Choosing and Sharing

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April 2009

Online at http://mpra.ub.uni-muenchen.de/14929/
MPRA Paper No. 14929, posted 1. May 2009 05:15 UTC
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April 25, 2009

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*We are grateful for stimulating conversations with Francis Bloch, Lars Ehlers, Bettina Klaus, Pierre Lasserre, Hervé Moulin, Nicolas Salhuguet, François Salanié, Larry Samelson and Bernard Sinclair-Desgagné as well as for feedback from participants of the Canadian Economics Theory Conference, the Canadian Economics Association Conference, the Montreal Natural Resources and Environmental Economics Workshop and the Economics Workshops at GATE and TSE.

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Abstract: Implementing a project, like a nationwide nuclear waste disposal, which benefits all involved agents but brings major costs only to the host is often problematic. In practice, revelation issues and redistributitional concerns are significant obstacles to achieving stable agreements. We address these issues by proposing the first mechanism to implement the efficient site (the host with the lowest cost) and share the exact cost while retaining total control over realized transfers. Our mechanism is simple and in the vein of the well-known Divide and Choose procedure. The unique Nash equilibrium outcome of our mechanism coincides with truth-telling, is budget-balanced, individually rational and immune to coalitional deviations. More generally, our mechanism can also handle the symmetric case of positive local externalities (e.g., Olympic Games) and even more complex situations where the usefulness of the project—regardless of its location—is not unanimous.

Keywords: Public goods; local externalities; NIMBY; implementation; mechanism design; VCG mechanisms.
1 Introduction

The search for hazardous waste landfills and nuclear waste repositories in the United States and in many other countries has proven to be a difficult task. Even when offered monetary compensation, very few communities have accepted to host such facilities. Since the mid-70s only one small radioactive waste disposal facility and a single hazardous waste landfill (fittingly located in Last Chance, Colorado) have been sited in the United States (see Gerrard, 1994). Consequently, the U.S. nuclear industry still faces a major problem. Several other similar programs face social rejection from local populations: noxious facilities, prisons, airports, etc. These public goods are socially necessary but come with local externalities (noise, pollution, noxious odors, etc.) or bear a negative connotation. Different factors can generate such rejection: the loss in the economic value of property, the loss in the perceived quality of life or the fear of health hazards, etc. In economic terms, these public goods have a private-bad aspect to them which creates a siting problem (the so-called "NIMBY" problem, for "Not In My Backyard"): all communities benefit from the project but only one—the host—bears the cost.

Similarly, choosing a host for a project which generates desirable local externalities, such as a major international research project (like the International Thermonuclear Experimental Reactor, ITER) or a major sporting event (the Olympic Games), is no easier task to accomplish. Here, benefits accrue to all (as in the "local bad" case) but the host obtains an additional localized surplus. Consequently, all communities compete to be the host. The selection process is then a long and costly one where each participant tries to prove that it is the best candidate.

For the sake of exposition, we consider the case of a project generating negative externalities throughout the body of the paper and postpone the issue of hosting a locally desirable project until the Appendix. The overall cost of the project being tied to the identity of the host, efficiency asks that the host be the community with the lowest hosting cost. In practice, a difficulty one faces when implementing such
a project is that the planner typically has access to much less information than the involved parties, which causes a revelation problem. Secondly, even when the efficient host is identified, there still could exist strong opposition (from the host or other participants) preventing the efficient outcome from occurring. As pointed out in Easterling (1992) and Frey et al. (1996), the structure of the compensation itself could result in the rejection of the project. Therefore, it is crucial that the planner be able to select a specific sharing outcome, which may take into account the voluntary participation of involved communities (no community should pay more than the benefits it derives from the project), their budget constraints, the relative size of their population, their respective involvement in the project or any other relevant characteristic. Taking into account such redistribution issues helps ease the siting process itself and reinforces the stability of the agreement. Unfortunately, no existing mechanism can handle the revelation and redistributinal aspects simultaneously.

Our main contribution is to propose a simple mechanism which gives the planner control over cost shares while not sacrificing efficiency. Our mechanism is in the vein of the well-known Divide and Choose procedure and does not rely on "doomsday" threats: communities jointly propose a monetary compensation for the host, which any community can accept—thus becoming the host—or reject. If no community accepts the compensation, the most fervent proponent of the project is selected to be the host instead.

A well-known class of outcome-efficient mechanisms is that of Vickrey-Clark-Groves (VCG) mechanisms. Announcing one’s true cost is a dominant strategy (thus, the efficient outcome is chosen) but these mechanisms fail to balance the budget and generate a surplus which cannot be redistributed between the agents while preserving the strategic properties of the method (see Moulin, 2007, Section 4, for a VCG treatment of the NIMBY problem). In fact, it is a well-known fact that an efficient and budget-balanced outcome cannot be implemented in dominant strategy\footnote{See Green and Laffont, 1979.}—i.e., where
agents need only know their own characteristics to behave optimally—for direct revelation mechanisms. Hence, one must impose some informational structure.

One way out of the efficiency/incentive compatibility trade-off is to allow the social planner to use statistical information about agents’ valuations and to replace dominant strategy equilibrium by Bayesian incentive compatibility. D’Aspremont and Gerard-Varet, 1979, elaborate a mechanism which is Bayesian incentive compatible, efficient, and budget-balanced. However, individual rationality is not satisfied. Mailath and Postlewaite, 1990, show that if individual rationality is required along with Bayesian incentive compatibility, then it is generally impossible to achieve efficiency. Moreover, they show that as the number of involved agents grows, the probability of undertaking the efficient project goes to zero.

For the problem at hand, O’Sullivan (1993) investigates Bayesian-Nash equilibrium behavior under a sealed-bid auction where the community submitting the lowest bid hosts the project and receives the highest bid as compensation (thus failing to balance the budget). In the same vein, Minehart and Neeman (2002) propose a method adapted from a second-price auction: the host is the community with the lowest cost or, if not (compensation induces misrepresentation of costs), the authors argue that the efficiency loss is small when the number of agents is large. The procedure is self-financed but the host obtains an arbitrary surplus because it is compensated with the second lowest bid, as in a second-price auction.

Given the likely nature of the application—the siting of a single large and long-lived facility between a handful of neighboring communities—we opted for a different informational context where agents have specific information on others’ characteristics. The restrictiveness of our analysis, and of the non-Bayesian implementation literature in general, is in terms of information held (or acquired) by agents: agents must have some information about the preference profile. This is the price to pay for dealing with "universal" mechanisms which do not use any statistical information about the distribution of agents’ characteristics. Part of the literature on non-cooperative
implementation under complete information has produced general characterization theorems (see Peleg, 1978, Maskin, 1999, or Jackson, 2001, for a survey). The resulting mechanisms are very abstract and so general that they often cannot take into account specific restrictions on preferences or specific environments and, thus, are of little help for the siting problem at hand in practice.

Another strand of the literature on non-Bayesian implementation takes advantage of the structure of specific environments in order to produce simpler mechanisms. For the problem at hand, Perez-Castrillo and Wettstein (2002) require that agents be fully informed about the preference profile and design a mechanism which leads to efficient siting with budget-balanced transfers. However, their mechanism does not allow for any control over transfer payments. In fact, while their mechanism allows for much flexibility in sharing the surplus between the aggregate bid and the hosting cost, this flexibility does not apply to the actual hosting cost as the surplus is always equal to zero in equilibrium. By contrast, our mechanism allows for the Nash implementation of any individually rational division of the whole hosting cost. Moreover, our mechanism only requires that communities know which community has the lowest hosting cost and not the whole cost profile. We assume that the planner has no information on the cost profile. We show that the unique Nash equilibrium outcome of our mechanism coincides with truth-telling (the host’s true cost is revealed). Moreover, the host chosen in equilibrium is an efficient one whenever carrying out the project turns out to be efficient, the outcome is budget-balanced, individually rational and the planner has total control over realized transfers.

Regarding the issue of accurately allocating costs in a non-Bayesian context, Jackson and Moulin (1992) jointly handle a revelation problem and redistribution concerns for the traditional case of a pure public good. As such, their mechanism considers only one possible project and therefore cannot address the issue of host selection which is

\footnote{Ehlers (2007) considers a variant of Perez-Castrillo and Wettstein’s mechanism. He shows that his natural interpretation puts severe restrictions on the existence of Nash equilibria. In particular, no equilibrium exists when only one project is efficient.}
central here. Moreover, while the non-Bayesian informational context is crucial for our results to hold, it should be pointed out that, even in perfect information, second price auctions à la Minehart & Neeman (2002) would admit a continuum of equilibria between the lowest and the second lowest hosting cost, thus preventing precise redistribution of costs.

The mechanism we propose proceeds in two stages. In the first stage, each community announces the lowest hosting cost (the community announcing the lowest such cost will be referred to as the "optimist"). In the second stage, each community other than the "optimist" announces its own cost of hosting the project. The host is then selected among those communities which announce the lowest cost in the second stage, provided it accepts the compensation announced by the optimist. If not, the optimist becomes the host. This Divide and Choose structure guarantees truthful revelation from the optimist: the optimist does not want to overstate the efficient cost because her cost share depends on it. But also, reporting a too low value puts her at risk of becoming the host. Contrasting with Perez-Castrillo and Wettstein (2002), whose one-stage game is unable to implement pre-determined cost shares, our two-stage structure is essential to allow the planner to have control over transfer payments.

The siting problem is related to the framework of King Solomon’s dilemma where a single indivisible prize must be awarded to one of several agents. The prize must be allocated to the agent which values it the most at no cost for him. Glazer and Ma (1989) implements that outcome in subgame-perfect Nash equilibrium by assuming perfect information among the agents. The mechanism developed in Perez-Castrillo and Wettstein (2002) also implements the efficient outcome by assuming perfect information. Perry and Reny (1999) relax the perfect information assumption by assuming that all agents know only who values the prize the most. They provide a mechanism which achieves the desired outcome in iteratively weakly dominated strategies. Comparatively, our mechanism would realize an efficient outcome in unique Nash equilib-
rium outcome with the same information structure. However, as in Perez-Castrillo and Wettstein (2002) our mechanism would impose a monetary cost on the winner but, due to the fact that we allow for total control over transfer payments, this cost could be mitigated.

In addition to being able to handle King Solomon’s dilemma and the symmetric counterpart to the NIMBY problem, where benefits accrue to all and the host obtains an additional benefit (e.g., as in the case of hosting the Olympic Games), our mechanism can be used to handle a much larger class of situations. For instance, certain categories of people may obtain negative benefits if a given project is carried out, independently of where it is sited (e.g., some communities may frown upon the use of nuclear energy in general). Our mechanism allows for these communities to be compensated by those in favor of the project.

2 The Model

Let \( N = \{1, \ldots, n\} \) be the set of communities, with \( n \geq 2 \). Each community \( i = 1, \ldots, n \) obtains a benefit, \( b_i \in \mathbb{R} \), if the project is carried out—regardless of its location—and incurs a cost, \( c_i \in \mathbb{R}_+ \), if it is selected to host the project\(^3\). We take the view that \( c_i \) encompasses the actual construction cost of the project plus the disutility of community \( i \) from hosting the project, both of which are localized and community-specific. Let \( b = (b_i)_{i \in N} \) be the profile of benefits and \( c = (c_i)_{i \in N} \) be the cost profile. Without loss of generality we rank communities from lowest to highest hosting cost: \( c_1 \leq c_2 \leq \ldots \leq c_n \). The profile of benefits is known to the planner\(^4\). The cost profile is unknown to the planner but it is common knowledge that each community \( i \) knows \( c_i \) and its own \( c_i \). The total payoff of community \( i \) if the project is carried out is given

\(^3\)For clarity, we use a benefit interpretation for the \( b_i \)’s, but these parameters can be negative for some communities, indicating that they are against the project altogether.

\(^4\)Note that, if the profile of benefits is not immediately accessible to the planner, one could readily embed the mechanism proposed in Jackson and Moulin (1992) in ours to elicit this profile as well.
by:

\[ u_i = b_i - \mathbb{I}(i = \text{host})c_i - t_i \]  

(1)

where \( t_i \) are transfers paid by community \( i \) and \( \mathbb{I}(i = \text{host}) \) is the indicator function equal to 1 if \( i \) is the host and 0 otherwise. Efficiency requires the good to be built if \( \sum_N b_i \geq c_1 \) and hosted by a community \( h \) such that \( c_h = c_1 \). If \( \sum_N b_i < c_1 \), the project should not be built.

We design a mechanism which selects a host community and assigns cost-shares \( \alpha_i(\theta)c_h \) to each community \( \mathbb{P} \), where \( \theta \) represents a number of exogenous (and known to the planner) characteristics relevant for sharing the cost \( \mathbb{P} \) and where \( \sum_N \alpha_i(\theta) = 1 \). We view it as the planner’s responsibility to ensure that the sharing method induces individual rationality (i.e., \( b_i - \alpha_i(\theta)c_1 \geq 0 \) for all \( i \)). Many well-known rules meeting such a requirement exist (see, e.g., Moulin, 2002). In particular, in a companion paper (Laurent-Lucchetti and Leroux, 2009), we discuss the appealing properties of Lindahl prices in this context (\( \alpha_i \equiv \frac{b_i}{\sum_N b_i} \)). Note that \( \alpha_i \) can be either positive or negative, making it possible to compensate communities which are wholeheartedly against the project regardless of its location (i.e. \( b_i < 0 \)).

\(^5\)The careful reader will notice that we slightly abuse notation because the communities are indexed in increasing order of \( c_i \)’s, which is unknown to the planner. In fact, the cost shares are attached to the agents names. In other words, the planner announces a vector of \( \alpha_{\pi(i)}(\theta) \)’s, where \( \pi : N \to N \) is the (unknown) permutation from the agents’ cost ranks to their names. The same notational shortcut applies to the vector of benefits.

\(^6\)It will be clear from the proof of our results, that if the planner wishes to include the cost profile \( c \) in \( \alpha \), \( \alpha_i(.) \) must be non-decreasing in \( c_i \) to maintain the strategic properties of the procedure. If the planner wishes the outcome of the mechanism to also be immune to coalitional deviations, then \( \alpha_i(.) \) should not depend on the costs of other communities, \( c_{-i} \).
3 The Mechanism

Our mechanism proceeds in two stages after the planner announces the value of $\sum N b_i$.

**Stage 1:** Each community is asked to announce the lowest hosting cost in the profile, we denote by $c^i$ agent $i$’s announcement. Define $\zeta = \min(c^i)$. The community which announces $\zeta$ will be referred to as community $i^*$ (the "optimist"). If there are more than one "optimist" then any tie-breaking rule can be applied to select $i^*$. If $\sum N b_i \geq \zeta$, proceed to Stage 2, otherwise stop and the project is not carried out.

**Stage 2:** Each community other than $i^*$ is asked to announce its own hosting cost: $\gamma_i$. Define $\gamma_h = \min_{i \neq i^*}(\gamma_i)$. The host community ("$h$") will be randomly chosen among those announcing $\gamma_h$.

If $\sum N b_i \geq \min \gamma_j$, the following transfers are implemented:

$$
\begin{align*}
&\begin{cases} 
  t_h = -\zeta + \alpha_h(\theta)\zeta \\
  t_i = \alpha_i(\theta)\zeta & \text{for } i \neq h
\end{cases}
\end{align*}
$$

If $\sum N b_i < \min \gamma_j$, the optimist becomes the host ($h = i^*$) and the above transfers are carried out.

The intuition is very simple: communities jointly propose a monetary compensation for the host, $\zeta$, which any community can accept by announcing the lowest $\gamma_i$ of all—thus becoming the host—or reject by announcing a large $\gamma_i$. If no community accepts the compensation (i.e., if $\min \gamma_j < \sum N b_i$), the most fervent proponent of the project (the "optimist") is selected to be the host instead. As we shall see, this Divide and Choose structure guarantees truthful revelation from the optimist.
Note that if the project is carried out the payoffs of communities are:

\[
\begin{align*}
  u_h &= b_h - \alpha_h(\theta)\zeta + (\zeta - c_h) \\
  u_i &= b_i - \alpha_i(\theta)\zeta \quad \text{for } i \neq h
\end{align*}
\]

**Theorem 1.** The unique Nash equilibrium outcome of the mechanism coincides with truthful revelation, \(\gamma_h = \zeta = c_1\), whenever it is efficient to carry out the project. Otherwise the project is not carried out. The outcome is efficient, budget-balanced and individually rational.

**Proof.** We show the existence of an equilibrium by determining best response strategies for all agents \(i \neq i^*\) in Stage 2, and show that, given these best responses, all agents announce \(c^i = c_1\) in Stage 1.

**Stage 2:** Best responses for \(i \neq i^*\)

- If \(\zeta > c_i\): Announce \(\gamma_i = 0\) (so as to maximize her chances of becoming the host).
- If \(\zeta < c_i\): Announce \(\gamma_i = \sum_N b_j + \epsilon\) (or anything higher so as to be sure not to become the host).
- If \(\zeta = c_i\): Becoming the host or not is payoff equivalent. Agent \(i\) can do no better than by announcing \(\gamma_i = c_i\).

**Stage 1:**

Best responses for \(i\) given the above best responses in Stage 2:

- If \(\min_{j\neq i} c^j > c_1\): Announce \(c^i = c_1\), in order to become the optimist and pay a lower cost share (recall that \(\zeta = \min_j c^j\) becomes the cost in Stage 2).
• If \( \min_{j \neq i} c^j < c_1 \): Announce \( c^i = c_1 \), in order to not become the optimist and, in turn, avoid becoming the host.

• If \( \min_{j \neq i} c^j = c_1 \): Agent \( i \) can do no better than announcing \( c^i = c_1 \).

We now show the uniqueness of the Nash Equilibrium outcome by establishing that in any Nash equilibrium \( \underline{c} = c_1 = c_h \):

**Step 1:** \( \underline{c} = c_1 \)

• Suppose a Nash equilibrium existed where \( \underline{c} > c_1 \). Then \( i \neq h \) could obtain a lower cost share by announcing a lower \( c^i \) in Stage 1 so as to lower \( \underline{c} \) and, in turn, her cost share. Thus, \( \underline{c} > c_1 \) cannot be part of a Nash equilibrium of the mechanism.

• Suppose a Nash equilibrium existed where \( \underline{c} < c_1 \).
  a) If \( i^* = h \) (because \( \min_N \gamma_i > \sum_N b_i \)): \( i^* \) could obtain a higher payoff by announcing a higher \( c^i \) in Stage 1 so as to obtain a higher compensation or to no longer be the optimist.
  b) If \( i^* \neq h \): \( h \) could obtain a higher payoff by announcing \( \gamma_h = \sum_N b_j + \epsilon \) so as to no longer be the host.

Thus, \( \underline{c} < c_1 \) cannot be part of a Nash equilibrium of the mechanism.

**Step 2:** \( c_h = \underline{c} = c_1 \)

• Suppose a Nash equilibrium existed where \( c_h > \underline{c} = c_1 \). Then, \( h \) could obtain a higher payoff by announcing \( \gamma_h = \sum_N b_j + \epsilon \) so as to no longer be the host.

Thus, \( c_h > \underline{c} = c_1 \) cannot be part of a Nash equilibrium of the mechanism. Hence, because \( c_h \geq c_1 \) (with our notation), it must be that \( c_h = \underline{c} = c_1 \).

\[ \square \]
Hence, the unique Nash equilibrium outcome exists and coincides with truth-telling\footnote{Note that throughout the proof, knowledge of the identity \textit{per se} of the host is not required, only his cost. However, situations where agents only know the hosting cost but not the host’s identity seem unlikely in practice.}: whenever it is efficient to build, the project is carried out, the host is an efficient one, and the cost to be shared is the true cost. An important concern regarding the selection of an efficient outcome is its robustness to coalitional deviations, especially in a context of possible proximity of agents (e.g., neighboring communities) which is consistent with our informational structure. It turns out that the unique Nash equilibrium outcome of our mechanism is also immune to coalitional deviations.

\textbf{Theorem 2.} The unique Nash equilibrium outcome of the mechanism is immune to coalitional deviations.

\textit{Proof.} We denote by \( \Delta x \) the variation of a variable \( x \); recall that \( b, \theta \) and \( \alpha \) are exogenously given. Denote by \( S \subseteq N \) a deviating coalition.

\begin{itemize}
  \item \textbf{Step 1: The grand coalition:} \( S = N \)

  The unique SPNE outcome of our mechanism is efficient: the host is one of the communities with the lowest cost, which maximizes the sum of all payoffs. Thus, the grand coalition cannot increase its aggregate payoff by a joint deviation.

  \item \textbf{Step 2:} \( h \in S \).

  Consider the joint payoff:

  \[
  \sum_s u_i(\gamma_h, c) = \sum_s b_i - \sum_s \alpha_i(\theta) c + (c - c_h)
  \]

  A positive variation in \( \gamma_h \) (so that \( h \) is still the host) does not change the joint payoff. If \( \gamma_h + \Delta \gamma_h > \min_{j \neq h} \gamma_j \), roles changes (\( h \) "becomes" an agent \( i \)) and
the joint payoff does not change (recall that $c = c_h$ in equilibrium). A negative variation in $\gamma_h$ does not change the joint payoff either.

A negative variation in $c$ decreases the coalition payoff by $\left(1 - \sum_i \alpha_i(\theta)\right)|\Delta c| > 0$. A positive variation is impossible given the best responses: all other communities announce $c^i = c_1$ in the first stage.

Coordinated deviations from $c$ and $\gamma_h$: if the variations are in opposite directions (positive for $\gamma_h$ and negative for $c$) or both negative the payoff decreases in either case by $\left(1 - \sum_i \alpha_i(\theta)\right)|\Delta c| \geq 0$.

Thus, $\sum_S u_i(\gamma_h, c) \geq \sum_S u_i(\gamma_h + \Delta \gamma_h, c + \Delta c)$ for any admissible values of $\Delta c$ and $\Delta \gamma_h$.

- **Step 3**: $h \notin S$.

   Deviations not including $h$ cannot be profitable for the member of $S$ because the only payoff-altering deviation would be to announce a lower $c$ in Stage 1 or a lower $\gamma_{i \neq h}$ in Stage 2. These deviations are inefficient ones where an agent in $S$ becomes the host, which cannot increase the aggregate payoff of coalition $S$.

\[
\square
\]

## 4 Conclusion

We developed a simple and general mechanism in the same vein as the Divide and Choose procedure to select a host for a project generating local externalities. To the best of our knowledge, our mechanism is the first to select an efficient site and implement any individually rational redistribution scheme. It is simple and its unique Nash equilibrium outcome coincides with truth-telling, is efficient, budget-balanced and immune to coalitional deviations. Regarding the informational structure, it requires that each community knows the community with the lowest cost. This assumption, while restrictive, reasonably approximates environments where the involved agents
have much more information about their respective characteristics than the planner (i.e., communities in a given region are typically better informed about their mutual characteristics than the federal state). Moreover, our mechanism can even be used to handle cases where communities disagree about the usefulness of the project (regardless of its location).

In a companion paper, Laurent-Lucchetti and Leroux (2009) propose a specific distribution of costs, \( \alpha_i(.) \), based on normative considerations. At the heart of the argument is the mitigation of the physical asymmetry of the siting problem: the fact that the cost incurred by a single community (the host) determines the cost to be shared by all. This approach characterizes a unique sharing rule which turns out to be Lindahl Pricing.

Our model could be extended by allowing for the disutility of other communities than the host to be positive (e.g., the neighbors of the host could also suffer a local disutility). We chose not to address this possibility in the present model, implying that communities are defined in a wide enough sense so that the externalities generated by the project stay contained in it. Nonetheless, the question is an interesting one, which we leave for further research.
References


A Appendix: The case of local positive externality

The model is identical to that in the body of the paper except that net hosting costs can be negative: \( c_i \in \mathbb{R}_- \) for all \( i \). Implicitly, a negative cost amounts to assuming that the host of the project enjoys a localized benefit which is larger than the construction cost.

Our mechanism must be modified as follows: in Stage 2 the \( \gamma \)'s are restricted to be no less than \( M \), a number fixed by the planner between stages such that \( M < \zeta \). The purpose of \( M \) is to prevent bidding wars \textit{ad infinitum} in Stage 2 should the surplus announced by the optimist be too low (note that this is an off-equilibrium consideration). In fact, this version of the mechanism can be applied to \( c_i \in \mathbb{R}_+ \) as well (actually, no restrictions are needed on the \( c_i \)'s) but we chose to present the simpler version in the body of the paper for the sake of exposition.

**Theorem 3.** The unique Nash equilibrium outcome of the mechanism coincides with truthful revelation, \( \gamma_h = \zeta = c_1 \), whenever it is efficient to carry out the project. Otherwise the project is not carried out. The outcome is efficient, budget-balanced and individually rational.

**Proof.** We show the existence of an equilibrium by determining best response strategies for all agents \( i \neq i^* \) in Stage 2, and show that, given these best responses, all agents announce \( c_i = c_1 \) in Stage 1.

\textit{Stage 2:} Best responses for \( i \neq i^* \)

- Case 1: if \( \zeta > c_i \): Announce \( \gamma_i = M \) (so as to maximize her chances of becoming the host).

- Case 2: if \( \zeta < c_i \): Announce \( \gamma_i > \sum b_i \) (so as to be sure not to become the host).

- Case 3: if \( \zeta = c_i \): Becoming the host or not is payoff equivalent. Agent \( i \) can do no better than to announce \( \gamma_i = c_i \).

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Stage 1:

Best responses for $i$:

- If $\min_{j \neq i} c^j > c_1$: Announce $c^i = c_1$, in order to become the optimist and obtain a higher share of the surplus (recall that $\zeta = \min_{j} c^j$ becomes the surplus to be shared in Stage 2).

- If $\min_{j \neq i} c^j < c_1$: Announce $c^i = c_1$, in order to not become the optimist and, in turn, avoid becoming the host given the best response profile in Stage 2.

- If $\min_{j \neq i} c^j = c_1$: Agent $i$ can do no better than announcing $c^i = c_1$ given the best response profile in Stage 2.

We now show the uniqueness of the Nash Equilibrium outcome by establishing that in any Nash equilibrium $\zeta = c_1 = c_h$:

Step 1: $\zeta = c_1$

- Suppose a Nash equilibrium existed where $\zeta > c_1$. Then $i \neq h$ could obtain a higher payoff by announcing a lower $c^i$ in Stage 1 so as to lower $\zeta$. Thus, $\zeta > c_1$ cannot be part of a Nash equilibrium of the mechanism.

- Suppose a Nash equilibrium existed where $\zeta < c_1$.

  a) If $i^* = h$ (because $\min_{N} \gamma_h > \sum_{N} b_i$): $i^*$ could obtain a higher payoff by announcing a higher $c^i$ in Stage 1 so as to redistribute a lower surplus or to no longer be the optimist.

  b) If $i^* \neq h$: $h$ could obtain a higher payoff by announcing $\gamma_h > \sum_{N} b_i$ so as to no longer be the host.

Thus, $\zeta < c_1$ cannot be part of a Nash equilibrium of the mechanism.

Step 2: $c_h = \zeta = c_1$
• Suppose a Nash equilibrium existed where $c_h > \zeta = c_1$. Then, $h$ could obtain a higher payoff by announcing $\gamma_h > \sum_N b_i$ so as to no longer be the host.

Thus, $c_h > \zeta = c_1$ cannot be part of a Nash equilibrium of the mechanism. Hence, because $c_h \geq c_1$ (with our notation), it must be that $c_h = \zeta = c_1$.

\[\square\]

**Theorem 4.** The unique Nash equilibrium outcome of the mechanism is immune to coalitional deviations.

**Proof.** Follows directly from the proof of Theorem 2. \[\square\]