Threshold Effect and Financial Intermediation in Economic Development

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28. April 2009

Online at http://mpra.ub.uni-muenchen.de/14905/
MPRA Paper No. 14905, posted 29. April 2009 07:24 UTC
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ABSTRACT

This paper analyzes the importance of financial intermediation on economic growth. Using the Neoclassical growth framework, we raise a new issue where our model has multiple stationary states with threshold effect. We further confirm that financial intermediation is better than self-financing system in order to ensure the existence and uniqueness of long-run steady state equilibrium of capital stock, as well as to decrease threshold level. The presence of threshold effect is an important finding in studying the finance-growth nexus, since it prevents the economy to raise sufficient initial capital.

Keywords: Threshold Effect, Financial Intermediation, Economic Growth, Developing Countries

JEL Classification: C61, C62, O16

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1 This paper has been presented in 25th GDR Symposium on Money, Banking and Finance at the University of Luxembourg, 20-22nd June 2008.
1. Introduction

Since the last two decades, many literatures analyze the nexus between financial development and economic growth, but the findings are still subject to relevant debate until nowadays\(^2\). In developing countries study, particularly, financial development is associated with banking sector development, since financial market is underdeveloped. However, the more recent literature suggests that financial market should be also taken into account to spur economic growth, even in developing countries. Using a very large cross-country sample incorporating both developed and developing countries, Levine and Servos (1998) show that stock market liquidity leads to faster rate of growth, productivity improvement, and capital accumulation\(^3\). This result is also theoretically supported by Levine (1991) and Bencivenga et al (1995), where stock market liquidity also facilitates long-term investment, since investors can easily sell their stake in the project if they need liquidity before their project matures. Enhanced liquidity and long-term investment, therefore, increase higher-return projects that boost productivity growth.

Meanwhile, it is also well accepted that financial market suffers from asymmetric information problems and thus, financial liberalization fostering stock market liquidity or banking sector development is often blamed for macroeconomic downturn, as well as banking vulnerability and crisis (Bihde, 1993; Detagriache et al, 1999). This is because stock market liquidity reduces shareholder’s incentive to undertake the costly task of monitoring managers. In turn, weaker corporate governance relating to unchecked asymmetric information impedes effective resource allocation and slows productivity growth. Thus, the adverse effect of market-based financial system appears. This is why,

\(^2\)In empirical study see King and Levine (1993a, 1993b), Levine (1998); Rajan and Zingales (1998) for the country level study, and Fisman and Love (2002) at the industry level; or recently Demirgüç-Kunt and Maksimovic (2002) at the firm level. In theoretical study, see Bencivenga and Smith (1991), or recently Hung and Cothren (2002). Levine (2005) provide a comprehensive literature review.

\(^3\)Stock market liquidity refers to the less expensive cost of equities trading.
according to Diamond (1984), the presence of bank as financial intermediation is necessary, since banks have technology to gain information from investors which enhance investor’s rational decision based on their consumption profile.

Extending the previous literatures on the importance of financial intermediation, Bencivenga and Smith (1991) establish a general equilibrium model which shows that financial intermediation is better than self-financed system (financial market), in order to spur economic growth. In this literature, there are basic lists of bank activities such as deposits funded loans, holding liquid reserves against predictable withdrawal demands, issuing liabilities that are more liquid than their primary asset and reducing the need of self-financed investment. In formalizing their model, Bencivenga and Smith (1991) consider that there are two types of agent (entrepreneur and non-entrepreneur) who can invest in either liquid or illiquid assets. The main result of this model is that financial intermediation promotes the development of productive long-term investment rather than short-term ventures. Interestingly, the optimal amount of long-term investment is negatively related with the income of long-term investment itself and the fraction of entrepreneurs, but positively related with the income of short-term ventures and the fraction of non-entrepreneurs. Hence, despite the income of long-term investment is higher than the income of short-term ventures, it does not provide enough incentive to the agents to be entrepreneur. Thus, entrepreneurship is not always growth-enhancing factor unless the opportunity cost of being entrepreneur exceeds the certain value of constraint.

The aim of this paper is therefore to reevaluate the finance-growth nexus developed by Bencivenga and Smith (1991). In our model, we use the Neo-classical growth without externalities in an overlapping generation model with three periods.

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4 Liquid assets are short-term unproductive investments, but illiquid assets are long-term productive investments.
instead of drawing heavily Bencivenga and Smith’s (1991) model\textsuperscript{5}. Since our motivation is to determine the most appropriate system in developing countries, we consider that externalities changes due to human capital and technological innovation may be less important, so that they might not much play pivotal role in boosting economic growth in developing countries. Meanwhile, using the Neo-classical growth framework allows us to obtain more realistic growth rate, notably in developing countries, where the growth rate in consecutive years lies between zero and one, which depends on the degree of capital stock accumulation.

In our model, there are also two types of agents and one consumption goods. The first type of agent is called as non-entrepreneur who lives until the second period, while the second one is called as entrepreneur who lives until the third period (the period of production). Further stylized fact in our model is that we distinguish the behaviour vis-à-vis of risk between non-entrepreneur and entrepreneur. More precisely, the utility function of non-entrepreneur follows the constant relative risk aversion form (Bencivenga and Smith, 1991) and the utility function of entrepreneur follows linear form which is also used by Azariadis and Smith (1998)\textsuperscript{6}. Using these features, we provide some innovative findings.

\textsuperscript{5} Externalities changes due to human capital and technological innovation may be less important in developing countries, so that they might not much play pivotal role in boosting economic growth. Meanwhile, using the Neo-classical growth allows us to obtain more realistic growth rate, notably in developing countries, where the growth rate lies between zero and one.

\textsuperscript{6} The reason why we use this hypothesis is that entrepreneur’s behavior should be more risky than non-entrepreneur’s behavior. See Baumol (1990) who analyzes the riskiness of entrepreneurship activity which may be unproductive or even destructive. This fact should not be neglected by financial sectors whose role is to provide financing for entrepreneurship activity. Moreover, the construction of risk-neutral entrepreneurs following Azariadis and Smith (1998) allows us to consider private information in the side of entrepreneurs that may be a source of risk-shifting trigger from entrepreneurs to financial sectors, as exemplified by Stiglitz and Weiss (1981). However, we do not incorporate how asymmetric information problems affect economic growth.
First innovation, we find that entrepreneurship is always growth-enhancing factor, since the optimal amount of long-term investment is positively related with the fraction of entrepreneurs, the income of long-term investment and short-term ventures, as well as the agent’s savings (wage) rate. Despite the income of short-term ventures is positively related with the optimal amount of long-term investment, it does not necessarily mean that short-term ventures become a pivotal factor to increase long-term investment. This is because the income of short-term ventures is always lower than the income of long-term investments. Thus, entrepreneurship is always preferable to non-entrepreneurship. Moreover, agent’s savings variable does not appear in the optimal amount of long-term investment à la Bencivenga and Smith (1991), where it indicates that financial intermediation always has capacity to increase productivity without necessarily needs proportional agent’s savings as input. Thus, regarding to the recent emerging literatures on bank efficiency, Bencivenga and Smith’s (1991) model does not accommodate the potential agency problems within banking institution, which in turn may increase bank inefficiency and impedes economic growth. Conversely, our model indirectly builds a link between bank efficiency and economic growth, since higher agent’s savings are associated with an increase in the optimal amount of long-term investment. If bank efficiency is too low, then agent’s savings cannot directly increase productive long-term investment due to the problems of the choice of investment between bank shareholders and managers. Thus, our model implicitly assumes that financial intermediation is efficient. Second innovation, our model is characterized by the existence of multiple

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7 See amongst of them, Hasan et al (2007) who find that efficiency in banking boosts economic growth in European economic agglomeration regions, as well as Koetter and Wedow (2006) who analyze the importance of bank’s efficiency for economic growth in Germany.

8 Berger and Di Patti (2006) test the presence of agency problems in banking using the profit function efficiency approach. The profit function efficiency may measure how bank maximize their inputs to generate outputs.
stationary states with threshold effect which impedes the economy to raise initial capital. In this case, the presence of financial intermediation may decrease threshold point and ensure the existence of higher long-run capital stock accumulation. While there are some empirical literatures finding that the presence of threshold effect may adversely affect economic growth, at our best knowledge, there are no much attempts to build theoretical foundation on this issue. And our paper fulfills this gap.

The rest of this paper is then organized as follows. Section 2 describes the model set-up. Section 3 models the self-financed system through financial market. Section 4 models the bank-based financial system. Section 5 builds the study of capital stock dynamic and threshold effect. Section 6 concludes.

2. The Set-up

The model we use is one of overlapping generations with three periods. There are young generation, middle-age generation, and old generation. Each agent may live for two or three periods. Each generation is defined by a continuum of agents. The size of population in the period \( t \) is denoted by \( N_t = N \). Let \( t \) be the time index. At \( t = 0 \) an old generation is endowed with an initial per firm capital stock of \( k_0 \) units, as well as at \( t = 1 \) a middle-age generation is also endowed with an initial per firm capital stock of \( k_1 \) units. Each young agent is endowed with one unit of labour in the first period, where it is supplied inelastically and there is unique consumption good.

In this model, all agents of a generation are identical at the first period of life. At the beginning of the second period of life, there are two-period-lived agents and three-period-lived agents with probability \( (1 - \pi) \) and \( \pi \), respectively. We call that three-

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period-lived agents as entrepreneur and two-period-lived agents as non-entrepreneur. Thus, there are \((1 - \pi)N\) agents who will be non-entrepreneur at the second period of life and \(\pi N\) agents who will be entrepreneur at the third period of life. All young agents save entirely their labour income in the first period, so the consumption of agents is zero. Meanwhile, if agents are non-entrepreneur, they consume their savings and return in the second period \(c_{2t}\). But, if agents are entrepreneur, they consume the profit of production in the third period \(c_{3t}\). Thus, the liquidity need of agents will be different if they become non-entrepreneur or entrepreneur. The non-entrepreneur have higher liquidity need because they live only for two periods. Meanwhile, the young agents have incentive to be entrepreneur because the profit of long-term investment is relatively higher than the return of non-entrepreneur’s saving. Therefore, we assume that entrepreneurs are risk-neutral. Finally, whatever the type of agents, we can define the agent’s preferences by the following expected utility function.

\[
U(c_{1t}, c_{2t}) = \frac{(1 - \pi)}{-\gamma} (c_{1t})^{-\gamma} + \pi \phi c_{2t}
\] (1),

where \(c_{it}\) is the period \(i\) consumption of an agent who is born at \(t\). The constant relative risk aversion is denoted by \(\gamma > 0\). And, \(\phi\) is individual specific random variable realized at the beginning of period 2. Thus, the value of \(\phi\) is equal to 0 with probability \(1 - \pi\), or 1 with probability \(\pi\).

In order to complete this model, we characterize the production function and the entrepreneur’s behaviour. The entrepreneur’s production \(Y_t\) is realized by physical capital \(k_t\) and units of labour \(L_t\). For the sake of simplification, we use the Cobb-Douglas production function as follows

\[
y_t = Ak_t^{\beta} L_t^{1-\beta}
\] (2)
where $\theta \in [0,1]$ is the part of production that uses $k_t$ and $A$ is an arbitrary coefficient. For simplification, we assume that capital depreciates completely at the end of period. Furthermore, there is no endowment of capital at period $t > 0$ except for the initial old generation and the initial middle-age generation. In order to complete the entrepreneur program, the profit function must be established. The entrepreneur’s profit ($\Pi_t$) is the difference between the production and the cost of quantity units of labour defined as follows

$$\Pi_t(k_t, L_t) = Ak_t^\theta L_t^{1-\theta} - w_t L_t$$

(3)

At the equilibrium of labour market, labour demand ($L_t$) is equal to labour supply ($N_t$) which is obtained by maximizing the entrepreneur’s profit subject to $L_t$. Thus, we have

$$w_t = A(1-\theta)k_t^\theta \pi^\theta$$

and the maximized profit function at each period $t$ as much as

$$\Pi_t = A\theta \psi k_t^\theta ,$$

with $L_t^{1-\theta} = \pi^{\theta-1} = \psi$.

3. Self-Financed System

This system also refers to an economy without the presence of bank as financial intermediation or we may call financial market. In the first period, both agents divide their savings between two financial instruments (liquid and illiquid assets). Liquid assets are considered as inventory of consumption goods. One unit invested in liquid asset at $t$ will yield $n > 0$ units of consumption goods at both $t+1$ and $t+2$. In other hand, one unit invested in illiquid asset will yield $R$ units of capital goods at $t+2$. And, if illiquid assets are liquidated at $t+1$, it means that agents sell out this asset for the ”scrap value” of $x$ units of consumption goods, with $0 < x < n$.

In order to establish budget constraint, let $z_t^*$ and $q_t^*$ be the proportion of liquid asset and illiquid asset saved by agents at $t$, respectively. Hence, we have
Furthermore, the saving at \( t \) is denoted as \( S_t \), where \( S_t = \frac{w_t}{z_t^*} \). This saving may be divided to \( z_t^* s_t \) units of liquid asset and \( q_t^* s_t \) units of illiquid asset. And let \( i_L, i_L, i_s \) be the interest rate of liquid asset, illiquid asset, and sold-out illiquid asset from "scrap" value, respectively. Thus, the saving at \( t \) is denoted as \( S_t \), where \( S_t = \frac{w_t}{z_t^*} \). This saving may be divided to \( (1 - \pi) z_t^* s_t \) units of liquid asset and \( (1 - \pi) q_t^* s_t \) units of illiquid asset. Let \( \omega_t \) be the income of non-entrepreneur after one period, then 

\[
\omega_t = (1 + i_L) z_t^* s_t + (1 + i_s) q_t^* s_t \quad \text{or} \quad \omega_t = (n z_t^* + x q_t^*) s_t, \quad \text{where} \quad n = 1 + i_L \quad \text{and} \quad x = (1 + i_s).
\]

Since \( s_t = w_t \) then

\[
\omega_t = (n z_t^* + x q_t^*) w_t
\]  

If the agents are entrepreneur, then there is no consumption at period \( t \) and \( t + 1 \). Thus, at the beginning of the third period, the entrepreneurs sell their illiquid assets and re-invest it again into the financing of physical capital. Namely, they use their fund for production in the third period. Let \( \theta_2 \) be the income received by entrepreneurs after two periods, then 

\[
\omega_2 = (1 + i_L) z_t^* s_t + (1 + i_s) q_t^* s_t \quad \text{or} \quad \omega_2 = (n z_t^* + x q_t^*) s_t, \quad \text{where} \quad R = 1 + i_L, \quad \text{we have}
\]

\[
\omega_2 = n z_t^* s_t + R q_t^* s_t, \quad \text{where} \quad R q_t^* w_t = k_{t,2}
\]

and \( 0 < x < n < R \)  

Using budget constraints in the equation (4), (5) and (6.a), we now define the agent’s program when investment is self-financed in the following equation

\[
U(q_t^*) = \left( -\frac{(1 - \pi)}{\gamma} (x q_t^* w_t + n (1 - q_t^*) w_t) \right)^{-\gamma} + \pi \left( A \theta \psi (R q_t^* w_t)^{\theta} + (1 - q_t^*) n w_t \right)
\]  

Hence, an agent chooses \( q_t^* \) in order to maximize (7). From the first order condition, we obtain the optimal proportion of illiquid asset \( (\bar{q}_t^*) \) as follows.

\[
\bar{q}_t^* = \bar{q}_t^* (w_t) = \frac{n}{(n - x)} - \frac{(B)^{-\gamma - 1}}{w_t(n - x)}
\]
where \( B = \frac{z}{\pi - 1} \left( \frac{nw_t - AR^t w^t_0 \theta \pi}{w_t(n - x)} \right) \)

This result is different from Bencivenga and Smith (1991) in the sense that we define \( \tilde{q}_t^* \) as a function of \( w_t \), while in Bencivenga and Smith (1991), \( \tilde{q}_t^* \) does not depend on \( w_t \). It is straightforward to proof that \( \tilde{q}_t^* \) is an increasing concave function of \( w_t, n, R \) and \( \pi \) since all their first derivative value are positive. This means that higher motivation becomes entrepreneur (higher \( \pi \) ) enhances the young agent’s preference to invest their labour income \( w_t \) into illiquid assets. It is confirmed that \( \tilde{q}_t^* \) increases when \( w_t \) increases. Meanwhile, the income of illiquid investment \( (R) \) attracts the young agent to invest into the illiquid asset, since \( \tilde{q}_t^* \) is an increasing function of \( R \). Although \( \tilde{q}_t^* \) is also increasing along with \( n \), the amount caused by the augmentation of \( n \) is always lower than the augmentation of \( \tilde{q}_t^* \) caused by \( R \), as long as (6.b) is hold. In this examination, we assume that \( x = 0 \) in order to simplify the functional form. Beside that, the influence of \( x \) on \( \tilde{q}_t^* \) may be neglected due to (6.b), although it may probably increase \( \tilde{q}_t^* \).

4. Financial Intermediation

In this part, we build a model in which agent’s financial decisions are intermediated through banking system, where the agent’s budget constraints are identical with the case of self-financed economy. Hence, we can directly define the program of financial intermediation realized by an institution called as “bank”. We assume that bank is a coalition of young agents who can be either non-entrepreneur or entrepreneur. Let \( z_t \) and \( q_t \) be the proportion of liquid and illiquid investment realized by banks, respectively. Thus, we have

\[ z_t + q_t = 1 \]  \hspace{1cm} (9)
Banks ensure non-entrepreneur to receive $R_{i1}^b$ units of consumption goods at $t+1$ from each unit invested at $t$ as following\(^\text{10}\)

$$(1 - \pi) R_{i1}^b = a_{1i} z_i n + a_{2i} q_i x$$

where $a_{1i}$ and $a_{2i}$ are the part of liquid and illiquid asset liquidated at the second period, respectively. The bank chooses the values of $a_{1i}$ and $a_{2i}$. Moreover, banks also ensure entrepreneurs to receive $R_{i2}^b$ units of capital goods at $t+2$ from each unit of time $t$ illiquid investment and $\tilde{R}_{i2}^b$ units of time $t+1$ consumption goods from each unit liquid asset invested at $t$. For the withdrawal after two periods, there are $\pi$ entrepreneurs who must receive $R_{i2}^b$ units of capital goods from each unit of illiquid investment. Thus, $\pi R_{i2}^b$ factor must be equal to the rest of illiquid asset $(1 - a_{2i})$ multiplied by the income of investment $Rq_i$. Thus, the bank must provide capital goods for entrepreneurs as much as

$$\pi R_{i2}^b = (1 - a_{2i}) R q_i$$

(11)

In addition, entrepreneurs must also receive $\tilde{R}_{i2}^b$ units of consumption goods for each unit of liquid investment at $t$. The constraint $\pi \tilde{R}_{i2}^b$ must be equal to the rest of consumption goods $(1 - a_{1i})$ multiplied by $z_i n$. Thus, banks must provide consumption goods for entrepreneurs as much as

$$\pi \tilde{R}_{i2}^b = (1 - a_{1i}) z_i n$$

(12)

In the next step, we define the program of financial intermediation for two types of agent. Firstly, there are $(1 - \pi)$ non-entrepreneurs who will liquidate their investment at $t+1$. Thus, the bank must ensure the non-entrepreneur by holding $R_{i1}^b w_i$ units of consumption goods to be distributed at $t+1$. Secondly, there are also $\pi$ entrepreneurs who will liquidate their investment at the beginning of $t+2$. Thus, the bank must ensure

\(^{10}\) The index $b$ refers the banking interest factor $R^b$, where $-1 \leq R^b \leq \infty$.\n
11
entrepreneurs by holding $R^b_{2t}w_t$ units of capital goods and $\tilde{R}^b_{2t}w_t$ units of consumption goods to be distributed at $t+2$. Using budget constraints in the equation (10), (11), and (12) we define the program of financial intermediation in the following relation

$$U(c_{1t}, c_{2t}) = \frac{(1- \pi)}{\gamma} (R^b_{1t}w_t)^{-\gamma} + \pi (A\theta \psi (R^b_{2t}w_t)^{\psi} + \tilde{R}^b_{2t}w_t)$$

Note that in the third period ($t+2$), entrepreneurs will use their income of investment to finance physical capital and use it in the production. Hence, we have $R^b_{2t}w_t = k_{t+2}$. In order to simplify condition in the equation (13), we assume that the bank should provide the liquidity at $t+1$, since none of the capital assets is liquidated “prematurely”. Thus, the bank should fulfil the following liquidity constraint

$$A\theta \psi R > n$$

By this assumption, we can reduce some variables as follows. In the third period ($t+2$), the bank will only consider the existence of $\pi$ entrepreneur. From (11), we have

$$k_{t+2} = (1- a_{2t})Rq_{t+1}w_t$$

is individual capital. Since the entrepreneur realize the production to get the profit and fulfil $A\theta \psi R > n$, then their profit is superior to all income of liquid investment, so that

$$A\theta \psi |(1- a_{2t})| (R/\pi)q_{t+1}w_t > |(n/\pi)q_{t+1}w_t|$$

Equation (15.a) is fulfilled if and only if the bank set

$$a_{2t} = 0$$

Meanwhile, the bank also maximizes the expected utility of non-entrepreneur. It means that the bank will reallocate the non-entrepreneur’s illiquid assets into liquid assets at the beginning of $t+1$. For realizing this strategy, the bank will therefore set

$$a_{1t} = 1$$

Using (15.b) and (15.c), we simplify (10), (11) and (12), respectively, become
\begin{align*}
R_{t}^{b} &= \frac{z_t}{1 - \pi} n \tag{16} \\
R_{2t}^{b} &= \frac{R}{\pi} q_t \tag{17} \\
\tilde{R}_{2t} &= 0 \tag{18}
\end{align*}

Using (16), (17), and (18), and the budget constraint (9) we establish the program of financial intermediation as follows

\begin{equation}
U(q_t) = -\frac{(1 - \pi)}{\gamma} \left( \frac{1 - q_t}{1 - \pi} nw_i \right)^{-\gamma} + \pi \left( A\theta \psi \left( \frac{Rq_t w_i}{\pi} \right)^{\theta} \right) \tag{19}
\end{equation}

Hence, banks will choose \( q_t \) to maximize \( U(q_t) \). From the first order condition, we obtain the optimal proportion of illiquid asset (\( \bar{q}_t \)) as follows

\begin{equation}
\bar{q}_t = \bar{q}_t (w_i) = 1 - \frac{(1 - \pi)(B_1)^\frac{1}{\gamma - 1}}{nw_i} \tag{20}
\end{equation}

where

\begin{equation}
B_1 = \frac{A\pi \left( \frac{R}{\pi} \right)^{\theta} w_i^{\delta} \theta^{\frac{n}{\gamma}}}{nw_i}.
\end{equation}

It is also straightforward to prove that \( \bar{q}_t \) is increasing along with \( w_i, n, R \) and \( \pi \). From the equation (8) and (20), we may establish the following proposition.

**Proposition 1**

The optimal value of illiquid investment under financial intermediation is higher than the optimal value of illiquid investment under self-financed system. In other words, we prove that \( \bar{q}_t > \bar{q}_t^* \).

**Proof:**

For \( x = 0 \), we then show that \( (1 - \pi)(B_1)^\frac{1}{\gamma - 1}/nw_i < (B)^\frac{1}{\gamma - 1}/nw_i \). Thus, we examine if

\( B_1 < B \). From \( B_1 \) and \( B \), we only examine if
$$\left(1 - \pi\right) \left(A \pi \left(\frac{R}{\pi}\right)^{\frac{y}{\gamma}} w_i^\theta \gamma^\psi\right)^{-\frac{1}{1-\gamma}} < \left(\frac{\pi}{1-\pi} (AR^\theta w_i^\theta \gamma^\psi - nw_i)\right)^{-\frac{1}{1-\gamma}}$$

Let $D_1 = \left(A \pi \left(\frac{R}{\pi}\right)^{\frac{y}{\gamma}} w_i^\theta \gamma^\psi\right)^{-\frac{1}{1-\gamma}}$ and $D_2 = \left(\frac{\pi}{1-\pi} (AR^\theta w_i^\theta \gamma^\psi - nw_i)\right)^{-\frac{1}{1-\gamma}}$, then we simplify $(1-\pi)D_1 < D_2$. Since $q_i, q_i^* \in [0,1]$, then $\max\{D_1\} = \max\{D_2\} = 1$. Thus, the inequality $(1-\pi)D_1 < D_2$ is proved because $0 < (1-\pi) < 1$. Finally, Proposition 1 is proved.

5. Capital Stock Dynamic and Threshold Effect

Firstly, in comparing the level of steady state capital stock under self-financed system and financial intermediation, we establish this following proposition

**Proposition 2**

*The existence of banks in an economy enhances economic growth more significantly than the absence of banks.*

**Proof:**

In the case of bank-based system, economic growth is determined by the value of

$$\bar{k}_{\mu_2} = \frac{R \bar{q}_i w_i}{\bar{\pi}}.$$  

Meanwhile, in the case of self-financed system, economic growth is determined by the value of $\bar{k}_{\mu_2} = R \bar{q}_i w_i$. From Proposition 1, it is straightforward to find $\mu_2 > \mu_2^*$, where $\mu_2 = \frac{\bar{k}_{\mu_2}}{k_i}$ and $\mu_2^* = \frac{\bar{k}_{\mu_2}}{k_i^*}$, are the growth rate of bank-based and self-financed model, respectively. Proposition 2 is thus proved.
Since $\bar{q}_t$ and $\bar{q}_t^*$ are both concave functions, then $\bar{k}_{t+2}$ and $\bar{k}_{t+2}^*$ are also concave.

In order to illustrate the capital dynamics, we run a numerical example and the graphic is shown as follows\textsuperscript{11}.

![Figure 3. Capital Stock Dynamics](image)

**Corollary 1**

*In a bank-based economy, $k_1^*$ is a critical point of threshold and $k_2^*$ is a steady state equilibrium of capital stock if and only if $k_1^* < k_2^*$. The analogous corollary also works in a self-financed economy.*

Since the solutions of equation $k_{t+2} = k_t$ are quite complicated, we then examine the characteristic of equilibrium point $k_1^*$ and $k_2^*$ through the function study showing that threshold effect exists. However, we only study Corollary 1 in the case of bank-based economy, since we have proved the importance of financial intermediation in Proposition 2.

\textsuperscript{11} Numerical examples are available from authors on request.
Firstly, we know that $\bar{q}_t$ is an increasing concave function of $w_t$. Meanwhile, it is also straightforward to prove that $w_t = A(1 - \theta)k_t^\delta \pi^\theta$ is an increasing concave function of $k_t$, since \( \lim_{k_t \to 0} \frac{d w_t}{dk_t} = \infty \) and \( \lim_{k_t \to 0} \frac{d w_t}{dk_t} = 0 \). Thus, by definition, $\bar{q}_t$ is also an increasing concave function of $k_t$. Since $\bar{q}_t$ is an increasing concave function of $k_t$, thus $\bar{k}_{t+2} = \frac{R \bar{q}_t w_t}{\pi}$ is also an increasing concave function of $k_t$. Moreover, we establish the following relationship.

\[
\frac{dk_{t+2}}{dk_t} = -\frac{R(-1 + \theta)\theta D_t}{k_t n \pi (1 + \gamma)} \left( -1 + \pi + Ak_t^\delta n \pi^\theta (1 + \gamma) \left( \frac{D_t}{n} \right)^{1+\gamma} \right)^{-1}
\]

where \( D_t = \frac{A\pi \left( \frac{R}{\pi} \right)^\theta (-Ak_t^\delta \pi^\theta (-1 + \theta)^{-1+\theta} \psi)^{1+\gamma} - 1}{n} \).

Using (21) we establish the table of variation of $\bar{k}_{t+2} = \frac{R \bar{q}_t w_t}{\pi}$ as follows.

**Table 1. Function Study of $\bar{k}_{t+2} = \frac{R \bar{q}_t w_t}{\pi}$**

<table>
<thead>
<tr>
<th>$k_t$</th>
<th>$k_t \to a; k_t \to b; k_t \to c; c = 0$</th>
<th>$k_t \to a; k_t \to b; k_t \to c; c = 0$</th>
<th>$k_t \to +\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lim_{k_t \to c} \frac{dk_{t+2}}{dk_t}$</td>
<td>-∞</td>
<td>-∞</td>
<td>-∞</td>
</tr>
<tr>
<td>$\lim_{k_t \to a} \frac{dk_{t+2}}{dk_t}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lim_{k_t \to b} \frac{dk_{t+2}}{dk_t}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
It is straightforward to obtain \( \lim_{k_i \to c} \frac{dk_{i+2}}{dk_i} = -\infty \), where \( k_i \to c \); \( c \neq 0 \). Meanwhile, we also find that \( \lim_{k_i \to c} \frac{dk_{i+2}}{dk_i} < 1 \). Thus, \( k_{i+2} \) is an increasing concave function of \( k_i \). But, there is also a threshold point at \( k_i = a \), thereby \( k_{i+2} = 0 \) and \( \lim_{k_i \to a} \frac{dk_{i+2}}{dk_i} > 1 \). Unfortunately, in order to find the value of \( a \), we must run numerical simulations due to the functional form complexity\(^{12}\).

Here, our purpose is to analyze why threshold effect may worsen capital accumulation. Suppose that \( k_0 \) is the initial capital of an economy which lies below the threshold point of self-financed system (see market curve at Figure 3). In order to reach the long-run steady state capital, \( k_0 \) should be iterated by financial intermediation curve (see bank curve at Figure 3) which in turn may converge to \( k_2^* \). Contrary if \( k_0 \) is only iterated by the self-financed system curve (see market curve at Figure 3), the economy will disappear because the steady state capital stock tends to zero. Hence, we show that bank-based system is better than self-financed system in order to ensure the existence and uniqueness of long-run steady state capital stock, as well as to reduce threshold level. Long-run economic growth is thus improved by the presence of financial intermediation, as long as long-term productive investments increase and short-term ventures as the potential source of speculations can be minimized.

\(^{12}\) Since the proofs of function characteristic are all straightforward, we do not present in this paper. However, all proofs as well as numerical simulations are available from authors on request.
6. Conclusion

In providing further issue on the finance-growth nexus, we have reevaluated the model of self-financed economy and bank-based economy à la Bencivenga and Smith (1991). Our novelties are twofold. Firstly, in modelling the finance-growth nexus, we use the Neo-classical growth framework rather than the endogenous growth as developed by Bencivenga and Smith (1991). Secondly, while drawing the Bencivenga-Smith’s (1991) model, we distinguish the behaviour vis-à-vis of risk between non-entrepreneur and entrepreneur.

Using these features, we find that bank-based system is better than self-financed system (financial market) in order to ensure the existence and uniqueness of long-run steady state of capital stock which is a necessary condition to achieve long-run economic growth. Moreover, we found that any level of financial development (both through financial intermediation and financial market) may raise a threshold effect. But the presence of financial intermediation clearly reduces threshold level and boost higher long-run steady state of capital stock. The presence of threshold effect is a new finding, since it may capture the difficulty of raising initial capital. Thus, the presence of threshold effect should be taken into account in future research on the finance-growth nexus, notably in developing countries, where externalities due to human capital and technological innovations are not yet well-improved.

References


