The Almost Ideal and Translog Demand Systems

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Abstract

This chapter reviews the specification and application of the Deaton and Muellbauer (1980) Almost Ideal Demand System (AIDS) and the Christensen, Jorgenson, and Lau (1975) tranlog (TL) demand system. In so doing we examine various refinements to these models, including ways of incorporating demographic effects, methods by which curvature conditions can be imposed, and issues associated with incorporating structural change and seasonal effects. We also review methods for adjusting for autocorrelation in the model's residuals. A set of empirical examples for the AIDS and a the log TL version of the translog based on historical meat price and consumption data for the United States are also presented.

Keywords: Almost ideal demand system, Autocorrelation, Curvature, Meat Demand, Translog

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1 Introduction

The now classic paper by Deaton and Muellbauer (1980) established a standard for applied demand analysis in the "Almost Ideal Demand System" or AIDS model. The fundamental demand model established by this paper has realized very widespread application in consumption analysis. The Social Science Citation Index shows that this paper has been cited 822 times (as of January 23, 2009). A closely related alternative—the "Transcendental Logarithmic" or "translog" demand system has also realized widespread application in demand analysis. The classic paper by Christensen, Jorgenson, and Lau (1975) that introduced the translog consumer demand system has been cited 361 times according to the Social Science Citation Index. In both cases, the tabulated citations clearly understate the impact of these two pioneering demand models. As is often the case, such classics become standard, accepted practice and thus many practitioners fail to cite the papers that originated the methods and instead depend upon widespread recognition of the models and methods.

As we discuss in this chapter, both demand system models are often motivated within the context of "flexible functional forms" that provide certain advantages in terms of minimizing specification biases in representing demand systems of unknown forms. In addition to its flexibility properties as a first-order approximation to any demand system, the AIDS model also possesses certain nonlinear aggregation properties that make it "almost ideal" for applied work.

In this chapter, we provide a broad overview of the AIDS and translog demand systems. We discuss practical issues relating to their utilization in applied demand analysis. We also outline a number of closely related issues that typically arise in the application of these models. These issues include incorporation of seasonality and structural change, representation of autocorrelation within a singular system of equations of the sort inherent in the AIDS and translog models, and inequality restrictions that are implied by concavity of the underlying utility function. We illustrate these practical issues associated with applying each model through empirical examples using a set of quarterly U.S. meat demand data. The focus of this paper is on the practical and applied issues associated with the use of these demand models and thus we refer the reader to the original sources as well as to a very large collection of other applied papers for greater detail on many of the issues that we address.

2 Specification of the Almost Ideal Demand System

The basic AIDS model is developed from a particular cost (expenditure) function taken from the general class of "price-independent, generalized logarithmic" or PIGLOG cost functions. In the case of the AIDS the cost function is of the form

$$\ln C(p,U) = (1-U)\ln(a(p)) + U\ln(b(p)) \tag{1}$$

where p is a $nx1$ vector of unit prices, U denotes the utility index, $a(p)$ is a translog price index given by

$$\ln a(p) = \alpha_0 + \sum_k \alpha_k \ln(p_k) + \tfrac{1}{2}\sum_k \sum_j \gamma^*_{kj} \ln p_k \ln p_j, \tag{2}$$

and

$$\ln b(p) = \ln a(p) + \beta_0 \prod_k p_k^{\beta_k}. \tag{3}$$

As well, $k, j = 1, \ldots, n$. Note that the utility index can be scaled to correspond to cases of subsistence ($U = 0$) and bliss ($U = 1$), in which case, $a(P)$ and $b(P)$ can be interpreted as representing the cost of subsistence and bliss, respectively.

Application of Shephard's Lemma through differentiation of the logarithmic cost function with respect to a logarithmic price yields budget (expenditure) share equations for each good in the utility function. We can "uncompensate" the share equations to remove utility by noting that total expenditure, (y), for a utility-maximizing consumer will equal the value of the cost function. We may therefore invert the cost function and solve for U, the indirect utility function $v(p,y)$. Finally, $v(p,y)$ may be used to substitute for U in each share equation, thereby obtaining the share–equation forms of corresponding Marshallian demands.

Doing so yields share equations of the form:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i(\ln(y) - \ln(P)) \tag{4}$$

where $w_i = \frac{p_i q_i}{y}$, $i = 1, \ldots, n$, and P is a price index defined by

$$\ln P = \alpha_0 + \sum_k \alpha_k \ln(p_k) + \tfrac{1}{2} \sum_k \sum_j \gamma_{kj} \ln p_k \ln p_j, \tag{5}$$

and where $\gamma_{ij} = \frac{1}{2}(\gamma^*_{ij} + \gamma^*_{ji})$.

Linear homogeneity of the cost function, symmetry of the second–order derivatives, and adding–up across the share equations implies the following set of (equality) restrictions:

$$\sum_{i=1}^{n} \alpha_i = 1, \quad \sum_{i=1}^{n} \gamma_{ij} = \sum_{j=1}^{n} \gamma_{ij} = 0, \quad \sum_{i=1}^{n} \beta_i = 0, \quad \gamma_{ij} = \gamma_{ji}. \tag{6}$$

As required for a locally flexible functional form, there are $n(n-1)/2$ free parameters in the Slutsky matrix for the AIDS model.

Using the familiar result that uncompensated (Marshallian) price elasticities in any demand system are given by $-\delta_{ij} + \frac{\partial \ln w_i}{\partial \ln p_j}$, where δ_{ij} is the Kronecker delta term, the price elasticities in the AIDS model are given by

$$\eta_{ij} = -\delta_{ij} + \frac{\gamma_{ij} - \beta_i(\alpha_j + \sum_k \gamma_{jk} \ln p_k)}{w_i}. \tag{7}$$

Note here also that in practice the term given by $(\alpha_j + \sum_k \gamma_{jk} \ln p_k)$ may sometimes be replaced by the equivalent $(w_j - \beta_j \ln(y/P))$ in elasticity expressions. Expenditure (income) elasticities are given by:

$$\eta_{iy} = \frac{\beta_i}{w_i} + 1. \tag{8}$$

As always, Hicksian or compensated price elasticities may be obtained by using the familiar

Slutsky equation. In elasticity form this equation yields:

$$\eta_{ij}^{c} = \eta_{ij} + w_j \eta_{iy}, \tag{9}$$

where η_{ij}^{c} denotes the compensated price elasticity for i^{th} good with respect to the j^{th} price.

Deaton and Muellbauer (1980) proposed replacing the nonlinear AIDS price index in 5 with an appropriately specified price index that can be defined outside of the AIDS system, thus leaving a purely linear system of share equations. Specifically, they suggest Stone's share-weighted geometric mean price index as an obvious candidate:

$$\ln P^{*} = \sum_{i=1}^{N} w_i \ln(p_i). \tag{10}$$

This version became known as the "Linear–Approximate" AIDS model, or LA–AIDS. This suggestion by Deaton and Muellbauer (1980) led to much consternation and debate over, among other things, the appropriate specification of the LA–AIDS elasticities and the overall properties of the LA–AIDS model. For example, Moschini (1995) shows that the Stone index in (10) is not invariant to units of measurement, although normalizing all prices by their respective sample means does circumvent this problem. As well, Eales and Unnevehr (1988) note that when Stone's index is used that budget shares also appear on the right–hand side of the equations. Likewise, Buse (1998) comments on the "errors in variables" problems introduced by using the Stone index in lieu of the true translog index. Finally, Lafrance (2004) examines the integrability properties of the LA–AIDS, finding that very restrictive forms are implied for the underlying expenditure function when symmetry conditions are imposed. In any event, all issues pertaining to the specification, estimation, and interpretation of the LA–AIDS are rendered moot if instead the translog price index $a(p)$ in (5) is simply used in estimation. Even so, it is often useful to estimate the LA–AIDS for purposes of obtaining starting values in estimation of the AIDS model with the nonlinear price index (Browning and Meghir 1991).

3 Specification of the Translog Demand System

A closely related demand model is found in the "Transcendental Logarithmic" or "translog" (TL) demand system of Christensen, Jorgenson, and Lau (1975). The translog consumer demand system is usually derived by applying Roy's Identity to a quadratic, logarithmic specification of an indirect utility function written in terms of expenditure-normalized prices. Normalizing each price by dividing by total expenditures imposes homogeneity. The quadratic, logarithmic indirect utility function is given by:

$$\ln \Psi(p, y) = \alpha_0 + \sum_k \alpha_k \ln(p_k/y) + \tfrac{1}{2} \sum_k \sum_j \gamma_{kj} \ln(p_k/y) \ln(p_j/y), \tag{11}$$

$k, j = 1, \ldots, n$. It is relevant to note the similarity of the translog specification to the price index inherent in the AIDS model as specified in (2). Applying the logarithmic version of Roy's identity to the indirect utility function in (11) yields share equations of the form:

$$w_i = \frac{\alpha_i + \sum_k \gamma_{ik} \ln(p_k/y)}{\sum_m (\alpha_m + \sum_k \gamma_{mk} \ln(p_k/y)}, \tag{12}$$

where again $i = 1, \ldots, n$. Note that the denominator of the share equation in (12) is the sum of the numerators across all shares. This is often written in an equivalent form as:

$$w_i = \frac{\alpha_i + \sum_k \gamma_{ik} \ln(p_k/y)}{\alpha_M + \sum_k \gamma_{Mk} \ln(p_k/y)}, \tag{13}$$

where $\alpha_M = \sum_{i=1}^{M} \alpha_i$ and $\gamma_{Mk} = \sum_{i=1}^{M} \gamma_{ik}$ and where, of course, $M = n$. Note also that the parameters are given in ratio form and thus are only identified to scale. A common normalization to allow identification is to set $\alpha_M = \sum_i \alpha_i = -1$.

Homogeneity is necessarily guaranteed in the standard translog model in light of the use of expenditure-normalized prices. Symmetry requires $\gamma_{ij} = \gamma_{ji}$. In the case of the TL there are $n(n+1)/2$ free parameters in the Slutsky matrix, thereby implying that the translog has more parameters than are necessary to qualify as a second–order locally flexible functional form.

Uncompensated price elasticities in the translog are given by:

$$\eta_{ij} = -\delta_{ij} + \frac{\gamma_{ij}/w_i - \sum_j \gamma_{ij}}{-1 + \sum_k \gamma_{Mk} \ln(p_k/y)}. \tag{14}$$

Expenditure elasticities are given by:

$$\eta_{iy} = 1 + \frac{-\sum_j \gamma_{ij}/w_i + \sum_i \sum_j \gamma_{ij}}{-1 + \sum_k \gamma_{Mk} \ln(p_k/y)}. \tag{15}$$

As in the case of the AIDS, compensated (Hicksian) price elasticities may be determined by using (9).

There is a variant of the translog model that is often used in practice, the log tranlog (log TL) or aggregatable tranlog. See, Pollak and Wales (1992). The basic log TL may be derived from the TL by simply imposing the additional restriction that $\sum_k \gamma_{Mk} = 0$. Of course doing so reduces the number of free parameters in the model by one so that now there are $n(n+1)/2 - 1$ free parameters in the Slutsky matrix. In any event even the log LT has more free parameters then are required for the system to satisfy the properties of a second–order (locally) flexible functional form.

Finally, any discussion of the AIDS and translog models would be remiss without explicit mention of the fact that the two models are of a very similar analytical structure. This similarity has led to efforts to directly compare the two closely–related specifications. A notable example exists in the work of Lewbel (1989), who developed a demand model that nests both the AIDS and the translog demand systems. Lewbel (1989) develops a more general model that nests both the AIDS and the translog systems as special cases defined by parametric restrictions which can, in turn, be used to pursue nested specification testing of each alternative. Lewbel (1989) applied this model to aggregated U.S. expenditure data and found that the explanatory power and statistical fit of the alternative models were very similar in every instance. This finding is especially interesting in light of the nearly identical conclusions reached in our own empirical example presented below.

4 Issues in Applying the AIDS and Translog Models

A variety of issues and concerns may apply to any specific application of the AIDS and translog models. These issues may arise as a result of characteristics of the data. For example, issues relating to seasonality and structural change may arise in applications using data collected over time. Likewise, data aggregated across households may raise questions regarding the aggregation properties of a particular demand model. Methods for imposing the curvature constraints inherent in the aforementioned conditions required for concavity also raise important modeling questions. The treatment of autocorrelation in a singular system of share equations may be important in applications to time-series data. Both the AIDS and translog demand systems are nonlinear in the parameters, though practice has shown the AIDS model to be relatively straightforward to estimate while the degree of nonlinearity inherent in the translog model may result in additional estimation challenges.

4.1 Aggregation Properties

Empirical applications of demand system models typically proceed according to one of two approaches. The first uses data collected from individuals or, more commonly, from households. Such paths to analysis typically assume a common underlying structure for tastes and preferences and commonly apply various demographic shifters or adjustments to reflect differences across individuals. A second approach to empirical analysis involves the use of more aggregate data collected over groups of individuals or households and most often taken across multiple time periods. In this case, one must consider aggregation properties that reflect the extent to which the demand estimates accurately represent the underlying preference structure reflecting the optimizing choices of individuals making up the aggregate. We most commonly consider a structure that represents the aggregate or a representative consumer defined at average values of prices and income. In their text, Deaton and Muellbauer (1980) note that "...in general, it is neither necessary, nor necessarily desirable, that macroeconomic relations should replicate their microfoundations so that exact aggregation is possible."

It is common to assume that perfectly integrated goods markets result in a common price for a good of a certain defined (homogeneous) quality. However, when aggregating across households, differences in income will clearly exist and thus conditions for exact aggregation should be considered. One of the AIDS model's "almost ideal" properties relates to its aggregation properties. In particular, the AIDS model satisfies exact nonlinear aggregation because the cost function upon which it is based is of a specific functional form for underlying preferences known as a "price independent generalized logarithmic" (or PIGLOG) form. This allows one to work with a representative measure of household expenditure, defined as y_0.

Deaton and Muellbauer (1980) show that, in the AIDS model case, y_0 is given by $y_0 = \kappa_0 \bar{y}$, where $\kappa_0 = H/Z$ where H is the number of households or individuals making up the aggregate and Z is Theil's entropy measure of dispersion of income across individual units. Note further here that, if the number of households and the distribution of income is constant across individual aggregate measures (i.e., aggregated time-series observations), the AIDS model satisfies exact nonlinear aggregation without further adjustment. If either the number of households or the distribution of income varies across the sample of aggregated data, a straightforward adjustment to the average level of household income can be applied to maintain desirable aggregation properties. Some applications of the AIDS model to aggregate data apply Theil's entropy index correction to average income levels to ensure valid aggregation properties. One example can be found in Eales and Unnevehr (1988), who adjusted average income levels to account for changes in the distribution of income over time.

4.2 Flexibility and Model Extensions

The AIDS model is derived from an expenditure function that can be interpreted as a second-order approximation to an arbitrary unknown function. As Deaton and Muellbauer (1980) note, the AIDS demand system can thus be interpreted as a first-order approximation to any demand system. Having noted these desirable flexibility properties, other authors have considered amendments to the basic AIDS structure in an effort to improve or enhance its flexibility properties.

Banks, Blundell, and Lewbel (1997) introduced a quadratic version of the standard AIDS model that adds a quadratic logarithmic income term and nests the standard AIDS model specification. The resulting QUAIDS model is given by share equations of the form

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln(y/P) + \frac{\lambda_i}{\prod_i p_i^{\beta_i}} (\ln(y/P))^2 \tag{16}$$

where P is the AIDS price index.

Other approaches to adding flexibility to the standard AIDS model have been proposed in other research. Chalfant (1987) augmented the AIDS expenditure function by replacing the translog cost function terms with Fourier series expansion terms. This specification also nests a standard AIDS model and thus permits straightforward specification testing. As Gallant (1981) has shown, the Fourier flexible form may offer more global flexibility than is the case for the translog demand system.

Another extension to the standard AIDS model was proposed by Eales and Unnevehr (1994), who considered applications where the quantities tended to be fixed in the short-run and thus where prices adjusted to clear the market. Eales and Unnevehr (1994) discussed specific applications where exogenous quantities may be more reasonable than exogenous prices. Specific examples include short-run demand for perishable commodities, commodities which are subject to long production lags and thus are essentially in fixed supply in the short-run, and goods for which government policies set consumption quotas that effectively exogenously fix quantities.

The inverse AIDS model (IAIDS) is entirely analogous to the standard AIDS model. The model is defined using a logarithmic distance function that is specified in a manner that is entirely analogous to the expenditure function. Differentiation of the logarithmic distance function yields share equations that are expressed as a function of quantities and utility. Substitution of the inverted distance function uncompensates the functions and yields a system of inverse demand share equations of the form:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln q_j + \beta_i \ln(Q) \tag{17}$$

where Q is a quantity index defined by

$$\ln Q = \alpha_0 + \sum_k \alpha_k \ln(q_k) + \tfrac{1}{2} \sum_k \sum_j \gamma_{kj} \ln q_k \ln q_j. \tag{18}$$

As illustrated by Christensen and Manser (1977), an inverse counterpart to the translog model, the ITL, is also available. Specifically, the direct utility function as being of a translog form. Specifically, the utility function may be specified as:

$$-\ln U = \alpha_0 + \sum_k \alpha_k \ln(q_k) + \tfrac{1}{2} \sum_k \sum_j \gamma_{kj} \ln q_k \ln q_j, \tag{19}$$

where $\gamma_{ij} = \gamma_{ji}$. By maximizing (19) subject to the budget constraint $p'q = y$, and then applying the so–called Hotelling–Wold identity (Hotelling 1935), a system of inverse demand equations of the general form

$$w_i = \frac{\alpha_i + \sum_k \gamma_{ik} \ln(q_k)}{\alpha_M + \sum_k \gamma_{Mk} \ln(q_k)}, \tag{20}$$

obtains. As with the IAIDS, the ITL in (20) is in every respect analogous to the direct TL system in (13).

4.3 Seasonality, Demographics, and Structural Change

In applications of the AIDS and translog demand systems to time-series data, one may need to be concerned with the potential for exogenous shifts in the underlying structure of the economic relationships represented by the model. The tastes and preferences underlying observed demand relationships may be subject to temporary or permanent structural shifts. This may reflect a change in preferences or the arrival of new information that is embedded in prices. Alternatively, if one is working with quarterly data, seasonal patterns may characterize consumption. It is important to recognize that any such adjustment necessarily implies analogous changes in the underlying utility maximization behavior of agents.

It is common practice to include exogenous intercept shifters in the basic AIDS and translog share equations to capture the effects of structural change, seasonality, or other exogenous shifters. In doing so, one must be careful to pay attention to the adding up conditions represented above in order to ensure that the addition of such shifters does not violate adding up across equations. A common approach to representing a linear shift in expenditure shares that does not reflect changes in prices or income is to add a simple linear trend to the share equations, which is analogous to allowing the intercept term of each share equation to trend such that $\alpha_{it} = \alpha_{i0} + \alpha_{it}t$ where $t = 1, \ldots, T$. In such a case, one would recover the trend of the last omitted equation from the adding up condition that requires $\sum_k \alpha_{kt} = 0$. This same general intuition carries over to more complex shifters that allow the intercept to shift according to the quarter of the year of the observation or even other shifters such as demographic terms. We should note that an entire literature has developed to address the incorporation of demographic translation and scaling terms in empirical demand models. For our purposes here, it suffices to note that attention to adding up must accompany any amendments to the underlying AIDS or translog share equations intended to allow the structure to vary outside of price and income changes. In the case of adjustments for seasonality, one may add a series of indicator variables intended to capture the fixed, seasonal effects. For example, for quarterly data, the share equations may be amended to incorporate seasonality by expressing the intercept terms as $\alpha_{is} = \alpha_{i0} + \sum_{s=1}^{3} \delta_s S$ where S is one if the quarter is dated $s = S$ and is zero otherwise.

4.4 Imposing Curvature

One issue that arises frequently in estimation of demand systems is the imposition of curvature (negativity) conditions (Barten and Geyskens 1975). Basic microtheoretic results indicate that in order for integrability of the system to hold that the $n \times n$ Slutsky matrix will be symmetric and that it will be, at most, of rank $n-1$. As well, quasi–concavity of the utility function implies that the Slutsky matrix will be negative semi–definite. One immediate implication is, of course, that Hicksian (compensated) demands will be non–increasing in own price. In any event, whether or not curvature conditions are satisfied at all or even for any points in the sample data is a relevant issue, and one that should be examined in any empirical investigation.

As illustrated by Moschini (1998) and Ryan and Wales (1998), the Slutsky matrix for the AIDS and the TL models assume rather simple forms when evaluated at the points $p^* = \iota$ and $y^* = 1$, where ι is a $n \times 1$ unit vector. Specifically, the ij^{th} element of the Slutsky matrix for the AIDS model is given by

$$S_{ij} = \gamma_{ij} - (\alpha_i - \beta_i\alpha_0)\delta_{ij} - (\alpha_j - \beta_j\alpha_0)(\alpha_i - \beta_i\alpha_0) + \beta_i\beta_j\alpha_0, \qquad (21)$$

where δ_{ij} is the Kronecker delta term. Moreover, if α_0 is restricted to zero–a normalization that is often used in empirical applications of the AIDS model–the ij^{th} element of the Slutsky matrix in (21) reduces to

$$S_{ij} = \gamma_{ij} - \alpha_i\delta_{ij} - \alpha_j\alpha_i. \qquad (22)$$

In similar fashion the ij^{th} element for the TL Slutsky equation, again evaluated at the point

$p^* = \iota$ and $y^* = 1$, is given by

$$S_{ij} = -\gamma_{ij} + \alpha_i \alpha_j + \alpha_i \delta_{ij} - \alpha_i \sum_k \gamma_{kj} - \alpha_j \sum_k \gamma_{ik} - \alpha_i \alpha_j \sum_k \sum_j \gamma_{kj}. \tag{23}$$

Of course for the log TL model the ij^{th} element of the Slutsky equation simply reduces to

$$S_{ij} = -\gamma_{ij} + \alpha_i \alpha_j + \alpha_i \delta_{ij} - \alpha_i \sum_k \gamma_{kj} - \alpha_j \sum_k \gamma_{ik}. \tag{24}$$

The expressions in (21)–(24) can be used to construct the relevant $n \times n$ Slutsky matrix. From here it is then possible to calculate the eigenvalues of the Slutsky matrix and to determine whether or not the lead eigenvalue is zero at the point of approximation.

Aside from checking to determine whether or not the curvature conditions are satisfied, it may be desirable to impose the non–negativity conditions during estimation. There are various ways of accomplishing this task, including both classical and Bayesian approaches. For example, Gallant and Golub (1984) propose imposition of inequality constraints directly on eigenvalues in the context of a maximum likelihood estimation routine. Alternatively, Chalfant, Gray, and White (1991) build off of a Bayesian approach introduced originally by Geweke (1993) that uses importance sampling in conjunction with Monte Carlo simulation to impose curvature in an AIDS demand system. An essentially identical approach was adopted by Terrell (1996) for purposes of imposing curvature conditions in a TL cost function.

More recently, several authors have investigated ways of reparameterizing the Slutsky matrix so that curvature may be directly imposed at a point during estimation. All of these procedures essentially build on results presented initially by Diewert and Wales (1988a, 1988b) in the context of imposing curvature globally in their normalized quadratic (NQ) demand system. For example, Moschini (1998) and Ryan and Wales (1998) show how the Diewert and Wales approach may be adopted to impose curvature at a point in the AIDS model.

To illustrate, assume that the Slutsky matrix, S, may be rewritten as $S = -\tilde{A}\tilde{A}'$, where

$\tilde{A}$ is a $n \times n$ lower triangular matrix. Of course, due to the fact that Hicksian demands are homogeneous of degree zero in prices, all relevant price vectors lie in the null space of Slutsky matrix, S. If, for example, prices are normalized to have a unit mean, then it would necessarily be true that $S\iota = 0$ as well. The implication is that the Slutsky matrix can be written in terms of its Cholesky decomposition as

$$-S = AA' = \begin{pmatrix} \tilde{A}\tilde{A}' & \tilde{a} \\ \tilde{a}' & -\iota'\tilde{a}' \end{pmatrix} \tag{25}$$

where

$$\tilde{A} = \Big[a_{ij}\Big], \qquad a_{ij} = 0 \quad \forall j > i, \qquad i, j = 1, \ldots, n-1,$$

and where the k^{th} element of the $(n-1) \times 1$ vector $\tilde{a}$ is given by $\tilde{a}_k = \sum_j^{n-1} a_{kj}$. Now, let $(-\tilde{A}\tilde{A}')_{ij}$ denote the ij^{th} element of $-\tilde{A}\tilde{A}'$. Then (22), after solving for γ_{ij}, may be expressed alternatively as

$$\gamma_{ij} = (-\tilde{A}\tilde{A}')_{ij} + \alpha_i \delta_{ij} - \alpha_j \alpha_i. \tag{26}$$

Specifically, it is the coefficients in the $(n-1) \times (n-1)$ lower triangular matrix $\tilde{A}$ that are estimated in lieu of the γ_{ij} terms when curvature is imposed at the $p^* = \iota$ and $y^* = 1$ point.

As discussed originally by Diewert and Wales (1988b), if it is necessary to impose curvature by using the Cholesky decomposition, then the leading non–zero eigenvalue in $-AA'$, while negative, will often be near zero. Diewert and Wales (1988b) show that the rank of $-\tilde{A}\tilde{A}'$ can be restricted to some $K < (n-1)$ by setting $a_{ij} = 0$ for all $i > K$. In turn restricting $-\tilde{A}\tilde{A}'$ in this manner will, via (25), restrict the rank of S to $K < n-1$, resulting in what Diewert and Wales refer to as a semiflexible form when combined with their normalized quadratic demand system. An interesting feature of this approach is that once curvature is imposed the rank of the Slutsky matrix can be successively reduced until noticeable harm is done to model fit, perhaps as measured by the Akaike information criterion (AIC) or some other model selection criterion. In his 1998 study Moschini used these results in conjunction with (25) to develop the semiflexible almost ideal demand system.

Finally, Moschini (1999) show shtat similar procedures may be applied to reparmaterized versions of the TL and log TL models, again for purposes of imposing curvature at a point. Furthermore, Ryan and Wales (1998) illustrate that in many instances curvature conditions may be forced to hold at all sample points if the sample point for which the violation is most stringent is used as the point for data normalization, that is, the point for which prices and expenditure are normalized to have unit values.

4.5 Stochastic Specification and Autocorrelation

An area of key interest in demand system estimation, at least when time series data are employed, is the inclusion of autocorrelation terms in stochastic share equation specifications. Because share equations must necessarily satisfy adding–up properties at all data points, it is not possible to specify autocorrelation terms without imposing additional restrictions (Berndt and Savin 1975). Here we briefly review the unique requirements of autocorrelation specifications in systems of singular share equations along with several popular autocorrelation models used in practice.

To begin, we follow Barten (1969) who illustrates that, for system's of equations subject to adding–up conditions, that: (1) in estimation an equation must be deleted since the resultant $n \times n$ covariance matrix is not of full rank, and therefore an equation must be omitted in estimation; and (2) that iterative Seemingly Unrelated Regression (SUR) yields maximum likelihood estimates of the system's parameters that are, moreover, invariant with respect to the equation that is deleted. Specifically, let a superscripted n applied to a vector (matrix) denote an operator that deletes the last row (row and column) of a vector (matrix). In this case we might then represent the $n-1$ equation system to be estimated as:

$$w_t^n = f^n(z_t, \theta) + e_t^n, \quad t = 1, \ldots, T, \tag{27}$$

where w_t^n is an $(n-1)$–vector of shares at time t, z_t is a vector of explanatory variables, including prices and income, θ is a vector of unknown parameters to be estimated, and e_t^n is an $(n-1)$–vector of mean zero random error terms.

Now assume that the errors in (27), that is, the elements in e_t^n, are autocorrelated, as is often the case when time series data are employed. Moreover, assume that the autocorrelation follows a first–order vector autoregressive process. That is,

$$e_t^n = \overline{R}^n e_{t-1} + \varepsilon_t^n, \quad t = 2, \ldots, T, \tag{28}$$

where $\overline{R}$ is an $(n-1) \times (n-1)$ autocorrelation matrix and ε_t^n is a $(n-1) \times 1$ error vector such that $E(\varepsilon_t^n) = 0$, $E(\varepsilon_t^n(\varepsilon_t^n)') = \Omega$, and $E(\varepsilon_t^n(\varepsilon_s^n)') = 0 \ \forall t \neq s$. By substituting the vector autoregressive error process in (28) into (27) the following quasi-differenced equation system obtains:

$$w_t^n = f^n(z_t, \theta) + \overline{R}^n[w_{t-1}^n - f^n(z_{t-1}, \theta)] + \varepsilon_t^n, \quad t = 2, \ldots, T. \tag{29}$$

As outlined by Moschini and Moro (1994) and Holt (1998) there are at least five ways to parameterize the autocorrelation matrix $\overline{R}^n$ in (28), including the unrestricted specification outlined by Anderson and Blundell (1982) and the single–parameter specification suggested by Berndt and Savin (1975). In the empirical application that follows we utilize the positive semi–definite parametrization for $\overline{R}^n$ advanced by Holt (1998), which is often found to provide reasonable flexibility (relative to the single–parameter case) and yet maintains reasonable parsimony (relative to the unrestricted case). In short, the positive semi–definite set–up applies the same same procedures to the autocorrelation matrix that Diewert and Wales (1988a, 1988b) use to impose negativity on the Slutsky matrix of a demand system.

The positive semi–definite specification is defined as follows. Assume the $n \times n$ counterpart to $\overline{R}^n$ is specified by, say, R. Moreover, it is assumed that R is symmetric and positive semi–definite, subject to the restriction $R\iota = 0$, where as before ι is a $n \times 1$ unit vector. As

Holt (1998) describes how, with the foregoing restriction, the R matrix may be specified as:

$$R = \begin{pmatrix} \tilde{R} & \tilde{r} \\ \tilde{r}' & -\iota'\tilde{r}' \end{pmatrix} \tag{30}$$

where

$$\tilde{R} = TT', \quad T = \left[\tau_{ij}\right], \quad \tau_{ij} = 0 \quad \forall j > i.$$

A typical element in the $(n-1) \times 1$ vector $\tilde{r}$ is $r_k = \sum_{j=1}^{n-1} \tilde{R}_{jk}$. Moreover, as specified in (30) the $(n-1) \times (n-1)$ matrix T is a lower-triangular matrix; it is the elements in T that are estimated directly by using Holt's (1998) method.

Alternative methods for dealing with demand systems in the context of time series data have been explored by Anderson and Blundell (1982), Ng (1995), Attfield (1997), Karagiannisa and Mergos (2002), and Lewbell and Ng (2005). The basic idea is that prices, expenditure, and perhaps budget shares follow something akin to a unit root process, implying that the data should be first differenced as a prelude to estimation. Moreover, if the underlying demand system is linear in variables—as it is for the AIDS model when a Stone (or other) price index is used to replace the nonlinear price index $a(p)$ in (2)—then it is also possible to examine cointegration properties in the context of a demand system. Alternatively, Lewbell and Ng (2005) propose a variant of the translog model that can also be used to estimate demand systems in the context of nonstationary data. Although holding promise, one limitation of the time series approach to demand system estimation is that shares are, by definition, bounded on the unit interval, a result that is in turn inconsistent with unit root behavior and therefore with first differencing. See, for example, the discussion in Davidson and Teräsvirta (2002). Future work may therefore focus on the possibility that demand equations estimated with time series data are fractionally cointegrated.

5 An Empirical Example

In this section we provide a stylized application of the AIDS and translog demand systems in the context of time series data. Specifically, we illustrate the application of these models to aggregate meat demand data in the United States. Indeed, there is a long tradition of using various demand systems, including both the AIDS and the translog models, to investigate the properties of U.S. meat consumption. See, for example, Christensen and Manser (1977), Moschini and Mielke (1989), and Piggott and Marsh (2004), among others. It therefore is reasonable to illustrate the properties of these demand systems in the context of aggregate meat demand data in the United States.

5.1 Data

Quarterly data on consumption and retail prices for beef, pork, chicken, and turkey were collected from various USDA sources for the 1960–2004 period. Data prior 1997 were obtained from various sources described in some detail by Holt (2002). Data for pork and beef from 1997 through 2004 were obtained from the online version of the U.S. Department of Agriculture (2006b) *Red Meat Yearbook*. Likewise, data for chicken and turkey were obtained from the online version of the U.S. Department of Agriculture (2006a) *Poultry Yearbook*. Similar to Piggott and Marsh (2004), we aggregate the chicken and turkey categories to obtain a single "poultry" category. The retail price for poultry is derived by determining the share–weighted averages for chicken and turkey prices where the shares are with respect to total expenditures on chicken and turkey. Basic descriptive statistics for the meat data used in all subsequent econometric analyses are reported in Table 1.

5.2 AIDS and Translog Estimates of US Meat Demand

The data and procedures described above are used to obtain meat demand estimates for the AIDS and translog demand systems. Regarding the translog, we utilize the log translog

(log TL) version described in Section 3. We do this in part because in the present case the log TL contains exactly the same number of free parameters as does the AIDS model and therefore straightforward comparisons between the two specifications are facilitated. Prior to estimation, all quantity variables and total expenditure are normalized to have a mean of one. In both instances Holt's (1998) first–order vector autoregressive autocorrelation procedure, as described in Section 4.5, is used to handle issues with remaining serial correlation in the models' residuals. As well, because quarterly data are employed, and because there is substantial seasonal patterns for some of these variables, most notably for quantities, a set of seasonal dummy variables is appended to each model.

The final specification for the AIDS demand system is therefore:

$$w_{it} = \alpha_i + \alpha_{i1}D1_t^* + \alpha_{i2}D2_t^* + \alpha_{i3}D3_t^* + \sum_j \gamma_{ij} \ln p_{jt} + \beta_i(\ln(y_t) - \ln(P_t)) + e_{it}, \qquad (31)$$

where i =1 (Beef), 2 (Pork), and 3 (Poultry), and also where lnP_t is the price index give by

$$\ln P_t = \sum_k (\alpha_k + \alpha_{k1}D1_t^* + \alpha_{k2}D2_t^* + \alpha_{k3}D3_t^*) \ln(p_{kt}) + \tfrac{1}{2}\sum_k \sum_j \gamma_{kj} \ln p_{kt} \ln p_{jt}. \qquad (32)$$

In addition to the restrictions defined in 6, the additional restrictions $\sum_{j=1}^{3} \alpha_{jk} = 0,\ k = 1, 2, 3$ are required to ensure that adding–up holds. In (31) $D1_t^*$, $D2_t^*$, and $D3_t^*$ are quarterly dummy variables defined such that $DJ_t^* = DJ_t - D4_t,\ J = 1, 2, 3$, and where DJ_t is one for quarter J and zero otherwise. This dummy variable specification is used because it allows the intercept term, in this case, α_i to retain its original interpretation. See, for example, van Dijk, Strikhom, and Teräsvirta (2003). Also note that in (32) we have restricted the intercept term α_0 to zero, a common practice in estimation of the AIDS model.[1]

[1] Empirical experience with the AIDS model has suggested that the likelihood function tends to be quite flat with respect to the α_0 term, thus complicating estimation. One common practice is to evaluate the likelihood function for alternative fixed values of the intercept term and use the value giving the highest likelihood.

Likewise, the final specification for the log TL model, corresponding to (13), is

$$w_{it} = \frac{\alpha_i + \alpha_{i1} D1_t^* + \alpha_{i2} D2_t^* + \alpha_{i3} D3_t^* + \sum_k \gamma_{ik} \ln(p_{kt}/y_t)}{-1 + \sum_k \gamma_{Mk} \ln(p_{kt}/y_t)} + e_t, \tag{33}$$

where again i =1 (Beef), 2 (Pork), and 3 (Poultry) and where $\gamma_{Mk} = \sum_{i=1}^{M} \gamma_{ik}$, $M = 3$. As before the restrictions $\sum_{j=1}^{3} \alpha_{jk} = 0$, $k = 1, 2, 3$ are required to ensure that adding–up holds for the share equations in (33). Finally, and in keeping with the log TL specification, the additional restriction $\sum_k \gamma_{Mk} = 0$ is imposed in estimation.

The parameter estimates for both models are reported in Table 2. Along with parameter estimates and asymptotic standard errors, 90–percent bootstrapped confidence intervals for the estimated parameters are obtained by using the percentile–t method. Specifically, each parameter's t–statistic is used to obtain critical values for the usual t–statistic as an alternative to those provided by asymptotic theory. Empirically derived critical values are then used to construct 90–percent confidence intervals for each estimated parameter. In implementing the bootstrap 1,000 dynamic bootstrap draws are used to build the empirical distribution for each parameter.

Because it is difficult to directly interpret many of the estimated parameters in the AIDS and log TL models, it is generally more useful to obtain elasticity estimates. Results in Table 2 do indicate that (1) seasonality is a significant feature in meat consumption; and (2) serial correlation is a relevant feature of each model's estimated residuals. Regarding the latter, for both estimated models the dominant root for the estimated autocovariance matrix is real-valued and is slightly less than one.

System and individual equation measures of fit and performance are recorded in Table 3. As previously noted both models have the same number of free parameters (16). The estimated likelihood function value is slightly higher for the AIDS than for the log TL implying, of course, that the AIC is slightly lower for the AIDS. Even so, the system R^2 values are quite similar in both instances, near 0.995. Of interest is that the Slutsky matrix is found to be negative semi–definite at all data points for both models, a result that is somewhat surprising given the rather long time span used in estimation. Individually, each share equation for each estimated model appears to fit the data well, as indicated by individual equation R^2's, with the pork equation apparently fitting most poorly of the three (R^2 values of 0.85). Overall, both the AIDS and log TL models provide a good fit to the data and, moreover, both models provide a nearly identical fit to the data.

To obtain further insights into the implications of each estimated model for meat consumption, Marshallian, Hicksian, and expenditure elasticities are obtained by using the various formulae outlined in Sections 2 and 3. Elasticity estimates for the AIDS model, obtained at the means of the sample data, are reported in Table 4, while the comparable estimates for the log TL model are recorded in Table 5. As with parameter estimates, 90–percent bootstrapped confidence intervals are provided for each elasticity estimate.

The elasticity estimates recorded in Tables 4 and 5 are consistent with those reported elsewhere in the literature. See, for example, Piggott and Marsh (2004). Even so, several observations are in order. To begin, qualitatively the price and expenditure elasticity estimates are similar to each other for both models, which is perhaps not surprising given that both models have the same number of parameters and that both fit the data equally well. As well, all Marshallian own–price elasticity estimates are generally less than one in absolute terms, the sole exception being pork for the AIDS model. Moreover, both models imply that poultry demand is most inelastic—the point estimate of the Marshallian own–price elasticity estimate for the AIDS model is -0.370 while the comparable estimate for the log TL model is -0.474. In any event, neither model generates an own–price elasticity estimate for poultry that is significantly different from zero, as implied by the 90–percent confidence intervals. Indeed, the only price effect that is significantly different from zero for poultry demand is that for beef.

Regarding the expenditure elasticities, the AIDS model implies an expenditure elasticity for poultry demand that is significantly different from one (0.750); the corresponding point estimate for the log TL model is 1.084, which is not significantly different from one. Indeed, results in Table 5 reveal that none of the estimated expenditure elasticities is significantly different from one, implying that consumer preferences are consistent with a homothetic ordering. Indeed, this is the principle difference between the AIDS and the log TL models: the log TL model seems to imply expenditure elasticity estimates for meat demand that are consistent with homothetic preferences while the AIDS model does not.

6 Concluding Remarks

The AIDS and TL demand systems introduced by, respectively, Deaton and Muellbauer (1980) and Christensen, Jorgenson, and Lau (1975) have over the past two–three decades become primary workhorses in modern empirical demand analysis. Many refinements have been considered for both specifications, including ways of incorporating demographic effects, quadratic income terms, methods by which curvature conditions can be imposed, and issues associated with incorporating structural change and seasonal effects. We also review methods for adjusting for autocorrelation in the model's residuals. Finally, we present a set of empirical examples for the AIDS and a the log TL version of the translog based on historical meat price and consumption data for the United States. Because the properties of these models are now well understood and because they are relatively easy to implement, there is every reason to believe that the AIDS and TL demand systems, and the AIDS in particular, will remain as important tools in quantitative demand analysis for years to come.

References

Anderson, G. J., and R. W. Blundell (1982): "Estimation and Hypothesis Testing in Dynamic Singular Equation Systems," Econometrica, 50, 1559–1571.

Attfield, C. L. F. (1997): "Estimating A Cointegrating Demand System," Europen Economic Review, 41, 61–73.

Banks, J., R. Blundell, and A. Lewbel (1997): "Quadratic Engel Curves and Consumer Demand," The Review of Economics and Statistics, 79(4), 527–539.

Barten, A. P. (1969): "Maximum Likelihood Estimatiion of a Complete System of Demand Equations," Europen Economic Review, 1, 7–63.

Barten, A. P., and E. Geyskens (1975): "The Negaticity Condition in Consumer Demand," European Economic Review, 6, 227–260.

Berndt, E. R., and N. E. Savin (1975): "Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive Disturbances," Econometrica, 43, 937–957.

Browning, M., and C. Meghir (1991): "The Effects of Male and Female Labor Supply on Commodity Demands," Econometrica, 59(4), 925–951.

Buse, A. (1998): "Testing Homogeneity in the Linearized Almost Ideal Demand System," American Journal of Agricultural Economics, 80(1), 208–220.

Chalfant, J. A. (1987): "A Globally Flexible, Almost Ideal Demand System," Journal of Business and Economic Statistics, 5(2), 233–242.

Chalfant, J. A., R. S. Gray, and K. J. White (1991): "Evaluating Prior Beliefs in a Demand System: The Case of Meat Demand in Canada," American Journal of Agricultural Economics, 73(2), 476–490.

Christensen, L., and M. E. Manser (1977): "Estimating U.S. Consumer Preferences for Meat with a Flexible Utility Function," Journal of Econometrrics, 5, 37–53.

Christensen, L. R., D. W. Jorgenson, and L. J. Lau (1975): "Transcendental Logarithmic Utility Functions," The American Economic Review, 65(3), 367–383.

Davidson, J., and T. Teräsvirta (2002): "Long Memory and Nonlinear Time Series," Journal of Econometrics, 110, 105–112.

Deaton, A., and J. Muellbauer (1980): "An Almost Ideal Demand System," American Economic Review, 70(3), 312–326.

Deaton, A., and J. Muellbauer (1980): Economics and Consumer Behavior. Cambridge University Press.

Diewert, W. E., and T. J. Wales (1988a): "Normalized Quadratic Systems of Consumer Demand Functions," Journal of Business & Economic Statistics, 6(3), 303–312.

——— (1988b): "A Normalized Quadratic Semiflexible Functional Form," Journal of Econometrics, 37, 327–342.

Eales, J. S., and L. J. Unnevehr (1988): "Demand for Beef and Chicken Products: Separability and Structural Change," American Journal of Agricultural Economics, 70(3), 521–532.

——— (1994): "The Inverse Almost Ideal Demand System," European Economic Review, 38(1), 101 – 115.

Gallant, A. R. (1981): "On the Bias in Flexible Functional Forms and an Essentially Unbiased Form: The Fourier Flexible Form," Journal of Econometrics, 15(2), 211 – 245.

Gallant, A. R., and G. H. Golub (1984): "Imposing Curvature Restrictions on Flexible Functional Forms," Journal of Econometrics, 26, 295–321.

Geweke, J. (1993): "Bayesian Treatment of the Independent Student-t Linear Model," Journal of Applied Econometrics, 8, S19–S40.

Holt, M. T. (1998): "Autocorrelation Specification in Singular Equation Systems: A Further Look," Economics Letters, 58, 135–141.

——— (2002): "Inverse Demand Systems and Choice of Functional Form," European Economic Review, 46, 117–142.

Hotelling, H. (1935): "Demand Functions with Limited Budgets," Econometrica, 3(1), 66–78.

Karagiannisa, G., and G. J. Mergos (2002): "Estimating Theoretically Consistent Demand Systems Using Cointegration Techniques with Application to Greek Food Data," Economics Letters, 74, 137–143.

Lafrance, J. T. (2004): "Integrability of the Linear Approximate Almost Ideal Demand System," Economics Letters, 84, 297–303.

Lewbel, A. (1989): "Nesting the Aids and Translog Demand Systems," International Economic Review, 30(2), 349–356.

Lewbell, A., and S. Ng (2005): "Demand Systems with Nonstationary Prices," Review of Economics and Statistics, 87, 479–494.

Moschini, G. (1995): "Units of Measurement and the Stone Index in Demand System Estimation," American Journal of Agricultural Economics, 77, 63–68.

——— (1998): "The Semiflexible Almost Ideal Demand System," European Economic Review, 42, 349–364.

Moschini, G. (1999): "Imposing Local Curvature Conditions in Flexible Demand Systems," Journal of Business & Economic Statistics, 17(4), 487–490.

Moschini, G., and K. D. Mielke (1989): “Modelling the Pattern of Structural Change in U.S. Meat Demand,” American Journal of Agricultural Economics, 71, 253–261.

Moschini, G., and D. Moro (1994): “Autocorrelation Specification in Singular Equation Systems,” Economics Letters, 46, 303–309.

Ng, S. (1995): “Testing for Homogeneity in Demand Systems When the Regressors Are Non-stationary,” Journal of Applied Econometrics, 10, 147–163.

Piggott, N. E., and T. L. Marsh (2004): “Does Food Safety Information Impact U.S. Meat Demand?,” American Journal of Agricultural Economics, 86, 154–174.

Pollak, R. A., and T. J. Wales (1992): Demand System Specification and Estimation. Oxford University Press.

Ryan, D. L., and T. J. Wales (1998): “A Simple Method for Imposing Local Curvature in some Flexible Consumer–Demand Systems,” Journal of Business and Economic Statistics, 16, 331–338.

Terrell, D. (1996): “Incorporating Monotonicity and Concavity Conditions in Flexible Functional Forms,” Journal of Applied Econometrics, 11(2), 179–194.

U.S. Department of Agriculture (2006a): “Economic Resaerch Service. Poultry Yearbook,” Online, Stock No. 89007.

——— (2006b): “Economic Research Service. Red Meat Yearbook,” Online Publication, Stock No. 94006.

van Dijk, D., B. Strikhom, and T. Teräsvirta (2003): “The Effects of Institutional and Technological Change and Business Cycle Fluctuations on Seasonal Patterns in Quarterly Industrial Produciton Series,” Econometrics Journal, 6, 79–98.

Table 1: Descriptive Statistics for Meat Demand Variables, 1960–2004.

Variable	Average	Min	Max
Prices:			
Beef Price	2.064	0.735	4.164
Pork Price	1.594	0.546	2.877
Poultry Price	0.742	0.379	1.111
Quantities:			
Beef Quantity	18.639	15.000	24.300
Pork Quantity	14.022	11.300	17.900
Poultry Quantity	16.931	6.200	29.800
Shares:			
Beef Share	0.522	0.420	0.593
Pork Share	0.303	0.252	0.367
Poultry Share	0.175	0.106	0.267

Note: Prices are in dollars per pound. Quantities are in pounds *per capita*. Min denotes minimum value and Max denotes maximum value. There are 180 sample observations.

Table 2: AIDS and log Translog Model Parameter Estimates for Quarterly U.S. Meat Demand with Seasonal Dummy Variables and Autocorrelation Corrections, 1960–2004.

AIDS Model Parameter Estimates				log Translog Model Parameter Estimates			
Parameter	Estimate	Asy. Std. Error	90% CI	Parameter	Estimate	Asy. Std. Error	90% CI
α_1	0.461	0.067	[0.359 0.839]	α_1	-0.445	0.090	[-1.070 -0.318]
α_{11}	0.004	0.001	[0.002 0.005]	α_{11}	-0.003	0.001	[-0.004 -0.001]
α_{12}	0.011	0.001	[0.009 0.013]	α_{12}	-0.010	0.001	[-0.012 -0.009]
α_{13}	0.008	0.001	[0.006 0.009]	α_{13}	-0.008	0.001	[-0.009 -0.006]
γ_{11}	0.074	0.020	[0.041 0.108]	α_2	-0.271	0.034	[-0.453 -0.218]
γ_{12}	0.027	0.015	[-0.001 0.055]	α_{21}	-0.008	0.001	[-0.009 -0.006]
β_1	0.020	0.030	[-0.031 0.070]	α_{22}	0.010	0.001	[0.009 0.012]
α_2	0.274	0.029	[0.231 0.400]	α_{23}	0.010	0.001	[0.008 0.011]
α_{21}	0.008	0.001	[0.007 0.010]	γ_{11}	-0.094	0.038	[-0.157 -0.038]
α_{22}	-0.010	0.001	[-0.011 -0.008]	γ_{12}	-0.032	0.020	[-0.065 -0.002]
α_{23}	-0.010	0.001	[-0.011 -0.008]	γ_{13}	0.109	0.019	[0.079 0.139]
γ_{22}	0.010	0.019	[-0.022 0.043]	γ_{22}	-0.007	0.023	[-0.041 0.027]
β_2	0.041	0.030	[-0.003 0.089]	γ_{23}	0.036	0.016	[0.008 0.064]
τ_{11}	0.783	0.010	[0.766 0.794]	τ_{11}	0.780	0.010	[0.764 0.792]
τ_{12}	-0.344	0.021	[-0.375 -0.299]	τ_{12}	-0.334	0.020	[-0.368 -0.290]
τ_{22}	0.680	0.009	[0.665 0.690]	τ_{22}	0.674	0.011	[0.657 0.686]

Note: Asymptotic standard errors are in columns headed Asy. Std. Error. Columns titled '90% CI' contain 90–percent bootstrapped confidence intervals obtained by using the percentile t method over 1000 bootstrap draws. The poultry equation is omitted during estimation. Sample size is $T = 179$.

Table 3: Measures of Fit for Estimated AIDS and log Translog Models.

AIDS Model			**log Translog Model**		
No. of Parameters		16	No. of Parameters		16
Log Likelihood		1224.60	Log Likelihood		1222.79
System R^2		0.9950	System R^2		0.9948
System AIC		-19.180	System AIC		-19.160
Curvature Violations		0	Curvature Violations		0
Beef Equation	$\overline{R}^2$	0.951	Beef Equation	$\overline{R}^2$	0.952
	DW	2.438		DW	2.430
Pork Equation	$\overline{R}^2$	0.853	Pork Equation	$\overline{R}^2$	0.852
	DW	2.046		DW	2.023
Poultry Equation	$\overline{R}^2$	0.968	Poultry Equation	$\overline{R}^2$	0.968
	DW	2.373		DW	2.342

Note: $\overline{R}^2$ denotes the individual equation coefficient of determination adjusted for (average) degrees of freedom. AIC denotes the system Akaike information criterion. DW denotes the individual equation Durbin–Watson statistic.

Table 4: Estimated Marshallian, Expenditure, and Hicksian Elasticities for the Estimated AIDS Model.

	Marshallian Price Elasticities			**Expenditure Elasticities**	
	Beef	Pork	Poultry		
Beef	-0.868	0.044	-0.217	Beef	1.041
	[-0.783 -0.954]	[-0.020 0.095]	[-0.142 -0.255]		[0.948 1.126]
Pork	0.026	-1.004	-0.172	Pork	1.150
	[-0.100 0.150]	[-0.897 -1.113]	[-0.056 -0.234]		[0.993 1.280]
Poultry	-0.296	-0.084	-0.370	Poultry	0.750
	[-0.184 -0.639]	[-0.266 0.073]	[-0.506 0.163]		[0.365 0.929]

	Hicksian Price Elasticities		
	Beef	Pork	Poultry
Beef	-0.360	0.326	0.035
	[-0.270 -0.410]	[0.297 0.405]	[-0.080 0.049]
Pork	0.586	-0.692	0.106
	[0.511 0.704]	[-0.571 -0.785]	[-0.033 0.167]
Poultry	0.070	0.119	-0.189
	[-0.260 0.139]	[-0.077 0.235]	[-0.287 0.281]

Note: Numbers in box brackets are 90–percent bootstrapped confidence intervals. All elasticities are computed at the means of the sample data.

Table 5: Estimated Marshallian, Expenditure, and Hicksian Elasticities for the Estimated log Translog Model.

	Marshallian Price Elasticities			Expenditure Elasticities	
	Beef	Pork	Poultry		
Beef	-0.823	0.063	-0.204	Beef	0.965
	[-0.753 -0.903]	[0.005 0.110]	[-0.127 -0.250]		[0.882 1.053]
Pork	0.102	-0.976	-0.113	Pork	0.987
	[-0.022 0.205]	[-0.886 -1.093]	[-0.011 -0.187]		[0.856 1.142]
Poultry	-0.460	-0.150	-0.474	Poultry	1.084
	[-0.357 -0.918]	[-0.052 -0.369]	[-0.597 0.059]		[0.853 1.362]

	Hicksian Price Elasticities		
	Beef	Pork	Poultry
Beef	-0.356	0.323	0.033
	[-0.268 -0.409]	[0.296 0.399]	[-0.075 0.053]
Pork	0.580	-0.710	0.130
	[0.509 0.695]	[-0.579 -0.789]	[-0.012 0.182]
Poultry	0.065	0.142	-0.207
	[-0.287 0.135]	[-0.003 0.270]	[-0.326 0.222]

Note: Numbers in box brackets are 90–percent bootstrapped confidence intervals. All elasticities are computed at the means of the sample data.