Alliances Among Asymmetric Countries

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Abstract

We examine alliances between asymmetric countries. We find that the results depend on the nature of the equilibrium. If the equilibrium is an interior one then, with an increase in *asymmetry*, the level of the alliance-wide defense good *decreases* and the divergence between the first best and the equilibrium level of the defense good *increases*. In case the equilibrium involves a corner solution, these results are reversed though. It may be argued, however, that the interior equilibrium case is the more relevant one.

Key words: Alliance; Asymmetry; Public good; Defence.
JEL Classification: D74, P16.

1 INTRODUCTION

This paper was motivated by the diversity among the various transnational collectives, as well as the fact that many of them are undergoing rapid changes and expansions. The EU, for example, is debating the pros and cons of inducting new members, in particular Turkey into the union. In March, 1999, the NATO also inducted several ex-Warsaw countries, the Czech Republic, Hungary and Poland (see Sandler and Murdoch (2000)).

It might be argued, that in the EU, as well as the NATO, asymmetry among member countries has increased. While, on the average, the larger countries have gotten larger because of growth, the smaller countries have, on the average, gotten smaller because of the inclusion of several relatively smaller countries.

Given these trends, in this paper we analyze alliances between asymmetric countries. In particular, we examine the effects of an increase in asymmetry on the level of the public good being provided by such an alliance, in absolute terms, as well as relative to the first best level.

In recent years academics have shown a growing interest in transnational alliances and collectives. Such collectives, e.g. organizations like the UN, the WTO and the WHO, are concerned with peace-keeping in the world's hot spots, controlling environmental degradation and terrorism, promoting world health, eliminating trade barriers, etc. This interest is also partly fuelled by the recent troubles confronting organizations like the EU and the UN.

The theory of such collective organizations, in particular that of (military) alliances, was pioneered by Olson and Zeckhauser (1966) who analyze an alliance of countries aimed at providing a pure public good, deterrence. We refer the readers to Sandler and Hartley (2001) for a succinct survey of this, by now, large literature. Most importantly, the insights gleaned from this literature is applicable, not only to military alliances like the NATO, but to a broad set of collectives.[1]

One of the central aims of this literature is to study the effects of *asymmetry* among the alliance partners on the outcome. For example, Olson and Zeckhauser (1966) (as well as much of the subsequent literature) is motivated, among other things, by the *exploitation hypothesis*, which says that relatively larger countries contribute proportionately more to such alliances.[2] This paper extends this literature by focusing on the effects of an *increase* in asymmetry.

We analyze a model with one private good, and two defence goods, one a country specific defence good, and the other an alliance wide one. To begin with we provide sufficient conditions such that the exploitation hypothesis holds. Further, in the appendix, we provide an alternative definition of the exploitation hypothesis. We then examine the effect of an increase in asymmetry on the equilibrium outcome. One interesting insight is that the results depend on the nature of the equilibrium. If it is an interior one then, with an increase in asymmetry, the absolute level of the alliance wide defence good decreases and the difference between the first best and the equilibrium level of this good increases. In case the equilibrium is a corner one, these results are reversed though. It may be argued, however, that the first set of results are, perhaps, more relevant.

2 THE MODEL AND PRELIMINARIES

We consider an alliance between two countries, 1 and 2. Country i is characterized by a size parameter k_i, $k_1 > k_2 > 0$, that is related to its GNP and population. There is one private good, X, and two defence goods, one an alliance wide defence good Y and the other a country specific defence good Z.[3] The endowment of country i is given by $k_i w$, $w > 0$.

Every country spends a part of its endowment t_i ($\leq k_i w$) on defence activity, and the rest, $k_i w - t_i$, on the private good X. An expenditure of t_i on the defence input leads to the joint production of $C.t_i$ units of the country specific defence good Z_i and $f(t_i)$ units of the alliance wide defence good Y. Hence the total production of the alliance wide defence good Y is $f(t_1) + f(t_2)$. Thus, apart from the size parameter k_i, the two countries are assumed to be symmetric.[4]

For simplicity we consider a utility function that is an additively separable version of the one in Sandler and Hartley (2001). Thus

$$U_i(X_i(t_i), Y(t_1, t_2), Z_i(t_i)) = A.(k_i w - t_i) + k_i B(f(t_1) + f(t_2)) + k_i C t_i, \ i = 1, 2, \tag{1}$$

where $A > Ck_1, \ Ck_2 > 0$ and $C \geq 0$.

While the separability assumption is primarily for simplifying the analysis, it may not be be too unreasonable. It may be argued though, with some justification, that a lower level of defence consumption increases a country's threat perception, and hence lowers its utility from private consumption. Equation 1, however, can accommodate such concerns. Suppose, for example, that country i's utility is $U_i(X_i(t_i) + Y(t_1, t_2) + Z_i(t_i)) + U_i(Y(t_1, t_2), Z_i(t_i))$, where the first term represents its utility from private consumption, and the second term represents its utility from defence consumption. Such a formulation is clearly consistent with equation (1), while allowing for the fact that the country's utility from private consumption depends on its consumption of defence goods.

We assume that $f(t_i)$ and $B(f(t_1) + f(t_2))$ satisfy the following properties.

Assumption 1. $f : [0, \infty) \to [0, \infty)$ is twice differentiable. Also, for all $t > 0$, $f(t)$ is increasing without bound and concave.

Assumption 2. $B : [0, \infty) \to [0, \infty)$ is twice differentiable. Also, for all $x > 0$, $B(x)$ is increasing and concave.

Assumption 3 states that $f(t)$ and $B(f(t_1) + f(t_2))$ satisfy the Inada conditions.[5]

Assumption 3. (i) $\lim_{t \to 0} f'(t) \to \infty$ and $\lim_{t \to \infty} f'(t) = 0$.

(ii) $\lim_{x\to 0} B'(x) \to \infty$ and $\lim_{x\to\infty} B'(x) = 0$.

We solve for the set of pure strategy Nash equilibria of the game where the two countries simultaneously decide on their level of t_i, $0 \leq t_i \leq k_i w$.

To begin with we solve for the reaction functions of the two countries:

$$\frac{\partial U_1(X_1(t_1), Y(t_1,t_2), Z_1(t_1))}{\partial t_1} = k_1 B'(f(t_1)+f(t_2))f'(t_1) - (A - Ck_1) = 0, \quad (2)$$
$$\frac{\partial U_2(X_2(t_2), Y(t_1,t_2), Z_2(t_2))}{\partial t_2} = k_2 B'(f(t_1)+f(t_2))f'(t_2) - (A - Ck_2) = 0. \quad (3)$$

We can write down our first proposition.

Proposition 1. *Let Assumptions 1 and 2 hold. (i) There is a unique Nash equilibrium. (ii) If, moreover, Assumption 3 holds, then the equilibrium is an interior one.*

Proposition 1(i) follows from the contraction mapping approach developed by Cornes, Hartley and Sandler (1999). The proof for Proposition 1(ii) is routine, and hence omitted.[6]

Let the unique Nash equilibrium be denoted by (t_1^*, t_2^*).

Next, using the fact that $k_1 > k_2$, from (2) and (3) it follows that

$$k_1 f'(t_1^*) < k_2 f'(t_2^*). \quad (4)$$

Given that $k_1 > k_2$, from (4) we have that $f'(t_1^*) < f'(t_2^*)$. Since $f(t)$ is concave, it follows that $t_1^* > t_2^*$, i.e. the larger country spends more on defence compared to the smaller one.

Definition. The *exploitation hypothesis* holds if and only if $\frac{t_1^*}{k_1} > \frac{t_2^*}{k_2}$.[7]

We then provide some sufficient conditions under which the exploitation hypothesis holds. From equation (4) we can write that

$$\frac{t_2^*}{k_2} t_1^* f'(t_1^*) < \frac{t_1^*}{k_1} t_2^* f'(t_2^*). \quad (5)$$

Suppose $f'(t)$ is elastic, i.e. $-\frac{f'(t)/t}{f''(t)} > 1$. Then $tf'(t)$ is increasing in t, so that $t_2^* f'(t_2^*) < t_1^* f'(t_1^*)$. From equation (5) it then follows that $\frac{t_1^*}{k_1} > \frac{t_2^*}{k_2}$.

Proposition 2. *(i)* $t_1^* > t_2^*$.
(ii) If $f'(t)$ *is elastic i.e.* $-\frac{f'(t)/t}{f''(t)} > 1$, *then* $\frac{t_2^*}{k_2} < \frac{t_1^*}{k_1}$.

Proposition 2(ii) provides a sufficient condition for the exploitation hypothesis to hold even though $f(t)$ is concave. This is interesting since Weber and Wiesmeth (1991) provide an example with a concave $f(t)$ such that the exploitation hypothesis does not hold.

3 MAIN RESULTS

We next turn to the central theme of this paper, the effect of an increase in asymmetry between the two countries on Y, in absolute terms, as well as relative to the first best level.

The first task is to formalize the notion of an *increase* in asymmetry in this framework. In order to ensure that the change in the size of the larger country is, in some sense, commensurate with that of the smaller country, we consider increases in asymmetry such that the average size do not alter. We therefore consider mean preserving spread of k_1 and k_2 such that $k_1 + k_2$ equals some constant K.[8]

3.1

In this sub-section we examine the effect of a mean-preserving spread of k_1 and k_2 on the absolute level of Y.

For technical reasons we assume that, $\forall t,\ f''(t) = F$. Totally differentiating (2) and (3) with respect to k_1 and k_2, and using the fact that $k_1 + k_2 = K$, we obtain

$$k_1[B''(.)f'(t_1)^2 + B'(.)F]\frac{dt_1}{dk_1} + k_1 B''(.)f'(t_1)f'(t_2)\frac{dt_2}{dk_1} = -[B'(.)f'(t_1) + C], \quad (6)$$

$$k_2B''(.)f'(t_1)f'(t_2)\frac{dt_1}{dk_1} + k_2[B''(.)f'(t_2)^2 + B'(.)F]\frac{dt_2}{dk_1} = [B'(.)f'(t_2) + C]. \quad (7)$$

Simultaneously solving equations (6) and (7) we obtain

$$\frac{dt_1}{dk_1}|_{k_1+k_2=K} = \frac{N_1}{D}, \quad (8)$$

$$\text{and, } \frac{dt_2}{dk_1}|_{k_1+k_2=K} = \frac{N_2}{D}, \quad (9)$$

$$\begin{aligned}
\text{where, } N_1 &= -(k_1+k_2)B'(.)B''(.)f'(t_1)f'(t_2)^2 - k_2B'(.)^2f'(t_1)F \\
&- C[k_1B''(.)f'(t_1)f'(t_2) + k_2B''(.)f'(t_2)^2 + k_2B'(.)F] > 0, \\
N_2 &= (k_1+k_2)B'(.)B''(.)f'(t_2)f'(t_1)^2 + k_1B'(.)^2f'(t_2)F \\
&+ C[k_2B''(.)f'(t_1)f'(t_2) + k_1B''(.)f'(t_1)^2 + k_1B'(.)F] < 0, \\
\text{and, } D &= k_1k_2[B'(.)^2F^2 + B'(.)B''(.)F(f'(t_1)^2 + f'(t_2)^2)] > 0.
\end{aligned}$$

Clearly, $\frac{dt_1^*}{dk_1}|_{k_1+k_2=K} > 0$ and $\frac{dt_2^*}{dk_1}|_{k_1+k_2=K} < 0$.

Next, from (8) and (9) we have that

$$\frac{d(t_1+t_2)}{dk_1}|_{k_1+k_2=K} = \frac{N_{12}}{D}, \quad (10)$$

where

$$\begin{aligned}
N_{12} &= (k_1+k_2)B'(.)B''(.)f'(t_1)f'(t_2)[f'(t_1) - f'(t_2)] + B'(.)^2F[k_1f'(t_2) - k_2f'(t_1)] \\
&+ CB''(.)f'(t_1)f'(t_2)(k_2-k_1) + CB''(.)[k_1f'(t_1)^2 - k_2f'(t_2)^2] + CB'(.)F(k_1-k_2).
\end{aligned}$$

From (8) and (9) it also follows that

$$\frac{d(f(t_1)+f(t_2))}{dk_1}|_{k_1+k_2=K} = \frac{N}{D}, \quad (11)$$

where, $N = B'(.)^2F[k_1f'(t_2)^2 - k_2f'(t_1)^2] + B'(.)FC[k_1f'(t_2) - k_2f'(t_1)]$.

Given that $t_1^* > t_2^*$, it follows that $\frac{d(f(t_1^*)+f(t_2^*))}{dk_1}|_{k_1+k_2=K} < 0$. Thus a mean-preserving spread of k_1 and k_2 leads to a decline in the equilibrium level of the alliance wide defence good Y.

Summarizing the above discussion we obtain

Proposition 3. *Suppose that* $f''(t) = F$, $\forall t > 0$. *A mean-preserving spread of* k_1 *and* k_2 *leads to a decline in* $f(t_1^*) + f(t_2^*)$, *i.e. the level of the alliance wide defence good* Y.

With increasing asymmetry, the larger country invests more in defence, since it has more to gain from investing, whereas the smaller country invests less, since it has less to gain. Thus t_1^* increases, whereas t_2^* decreases. The decline in t_2^*, however, outweighs the increase in t_1^*. Hence the result.

Proposition 3 demonstrates that the nature of the technology is critical in determining whether an increase in asymmetry adversely affects the supply of Y or not. In case $f(t)$ is concave, it does. If $f(t)$ is linear, then, from (11), it does not. Note that Proposition 3 goes through even if $C = 0$. Further, it may go through even if $f''(t)$ is not constant.

Example 1. Let $f(t) = 1 - \mathrm{e}^{-t}$.[9] From (2) and (3), straightforward calculations yield that

$$[1 - (f(t_1^*) + f(t_2^*))]B'(f(t_1^*) + f(t_2^*)) = \frac{A(k_1 + k_2) - 2Ck_1k_2}{k_1k_2}.$$

A mean-preserving spread of k_1 and k_2 leads to an increase in the right hand side, so that there is a decrease in $f(t_1^*) + f(t_2^*)$.[10]

3.2

In this sub-section we compare the equilibrium level of Y with the first best level. Note that the aggregate utility

$$\begin{aligned} & U_1(X_1(t_1), Y(t_1, t_2), Z_1(t_1)) + U_2(X_2(t_2), Y(t_1, t_2), Z_2(t_2)) \\ = \; & A.[(k_1 + k_2)w - t_1 - t_2] + (k_1 + k_2)B(f(t_1) + f(t_2)) + C(k_1t_1 + k_2t_2). \quad (12) \end{aligned}$$

Let (t_1', t_2') maximize the aggregate utility $U_1(.) + U_2(.)$. (We can mimic Proposition 1 to argue that (t_1', t_2') is unique and interior.) Clearly (t_1', t_2') must satisfy the first order conditions

$$(k_1 + k_2)B'(f(t_1) + f(t_2))f'(t_1) \;\; = \;\; A - Ck_1, \qquad (13)$$

$$\text{and,} \quad (k_1 + k_2)B'(f(t_1) + f(t_2))f'(t_2) = A - Ck_2. \tag{14}$$

From (2), (3), (13) and (14), it follows that $(t_1^*, t_2^*) \neq (t_1', t_2')$, i.e. (t_1^*, t_2^*) is non-optimal.

We then examine the effect of a mean-preserving spread of k_1 and k_2 on (t_1', t_2'). Totally differentiating (13) and (14) with respect to k_1 and k_2, and using the fact that $k_1 + k_2 = K$, we obtain

$$K[B''(.)f'(t_1)^2 + B'(.)F]\frac{dt_1}{dk_1} + KB''(.)f'(t_1)f'(t_2)\frac{dt_2}{dk_1} = -C, \tag{15}$$

$$\text{and,} \quad KB''(.)f'(t_1)f'(t_2)\frac{dt_1}{dk_1} + K[B''(.)f'(t_2)^2 + B'(.)F]\frac{dt_2}{dk_1} = C. \tag{16}$$

Simultaneously solving equations (15) and (16) we obtain

$$\frac{dt_1}{dk_1}|_{k_1+k_2=K} = \frac{X_1}{Y}, \tag{17}$$

$$\text{and,} \quad \frac{dt_2}{dk_1}|_{k_1+k_2=K} = \frac{X_2}{Y}, \tag{18}$$

$$\begin{aligned} \text{where,} \quad X_1 &= -CK[B''(.)f'(t_1)f'(t_2) + B''(.)f'(t_2)^2 + B'(.)F] < 0, \\ X_2 &= CK[B''(.)f'(t_1)f'(t_2) + B''(.)f'(t_1)^2 + B'(.)F] > 0, \\ \text{and,} \quad Y &= K^2[B'(.)^2F^2 + B'(.)B''(.)F(f'(t_1)^2 + f'(t_2)^2)] > 0. \end{aligned}$$

Next, from (17) and (18), it follows that

$$\frac{d(f(t_1) + f(t_2))}{dk_1}|_{k_1+k_2=K} = \frac{X}{Y}, \tag{19}$$

$$\text{where,} \quad X = CKB'(.)F(f'(t_2) - f'(t_1)].$$

From (13) and (14) it follows that $t_1' > t_2'$. Thus, from (19), for $C > 0$, $\frac{d(f(t_1')+f(t_2'))}{dk_1}|_{k_1+k_2=K} < 0$.

We then examine the effect of a mean-preserving spread of k_1 and k_2 on the difference between the optimal and equilibrium level of Y, i.e.

$$f(t_1') + f(t_2') - f(t_1^*) - f(t_2^*). \tag{20}$$

For simplicity we restrict attention to the case where there is no country specific defence good. Formally, we assume that $C = 0$. Then, for a mean preserving spread in k_1 and k_2, $f(t_1') + f(t_2')$ remains constant (see (19)), whereas, from (11), $f(t_1^*) + f(t_2^*)$ decreases.

Summarizing the preceding discussion we have our final result.

Proposition 4. *(i)* $(t_1^*, t_2^*) \neq (t_1', t_2')$.

(ii) Suppose that $f''(t) = F$, $\forall t > 0$. *For* $C > 0$, *a mean-preserving spread of* k_1 *and* k_2 *leads to a decline in* $f(t_1') + f(t_2')$.

(iii) (a) Suppose that $f''(t) = F$, $\forall t > 0$. *There exists* $\overline{C} > 0$, *such that* $\forall C \leq \overline{C}$, *a mean-preserving spread of* k_1 *and* k_2 *leads to an increase in* $f(t_1') + f(t_2') - f(t_1^*) - f(t_2^*)$.

(b) Suppose that $f''(t) = F$, $\forall t > 0$, *and* $B(x) = x$. *There exists* $\overline{A} > 0$, *such that* $\forall A \geq \overline{A}$, *a mean-preserving spread of* k_1 *and* k_2 *leads to an increase in* $f(t_1') + f(t_2') - f(t_1^*) - f(t_2^*)$.

Proposition 4(iii), however, may go through even if C is large, or A is small.

Example 2. Let $f(t) = 1 - \mathrm{e}^{-t}$. From (13) and (14), it follows that

$$B'(f(t_1') + f(t_2'))[1 - (f(t_1') + f(t_2'))] = \frac{2A - C(k_1 + k_2)}{k_1 + k_2}.$$

Clearly, a mean-preserving spread of k_1 and k_2 does not affect $f(t_1') + f(t_2')$, whereas, $f(t_1^*) + f(t_2^*)$ decreases (Example 1). Thus $f(t_1') + f(t_2') - f(t_1^*) - f(t_2^*)$ increases.

4 CORNER EQUILIBRIUM

In this section we focus on corner solutions. Hence the Inada conditions, i.e. Assumptions 3(i) and 3(ii), are not imposed.

Let $\overline{t_1}$ solve

$$k_1 B'(f(t_1) + f(0)) f'(t_1) = A - ck_1, \tag{21}$$

and let $\underline{t_1}$ solve

$$k_2 B'(f(t_1) + f(0))f'(0) = A - ck_2. \tag{22}$$

Assumption 4. $\overline{t_1} > \underline{t_1}$.

It is easy to see that given Assumptions 1, 2 and 4, there exists a unique corner equilibrium, (t_1^*, t_2^*), such that $t_1^* > 0$ and $t_2^* = 0$. Consequently the exploitation hypotheses holds, i.e. $\frac{t_1^*}{k_1} > 0 = \frac{t_2^*}{k_2}$.

Hence, even in case of a corner equilibrium, analogues of Propositions 1 and 2 go through. Interestingly, unlike Proposition 2(ii), in this case the exploitation hypothesis does not require any elasticity condition on $f(t)$.

4.1

We first examine the effect of a mean-preserving spread of k_1 and k_2 on the absolute level of Y. Given that there is a corner solution, (t_1^*, t_2^*) solves

$$k_1 B'(f(t_1) + f(0))f'(t_1) = A - ck_1. \tag{23}$$

Our next proposition follows from equation (23).

Proposition 5. *A mean-preserving spread of* k_1 *and* k_2 *leads to an increase in* $f(t_1^*) + f(t_2^*)$*, i.e. the level of the alliance wide defence good* Y*.*

Comparing Propositions 3 and 5, we find that the effect of a mean-preserving spread of k_1 and k_2 on the level of Y is sensitive to the nature of the equilibrium. The level of Y *increases* if the equilibrium is a corner one, whereas it *decreases* if the equilibrium is an interior one. Further, unlike Proposition 3, Proposition 5 does not require any restrictions on $f''(t)$.

4.2

We then compare $f(t_1^*) + f(t_2^*)$, the equilibrium level of Y, with the first best level, $f(t_1') + f(t_2')$. In case the first best outcome is interior, (t_1', t_2') satisfies equations (13) and (14). Otherwise, $t'_2 = 0$ and t'_1 satisfies

$$(k_1 + k_2)B'(f(t_1) + f(0))f'(t_1) = A - ck_1. \tag{24}$$

The following Proposition follows from our earlier analysis and equation (24) above.

Proposition 6. *(i)* $(t_1^*, t_2^*) \neq (t'_1, t'_2)$.

(ii) For $C > 0$*, a mean-preserving spread of* k_1 *and* k_2 *leads to an increase in* $f(t'_1) + f(t'_2)$.

(iii) There exists $\overline{\overline{C}} > 0$ *such that* $\forall C \leq \overline{\overline{C}}$*, a mean-preserving spread of* k_1 *and* k_2 *leads to a decrease in* $f(t'_1) + f(t'_2) - f(t_1^*) - f(t_2^*)$.

Comparing Propositions 4(iii) and 6(iii) we find that the results are sensitive to the nature of the equilibrium. In case of a corner equilibrium, a mean-preserving spread of k_1 and k_2 leads to a *decrease* in the spread between the first best and the equilibrium level of Y. For an interior equilibrium the results are just the opposite.

Interestingly, Propositions 5 and 6(iii) are in the spirit of Itaya et. al. (1997). In the context of private provision of a public good, Itaya et. al. (1997) demonstrate that, under a corner equilibrium, an increase in income asymmetry increases welfare.

Finally, note that the results in this section generalize easily to the case where the number of countries is more than 1. Suppose that the equilibrium involves a corner solution, with country 1 being the only country to make a non-zero investment in defence. It is clear that equations (23) and (24) go through in this case also. Hence so do analogues of Propositions 5 and 6.

Whether the analysis in Section 3 generalizes in a similar fashion is an open question. Suppose however, that the countries in the alliance can be divided into two groups, one group consisting of relatively larger countries, and the other group consisting of relatively smaller ones. If, moreover, the groups are relatively homogeneous, then the two country model may, perhaps, provide a reasonable approximation to reality.

5 CONCLUSION

In conclusion, this paper makes some testable predictions regarding the effects of asymmetry on transnational collectives in general, and military alliances in particular.

The analysis in this paper suggests that the results depend on whether the equilibrium is an interior, or a corner one. In case the equilibrium is interior, we find that alliances that are more diverse, are less likely to perform well, at least as far as the supply of the public good is concerned (absolutely, as well as relative to the first best level). In case of a corner equilibrium, however, these results are reversed.

Whether the equilibrium is an interior, or a corner one, is, of course, an empirical question, and the answer will vary from case to case. It may, however, be argued that the interior equilibrium case may be the more relevant one.

From a theoretical point of view, it is doubtful whether an alliance will form at all if the resulting equilibrium is going to be a corner one. In that case, assuming that $f(0) = 0$, country 1 is indifferent between forming and not forming an alliance.[11] Furthermore, suppose that such an alliance involves some fixed cost for country 1. Such costs may arise because of perceived cultural differences between the two countries, historical animosities, etc. In that case country 1 would strictly prefer not to form an alliance at all. The only way such an alliance may form is if country 2 subsidizes country 1, something that may be politically infeasible.

Further, from an empirical point of view, while there is some support for the exploitation hypothesis (see endnote 2), it is rarely the case that the alliance partners contribute nothing at all. Thus, on both these grounds, the interior equilibrium case may be the more relevant one.

Finally, let us consider a situation where a relatively small country is in the process of being inducted into an alliance. Suppose that the existing alliance can be modeled, without too much violation of reality, as a two country one. This may be a sensible approximation when the alliance can be divided into two groups, one consisting of relatively larger, the other consisting of relatively smaller countries, and the groups are relatively homogeneous. In that case the induction of the new country may be modeled as a fall in the average size of the group of smaller countries. At the same time suppose that the larger countries are growing at a relative faster rate, so that their average size increases. Our analysis then suggests that, compared to the existing situation, post-alliance there will be a fall in the alliance wide public good, absolutely, as well as relative to the first best level.

It goes without saying though, that this is not to argue that such alliances should not take place, but rather that the justification for such alliances cannot be found in static gains in the supply of public goods. One possible justification could be in terms of dynamic effects. It is possible, for example, that as a result of this induction, the new member responds with a growth in income, leading to an increase in the *future* supply of the public good. However, while undoubtedly fascinating, a detailed examination of such dynamic considerations is beyond the scope of the present paper.

6 APPENDIX

Revisiting the Exploitation Hypothesis: The literature says that there is exploitation of country 1 whenever, for $k_1 > k_2$, $\frac{f(t_1^*)/k_1}{f(t_2^*)/k_2} = \alpha > 1$. Suppose, however, that even the first best level of t_1, t_2, i.e. (t_1', t_2') exhibits such exploitation, in the sense that $\frac{f(t_1')/k_1}{f(t_2')/k_2} = \beta > 1$. In that case it may be argued that the technology itself is biased in favor of such *exploitation*.

The following alternative definition takes such bias into account.

Definition. The *exploitation hypothesis* holds if and only if $\frac{f(t_1^*)/k_1}{f(t_2^*)/k_2} > \frac{f(t_1')/k_1}{f(t_2')/k_2}$, i.e. $\frac{f(t_1^*)}{f(t_1')} > \frac{f(t_2^*)}{f(t_2')}$.

For $\beta > 1$, the standard definition clearly over-estimates the extent of exploitation vis-a-vis the alternative one proposed here. Whereas for $\beta < 1$, the standard definition under-estimates the extent of exploitation.

7 END NOTES

1. This point was also made by Olson (1965), as well as Sandler and Hartley (2001).

2. In 1970 the USA contributed around 75% of the NATO's defence spending, while Germany, France and the UK each contributed less than 6%. Between 1990-1999 though, there is little evidence of such imbalance (Sandler and Murdoch (2000)).

3. Sandler (1977) and Strihou (1967), among others, argue that defence expenditures lead to multiple outputs with different degrees of publicness.

4. Olson and Zeckhauser (1967) and Wong (1991), among others, allow for other differences, e.g. the marginal cost of defence.

5. In Section 4 we examine the case where Assumption 3 does not hold, thus allowing for corner equilibria.

6. We assume that $k_i w$ is not too small in the sense that, in equilibrium $t_i < k_i w$, $\forall i$. A sufficient condition is that $g(k_i w) > \frac{A - ck_i}{k_i}$, $i = 1, 2$, where $g(t) = B'(f(0) + f(t))f'(t)$.

7. In the Appendix we provide an alternative definition of the exploitation hypothesis.

8. In the literature on risk and uncertainty also, one uses the idea of mean-preserving spreads to control for changes in the average income level.

9. $f(t) = 1 - \mathrm{e}^{-t}$ violates the Inada condition that $\lim_{t \to 0} f'(t) \to \infty$. However, since this condition is required for proving existence alone, this does not affect our argument.

10. The constancy of $f''(t)$, is, however, 'necessary' in the sense that Proposition 3 may fail if it is violated. Suppose $f(t) = t^{\frac{1}{2}}$. From (2) and (3), we find that $\frac{2(f(t_1^*) + f(t_2^*))}{B'(f(t_1^*) + f(t_2^*))} = \frac{A(k_1 + k_2) - 2Ck_1 k_2}{(A - Ck_1)(A - Ck_2)}$. A mean-preserving spread of k_1 and k_2 leads to an increase in the right hand side, so that there is an increase in $f(t_1^*) + f(t_2^*)$.

11. In this case, irrespective of whether a coalition forms or not, the first order condition for country 1 is given by $k_1 B'(f(t_1))f'(t_1) = A - Ck_1$.

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