Extracting, Using and Analysing Cyclical Information

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Abstract

Recent events suggest that the death of the business cycle has been exaggerated; the issue of how one learns about and monitors the business cycle remains centre stage. Advent of the Euro and the potential for tensions when sovereign nations subsume their monetary policy into a single response also makes monitoring the business cycle of particular interest for Euro area policy makers.

In this paper we summarize recent research on three questions relating to cycles in economic activity — how to extract cyclical information, how to analyse it, and how to enquire into what special difficulties might be encountered when using cyclical indicators.

This survey focuses on our own research which we view as a formalization of some of the procedures developed by Burns and Mitchell at the NBER. However, defence of our position goes beyond continuity with the past and is based on the view that the way in which these investigators defined the business cycle is a very natural one that connects with the way policy makers and commentators discuss the cycle.

Key Words: Business cycle; growth cycle, synchronization, turning points.
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1 Introduction

After some years in which there was a loud chorus that the business cycle was becoming extinct, the year 2001 has shown that its extinction is still some time off. Naturally such events have once again raised issues of how one monitors and learns about the cycle. Although the situation in Europe is currently somewhat different to the US, the advent of the Euro has also been the source of a renewed interest in the business cycle, since the monetary policy responses to cycles in sovereign countries are now to be subsumed under a general policy response to Euro Area wide aggregates. Hence, although all countries have regional differences that are often in conflict with macroeconomic aggregates and which create tensions in the monetary policy formation process, these are likely to be much larger when sovereign states have only recently conceded their powers over the use of monetary policy to achieve their domestic objectives.

This paper sets out to summarize recent research on three questions relating to cycles in economic activity - how to extract cyclical information, how to analyse it, and how to enquire into what special difficulties might be encountered when using cyclical indicators that are readily available or may have been constructed for some purpose. In writing this survey we have tended to focus upon our own recent work, although we do comment upon our differences with others and offer a defence of our own position. At its most primitive, our defence is simply one of continuity. We believe that our methodology is simply a formalization of the procedures that some of the earliest writers on the business cycle followed. Of course having such historical continuity does not mean that this is the best way to accomplish a task, but we also believe that the way in which these early investigators identified recessions and expansions was a very natural one and that it still exists implicitly or explicitly in a great deal of commentary on the business cycle today.

Section 2 of the paper sets out our definition of a recession as a sustained decline in the level of economic activity and then explains how one measures such a concept. This leads us to the description of rules that would identify recession and expansion states from a given series of observations on either single or multiple series. We look at various alternatives which have been proposed and contrast them with our position. In particular we address the vexing question of whether any components should be removed from a series measuring the level of economic activity before we begin a cyclical analysis.

Once identified by the methods described in section 2 of the paper, Section 3 turns to how one would use this information to describe the cycles in the Euro Area and six of the countries making it up. In particular we are interested in how synchronized the cycles would be.

The information supplied by the techniques of section 1 can be summarized by a binary random variable S(t) that takes the value unity in expansions and zero in contractions. Such binary random variables have been used in many contexts e.g. as regressands in attempts to predict recessions using observable indicators and as regressors when looking at whether the impact of policy actions differ according to the state of the economy. Little research has been done on whether there are any econometric difficulties raised by using the S(t) in such contexts. To answer that question it turns out that one needs to know the data generating process (DGP) of the S(t). In section 4 we therefore report on some research that we have performed which seeks to describe this DGP and we use it to shed light on some of the econometric difficulties one might encounter in using these variables. Some illustrations are given of methods that go at least part of the way towards making an allowance for these difficulties.

2 Measuring Cycles With Univariate Information

2.1 The Business Cycle

We begin with the issue of how to measure the business cycle. The classical definition of this used by the NBER refers to the determination of turning points in the *level* of economic activity. A single series Y_t might be regarded as summarizing the level of economic activity and its turning points would then be the local maxima and minima in its sample path. It is convenient to work with the turning points in $y_t = \ln(Y_t)$ rather than Y_t. Since these turning points are identical the transformation loses no information.

Common usage of the word "recession" identifies it with a sustained decline in the level of economic activity. Consequently, a contraction is initiated by a *peak* in the level of activity and we need some rule to recognise when a peak (or trough) occurs. Visualizing a peak in a series leads one to the idea that a local peak in y_t occurs at time t if y_t exceeds values y_s for $t - k < s < t$ and $t + k > s > t$, where k delineates some symmetric window in time around t. One can define a trough in a similar way. By making k large enough we also capture the idea that the level of activity has declined (or increased) in a sustained way. Of course we need to limit the window in time over which this test is applied when performing the test. It is this simple idea that is the basis of the NBER procedures summarized in the Bry and Boschan (1971) dating algorithm. In that program, designed for the analysis of monthly data, $k = 5$. However, because much analysis is conducted with quarterly data we will take y_t to be a quarterly series and set $k = 2$ as an analogue. One can make the appropriate substitutions if monthly data is being examined for turning points.

Based on the above discussion we will define turning points in the *business cycle* in the following way.

$$\begin{aligned} peak\ at\ t &= \{(y_{t-2}, y_{t-1}) < y_t > (y_{t+1}, y_{t+2})\} \\ trough\ \ at\ t &= \{(y_{t-2}, y_{t-1}) > y_t < (y_{t+1}, y_{t+2})\}. \end{aligned} \tag{1}$$

These definitions could be re-expressed as

$$\begin{aligned} peak\ at\ t &= \{(\Delta_2 y_t, \Delta y_t) > 0, (\Delta y_{t+1}, \Delta_2 y_{t+2}) < 0\} \\ trough\ \ at\ t &= \{(\Delta_2 y_t, \Delta y_t) < 0, (\Delta y_{t+1}, \Delta_2 y_{t+2}) > 0\} \end{aligned} \tag{2}$$

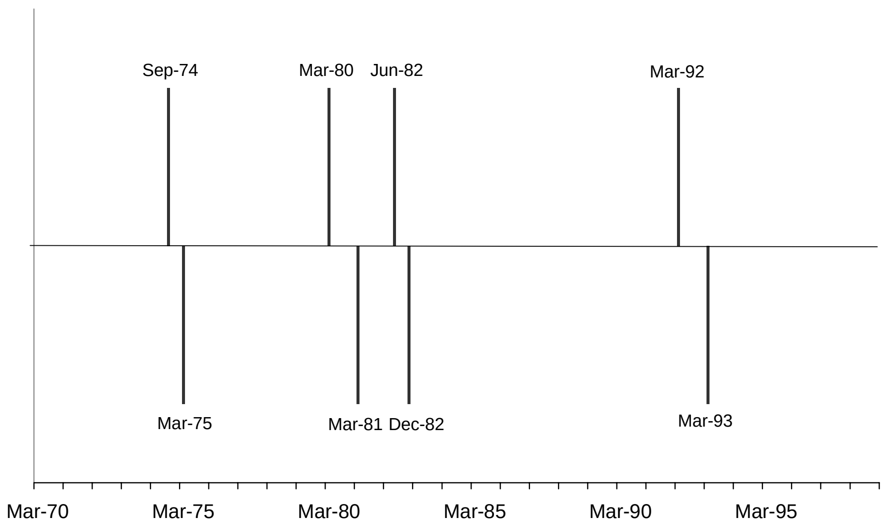
where $\Delta_2 y_t = y_t - y_{t-2}$. In words, a recession occurs if the level of economic activity declines for two quarters and an expansion if it increases for the same interval. In practice, the Bry and Boschan algorithm also applied some extra *censoring* procedures to the dates that emerged from applying the above rule. In particular the contraction and expansion phases must have a minimum duration of six months and a completed cycle must have a minimum duration of fifteen months. We emulate this by imposing two quarter and five quarter minima to the phase lengths and complete cycle duration respectively. Further details on the algorithms that are used to find turning points in this manner can be found in Harding and Pagan (2001a) where the computer program which implemented the above rules was termed BBQ. Applying the rules to Euro Area (EA) GDP we get the turning points shown in Figure 1 using quarterly data over the period 1970/1-1998/4[1].

One should think about this algorithm in the same way that we think about Taylor rules. It is not that a Taylor rule re-produces the actual decisions made by the Fed about the Federal Funds rate but that it is a good enough approximation to be a useful tool for summarizing their decisions. Thus, to assess the utility of the algorithm, we might compare the business cycle turning points established by applying the algorithm to quarterly US GDP data with what decisions were actually made about the turning points in economic activity by the NBER committee.[2] Figure 2 does this and it is apparent that the fit is very good. Because the NBER dating is based on monthly data it is quite possible that the two could differ by a single quarter and that is generally what happens. The only time when there is a discrepancy of greater than a quarter is the trough in 1970. Thus we conclude that this non-parametric dating rule seems to be a useful way of constructing business cycle information. It is a very simple

[1] Data for Euro Area GDP was taken from Fagan *et. al* (2001). Data on individual countries was taken from the OECD data base. In the case of Germany the statistics for unified Germany prior to unification were constructed by splicing the West German and unified Germany data in that database.

[2] In this instance the turning point dates remain the same whether the censoring rules are applied or not.

Figure 1: Classical cycle turning points in Euro area GDP

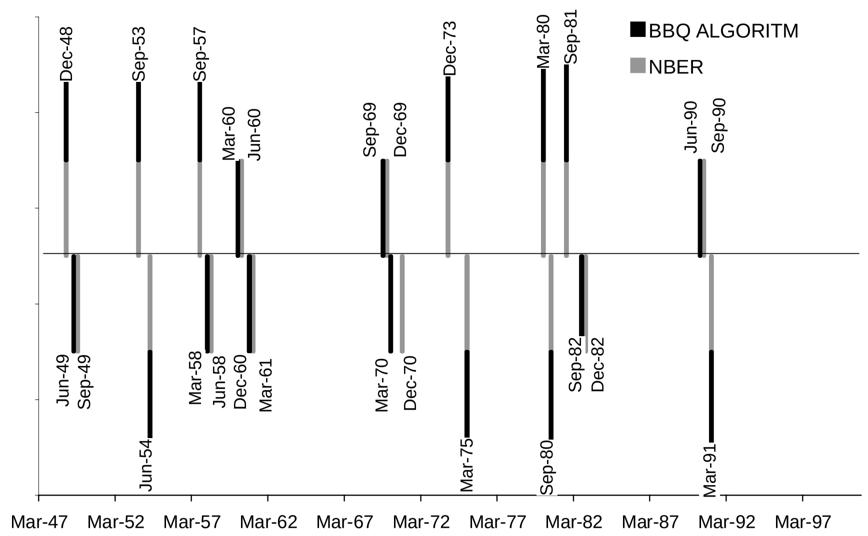


algorithm to apply and is very transparent. It is highly robust in that the dates would not change as one changed the sample of observations. It might not, of course, be robust to major changes in the window width indexed by k.

Now there are other claims made about how one should measure business cycles in the literature. In particular there is a strand of the literature that believes that y_t can be decomposed as $T_t+C_t+I_t$, where these are "trend", cycle and irregular terms respectively. Much effort is devoted to extracting $z_t = C_t + I_t$, inevitably devolving into extensive discussions of what are appropriate "trend removal" filters - see Hodrick and Prescott (1997)[3], Christiano and Fitzgerald (1998), Corbae et al (2001) and Baxter and King (1999). Since the business cycle relates to the turning points in the level of economic activity all such discussion is rather irrelevant since there is no need to remove any component from y_t. To us it is rather strange that these authors often justify their "trend" removal filters by citing business cycle information found by the NBER that pertains to the cycle in the levels of the series i.e. with T_t included in it. To some extent the confusion that has arisen over this issue comes from a semantic error and needs to be sorted out in order to understand why this "trend removal" literature is an unnecessary diversion for business cycle analysis. Fundamentally these filters are designed to remove a *permanent* component in a series (which is not unique as any **I**(0) series can be added on to the **I**(1)

[3] The working paper version of this paper was published in 1981.

Figure 2: Comparison of NBER and BBQ(GDP) turning points for the United States



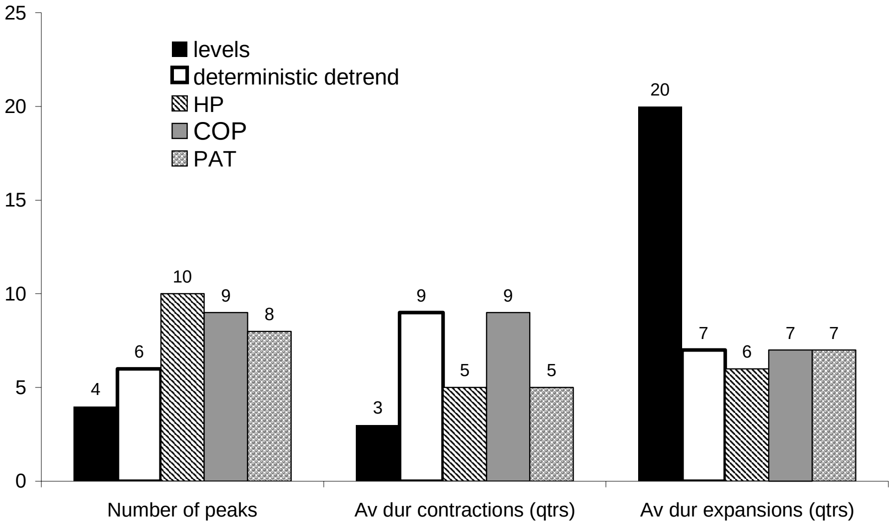
permanent component and still remain I(1)).[4] Then the permanent component is (invalidly) identified as "trend". What the RBC literature should have taught us was that the permanent component is a major cause of the business cycle so that removing it will mean we are no longer examining the business cycle. Indeed, the turning points in y_t and C_t can be very different. To see this, suppose we locate turning points in both the level of the European GDP series and in various series that result when it has been "detrended" by taking out a linear deterministic trend, the HP filter ($\lambda = 1600$), the Corbae, Ouliaris and Phillips (COP) filter and the phase-average trend filter (PAT) used by the NBER. This produces the summary statistics shown in Figure 3.

It is clear that the levels measure is, as expected, close to what we think of as the business cycle i.e. it produces a cycle with duration of around 23 quarters or 69 months. The others produce cycles whose durations range between 33 and 48 months and are therefore well away from the business cycle in their duration. The range of durations in the cycles of these filtered series reflect the differing degrees of serial correlation left over in z_t.[5] An important point

[4]Removing a permanent component does not mean removing the permanent shocks. This is most easily seen in the Beveridge-Nelson decomposition of a series like $\Delta y_t = e_t + \alpha e_{t-1}$ where the permanent component is $\Delta y_t^p = (1+\alpha)e_t$ and the transitory component is also a function of e_t and hence the permanent shock.

[5]Since all filters remove a deterministic trend and the turning points in z_t are obviously

Figure 3: Business cycle summary features compared for alternative filtering options



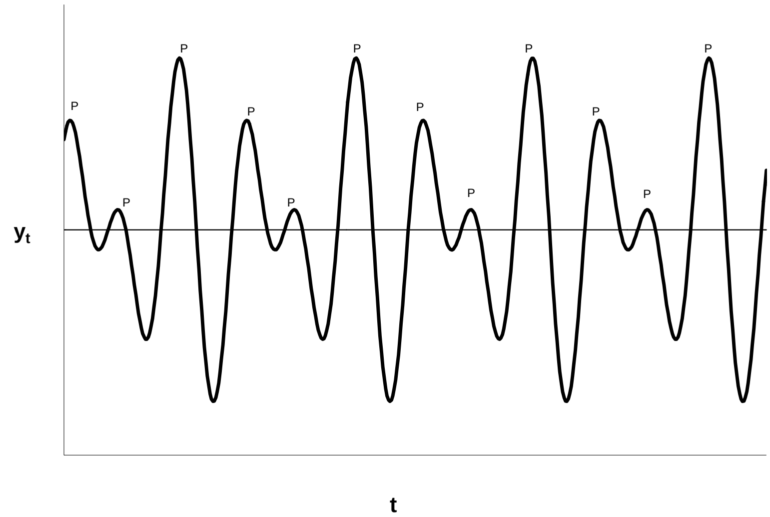
to notice here is that the cycle is defined through durations between turning points and not in terms of the spectral density of either z_t or Δy_t. Indeed, there is no clear connection with the definition of a cycle through its turning points with that derived from a spectrum. Even if y_t was composed only of a periodic cycle formed from deterministic cosine and sine waves it is clear that the period between turning points is not necessarily the period of the cycle in y_t as indicated by those functions. For example, consider $y_t = sin(t) + cos(1.5t)$. This generates cycles dated by NBER methods with (peak to peak) average duration of 4.2 periods (see Figure 4) but by construction y_t repeats itself every 4π periods.

Now it has long been held that the business cycle is not periodic and different ways to account for this appear in the literature. One is to regard the series as stochastic and then identify the cycle in it with the period derived from the frequency where the spectral density of z_t has a peak. But here again there may be no close connection between the period of the cycle identified by that definition and the cycle identified through the turning points in z_t. For example suppose we simulate quarterly data from the process

$$y_t = 1.4y_{t-1} - .57y_{t-2} + e_t$$

invariant to its standard deviation this leaves serial correlation in z_t as the sole remaining feature to determine the cycle characteristics.

Figure 4: Graph of $y_t = sin(t) + cos(1.5t)$ with turning points super imposed



where $e_t \tilde{} n.i.d.(0,1)$. This process has a spectral density peak at 23 quarters but application of the BBQ algorithm gives a turning point cycle of 11 quarters.[6] The fact that there is no connection between the two ways of defining a cycle was in fact noted by Sargent (1979, p 240) when he simulated data from an AR(1) process and observed that "This illustrates how stochastic difference equations can generate processes that "look like" they have business cycles even if their spectra do not have peaks...". Thus the emphasis placed on spectral quantities by writers such as Burnside (1998) and Cogley (2001) is mis-placed.[7]

A proponent of the HP filtered series as a way of measuring a cycle might argue that some of the turning points identified in a graph of its series are associated with phases that are too minor to be regarded as serious cycles.[8] This view would involve a censoring of the cycles according to some amplitude criterion. To analyse this suppose that we decompose T_t into a deterministic

[6] The latter is the equivalent of a population parameter as it is found by simulating hundreds of thousands of observations.

[7] Another example in the same vein is the cyclical component in Harvey and Jaeger (1993). Using the parameter values they give for this component for US GNP we simulate data and find that the average duration of the cycle is 13 quarters. This contrasts with a period of 22.2 quarters that spectral analysis would indicate.

[8] This may even be true of the business cycle turning points identified by a program such as BBQ. Some expansions might be regarded as too weak to qualify for that term. Thus an expansion in which (say) y_t rose just .5% in an economy which normally grew at (say) 3%pa might be regarded as too weak a recovery to be called that.

and stochastic part i.e. $T_t^d + T_t^s$ and define $z_t = y_t - T_t^d$. Then

$$\begin{aligned}\Delta_j y_t &= \Delta_j z_t + \Delta_j T_t^d \\ &= \Delta_j z_t + jb\end{aligned}$$

if $T_t^d = a + bt$. Hence application of the following dating rules to z_t

$$\begin{aligned} peak\ at\ t &= \{(\Delta_2 z_t, \Delta z_t) > (-b, -2b), (\Delta z_{t+1}, \Delta_2 z_{t+2}) < (-b, -2b)\} \\ trough\ \ at\ t &= \{(\Delta_2 z_t, \Delta z_t) < (-b, -2b), (\Delta z_{t+1}, \Delta_2 z_{t+2}) > (-b, -2b)\} \end{aligned} \tag{3}$$

produces the same turning points as if we had applied (1) to y_t. Thus imposing an amplitude constraint is equivalent to moving back to the turning points in the level of the series leading one to wonder why the filtering was done in the first place. It seems more sensible to us to simply work with the levels of the series i.e. not to bother doing any filtering. Nothing is lost by doing this. But if it is desired to remove a component then a deterministic trend is best since one can simply recover the business cycle information by adjusting the dating rules to (3). Moreover, this trend can be regarded as coming from the *long-run growth rate* as measured by $E(\Delta y_t)$. Most models have a deterministic trend around which steady state approximations are done i.e. they describe z_t and hence their predicted y_t can be recovered. If the model fails to do this it seems sensible to add back on to it the observed deterministic growth path, as we did in Harding and Pagan (2000a) in order to effect a comparison with business cycle statistics.

Once turning points have been found these can be used to segment the sample into periods of business cycle expansions and contractions. We identify these periods with a binary random variable S_t that takes the value unity in expansions and zero in contractions. The method we use to produce realizations of the random variable S_t from y_t is essentially non-parametric in nature. Other suggestions have been made to construct analogues of the S_t that are based on parametric statistical models. Chief among these is the class of methods associated with Markov Switching (MS) models. In this approach a series such as GDP is modelled as

$$\Delta y_t = \mu_j \zeta_t + \sigma_j \zeta_t e_t, j = 0, 1 \tag{4}$$

where $e_t\tilde{}n.i.d.(0,1)$ and ζ_t is a Bernoulli random variable. The evolution of ζ_t is that of a Markov Chain with transition probabilities $P(\zeta_t = j|\zeta_{t-1} = k) = p_{kj}$. Segmentation of the sample is performed with $\xi_t = 1(\Pr(\zeta_t = 1|F_t) > .5)$ where F_t is composed of either $\{\Delta y_{t+s}\}_{s=0}^{\infty}$ or $\{\Delta y_{t+s}\}_{s=-\infty}^{\infty}$ depending on whether one wants filtered or smoothed estimates of the probability. Thus the binary random variable is then said to be in a recession state when ξ_t takes the value zero and in expansion when it takes the value unity.

What can one say about this method of producing business cycle information? First, it is simply another method of performing a sample segmentation so that comments like that of Diebold and Rudebusch (2001, p6)

> "..because it is only within a regime switching framework that the concept of a turning point has intrinsic meaning...One can of course define turning points in terms of features of sample paths, but such definitions are fundamentally as hoc",

seems rather misleading. All methods which locate turning points involve ways of segmenting the sample into expansion and contraction states. The proof of the pudding is in the eating and we really need to ask whether the method used to perform the sample segmentation is one that appeals as a sensible way to proceed. Our argument is that doing the segmentation with (2) to produce S_t makes sense since these are consistent with a widely accepted definition of what constitutes a recession. In contrast the criterion used for sample segmentation with MS models to generate ξ_t is much more obscure. Harding and Pagan (2001b) looked at a simple example in the US context to argue that the MS dating rules that produce ξ_t effectively involve a combination of past and future values of Δy_t but there was no connection between these rules and any popular idea of what constitutes a recession. Of course it may well be that ξ_t and S_t are highly correlated, as they were when the idea was first used by Hamilton (1989), but one can devise situations where this will not be so.

Second, the introduction of a latent state has created a lot of confusion. For example it is often concluded that the average duration of a recession is $\frac{1}{1-p_{00}}$.However, it is ξ_t and not ζ_t which are expansion and recession states and so the average duration of a recession is $\frac{1}{\Pr(\xi_t=0|\xi_{t-1}=0)}$ and the random variables ξ_t and ζ_t may have quite different properties.[9] Finally, one wonders about the wisdom of using parametric models for a primary data summary. It is worth thinking of this in terms of the following analogy. Suppose we want to measure the mean and variance of a time series. We could just compute sample moments. However, an alternative is to fit an AR(1) process with μ as the intercept, ρ as AR(1) parameter and σ^2 as the variance of the innovations and then infer these quantities from $\frac{\mu}{1-\rho}$ and $\frac{\sigma^2}{1-\rho^2}$. If the AR(1) is a good description of the data then this would work well; otherwise it would not be very sensible. Indeed it would seem as if one would want to check that the AR(1) was a good fit to the data before engaging in such an exercise. Now the example above in which a parametric statistical model was used to summarize data is exactly what is being done with MS models when they are used as ways of dating the business cycle. Therefore, it does not seem sensible to regard the ξ_t as primary data summaries without checking that the MS model does fit the data well. It's probably significant that often ones sees investigators comparing ξ_t and S_t. But if all one wants is business cycle dates then why engage in an additional level of complexity by fitting an MS model? It's only if one wants a parametric statistical model of Δy_t for some purpose other than business cycle

[9] Indeed investigators have often argued that ζ_t should take three possible states but then they date the cycle and identify ξ_t as only having two values e.g. Krolzig and Toro (2000).

dating, say forecasting, that it makes sense to fit an MS model The ability to reproduce the S_t may then be thought of as a very useful diagnostic device since it focuses directly upon a quantity that is very meaningful to many consumers of the output.

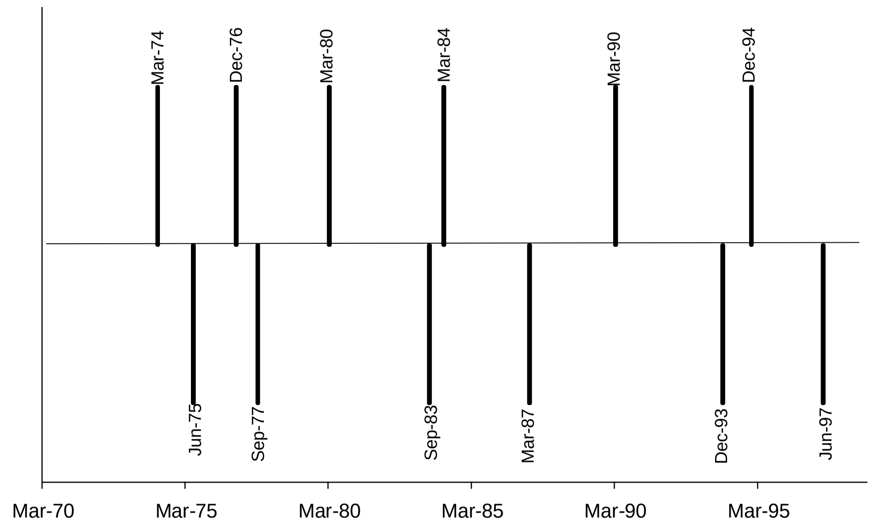
2.2 The Growth Cycle

Although there is little point in moving from y_t to z_t when one is trying to measure the business cycle there may be independent interest in forming some z_t. Initially the impetus for this came from the fact that, with a strong deterministic trend in the data, it would be rare to find a case where y_t declined, and so there would be no turning points in it. This situation described Germany and Japan over various parts of the post-WW2 period and was also true of some Asian economies prior to 1978. In those instances it was clearly of more interest to study turning points in a series from which a deterministic growth path had been removed. There were good economic reasons for this as well since it was the occurrence of low growth relative to the expected amount that was likely to cause stress in those economies since the institutions had adapted to high rates of long-run growth. In economies with relatively low long-run growth rates of the order of 2-3.5% p.a. it is a period of negative growth which creates stress since it is an extreme event. Hence the NBER rules and those which involve using two quarters of negative growth to mark a recession make sense for economies in which the long-run growth rate is relatively low but are not so useful if it isn't. But the fact that there may be some good arguments for subtracting a deterministic trend from y_t does not mean that there is a case for subtracting off a permanent stochastic component.

A second argument for investigating cycles in z_t is that these quantities often appear in applied macro-economic models as "output gaps" or measures of "disequilibrium" and so may be important in connecting the nominal and real sides of the economy. In particular, suppose that the inflation rate in country j depended on its output gap as measured by z_{jt}. Then, if the z_{jt} are not synchronized and do not have the same amplitude, one might expect that it will be difficult to engage in a common monetary policy without some stresses. Essentially the problem is that wage setting mechanisms may be very different in different countries and so the impact of z_{jt} upon inflation in the $j'th$ country can be very different To get a common inflation rate would then require very different values for z_{jt}. In this perspective the "right" measure for z_t, and so the "right" "detrending" methods presumably follow from what produces the best fit for the model relating inflation rates to output gaps z_t. It is important to observe that the argument for working with z_t is not specifically related to either the business or growth cycles since the focus of attention is really upon whether $z_t \gtrless 0$ and not whether $\Delta z_t \gtrless 0$ or $\Delta y_t \gtrless 0$. Nevertheless, there may still be a role for the latter information and we will return to this question in the final section of the paper.

Figure 5 presents the turning points for the EA growth cycle when a linear deterministic trend has been removed from the log of EA GDP. Comparing the results with Figure 1 one sees that there are 50% more turning points.

Figure 5: Growth Cycle Turning Points in Euro Area GDP



3 Measuring Cycles with Multivariate Information

3.1 The Business Cycle

In the above we have looked at the measurement of cycles of various types in a number of different countries and for the Euro area as a whole using a single series such as GDP. This fact brings up an objection that is often made to working with the turning points in a single series like GDP, namely that the business cycle is about "co-movement". There are probably close to an uncountable number of papers which make this claim. There is some merit in this position and we are agnostic about whether one should use a single series such as GDP or more information. In fact the NBER style of dating the cycle does use a lot of information other than output measures. The fundamental problem is of course the difficulty in identifying the "level of economic activity" at the frequency one wishes the cyclical information at. Although a single series

GDP has the advantage that it incorporates many sectorial output series it does weight these in a particular way. Apart from the weighting issue its other disadvantages are the frequency of collection and the difficulty in getting a long historical record at the desired frequency.[10]

There are two key issues in using multivariate information to provide cycle information. Neither revolves around how many series should be used but rather how one uses whatever number is selected to come up with a single measure of the business cycle. In the NBER strategy a number of series y_{jt} are selected and the methods described previously are used to find the turning points in each of these, leading to S_{jt}. Subsequently, the S_{jt} were combined to produce a series that represents the phase states S_t^a in the aggregate level of economic activity y_t^a. We refer to this procedure as the *aggregation of turning points*. It leads to the NBER's *reference cycle.*

A different approach is to aggregate the y_{jt} to produce a y_t^a and then to segment this sample to produce turning points We refer to this as locating *turning points in an aggregate.*

3.1.1 Turning Points in an Aggregate

Methods to combine the y_{jt} together tend to fall under the rubric of factor analysis. A common factor representing aggregate economic activity is taken to drive all the y_{jt} and the issue is how to extract it.[11] Quite a few proposals exist in this vein depending on how non–parametric one wants to be. Forni *et al.* (2000) tend to follow non-parametric methods whereas others e.g. Krolzig and Toro (2000) use a parametric statistical model for the factor.

Suppose we have the factor format

$$\frac{\Delta y_{jt} - \mu_j}{\sigma_j} = \gamma_j \Delta f_t + v_t, j = 1, .., N$$

where f_t is the common factor and there are N series for p countries. Forni *et al* first extract the common factor Δf_t using dynamic principal components of the Δy_{jt}. Then, if the first p series are GDP for p countries, the quantities $\gamma_j \Delta f_t$ are the common factor contributions to normalized GDP growth in each

[10] Burns and Mitchell (1946) said this about the use of a single series "Aggregate [economic] activity can be given a definite meaning and made conceptually measurable by identifying it with gross national product." Their reasons for choosing to use a larger number of series as inputs into the dating process resided in the fact that GDP was not available at the monthly frequency and, even for quarterly series, did not extend far back in history.

[11] There may be more than one common factor but these can be aggregated to make a single one after they have been isolated.

of these countries. Letting country k have weight ϕ_k in an overall GDP index the factor contribution to the growth in aggregate GDP will be

$$\sum_{k=1}^{p} \phi_k \mu_k + (\sum_{k=1}^{p} \phi_k \gamma_k \sigma_k) \Delta f_t.$$

Hence one could then find business cycle information by applying dating techniques to the *level* of this series i.e. to

$$(\sum_{k=1}^{p} \phi_k \mu_k) t + (\sum_{k=1}^{p} \phi_k \gamma_k \sigma_k) f_t.$$

Forni *et al* (2000) perform their analysis over a relatively short sample so it is hard to compare their business cycle turning points with those found using the NBER dating methods. They focus on the growth cycle[12] which is discussed in section 2.2. However, using their Figure 2, which shows their coincident index with the trend added back in, it is possible to discern a classical cycle peak at 1992/3 followed by a trough at 1993/2. These dates correspond to dates of 1992/1 for the peak and 1993/1 for the trough obtained by applying NBER dating methods applied to Euro GDP.

In the analysis above the nature of the factor is not specified. One might however specify a parametric model for it. Once estimated this model could then be used to extract the factor and then standard dating methods might be applied to the estimated f_t. Although many processes for f_t might be used a Markov switching process has been popular. Thus Krolzig and Toro (2000) proceed in this way although they actually have ζ_t in (4) following a three state process so that the rule for constructing ξ_t has to be modified. They compare their ξ_t to S_t found by using the BBQ approach and find a close correspondence.

3.1.2 Aggregation of Turning Points

The alternative approach is to aggregate the S_{jt}. This is effectively what the NBER does in producing a reference cycle. We illustrate this point using data on turning points in the level of activity of six countries forming part of EMU. Turning points in the business cycle were established for all six countries using the BBQ program applied to quarterly GDP data found from the OECD data base over the period 1970/1-1998/4. [13] There is clearly a good deal of similarity in the business cycles although some of the small countries seem to have extra

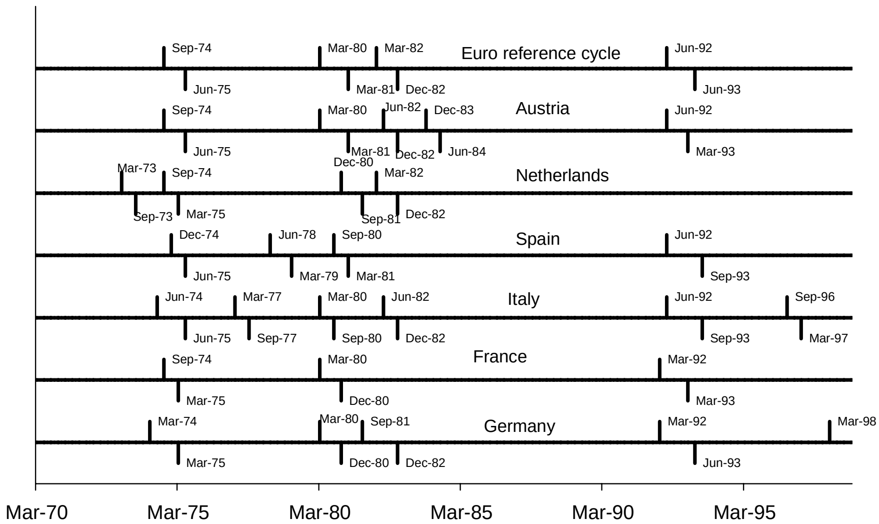
[12] Forni et al (2000) locate two completed growth cycles in their coincident index. Their turning points are 1990/1 (P), 1994/1 (T), 1995/1 (P) and 1996/2 (T). These correspond to dates of 1990/3 (P), 1993/4 (T), 1994/4 (P) and 1997/2 (T) foundby applying the NBER daying methods to the deviation of Euro GDP from a linear trend.

[13] Germany poses problems in getting a long historical series due to unification. We simply scaled up the West German series by a multiple found from the index values at an overlapping date.

cycles. The turning points were then aggregated to obtain a reference cycle using the algorithm described in Harding and Pagan (2000b). This algorithm is based on the NBER procedures as documented by the late Geoffrey Moore. The clearest description of those procedures are in Boehm and Moore (1984) who obtain a NBER-like reference cycle for Australia.

The specific cycles for individual Euro countries and the reference cycle obtained from aggregating the turning points are shown in Figure 6 below. It is evident that the reference cycle obtained from aggregating turning points (Figure 6) corresponds closely to the specific cycle in Euro GDP (Figure 1). That is, for the Euro case the business cycle is robust to whether it is measured via turning points in the aggregate or the aggregation of turning points. We have also found this robustness result to hold for the United States and Australia where the specific cycle in GDP yields a close approximation to the reference cycle.

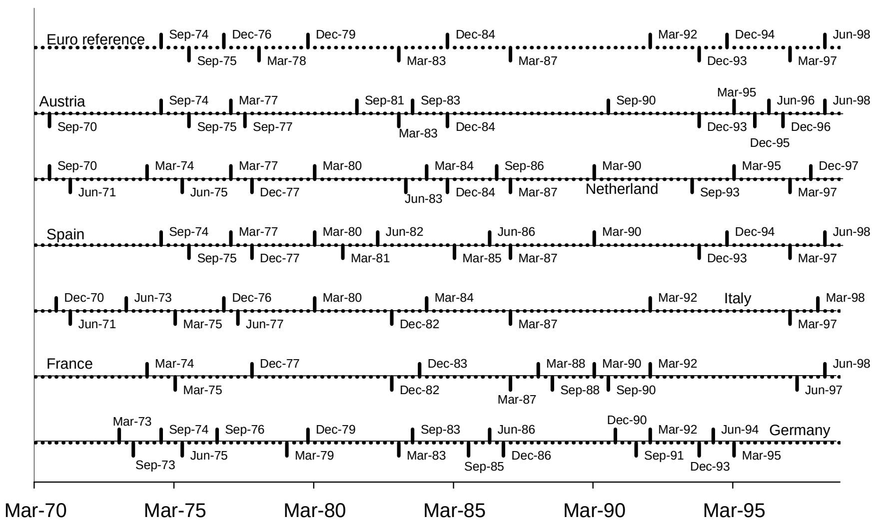
Figure 6: Euro area classical reference cycle and specific cycles in GDP for individual Euro countries



3.2 The Growth Cycle

We can also compute the growth cycle for the same six countries as in figure 6. We have subtracted off a first order linear trend in the log of GDP in each case. Figure 7 gives the turning points in each over the period 1970/1-1998/4. We use the BBQ rules to locate the turning points but, since these series are not as smooth, one might argue that the window width could be made larger. In Dungey and Pagan (2000) k was set to 3 when determining what the growth cycle implications were for an SVAR model of the Australian economy.

Figure 7: Turning points in the growth cycle for selected Euro countries and the Euro reference cycle



4 Analysing Cyclical Information

4.1 Analysing Information On Single Series

Once the states S_t have been constructed by some method we then wish to utilize the information $\{y_t, S_t\}$ to give us a picture of the characteristics of the cycle. The S_t can be used to produce measures of the durations of expansions and contractions while y_t can be used in conjunction with S_t to derive the amplitudes of phases and other quantities. All measures are sample means that estimate some population characteristic e.g. the average duration of an expansion will estimate $1/E(1 - S_t)$ where S_t is the binary random variable describing which phase of the cycle is being observed at time t and which is constructed from y_t using NBER-type rules. A complete description of the DGP of S_t is left until the next section.

As an example of the type of information that can be generated and how it relates to the underlying random variables consider estimating the number of peaks (hence expansions). An estimator of that quantity would be given by $NTP = \sum_{t=1}^{T-1}(1-S_{t+1})S_t$, since the series $(1-S_{t+1})S_t$ equals unity when a peak occurs at time t i.e. when $S_t = 1, S_{t+1} = 0$. Because the total time spent in expansions is $\sum_{t=1}^{T} S_t$ the average duration of an expansion will be[14]

$$\hat{D} = NTP^{-1}\sum_{t=1}^{T} S_t.$$

The average amplitude of expansions will be

$$\hat{A} = NTP^{-1}\sum_{t=1}^{T} S_t \Delta y_t.$$

However it is often desirable to go past such simple measures in order to capture the "shape" of the cycle. There is a large literature concerned with shape issues described in phases such as "steepness", "deepness" and "sharpness" - see Sichel(1993), Neftci(1984), Clements and Krolzig (2000). Sometimes this literature fails to grapple with the question in terms of the cycles that we are familiar with e.g. Neftci (1984) defines S_t as $S_t = 1(\Delta y_t > 0)$ which would produce far too many expansions and contractions when applied to a series like GDP. Moreover there is often no direct connection with the concept and the suggested test statistics e.g. Sichel(1994) looks at skewness in Δy_t as a test of steepness. The best way to think about these ideas is to follow the triangle diagram in Harding and Pagan (2001a) and think of a phase as a triangle which has y_t on the y axis and time on the x axis. In this triangle the base is the duration of the phase, D, while the height is the amplitude A. Then steepness of a phase is naturally measured by the angle of the triangle where the hypotenuse intersects the base and the tan of this is A/D, leading us to advocate $STEEP = A/D$ as a suitable index. The deepness of a phase is naturally measured by the height of the triangle and of course this is just the amplitude A. To measure the average degree of steepness over all (expansion) phases we would have

$$STEEP = \frac{\hat{A}}{\hat{D}} = \frac{\sum_{t=1}^{T} S_t \Delta y_t.}{\sum_{t=1}^{T} S_t}.$$

Thus a comparison of the steepness of expansions verses contractions in the average cycle would involve (due to a negative sign on the amplitude of a con-

[14] Some difficulties can arise with these formulae due to the possibility of incomplete phases at the both ends of the sample. In this paper we have estimated statistics based on both complete and incomplete phases If one used *completed* phases the summation should run from the beginning of the first completed phase until the end of the last one rather than over $1, ..., T$. Of course the asymptotic theory for the statistics would not be affected by that modification.

traction)[15]

$$
\begin{aligned}
COMP &= \frac{\sum_{t=1}^{T} S_t \Delta y_t.}{\sum_{t=1}^{T} S_t} + \frac{\sum_{t=1}^{T}(1-S_t)\Delta y_t}{\sum_{t=1}^{T}(1-S)_t} \\
&= \hat{p}_c^{-1} T^{-1} \sum_{t=1}^{T} S_t \Delta y_t + (1-\hat{p}_c)^{-1} T^{-1} \sum_{t=1}^{T} (1-S_t)\Delta y_t.
\end{aligned}
$$

where p_c is the proportion of time spent in contractions. The asymptotic distribution of $\sqrt{T}COMP$ is readily obtained[16]. It is worth dwelling on the relation of our proposed measure to that of Sichel's. Suppose that $S_t = 1(\Delta y_t > 0)$. Then our test would come down to comparing the sample means of the absolute values of the positive and negative values of Δy_t and so is a test for symmetry in Δy_t. Sichel's test also does this but using the cubes of Δy_t for the comparison. Its advantage seems to be that it measures and tests the notion of steepness in a direct rather than indirect way. It also generalizes to other ways of forming S_t. Certainly the simple rule that $S_t = 1(\Delta y_t > 0)$ would be inadequate as a description of most business cycle dates (unless one was using yearly data).

Sharpness was a concept introduced in McQueen and Thorley (1993) which looks at the extent to which peaks and troughs are rounded. The series $k^{-1}\Delta_k y_t(1-S_{t+1})S_t$ will be the average growth rate for the level of activity for the k quarters leading up to the peak at time t and $k^{-1}\Delta_k y_{t+k}(1-S_{t+1})S_t$ will be the average growth rate for the k quarters that follow the peak. McQueen and Thorley suggested that one form the difference of the two as a test statistic ie.[17]

$$
MT = k^{-1} \sum_{t=1}^{T} \Delta_k (y_{t+k} + y_t)(1-S_{t+1})S_t
$$

Their test is then a t-test that MT is zero. Although details are not given by McQueen and Thorley it seems as if the test is based on the assumption that $\Delta_k(y_{t+k}+y_t)(1-S_{t+1})S_t$ is *i.i.d.* In fact the use of the operator Δ_k will mean that there is serial correlation to be taken into account and, as we will see later, the presence of S_t will also induce serial correlation and heteroskedasticity. They comment that "The significance at which the null hypothesis of equal turning

[15]There is always an issue of whether we wish to consider measures for the *average cycle* or whether we want to take *the average of each statistic across cycles.* Thus the latter would measure the steepness of an expansion by $\frac{1}{\#peaks}\sum_{k=1}^{\#peaks}\frac{A_i}{D_i}$. The average characteristics we use are in fact a wighted average of the individual ones i.e. $\frac{A}{D} = \sum_{k=1}^{\#peaks}\frac{D_i}{D}\frac{A}{A_i}\frac{A_i}{D_i} = \sum_{k=1}^{\#peaks} w_i \frac{A_i}{D_i}$,where $w_i = \frac{D_i}{D}\frac{A}{A_i}$.

[16] Since $S_t(1-S_t) = 0$ the two elements making up $COMP$ are uncorrelated so the variance of $COMP$ will just be the difference in the variances of each .

[17]They actually compare the absolute value of one of the two quantities to the other. But since they choose $k = 6$ in order to use monthly data it should be the case that one quantity will always be negative and the other positive provided the turning points have been dated with NBER-type rules.

point sharpness is rejected is surprising given the low test power that results from the small sample size". It's possible that this is due to the serial correlation. It is rather difficult to apply this test to the business cycle dated with quarterly data since it is effectively based on using only $2 * k * (\#peaks)$ of data and for the EA cycle this would be 8 observations. For what it is worth there seems to be no evidence of any sharpness.

Rather than focus on the shape of phases around their turning points one might want to compare the shape in the early part of the phase with what it is in the later part. A simple way of capturing the shape of the cycle in this regard is to consider a phase such as an expansion and to compare the average growth over its first half with the average growth over the second.[18] Then simple models like a random walk will produce much the same average over the two half-phases and so the difference between these average growth rates will be zero. Some non-linearity is needed to produce a non-zero differential.

Let i be the $i'th$ phase and d_i be the duration of this phase, starting in time t_i, then the quantity we have just described can be represented as (if d_i is even; if d_i is odd we will replace $d_i/2$ by $(d_i/2)+1$ and adjust the average growth rate of the second half accordingly)

$$v_i = \frac{1}{d_i/2}[\sum_{k=1}^{d_i/2}(y_{k+t_i} - y_{k+t_i-1}) - \sum_{k=d_i/2+1}^{d_i}(y_{k+t_i} - y_{k+t_l-1})].$$

This is a measure of the shape of a single phase. Thus if an expansion has an early fast growth phase this measure will be positive. Generally however we want some overall measure across all phases. We could get this by summing v_i across all the relevant phases. However there is some argument for weighting the v_i with $w_i = d_i/N$,where N is the total period of time spent in the phases being focussed on. This weighted average gives more weight to longer phases, but, since the sum of w_i across all phases is unity, $E(\sum_i w_i v_i)$ should still be zero if $E(v_i) = 0$. Hence the aggregate test we propose is based on

$$A = \frac{1}{N}\sum_i (d_i/N)v_i.$$

This test is related to the "excess" test introduced in Harding and Pagan (2000a) but is likely to be more precise in that the older test gives equal weight to each phase and so may be affected by a very imprecisely estimated d_i if that particular

[18] In a similar way to the Goldfeld-Quandt test for heteroskedasticity one might even consider splitting the phase into the first and last thirds and comparing growth over these segments. For expansions this may be satisfactory as they tend to be lengthy, but, for contractions, which rarely go past four quarters, there would be a major loss of information. In any case contractions of the business cycle are really too short to gain much useful information about their shape. Things are different for the growth cycle where contractions and expansions are of much the same duration.

phase is short. Indeed, we suspect that this is a problem with the test based on simulations in Athanasopoulos *et al.* (2001).

4.1.1 The Business Cycle

In Table 1 we summarize the nature of the business cycles in different countries using the measures suggested above.[19] The US is included as a benchmark that we have studied previously over a much longer period of time.[20]

Table 1: Business Cycle Characteristics of selected countries, March 1970-December 1998

	GE	FR	IT	SP	NE	AU	EA	US
Duration								
PT	4.25	3.00	2.83	3.00	2.50	2.80	3.00	3.50
TP	19.75	32.50	14.80	21.00	9.67	15.00	20.67	16.75
Amplitude								
PT	-2.25	-1.56	-1.57	-1.00	-2.37	-1.18	-1.43	-2.52
TP	14.44	21.29	11.29	14.59	7.85	12.14	15.27	18.03
Excess								
PT	-0.03	.0.03	-0.06	-0.04	-0.03	-0.04	-0.00	-0.13
TP	-0.38	-0.37	0.33	-0.29	1.02	-0.25	-0.18	1.25

There are clear differences in the business cycles across countries. France stands out as having very long expansions and the Netherlands rather shorter ones, although figure 6 points to this as having been more because the Netherlands has a long uncompleted expansion phase in the 1980s. In general the EA tends to have a longer cycle than the US although the differences are not great. The amplitude of US expansions are somewhat greater than for Europe but the difference is not particularly remarkable. A recurrent theme in US business cycle research, that expansions are very strong in their early stages, is found to hold for Italy and the Netherlands only.

To understand some of these outcomes consider the following simple models for the growth rate in GDP.

$$\Delta y_t = \mu + \sigma e_t$$

$$\Delta y_t = \rho_1 \Delta y_{t-1} + \rho_2 \Delta y_{t-2} + v_t$$

[19] The countries considered comprise Germany (GE), France (FR), Italy (IT), Spain (SP), Netherlands (NE), Austria (AU), Euro area (EA) and United States (US).

[20] The US data is taken from the Bureau of Economic Analysis home page, it is in chained 1996 dollars.

Since S_t is constructed from Δy_t we know that the business cycle characteristics come from the magnitudes of μ/σ and the nature and extent of serial correlation. Exactly how these interact is quite complex but the general outcomes for the cycle can often be interpreted from a knowledge of the magnitude of these quantities. Table 2 presents these. It is worth noting that in no case are there complex roots in the polynomial $(1 - \rho_1 L - \rho_2 L^2)$.

Table 2: GDP Growth Characteristics

	GE	FR	IT	SP	NE	AU	EA	US
μ	0.54	0.61	0.58	0.73	0.64	0.65	0.61	0.78
σ	0.77	0.64	0.85	0.64	1.09	1.23	0.63	0.92
ρ_1	0.22	0.21	0.52	0.88	-0.17	-0.26	0.23	0.23
ρ_2	0.10	0.18	-0.03	-0.08	-0.02	-0.02	0.19	0.14

Thus despite the fact that Spain has a very high trend rate of growth compared to the Euro Area average, it has much the same business cycle characteristics as the EA. The resolution of this difference comes from the extremely strong positive serial correlation in the growth rates of output since these make it more likely that one will see a second negative growth rate after encountering the first and that signals a higher probability of a recession than for the EA where the serial correlation is much lower. The effect of this strong positive serial correlation is to reduce the cycle length. By the same argument negative serial correlation, as in the Austrian statistics, will tend to lengthen the cycle, although the larger volatility of growth rates for that economy will pull in the opposite direction There is so much extra volatility that it has a dominant effect upon the cycle characteristics.

4.1.2 The Growth Cycle

Growth cycle characteristics are summarized in Table 3 below. In this case z_t will have a zero mean by construction and the volatility of z_t is irrelevant since the turning points in $std(z_t)^{-1}z_t$ and z_t are the same. All that matters is the degree of serial correlation in z_t. All series have close to a unit root in z_t but some also have high serial correlation in growth rates, see Table 2 above. Simulations show that, for an AR(1) with coefficient of .2 for Δy_t and normally distributed growth rates, the average cycle length is 12 quarters while, when the AR(1) coefficient becomes .8, this rises to 20 quarters, although if the shocks are symmetric then the expansions and contractions will both be 10 quarters long. Hence one can understand the longer expansions for the Spanish growth cycle but the asymmetry must come from some asymmetry in the growth rate process. It is interesting to note that the AR(2) coefficient fitted to the growth rate in Spanish GDP is not significant but inspection of the autocorrelation function shows that it fluctuates and that it is hard to capture with simple linear stationary models.

In general the growth cycles are reasonably similar across European countries both in their duration and amplitude. Certainly they resemble one another much more closely than they resemble the US growth cycle. The shape effect that is evident in the US growth cycle is not so evident in the European growth cycle.

Table 3: Growth Cycle Characteristics

	GE	FR	IT	SP	NE	AU	EA	US
Duration								
PT	9.00	7.00	6.38	4.86	5.56	10.50	9.33	5.38
TP	9.17	6.57	7.25	10.14	4.75	5.67	7.20	6.88
Amplitude								
PT	-4.05	-3.12	-4.05	-2.06	-3.21	-4.21	-3.35	-3.82
TP	4.13	2.56	4.04	3.00	3.30	3.34	2.95	4.02
Excess								
PT	0.03	-0.10	.0.06	0.07	0.10	-0.03	0.04	0.22
TP	-0.18	0.14	-0.13	-0.01	0.18	0.10	0.01	0.21

4.2 Cyclical Relations Between Series

As mentioned earlier the fact that the Euro area is composed of a number of countries for which data on individual country cycles is often available means that the study of the business cycle in the Euro Area will always be one that monitors country specific information. The data is of varying quality and may not go far back into time, at least for a quarterly or monthly frequency. However its presence brings forth the issue of how closely the cycles of individual countries are synchronized with the EA cycle. Accordingly, we need to be able to address that issue, and the most popular index that has been proposed for this task is the *index of concordance,* which measures the fraction of time spent in the same phase. If T is the period over which synchronization is being checked then the concordance index between the cycle in country j and the Euro Area cycle is:

$$I_j = \frac{1}{T}\sum_{t=1}^{T}\{S_i^a S_{jt} + (1 - S_t^a)(1 - S_{jt})\}.$$

This is a variant of the contingency table test of Pearson. The latter has been used by Artis *et al* (1997) while the concordance index stated above was used by Harding and Pagan (2000b) and Cashin *et al.* (1998). The index has wider use than just associating country and Euro Area cycle information. In particular, S_{jt} may be some indicator of the Euro Area cycle such as industrial production.

Then, by lagging the states, say by using S_{jt-k} in place of S_{jt}, one could compute an index which assesses whether the y_{jt} from which S_{jt} was derived is a leading indicator of the Euro Area cycle.

It is not clear whether this index is the best measure of synchronization. Suppose that S_t^a and S_{jt} were independent. Then $E(I_{jt}) = 1+2E(S_{jt})E(S_t^a) - E(S_t^a) - E(S_{jt})$ and so one might have a high value of I_j simply because $E(I_j)$ is high owing to the large fraction of time spent in expansions. Thus it may be desirable to mean correct I_j and this points to the computation of a correlation coefficient between S_{jt} and S_t^a as a natural measure of "co-movement" or synchronization.

After mean correction the new index can be shown to be proportional to the regression coefficient estimate attached to S_t^a in the regression of S_{jt} against a constant and S_t^a. In turn this is proportional to the correlation coefficient between S_t^a and S_{jt}. Note that there is a complex relation between this correlation and that between either $(\Delta y_{jt}, \Delta y_t^a)$ or (z_{jt}, z_t^a) where the latter are "detrended" quantities. Estimates of this correlation and the expectation of I_j if S_{jt} and S_t^a are independent i.e.$E[S_t^a]E[S_{jt}] + E[(1 - S_t^a)]E[(1 - S_{jt})]$ are given in Tables 4 and 5 for both the business and growth cycles. Mostly the EA countries on which we have data show good correlations with the EA GDP cycle, at least in comparison with the U.K which is presented as a benchmark for comparison. Of course a cynic might say that what matters politically is concordance and not correlation- it is only if one economy is in recession while the Euro Area is in expansion that there would be opposition to contractionary monetary policies.

Table 4: Concordance and Correlation of Euro and Individual Country Business Cycle States

	$\hat{I}_j$	$\hat{E}(I_j)$	corr
Germany	0.91	0.76	0.67
France	0.97	0.84	0.85
Spain	0.91	0.78	0.57
Italy	0.90	0.81	0.44
Netherlands	0.90	0.83	0.39
Austria	0.97	0.80	0.83
UK	0.84	0.77	0.31

A final use of the concordance statistic is not for business cycles *per se* but to test synchronization of output gaps something that is also done by Scott (2000).[21] Table 6 gives this where the states are now $S_{jt} = 1(z_{jt} > 0)$. One

[21] Scott's tests are, however, invalid as they are not made robust to the serial correlation that is evident in the output gap states.

Table 5: Concordance and Correlation of Euro and Individual Country Growth Cycle States

	$\hat{I}_j$	$\hat{E}(I_j)$	corr
Germany	0.67	.50	0.35
France	0.75	.50	0.50
Spain	0.79	.50	0.59
Italy	0.84	.50	0.70
Netherlands	.0.84	.50	0.69
Austria	0.72	.50	0.47

might think that such synchronization is important since, if domestic Phillips curves depend on domestic output gaps, a lack of synchronization of output gaps across the EA will potentially cause political ferment. Again there seems to be a reasonable degree of synchronization.

Table 6: Concordance and Correlation of Euro and Individual Country Output Gap States

	$\hat{I}_j$	$\hat{E}(I_j)$	corr
Germany	.91	.50	.83
France	.72	.50	.43
Italy	.82	.50	.63
Spain	.73	.50	.48
Netherlands	.84	.50	.68
Austria	.81	.50	.63

5 Using Cyclical Information

5.1 The Nature of Cyclical State Variables

Cyclical state indicators S_t have the important feature that they are "constructed" random variables, to use the terminology in Harding and Pagan (2001c). They are the product of the interaction of the nature of the process y_t which they are derived from with the rules that are used to form them. Their properties crucially depend upon this interaction. It is important to realize that, because they are constructed variables, they cannot be treated in the same way as they would be in micro-econometric work where information is directly available (say) on whether a person is unemployed or not. To appreciate why this is so consider how modelling typically proceeds in the latter case. First it is assumed that there is some latent variable process y_t^* that is a linear function of a single index $x_t'\beta$ of the form

$$y_t^* = x_t'\beta + u_t^*, \tag{5}$$

where u_t^* are $i.i.d.$ Secondly, a density for u_t^* is prescribed and $S_t = I(y_t^* > 0)$ is taken to be the rule for generating the S_t. With these assumptions it follows that

1. $\Pr(S_t = 1) = F(-x_t'\beta)$ where $F(.)$ is the cumulative distribution function of u_t^*.

2. The S_t are independently distributed.

Neither of these assumptions remains true when S_t are constructed, even when the latent variable model is as in (5) and the censoring rule is just $S_t = 1(y_t^* > 0)$. It is this fact that makes it much harder to correctly utilize constructed variables such as cyclical indicators.

Since the impacts differ depending on whether S_t are used as regressors or regressands we will arrange our discussion under those headings. Our example will be the business cycle and so we will want to use a rule that produces states S_t that are close to those found with NBER methods. In general it is hard to do analytic work with the NBER type of rule that is embodied in BBQ rules and so we have often resorted to working with simpler rules for illustrative purposes. These simpler rules are stated using a general approach to dating methodologies set out in Harding and Pagan(2001a) which works with the concepts of Expansion (Contraction) Terminating Sequences ($E(C)TS$). Here an ETS signals a move from the state $S_{t-1} = 1$ to $S_t = 0$ while a CTS governs the movement from $S_{t-1} = 0$ to $S_t = 1$. In this lexicon a "calculus rule" for dating turning points involves an $ETS = \{\Delta y_t < 0 | S_{t-1} = 1\}$ and its associated $CTS = \{\Delta y_t > 0 | S_{t-1} = 1\}$. Consequently, when the calculus rule is adopted, $S_t = 1(\Delta y_t > 0)$, where $1(.)$ is the indicator function taking the value unity when the event in brackets is true. Our approximation to NBER type rules is the "Extended Okun Rule" defined as $ETS = \{\Delta y_t < 0, \Delta y_{t+1} < 0 | S_{t-1} = 1\}$ and a $CTS = \{\Delta y_t > 0, \Delta y_{t+1} > 0 | S_{t-1} = 0\}$. As mentioned earlier the dates selected with the rules above have to be subjected to a further round of consolidation through the imposition of minimum phase and cycle lengths to meet NBER restrictions. Generally BBQ produces an identical number of peaks (six) to that given by the NBER for the post-WW2 US business cycle while the extended Okun rule has one fewer at five and the "raw" calculus rule behaves badly with some 14 peaks. Nevertheless, after censoring, the latter produces much the same outcomes as BBQ.

In the case of the NBER dating process, where a committee determines the business cycle dates, we do not know exactly which primitive series are used as inputs into the dating process. Nevertheless, for analytical purposes we might presume that there is a single series y_t^* which is used by them for dating cycles but which is unknown to the econometrician. We then make some assumptions about the dating rule and the DGP of y_t^* and thereby derive the DGP of S_t. In its simplest incarnation we might assume that y_t^* is a random walk with a drift μ_t and normally distributed increments i.e. Δy_t^* is $i.d$ $N(\mu_t, \sigma^2)$. The latter assumption is a good approximation to many real series such as GDP. The time variation in the drift rate would arise if it is a function of some exogenous variable x_t, for example, $\mu_t = a + bx_{t-k}$, It will be convenient to let $\Im_t = \{x_{t-i}\}_{i=0}^{\infty}$ represent the history of this exogenous variable.

In Harding and Pagan(2001c) we show that, when the Extended Okun rule is used as the dating method,

(i) The S_t follow a process that is at least second order Markov.

(ii) $Pr(S_t = 1|\Im_t) = E(S_t|\Im_t)$ is a function of all elements of $\{a+bx_{t-j}\}_{j=k-2}^{\infty}$.

As we will detail later there are important implications of (i) for accurate inference. However the result also implies another feature, namely that *there must be duration dependence in states found using NBER type rules* since the hypothesis which is being tested is whether S_t follows a first order Markov process. To some extent the reason for S_t following a higher order Markov process is the minimum phase restriction that expansions and contractions must last at least two quarters. Some tests for duration dependence e.g. Diebold and Rudebusch (1990) try to make an allowance for this by recognizing that, if S_t is a first order Markov process, then the censoring means that the density of durations from these states would be a left censored exponential and the test for a first order Markov property is done with a censored exponential density as the null. Whether this adjustment is sufficient to overcome the constructed nature of S_t remains to be seen. One argument for why it may not be is as follows. Suppose we abstract from censoring by adopting the "calculus" rule for dating the cycle i.e. $S_t = 1(\Delta y_t^* > 0)$. Then Kedem (1980) shows that S_t follows a higher order Markov process whenever there is serial correlation in the Δy_t^*. Since there can be very few real series which are pure random walks it is hard to see how S_t would not be higher order Markov and so one should conclude that there is duration dependence in them. Failure to find it probably says more about the power of the duration based tests than anything else.

We should note here another confusion in the literature over this issue which again arises from assuming that Δy_t follows a Markov switching process as in (4). There are a number of papers which check for duration dependence in the transition matrix of the ζ_t e.g. Durland and McCurdy (1994). However this is not testing for duration dependence in either the ξ_t or the S_t. Indeed if the constant transition probability MS model is a good description of Δy_t then it must be the case that Δy_t exhibits serial correlation, see Timmermann (2000), and hence the ξ_t (and S_t) states must feature duration dependence.

5.2 Cyclical State Indicators as Regressands

In a good deal of the literature the S_t are sometimes either explicitly or implicitly made a function of some regressors x_t. One such approach is the latent variable described in the introduction, which in this context has the form

$$\Delta y_t^* = a + bx_{t-k} + u_t^*, \tag{6}$$

where u_t^* is $n.i.d.(0,1)$. Estimation of a and b is then done under the assumption that $E(S_t|\Im_t) = \Phi(-a - bx_{t-j})$, see for example Estrella and Mishkin (1998). Now the problem with this strategy is that the conditional mean in (6) is incorrect, since we observe that an implication of (ii) when the S_t are constructed is that $E(S_t|\Im_t) = \Pr(S_t = 1|\Im_t)$ is a function of the *complete history* of x_t and not just x_{t-k}. Failure to recognize this means a mis-specified log likelihood and so inconsistent estimators of a and b. Thus many studies which utilize Probit estimation with the NBER states, such as Estrella and Mishkin (1998), are in error. Others, such as Chin *et al.* (2000) and Birchenhall et al. (1999), are explicit about the fact that S_t is a constructed variable in time series but then ignore the method of construction when specifying and estimating the models that seek to explain S_t.

Even if the $\Pr(S_t|\Im_t)$ could be well approximated by $a + bx_{t-j}$ hypothesis tests about b and a would need to account for the fact in (i) that S_t is serially correlated. To assess the likely degree of this we note that S_t is a binary random variable and so we can always write it as

$$S_t = p_{01}^s + (1 - p_{01}^s - p_{10}^s)S_{t-1} + \eta_t, \tag{7}$$

where $p_{ij}^s = \Pr(S_t = j|S_{t-1} = i)$ and η_t is discrete and conditionally heteroskedastic since it depends upon S_{t-1}. Consider the case where Δy_t has no serial correlation. Then the magnitudes of p_{ij}^s are determined solely by the dating rule employed. If the calculus rule is used to derive S_t we would have $p_{01} + p_{10} = 1$ and so there is no serial correlation in S_t. In contrast, when the extended Okun rule is adopted there is serial correlation. In fact, for US data Harding and Pagan (2001c) show that one would predict that the equation for the evolution of the states would be

$$S_t = .35 + .62S_{t-1} + \eta_t. \tag{8}$$

Fitting an AR(1) to the "NBER states" (found from their web page) over the

same period yielded

$$S_t = .29 + .67S_{t-1} + \eta_t, \tag{9}$$

which shows that the predicted serial correlation does materialize. The equivalent regression for the EA GDP states is

$$S_t = .33 + .63S_{t-1} + \eta_t.$$

Of course this is just meant to illustrate the fact that the states have reasonable amounts of serial correlation in them and not that the DGP is a first order Markov process. In fact, as (i) claims, the DGP must be least second order. Therefore, fitting an AR(2) process to the EA states produces

$$S_t = .41 + .77S_{t-1} - .23S_{t-2} + \eta_t,$$

with heteroskedastic robust t ratios for the AR parameters of 7.7 and 2.3 respectively

5.3 Cyclical State Variables as Regressors

It may also be the case that one wishes to use a cyclical state indicator S_t as a regressor i.e. we might fit relations such as

$$y_t = a + bx_t + cS_t + dx_tS_t + e_t$$

where the relation between y_t and x_t changes according to the state of the cycle. Thus y_t might be output and x_t might be an interest rate or y_t might be inflation and x_t an output gap. Such possibilities are often mentioned. In particular there have been tests for the asymmetric effects of monetary policy e.g. Cover (1992) but in the past these have $S_t = 1(w_t > 0)$ where w_t might be e_t or Δy_t. Clearly such tests do not effectively address whether the impact of monetary policy is different in different phases of the cycle since the resulting S_t do not delineate the business cycle phases.

Since we know that

$$\Pr(S_{t+1} = 0, S_t = 1) = E1(\Delta_2 y_t > 0, \Delta y_t > 0, \Delta_2 y_{t+2} < 0, \Delta y_{t+1} < 0)$$

for the BBQ rule it is clear that we cannot use S_t as a regressor since it is a function of e_{t+2}, e_{t+1} and e_t. It would however be possible to use S_{t-3} as an instrument for S_t. Even then this would not be a good equation for prediction purposes (since S_t would be unknown) and this points to the estimation of relations such as

$$y_t = a + bx_t + cS_{t-3} + dx_tS_{t-3} + e.$$

As an illustration of this approach we take a simple Phillips curve estimated by Gali et al. (2001)(GGLS) in which y_t is the quarterly inflation rate, $\ln P_t - \ln P_{t-1}$,x_t includes an output gap (formed from subtracting a linear deterministic trend from the log of EA GDP) and four lags of y_t.We modify their basic equation in a number of ways. First we use the annual inflation rate $\ln P_t - \ln P_{t-4}$ as the dependent variable. Since four lags of y_t appear on the RHS of the equation this is just a re-parameterization that makes the dependent variable the one of policy interest.[22]Secondly we consider augmenting the equation with the change in the output gap to capture the well known fact that rapid movements

Table 7: European Inflation Equations

	GGLS		Modified	
	Est	*t*	*Est*	*t*
c	.0006	.95		
y_{-1}	1.52	16.95	1.44	16.97
y_{-2}	1.23	11.98	1.25	13.18
y_{-3}	.93	9.16	.98	10.45
y_{-4}	.25	2.81	.33	3.90
gap_{-1}	.05	3.17	.03	2.16
Δgap_{-2}			.13	2.80
$(\Delta gap_{-1})S_{t-3}$			.18	3.71

in demand pressure have stronger impacts upon inflation outcomes. Table 7 contains estimated results.

The F test for the deletion of the extra variables Δgap_{-2} and $(\Delta gap_{-1}S_{t-3})$ which were not in the GGLS equation is 11.75, which is highly significant. Thus the equation indicates that, when the output gap is increasing and the economy is in an expansion, there are much stronger inflation effects then when the output gap is increasing in a recession. Thus the state of the business cycle can be a useful adjunct to the output gap as a factor in influencing inflation outcomes.

5.4 Cyclical State Variables as Regressand and Regressor

Earlier we indicated that a suitable measure of synchronization was the concordance index. Often however we want to test if two cycles represented by S_{yt} and S_{xt} (and derived from underlying random variables y_t and x_t) are independent If we form the linear regression

$$S_{yt} = a + bS_{xt} + \eta_t$$

then the estimate of b from this regression is a multiple of the correlation coefficient between S_{yt} and S_{xt} and so we can test for independence by testing if b is zero. But under the null hypothesis it is clear that the error term η_t inherits the serial correlation properties of S_{yt} and so this must be allowed for in any hypothesis tests about b. It is also the case that S_{yt} is heteroskedastic and so we need to form t ratios that are robust to both heteroskedasticity and serial correlation Table 8 therefore conducts tests for independence of the business and growth cycles in Europe using the HAC robust standard errors in Hamilton (1994) with serial correlation of order 10. Also presented are the t ratios before an HAC adjustment. It is clear from these results that one needs to be very careful to conduct inferences with the robust t ratios as the answers are

[22]Gruen et al (1999) use this idea.

critically dependent upon which t ratio one uses. It is possible that the use of asymptotic results here might not work as well as they normally do owing to the binary nature of the regressand and regressor and one needs to engage in some simulation work to assess the utility of such an adjustment.

Table 8: Tests of Synchronization

	Bus Cycle			Growth Cycle		
	b	*t*	*robt*	*b*	t	*robt*
Germany	0.83	9.6	2.5	0.35	4.0	2.1
France	0.75	17.5	3.0	0.50	6.1	2.4
Italy	0.67	7.6	2.7	0.59	7.8	2.8
Spain	0.44	5.3	1.8	0.70	10.5	3.0
Netherlands	0.37	4.7	1.3	0.69	10.3	3.0
Austria	0.89	15.9	3.1	0.45	5.7	2.3

Taken at their face value the results indicate that there is rather weak synchronization of the business cycles in Europe but somewhat stronger evidence for a synchronized growth cycle. It is useful to think about this in terms of the factor models earlier since the lack of synchronization may suggest that there is no common factor. If the growth rates and volatility of quarterly GDP were equal, and there was no idiosyncratic variation in any country, there must be perfect synchronization, but the converse need not hold since there may be substantial variation across countries in the features that have just been held constant in the conceptual experiment. Since the growth cycle abstracts from trend growth, and volatilities are irrelevant to turning points, it is dis-similarity in serial correlation patterns that may cause divergence, and we actually see this in the cases of Spain, Austria and the Netherlands which differ from the serial correlation in the EA output gap. Nevertheless, the impact of serial correlation upon growth cycle characteristics is rather muted, at least in comparison to that of trend growth and volatility upon the business cycle.

Table 9: Tests of Synchronization of Output Gaps

	b	*t*	*robt*
Germany	0.62	8.5	2.7
France	0.84	16.0	3.1
Italy	0.48	6.0	1.9
Spain	0.69	10.2	2.8
Netherlands	0.44	5.2	2.0
Austria	0.64	8.8	3.0

Table 9 tests for synchronization of output gaps. Although previously the concordance indices seemed of reasonable magnitude it is clear that the synchronization is a lot weaker than one would like. In particular the weak evidence for synchronization of the Italian and EA output gaps is a potential worrisome problem for the ECB monetary policy makers.

6 Conclusions

We have summarized NBER-like methods that can be used to extract cyclical information on the business cycle. Here we find that the NBER peak and trough dating methods can be formalized into simple algorithms that provide useful tools for segmenting series into recurring patterns of expansions and contractions. We recommend applying these methods to the level of the series to obtain the business cycle (S_t) and to the series after a deterministic trend has been removed to study the growth cycle (S_t^g). From studying S_t we can learn about the interaction between long run growth abd the cycle while study of S_t^g yields information about serial correlation in the data generating process.

An important practical issue is whether to locate the turning points in some aggregate measure of economic activity such as real GDP or whether to aggregate turning points in several series as in the NBER reference cycle. We have produced tools for both approaches and compare the cycles so obtained. The results for the Euro are that the cycles located in the aggregate (Euro GDP) concord closely with the cycles obtained by aggregating the turning points for individual countries' GDP. We have found a simmilar result for both the United States and Australia in that the reference cycles for those countries obtained by aggregating the turning points in many series correspond closely to the turning points located in GDP.

Once the states have been constructed we can combine $\{S_t, y_t\}$ to create statistics that characterise the cycle. These include duration, amplitute and measures designed to capture the shape of phases. We find that there are differences among Euro countries in both their classical and growth cycles. These differences point to tensions that will be experienced as the European Central Bank tries to manage these cycles within a single monetary policy.

Policy interest centres on the synchonization of cycles within the Euro area. A number of people have sought to measure synchronization via an index of concordance that measures the proportion of time that two cycles are in the same state. We point to some statistical problems with this statistic as a measure of the degree of synchronization and suggest some alternative measures. We apply these alternative measures and show how the conclusions are modified. We find rather weak synchronization between the aggregate Euro cycle business cycle and the cycles of individual Euro countries. Evidence for synchronization of the Euro growth cycles is somewhat stronger than for the business cycle.

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