Forward-Looking Beta Estimates: Evidence from an Emerging Market

Ibrahim Onour

Arab Planning Institute

10. February 2008

Online at http://mpra.ub.uni-muenchen.de/14992/
MPRA Paper No. 14992, posted 4. May 2009 00:46 UTC
Forward-Looking Beta Estimates: Evidence From An Emerging Market

Ibrahim A. Onour
Research Economist
Arab Monetary Fund
P.O.Box:2818 Abu-Dhabi
UAE
ibonour@hotmail.com
Forward-Looking Beta Estimates: Evidence From an Emerging Market

Abstracts
Results in this paper support evidence of time-varying beta coefficients for five sectors in Kuwait Stock Market. The paper indicates banks, food, and service sectors exhibit relatively wider range of variation compared to industry and real estate sectors. Results of time-varying betas invalidate the standard application of Capital Asset Pricing model that assumes constant beta. In terms of risk exposure, banks and industrial sectors reflect higher risk as their average betas exceed the market beta, which is a unit.

Keywords: Beta, CAPM, GARCH, Volatility, Asymmetry
JEL: C10, C50, G10
1-Introduction:
How should a rational investor measure the risk of stock market investments? The search for an answer to this question became the major task in financial economics and that led to the development of Capital Asset Pricing Model (CAPM) which become the centre piece in modern finance textbooks. The CAPM decompose risk valuation into risk size (risk premium) and risk price (beta\(^1\)). According to CAPM the required rate of return on a company’s stock (or the cost of equity capital) depends on three components among which the stock’s equity beta which measures the risk of company’s stock relative to the market risk; or putting it differently, the risk each dollar invested in equity i contributes to the market portfolio. CAPM predict low beta stocks should offer low stock returns and higher beta stocks should offer higher stock returns. This imply stocks with higher risks should yield higher returns to compensate for the additional risk borne.

Since the empirical findings of Fama and French (1992, 1993, 1995, 1996, and 1997) the traditional application of CAPM that assume constant beta has been invalidated, and since then research efforts directed towards time-varying beta estimates. In theory beta estimates should reflect investors’ uncertainty about future cash flows to equity, which in turn requires time-varying and thus forward looking beta estimates.

In pursuit for obtaining better beta estimates research focused on the use of time varying volatility models. Recent such work supporting time-varying beta include Mckenzie et al (2000) for U.S., banks; Lie, Brooks and Faff

---

\(^1\) Beta also called systematic risk, which is the risk that cannot be reduced via diversification strategy.

An appropriate specification of time-varying volatility depends on what empirical regularities the model should capture. An important phenomena that characterize volatility of asset returns is the so-called “leverage effect” which refers to the different response of volatility to bad news as compared to good news. To account for asymmetric effect of news on traded asset returns’ volatility in this paper Glosten, Jagannathan, and Runkle (1993) specification of GARCH model is adopted. GJR-GARCH specification separates the effect of negative news on volatility from that of positive news.2

While there is a considerable amount of research in this area for industries in developed and in some emerging stock markets, similar work on GCC and less developed stock markets is lacking. Constrained by the lack of suitable time series data availability for other GCC stock markets, investigation in this paper has been limited on Kuwait stock market using data on six key sectors in Kuwait economy. This paper contributes to the existing literature by taking into account leverage effect, and skewed-fat-tailed aspects of volatility when estimating beta coefficients.

---

2 Engle and Ng (1993) report evidences that, of many GARCH specifications the GJR asymmetric GARCH model provides the best forecast of volatility.
The reminder of the paper is structured as follows. The next section includes summary statistics. Section three outlines beta and volatility modeling approach. Section four includes estimation results. The final section concludes the study.

2. Data and Summary Statistics:

Data employed in this study are daily stock price indices related to five sectors in Kuwait stock market, beside the aggregate stock index. The sample period covers from June-17-2001 to January-16-2007, including 1425 observations. The sectors included in this study are, banks; food; industrial; real estate; and the service sectors. Results in table (1) indicate that all sectors yield positive mean returns. The high values of kurtosis statistics indicate the stock price returns distribution is characterized by high peakness (fat tailedness). The negative skewness results indicate that Kuwait portfolio industry exhibit a higher probability for investors to get a negative returns, which is similar to the case in some developed and emerging markets as indicated by Harvey and Siddique (1999).

The Jarque-Bera (JB) test statistic provides clear evidence to reject the null-hypothesis of normality for the unconditional distribution of the daily stock price changes for all sectors. The sample autocorrelation statistic indicated by Ljung-Box, Q statistic, show the Q(5) test statistic reject the null hypothesis of uncorrelated price changes for five lags for all sectors. The high values for $Q^2(5)$ test statistic suggest that conditional homoskedasticity can be rejected for all sectors. To test the presence of hetroskidasticity more formally the LM test is employed. Results of LM statistics for ARCH(1) and ARCH(5) error terms confirm the significance of ARCH effects in the data.

Table (1): Summary statistics of log differenced stock returns
### 3- Methodology

**3.1: Beta and Volatility modeling:**

Although the simple GARCH specification is widely used in the empirical research of finance, there are substantial evidences that volatility of asset returns characterized by time varying asymmetry (Glosten, Jagannathan and Runkle (1993). As a result, to avoid misspecification of the conditional variance equation, a leverage term in the GARCH specification is included. The GARCH-type specification introduced by Glosten, et al, (1993) allows a quadratic response of volatility to news with different coefficients for good

<table>
<thead>
<tr>
<th></th>
<th>Banks Sector</th>
<th>Food Sector</th>
<th>Industrial Sector</th>
<th>Real Estate Sector</th>
<th>Service Sector</th>
<th>Market Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.1E-2</td>
<td>0.7E-3</td>
<td>0.9E-3</td>
<td>0.9E-3</td>
<td>0.1E-2</td>
<td>0.1E-2</td>
</tr>
<tr>
<td>Excess kurtosis</td>
<td>704</td>
<td>594</td>
<td>705</td>
<td>673</td>
<td>687</td>
<td>694</td>
</tr>
<tr>
<td>JB test p-value</td>
<td>3856 (0.00)</td>
<td>2485 (0.00)</td>
<td>4348 (0.00)</td>
<td>3363 (0.00)</td>
<td>4400 (0.00)</td>
<td>5527 (0.00)</td>
</tr>
<tr>
<td>Q(5) p-value</td>
<td>356 (0.00)</td>
<td>319 (0.00)</td>
<td>353 (0.00)</td>
<td>338 (0.00)</td>
<td>346 (0.00)</td>
<td>351 (0.00)</td>
</tr>
<tr>
<td>Q2(5) p-value</td>
<td>355 (0.00)</td>
<td>354 (0.00)</td>
<td>352 (0.00)</td>
<td>354 (0.00)</td>
<td>355 (0.00)</td>
<td>355 (0.00)</td>
</tr>
<tr>
<td>LM ARCH(1) p-value</td>
<td>141 (0.00)</td>
<td>1.50 (0.47)</td>
<td>226 (0.00)</td>
<td>8.77 (0.01)</td>
<td>32.3 (0.00)</td>
<td>27.3 (0.00)</td>
</tr>
<tr>
<td>LM ARCH(5) p-value</td>
<td>208 (0.00)</td>
<td>567 (0.00)</td>
<td>579 (0.00)</td>
<td>567 (0.00)</td>
<td>539 (0.00)</td>
<td>568 (0.00)</td>
</tr>
</tbody>
</table>
and bad news, but maintains the assertion that the minimum volatility will result when there is no news\(^3\).

Given the capital market model,

\[
R_i = \eta_i + \beta_i R_{mt} + e_i
\]

where \(e_i = \sigma_i z\)

\[
z_i \sim f(\omega; 0,1)
\]

\[
\sigma^2_i = \alpha_0 + \sum_{q=1}^{Q} \alpha_q e^2_{i-q} + \sum_{p=1}^{P} \delta_p \sigma^2_{i-p} + \epsilon_i
\]

where \(R_{mt}\) is the return on market portfolio, and \(R_{it}\) is the return on sector \(i\), and \(\eta_i\) and \(\beta_i\) are the associated portfolio mean, and beta respectively.

Beta coefficient in (1) reflect the sensitivity of industry return to change in market return. Thus, a portfolio of beta greater than one is considered more sensitive to market conditions\(^4\). \(f(.)\) is the density function of the standardized residuals, \(z_i\), where \(E(z_i) = 0, \nu(z_i) = 1\), and \(\omega\) is a vector of the parameters reflecting skewness and kurtosis parameters. In GARCH-type models the variance-covariance matrix of the different portfolios and the market index returns are not constant over time. In this case Beta defined as:

---

\(^3\) Any selection of an appropriate ARCH/GARCH model requires having a good idea of what empirical regularities the model should capture. Among documented other regularities in the literature are thick tails that characterize asset returns, and volatility clustering, which refers to the phenomena that large changes in volatility tend to be followed by large changes of either sign, and small changes to be followed by small changes.

\(^4\) An important implication of Market Model represented by (1) is that the average beta for all sectors is equal a unit. To see this note that the average market index is given as:

\[
\overline{R}_{mt} = (1/N) \sum_{i=1}^{N} R_{it} = \sum_{i=1}^{N} (1/N)(\overline{\eta}_i + \beta_i \overline{R}_{mt} + \overline{e}_i)
\]

\[=
\overline{\eta} + \overline{\beta} \overline{R}_{mt} + \overline{\epsilon}
\]

\[\rightarrow \overline{\eta} = 0, \overline{\beta} = 1
\]
\[
\beta^{GARCH}_{it} = \frac{\text{cov}(R_{it}, R_{mt})}{\text{var}(R_{mt})}
\]  

(2)

so that equation (1) becomes,

\[
R_{it} = \eta_{i} + \beta^{GARCH}_{it} R_{mt} + e_{it}
\]  

(3)

One approach to estimating \( \beta^{GARCH}_{it} \) is to estimate conditional covariance, \( \text{cov}(R_{it}, R_{mt}) \) and conditional market variance \( \text{v}(R_{mt}) \). Adopting asymmetric GARCH-type model the problem can be reduced to estimating the following specifications of variance and covariance equations:

\[
\text{var}(R_{it}) \equiv \sigma_{it}^2 = \omega + \sum_{j} [\alpha^+_{j} I(e_{i,t-j} > 0) |e_{i,t-j}|^2 + \alpha^-_{j} (1) |e_{i,t-j} \leq 0) |e_{i,t-j}|^2 ]
\]

\[
+ \sum_{j=1}^{\delta_{j}} \sigma_{i,t-j}^2
\]

(4)

where I denotes indicator function taking on the values of 1 when \( e_{i,t} > 0 \), and 0 otherwise. The threshold ARCH (TARCH) model of Zakoian (1993) corresponds to equation (4) with \( \lambda = 1 \), whereas GJR–GARCH–type specification treats equation (4) with \( \lambda = 2 \), to allow for quadratic response of volatility to news with different coefficients for good and bad news, while maintaining the possibility that minimum volatility occur when there is no news. Similarly, the variance of market portfolio from equation (4) hold with the change of the subscript from i to m.

The situation that \( \alpha^+ > 0 \), captures the asymmetric relationship between news \( (e_i) \) and volatility. For example, when \( e_{a-j} > 0 \), then I=1 and the conditional variance becomes,
\[ \sigma^2_{it} = \omega + \sum_{j,q} \alpha_i^+ e_{i,t-j}^2 + \sum_{j,p} \delta_j \sigma_{i,t-j}^2 \]

and when \( e_{i,t-j} < 0 \), then \( I=0 \) and the conditional variance becomes

\[ \sigma^2_{it} = \omega + \sum_{j,q} \alpha_i^- e_{i,t-j}^2 + \sum_{j,p} \delta_j \sigma_{i,t-j}^2 \]

Therefore, the negative news result in a variance level different from that associated with positive news. This type of investors behavior imply risk aversion attitudes depend on the magnitude of risk investors expecting to face.

Since it can be verified from (4), that \( E(R_i - E(R_i))^2 = E(e_i^2) \) then

\[ \nu(R_i) = \nu(e_i), \nu(R_m) = \nu(e_m). \]

Then the conditional covariance of industry and market portfolio can be computed by:

\[ \text{cov}(R_{it}, R_{mt}) = \rho_{im} \sqrt{\sigma_{it}^2 \sigma_{mt}^2} \] (5)

where \( \rho_{im} \) is the correlation coefficient between \( R_{it} \) and \( R_{mt} \).

3.2: Skewed distribution:

It is well documented that even asymmetric GARCH models fail to fully account for skewness and leptokurtosis of high frequency financial time series when they are assumed to follow normal or symmetric student’s t-distributions. This has led to the use of asymmetric non-normal distributions to better specify conditional higher moments. An important candidate in this respect is Hansen’s (1994) skewed t-distribution. Despite there are other distributions that allow for skewness and excess kurtosis we choose
Hansen’s distribution due to its simplicity and its superiority in empirical performance (Patton, 2004).

Given the standardized errors $\frac{\varepsilon_i}{\sqrt{\sigma^2_i}} = z_i$, with mean zero and variance one, then Hansen’s (1994) autoregressive conditional density model with skewed error terms specified as:

$$skt(z \setminus \phi, \theta) = \begin{cases} 
bc \left(1 + \frac{1}{\theta - 2} \left(\frac{bz + a}{1 - \phi}\right)\right)^{-(\theta+1)/2} & \text{if } z < -a/b \\
b \left(1 + \frac{1}{\theta - 2} \left(\frac{bz + a}{1 + \phi}\right)\right)^{-(\theta+1)/2} & \text{if } z \geq -a/b 
\end{cases}$$

where $\Gamma$ denotes gamma function, and

$$a = 4\phi \frac{\theta - 2}{\theta - 1}, \quad b = 1 + 3\phi^2 - a^2, \quad c = \frac{\Gamma(\theta + 1)/2}{\sqrt{\pi(\theta - 2)\Gamma(\theta/2)}}$$

Specification of conditional distribution of the standardized residuals, $Z_t$, in equation (6) is determined by two parameters, Kurtosis ($\theta$) and the skewness parameter ($\phi$). The two parameters are restricted to $\theta > 2$, and $-1 < \phi < 1$.

When $\phi = 0$, the skewed t-distribution tend to symmetric t-distribution, and when $\theta \to \infty$, tend to standardized normal distribution.

Hansen’s skewed t-distribution is fat tailed, and skewed to the left (right) when $\phi$ is less (greater) than zero. Similar to the case of Student’s t-distribution, when $\theta > 2$, Hansen’s skewed t-distribution is well defined and
its second moment exist, while skewness exist if $\phi \neq 0$, and kurtosis is defined if $\theta > 4$. The formulas for the third and fourth moments of Hansen’s skewed distribution are given as:

$E(Z^3) = (m_3 - 3am_2 + 2a^3)/b^3$
$E(Z^4) = (m_4 - 4am_3 + 6a^2m_2 + 6a^2m_2 - 3a^4)/b^4$

where

$m_2 = 1 + 3\phi^2$

$m_3 = 16c\phi(1 + \phi^2) - (\theta - 2)^2 \quad \text{if} \quad \theta > 3$

$m_4 = 3\frac{\theta - 2}{\theta - 4} (1 + 10\phi^2 + 5\phi^4) \quad \text{if} \quad \theta > 4$

(for proof see Hansen, 1994, and also Jondeau, and Rockinger 2000). The log-likelihood function of the GJR-skts is defined as:

$L(\Omega; \Psi_{t-1}) = \sum_{t=2}^{T} \ln[SKt(z \setminus \theta, \phi; \Psi_{t-1})]$

The maximum likelihood estimator for $\Omega$ is the solution of maximizing the log likelihood function stated above.

### 3.3: Performance Evaluation:

In the following the predictive power of volatility forecast is utilized to evaluate the performance of the two models. To compute s-step ahead forecast for the conditional variance in equations (1) - (4), we need first to simplify equation (4) by assuming:

$E(I(e_t > 0)) = p(e_t > 0) = 0.5$
$E(I - 1)(e_t \leq 0) = p(e_t \leq 0) = 0.5$

and

$E(e^2 \setminus \Omega_t) = \sigma^2_t$
Since $\epsilon_t^2$ and the indicator function $I_r(e_t)$ are uncorrelated, then s-step ahead forecast can be stated as:

$$\hat{\sigma}^2_{t+s|t} = w + [(0.5\alpha^+ + 0.5\alpha^-) + \delta]\sigma^2_{t+s-1|t} \quad (8)$$

The parameters of the two models estimated using the sample data up to three days before the end of the sample date (Jan/13/2007). And then a forecast of one day ahead (Jan-14 observation) is computed. Using the estimated parameters and the one day-ahead forecast value of volatility a new forecast for volatility of Jan-15, is computed from equation (8) to obtain two days ahead forecast value. This procedure is repeated until we exhaust the actual realized values.

To test the predictive power of the two competing models (GJR-N, and GJR-sk) the Root Mean Squared Error (RMSE) employed, which is computed by comparing the forecast values $F_{t+j}$ with the actually realized values, $A_{t+j}$, or

$$RMSE(k) = \sqrt{\frac{\sum_{j=0}^{N-1}(F_{t+j+k} - A_{t+j+k})^2}{N_k}}$$

Where $k=1,2,3$ denotes the forecast step, $N_k$, is total number of k-steps ahead forecasts.

Diebold and Mariano (1995) (DM) test has been employed to compare the accuracy of forecasts. When comparing forecasts from two competing models; model A, and model B, it is important to verify that prediction of model A is significantly more accurate, in terms of a loss function, DM(d), than the prediction of model B. The Diebold and Mariano test aims to test the null hypothesis of equality of forecast accuracy against the alternative of different forecasts across models. The null hypothesis of the test can be written as:

$$d_t = E(h(e_t^A) - h(e_t^B)) = 0 \quad (9)$$
where \( h(e'_i) \) refers to the forecast error of model \( i = A, B \), when performing \( k \)-steps ahead forecast. The Diebold and Mariano test uses the autocorrelation-corrected sample mean of \( d_i \) in order to test significance of equation (9). If \( N \) observations available, the test statistic is:

\[
DM = \left[ \hat{w}(\bar{d}) \right]^{-1/2} \bar{d}
\]

where

\[
\hat{w}(\bar{d}) = \frac{1}{N} \left\{ \hat{\rho}_0 + 2 \sum_{h=1}^{K} \hat{\rho}_h \right\}
\]

and

\[
\hat{\rho}_h = \frac{1}{N} \sum_{t=h+1}^{N} (d_t - \bar{d})(d_{t-h} - \bar{d})
\]

Under the null hypothesis of equal forecast accuracy, \( DM \) is asymptotically normally distributed.

4: Estimation Results

Estimation of beta coefficient based on conditional volatility of stock returns, assuming asymmetric GARCH specification under Normal distribution (GJR-N); and Skewed t-distribution (GJR-skt) of error terms is reported in table (2). Table (A1) in the appendix include estimation results of the parameters of equations (4), (6), and (7). The significance of the asymmetry coefficient (\( \alpha^+ \)) for the Normal distribution for all sectors indicate positive shocks (or good news) have more significant effect on volatility than the effect of bad news. This result indicate, since investors in
stock markets seek short term profit gains they attempt to benefit from positive news they seize often, but adjusting portfolios to negative shocks depends on the size of the shock, because portfolio adjustment to adverse shocks require hedging aspects that entails additional cost. Results of the skewed t-distribution also indicate significance of the Kurtosis coefficient \( \theta \) for all sectors, which suggest fat-tailed student t-density is needed to fully model the distribution of return. Despite the significance of the Kurtosis coefficient \( \theta \) for all sectors, the log-likelihood values strongly suggest that the Normal distribution (GJR-N) outperform, the skewed-t distribution GARCH/ARCH model. This imply that estimation of parameters in table (A) has QML features. It is apparent from the range of beta values both models support evidence of time-varying beta values for all sectors. Based on the Normal distribution results, Beta values for Banks, Food, and Service sectors exhibit wider range compared to the remaining other sectors. The, Industry and Real estate sectors, show relatively stable beta variation. The high values of the correlation coefficient values confirm strong association between volatilities of the sectors and the market volatility.
### Table (2): Estimate of Beta Coefficients

<table>
<thead>
<tr>
<th>sectors</th>
<th>GJR-GARCH Normal dist.</th>
<th>GJR-GARCH Sk-t dist.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Banks (low/high)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>24.4</td>
<td>3.8</td>
</tr>
<tr>
<td>ρ</td>
<td>0.75</td>
<td>0.69</td>
</tr>
<tr>
<td><strong>Food (low/high)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>7.3</td>
<td>2.3</td>
</tr>
<tr>
<td>ρ</td>
<td>0.78</td>
<td>0.88</td>
</tr>
<tr>
<td><strong>Industry (low/high)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>1.01</td>
<td>3.6</td>
</tr>
<tr>
<td>ρ</td>
<td>0.79</td>
<td>0.62</td>
</tr>
<tr>
<td><strong>Real Estate (low/high)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>0.6</td>
<td>4.9</td>
</tr>
<tr>
<td>ρ</td>
<td>0.89</td>
<td>0.95</td>
</tr>
<tr>
<td><strong>Service (low/high)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>8.8</td>
<td>2.5</td>
</tr>
<tr>
<td>ρ</td>
<td>0.78</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Note: The first row entries are mean values of Betas. Range statistics refer to the difference between high and low values. \( \rho \) denotes correlation coefficient between volatilities of market index and sector portfolio.

Root Mean Square Error (RMSE) and Diebold-Mariano (1995) test results in table (3), indicate the GJR-Normal distribution model yield the lowest values of the RMSE loss functions for all sectors compared to GJR-t skewed distribution model.
DM test statistic confirm that the predictive power of the two models are significantly different for all sectors; implying that GJR-Normal distribution model yield superior forecast performance for forward-looking beta values.

Table (3): RMSE Loss functions and Diebold & Mariano test.

<table>
<thead>
<tr>
<th>Sector</th>
<th>RMSE Loss Functions</th>
<th>D&amp;M statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GJR-N</td>
<td>GJR-sk(t)</td>
</tr>
<tr>
<td>Banks</td>
<td>0.14</td>
<td>0.25</td>
</tr>
<tr>
<td>Food</td>
<td>0.024</td>
<td>0.24</td>
</tr>
<tr>
<td>Industry</td>
<td>0.11</td>
<td>0.18</td>
</tr>
<tr>
<td>Real estate</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>Service</td>
<td>0.06</td>
<td>0.30</td>
</tr>
</tbody>
</table>

*The loss functions are based on three days ahead forecast errors.
5: Concluding remarks:
Taking into account empirical regularities that characterize asset returns in emerging markets in this paper time-varying beta coefficients for the major sectors in Kuwait economy estimated. Among the regularities that characterize asset markets are the “leverage effect” which refers to the different response of volatility to bad news as compared to good news, and skewness and fat-tailedness of stock returns distribution. To account for the asymmetric effect of news on asset returns’ volatility in this paper Glosten, Jagannathan, and Runkle (1993) specification is adopted under two alternative assumptions about stock returns distribution, the Normal distribution and skewed t-distribution specification.
Results of predictive power performance and log-likelihood values support overwhelmingly, evidence of GJR-Normal distribution model outperforming GJR-skewed t-distribution specification when modeling volatility in Kuwait stock market.
The findings in the paper also support evidence of time-varying beta values for all sectors included in the study. However, banks, food, and service sectors exhibit relatively wider range of beta coefficients compared to the beta values of industry and real estate sectors. Results of time-varying beta values invalidate the standard application of Capital Asset Pricing model that assumes constant beta. The implication of relatively wider range of beta variation for banks and industrial sectors reflect higher risk for the securities of these two sectors as their average betas exceed the market beta, which is equal to a unit.
References


## Appendix

Table (A1): GARCH(1,1)/ARCH(q) parameter estimates

<table>
<thead>
<tr>
<th></th>
<th>Banks GARCH(1,1)</th>
<th>Food ARCH(1)</th>
<th>Industry ARCH(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GJR-t skew</td>
<td>GJR-Normal</td>
<td>GJR-t skew</td>
</tr>
<tr>
<td><strong>ω (p-value)</strong></td>
<td>0.11 (0.00)</td>
<td>0.09 (0.00)</td>
<td>0.03 (0.00)</td>
</tr>
<tr>
<td><strong>δ (p-value)</strong></td>
<td>0.36 (0.00)</td>
<td>0.42 (0.00)</td>
<td>0.29 (0.02)</td>
</tr>
<tr>
<td><strong>α⁺ (p-value)</strong></td>
<td>0.00 (0.64)</td>
<td>0.26 (0.00)</td>
<td>0.26 (0.30)</td>
</tr>
<tr>
<td><strong>α⁻ (p-value)</strong></td>
<td>0.47 (0.85)</td>
<td>-0.00 (0.99)</td>
<td>-2.5 (0.59)</td>
</tr>
<tr>
<td><strong>ϕ (p-value)</strong></td>
<td>0.46 (0.15)</td>
<td>--</td>
<td>0.99 (0.16)</td>
</tr>
<tr>
<td><strong>θ (p-value)</strong></td>
<td>2.96 (0.00)</td>
<td>--</td>
<td>3.91 (0.00)</td>
</tr>
<tr>
<td><strong>LnL</strong></td>
<td>1104</td>
<td>3931</td>
<td>2856</td>
</tr>
</tbody>
</table>

*The lag parameters (p,q) determined based on stationarity restrictions. An examination of the coefficients in GARCH specification in table (A1) and (A2) reveals that \( h_t \) for Banks, Industrial, Real Estate, and Service sectors follow stationary ARCH(1), whereas Food sector follows ARCH(3); that is, the condition \( |α_i| < 1 \) is satisfied for all sectors. Results in table(A1) also reveals market portfolio index follows GARCH(1,1) process, and the stationarity conditions, \( [α_i > 0, \ δ > 0, (α + δ) < 1] \), are satisfied.

** GJR-skt model generates only negative error terms.
Table (A2): GARCH(1.1)/ARCH(q) parameter estimates

<table>
<thead>
<tr>
<th></th>
<th>Real estate</th>
<th>Service</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ARCH(1)</td>
<td>GARCH(1,1)</td>
</tr>
<tr>
<td>$\omega$ (p-value)</td>
<td>0.04 (0.00)</td>
<td>0.03 (0.00)</td>
</tr>
<tr>
<td>$\delta$ (p-value)</td>
<td>0.31 (0.00)</td>
<td>0.49 (0.00)</td>
</tr>
<tr>
<td>$\alpha^+$ (p-value)</td>
<td>0.52 (0.30)</td>
<td>0.13 (0.00)</td>
</tr>
<tr>
<td>$\alpha^-$ (p-value)</td>
<td>0.20 (0.00)</td>
<td>-0.00 (0.99)</td>
</tr>
<tr>
<td>$\phi$ (p-value)</td>
<td>0.99 (0.15)</td>
<td>--</td>
</tr>
<tr>
<td>$\theta$ (p-value)</td>
<td>3.9 (0.00)</td>
<td>--</td>
</tr>
<tr>
<td>LnL</td>
<td>2853</td>
<td>4003</td>
</tr>
</tbody>
</table>

** GJR-skt model generates only negative error terms.