Overborrowing and Systemic Externalities in the Business Cycle

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Overborrowing and Systemic Externalities in the Business Cycle

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Abstract
Credit constraints that link a private agent’s debt to prices embody a systemic credit externality that drives a wedge between competitive and (constrained) socially optimal equilibria, which induces private agents to “overborrow”. We quantify the effects of this externality in a two-sector DSGE model of a small open economy calibrated to emerging markets. Debt is denominated in units of tradable goods, and is constrained not to exceed a fraction of income, including nontradables income valued at the relative price of nontradables. The externality arises because agents fail to internalize the price effects of their individual borrowing, and hence the adverse debt-deflation amplification effects of negative income shocks that makes the credit constraint binding. Quantitatively, the credit externality causes a modest increase in average debt, of about 2 percentage points of GDP, but it triples the probability of financial crises and doubles the average current account and consumption reversals caused by these crises. On the normative side, we compute a schedule of state-contingent taxes on capital flows that attains the constrained social optimum.

Keywords: Financial Crises, Business Cycles, Amplification Effects, Sudden Stops, Systemic Externalities

JEL Classification Numbers: D62,E32,E44,F32,F41

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1 Introduction

Recent episodes in the financial markets have shown that economies can experience deep recessions which differ from typical business cycles. In emerging markets crises are often preceded by tranquil times during which the economy experiences an increase in debt, before it suddenly loses access to the international financial market. The disruption associated with these episodes has lead to a concern in academic and policy circles that countries may borrow excessively. If agents take borrowing decisions from a privately optimal standpoint, why should the aggregate level of debt not be socially optimal? If rational agents take decisions that leave them vulnerable to the occurrence of a crisis, why should the incidence and the severity of financial crises not be socially optimal? Is there a role for financial regulation to prevent financial crises?

This paper address these questions in a two-sector dynamic stochastic general equilibrium small open economy model (DSGE-SOE) where a credit market externality drives a wedge between competitive and (constrained) socially optimum equilibria. Agents do not internalize the fact that additional borrowing entails amplification effects that increase both the incidence and the severity of financial crises. Quantitatively, in a calibrated version of our model, the private economy’s debt exceeds the socially optimal level by two percentage points of GDP. The overborrowing effect may seem modest, but the credit externality increases the probability of a financial crises from two to seven percent and also the severity of these episodes. In the the most severe crises episodes, capital flight increases by an average of four percentage points of GDP and consumption decreases by ten percent when the externality is not present. In the presence of the externality, however, the private economy experiences average capital flights of 14 percentage points of GDP and a drop in consumption of 17 percent.

Our model features a credit constraint that requires debt, denominated in the international unit of account (i.e., tradable goods), not to exceed a fraction of income composed of tradables and nontradables, as in Mendoza (2002). Because debt is partially leveraged in nontradables income which is valued at a different relative price, changes in the price of nontradable goods can induce sharp and sudden adjustments in access to foreign financing.
When a negative shock to income makes the credit constraint binding, tradables consumption fall which reduces the price of nontradables. The drop in the price of nontradables leads to a further tightening of the credit constraint, setting in motion Fisher’s debt deflation channel by which drops in consumption demand, price of nontradables and access to foreign financing mutually reinforce each other. A financial crisis, therefore, is rationalized in our model with a binding credit constraint. Households will self-insure against these episodes by holding precautionary savings but the economy will occasionally face crises as an endogenous response to negative shocks when leverage is sufficiently high. In this way, typical business cycles coexist with financial crisis.

The systemic credit externality arises because agents fail to internalize the price effects of their individual borrowing, and hence the adverse debt-deflation amplification effects of income shocks when the credit constraint binds. Private agents form rational expectations about the evolution of macroeconomic variables, in particular the price of nontradables, and correctly perceive the risks and benefits of their consumption-savings decisions. Nevertheless, they do not internalize the general equilibrium effects of their consumption demand on the price of nontradables. This is a pecuniary externality that would not have welfare effects in the absence of the adverse debt-deflation amplification effects. By reducing the amount of debt, however, a social planner mitigates the downward spiral in the price of nontradables and the borrowing capacity of the economy during a crisis, which results in higher consumption smoothing and welfare gains.

On the normative side, the inefficiency result implies that there is a role for financial regulation taking as given the financial frictions considered. In our model, a schedule of state-contingent taxes on debt can attain the constrained social optimum. The rationale for this policy is to discourage leverage and reduce the economy’s vulnerability to financial crises. In the calibrated version of our model, the tax on debt necessary to correct the externality is 3 percent on average, and increases with leverage and the probability of a future financial crisis. The welfare gains from correcting the externality are associated with a smoother business cycle and the magnitudes and vary quite strongly depending on the state of the economy. As a result of the macro-prudential nature of optimal policy, the benefits of the introduction of financial regulation to correct the externality are maximized outside crises.
states, particularly in the runup to a financial crash.

Our work builds on extensive research on the financial accelerator that started with Fisher (1933) and developed further with the contributions of Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1998) and Aiyagari and Gertler (1999). In the emerging markets literature, Mendoza (2002, 2008) has shown that the quantitative predictions of a DSGE model with Fisher’s debt deflation channel are consistent with key features of actual Sudden Stops. Our contribution to this literature is twofold. First, we show that the financial accelerator creates an externality by which excessive overborrowing leads to a level of amplification that can differ from the optimal in a quantitatively significant way. Secondly, we propose a policy that achieves the constrained optimal by reducing the amount of debt.

Previous studies of pecuniary externalities inducing distortions in borrowing decisions have been primarily static and designed to address qualitative effects (see for example Caballero and Krishnamurthy (2001) and Korinek (2008a) in the emerging market literature and Lorenzoni (2006) in the closed economy macroeconomic literature). \(^1\) The quantitative significance of pecuniary externalities which is important to determine to what extent is regulation desirable to correct the externalities remains, therefore, an open question. This paper fills in this gap by providing a dynamic stochastic general equilibrium framework where macroeconomic externalities associated with the financial accelerator can be analyzed and assessed quantitatively.

The existent literature has offered different explanations of why countries could borrow excessively. One explanation is moral hazard: banks could lend excessively to take advantage of some form of government guarantees (see e.g. McKinnon and Pill 1996; Schneider and Tornell 2004). Another explanation suggested in policy debates is that emerging markets tend to overborrow when the lending decisions of foreign investors are guided by macroeconomic indicators and not by an evaluation of individual borrowers’s abilities to repay. This explanation has not been supported by a formal model developed by subsequent research. In fact, Uribe (2006) finds that an equilibrium with an aggregate borrowing constraint and an equilibrium with an internalized borrowing constraint deliver the same borrowing decisions.

\(^1\) The externality result of these papers are an application of a general proposition that economies with incomplete markets and multiple goods are generically inefficient. See Geanakoplos and Polemarchakis (1984) and Stiglitz (1982).
We will show, however, that in the presence of multiple goods, because a social planner internalizes the adverse debt-deflation effects of individual borrowing, the private economy exhibits “overborrowing”.

Our normative analysis is related to Christiano, Gust, and Roldos (2004) and Benigno, Chen, Otrok, Rebucci, and Young (2008) who show that stabilization plans can be optimal when economies face Sudden Stops. While the policy implications of the two previous papers suggest the need of a "reactive" policy to reduce the severity of a Sudden Stop once it has taken place, our normative analysis suggest the need of a “prudential” policy to reduce the incidence of arriving at a Sudden Stop. The fact that many stabilization plans have fallen during recent Sudden Stops may lend support to prudential regulation that help prevent the economy from exposing to these episodes. Our paper provides a quantitative framework to analyze policies that have this objective.

Our results are also related to the ongoing debate on the desirability of financial globalization. Standard neoclassical theory predicts that the liberalization of the capital account will generate international risk sharing and efficient capital reallocation. Critics like Stiglitz (2002) and Bhagwati (2004) have argued against financial globalization on the grounds that it makes emerging markets vulnerable to financial crises. Recently, Korinek (2008b) develops this argument further in a theoretical model and shows how taxes on particular capital flows can be welfare improving. Our paper contributes to this debate by undertaking a quantitative investigation of how a state contingent tax on capital flows can be effective in reducing the vulnerability to financial crises while still allowing the benefits from access to the global financial market.

The remainder of the paper is organized as follows. Section 2 presents the analytical framework. Section 3 characterizes the decentralized equilibrium and the social planner’s problem and demonstrates the externality result of the paper. Section 4 analyzes the quantitative importance of the externality. Section 5 discusses welfare and policy implications and section 6 concludes.
2 Analytical Framework

Consider an endowment version of the representative-agent two sector DSGE-SOE model. We assume that debt is denominated in units of tradables with a credit constraint that links credit-market access to income, including non tradable income. This setup, proposed by Mendoza (2002), captures the phenomenon called “liability dollarization” in a flexible-price environment setting, which the empirical literature has emphasized as an important transmission mechanism of financial crisis in emerging markets.\(^2\)

2.1 Households

There is a continuum of identical infinitely lived households of measure unity. Preferences of the representative household are given by:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}
\]

In this expression, \(E(\cdot)\) is the expectation operator and \(\beta\) is the discount factor. The period utility function \(u(\cdot)\) has the constant-relative-risk-aversion (CRRA) form. The consumption basket \(c_t\) is an Armington-type CES aggregator with elasticity of substitution \(1/(\eta + 1)\) between tradable \(c^T\) and non tradable goods \(c^N\) given by:

\[
c_t = \left[ \omega (c^T_t)^{-\eta} + (1 - \omega) (c^N_t)^{-\eta} \right]^{-\frac{1}{\eta}}, \quad \eta > -1, \omega \in (0, 1)
\]

Households receive a random endowment of tradables \(y^T_t\) and a constant endowment of nontradables \(y^N\). The tradable endowment shock has a compact support and follows a finite-state stationary Markov process. Foreign assets are restricted to a one period non-contingent bond \(b_t\), denominated in units of tradables, that pays a fixed interest rate \(r\) determined in the world market. As is standard in incomplete market models with non-contingent debt, we assume that the discount factor and the interest rate are such that \(\beta(1 + r) < 1\), which

ensure a well-defined stochastic stationary equilibrium.\footnote{When $\beta(1+r) < 1$, $\beta^t(1+r)^t u'(c_t) < 1$ is a nonnegative supermartingale that converges almost surely to a nonnegative random variable (see Chamberlain and Wilson (2000)). Hence the long-run averages of assets and consumption also remain finite. In contrast, with $\beta(1+r) \geq 1$, assets diverge to infinity in the long run because the marginal utility converges to zero almost surely.}

Normalizing the price of tradables to 1 and denoting the price of nontradable goods by $p^N$, the budget constraint is:

$$b_{t+1} + c_t^T + p_t^N c_t^N = y_t^T + b_t(1 + r) + p_t^N y^N$$

(2)

Households can borrow up to a fraction $\kappa$ of their income, so that:

$$b_{t+1} \geq -\kappa (p_t^N y^N + y_t^T)$$

(3)

This type of borrowing constraint may arise in optimal contracts where a higher level of income supports debt repayment incentives under limited commitment.\footnote{For examples on how to derive credit constraints within an optimal contract framework see e.g. Holmstrom and Tirole (1998).} Borrowing constraints of this form are widely used by lenders in determining maximum loan amounts and setting borrowers’ qualification criteria, particularly in mortgage loans and consumer debt contracts (see Arellano and Mendoza (2003) for a discussion).\footnote{While many credit markets are characterized by a constraint that is linear in current income, our results do not depend on this particular specification, which we assume here for simplicity.}

Given a stochastic sequence of prices, the consumer problem consists of maximizing the expected present discounted value of utility (1) subject to the sequence of borrowing and credit constraints given by (2) and (3). The optimization problem for a typical household of this small open economy is analogous to the optimization problem of a single household in the heterogenous-agents models of precautionary savings (e.g., Aiyagari, 1994 or Hugget, 1993). As in those models, CRRA utility implies a strong aversion to a zero consumption basket. In addition, since the CES aggregator implies that $\lim_{c_T \to 0^+} u'(c) c_T(c_T^T, c_N^T) = \lim_{c_N \to 0^+} u'(c) c_N(c_T^T, c_N^T) = +\infty$ agents will self-insure to rule out stochastic sequences that could expose them to zero consumption of tradables or nontradables at any date at any state of nature.
2.2 Decentralized Competitive Equilibrium Conditions

The optimality conditions of the household problem are:

\[ \lambda_t = u'(t)c_T(t) \]  \hspace{2cm} (4)

\[ p_t^N = \left( \frac{1 - \omega}{\omega} \right) \left( \frac{c_T}{c_N} \right)^{\lambda+1} \]  \hspace{2cm} (5)

\[ \lambda_t = \beta E_t \lambda_{t+1}(1 + r) + \mu_t \]  \hspace{2cm} (6)

\[ 0 = \lambda_t[b_{t+1} + \chi (p_t^N y^N + y_T)] \]  \hspace{2cm} (7)

where \( \lambda \) is the multiplier associated with the budget constraint and \( \mu \) is the multiplier associated with the credit constraint. The optimality condition (4) equates the marginal utility of tradables consumption to the shadow value of current wealth. Condition (5) is a static condition linking the marginal rate of substitution of two goods, tradables and nontradables, to their relative price. Equation (6) determines the intertemporal allocation of wealth and consumption. When the credit constraint is binding, there is a wedge between the current shadow value of wealth and the expected value of reallocating wealth to the next period given by the shadow price of relaxing the credit constraint \( \mu_t \). Condition (7) is the standard complementary slackness condition.

Market clearing conditions are given by:

\[ c^N = y^N \]  \hspace{2cm} (8)

\[ c^T = y^T + b_t(1 + r) - b_{t+1} \]  \hspace{2cm} (9)

Note from (5) that when a negative shock occurs, the reduction in \( c^T_t \) that follows generates on equilibrium a reduction in \( p_t^N \) which by equation (3 reduces the collateral value. Therefore, the borrowing constraint produces asymmetric responses of the economy to shocks, as in Mendoza (2002).\(^6\) Binding borrowing constraints amplify the drop in consumption in response to negative shocks to tradables output relative to perfect capital markets, but there are no amplification effects when borrowing constraints are slack. As in a standard

\(^6\)See Mendoza (2006) for an illustrative deterministic example of this mechanism.
model of insurance, the demand for borrowing is decreasing with current income $y^T_t$. As a consequence, for a given $p^N_t$ there is a threshold $\tilde{y}^T_t$ such that for any $y^T_t < \tilde{y}^T_t$ the constraint becomes binding.

Given an initial foreign asset position $b_0$, the stochastic process for $y^T$ and the world real interest rate $r$, the competitive equilibrium is fully characterized by a set of stochastic sequences $\{c^T_t, c^N_t, b_{t+1}, p^N_t, \lambda_t, \mu_t\}_{t=0}^{\infty}$ such that (3)-(9) are satisfied.

In the environment just described, the borrowing constraint sets a limit on individual borrowing. When foreign creditors monitor borrowers based on a set of macroeconomic indicators such as GDP, an economy may become subject to an aggregate borrowing constraint. Uribe (2006) shows that if households fail to internalize the aggregate debt limit, the borrowing decisions are still equivalent to those where the debt limit is internalized.\textsuperscript{7} The reason for this equivalence is that market incentives conveyed by the interest rate induce individual borrowing decisions to behave as if they were internalizing the aggregate debt constraint. This result represents a form of “no-overborrowing”. We will show below, however, that in the presence of multiple goods, the private economy displays “overborrowing” relative to a social planner who internalizes the price effects of individual decisions.

3 Recursive Problem

3.1 Decentralized Competitive Equilibrium

We start by describing the optimization problem of a typical agent in recursive form. A crucial feature of this setup is that aggregate bond holdings, denoted by $B$, constitute a state variable in this problem. This feature arises because the representative agent needs to forecast aggregate variables, beyond his control, in order to form expectations of future prices. In particular, he needs to forecast aggregate tradable and nontradable consumption to predict the price of nontradables. Because the endowment of nontradables is fixed, the forecast of nontradables consumption is simply $y^N$. In order to forecast tradable consumption, the representative agent must conjecture a law of motion for the aggregate foreign asset position.

\textsuperscript{7}A proof for this particular setup is available in an appendix upon request.
We denote by $\Gamma(\cdot)$ the perceived law of motion for the aggregate foreign asset position which is a function of the current aggregate state $(B, y^T)$. Using (5), the forecast price for nontradables at time $t$ can be expressed as:

$$p^N(B_t, y^T_t) = \left(\frac{1 - \omega}{\omega}\right) \left(\frac{y^T_t + B_t(1 + r) - \Gamma(B_t, y^T_t)}{y^N} \right)^{\eta + 1}$$

(10)

The optimization problem of a typical agent is:

$$V(b, B, y^T) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + \beta E_t V(b', B', y^{T'})$$

subject to

$$b' + p^N(B, y^T)c^N + c^T = y^T + b(1 + r) + p^N(B, y^T)y^N$$

$$b' \geq -\kappa \left(p^N(B, y^T)y^N + y^T\right)$$

$$B' = \Gamma(B, y^T)$$

(11)

where we have followed the convention of denoting current variables without subscript and denoting next period variables with the prime superscript.

The solution to the household problem provides decision rules for individual bond holdings $\hat{b}(b, B, y^T)$, tradables consumption $\hat{c}^T(b, B, y^T)$ and nontradables consumption $\hat{c}^N(b, B, y^T)$. The household optimization problem induces a mapping from the perceived law of motion for the aggregate foreign asset position $\Gamma(\cdot)$ to an actual law of motion, given simply by the representative agent’s choice $\hat{b}(B, y^T)$. In a rational expectations equilibrium, as defined below, these two laws of motion must coincide.

**Definition 1 (Recursive decentralized competitive equilibrium)**

A recursive decentralized competitive equilibrium for the SOE is a pricing function $p^N(B, y^T)$, a perceived law of motion $\Gamma(B, y^T)$ and decision rules $\{\hat{b}(b, B, y^T), \hat{c}^T(b, B, y^T), \hat{c}^N(b, B, y^T)\}$ such that the following conditions hold:

1. **Household optimization**: taking as given $p^N(B, y^T)$ and $\Gamma(B, y^T)$, $\{\hat{b}(b, B, y^T), \hat{c}^N(b, B, y^T), \hat{c}^N(b, B, y^N)\}$ solve the optimization problem of the household.

2. **The perceived law of motion is consistent with the actual law of motion**:
\[
\Gamma(B, y^T) = \hat{b}(B, B, y^T), \quad \text{(rational expectation condition)}.
\]

3. Markets clear: \( y^N = \hat{c}^N(B, B, y^T) \) and \( \Gamma(B, y^T) + c^T = y^T + b(1 + r) \).

### 3.2 Social Planner

We previously described the equilibrium achieved when agents take aggregate variables as given, particularly the price of nontradables. Consider now a benevolent social planner (SP) who directly chooses allocations subject to the collateral constraint and competitive market clearing in order to maximize a representative agent’s utility. As opposed to the representative agent, a social planner internalizes the effects of agents’ decisions on the price of nontradables. Critically, the social planner realizes that a lower debt level in a crisis event reduces the adjustment of the price of nontradables, and therefore decreases the subsequent drop in consumption needed to satisfy the credit constraint. As a result, we will show that the decentralized equilibrium allocation is not constrained efficient, as defined below.

**Definition 2 (Constrained efficiency)**

Let \( \{\hat{b}(b, B, y^T), \hat{c}^T(b, B, y^T), \hat{c}^N(b, B, y^T)\} \) be the solution to the competitive equilibrium yielding utility \( \hat{V} \). The competitive equilibrium is constrained efficient if a planner who directly chooses allocations, subject to the collateral constraint and competitive markets clearing, cannot improve welfare for the consumer over \( \hat{V} \).

The planner’s optimization problem can be stated as:

\[
V(b, y^T) = \max_{b', c^T} \left[ u(c(c^T, y^N)) + \beta E_t V(b', y^T) \right]
\]

subject to

\[
b' + c^T = y^T + b(1 + r) \]

\[
b' \geq -\kappa \left( \frac{1 - \omega}{\omega} \left( \frac{c^T}{y^N} \right)^{\eta+1} y^N + y^T \right)
\]

---

\( ^8 \)Note that we substitute \( p^N \) with the equilibrium price \( \left( \frac{1 - \omega}{\omega} \right) \left( \frac{c^T}{y^N} \right)^{\eta+1} \) in the collateral constraint and \( c^N = y^N \) in the resource constraint.
Using superscript "SP" to distinguish the multipliers of the social planner from the decentralized equilibrium, this results in the following first order conditions for the social planner:

\[
\lambda_{t}^{SP} = u'(t)c_T(t) + \mu_{t}^{SP}\Psi_t \tag{13}
\]

\[
\lambda_{t}^{SP} = \beta E_t\lambda_{t+1}^{SP}(1 + r) + \mu_{t}^{SP} \tag{14}
\]

where \(\Psi_t = \kappa \frac{\partial p^N}{\partial c_T} y^N = \kappa \frac{1-\omega}{\omega} \left( \frac{c_T^T}{y^N} \right)^\eta (1 + \eta)\) represents how the value of the collateral changes in equilibrium when there is a marginal change in tradables consumption. This variable is not present in the corresponding equation for the decentralized equilibrium (4) since decentralized agents take the price as given. Condition (13) states that at the social optimum the value of a marginal increase in current wealth, equals the marginal utility of consumption plus the value of relaxing the credit constraint that arises from the marginal increase in consumption, (i.e, \(\mu_{t}^{SP}\Psi_t\)). Condition (14) is identical to the decentralized equilibrium condition (6). If the constraint does not bind in in the stochastic steady states of the decentralized equilibrium and the social planner, it is easy to see that the allocations of the two environments coincide.

Consider an initial situation where the constraint is satisfied with equality, but is not binding, and a negative shock tightens the borrowing constraint by one unit. Because atomistic agents take prices as given, the perceived decrease in tradables consumption is one unit. On the other hand, the social planner internalizes the fact that a decrease in tradables consumption decreases the price of nontradables, making the constraint even more binding as captured by \(\Psi_t\). A direct consequence of the debt-deflation effect is summarized in the following proposition that will prove useful to show that the decentralized equilibrium is constrained inefficient.

**Proposition 1 (Wealth undervaluation)**

Assume the credit constraint binds at time \(t\) and let \((b_t, y_T^T)\) be the initial state and \((c_T^T, c_N^T, b_{t+1})\) be the allocations of the decentralized equilibrium and the social planner. The social planner’s shadow value of wealth is strictly higher than the decentralized agents’s shadow value of wealth.
Proof. The proof follows simply by comparing (4) and (13), which yields that the social shadow value of wealth exceeds the private value by $\mu_t^{SP}\Psi_t$. ■

As explained above, this result stems from the fact that the social planner internalizes that a marginal increase in wealth would allow a marginal increase in consumption that will relax the credit constraint by $\Psi_t$ with a marginal utility of $\mu_t^{SP}$.

Suppose that at time $t$ the constraint is not binding but it may become binding constraint in the next period if a sufficiently high leverage is chosen. Using (6) and (4), the Euler equation for consumption for the decentralized equilibrium is simply:

$$u'(c_t) = \beta u'(c_{t+1})(1 + r) \quad (15)$$

Using (14) and (13), the Euler equation for consumption for the social planner becomes:

$$u'(c_t) = \beta \left[ u'(c_{t+1}) + \mu_{t+1}^{SP}\Psi_{t+1} \right] (1 + r) \quad (16)$$

Consider now a reallocation of wealth by the social planner starting from the privately optimal allocations in the decentralized equilibrium. In particular, consider a reduction of one unit of debt. Because decentralized agents are at the optimum, (15) shows that the first order welfare benefits of an additional marginal unit of savings $\beta u'(c_{t+1})(1 + r)$ are equal to the first order welfare losses $u'(c_t)$. Using (16), the social planner has the same marginal cost of reducing debt by a marginal unit but face higher marginal benefits given by $\beta E_t \left[ u'(c_{t+1}) + \mu_{t+1}^{SP}\Psi_{t+1} \right] (1 + r)$. The last term corresponds to the expectation on how an additional marginal unit of savings helps relax the credit constraint when it becomes binding due to the debt-deflation channel, a fact that is not internalized by the representative agent. This yields our externality result.

**Proposition 2 (Constrained inefficiency)**

Assume the credit constraint binds in some state of nature in the decentralized equilibrium. Then, the decentralized equilibrium is constrained inefficient.

Proof: See appendix
4 Quantitative analysis

4.1 Calibration

Section 3 showed analytically that the decentralized equilibrium is not constrained efficient. In particular, it showed that a social planner has a higher incentive to save than an individual agent in the decentralized equilibrium when it is likely that the credit constraint will become binding in a future state of nature. In this section, we study the quantitative implications of this pecuniary externality by comparing the results from simulating the social planner’s solution and the decentralized equilibrium. We are particularly interested in how the pecuniary externality affects the aggregate level of debt and and the vulnerability of a country to financial crises.

The values assigned to all model’s parameters are listed in Table 1. The model frequency is annual. The risk aversion parameter $\sigma$ of the CRRA utility function is set to 2 and the world interest rate $r$ is set to 4 percent, standard values in quantitative business cycle analysis for open economies. For the discount factor, we choose the annual equivalent of the simple average of two well-known SOE models and . We choose to calibrate the model to Argentina as an example of an emerging market whose business cycle has been studied extensively. The reference period is 1980-2007, the usual sample for the study of emerging market crises. We assume that the endowment for tradables follows a log AR(1) process such that $\ln y_t^T = p \ln y_{t-1}^T + \varepsilon_t$ and $\varepsilon_t \sim N(0, \sigma^2_{\varepsilon})$. This process is estimated using the sectorial data from the World Development Indicators (WDI) from 1980 to 2007. We classify manufacturing industry and primary products as tradables, following the standard methodology. After detrending the data with the HP filter with a parameter of 100, the standard value for annual data, the estimation of the cyclical component of the tradable output process yields $\sigma^T = 0.067$ and $\rho = 0.60$. The shock is discretized into a finite first-order Markov process with five grid points using the quadrature based procedure of Tauchen and Hussey (1991). The endowment of nontradables $y^N$ is normalized to one.

There is some range on the value assigned to the elasticity of substitution in SOE model.

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9See for example Garcia (2008) and Mendoza and Terrones (2008)
### Table 1: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>( r = 0.04 )</td>
<td>Standard RBC value</td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>( \sigma = 2 )</td>
<td>Standard RBC value</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>( 1/(1 + \eta) = 0.4 )</td>
<td>Indep. estimation Gonzalez-Neumeyer(2004)</td>
</tr>
<tr>
<td>Non tradable endowment</td>
<td>( y^N = 1 )</td>
<td>Normalization</td>
</tr>
<tr>
<td>Stochastic Structure</td>
<td>( \sigma_{yt} = 0.067 ), ( \rho = 0.60 )</td>
<td>Argentina’s economy</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calibration</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor</td>
<td>( \beta = 0.90 )</td>
<td>Trade Balance Output ratio=0.9%</td>
</tr>
<tr>
<td>Weight on Tradables in CES</td>
<td>( \omega = 0.32 )</td>
<td>Share of Tradable Output=33%</td>
</tr>
<tr>
<td>Credit Coefficient</td>
<td>( \kappa = 0.22 )</td>
<td>Frequency of Sudden Stops 7.4%</td>
</tr>
</tbody>
</table>

Empirical studies estimate the elasticity of substitution to be less than 0.5. For Argentina, a comprehensive empirical research of the Argentine economy by Gonzalez and Neumeyer (2004) reports an elasticity of substitution \( 1/(1 + \eta) \) equal to 0.40. We use this estimation which is also very close to the estimation of 0.46 for Uruguay of ? and the estimation of 0.44 of Stockman and Tesar (1995) (1995), who found an average value of 0.44 for developed countries.

The remaining parameters \( \{ \beta, \omega, \kappa \} \) are calibrated to generate long run averages of the decentralized equilibrium that mimic the respective unconditional means in our reference period. The discount factor \( \beta \) is calibrated to match a trade-balance-to-output ratio of 0.08 percent. The weight of tradables consumption in the CES aggregator \( \omega \) is calibrated to match a share of tradables production of 33 percent. This is a reasonable approach to calibrate \( \omega \) since manipulating (5) yields that the ratio nontradables-tradables GDP denoted by \( \frac{p_n}{p_t} \) equals \( \left( \frac{1-\omega}{\omega} \right)^{\frac{\eta}{\zeta}} \). The credit coefficient \( \kappa \) is calibrated so that the chances of a binding credit constraint are 7.4 percent, which is the estimated frequency of Sudden Stops for Argentina in the panel dataset of Eichengreen, Gupta, and Mody (2006).

### 4.2 Solution Method

The non-linearities introduced by the borrowing constraint require a global method to solve the planner’s problem and the decentralized equilibrium. To solve for the planner’s problem, we use the algorithm described in Mendoza and Arellano (2004). This method
is based on value function iteration and respects the fact that the credit constraint is only occasionally binding. The key difference between this method and the one necessary to solve for the decentralized equilibrium is that atomistic agents must take prices as given. We propose a method to solve for the recursive competitive equilibrium employing a nested fixed point algorithm.

For any given belief of the evolution of aggregate bonds, there is an “inner” fixed point algorithm that solves the Bellman equation. Once the Bellman equation is solved, the actual law of motion \( \hat{b}(B, B, y_T) \) can be derived and the outer fixed point algorithm checks whether it approximately coincides with the perceived law of motion for aggregate bonds \( \Gamma(B, y_T) \).\(^{10}\) If these laws of motions do not coincide, a new trial is obtained and fed into the Bellman Equation again. The new trial is based on a Gauss-Seidel algorithm that assigns a decreasing weight to the effective law of motion. It is important to adjust slowly the new guess since iterations on the perceived laws of motions do not follow a contraction mapping and may not converge due to the well-known problem of unstable hog cycles. To obtain convergence in the outer fixed point algorithm, it is also important to allow the agents in their optimization problem to choose a vale of foreign assets which is outside the grid, as in Krusell and Smith (1997). The algorithm to solve the model for particular values of parameters follow these steps:

1. Generate a discrete grid for the economy’s asset of size \( NB \) and the shock state space \( NS \).

2. Conjecture a law of motion for aggregate foreign assets \( B' = H(B, y_T) \) at each point of the state space, using \( Y^N = C^N \) this implies a price given by (10).

3. Solve for the policy functions \( \{ \hat{b}(b, B, y_T), c^T(b, B, y_T), c^N(b, B, y_T) \} \) via value function iteration in the bellman equation (11).

4. Using the decision rules in the previous step, calculate the implied transition of aggregate bonds \( \sigma^k(B, B, y_T) \) and evaluate the previous guess. If \( \text{sup}_{B, y_T} \| \hat{b}(B, B, y_T) - \Gamma(B, y_T) \| < \epsilon \), the recursive competitive equilibrium is found. Otherwise, update \( H(B, y_T) \) with a Gauss-Seidel algorithm and go to step 3.

\(^{10}\)By Walras Law, when these laws of motion coincide, markets clear in the nontradable sector.
The solution method is described in more detail in the appendix.  

4.3 Simulation Results

4.3.1 Policy Functions

We start by showing how the policy functions of the social planner and the decentralized equilibrium differ and then analyze the consequences for the average debt and for the probability and the severity of a financial crisis. Figure 1 shows how the policy function for the social planner differs from the decentralized equilibrium policy function when the economy is hit by a roughly one standard deviation negative shock to tradables output. Since the mean value of tradables output is 1, we can interpret all results in terms of this variable. Without the endogenous borrowing constraint, the policy function for next period’s foreign assets would be monotonically increasing in current assets. However, when the actual model economy is hit by the negative shock and the borrowing constraint binds, a lower current asset position reduces the level of tradables consumption consistent with the borrowing constraint, and thereby reduces the price of nontradables. Because the borrowing constraint depends endogenously on the price of nontradables, this means that the borrowing constraint becomes tighter. Therefore, the economy displays a policy function for next period foreign assets that is decreasing in current foreign assets in the range for which the borrowing constraint binds. In terms of Figure 1, this range corresponds approximately to a level of foreign assets lower than -0.540 for the social planner but lower than -0.535 for the decentralized equilibrium. Because atomistic agents do not internalize the price effects of their actions, they continue to borrow enough to trigger the binding credit constraint over some range of current assets for which the social planner would choose a lower borrowing level that is monotonically increasing in current debt.

We can distinguish three regions in Figure 1 according to the level of current foreign assets: a “constrained region”, a “high externality region” and a “low externality region”. For low levels of net foreign assets, the allocations of future bonds coincide since both the

11See Rust(2008) for an extensive treatment of numerical methods to solve competitive equilibria and dynamic games.

12By policy function in the decentralized equilibrium, we mean the equilibrium law of motion \( \dot{b}(B, B, y^T) \).

13Unless otherwise noted, all figures are expressed as percentages of mean tradables output.
planner and the atomistic agents in the decentralized equilibrium choose the maximum they can borrow. This constitutes the constrained area. At moderately higher current levels of net foreign assets, the economy is still highly leveraged and a future financial crisis is relatively likely. Consequently, the social planner increases future assets relative to decentralized agents who do not consider the general equilibrium effects of their actions. This high externality region ends approximately when foreign assets reach -0.42. Finally, as net foreign assets further increase and the probability of a financial crisis becomes relatively low, the policy functions become similar to each other. Comparing the policy functions against to the 45 degree line also shows that for relatively low current levels of bonds, the economy increases savings in response to a negative shock, resulting in capital outflows.
4.3.2 Long Run Distribution of Debt

It is clear from Figure 1 that the decentralized equilibrium will display higher levels of debt than what the social planner would command. Figure 2 confirms this result by illustrating the ergodic distribution of net foreign assets for both environments. In fact, the social planner’s distribution of foreign assets first order stochastic dominates the decentralized equilibrium distribution. While the maximum level of debt occurring with positive probability for the planner is 0.49, in the decentralized equilibrium the economy exceeds this level of debt about 80 percent of the time. The maximum level of debt that is supported with positive probability is 0.54 in the decentralized equilibrium. These results are critical to understanding the different levels of amplification that the two sets of allocations deliver, as will be shown below. For chosen levels of debt higher than 0.46, there is a positive probability under both environments that a sufficiently negative shock will trigger the credit constraint in the next period. Moreover, the higher is the existing leverage, the greater the adjustment in tradables consumption needed in order to satisfy the borrowing constraint. Consequently, Figure 2 shows that the planner reduces the exposure to states in which the contraction in consumption needed to satisfy the collateral constraint is large. Even if an atomistic agent foresees the probability of a big reduction in consumption in a crisis event, she does not internalize the impact that her actions have on the balance sheet of the rest of the agents.
The quantitative significance of the distortion can also be assessed by comparing the long run averages of debt. Table 2 shows how average debt levels differ in the two cases. The Debt-GDP ratio in the decentralized equilibrium is about 14 percent higher than what a social planner would find optimal on average. The mean Debt-GDP ratio is 17.3 percent for the private economy and 15.2 for the social planner. The private economy’s debt to tradable output ratio exceeds the social optimum by 5.3 percentage points. Conditional on a financial crisis, the decentralized equilibrium debt to GDP is 3 percentage points higher than in the social optimum.

4.3.3 Incidence and Severity of SS

In this subsection, we establish that this overborrowing in the decentralized equilibrium leaves the economy vulnerable to more frequent and more severe financial crises. To demon-
Table 2: Average Debt Levels

<table>
<thead>
<tr>
<th>Mean Ratios</th>
<th>Social Planner</th>
<th>Decentralized Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt to Tradable Output</td>
<td>45.0%</td>
<td>50.3%</td>
</tr>
<tr>
<td>Debt to GDP</td>
<td>15.2%</td>
<td>17.3%</td>
</tr>
<tr>
<td>Debt to GDP conditional on a crisis</td>
<td>24.1%</td>
<td>27.0%</td>
</tr>
</tbody>
</table>

To capture the different amplification levels in the decentralized equilibrium and social optimum in the most severe crises, we take those SS where the decline of capital inflows exceeds the median SS decline and compare the mean of the time series centered on these events, which we call SS Collapses. Results are similar if we define collapses as SS with contractions in output larger than the median as in Calvo, Izquierdo and Talvi (2006). As Figure 2 shows the drops of consumption, the real exchange rate and capital flows following the onset of a SS collapse are more pronounced in the decentralized equilibrium. The decentralized equilibrium experiences capital outflows which are 3.6 times larger than in the social planner allocations. The real exchange rate (RER) depreciates by 52 percent for the social planner and 93 percent for the decentralized equilibrium, indicating a much larger decline.
in the price of nontradables in the decentralized case.\textsuperscript{14} As panel c) of Figure 2 shows, the consumption bundle falls by less than 10 percent under the social planner compared to 17 percent in the presence of the credit externality.

The differences in the severity of a financial crisis are a consequence of the higher leverage of the decentralized equilibrium, shown above. A higher initial level of leverage requires a more drastic reduction in tradables consumption in response to a negative shock and therefore makes the constraint binding. In equilibrium, this implies a lower price of non tradables, a depreciation of collateral and larger capital outflows. By leveraging their position decentralized agents contribute to the feedback loop between the real exchange rate and economic activity but they do not internalize that their additional borrowing entail amplification effects when a negative income shocks triggers the credit constraint.

\textsuperscript{14} We define a real exchange rate depreciation as $-\Delta P/P$ where $P$ is the aggregate CES price index of a unit of the consumption bundle in units of tradable consumption that equal $P = \left[ \omega \pi^{\frac{1}{\sigma}} + (1 - \omega) \pi^{\frac{1}{\sigma}} p^N \pi^{\frac{1}{\sigma}} \right]^{\frac{\sigma+1}{\sigma}}$. Consistently with this definition, a real exchange rate depreciation implies a higher real exchange rate.
Figure 2: Event Analysis-Sudden Stop Collapse

Note: Real Exchange Rate Depreciation (RER) and Consumption Drop are expressed in percentage deviations from long run values. The Current Account is expressed in percentage of GDP.
5 Welfare and Policy Implications

First best policy measures are necessary to address the root of the externality: the borrowing constraint. Examples of such policies are enhancing the enforcement of contracts and increasing transparency. We focus here on second best policy measures. The questions that we address in this section are what policies should the government take to correct the externality and the welfare gains associated with these policies.

Theoretically, a wide range of policy measures can improve welfare from the decentralized equilibrium outcome; our focus is on how state contingent taxation on debt can correct the systemic externality. The following proposition summarizes our results:

**Proposition 3 (Optimal Policy)** Consider the social planner’s solution be \( \{ c_t^T, c_t^N, b_t^T, p_t^N, \lambda_{SP}^t, \mu_{SP}^t \} \) \( t=0 \) to infinity. Let the linear tax rate on debt contingent on the level of borrowing chosen at time \( t \) and collected at time \( t+1 \) be:

\[
\tau_{t+1}^* = \begin{cases} 
(1 + r) \frac{E_t \mu_{SP}^t \Psi_{t+1}}{E_t u_T(t+1)} & \text{if } \mu_{SP}^t = 0 \\
0 & \text{if } \mu_{SP}^t > 0
\end{cases}
\]

which is returned to households in the form of lump sum transfers \( T_{t+1} = b_{t+1} \tau_{t+1}^* \). Then the social planner’s allocations \( \{ c_t^T, c_t^N, b_t^T \} \) and the price \( \{ p_t^N \} \) constitute a decentralized equilibrium.

**Proof**: See appendix

The intuition for this expression is straightforward: (17) represents the non-internalized marginal cost of raising debt for next period. If the constraint is triggered next period, a marginal increase in debt requires a further tightening of the borrowing constraint of \( \Psi_{t+1} \), which has a marginal utility cost of \( \mu_{SP}^t \). The uninternalized marginal cost of raising debt is then given by the expected marginal utility cost of tightening the borrowing constraint \( E_t(1 + r) \mu_{SP}^t \Psi_{t+1} \) normalized by the expected marginal utility \( E_t u_T(t+1) \).

For a given state, the tax that will be paid next period depends on the level of borrowing chosen today. When the level of borrowing is relatively low, the economy is not vulnerable to a financial crisis in the next period and no tax is charged. As the economy increases
borrowing, it becomes likely that a financial crisis will take place so a tax is charged. The tax increases with the terms $\Psi_{t+1}$ and $\mu_{t+1}$ for the reasons explained above. The tax dependence on the expectation operator implies that the tax rate should be higher the more likely is a financial crisis next period.\(^{15}\)

How large must the tax be in order to correct the externality? Note that because the tax is contingent on the level of borrowing, there is a subset of taxes that are not paid in equilibrium. We denominate effective tax rates, those taxes that are actually paid in equilibrium. The maximum effective tax rate that decentralized agents pay with positive probability is 16 percent and the average effective tax rate is 3.5 percent.

Figure 3 shows the effective tax rate when tradables output is one standard deviation below its mean. There is a parallel between the effective tax rate and the three regions described in Figure 1. In the constrained region, when the crisis is already underway, taxing borrowing is ineffective. In the high externality region, where decentralized agents take on a level of debt that leaves the economy substantially more vulnerable to a financial crises than the social optimum, a tax must be imposed to discourage borrowing. Finally, in the low externality region, borrowing in the private economy is low enough to avoid future financial crises and no tax is needed.

It is worth noting that correcting the externality reduces the vulnerability to financial crises but does not rule them out in the long run. In some unfortunate states of nature, it is still socially optimal to experience a crisis. The trade-off of the optimal policy should be clear: discouraging leverage reduces systemic risk, but on the other hand postponing consumption is not free. Our policy trade-off resembles the dilemma in Bordo and Jeanne (2002) about how optimal monetary policy should respond to a credit boom: preventing a credit boom reduces the probability of a future bust, but this entails immediate costs in terms of output. In broader terms, our analysis can be seen as a case for macro-prudential regulation or prudential taxation (see Jeanne (2008)).

We also analyze different measures of welfare gains associated with correcting the systemic externality. It is well-known that the welfare cost of business cycles are typically small.

\(^{15}\)Note that the tax is not dependent on next period state, so the Government is not effectively introducing an additional set of securities to correct the externality, (see Stiglitz,1982).
Since the welfare gains from correcting the credit externality are associated with a smoother business cycle, they benefits from correcting the externality are also quite small. Following Lucas (1987), we first calculate the increase in consumption across all states of nature in the decentralized equilibrium necessary to make a consumer indifferent between living in the decentralized equilibrium (without government intervention) and correcting the externality starting at a particular state. Formally, for an initial state \((b, y)\) the welfare gain is \(\gamma\) such that:

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_{t}^{DE}(1 + \gamma)) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{t}^{SP})
\]  

(18)

where \(c_{t}^{DE}\) is the agent’s consumption in the decentralized equilibrium and \(c_{t}^{SP}\) is the agent’s consumption in the social planner when starting at an initial state \((b, y)\).\(^{16}\)

For a given initial negative shock, the welfare gains display a hump shape; Figure 4 shows the welfare gains of correcting the externality when the shock is one standard deviation below the mean. For high values of leverage, the economy is already constrained so the welfare gains from correcting the externality occur in the distant future. At higher levels of current

\(^{16}\) Because of the homotheticity of the utility function, (18) can be written as: 

\((1 + \gamma (b, y))^{1-\sigma} V^{DE} (b, y) = V^{SP} (b, y)\)
foreign assets, the economy is vulnerable to a financial crises in the next period. Since the social planner acts in a precautionary way, the welfare gains from correcting the externality increase. Welfare gains may reach 0.1 percent in the high externality region when the shock is two standard deviation below the mean. Finally, in the low externality region, financial crises become less likely and the present discounted value of correcting the externality is reduced. On average the welfare gains from correcting the externality are 0.05 percent, about 3 times as large as the benefits from introducing indexed bonds (see Durdu, 2008), which has been suggested as a policy to address Sudden Stops (see Caballero (2002)).

Our welfare analysis provides insights on how the benefits of the introduction of new financial regulation can be linked to the business cycle. There is some concern among policymakers that the introduction of financial regulation during the early stages of a financial crisis can make this regulation counterproductive. The results in this paper suggest that financial reform delivers the maximum benefits neither in crisis times nor in good times, but during periods just prior to a financial crisis.

Figure 4: Welfare Gains from Taxation of Capital Flows

The plot corresponds to 1 SD negative tradable output

Our normative analysis complements Christiano, Gust, and Roldos (2004)) and Benigno, Chen, Otrok, Rebucci, and Young (2008) who study optimal policy in a setup where credit-market access is linked to the nontradables output as in this paper. Christiano et al. (2004)
studies optimal monetary policy in response to a Sudden Stop and shows that raising the
interest rate may be optimal. The argument for this policy is to limit the depreciation of the
domestic currency that makes the borrowing constraint tighter. Benigno et al. (2008) shows
that a subsidy on nontradable goods during a Sudden Stop is welfare improving since it
helps to mitigate real exchange rate depreciation. Our optimal policy analysis differs in two
key aspects. First, we focus on how regulation should correct the credit externality. Second,
our normative analysis results suggest that optimal policy possesses a “prudential” motive
designed to reduce the incidence and severity of arriving at a Sudden Stop. On the other
hand, the policy implications of the two previous papers suggest the need of a “reactive”
policy to reduce the severity of a Sudden Stop once it has taken place.

In the context of the debate of the desirability of financial globalization. Mishkin (2006)
among others argues that domestic financial development is necessary to reap the benefits
of financial globalization. Our analysis suggests that when financial development is not
complete, countries should integrate with the global financial market, but impose a state
contingent tax on capital inflows to reduce the vulnerability to financial crises. In the long
run, however, by improving the contractual environment of credit markets, countries that
address the underlying root of the externality, i.e. the credit constraint, could obtain higher
welfare gains.

Our rationale for taxing debt differs from the argument that agents do not internalize
the fact that their private debt levels increase the country’s average cost of funds because of
its monopoly power in the financial market. Tobin (1978) also proposed a tax on capital
flows, but he claimed it was beneficial solely on the grounds that it reduced volatility per
se. In our model, our policy measure is justified as a response to a well specified externality.
Our results complement Korinek (2008b) who uses a 3 period model to illustrate how it can
be optimal to tax securities that deliver capital outflows in constrained states. While the
regulation target in Korinek (2008b) is to alter the financial structure of a given amount
of debt, our regulation target is to reduce the absolute level of debt. Interestingly, the

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17 See also Edwards (1999), Mendoza, Quadrini and Rios-Rull (2007)
18 Chile’s experience with taxes on capital inflows is generally seen as successful, see Eichengreen and
19 See Harberger (1985) and Bardhan (1967).
empirical estimations of the optimal taxes on non-contingent dollar debt in Korinek (2008b) are of similar orders of magnitudes as the ones we obtain in our DSGE framework.

6 Conclusions

Credit constraints that link a private agent’s debt to market-determined prices embody a systemic credit externality that magnifies the incidence and the severity of financial crises. When a negative shock triggers the credit constraint, agents are forced to cut consumption which reduces the price of nontradables and erodes the collateral backed by nontradables income. This in turn leads to a further decrease in consumption which tightens credit constraints even further. Consequently, the economy is plunged in a downward spiral where the drops in consumption, in the real exchange rate and in the value of collateral mutually reinforce each other. Although agents are fully rational, they fail to internalize the price effects of their individual borrowing, and hence the adverse debt-deflation amplification effects resulting from income shocks that trigger a binding credit constraint.

In our quantitative analysis, the credit externality produces a modest level of overborrowing of about 2 percentage points on average, but triples the probability of financial crises and roughly doubles the average current account and consumption reversals during the most severe episodes. These conclusions are obtained in the context of a two sector DSGE-SOE model with occasionally binding endogenous credit constraints and an event analysis of financial crises that compares the economy with and without the systemic credit externality.

Although our financial accelerator stems from a feedback loop between the real exchange rate and output, our results suggest that systemic externalities are likely to play an important role for closed economy macroeconomic models as well. A similar mechanism linking asset pricing and economic activity offers promising results to account for the fluctuations observed in the current global financial crises. We leave this issue for future research.
References


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Appendix 1

Proposition 1 Proof.

This is a proof by contradiction. Suppose the decentralized equilibrium is constrained optimal. Because the utility function is strictly concave, the social planner has a unique maximum which coincides with the decentralized equilibrium allocations. Then, using (4) and (13) yields:

$$\lambda_t^{DE} = \lambda_t^{SP} + \mu_t^{SP} \Psi_t$$  \hspace{1cm} (19)

where we denote with superscript ‘SP’ the multipliers of the social planner and with ‘DE’ those of the decentralized equilibrium. Updating this equation one period forward and taking conditional expectations at time $t$:

$$E_t \lambda_{t+1}^{DE} = E_t \lambda_{t+1}^{SP} + E_t \mu_{t+1}^{SP} \Psi_{t+1}$$  \hspace{1cm} (20)

Suppose that at time $t$, the borrowing constraint does not bind. Combining (6) and (14) we obtain:

$$E_t \lambda_{t+1}^{DE} = E_t \lambda_{t+1}^{SP}$$  \hspace{1cm} (21)

Because at time $t + 1$ the borrowing constraint binds with positive probability, comparing (21) and (20) yields a contradiction:  \hspace{1cm} $\blacksquare$

Proposition 2 Proof.

When a state contingent tax is placed on borrowing in the decentralized equilibrium, the only first order condition that changes is the Euler intertemporal condition (6) that becomes

$$\lambda_t^{DE} = \beta E_t \lambda_{t+1}^{DE}(1 + r + \tau_{t+1}) + \mu_t^{DE}$$  \hspace{1cm} (22)

Combining (13) and (14) yields:

$$\frac{u_T(t)}{\beta E u_T(t+1)} = (1 + r) + (1 + r) \frac{E_t \mu_{t+1}^{SP} \Psi_{t+1}}{E u_T(t+1)} + \frac{\mu(1 - \Psi_t)}{E u_T(t+1)}$$  \hspace{1cm} (23)
When the constraint binds for the social planner, it also binds for the decentralized equilibrium and allocations coincide. If the constraint does not bind, it follows then by construction of the tax rate (17) that combining (22) and (4) yields an equations identical to (23). 

**Appendix 2 - Numerical Solution Method**

The algorithm employed to solve for the recursive competitive equilibrium is the following:

1. Start with an initial set of Parameters \(\{\beta, \omega, \kappa\}\)

2. Generate a discrete grid for the economy’s asset of size \(NB\) and the shock state space \(NS\).

3. Conjecture a law of motion for aggregate foreign assets \(B' = H(B, y^T)\) at each point of the state space, using \(Y^N = C^N\) this implies a price given by (10)

4. Solve for the policy functions \(\{\hat{b}(b, B, y^T), \hat{c}^T(b, B, y^T), \hat{c}^N(b, B, y^T)\}\) via value function iteration in the bellman equation (11).

5. Using the decision rules in the previous step, calculate the implied transition of aggregate bonds \(\sigma^k(B, B, y^T)\). and evaluate the previous guess. If \(\sup_{B,y^T}\|\hat{b}(B, B, y^T) - \Gamma(B, y^T)\| < \epsilon\), the recursive competitive equilibrium is found. Otherwise, update \(H(B, y^T)\) with a Gauss-Seidel algorithm and go to step 4.

6. Simulate the model. If the defined model’s moments match the data, we stop. Otherwise, adjust parameters and go to step 3.

We use 80 points in the grid for individual and aggregate bonds with more points located at the beginning of the grid where the value function displays more curvature. To solve the optimization problem in step 4, we do not restrict the choice of assets to lie in the grid. We use various interpolation schemes to compute the value function at points not on the grid. Linear interpolation and Schumaker spline preserving shape do not deliver noticeable differences. In order to solve the constrained optimization problem, we first assume that the constraint is binding and calculate the marginal utility of relaxing the constraint. If
the marginal utility is positive, the solution is at the boundary. Otherwise, we solve the unconstrained optimization problem. Because it is a strictly concave optimization problem, the method guarantees that the global maximum is achieved. To accelerate the algorithm, we use Howard improvement steps and Chow and Tsitsiklis (1991). We start with a coarse grid, that we extend to a finer grid (with linear interpolation to fill the unknown values), and recompute the value function. Our results do not change noticeably with 150 grid points.