is an NE, then $x_2 > q_2^1$ and by A1 (ii)(a)-(b), $x_1 = b^{c_1}(x_2)$ and $x_2 = b^{p_0}(x_1)$. The unique solution of this system has $x_1 = x_1^{\mathbb{C}}(p_0)$ and $x_2 = x_2^{\mathbb{C}}(p_0)$, completing the proof.

(i)(b) Let $x_2^{\mathbb{C}}(p_0) \leq q_2^1 \leq x_2^{\mathbb{C}}(0)$. Since for $c_2 \in \{0, p_0\}$, $x_2 \lesseqgtr x_2^{\mathbb{C}}(c_2) \Leftrightarrow x_2 \lesseqgtr b^{c_2}(b^{c_1}(x_2))$, we have $b^{p_0}(b^{c_1}(q_2^1)) \leq q_2^1 \leq b^0(b^{c_1}(q_2^1))$ and by A1(i) it follows that $(b^{c_1}(q_2^1), q_2^1)$ is an NE. The uniqueness follows from A1(ii)(a)-(d) by noting that for this case there is no NE where $x_2 \neq q_2^1$.

(i) (c) Let $q_2^1 > x_2^{\mathbb{C}}(0)$. First we show that $(x_1^{\mathbb{C}}(0), x_2^{\mathbb{C}}(0))$ is an NE. Clearly $x_1^{\mathbb{C}}(0)$ is the unique best response of firm 1 to $x_2^{\mathbb{C}}(0)$. Since $b^0(x_1^{\mathbb{C}}(0)) = x_2^{\mathbb{C}}(0) < q_2^1$, by (i)(b), $x_2^{\mathbb{C}}(0)$ is the unique best response of firm 2 to $x_1^{\mathbb{C}}(0)$.

To prove the uniqueness, note by A1(iii)(b) that if (x_1, x_2) is an NE, we must have $x_2 \leq q_2^1$.

If (x_1, q_2^1) is an NE, then by A1(ii)(a) and (d), $x_1 = b^{c_1}(q_2^1)$ and $q_2^1 \leq b^0(x_1) = b^0(b^{c_1}(q_2^1))$. Since $x_2 \lesseqgtr x_2^{\mathbb{C}}(0) \Leftrightarrow x_2 \lesseqgtr b^0(b^{c_1}(x_2))$, we have $q_2^1 \leq x_2^{\mathbb{C}}(0)$, a contradiction.

Hence if (x_1, x_2) is an NE, then $x_2 < q_2^1$ and by A1(ii)(a) and (c), $x_1 = b^{c_1}(x_2)$ and $x_2 = b^0(x_1)$. The unique solution of this system has $x_1 = x_1^{\mathbb{C}}(0)$ and $x_1 = x_2^{\mathbb{C}}(0)$, completing the proof.

(ii) If $p_0 \geq (a + c_1)/2 > c_1$, then $q_1^0 = 0$ and in the NE of $G(p_0, p_1, q_2^1)$, $q_2^0 > 0$ only if $q_2^1 \in [0, x_2^{\mathbb{C}}(p_0))$ [part(i)]. Since $x_2^{\mathbb{C}}(p_0) = 0$ for $p_0 \geq (a + c_1)/2$ [by (11)], we have $q_1^0 + q_2^0 = 0$ for $p_0 \geq (a + c_1)/2$, yielding zero payoff for firm 0. This proves (ii). ∎

Proof of Lemma 3 (i) Observe by (13) that firm 2 chooses $q_2^1 \leq x_2^{\mathbb{C}}(0)$ for any $p_1 > 0$. By lemmas 1 and S3, $p_0 \in [c_1, (a + c_1)/2]$. If $p_0 = (a + c_1)/2$, then $x_2^{\mathbb{C}}(p_0) = 0$ and the result is immediate. So let $p_0 < (a + c_1)/2$. Then $x_2^{\mathbb{S}}(p_1) > x_2^{\mathbb{C}}(p_0) > 0$ for $p_1 \leq p_0$. As the unconstrained maximum of $\Pi_2^p(q_2^1)$ over $q_2^1 \in [x_2^{\mathbb{C}}(p_0), x_2^{\mathbb{C}}(0)]$ is attained at $q_2^1 = x_2^{\mathbb{S}}(p_1)$, the constrained maximizer is $q_2^1 = \widetilde{x}_2^{\mathbb{S}}(p_1)$ and $\Pi_2^p(\widetilde{x}_2^{\mathbb{S}}(p_1)) > \Pi_2^p(x_2^{\mathbb{C}}(p_0))$. Noting that for $q_2^1 \leq x_2^{\mathbb{C}}(p_0))$, $\Pi_2^p(q_2^1)$ is either increasing (if $p_1 < p_0$) or constant (if $p_1 = p_0$), it follows that the unique global optimal choice for firm 2 in Stage 2 is $q_2^1 = \widetilde{x}_2^{\mathbb{S}}(p_1)$.

(ii) If $p_1 \leq c_1$, then firm 1 does not obtain any positive profit as a supplier of η. We will show now that if $p_1 \leq c_1$, firm 1's profit in the final good market α is less than $\kappa_1(c_0)$. Then the result will follow from Lemma 2.

By Lemma 1, $p_0 \geq c_1$. If $0 < p_1 \leq c_1 \leq p_0$, then by (i), it is optimal for firm 2 to choose $q_2^1 = \widetilde{x}_2^{\mathbb{S}}(p_1) = \min\{x_2^{\mathbb{S}}(p_1), x_2^{\mathbb{C}}(0)\}$. If $p_1 = 0$, then by (13), it is optimal to choose either $q_2^1 = x_2^{\mathbb{S}}(p_1)$ or any $q_2^1 \geq x_2^{\mathbb{C}}(0)$.

If $q_2^1 = x_2^{\mathbb{S}}(p_1)$, 2 will supply $x_2 = x_2^{\mathbb{S}}(p_1)$ in the market α (Lemma S3) and 1's profit there would be $f(p_1) < \kappa_1(p_1) < \kappa_1(p_0) \leq \kappa_1(c_0)$. If $q_2^1 \geq x_2^{\mathbb{C}}(0)$, then 2 will supply $x_2 = x_2^{\mathbb{C}}(0)$ (Lemma S3) and firm 1's profit from the market α would be $\kappa_1(0) < \kappa_1(c_0)$. ∎

Proof of Lemma S2 First we prove (ii)-(iv). Part (i) follow immediately from (ii)-(iv).

To prove (ii)-(iv), note that in any SPNE of $G(p_0, p_1)$, $q_1^0 = 0$ and $x_1 = q_1^1 + q_2^1$. Denote $A_1(p) = [0, x_2^{\mathbb{C}}(p_0)]$ and $A_2(p) = [x_2^{\mathbb{C}}(p_0), x_2^{\mathbb{C}}(0)]$. Let $A_t^*(p)$ be the set of all maximizers of

$\Pi_2(q_2^1)$ over $q_2^1 \in A_t(p)$ and let $A^*(p)$ be the set of all $\Pi_2(q_2^1)$ over $q_2^1 \geq 0$. We know that for $p_0, p_1 \in [c_1, (a+c_1)/2]$, $A^* \subseteq A_1^* \cup A_2^*$. We prove (ii) by showing that $A^*(p) = \{0\}$ if $p_1 < \tau(p_0)$, $A^*(p) = \{\widetilde{x}_2^{\mathbb{S}}(p_1)\}$ if $p_1 > \tau(p_0)$ and $A^*(p) = \{0, \widetilde{x}_2^{\mathbb{S}}(p_1)\}$ if $p_1 = \tau(p_0)$.

By Lemma 3, $A^*(p) = \{\widetilde{x}_2^{\mathbb{S}}(p_1)\}$ if $p_1 \leq p_0$. So consider $p_1 > p_0$. Then we know that $A_1^* = \{0\}$. Let $g(p_0) := (2/3)p_0 + (1/3)(a+c_1)/2$. If $p_0 = (a+c_1)/2$, then $x_2^{\mathbb{C}}(p_0) = 0$ and $A = A_2$, so that $A^* = A_2^* = \{\widetilde{x}_2^{\mathbb{S}}(p_1)\}$. Since $g(p_0) = \tau(p_0) = (a+c_1)/2$ for $p_0 = (a+c_1)/2$, the proof for this case is complete. Now let $p_0 < (a+c_1)/2$, so that $g(p_0) > p_0$. For $p_1 \geq g(p_0)$, we have $\widetilde{x}_2^{\mathbb{S}}(p_1) \leq x_2^{\mathbb{C}}(p_0)$. Hence $A_2^* = \{x_2^{\mathbb{C}}(p_0)\}$. Thus, $x_2^{\mathbb{C}}(p_0) \in A_2^* \cap A_1$ but $x_2^{\mathbb{C}}(p_0) \notin A_1^* = \{0\}$. Hence $A^* = A_1^* = \{0\}$.

Finally let $p_1 \in (p_0, g(p_0))$. Then $A_1^* = \{0\}$ and $A_2^* = \{\widetilde{x}_2^{\mathbb{S}}(p_1)\}$. Hence $A^* \subseteq \{0, \widetilde{x}_2^{\mathbb{S}}(p_1)\}$. Note that $\Pi_2^p(0) = \kappa_2(p_0) = (a+c_1-2p_0)^2/9$ and $\Pi_2^p(\widetilde{x}_2^{\mathbb{S}}(p_1)) = \widetilde{\ell}(p_1)$. By (14), (15) and (16) we have

$$\widetilde{\ell}(p_1) = \begin{cases} \widehat{\ell}(p_1) = (a+c_1)^2/9 - p_1(a+c_1)/3 \text{ if } c_1 < a/2 \text{ and } p_1 \leq (a+c_1)/6, \\ \ell(p_1) = (a+c_1-2p_1)^2/8 \text{ otherwise.} \end{cases} \qquad (28)$$

Comparing $\kappa_2(p_0) = (a+c_1-2p_0)^2/9$ with $\widehat{\ell}(p_1)$ and $\ell(p_1)$ we have the following where τ_1, τ_2 are given in (19).

$$\widehat{\ell}(p_1) \gtreqqless \kappa_2(p_0) \Leftrightarrow p_1 \lesseqqgtr \tau_1(p_0) \text{ and } \ell(p_1) \gtreqqless \kappa_2(p_0) \Leftrightarrow p_1 \lesseqqgtr \tau_2(p_0) \qquad (29)$$

There are following possible cases, where $\bar{\theta}(c_1)$ is given by (20).

Case 1(a) If $c_1 < a/2$ and $p_0 \geq (a+c_1)/6 > \bar{\theta}(c_1)$, then by (21), $\tau(p_0) = \tau_2(p_0)$. Since $p_1 > p_0$, under this case we have $p_1 > (a+c_1)/6$.

1(b) If $c_1 \geq a/2$, then by (20), $\bar{\theta}(c_1) < c_1 \leq p_0$ and again $\tau(p_0) = \tau_2(p_0)$.

Observe by (28) that if either (a) or (b) holds, then $\widetilde{\ell}(p_1) = \ell(p_1)$. Hence by (29), $\widetilde{\ell}(p_1) \gtreqqless \kappa_2(p_0) \Leftrightarrow p_1 \lesseqqgtr \tau_2(p_0) = \tau(p_0)$ proving the result for Case 1.

Case 2 $c_1 < a/2$ and $p_0 < (a+c_1)/6$:

2(a) If $p_0 \leq \bar{\theta}(c_1)$, then $\tau(p_0) = \tau_1(p_0) \leq (a+c_1)/6$ and $\tau_2(p_0) \leq (a+c_1)/6$ [by (21)].

(i) If $p_1 \in (p_0, (a+c_1)/6]$, then by (28), $\widetilde{\ell}(p_1) = \widehat{\ell}(p_1)$. Hence by (29), $\widetilde{\ell}(p_1) \gtreqqless \kappa_2(p_0) \Leftrightarrow p_1 \lesseqqgtr \tau_1(p_0) = \tau(p_0)$.

(ii) If $p_1 \in ((a+c_1)/6, g(p_0)]$, then by (28), $\widetilde{\ell}(p_1) = \ell(p_1)$. Hence by (29), $\widetilde{\ell}(p_1) < \kappa_2(p_0)$ for $p_1 > (a+c_1)/6 \geq \tau_2(p_0)$.

The result for Case 2(a) follows by (i) and (ii) above.

2(b) If $p_0 \in (\bar{\theta}(c_1), (a+c_1)/6)$, then $\tau(p_0) = \tau_2(p_0) > (a+c_1)/6$ and $\tau_1(p_0) > (a+c_1)/6$ [by (21)].

(i) If $p_1 \in (p_0, (a+c_1)/6]$, then by (28), $\widetilde{\ell}(p_1) = \widehat{\ell}(p_1)$. Hence by (29), $\widetilde{\ell}(p_1) > \kappa_2(p_0)$ for $p_1 \leq (a+c_1)/6 < \tau_1(p_0)$.

(ii) If $p_1 \in ((a+c_1)/6, g(p_0))$, then by (28), $\widetilde{\ell}(p_1) = \ell(p_1)$. Hence by (29), $\widetilde{\ell}(p_1) \gtreqqless \kappa_2(p_0) \Leftrightarrow p_1 \lesseqqgtr \tau_2(p_0) = \tau(p_0)$.

The result for Case 2(b) follows by (i) and (ii) above. ∎

Proof of Lemma 4 Parts (i)-(ii) follow from (18) by noting that $x_2^{\mathbb{C}}(p_0) = (a+c_1-2p_0)/3$ for $p_0 \in [c_1, (a+c_1)/2]$.

(iii) Noting that $\theta_0 > (a + c_1)/6$, by (17), we have $F(p_1) = f(p_1) + (p_1 - c_1)x_1^{\mathbb{S}}(p_1)$. As $c_1 < c_0 < \theta_0 < (a+c_1)/2$, by (14), $F(\theta_0) = (3a+2c_0-5c_1)^2/64+(a+2c_0-3c_1)(a+c_1-2c_0)/16$. As $\kappa_1(\theta_0) = (5a + 2c_0 - 7c_1)^2/144$, we have $F(\theta_0) - \kappa_1(\theta_0) = (17a + 62c_0 - 79c_1)(a + c_1 - 2c_0)/576 > 0$ since $a > c_0 > c_1$. This proves (iii).

(iv) Follows by standard computations by using (14) and (17).

(v) First let $p_0 \geq \bar{\theta}(c_1)$. Then by (21), $\tau(p_0) = \tau_2(p_0) \geq (a + c_1)/6$. Hence by (14) and (17), $F(\tau(p_0)) = (a - 3c_1 + 2\tau(p_0))^2/16 + (\tau(p_0) - c_1)[(a + c_1)/2 - \tau(p_0)]$. Comparing it with $\kappa_1(p_0) = (a - 2c_1 + p_0)^2/9$, we have the following where $\hat{\theta}_2(c_1)$ is given by (23).

$$\text{For } p_0 \geq \bar{\theta}(c_1), \;\; \kappa_1(p_0) \gtreqqless F(\tau(p_0)) \Leftrightarrow p_0 \lesseqqgtr \hat{\theta}_2(c_1) \tag{30}$$

Next observe that for $p_0 < \bar{\theta}(c_1)$,

$$\bar{\theta}(c_1) \gtreqqless c_1 \Leftrightarrow c_1 \lesseqqgtr \bar{\rho}a \text{ where } \bar{\rho} \equiv 1/(3 + 2\sqrt{2}) \in (0, 1/2) \text{ and } \bar{\rho} > \underline{\rho}. \tag{31}$$

Case 1 $c_1 \geq \bar{\rho}a$: Then by (31), $[c_1, (a + c_1)/2) \subseteq [\bar{\theta}(c_1), (a + c_1)/2)$. As $\underline{\rho} < \bar{\rho}$, for this case $\hat{\theta}(c_1) = \hat{\theta}_2(c_1)$ [by (25)] and the result follows by (30).

Case 2 $c_1 < \bar{\rho}a$: Then by (31), $[c_1, (a + c_1)/2) = [c_1, \bar{\theta}(c_1)) \cup [\bar{\theta}(c_1), (a + c_1)/2)$.

If $p_0 \in [c_1, \bar{\theta}(c_1)]$, then by (21), $\tau(p_0) = \tau_1(p_0) < (a + c_1)/6$. Since $c_1 < \bar{\rho}a < a/2$, by (12) and (17), $F(\tau(p_0)) = (a - 2c_1)^2/9 + (\tau(p_0) - c_1)(a + c_1)/3$. Comparing it with $\kappa_1(p_0) = (a - 2c_1 + p_0)^2/9$, we have

$$\kappa_1(p_0) \gtreqqless F(\tau(p_0)) \Leftrightarrow w(p_0) \gtreqqless 0 \text{ where } w(p_0) := 5p_0^2 - 2(a + 4c_1)p_0 + 3c_1(a + c_1). \tag{32}$$

Noting that (i) $w(p_0)$ is strictly decreasing for $p_0 \in [c_1, \bar{\theta}(c_1)]$, (ii) $w(c_1) > 0$ and (iii) $w(\bar{\theta}(c_1)) \gtreqqless 0 \Leftrightarrow c_1 \gtreqqless \underline{\rho}a$, we have the following two subcases.

Subcase 2(a) $c_1 \in [\underline{\rho}a, \bar{\rho}a)$: Then for all $p_0 \in [c_1, \bar{\theta}(c_1))$, $w(p_0) > 0$ and hence by (32), $\kappa_1(p_0) > F(\tau(p_0))$. Since for this case $\hat{\theta}(c_1) = \hat{\theta}_2(c_1) \geq \bar{\theta}(c_1)$ [(23) and (26)], the result follows by (30).

Subcase 2(b) $c_1 < \underline{\rho}a$: Then $\hat{\theta}_2(c_1) < \bar{\theta}(c_1)$ [by (24)]. Hence by (30), $\kappa_1(p_0) < F(\tau(p_0))$ for $p_0 \in [\bar{\theta}(c_1), (a + c_1)/2)$. For $p_0 \in [c_1, \bar{\theta}(c_1))$, $w(c_1) > 0 > w(\bar{\theta}(c_1))$ and $\exists \hat{\theta}_1(c_1) \in (c_1, \bar{\theta}(c_1))$ [given by (23)] such that $\kappa_1(p_0) \gtreqqless F(\tau(p_0)) \Leftrightarrow p_0 \lesseqqgtr \hat{\theta}_1(c_1)$. Noting that $\hat{\theta}(c_1) = \hat{\theta}_1(c_1)$ for this case [by (25)], the proof is complete. ∎

Proof of Lemma S1 (i)(a) Follows from Lemma 3.

(i)(b) If $p_1 < \tau(p_0)$, then firm 1 obtains $F(p_1)$. As F is monotonic (Lemma 4), 1 can deviate to $p_1' \in (p_1, \tau(p_0))$ to obtain $F(p_1') > F(p_1)$. So we must have $p_1 \geq \tau(p_0)$.

If $p_1 > \tau(p_0)$, firm 0 obtains $\Psi(p_0)$. If p_0' is marginally higher or lower than p_0, we will have $p_1 > \tau(p_0')$ and 0 would obtain $\Psi(p_0')$ by deviating to p_0'. As Ψ is strictly increasing for $p_0 \in [c_1, \theta_0)$ and strictly decreasing for $p_0 \in [c_1, \theta_0)$ (Lemma 4), there are gainful deviations for 0 if $p_0 \neq \theta_0$.

Now let $p_0 = \theta_0$ and $p_1 > \tau(\theta_0)$. Then firm 1 obtains $\kappa_1(\theta_0)$. By deviating to $p_1' = \theta_0 < \tau(\theta_0)$, firm 1 would obtain $F(\theta_0) > \kappa_1(\theta_0)$ (Lemma 4). This completes the proof of (b).

(i)(c) If the claim is false, then $p_0 \in (\theta_0, (a + c_1)/2]$. Since $p_1 = \tau(p_0)$ [by (i)(b)], firm 0 obtains either 0 (the Cournot outcome) or $\Psi(p_0)$ (the Stackelberg outcome). Let 0 deviate

to $p_0' = \theta_0 < p_0$ so that $\tau(\theta_0) < \tau(p_0) = p_1$. Then 0 would obtain $\Psi(\theta_0)$ which is positive and more than $\Psi(p_0)$ (Lemma 4), proving the result.

(ii) Part (a) follows by (i)(a) and part (b) follows from the first paragraph of the proof of (i)(b) by taking $p_0 \equiv c_0$.

(iii)(a) The "if" part: Let $p_1 = \tau(p_0)$, $p_0 \in [c_0, \theta_0]$ and $p_0 \leq \widehat{\theta}$. Then the Cournot outcome is played in an SPNE of $G(p_0, p_1)$, where 0 obtains $\Psi(p_0) \geq 0$ (since $p_0 \geq c_0$) and 1 obtains $\kappa_1(p_0)$. We prove the result by showing that neither 0 nor 1 has a gainful unilateral deviation.

Clearly a deviation to $p_0' < c_0$ is not gainful for firm 0. If it deviates to $p_0' > p_0$, then $\tau(p_0') > \tau(p_0) = p_1$ and it would obtain $0 \leq \Psi(p_0)$. If it deviates to $p_0' \in [c_0, p_0)$, then $\tau(p_0') < \tau(p_0) = p_1$ and it would obtain $\Psi(p_0') < \Psi(p_0)$ (by Lemma 4, since $p_0 \leq \widehat{\theta} < \theta_0$).

Now consider firm 1. If it deviates to $p_1' > p_1 = \tau(p_0)$, 1 would still obtain $\kappa_1(p_0)$. If it deviates to $p_1' < p_1 = \tau(p_0)$, 1 would obtain $F(p_1') < F(\tau(p_0))$ (by the monotonicity of F). Since $F(\tau(p_0)) \leq \kappa_1(p_0)$ for $p_0 \leq \widehat{\theta}$ (Lemma 4), we have $F(p_1') < \kappa_1(p_0)$. This completes the proof of the "if" part.

The "only if" part: By (i), $p_1 = \tau(p_0)$ and $p_0 \in [c_1, \theta_0]$ in any SPNE. Under the Cournot outcome, firm 0 obtains $\Psi(p_0)$ and firm 1 obtains $\kappa_1(p_0)$. If $p_0 < c_0$, then $\Psi(p_0) < 0$. As 0 can deviate to $p_0' = (a + c_1)/2$ to obtain zero payoff, if the Cournot outcome is played in an SPNE of Γ, we must have $p_0 \geq c_0$.

By deviating to $p_1' > p_1 = \tau(p_0)$, firm 1 obtains $F(p_1')$ which can be made arbitrarily close to $F(\tau(p_0))$ by choosing p_1' close to p_1. To ensure that firm 1's deviation is not gainful *for any* $p_1' \in (0, p_1)$, we need $\kappa_1(p_0) \geq F(\tau(p_0))$, so by Lemma 4 we must have $p_0 \leq \widehat{\theta}$.

(iii)(b) The "if" part: Let $p_1 = \tau(p_0)$, $p_0 \in [c_1, c_0]$ and $p_0 \geq \widehat{\theta}$. Then the Stackelberg outcome is played in an SPNE of the game $G(p_0, p_1)$, where 0 obtains zero payoff and 1 obtains $F(p_1)$. We prove the result by showing that neither 0 nor 1 has a gainful unilateral deviation.

If 0 deviates to $p_0' > p_0$, then $p_1 = \tau(p_0) < \tau(p_0')$ and it would still obtain zero payoff. If it deviates to $p_0' < p_0 \leq c_0$, then it would obtain at most zero and such a deviation is also not gainful.

Now consider firm 1. If it deviates to $p_1' > p_1 = \tau(p_0)$, it would obtain $\kappa_1(p_0) \leq F(p_1)$ (by Lemma 4, since $p_0 \geq \widehat{\theta}$). If firm 1 deviates to $p_1' \in [0, c_1)$, it would obtain less than $\kappa_1(c_1)$ (see the proof of Lemma 3(ii)) which is at most $\kappa_1(p_0) \leq F(p_1)$. Finally if it deviates to $p_1' \in [c_1, p_1)$, it would obtain $F(p_1')$ and by the monotonicity of F, such a deviation is also not gainful.

The "only if" part: By (i), $p_1 = \tau(p_0)$ and $p_0 \in [c_1, \theta_0]$ in any SPNE. Under the Stackelberg outcome, firm 0 obtains zero and firm 1 obtains $F(p_1) = F(\tau(p_0))$. If $p_0 > c_0$, let firm 0 deviate to $p_0' \in (c_0, p_0)$. Then $p_1 = \tau(p_0) > \tau(p_0')$ and it would obtain $\Psi(p_0') > 0$, so we must have $p_0 \leq c_0$. If firm 1 deviates to $p_1' > p_1 = \tau(p_0)$, it would obtain $\kappa_1(p_0)$. To ensure that this deviation is not gainful, we need $F(\tau(p_0)) \geq \kappa_1(p_0)$, so by Lemma 4 we must have $p_0 \geq \widehat{\theta}$.

(iv) As firm 0 does not play any strategic role in $\widetilde{\Gamma}$, we only check for unilateral deviations of firm 1. Then the results follow from part (iii) by taking $p_0 \equiv c_0$. ∎

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