“Matching Auctions” for Hostile Takeovers: A Model with Endogenous Target

Antonio Rosato

26. June 2008

Online at http://mpra.ub.uni-muenchen.de/15083/
“Matching Auctions” for Hostile Takeovers: A Model with Endogenous Target

Antonio Rosato*

Abstract

In this paper we analyze incentives for a potential entrant to get into an oligopolistic Cournot-like market by taking over one of the incumbents and we derive the conditions under which the hostile merger is possible and profitable. The key-feature is that the target of the takeover is endogenously determined and this is also the main difference with respect to the previous literature on this topic. Actually, the main objective of our analysis is that of determining why and how the buyer chooses as target this firm rather than that one. The takeover game is modeled as a “matching auction” in which the potential entrant has to make a first and final offer and the other bidders are asked to match this offer. We find different types of SPNE depending upon the values of the parameters. Whenever entry takes place it reduces incumbents’ profits and raises consumers’ welfare at the same time.

1 Introduction

At a first glance mergers and takeovers (or acquisitions) look like very similar corporate actions: they combine two previously separate firms into a single legal entity. Such actions can provide significant operative advantages and, in fact, the goal of most mergers and acquisitions is to improve a firm’s performance and shareholders’ value over the long-run.

Therefore, the motivation to pursue a merger or a takeover can be considerable; a firm combining itself with another one can experience economies of scale, greater sales revenue and market share, broader products’ diversification and increased tax efficiency. Nevertheless, the underlying motivation and methodology for mergers and takeovers are substantially different.

A merger involves the mutual decision of two firms to combine and become one entity; so, it can be seen as a decision made by two equal actors. The combined business that results, through structural and strategical advantages secured by the merger, can cut costs, increase profit and boost shareholders’ value for both groups. In other words, a typical merger involves two companies that

*This paper has been submitted in partial fulfillment of the requirements for the degree of Laurea Specialistica in Scienze Economiche at University of Pisa. I wish to thank my personal supervisor, Neri Salvadori, for his patient guidance and his helpful remarks. Large part of this work has been carried out during my visiting position at ENS-Paris School of Economics where I had the chance of working with Philippe Jehiel whom I gratefully acknowledge for his invaluable comments and suggestions. Finally, the numerous discussions with Marco Pagnozzi and Paolo Zacchia have been an important source of improvements. I alone am responsible for all errors and omissions.
spontaneously combine each other in one legal entity with the goal of producing a company which is worth more than just the sum of its parts.

A takeover, or acquisition, on the other hand, is characterized by the purchase of a firm by another one.\textsuperscript{1} This combination of unequal firms could produce the same benefits as a merger, but it does not necessarily have to be a mutual decision. Unlike in a merger, in a takeover the acquiring firm usually offers a cash price per share to the target firm’s shareholders.\textsuperscript{2}

Since mergers and takeovers are very different in practice, different is also the approach by which the economic literature has investigated them. Mergers have been studied mainly in Industrial Organization and Competition Policy frameworks,\textsuperscript{3} whereas takeovers have been the key argument of many of the most recent works in Auction Theory and Corporate Finance.\textsuperscript{4} In this paper we are going to discuss horizontal mergers and takeovers both from an Industrial Organization and Auction Theory point of view.\textsuperscript{5}

The Industrial Organization literature on mergers assume that managers are rational and profit-maximizing. Under these assumptions, this literature focuses upon the effect of mergers on product market competition, ex post profits and total welfare changes. Large part of the literature deals with an oligopoly model to examine the incentives to merge in cases of either Cournot or Bertrand competition. Salant, Switzer and Reynolds (1983) have found that in a symmetric Cournot model with linear demand and identical constant average costs, mergers are unprofitable as long as they do not encompass at least 80\% of the market.\textsuperscript{6} The profits earned by the merged firm will be less than the sum of the profits of the merging partners before the merger.

On the other hand, Deneckere and Davidson (1983) have shown that if firms are engaged in a price competition with some degree of products’ differentiation, then mergers of any size are beneficial. Perry and Porter (1985) considered a more general market structure and symmetric firms with different convex cost functions and they found a stronger incentive to merge than Salant et al. (1983). Levin (1990) relaxed the assumption that the merged firm competes in a Cournot fashion even after the merger and found that if firms with collective market shares of less than 50\% merge, then any contraction of their output will cut their profits and increase welfare. Farrell and Shapiro (1990) provide a more general model and prove that mergers can be privately profitable in a static Cournot framework only if they generate synergies.\textsuperscript{7} For what concerns preemptive incentives, they can happen both in Cournot and Bertrand competition with differentiated product markets.

The other line of research connected to the model of this paper is the preemptive patent licensing and the auctions with externalities literature. Arrow (1962) discussed how the value of a patent on a cost-reducing innovation could depend on the underlying market structure; he found that licensing to

\textsuperscript{1} Usually, the acquiring firm is much larger than the acquired one; but this does not need to be always true.

\textsuperscript{2} The acquiring firm can also offer its own shares in exchange of the target’s ones according to a specified conversion rate.

\textsuperscript{3} An excellent starting point for the study of mergers and takeovers as well as for general Competition Policy topics is Motta (2004); Tirole (1988) provides a more analytically rigorous treatment.


\textsuperscript{5} Horizontal mergers are those carried out by firms operating at the same stage of the production process, while vertical mergers apply to firms operating at successive stages of the production process.

\textsuperscript{6} This number, 80\%, strongly relies on the linear demand and constant marginal cost assumption.

\textsuperscript{7} However, they restrict their analysis to the cases in which acquirers do not lose.
a member of a downstream competitive industry yields more revenue than licensing to a monopolist. Gilbert and Newbery (1982) use an auction model to study whether a monopolist incumbent or a potential entrant is willing to pay more for an innovation and their main result is that the monopolist will take into account the potential negative externalities of losing the auction and will pay more to preempt entry. Kamien and Tauman (1984, 1986) and Katz and Shapiro (1985, 1986) re-examine the results of Arrow in the case when the downstream industry is oligopolistic. They study several variations of a basic model which has the following structure: in the first stage the patentee decides the format of the procedure used to sell the licenses and the number of licenses he wants to sell; in the second stage firms decide simultaneously whether or not to purchase a license and how much to bid for it; in the third stage the firms in the oligopoly compete either through quantities or prices. The main result is that from the sellers point of view an auctions dominates both fixed fees and royalties contracts. Hoppe, Jehiel and Moldovanu (2006) study a license auction with a free-rider problem among incumbents. Their paper focuses on determining the optimal number of licenses from a regulator perspective.

In all papers mentioned above, all the relevant information is common knowledge for the participants. Differently, Jehiel and Moldovanu (2000) consider a bidding model with incomplete information where the outcome of the auction affects future downstream competition via a reduced-form externality. Molnar (2000a) considers a merger game with asymmetric information where bids can convey information. If after the auction the information is still asymmetric, bids can influence the payoffs of the second stage game. Depending on whether the actions of the firms are strategic substitute or complements in the second stage and on the format of the auction, firms want to bid more or less than in the case of Jehiel and Moldovanu.

Our model distances itself from the main literature on auctions and takeovers by relaxing the assumption about the exogeneity of the target; indeed, our main purpose is exactly that of studying the reasons that induce the bidders to choose the target to take over and that of modeling the methods through which this selection occurs. It is commonly accepted that takeovers can be modeled in a very effective way as an ascending auction with significant participation (sunk) costs: in this paper we explicitly relate the extent and form of these sunk costs to the choice of the “right” target and to the features of the optimal bid. Moreover, since in practice takeover battles begin only after a potential buyer has expressed an interest in a target and evolve following not a fixed set of auction rules but rather as bidders and bids arrive, we model the takeover as a “matching auction”; that is, an auction in which a particular bidder has to make a first and final offer and the other bidders are asked to match this offer. In our humble opinion this type of selling procedure, more than the standard ascending or first-price auctions, is fit to effectively model a takeover contest. This effectiveness becomes even more evident in our model where the bidder making the first move is a potential entrant that can enter an oligopolistic market only by taking over one of the incumbents and where the other incumbents are given a sort of preemption right such that if one of them matches the initial bid it is declared as winner. From this perspective our model is related also to the literature on the Barriers to Entry; however, we will not go into this topic thoroughly since

---

8On this point, Bulow and Klemperer (1996) derived a more general result on the supremacy of an auction with \( N+1 \) symmetric bidders with respect to any other possible negotiation mechanism that involves only \( N \) bidders. However, the above mentioned result strongly relies on the assumption of symmetry among bidders.
the only way through which the incumbents can block the entry is that of winning the auction and therefore merging with the target.9

In order to isolate and study the motives of the selection of a target there must be some heterogeneity among the incumbents, at least from the entrant viewpoint. This can be achieved by making the incumbents different from the perspective of the efficiency for the entrant; that is, by assuming that the entrant can realize different productive synergies with respect to the incumbent with which it merges. Of course, this in turn implies that with respect to the measure and the kind of these synergies the entrant firm could impose a negative externality on the other firms competing in the same market; and, the bigger is this externality, the more the incumbents will be willing to preempt the entry.

Thus, the aim of our paper is that of explaining how the choice of the target to (eventually) take over takes place in a particular set up: that where a takeover is the only possible way to enter in an oligopolistic market. And we pursue this objective by relating this choice to three crucial variables: the participation cost, the cost synergies and the negative externality imposed on the incumbents.

The model of this paper is well suited to describe regulated markets where the number of operating firms is subject to some restriction i.e., TV rights, electricity pools, oil-draining rights, telecommunications, airlines, railways and highways. It is commonly accepted that the firms operating in these industries are able to extract significant rents to consumers’ disadvantage thanks to the reduced competition they have to face; and, indeed, this explains why the word “de-regulation” has become an habitual attender of many public speeches and debates.

Although the measures intent to promote the openness of the markets and to increase competition seem like the most attractive and effective ones, they are usually not easy to carry out because of institutional as well as political hindrances.

The results of our model point out that in those industries subject to regulations there could be significant sources for improvement besides the issue of de-regulation and also keeping still constant the number of the firms operating in such industries. This may happen, for example, if there is a more efficient firm that is left out of the regulated market and cannot enter in because of bureaucratic costs or of the resistance of the incumbents which obviously want to preserve their situation rents.10 If this is the case, the entry of the more efficient outsider should be not only strongly encouraged but also tangibly supported by Governments and Competition Authorities. To some extent, this is exactly what the Italian Government failed to realize about the “affaire” Alitalia-AirFrance.

The rest of this paper is organized as follows. Section 2 deals in detail with takeovers - that is, hostile mergers - and describes the tools through which Auction Theory has modeled these topics and improved our understanding of them. This section point out that acquisitions work basically as a particular kind of auction and therefore that most of the results achieved by Auction Theory apply to these issues too. In section 3 a simple model is described to study the situation in which an outsider firm can enter an oligopolistic Cournot-like market only by taking over one of the actual incumbents. The takeover is modeled process as a “matching auction” in which the potential entrant firm has to submit a first and final offer, the incumbent chosen as target must decide whether

---

9The standard IO literature on entry barriers deals mainly with entry deterrence strategies such as quantity and/or price reactions.
10In other words, not only the number of the players, but also their quality or ability do matter.
accepting or refusing the offer and, if it accept, the other incumbents are asked to match this offer. Section 3.1 describes the matching stage equilibria. Section 3.2 derives the condition for which the target accepts the bid. Section 3.3 analyzes the bidding stage and describes the bidding and entry strategy equilibrium. Section 3.4 deals with the target’s selection which is made endogenous to the model itself and, at the best of our knowledge, this is a very original feature of our model because in almost all the literature about takeovers and mergers the target firm is usually considered as exogenous. Section 3.5 summarizes and discusses the results of the model. Section 4 concludes and gathers final remarks.

2 Auction theory and takeover battles

Since making an offer in order to acquire a firm is merely as similar as making an offer for purchasing a standard good, takeovers have been mainly studied as auctions and many are the results that have been established in auction theory and that play a sensitive role in takeover games.

Under the Delaware law (the predominant corporate law in the US), when a potential acquirer makes a serious bid for a target, the target’s board of directors is required to act as would “auctioneers charged with getting the best price for the stockholders at a sale of the company”. The target’s board may not use defensive tactics that destroy the auction process and must attempt to seek higher bids. Similarly, the Williams Act requires takeover bids to remain open for at least 20 business days on the grounds that the delay facilitates auctions. This preference for auctions follows from the view that auctions maximize shareholder returns. In addition, auctions promote efficiency by shifting corporate assets into the hands of those that value them most highly. And auctions mitigate the collective action of target shareholders by requiring the target board to seek the highest bid.

However, takeover auctions differ from traditional auctions in several relevant aspects. In a traditional auction, the seller describes what is being sold and states the auction rules in a public announcement. Takeover auctions are instead prompted by a potential buyer and there exist not an explicit set of rules that carry on this kind of auctions. Only after a potential buyer has expressed an interest in the target are bids from others sought. The whole process is governed not by a fixed set of rules specified by the seller, but rather by complex takeover regulations, which give the target board some latitude in the process which, in turn, is typically not stated in advance but evolves as bidders and bids arrive. For the most part, the regulations provide ways to defend against hostile takeovers with poison pills, greenmail and other tactics. This is why, usually, takeovers are considered auctions by practice but not by name. However, despite these differences, to a first approximation takeovers are well modeled as an ascending auction with significant participation costs.

Nowadays some authorities have started to consider the idea of implementing real auction procedures (that is, auctions by name) for regulating takeovers. In this way one could look at the decision undertaken in January 2007 by the UK Regulation Agency in order to carry on the acquisition of Corus Group. This is one of the fewest examples of a real auction designed just to arrange a takeover contest; since no bidder had formulated yet a definitive bid for Corus, the Executive Panel, after having discussed with all parties and according to the section 32.5 of the Takeover Code, decided to arrange a formal auction.
It is useful to distinguish between two ideal auction settings: common and private value auctions. In a common value auction, every bidder has the same value for the target, but the bidders have private estimates of this uncertain value. In a private value auction, each bidder knows its own value, but not the values of other bidders. However, in terms of value, it is not easy to say if the takeovers are auctions with private or common values. For instance, a takeover to displace bad management in which all bidders have similar plans for the target is more like a common auction; a takeover motivated by synergistic gains particular to each bidder is closer to a private value auction. In practice, however, both private and common value’s elements are likely to be in place.

The first strategic issue to be taken into account in a common value auction is the winner’s curse. Bids are based on some kind of estimate of the real (and common) value of the good (or firm) that is the object of the auction and, ceteris paribus, there is a tendency for the bidder who overestimates value the most to win. However, sophisticated bidders recognize the winner’s curse problem and adjust downward their bids to account for the bad news about the value that winning conveys. Empirical studies suggest that targets capture nearly all of the gains from trade in takeover auctions. This is consistent either with sophisticated bidders with limited private information about the value (so information rents are small), or some amount of naïve bidding which consumes the rents to private information.

Auction Theory has provided invaluable contributions also by highlighting the most common strategies used by bidders for augmenting their profits as well as the techniques that the target can undertake in order to augment the number of participants and increase its revenue. Preventive bids and toeholds belong to the first group whereas stock lockups and breakup fees to the second.

Preemptive bidding is a strategic issue in both private value and common value takeover auctions. Preemptive bids are especially relevant in the takeover context, because of the substantial costs associated with making a bid. The initial bidder can find the target, do the investigation, and then make a high initial bid. The bidder is saying, “I want this target and I have a high value. Attempts to compete with me will fail.” In the private value context, the high initial bid is a credible signal of high value (Fishman 1988). Firms with low values find unprofitable to make high bids; whereas, high-value firms have an incentive to make high bids if it deters other firms from entering the auction. Since the other potential bidders have not yet sunk the investigation cost, the high initial bid may stand. In the common value setting, the high initial bids can signal a high estimate of value. The bids serve to warn the other potential bidders to stay out or else suffer from the winner’s curse.

It is difficult to assess how common preemptive bids are. Certainly, there are instances of successful initial bids, but competition is more common. Sometimes the same effect can be accomplished by a ratcheting bid, in which the initial bidder announces that it will top any other bid in the field. However, target boards often refuse to entertain ratcheting bids, and it is difficult to make ratcheting bids credible.

The probably most successful strategy for a firm seeking to acquire another one is to buy a certain amount of the other firm’s stocks, that is a toehold. Indeed, all hostile takeovers do not

---

11 Although in many hostile takeovers a key role is played by the common value element, in this paper we are going to keep our attention only on auctions with private firm-specific strategic values.

12 If the winner’s curse occurs, hence, the winning bidder has not a generic biased estimation, but an upper-biased estimation.
start with an official offer, but with a rival firm buying as many as possible target’s stocks before its real intentions are revealed. Toeholds play a key-role for two reasons. Firstly, the acquirer’s profits heavily rely on those target’s stocks purchased at a relatively low price before the market realizes. Secondly, toeholds can dramatically affect the bidding strategies; if a bidder owns a toehold, he will be able to bid in a more aggressive way. Without a toehold, a higher bid simply means a higher price for acquiring the target; but, for a bidder with a toehold a higher bid means also an higher price for the stocks he owns. Therefore, who owns a toehold enjoys both the owner’s and the bidder’s advantages at the same time.

But this is only half the battle! The strategic role of toeholds is magnified in ascending common value auctions. In fact, a bidder with a toehold will play in a more aggressive way; in turn, this augments the winner’s curse for the bidders without toehold so that they will play in a less aggressive way; in turn, this reduces the winner’s curse for the bidder with the toehold whose bids will be even more aggressive; and so on and so far... That is, the combined effect of a toehold and the winner’s curse enable the bidder with the toehold to always win. Furthermore, toeholds can even prevent some firms from taking part in the auction since they know that is almost impossible to win against a bidder who owns a significant amount of stocks of the target firm.13

In light of these results, it is perhaps surprising that initial bidders do not always acquire toeholds (Cabrerizo 1997). Although toeholds are common there is a significant fraction of acquirers that do not purchase stock prior to the initial tender. An explanation is that the size of the toehold signals (at least partially) to the market the value of the acquisition. Thus, a bidder may choose not to acquire shares if it feels that the toehold would leak information that would force the bidder to pay a larger takeover premium.

As claimed before, targets are very interested in supporting participation and stimulating competition among bidders and they often use some lockup clauses like breakup fees and stock lockups.14 Breakup fees are just money that target’s executives promise to a potential and motivated bidder, if he loses, in order to make him participating to the auction (by submitting the first bid or by raising the actual one). Actually, this device can be used either for increasing the competition or for destroying it; and, usually the latter effect dominates the former. In fact, by offering such an incentive to only one bidder, the target ends up with reducing its value for other potential buyers of the same amount as the breakup fee.

Stock lockups are just slightly different: the target and a potential bidder agree that if the latter takes part to the takeover contest (by submitting the first bid or by raising the actual one) and does not win, he will be rewarded with the possibility of buying a certain amount of stocks of the target at an agreed, and usually favorable, price. So, the main difference between these two lockup clauses is the type of reward that is offered; indeed, target stocks’ value might vary quite much among the different bidders. For instance, the target can use stock lockups to attract more buyers and then bargain with those who have the highest valuation for the target. In that way they can be efficient as well as remunerative. Nevertheless, because of the ambiguity of their effects on participation and competition lockup clauses have not always been well accepted by regulation authorities.

Another different way trough which the target could boost its revenue is that of discriminating

13For an exhaustive treatment on almost common value auction see Klemperer (1998).
14See also Che and Lewis (2007).
among bidders, especially against the stronger one. For instance, just by imposing an order of moves on the bidding firms and by asking the stronger bidder to make a first offer. This is exactly what Riley and Samuelson (1981), Berkovitch and Khanna (1990) and Dasgupta and Tsui (2003) do by modeling the takeover context as a “matching auction”; the former two works in a private values environment and the latter in a common values one. As this is also the selling mechanism used in our model, it is worth to spend some words on it.

In general, the “matching auction” works as follows. After the bidders show an indication of interest in the target, and a Special Committee is formed to secure the best price for the target, the Committee asks one of the bidders to submit its “best bid”. This is an open bid, and if it is matched by a second bidder, the latter is declared the winner. If it is not matched, the first bidder is declared the winner and its bid is the winning bid.

Previous studies have almost entirely considerate the optimality of discrimination in the presence of bidder heterogeneity in private values settings. Myerson (1981), McAfee and McMillan (1989), Naegelen and Mougeot (1998) found that the bidder with the more favorable distribution of private values should be discriminated against. More recently, Dasgupta and Tsui (2003) show that a matching auction in which the stronger bidder is asked to bid first and the second bidder is asked to match that bid (and declared the winner if it does match) can generate higher expected revenue for the target shareholders than both the first-price and the second-price auction when asymmetries between bidders are significant.

Recently, a huge amount of empirical studies has examined the extent to which takeovers improve target and bidder firms’ value. The evidence indicates that the acquired firm’s shareholders benefit, and that the acquiring firm’s shareholders are either worse off or at best unaffected (Jensen and Ruback, 1983). These facts seem to be inconsistent with the neoclassical assumptions of rationality and profit-maximization behavior. If mergers are driven by profit-maximization, then why do executives of the acquiring firms often pay more for the target than is justified by its market value?

Roll (1986) proposes the “hubris” hypothesis to explain why many acquiring firms incur losses. Suppose that there are no possible gains in a takeover. According to Roll, managers trust their own valuations more than they trust the market valuation and they will attempt a takeover only when they believe they have found an undervalued firm. If market and managers (on average) are both correct, takeover attempts will occur only in firms where managers’ estimates are overly optimistic and so above the average. As a result, they will incur losses in takeovers. According to this theory, managers, because of their hubris, refuse to account for the biases in their estimates. This behavior parallels that of unsophisticated bidders in a common value auction who do not adjust for the winner’s curse.

Agency problems have been identified as another possible explanation for loss-incurring acquisitions. According to this theory, managers of the acquiring firm may not be acting in the best interest of shareholders while making acquisition decisions. Several motives have been assigned to managers for going on an acquisition binge; some examples are: diversification of managers’ personal portfolio (Amihud and Lev, 1981), the use of free cash flow to increase the size of the firm (Jensen, 1986) and a desire to increase the size of the firm so that it becomes more dependent upon its management (Shleifer and Vishny, 1989).
In the most recent literature on mergers and takeovers, a lot of credit has been awarded to the so called “preemption theory”; a new theory of why acquiring firms pursue value-decreasing horizontal mergers even if the managers are rational and they try to maximize shareholders’ value. If a firm fears that one of its rivals will gain large costs savings or efficiencies or increase its own market power from taking over some third firm then it can be rational for the first firm to preempt this merger with a takeover attempt of its own. By preempting the rival’s bid, the first firm avoids the reduced profits it would have suffered if its own rival would have been successful; but, its post-merger profits could still decrease compared to its pre-merger ones.

Furthermore, this preemptive behavior can be optimal even if it requires the first firm to overpay relative to the increase in the joint profits of the new combined firm. In this case, preemption decreases the net profit of both the rival and the merged firms. In an efficient stock market, stock prices reflect current and expected future net profits: if the discounted cash flow falls, the stock price must also fall (other things being equal).

Therefore, an empirically testable prediction of the preemption theory is that when the acquiring firm loses value as a consequence of the transaction, the rivals should lose as well. This prediction cannot be reconciled with those of the earlier theories that tried to explain why value-decreasing takeovers occur. In fact, if acquiring firm’s managers are irrational and/or self-interested this should benefit the rivals or at least leave them unaffected. Fridolfsson and Stennek (2000) deeply analyzes the argument of preemptive mergers and Molnar (2000b), besides discussing the subject theoretically, also provides some empirical tests for the theory. Moreover, he found that with a slight modification to the benchmark model, called “acquisition probability hypothesis”, preemption theory can also account for for those rivals whose stock price increased when the acquiring firm lost. He concludes claiming that the predictions of the preemption theory fit the data much better than either the simple efficiency or hubris and agency models.

Besides the standard conditions of stability (that is, whether or not insiders benefit from merging their firm instead of staying independent), at the heart of the literature on mergers and takeovers there is also a public good problem among potential buyers. That consolidating an industry can give rise to a public good problem was already pointed out by Grossman and Hart (1980) and Stigler (1990), who observed that such a problem should exist if outsiders gain more than insiders. Typically, outsiders gain from an increase in concentration; moreover, as we have explained in the previous section, it is not uncommon that outsiders gain even strictly more than insiders, implying that each firm would prefer the takeover to happen but the target to be acquired by another firm. Inderst and Wey (2004) show that, even if there are substantial gains for the insiders, when the target sets an optimal reserve price it always creates and exacerbates the public good problem and in the end, the acquirer is worse off than all the other firms.

Fishman (1988) also studies a preemptive motive for merger. However, in his model the preemption happens in the bidding phase and he does not consider the strategic effect of the downstream competition.
3 The Model

Consider three firms (A, B and C) that produce a homogeneous product and compete in a Cournot-like market. Let $P$ and $Q$ denote respectively the price and the overall quantity produced by all the firms operating in the market. Suppose that the demand price is given by

$$P = a - Q.$$  \(1\)

We assume that each firm has a constant, positive marginal cost $c$ with no fixed costs. The superscript $SQ$ (for STATUS QUO) will be used to denote market outcomes when the three incumbents compete in the market. These are given by

$$\pi^i_{SQ} = \frac{(a - c)^2}{16} \quad (2)$$

for $i = A, B, C$.

Suppose there is another firm E that can enter the market only by taking over one of the three actual incumbents. We could think of E either as a foreign firm that wants to enter in a domestic-market or as a firm operating in some other industry and that wants to (horizontally) expand its market.

Let’s now define the subset $S \equiv [0, c] \subseteq \mathbb{R}_+$. Once entered the market firm E can produce with no fixed costs and with a constant marginal cost $c - s_i$ with $s_i \in S \forall i$ and with $c > s_A > s_B > s_C > 0$. The remaining firms would continue to have cost $c$.

These parameters $s_i$ represent the level of cost synergies resulting from the merger.

Firm E has to take a decision on whether entering or not, which target it wants to take over and the exact bid to submit. So, it has to choose a vector of bids, $b = (b^A, b^B, b^C)$, one for each possible target with at least two of them equal to zero (this is because we are assuming that the entrant can submit only one bid).

However, in order to submit a bid and enter the market firm E has to incur a participation (or investigation) cost $F : S \rightarrow \mathbb{R}_+$ with $F(0) = 0$ and $F(s) > 0 \forall s \in S$. Note that this investment is a “sunk” cost: once the bid is submitted, the cost is suffered no matter whether the bid will be successful or not. The cost $F(s)$ can either represent expenses for technical reasons, as for industrial reorganization reasons, or expenses due to legal advices, to the identification of the “right target” or to the resistance of target’s managers and employees. The idea of a participation or search cost that is positively related to the level of synergy looks like rather realistic: the better is the performance achievable in the downstream market, the bigger the cost firm E has to suffer in order to exploit this synergy. This relationship gives rise to two different but consistent interpretations: one in terms of the sophistication of the production process and the other in terms of effort. For the first one, we can think of the synergy parameter as a measure of the efficiency or sophistication of production; in this perspective it is rather obvious that the bigger the sophistication, the bigger is the sunk cost.

\footnote{These synergy parameters allow for several different interpretations: either they can be viewed as a measure of some sort of technological compatibility between the merging firms; or they can represent the ability of managers of the buying firm in successfully carrying out the merger; or, under the idea that firms experiencing some kind of financial distress or default are more easy to be taken over, we can also think of $s_i$ as a parameter which indicates the level of financial distress or crisis of the target and so the easiness of this firm to be taken over.}
for achieving that level of sophistication.\textsuperscript{17} On the other hand, we can look at the participation cost as a measure of the effort the entrant exercises for discovering targets with which it can experience better synergies and for placing a serious bid; thus, it is reasonable that more effort implies a bigger cost.

The bidding stage is modeled as a matching auction: first E submit its bid; then the target decides whether refusing this bid (in which case the STATUS QUO remains) or accepting it. When an initial bid is accepted, the other two incumbents are given the opportunity to match that bid; however, they must answer at the same time. If at least one incumbent matches the bid it is declared as winner, merges with the target, pays the price offered by E and the entry is blocked (when both incumbents match, ties are broken by the usual rule: each of the incumbents wins the auction half of the times). If no incumbent matches the bid, firm E wins the auction, pays its bid, merges with the target and the entry occurs. Firm E’s payoff when it chooses to stay out of the market is normalized to zero.

When potential bidders for a target are heterogeneous, standard auction methods for selling the firm are not generally optimal, as they treat the bidders symmetrically. So, one way to discriminate against the stronger bidder is just that of imposing a precise order of moves. A “matching auction” can achieve this objective, in which the stronger bidder is asked to make a first and final offer, and the other bidders are asked to match this bid.

It is easy to show how this type of mechanism yields a discrimination among the bidders; actually, the same auction represents a first-price auction for the potential entrant firm and a second-price auction for the incumbents so that the first bidder suffers a first mover disadvantage. This can be easily justified either just by recalling that the actual takeover regulations provide that target’s executives have to act in order to maximize shareholders’ value; or by assuming, in a rather realistic way, that incumbents benefit of some preemption rights. Since courts are more concerned about shareholder value than whether the playing field is leveled or not, it is unlikely that the matching auction will run into troubles on the grounds that it does not treat bidders symmetrically.\textsuperscript{18}

We further assume that when two incumbents merge in order to preempt the potential entry, the new market structure is a symmetric duopoly in which the two firms have the same constant marginal cost as before $c$ and earn the same profit; that is, the merger between two incumbents does not yield any synergy or efficiency gain. This is essentially due to the fact that if a merger between incumbents occurs, it has the only purpose of preemption and entry avoidance: the incumbents do not put any effort - that is, they do not suffer any sunk cost - in searching for targets and therefore they do not exploit any synergy.

Thus, the superscript $M$ (for Merged) will be used to denote values of the merged firm while the superscript $U$ (for Un-merged) will be used to denote those of the un-merged ones and the superscript $D$ (for Duopoly) will be used to denote the profits of the incumbents in the duopolistic set up. These are given by:

\textsuperscript{17}If the most sophisticated production process is also the cheapest one the choice of the target would become trivial.\textsuperscript{18}Indeed, it is well established that under some circumstances the matching auction results in a higher expected selling price than the standard auctions when the asymmetry among the bidders is sufficiently large. Possible sources of bidder heterogeneity can be: differences in the interval from which the values are drawn or differences in the bidding function among bidders in the private values case and differences in initial toeholds or asymmetric effects of the bidder’s private signals on the value in the common values one.
\[
\pi^M_E = \left( \frac{a - c + 3s_i}{4} \right)^2 \tag{3}
\]
\[
\pi^U = \left( \frac{a - c - s_i}{4} \right)^2 \tag{4}
\]
\[
\pi^D = \left( \frac{a - c}{3} \right)^2 \tag{5}
\]

for \( i = A, B, C \). All these values and rules are common knowledge.\textsuperscript{19}

It is quite trivial to show that the following comparative statics results hold

\[
\frac{\partial \pi^M_E}{\partial s_i} > 0 > \frac{\partial \pi^U}{\partial s_i} \quad \forall s_i \in S.
\]

Furthermore, we assume:

\[
\pi^M_E > F(s_i) \quad \forall s_i \in S;
\]

otherwise the entrant would not have any incentives to enter the market.

The game unfolds in four steps. At the first step, firm E places its bids for each of the incumbents. The incumbent chosen as target, at the second stage, decides upon the reliability of the offer received from the outsider and has two possible actions: either refusing the bid or asking its two peers whether or not they would match it. At step three, the two incumbents - that have not become target - simultaneously decide whether matching or not. In the last stage the market opens and prices, quantities and profits are determined as described above, depending upon whether or not the entry has occurred. \textit{Figure 1} shows the whole game in its extensive form.\textsuperscript{20}

\textsuperscript{19}Here we consider only the case where both the merged and the un-merged firms produce non-negative quantities and earn a non-negative profit. That is, the case when \( s_i < a - c \ \forall i \).

\textsuperscript{20}Actually \textit{Figure 1} is slightly incorrect; indeed, from a rigorous viewpoint we would have infinite possible real offers for each possible target and so infinite possible sub-games and branches. However, for the sake of simplicity we represent the selection of the target to take over with a single branch and we derive for a given target the best bid among all the possible ones; then, we compare the net downstream profits attainable with each target - placing the best bid for each of them - with the profit attainable by choosing to stay out of the market.
In sub-sections 4.1-4.3 we are going to analyze a simplified version of the more general model in which we assume the target to be taken over to be exogenously identified and we derive the SPNE under this assumption. Then, in sub-section 4.4 we solve the model in its more general fashion where the choice of the target to take over becomes endogenous.

3.1 Equilibrium of the matching stage

We start our analysis by looking at the last sub-game, that is the matching stage. Suppose firm E submitted a bid, $b_A$, for firm A (from now on and unless differently specified we will work with $i = A$; it is just an example, but the analysis that follows applies to all cases in which firm E submits a serious bid for one of the three possible targets). The strategies available to firms B and C are “to match” and “not to match”. The payoffs of the two incumbents will of course depend on the bid submitted by E.

Figure 2 illustrates in strategic (or normal) form the matching (sub-)game when A is the chosen target.

![Diagram](image)

**Figure 1**: The Takeover-Entry Game

<table>
<thead>
<tr>
<th>B\C</th>
<th>“to match”</th>
<th>“not to match”</th>
</tr>
</thead>
<tbody>
<tr>
<td>“to match”</td>
<td>$\pi_D - \frac{b_A^2}{2}$; $\pi_D - \frac{b^2}{2}$</td>
<td>$\pi_D - b_A^2$; $\pi_D$</td>
</tr>
<tr>
<td>“not to match”</td>
<td>$\pi_D^U (s_A)$; $\pi_D^U (s_A)$</td>
<td>$\pi_D^U (s_A)$; $\pi_D^U (s_A)$</td>
</tr>
</tbody>
</table>

**Figure 2**: The Matching sub-game

21 According to the Auction Theory literature, we call a bid serious if it is strictly positive.
We can distinguish two possible cases:

i) \( b^A \geq \pi^D - \pi^U (s_A) \)

ii) \( b^A < \pi^D - \pi^U (s_A) \).

Lemma 1 describes the equilibria in both cases.

**Lemma 1** In case i) there is just one pure strategy NE in which both the incumbents choose “not to match” so that the entry cannot be avoided.\(^{22}\) In case ii), on the contrary, there are two pure strategy NE: firm B will match the bid submitted by firm E and firm C will not, or vice versa in the other equilibrium; and one mixed strategy NE in which both the players are mixing between matching and not matching with probability \( p^* (b^A) \) and \( 1 - p^* (b^A) \) respectively, where \( p^* (b^A) = \frac{2[\pi^D - \pi^U (s_A) - b^A]}{2\pi^D - 2\pi^U (s_A) - b^A} \).

**Proof:** Actually the proof of this Lemma is quite trivial. Assume firm B follows the above described strategy; we show that firm C has no incentive to deviate from that strategy. Let’s start from case i). In this case, it is easy to check that “not to match” is a weakly dominant strategy. Indeed, by matching the offer made by E, firm C’s payoff would be \( \pi^D - b^A \) which is lower than or, at the most, equal to \( \pi^U (s_A) \).

Now we turn over case ii). In this situation each player would strictly prefer its peer to match the bid; however, if player C does not match, player B’s best response is to match and vice versa. So, if firm B is going to match the bid, firm C’s best response is not to match; given that firm C is not going to match E’s bid, firm B’s best response is to match that bid.

Furthermore, in case ii) there is also the mixed strategy equilibrium in which the strategy is “to match” with probability \( p^* (b^A) \). This probability makes firm C indifferent between matching or not. Given that firm B is playing that mixed strategy, the expected payoff for firm C arising from the same mixed strategy dominates that referable to other mixed or pure strategies.

Finally, it is easy to show that a situation in which both incumbents match firm E’s bid can never constitute a NE; indeed, given that B is matching, C’s best response is not to match at all in case i) as well as in case ii). \( Q.E.D. \)

### 3.2 Equilibrium of the acceptance stage

\( \star \) The target, once received an offer from the potential entrant, can take two possible actions: either rejecting it or asking the two incumbents whether they are willing to match that bid; however, by asking the incumbents to match the bid submitted by E the target automatically reveals its willingness to accept that bid and this means that a merger will take place for sure. Here we consider only the case in which the target firm has not a share in the downstream profits of the

\(^{22}\)Actually, in case i) “Not to match” is a (weakly) dominant strategy for both the incumbents.
firm resulting from the merger; that is, if the takeover takes place, the target just takes the money and drops out of the market. This allows us to leave out the cases of joint ownership and profits’ sharing.

Therefore, it will accept only a bid higher than, or equal to, the reserve price; where the reserve price is represented by the best “outside” option for the target.\footnote{For a general and more rigorous discussion on the optimality conditions of a reserve price, see Riley (1999).}

Given the structure of the matching auction, it is pretty clear that each plausible target’s reserve price is just the STATUS QUO profit, $\pi^{SQ}$. Indeed, this is the profit the target would get by rejecting firm E’s offer.

So, the condition about the minimum bid the target (in our example, firm A) could accept is as follows:

$$b^A \geq \pi^{SQ}.$$ \hspace{1cm} (acceptance condition)

A thing that is worthy to note is that the acceptance condition does not depend upon the synergy parameter $s_i$ and does not vary across the three incumbents; as we shall see, this could play a non trivial role in the derivation of the optimal bidding strategy for the entrant. We turn now over the bidding stage.

3.3 Equilibrium of the bidding stage

\hspace{1cm} \triangleright \hspace{1cm} Up to now, by using standard backward induction principles we have solved the last two stages of the whole game. In this section we are going first to derive the winning bidding strategy for firm E assuming that the entry is profitable; then, we will look at the conditions under which the entry is profitable.\footnote{We have also carried on our analysis keeping the assumption that firm E chooses A as target. In the next section we will show the general and final equilibrium where firm E has to decide, at the same time, not only whether entering or not and the optimal bid, but also which incumbent it wants to take over.}

In order to find firm E’s optimal bid at the first stage, we must distinguish two possible cases.

I) $\pi^D - \pi^U (s_A) < \pi^{SQ}$

II) $\pi^D - \pi^U (s_A) \geq \pi^{SQ}$.

The first case arises when, from the viewpoint of the entrant, the acceptance condition is a binding constraint; that is, if a bid satisfies the acceptance condition, it is also a “winning” bid such that no incumbent will never match it. On the other hand, the second scenario takes place when the acceptance condition is actually not binding; that is, a “winning” bid, such that all rivals choose “not to match”, already satisfies the acceptance condition. The following Lemma describes the equilibrium bids in both cases.
Lemma 2  The optimal bidding strategy for firm $E$ is as follows:

$$b^A = \max \{ \pi^S \mu, \pi^D - \pi^U (s_A) \}$$

Proof: Let’s start from case I). If this is the case, firm $E$ cannot be better off than by bidding just $\pi^S$; indeed, at $b^A = \pi^S$ the entrant wins the auction, and realizes a net profit of $\pi^M_E (s_A) - F (s_A) - \pi^S$. Now let’s consider a small $\varepsilon > 0$. By bidding $b^A = \pi^S + \varepsilon$ firm $E$ is worse off because it still wins the auction but pays an higher price for the target; on the other hand, by bidding $b^A = \pi^S - \varepsilon$ firm $E$ is worse off again because it loses the auction and attain a net profit of $-F (s_A)$ since the target will refuse this offer.

Case II) is slightly more tricky. As pointed out in Lemma 1 the equilibrium of the matching sub-game depends on the relation between $b^A$ and $\pi^D - \pi^U (s_A)$. When $b^A \geq \pi^D - \pi^U (s_A)$, firm $E$’s net profit is a decreasing function of $b^A$: 

$$\Pi_E \equiv \pi^M_E (s_A) - F (s_A) - b^A.$$ 

On the contrary, when $b^A \leq \pi^D - \pi^U (s_A)$ we are in the mixed strategy’s space of the matching sub-game. In this situation, firm $E$’s net expected profit is given by

$$E (\Pi_E) = \left[ \frac{b^A}{2\pi^D - 2\pi^U (s_A) - b^A} \right]^2 \left[ \pi^M_E (s_A) - b^A \right] - F (s_A).$$

So, once defined firm $E$’s generalized profit as

$$G_E \equiv \left\{ \begin{array}{ll} \Pi_E & \text{if } b^A \geq \pi^D - \pi^U (s_A) \\ E (\Pi_E) & \text{if } b^A \leq \pi^D - \pi^U (s_A) \end{array} \right.$$ 

to complete the Proof we have to show that $\max_{b^A} G_E$ is actually attained at

$$b^A = \pi^D - \pi^U (s_A).$$

Since $\Pi_E$ is a decreasing monotonic function of $b^A$ and is defined only for $b^A \in [\pi^D - \pi^U (s_A), +\infty[$ we trivially have that $\arg \max (\Pi_E) = \pi^D - \pi^U (s_A)$. Now we have to prove that $\arg \max (E (\Pi_E)) = b^A \in \{ \pi^D - \pi^U (s_A) \}$
\[ \pi^D - \pi^U(s_A). \] To this end, bearing in mind that \( E(\Pi_E) < 0 \) for \( b^A = 0 \) and that \( E(\Pi_E) > 0 \) for \( b^A = \pi^D - \pi^U(s_A) \), it is enough to show that \( E(\Pi_E) \) is a monotonically increasing function; that is: \( \frac{\partial E(\Pi_E)}{\partial b^A} > 0 \forall b^A \in ]0, \pi^D - \pi^U(s_A)] \).

We have that

\[
\frac{\partial E(\Pi_E)}{\partial b^A} \equiv \frac{b^A}{[2\pi^D - 2\pi^U(s_A) - b^A]^2} \cdot \left\{ 2\left[ \pi_E^M(s_A) - b^A \right] \cdot \frac{2\pi^D - 2\pi^U(s_A)}{2\pi^D - 2\pi^U(s_A) - b^A} - b^A \right\} > 0
\]

only if

\[
2\left[ \pi_E^M(s_A) - b^A \right] \cdot \frac{2\pi^D - 2\pi^U(s_A)}{2\pi^D - 2\pi^U(s_A) - b^A} - b^A > 0.
\]

(6)

Since \( b^A < 2\pi^D - 2\pi^U(s_A) \) inequality (6) is equivalent to

\[
(b^A)^2 - 6\left[ \pi^D - \pi^U(s_A) \right] b^A + 4\pi_E^M(s_A)\left[ \pi^D - \pi^U(s_A) \right] > 0.
\]

(6 bis)

Since \( \pi_E^M(s_A)\left[ \pi^D - \pi^U(s_A) \right] > 0 \), inequality (6 bis) holds whenever \( \Delta \leq 0 \), where \( \Delta = 9\left[ \pi^D - \pi^U(s_A) \right]^2 - 4\pi_E^M(s_A)\left[ \pi^D - \pi^U(s_A) \right]. \)

Further, if \( \Delta > 0 \) inequality (6 bis) holds for either \( b^A < b_1^A \) or \( b^A > b_2^A \) where

\[
b_1^A = 3\left[ \pi^D - \pi^U(s_A) \right] - \sqrt{9\left[ \pi^D - \pi^U(s_A) \right]^2 - 4\pi_E^M(s_A)\left[ \pi^D - \pi^U(s_A) \right]}
\]

and

\[
b_2^A = 3\left[ \pi^D - \pi^U(s_A) \right] + \sqrt{9\left[ \pi^D - \pi^U(s_A) \right]^2 - 4\pi_E^M(s_A)\left[ \pi^D - \pi^U(s_A) \right]}
\]

Since

\[
3\left[ \pi^D - \pi^U(s_A) \right] + \sqrt{9\left[ \pi^D - \pi^U(s_A) \right]^2 - 4\pi_E^M(s_A)\left[ \pi^D - \pi^U(s_A) \right]} > 3\left[ \pi^D - \pi^U(s_A) \right] > \left[ \pi^D - \pi^U(s_A) \right]
\]

\( ^{25}\)Note that \( \left[ \pi^D - \pi^U(s_A) \right] > 0 \forall s \in S \)
we have that the solution $b^A_2$ is on the right of the subset on which $E(\Pi_E)$ is defined.

To complete the proof we must show that $b^A_1 > \pi^D - \pi^U(s_A)$, that is

$$2 \left[ \pi^D - \pi^U(s_A) \right] > \sqrt{9 \left[ \pi^D - \pi^U(s_A) \right]^2 - 4\pi^M_E(s_A) \cdot \left[ \pi^D - \pi^U(s_A) \right]}$$

which is equivalent to

$$\pi^M_E(s_A) > \frac{5}{4} \left[ \pi^D - \pi^U(s_A) \right]$$

and, because of expressions (3)-(5) to

$$\left( \frac{a - c + 3s_A}{4} \right)^2 > \frac{5}{4} \left[ \left( \frac{a - c}{3} \right)^2 - \left( \frac{a - c - s_A}{4} \right)^2 \right]$$

that is

$$(a - c)^2 + 126(a - c)s_A + 369s_A^2 > 0$$

which is always true.

So, we have proved that in the range $\left[ 0, \pi^D - \pi^U(s_A) \right] E(\Pi_E)$ monotonically increases; on the other hand, $\Pi_E$ is monotonically decreasing over $\left[ \pi^D - \pi^U(s_A), +\infty \right)$. Thus $G_E$ reaches its maximum at $b^A = \pi^D - \pi^U(s_A)$. $Q.E.D.$

*Figure 3* in the following page illustrates $G_E$ as function of $b^A$. 

18
From the Lemmas analyzed so far we can derive the following, important, Corollary.

**Corollary 1** *The strategy profile

\[
\{b^A = \max \{\pi^{SQ}, \pi^D - \pi^U(s_A)\}\} \text{ "accept"; "not to match"; "not to match"}
\]

constitutes a SPNE for case I) and II) respectively. And this also the only SPNE in which the entry occurs.*

**Proof:** To show this, we must recall and put together the results provided by Lemmas 1 and 2 and the *acceptance condition*. Indeed, each of them has furnished us with a NE for each informative set (the matching stage, the acceptance stage and the bidding stage). Therefore, the strategy profiles mentioned in Corollary 1 constitute a SPNE for case I) and II) respectively because the strategy played by each player at each informative set is a best response to the strategies played by the other players at all the previous and subsequent informative sets.

Moreover, if firm E bids less than \(b^A = \max \{\pi^{SQ}, \pi^D - \pi^U(s_A)\}\) the entry would not happen; on the other hand, we have already shown in Lemma 2 that bidding more than that amount would not be optimal; thus, \(b^A = \max \{\pi^{SQ}, \pi^D - \pi^U(s_A)\}\) is the only plausible bid when the entry is profitable in both cases I) and II). And the incumbents are not willing to play a strategy that is
different from that specified by the Corollary because they would be worse off. Therefore, this is also the only SPNE when the entry turns out to be profitable.  \( Q.E.D. \)

Of course the bidding equilibrium strategy analyzed in Lemma 2 is valid only when the entry turns out to be profitable for the potential entrant firm. Therefore, firm E compares the profit attainable by entering the market with the reserve profit he could attain by remaining out of the market and that we normalize to zero for the sake of simplicity.

We conclude this section with the following Lemma that describes the conditions for which the entry is profitable.

**Lemma 3**  Firm E’s entry strategy is:

\[
\begin{align*}
\text{sub case I)} \\
b^A &= \begin{cases} \\
\pi^{SQ} & \text{if } \pi^M_E(s_A) - F(s_A) - \pi^{SQ} \geq 0 \\
0 & \text{if } \pi^M_E(s_A) - F(s_A) - \pi^{SQ} < 0 
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\text{and sub case II)} \\
b^A &= \begin{cases} \\
\pi^D - \pi^U(s_A) & \text{if } \pi^M_E(s_A) - F(s_A) - \pi^D - \pi^U(s_A) \geq 0 \\
0 & \text{if } \pi^M_E(s_A) - F(s_A) - \pi^D - \pi^U(s_A) < 0 
\end{cases}
\end{align*}
\]

**Proof:** It is rather trivial to note that the entrant will submit a serious bid only when its net final profit is at least equal to its reserve profit, that is zero. When the entrance is profitable, in case I) as well as in case II), firm E submits that bid that we have previously shown in Lemma 2 to be the net profit-maximizing bid.  \( Q.E.D. \)

### 3.4 General solutions of the game and selection of the right target

In the previous sections we have derived the SPNE and the entry conditions on the assumption that firm A was the only possible target. Now we use the results obtained so far in order to extend our model by making the choice of the target endogenous to the model itself. This allows us to give a more realistic picture of the phenomenon and to better highlight the relationship between the synergy parameters, the optimal bid and the entry conditions.

Thus, at the first step firm E has not to choose only one bid anymore, but instead a vector of bids \( b \equiv (b^A, b^B, b^C) \) with at least two elements out of three of the vector being equal to zero. Furthermore, we consider the case in which E chooses to play \( b = b^0 \), that is the vector of bids where all the elements are zero, as the decision to stay out of the market.

Recalling our assumptions about the different \( s_i \) and about the sign of \( \frac{\partial \pi^U(s_i)}{\partial s_i} \) for \( i = A, B, C \) we can have now four different cases:
1) \( \pi^{SQ} < \pi^{D} - \pi^{U} (s_c) \)
2) \( \pi^{D} - \pi^{U} (s_C) \leq \pi^{SQ} < \pi^{D} - \pi^{U} (s_B) \)
3) \( \pi^{D} - \pi^{U} (s_B) \leq \pi^{SQ} < \pi^{D} - \pi^{U} (s_A) \)
4) \( \pi^{D} - \pi^{U} (s_A) \leq \pi^{SQ} \).

which, recalling expressions (3)-(5) and keeping in mind that \( a - c > s_A > s_B > s_C > 0 \), are equivalent to:

1) \( s_A, s_B, s_C \in \left[ \frac{(a-c)(6\sqrt{1-t}-18)}{18}, a-c \right] \)
2) \( s_A, s_B \in \left[ \frac{(a-c)(6\sqrt{1-t}-18)}{18}, a-c \right] \) and \( s_C \in \left[ 0, \frac{(a-c)(6\sqrt{1-t}-18)}{18} \right] \)
3) \( s_A \in \left[ \frac{(a-c)(6\sqrt{1-t}-18)}{18}, a-c \right] \) and \( s_B, s_C \in \left[ 0, \frac{(a-c)(6\sqrt{1-t}-18)}{18} \right] \)
4) \( s_A, s_B, s_C \in \left[ 0, \frac{(a-c)(6\sqrt{1-t}-18)}{18} \right] \).

Case 1) relates to the situation in which the acceptance condition is never binding for any of the incumbents while case 4) refers to the just opposite one; that is when the acceptance condition is always binding. Cases 2) and 3), on the other hand, take place when the acceptance condition is indeed binding for some of the targets but not for all of them.

Let’s now introduce the following expressions:

\[
\frac{\partial \pi^M_i}{\partial s_i} - F' (s_i) \quad (7)
\]

and

\[
\frac{\partial \pi^E_i}{\partial s_i} - F' (s_i) + \frac{\partial \pi^U_i}{\partial s_i} \quad (8)
\]

for \( i = A, B, C \). These expressions are nothing but the partial derivatives of the net-profit function \( \pi^M_i (s_i) - F (s_i) - b^i \) where \( b^i = \max \{ \pi^{SQ}, \pi^{D} - \pi^{U} (s_i) \} \) and their sign will play a key role in the process of selection of the best target. However, we do not have an explicit expression for \( F (s_i) \) because we did not want to restrain our analysis to a particular form of function; so, we consider only those functions \( F (s_i) \) such that the signs of (7) and (8) are invariant, that is only the cases in which the profit function increases or decreases monotonically.

To this end, we solve the differential equations \( F' (s_i) = \frac{3(a-c+3s_i)}{8} \) and \( F' (s_i) = \frac{a-c+5s_i}{4} \) obtained by setting equal to zero expressions (7) and (8) respectively. From the first (second) one we get \( \alpha (s_i) = \frac{3}{8} (a - c) s_i + \frac{9}{16} s_i^2 + k \) (\( \omega (s_i) = \frac{a-c}{4} s_i + \frac{5}{8} s_i^2 + k \)) where \( k \) is a not specific constant.

Thus we have that (7) \( (\forall s_i \in \mathbb{R}) \Leftrightarrow F (s_i) < (>) 0 \forall s_i \Leftrightarrow (\forall s_i) \Leftrightarrow F (s_i) < (>) 0 \forall s_i \Leftrightarrow (\forall s_i) \).

The following Proposition describes the only SPNE of the whole game.

\(^{26}\) It is very easy to verify that (8) \( \Leftrightarrow (\forall s_i) \).

\(^{27}\) We prefer the assumption on monotonicity over the one of setting an explicit expression for the sunk cost because, in our humble opinion, the former appears a good arrangement between generality of results and analytical tractability.
Proposition 1  The optimal strategy for firm E is to choose the vector \( b^* \equiv (b^A, b^B, b^C) \) where two elements out of three are equal to zero and the only non-negative element represents the bid placed for the target i whose \( s_i \) is such that maximizes the net downstream profit if \( \pi^M_E(s_i) - F(s_i) - b^i \geq 0 \) where \( b^i = \max \{ \pi^{SQ}, \pi^D - \pi^U(s_i) \} \) for \( i = A, B, C \) and \( b^* = b^0 \) otherwise, with \( b^0 \equiv (0, 0, 0) \). When \( b^* \neq b^0 \) the incumbent selected as target will ask its peers to match the entrant’s bid but they will not and the entry will occur.

Proof: We have already proved through Lemmas 1-2 that, for any given target \( i \), the optimal winning bid is \( b^i = \pi^D - \pi^U(s_i) \) when the acceptance condition is not binding at all and \( b^i = \pi^{SQ} \) when the acceptance condition is actually binding. Furthermore, we have also shown by Lemma 3 that, given an optimal bidding strategy, entry occurs only when it yields a final net profit greater or equal than zero. So, for any of the four cases 1)-4) firm E will choose to take over the target which maximizes its ex-post profit if this is at least non-negative; otherwise, it will not enter the market. Q.E.D.

The proof of the previous Proposition is indeed quite trivial: if firm E can choose its target, it is rather obvious that it will select the “best” one; that is, the one that maximizes its net downstream profit. But, what are the features and the elements that characterize the “best” target?

If we had an explicit expression for \( F(s_i) \) we could have calculated the exact values of the profit function for each \( i \) and then selected the highest one; moreover, if we had also a continuum of plausible targets we could have found the optimal target just by solving the FOC for either (7) or (8). However, we do not have a continuum of incumbents - which we think would be pretty unrealistic - and we do not want to restrain our analysis to a precise expression for \( F(s_i) \). So, what we do is looking at the best target and its features when both (7) and (8) are monotonic. This leads to the taxonomy showed in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Case 1)</th>
<th>Case 2)</th>
<th>Case 3)</th>
<th>Case 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((7), (8) &lt; 0)</td>
<td>(C)</td>
<td>(C)</td>
<td>(C)</td>
<td>(C)</td>
</tr>
<tr>
<td>((7), (8) &gt; 0)</td>
<td>(A)</td>
<td>(A)</td>
<td>(A)</td>
<td>(A)</td>
</tr>
<tr>
<td>((7) = 0, (8) &lt; 0)</td>
<td>(C)</td>
<td>(C)</td>
<td>(B ~ C)</td>
<td>(A ~ B ~ C)</td>
</tr>
<tr>
<td>((7) &gt; 0, (8) = 0)</td>
<td>(A \sim B \sim C)</td>
<td>(A \sim B)</td>
<td>(A)</td>
<td>(A)</td>
</tr>
<tr>
<td>((7) &gt; 0, (8) &lt; 0)</td>
<td>(C)</td>
<td>(C or B)</td>
<td>(B or A)</td>
<td>(A)</td>
</tr>
</tbody>
</table>

Table 1: A Taxonomy for the cases in which the downstream net profit functions are monotonic.

In Table 1 each column corresponds to a different case with respect to the relationship between the winning bid and the acceptance condition, whereas each row correspond to a different condition on the signs of expressions (7) and (8); finally, each cell contains the index of the best target for that pair of conditions.
We have already discussed the meaning of cases 1)-4). With respect to the conditions on the rows it is quite trivial to show that the sign of (7) restrains, to some extent, that of (8); indeed, since \((8) < (7) \forall s_i\) we have that if \((7) \leq 0\) it follows that \((8) < 0\) must hold. On the other hand, when \((7) > 0\) there are three different all possible cases for the sign of (8).

The Proposition of this Section concludes our model. In the next section we are going to discuss the results of the model and its policy implications together with some examples.

### 3.5 Results of the model

- Propositions 1 along with the assumptions on which it relies provided us with clear economic results. However, in order to discuss its implications we must have a look at the insights that arise from the analysis of each of the stages of the whole sequential game.

At the matching stage we have identified several different possible equilibria (the asymmetric one which refers to the pure strategy set-up and the symmetric ones which in turn apply to both the pure and mixed situations). Of course the more interesting states are those in which the entry cannot be blocked. Apart from the simple case in which “not to match” is a weakly dominant strategy for both the incumbent - that is if \(b' \geq \pi^D - \pi^U (s_i)\) - we have found that the entry can happen also if \(b' < \pi^D - \pi^U (s_i)\). In this situation, in fact, a sort a coordination problem arises between the incumbents: each firm would like the entry to be avoided, but would prefer its rival to match the bid. This leads to the existence of a NE in mixed strategy which, in turn, makes the entry still possible. We have shown that the net profit-maximizing bid for firm E is actually \(b' = \pi^D - \pi^U (s_i)\); this happens essentially because the derivative with respect to \(b'\) of firm E’s generalized profit \(G_E \equiv \Pi_E + E (\Pi_E)\) is positive in the space of the mixed strategy (because the higher the bid the higher the probability of not matching and - therefore of entry - is) and negative over the pure strategy’s space.

We can give an interpretation of the matching stage also in terms of the negative externalities that the entrant imposes on the incumbents. In fact, because of the synergy parameter \(s_i\) the entrant is able to produce with a more efficient technology so that the oligopoly from being symmetric (in the STATUS QUO) becomes asymmetric; the measures of this negative externality is given by \(\pi^{SQ} - \pi^U (s_i)\) for \(i = A, B, C\). From a comparative statics viewpoint, it is easy to verify that the grater the synergy gains are, the greater the externality is.

Therefore, \(\pi^D - \pi^U (s_i)\) really represents the gain the incumbents would attain by avoiding firm E’s entry and the bidding price means the cost they have to suffer in order to avoid the negative externalities related to the entry. As long as the price that an incumbent has to pay in order to avoid the entry is smaller than the gain, the entry can be blocked. When, however, the price exceeds this sort of threshold the incumbents are not willing to block the entry anymore.

The acceptance stage has provided us with another condition, namely the acceptance condition, about firm E’s optimal bid. The intuition is quite obvious: all the incumbents that can be selected as target have a reserve price that is equal to the profit they would get by refusing the takeover bid; in our model this profit is \(\pi^{SQ}\). Though obvious, this acceptance condition heavily affects the prediction of the model. Indeed, according to whether or not this condition is more binding than the one on the “winning” bid we have different bidding equilibria. More precisely, the winning price of
the matching auction is directly affected by the *acceptance condition* while the choice of the target also depends on it but only in an indirect way.

The role played by the *acceptance condition* becomes clearer when solving for the equilibrium bidding functions in the simplified version of the model in which the target is exogenously given. In fact, it is easy to note that in the case in which the target’s reserve price is not binding the bid that arises in equilibrium is higher than the one arising when the *acceptance condition* is binding.\(^{28}\)

Of course, with respect to the entrant strategy, in order to decide whether to enter the market or not firm E must take into account the net downstream profit \(\pi^M_E (s_i) - F (s_i) - b^i\) where \(b^i = \max (\pi^Q, \pi^D - \pi^U (s_i))\). Thus, another important variable to take into account is the sunk cost. Of course the smaller the sunk cost, the more likely is that firm E enters the market. However, given the assumptions on \(F (\cdot)\) there is a clear trade off because the bigger the synergies are, the bigger the downstream profit but the bigger the sunk cost too.

Therefore, the model studied in its simplified version characterizes the optimal bidding and entry strategy for the entrant; however, we can not draw any conclusions about how and why the entrant chooses the target to take over.

The frame of the general model, where the target is endogenously selected, is rather richer. In fact, in this way we can relate the sunk cost, the synergies and the bidding strategies not only to the decision about the profitability of entering the market, but also to the selection of the optimal target to take over.

We can identify two main effects that affect the choice of the “best” target. One, that we can call as *participation effect* and that is given by \(-F (s_i)\), is essentially due to the sunk cost suffered to start the takeover battle; the other, namely the *competition or externality effect* is given by \(\pi^U (s_i)\) and relates to the assessment of the equilibrium winning bid (that is a bid such that it will not be matched). The direction as well as the magnitude of these two effects do heavily matter.

In this context, a crucial role is assigned to the sign of expressions (7) and (8). Indeed, when (7) and (8) have concordant sign, that means that the *competition effect* is weaker than the *participation* one, the model yields the results that after all were to be expected: if non-negative, when the net profit of the entrant firm (monotonically) increases with the level of synergies the best incumbent to take over is actually the one with which the entrant can experience the greatest synergies (firm A in our model) and when instead the net profit (monotonically) decreases with the level of synergies the best target is the firm with which the entrant can experience the smallest synergies (firm C in our model).\(^{29}\) This happens essentially because the search costs affect the downstream profits of the merged firm more than the bid paid by the entrant to carry out the merger. Of course the price paid by E in equilibrium will be different depending on whether or not the *acceptance condition* is binding (that is, depending on which column in Table 1 we actually are); but the target is unambiguously determined.

Rather more original and interesting are the results offered by the model when (7) and (8) have different sign. In this situation, not only the equilibrium bid differs with respect to whether the *acceptance condition* is binding or not, but the incumbent chosen as target changes too.

For instance, in the third column of Table 1 we have that (7) = 0 and (8) < 0; this actually

\(^{28}\)Whether the reserve price is binding or not depends on the values of the synergy parameters \(s_i\).

\(^{29}\)Note that these two cases correspond to the first two rows in Table 1.
means \( \frac{\partial \pi^M}{\partial \pi^i} = F'(s_i) \) \( \forall s_i \) and therefore that by moving from a less efficient target to a more efficient one, the gain in terms of efficiency is totally outweighed by the higher sunk cost. This in turn implies that in case 4), where the winning bid does not vary across targets, for firm E each target is the same. But this indifference is reduced in case 3) where firm E considers \( A \prec B \sim C \) and disappears in cases 1)-2) where the best target is \( C \); this happens because in cases 1)-3) the winning bid is target-specific.

The example discussed above is perfectly mirrored by the situation described in the fourth row of Table 1. Here we have that \( (7) > 0 \) and \( (8) = 0 \); thus, while in case 1) all the targets are the same from the entrant's perspective, in case 2) \( A \sim B \succ C \) and \( A \) becomes the best target in both cases 3) and 4). In this case we have that the difference between the downstream profit and the sunk cost increases with \( s_i \) but the competition effect perfectly outweighs this difference.

The last case, which looks like the more interesting one, is depicted in the fifth row of Table 1. Here we have that \( (7) > 0 \) and \( (8) < 0 \) which in turn implies that the competition effect has a bigger magnitude and works in a different direction than the participation effect; in fact, expression \( (7) \) in which the competition effect plays no role at all is positive while expression \( (8) \) is negative. This yields two unambiguous cases that are 1) and 4) and two ambiguous ones that are 2) and 3).

In case 4), when the acceptance condition is actually binding for all the three incumbents, and if the entry is profitable the entrant cannot be better off than by bidding for firm A (that is, for the best firm from the synergy point of view). On the other hand, in case 1) the best possible target coincides with the incumbent with which the entrant would experience the smallest synergies (firm C in our model).

With respect to cases 2) and 3), unless we introduce some other assumptions, we are able only to rule out one of the incumbents from the set of the plausible optimal targets (firm A in the second case and firm C in the third one). More precisely, we would need some more restrictive assumption about the distance \( s_i - s_j \) for \( i = A, B, j = B, C \) and \( i \neq j \). However, firm E will choose its target among two incumbents that are “contiguous” in terms of synergies; and even without knowing the precise distances between the \( s_i \) we can identify a sort of threshold in both cases. Indeed, there must exist two distances \( (s_B - s_C)^* \) and \( (s_A - s_B)^* \) such that firm E is indifferent to bid either \( b \equiv (0, 0, \pi^{SQ}) \) or \( b \equiv (0, \pi^D - \pi^U(s_B), 0) \) and to bid either \( \bar{b} \equiv (0, \pi^{SQ}, 0) \) or \( \bar{b} \equiv (\pi^D - \pi^U(s_A), 0, 0) \) for cases 2) and 3) respectively. Therefore, for instance, in case 2) firm E will bid for firm C if \( s_B - s_C < (s_B - s_C)^* \) and for firm B otherwise; the same discussion applies for case 3).

The economic rationale underlying these last two cases is that as long as the synergy difference \( s_i - s_j \) for \( i = A, B \) \( j = B, C \) and \( i \neq j \) is sufficiently small, the entrant will take over the “worst” incumbent in order to pay a lower price; on the other hand, when the difference term mentioned above becomes sufficiently large the synergy gain outweighs the price difference so that firm E will take over the incumbent with which it can experience the highest synergies.

In our opinion these results have some interesting implications from the regulation point of view as well as from that of the efficiency and of the welfare of the society as a whole.

The main point is that, when the acceptance condition is binding for all the incumbents the price that firm E has to bid in order to enter the market does not vary across the targets and does not neither depend on the externality its entry impose on the rivals; therefore, in this situation the
choice of the target is completely driven by the participation effect.

Dramatically different is the story when the acceptance condition is not binding at all for any of the incumbents; in this case, the price at which firm E wins the auction, and therefore entries, is target-specific and strongly related to the negative externalities the other incumbents would suffer. Thus, if the competition effect is big enough it can happen that the net downstream profit for firm E is a decreasing function of the level of the synergies so that the firm with the lowest \( s_i \) becomes the best target. This can be explained by noting that the higher is \( s_i \) the higher is the negative externality suffered by the un-merged incumbents and, thus, the higher is the bid firm E has to submit in order to win the takeover.

It should be clear now that the choice of the target in a takeover strongly relies on the type and extent of the sunk costs that the bidder has to suffer in order to submit its bids; indeed, these costs strongly affect the ability of the bidder in placing a successful bid. However, in the prospect of the society as whole and of the efficiency it is rather obvious that the equilibria in which firm A is selected as target strongly dominates the others equilibria.

Therefore, the main implication that arises from our model is that too high sunk costs can give spring to less efficient mergers - where the efficiency is considered from the synergies’ viewpoint - and, to some extent, prevent a merger that could turn out to be more efficient and desirable from the consumers’ perspective as well as from that of the society as a whole.

The market considered in our model can also resemble as a regulated industry in which there cannot be more than three firms. It is well-known and almost unanimously accepted that the higher is the number of firms operating in a given market, the less this market is concentrated; and this, in turn, implies that a higher degree of participation and competition benefits the consumers surplus as well as the total welfare.

Our model claims that the issue is not only that of favoring participation and, therefore, enlarging the number of firms operating in the market; we show that also by keeping constant the number of firms there is still space for improvements i.e., there could be some more efficient firm which is left out of the market. However, the high participation costs and the preempting strategy - finalized to avoid negative externalities due to the entry of a dangerous competitor - pursued by the incumbents can either bar the entry at all of a more efficient firm or give rise to less efficient mergers.

4 Conclusions

A main policy question on takeovers is about whether or not takeover regulations should be structured to encourage or discourage auctions.

One argument against auctions is that they weaken incentives to search for under-performing target firms (Schwartz 1986). Search is expensive and auctions reduce the gains from search. However, some participants in the takeover boom of 1980s judge that too much takeover research was done - that much of the research was duplicated and undertaken by investment bankers and other financial professionals in search of big contingent fees (Herzel and Shepro 1990, 1992). By dampening the incentives to search, auctions may reduce this overfishing problem.

Cramton and Schwartz (1991) argue that the desirability of takeover auctions hinges on three
factors: the objective of the authority in setting policy, the auction environment, and whether the target managers are trustworthy. Two policy objectives are considered: efficiency and revenue maximization. Under efficiency, the authority sets takeover policy to maximize the social welfare. Under revenue maximization, policy is set to maximize the gain to target shareholders after an initial bid is made. The rationale for a goal of revenue maximization is weak. It appears in the law, because parties commonly sue target’s board for a failure to fulfill its fiduciary duty. In the context of takeovers, this duty to shareholders is to maximize revenue to target shareholders. Hence, the courts have focused on the question did the board maximize revenues? However, there is little reason to favor target shareholders over society as a large, and thus efficiency is the better objective.

Under the efficiency objective, auctions should be discouraged in common value settings. However, when values differ across bidders, there are efficiency gains to holding an auction. When values differences are large, auctions should be encouraged.

The case for auctions is much stronger under a revenue maximization objective. Auctions generally dominate negotiation in terms of revenue. Ascending auctions are especially desirable when the target’s board is unfaithful, as should be presumed. Ascending auctions provide a process of price discovery. Value is socially determined through the escalation of bids. The bidders learn from each other’s bidding, adjusting valuations throughout the process. This process is important when resale is a possibility or more generally when others have information relevant in assessing the target’s value This open competition gives ascending auctions a legitimacy that is not shared by other auctions. Throughout the auction, every bidder is given the opportunity to top the high bid. The auction ends when no bidder is willing to do so. The winner can say, “I won because I was willing to pay a bit more than the others”. Losers are given every opportunity to top the winning bid. Their loss stems solely from their failure to do so.

Boards should be limited in their ability to take actions to favor particular bidders or to discourage entry. Discouraging entry is rarely in the shareholders interest. Favoring particular bidders may be in the shareholders interest provided the favoring is done to level an uneven playing field. However, with unfaithful boards, there is no guarantee that this will be the case.

Under the presumption that boards are unfaithful, open ascending auctions should be favored. Such a rule performs well with respect to both the revenue maximization and efficiency objectives, and is the least vulnerable to manipulation by an unfaithful board.

In our model we can correctly assume that the board of directors of the target firm is interested in maximizing shareholders' value: actually, here the target has not a share in the profits of the firm resulting from the merger so that the board’s only goal is that of maximizing the revenue, that is the selling price. However, in a matching auction the issue about the ability of the board to make a successful commitment arises.

The matching auction requires the board to be able to commit to sell the firm to the second bidder should he match the first bidder’s offer, even though the latter subsequently makes a higher bid. It is worth noting that this commitment problem is not different than the one the board faces if it conducts a sealed-bid auction, or indeed, even when it negotiates with a single bidder (since prospective bidders may incur significant investigation and bid-preparation costs). There are several ways around this problem, however. The most common practice is to enter into a lockup
arrangement with the declared winner, together with an agreement to pay a break-up fee should the sale be terminated.\(^{30}\) Another possibility is for the target board to refuse to rescind poison pills for any but the declared winning bidder. While it is unclear whether the courts will allow such poison pills to stand, the legal costs of challenging the poison pills and the possibility that the board might switch to an ascending auction (so that the challenger is no means assured of winning the contest) may deter further challenge from a losing bidder.

One way to understand the matching auction is to think of it as a discriminatory selling scheme in which the seller discriminates against the stronger bidder (i.e., the bidder with either the highest valuation or the larger toehold or the most informative signal). This discrimination takes a particularly simple form: the seller simply imposes an order of moves on the bidding firms and asks the stronger bidder to make a first offer. Since courts are more concerned about shareholder value than whether the playing field is level or not, it is unlikely that the matching auction will run into troubles on the grounds that it does not treat bidders symmetrically.

Selling procedures equivalent or similar to the matching auction are often observed in practice. For example, considerations of bidding costs may imply that an initial bidder will bid only once in an open auction. Alternatively, the seller may negotiate with a buyer by gradually raising the asking price. Once a price is reached such that the prospective buyer is not willing to go any higher, a second bidder may be offered the chance to match this price. Most importantly, many ascending auctions may eventually end in a matching auction. After several rounds of open bidding, the seller may announce one last round of open bids; this last round is then similar to a matching auction.

Looking at the policy implications, our model points out the existence of two main reasons which could prevent efficient mergers from happening: a preemptive behavior of the incumbents aimed to avoid the bad consequences of a tougher competition and a too high participation cost. Since the former is a rational and self-interested practice, regulation authorities should concentrate their effort on the latter. Indeed, most of the times these costs are the resultants of bureaucratic inefficiencies or informative imperfections and symmetries that come up on the market and that an external agency is asked to remove.

There are several directions left for future research. First of all, we could extend our model to the presence of more than just one possible entrant and this would give rise to a simultaneous bidding stage at the first step. Several possible extensions can be carried out by modifying the type of competition in the downstream market after the merger has eventually taken place (i.e., price competition or perfect competition).

Another issue is represented by the demand and cost features. Following the traditional literature on the topic, we have assumed a linear demand function, a constant marginal cost and no fixed costs; however, the results we got in our basic model do not undergo any change if we modify the demand’s features and/or the costs’ structure; at least as long as we keep the symmetry among the incumbents. Actually, another possible line of research could consist in relaxing this symmetry among incumbents either by allowing different cost functions or by modifying the field of competition (for instance, with a model à la Stackelberg).

\(^{30}\)For example, Viacom’s initial offer for Paramount in 1993 was associated with (a) an option to buy 20\% of Paramount’s outstanding shares and (b) a termination fee of $150 million plus expenses, should the transaction not be concluded.
Other interesting scenarios could arise if the entrant firm could simultaneous submit more bids for different targets. Finally, in our model the incumbents are able to merge only in order to avoid the entry of an outsider, but we could think of a more general model where incumbents can make the first move too.
References


