Firm Size and Pricing Policy

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Abstract: We relate the pricing policy of the firms to their size, where firm size is interpreted as the size of the clientele served by the concerned firm. We argue that a firm with a large clientele faces a more severe reputational backlash if it reneges. This allows the firm to effectively commit to its offers, leading to a unique equilibrium without delay, where the firm extracts the whole of the surplus. For smaller firms, however, the reputational effects are much less intense and, consequently, the equilibria involve reneging possibilities. In this case the equilibria are non-unique, and may involve delays as well.

Key words: Firm size, reneging, reputation.

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1 Introduction

It is seen that the pricing policies of firms vary to a large extent. Some firms charge a high, but unique price, whereas other firms charge different prices from different customers. We also find that some firms stick to one single offer which is immediately accepted, so that there is no bargaining or haggling in the usual sense. In many other cases, however, the initially quoted offer is not the final price at which the deal goes through, leading, naturally, to some delays in reaching an agreement as well. In this paper we try to provide a framework that is capable of providing a unified explanation for these phenomena.

The explanation we offer runs in terms of the size of the firms, where we interpret firm size as the number of customers serviced by the firm. We argue that a possible explanation can be built around two facts - first, that the firms may renege on their offers, and second, that the firms operate in inter-related markets and therefore such reneging may affect the reputations of the firms in the other markets where they operate. In real life bargaining, reneging is an important strategic option. As Muthoo (1990) points out "there is no sacred rule (or law) which says that if a proposal is accepted, then it must be implemented." Allowing for such reneging essentially introduces a lack of commitment. We want to model the idea that such commitment problems are much less for larger firms, and it is this difference in the ability to commit that creates the difference in the pricing behavior.

The basic idea is that reneging will create negative effects on the reputation of the firm, which clearly are larger for larger firms as it will operate on a larger number of customers. For a large enough firm this reputational effect may be large enough for the firm to effectively commit to its offers. For a smaller firm the possible reputational backlash is small, and thus reneging is possible. This opens up the possibility of sustaining different prices using a hierarchy of punishments. Finally, the non-uniqueness of the outcomes allows us to construct outcomes that involve delay as well.

We then relate our work to the existing literature. There appears to be very little work in the industrial organization literature that relates the pricing policies of the firms to their size. The works that are closest to our own are by Muthoo (1989, 1990). Muthoo (1989) examines the implications of introducing reneging in a model of one-sided offers bargaining. He shows that under such a setup, all possible splits of the cake can be sustained as subgame perfect equilibria. Muthoo (1990) carries out a similar exercise for the alternating-offers model.

The rest of the paper is organized as follows. In section 2 we describe the model. Section 3 contains the analysis and the results. Section 4 concludes. Finally, most of the formal proofs are collected together in the appendix.

2 Description of the Model

The model comprises of a single firm facing $(N+1)$ buyers, where N is some non-negative integer, possibly 0. The buyers are divided into two groups, the first group consisting of a single buyer who we call the A-agent, and the second group comprising of N buyers, denoted the B-agents. Our main interest lies in analyzing the dynamic interaction between the firm and buyer A, when the firm can renege on its offer. We assume that the firm and buyer A are bargaining over the price of a good which the firm values at 0 and the buyer values at 1. Thus in the language of bargaining theory they are essentially bargaining over a cake of size 1.

The bargaining process is modelled as an infinite horizon one-sided offers game with reneging, where only the firm is allowed to make offers. We assume that time is discrete and continues for ever, so that $t = 1, 2, \cdots$. The firm and the buyer A has a common discount factor δ. For technical reasons we assume that $1 > \delta \geq \frac{1}{2}$.[1]

[1] The fact that $\delta \geq \frac{1}{2}$ is required while proving Propositions 1 and 2.

The game between the firm and the buyer can now be described as follows. We first describe the case where $N = 0$. In each period the firm makes an offer $(x, 1 - x)$ to the buyer A, where x denotes the share of the firm and $(1 - x)$ that of the buyer. The buyer can either accept the offer, or reject it. If he rejects the offer, then the game moves on to the next period. If, on the other hand, the buyer accepts the offer, then there are two possibilities. If the firm rejects the acceptance by the buyer, then the game goes to the next period where the firm makes another offer. If the firm accepts the acceptance, then the game between the firm and buyer A ends with the implementation of this offer. If the firm and buyer A agrees on a split of $(\alpha, 1 - \alpha)$ at nth period, then the present discounted value of the payoffs are $\alpha\delta^n$ and $(1 - \alpha)\delta^n$ for the buyer.

We then model the case where $N \geq 1$. Thus there is at least one B-agent in this model. For the sake of analytical convenience, the game between the firm and the B-agents is modelled somewhat differently to that between the firm and the A-agent. This is because the two kinds of agents play two different kinds of roles in the analysis. The idea behind our formulation is to examine the outcome of the bargaining process between the firm and buyer A, in the presence of reputational effects where the reputational effects arise out of the interactions between the firm and the B-agents. We therefore assume that the B-agents are not sure about whether the firm is a reneger or not, and only has a probability distribution about it. Furthermore, this distribution is not fixed over time, and is affected by the number of reneging that occurs in the game between the firm and buyer A. Thus it is through the probability distribution, $\lambda(n)$, that reputational effects enter the analysis.

The introduction of the B-agents, however, complicates the analysis enormously and forces us to look for some simplifications. To begin with, we assume that the discount factors of the B-agents have a common value of zero. This implies that the B-agents are only concerned about the present value of their payoffs, and are not concerned about the future. We also assume that whether the firm reneges with a B-agent or not, does not affect the probability distribution of the other B-agents. In case of any reneging, the affected B-agent immediately leaves the market and is replaced by another B-agent. The result of the above two assumptions is that the game between the firm and a B-agent can be treated as an one-shot game, that has no repercussions on the future play. Thus we have a model where the bargaining between the firm and the A-agent affects the future play, through the probability distribution, but the bargaining between the firm and the B-agents does not affect the future play. This simplifies the analysis to a great extent which, even then, is complex enough.

After the informal discussion above, we are now in a position to resume the formal description of the model. Consider some period t, where the number of reneging so far is n. Clearly, there are two possible cases,
(a) where the firm has not yet reached an agreement with agent A, and $n \leq t-1$, and
(b) where the firm reached an agreement with the A-agent at some time $s < t$, and $n \leq t-2$.

Case (a). Assume that there is a pool of B-agents out of which nature selects, randomly and in every period, N agents to play with the firm. The game starts off with the bargaining between the firm and the A-agent, where this is identical to that in the case when $N = 0$. After the firm and the A-agent complete their negotiations (whether successfully or not), the firm (in the same period) simultaneously plays N games with the N agents chosen by nature. We assume that the firm and a typical B-agent bargain over the price of a good which the firm values at zero, and the buyer B values at P, where $P > 0$, so that they are effectively bargaining over a cake of size P.

We then describe the structure of the game between the firm and a typical B-agent. Nature moves first and selects the type of firms. The B-agent assumes that the firm is a reneger with probability $\lambda(n)$, and that he is not a reneger with probability $1 - \lambda(n)$, where n denotes the number of times that the firm has reneged with the A-agent. At the start of the one-shot game the firm makes an offer of x to the B-agent when the B-agent decides whether to accept the offer or not. If he rejects the offer then the game ends here and the B-agent goes back to the pool of potential partners for the firm. If he accepts the offer, then he makes an irreversible investment of ϵ.[2]

In case the firm is not a reneger, the game ends immediately and the offer is implemented. Thus the net payoff of the firm is x, and that of the B-agent is $P - x - \epsilon$. In case the firm is a reneger it can renege on his earlier offer, and offer another price of x' to the B-agent. If the B-agent accepts, then the game ends with a payoff of x' for the firm, and $P - x' - \epsilon$ for the B-agent. Otherwise, the net payoff vector is $(0, -\epsilon)$. In either case, once the firm reneges, the concerned B-agent leaves the market.[3]

Case (b). In this case in period t, nature selects N of the B-agents to play with the firm. The firm then plays N simultaneous bargaining games with these B-agents. The structure of these games are identical to that described in case (a) above, and hence omitted.

We now make some assumptions regarding the function $\lambda(n)$. It is in-

[2]The irreversibility can be attributed to relation specific investments discussed in the literature on asset specificity, where asset specificity refers to the degree to which an asset can be redeployed in alternative uses. Among examples of such asset specificity we can mention physical asset specificity, human asset specificity, site specific asset specificity, etc. For a discussion of these issues in the context of vertical integration by firms we refer the readers to Klein et. al. (1983).

[3]This assumption ensures that the outcome in the one-shot game has no effects on the probability distribution. An alternative formulation could have been where the B-agent stays in the market, but he can affect the probability distribution of only a finite number of the other agents, say his neighbors. Thus, under a random selection mechanism, these agents will be selected with a zero probability and we have the same outcome under this formulation as well.

tuitively clear that this function should be increasing in n, the number of reneging. Our next assumption is a crucial one that drives much of the subsequent analysis, specifically Proposition 3. This assumption states that if the firm reneges a sufficiently large number of times, then the B-agents become convinced that the firm is a reneger.

Assumption 1. $\lim_{n \to n} \lambda(n) = 1$.

Notice, however, that we make no assumptions about the rate at which $\lambda(n)$ increases. In fact, the probability distribution $\lambda(n)$ can be constant in n, the number of reneging, for *any* finite number of times at the beginning of the game. All we require is that *ultimately*, and in the face of *overwhelming* evidence, the B-agents become convinced as to the real nature of the firm.

In the next section we provide a formal analysis of this game.

3 Analysis of the Model

We analyze the two cases where $N = 0$, and where $N \geq 1$, separately in two different sub-sections.

3.1 $N = 0$

To begin with, we specialize to the case where $N = 0$. In this case the problem reduces to one of bargaining between the firm and a single agent, buyer A. Our first proposition shows that any price p, where $p \in [0, 1]$, can be supported as a subgame perfect equilibrium of this game. The proof essentially follows from Propositions 1, 2 and 3 in Muthoo (1989) and can be found in the appendix. Here we just provide a sketch of the proof. The first step is to demonstrate that both the prices 1 and 0 can be sustained as subgame perfect equilibria. The strategies used in sustaining these equilibria employ a hierarchy of punishments where, if the firm deviates from the equilibrium offer, then the buyer rejects this offer and punishes him for the deviation, because otherwise the firm will penalize the buyer for not punishing him. These extreme prices are then used as threat outcomes to sustain any other price as subgame perfect equilibria.

Proposition 1. For $N = 0$, any price p, where $p \in [0, 1]$, can be sustained as subgame perfect equilibria.

Clearly, at a first glance the above proposition appears counter-intuitive. After all it is well known that in the one-sided offers game without reneging there is a unique outcome where the firm obtains the whole of the surplus. And here we have a game where the strategy space of the firm is larger in the sense that he can renege on his offer something he was not allowed to do in the game without reneging. Therefore, we may argue, since his strategic position is no worse, his payoff should also be no worse. But as the proposition demonstrates, the payoff of the firm may be strictly less than 1, and even zero in the reneging game. The key to this apparent paradox lies in the observation that the firms are not really in a stronger bargaining situation in this game. What the introduction of the reneging option does is to take away the ability of the firm to commit to an offer, thus *weakening* its bargaining power. It is this loss of commitment ability that explains the fact that in this game the firms may obtain less than what they were getting before.

We then show that we can sustain an outcome that involves a delay of one period, where the delay arises because the firm refuses to accept an acceptance by the buyer. Notice that since $\delta > \frac{1}{2}$, the interval $(\frac{1-\delta}{\delta}, 1]$ in Proposition 2 below is well defined.

Proposition 2. We can sustain any price p, where $p \in (\frac{1-\delta}{\delta}, 1]$, with a *delay of one period. In this case the first period outcome involves the firm making an offer which the buyer accepts but the firm refuses to accept the acceptance.*

The formal proof can be found in the appendix. One way to interpret the result is as follows. It is as if the firm is not certain about the price that the consumer can bear. The firm therefore starts out by offering a relatively modest price for the good. Once the offer is accepted, however, the firm gains in confidence and, after reneging on its earlier offer, demands a much higher price at which the deal is closed. Of course, from a social viewpoint such an outcome leads to a loss arising out of the delay in implementation. We can interpret this loss as a manifestation of the coordination problems intrinsic to games with multiple equilibria.

3.2 $N \geq 1$

We then move on to the case where the firm has a relatively large clientele.

We begin by analyzing the game between the firm and a typical B-agent.

Working backwards, we find that the expected profits of a B-agent, if he accepts an offer of $(x, P - x)$ is $(1 - \lambda(n))(P - x - \epsilon) - \lambda(n)\epsilon$, where recall that the B-agents attach a probability of $\lambda(n)$ that the firm is a reneger . Thus he accepts the offer if and only if

$$(1 - \lambda(n))(P - x - \epsilon) - \lambda(n)\epsilon \geq 0,$$

i.e. $P - x \geq \frac{\epsilon}{1-\lambda(n)}$.

Thus the firm initially offers a price of x^*, where $P - x^* = \frac{\epsilon}{1-\lambda(n)}$, which the B-agent accepts. Once the B-agent makes the investment, however, the firm reneges and makes an offer of P, which in equilibrium the B-agent accepts. Thus the net payoff of the firm is P and that of the B-agent is $-\epsilon$. This case holds whenever $\lambda(n)$ is not too high. If $\lambda(n) > \lambda^* = 1 - \frac{\epsilon}{P}$, then there does not exist any price for which the B-agent accepts the offer and the firm makes a non-negative payoff. The outcome thus involves the firm making an unacceptably high offer, which the B-agent rejects.

Next notice that with a large number of agents the reputational effect is likely to be large so that it may be possible for the firm to commit to its offer. In fact, Proposition 3 shows that if the size of the clientele, N, is large enough then a unique equilibrium exists. In this equilibrium there is no delay and the firm extracts the whole of the surplus.

Proposition 3. There exists an $\hat{N}$ *such that, for all* $N \geq \hat{N}$*, the game between the firm and the buyer* A *has a unique outcome where there is no delay, the firm makes an offer of* $(1, 0)$ *which the buyer accepts, and the firm accepts the acceptance.*[4]

The basic idea of the proof is quite simple. Consider a stage of the game where the firm has reneged often enough so that if the firm reneges once more, then he loses all the B-agents as clients. This threat is sufficient to ensure that the firm is not going to renege any more, and thus the firm can, effectively, if not in reality, commit to its offers. We can use this fact to construct an argument which shows that the outcome in this particular subgame is going to be unique and involves the firm obtaining the whole of the surplus. This allows us to work backwards from this particular stage, and finally an inductive argument shows the outcome is unique at all previous stages as well.

Finally, notice that we do not provide a complete characterization of the outcomes in terms of the size of the clientele. We conjecture that there is an N^*, such that for all $N \leq N^*$, the outcome is non-unique whereas, for

[4] Notice that the proof of Proposition 3 does not depend on the fact that $\delta \geq \frac{1}{2}$. This is in contrast to Propositions 1 and 2.

all $N > N^*$, the outcome is unique. However, since such a characterization promises little further economic intuition, we refrain from attempting such an exercise.

4 Conclusion

In this paper we relate the pricing policy of the firms to their size, where firm size is interpreted as the size of the clientele served by the concerned firm. We argue that a firm with a large clientele faces a more severe reputational backlash if it reneges. This allows the firm to effectively commit to its offers, leading to a unique equilibrium without delay, where the firm extracts the whole of the surplus. For smaller firms, however, the reputational effects are much less intense and, consequently, the equilibria involve reneging possibilities. In this case the equilibria are non-unique, and may involve delays as well. Most of these results appear to parallel our real life experience, and thus serves to deepen our understanding of these phenomena.

We also take this opportunity to briefly return to the question of why the A- and the B-agents are treated differently in our model. In the light of our analysis it is clear that the important assumption that drives the argument is that the firms operate in more than one markets, and what happens in one market may affect the outcomes in the other markets, thus affecting the behavior of the firms in the first market itself. Here we have chosen to represent this other market as another set of customers who are also purchasing some goods from the firm. It should be possible to conduct the analysis in terms of other inter-related markets where the firm operates, say finance, or the raw materials market, and come up with substantially the same answers.

The results derived in this paper are also of interest from a purely game theoretic viewpoint. As Muthoo (1989, 1990) demonstrates, in the presence of reneging effects, the uniqueness of subgame perfect equilibrium, on of the main reasons for the interest commanded by the bargaining literature, is lost. (In fact Muthoo (1995) demonstrates that as long as there is some positive probability that the offers may be retracted the non-uniqueness result goes through.) Our analysis shows that the non-uniqueness problem is intimately tied up with the reputational effects of reneging, and, under some conditions at least, the uniqueness of perfect equilibrium can be restored. Of course, if the reputational effects are not very strong, then the non-uniqueness problem persists.

5 Appendix

Proof of Proposition 1. The proof proceeds in three steps.

Step 1. We begin by showing that the price $p = 1$ can be sustained as a perfect equilibrium outcome.

Consider the following strategies. For any history of the game, the firm offers the price $p = 1$, the buyer accepts any price $p \leq 1$ and the firm accepts those acceptances of the buyer such that the price offer is $p \geq \delta$.

Step 2. We then show that a price of $p = 0$ can be sustained in the equilibrium.

We then use the language of 'states' and 'transitions' to describe the perfect equilibria supporting a price of 1, where an agent's action at any node is specified as a function of the state prevalent at that node. Let us define two states s_a and s_b as follows, where state s_b is absorbing. We then describe the transition rules as follows.

Transition to s_a occurs if either $t = 1$, or s_a prevails, and one of the following happens:
(i) an offer of $p = 0$, or $p \geq 1 - \delta$ is either rejected by the buyer, or accepted by the buyer and rejected by the firm, or
(ii) an offer of $0 < p < 1 - \delta$ is rejected by the buyer.

Transition to s_b occurs if an offer of $0 < p < 1 - \delta$ is accepted by the buyer and rejected by the firm.

The strategies are now defined as dependent on the prevailing states.

In state s_a, the firm offers a price of $p = 0$, and accepts an acceptance by the buyer if and only if $p = 0$, or $p \geq 1 - \delta$. The buyer accepts an offer if and only if $p = 0$.

In state s_b, the two agents play the strategies supporting a price of 1.

The reason we require that $\delta \geq \frac{1}{2}$ is the following. Suppose a price $0 < p < 1 - \delta$ is offered and accepted by the buyer. The strategy of the buyer suggests that the firm should reject this acceptance by the buyer. By doing so the game goes into state s_b, and the firm obtains a payoff of δ. If it deviates and instead accepts the acceptance by the buyer, the payoff of the firm would equal p where $1 - \delta > p > 0$. Since $\delta \geq \frac{1}{2}$, and $p < 1 - \delta$, it follows that $p < \delta$. Therefore, deviating from its strategies is not sensible for the firm.

Step 3. We then show that any price $\hat{p} \in (0, 1)$ can be sustained as perfect equilibria.

We define the three states S_A, S_B and S_C, where S_B and S_C are absorbing. The transitions can be defined as follows.

Transition to S_A occurs if $t = 1$, or the state S_A prevails and one of the following happens:

(i) an offer of $p \leq \hat{p}$ is either rejected by the buyer, or accepted by the buyer and rejected by the firm, or

(ii) an offer of $\delta \geq p \geq \hat{p}$ is rejected by the buyer.

Transition to S_B occurs if an offer of $p > \max[\delta, \hat{p}]$ is rejected by the buyer.

Transition to S_C occurs if an offer of $p > \hat{p}$ is accepted by the buyer, but rejected by the firm.

The strategies are now defined as follows.

In state S_A the firm offers a price of $\hat{p}$, and accepts an acceptance by the buyer if and only if either $p > \max[\delta, \hat{p}]$, or $\hat{p} \geq p \geq \delta\hat{p}$. The buyer accepts price offers such that $\delta\hat{p} \leq p \leq \min[\hat{p}, 1-\delta(1-\hat{p})]$ and rejects otherwise.

In state S_B the two agents play the strategies supporting the price $p = 0$.

In state S_C the two agents play the strategies supporting the price $p = 1$.

It is entirely straightforward to check that these strategies, in fact, constitute subgame perfect equilibria of our game. ■

Proof of Proposition 2. The proof consists of writing down the strategies supporting the given outcome. Consider some $\overline{p} \in (\frac{1-\delta}{\delta}, 1]$. We claim that the following strategies support this price with a delay of one period.

In period 1.

The firm offers a price of y in period 1, where $y < \delta\overline{p}$. The buyer accepts an offer of x if and only if $x < \delta\overline{p}$. The firm accepts the acceptance of an offer x by the buyer if and only if $x > \delta$.

In period 2. In period 2 three possible states S^A, S^B and S^C, can occur.

State S^A occurs if the preceding price was $p < \delta\overline{p}$, which the buyer accepts and the firm rejects.

State S^B prevails if either (i) the preceding price $p < \delta\overline{p}$ was rejected by the buyer, or (ii) the preceding price $p \geq \delta\overline{p}$ was accepted by the buyer and rejected by the firm.

State S^C prevails if the preceding price $p \geq \delta\overline{p}$ was rejected by the buyer.

The strategies are as follows. In state S^A the two agents play the strategies supporting a price of $\overline{p}$. In state S^B they play the strategies supporting a price of 1, and in state S^C they play the strategies supporting a price of 0.

We then check that the above strategies do form a subgame perfect equilibrium. For this purpose it is clearly sufficient to consider the strategies in period 1. (That the strategies in period 2 constitute perfect equilibria follow from Proposition 1.)

The firm's offer: Suppose that the firm offers a price of x, where $x \geq \delta\overline{p}$.

The buyer rejects the offer, the game goes to state S^C, and the present discounted value of the firm's payoff is 0. Whereas, if the firm offers a price of x, where $x < \delta\overline{p}$, then the present discounted value of its payoff is $\delta\overline{p} > 0$.

The buyer's decision: If the buyer accepts a price of x, where $x \geq \delta\overline{p}$, then the firm accepts and the buyer's payoff is $1 - x \leq 1 - \delta\overline{p}$. If he rejects, however, then the game goes to state S^C and the present discounted value of his payoff is δ. Since $\delta > 1-\delta\overline{p} \geq 1-x$, it is better for the buyer to reject. We then consider the response of the firm to a price of x, where $x < \delta\overline{p}$. If he accepts, then the firm rejects and with the game going to state S^A, the present discounted value of the buyer's payoff is $\delta(1 - \overline{p})$. If he rejects then the game goes to S^B and his payoff is 0. Thus it is better for the buyer to accept.

The firm's acceptance-rejection decision: Suppose that the firm accepts a price of x. There are now three cases to consider.

(1) $x < \delta\overline{p}$. If the firm accepts the acceptance then its payoff is x. Whereas if it rejects then the game goes to state S^A and the present value of its payoff is $\delta\overline{p} > x$.

(2) $\delta\overline{p} \leq x < \delta$. If the firm accepts the acceptance then its payoff is x. Whereas, if it rejects, then the game goes to state S^B, and it obtains a discounted payoff of $\delta > x$.

(3) $x \geq \delta$. Clearly, by rejecting the acceptance, the most that the firm can obtain is δ, and it is therefore better to accept. ■

Proof of Proposition 3. Let $\hat{N}$ be such that $\frac{P\hat{N}}{1-\delta} > 1$. Then the number of B-agents at any given period is $N \geq \hat{N} > \frac{1-\delta}{P}$.

The proof proceeds via induction. We first consider the case where the firm has already reneged n times, where n is such that $\lambda(n) \leq \lambda^* < \lambda(n+1)$. Now suppose that the supremum of the A-agent's payoff in any subgame perfect equilibrium of this game is z, and suppose that $z > 0$. We then consider two cases depending on whether the supremum is being reached or not.

Case 1. Suppose that the supremum is being reached. To begin with, observe that no subgame perfect equilibrium in the subsequent game can involve the firm reneging on an offer to buyer A. If it does renege, then the loss in payoff from doing so is at least $\frac{NP}{1-\delta}$, which outweighs any possible gains from such an action.

In this case there is an equilibrium where the firm offers a price of $1-z$ and the A-agent accepts. We show that there is an offer that the buyer will accept and the firm gains as well, thus leading to a contradiction. Consider the case where the firm offers a price of $1-q$ such that

$$q > \delta z, \tag{1}$$

$$1-q > 1-z. \tag{2}$$

Clearly, from equation (1) the A-agent accepts and from equation (2) the firm gains as well.[5]

Case 2. Suppose that the supremum is not being reached. In that case we can always find an equilibrium where there is no delay, no rejection by the firm of an accepted offer, and where the payoff of the A-agent, say s, is close enough to z, or to be more precise where $s > \delta z$. We can now apply the argument given in case 1 above to arrive at a contradiction. Thus $z = 0$. Now if the firm offers a price of $p = 1$, the A-agent is going to accept.

The strategies supporting the above equilibrium are as follows. The firm offers a price of 1 at every period, and accepts all acceptances by the A-agent provided the payoff of the firm is at least δ. The A-agent accepts all offers by the firm.

We then use an induction hypothesis to complete the proof.

The Induction Hypothesis: If the firm has already reneged for x times

[5]Notice that such an offer exists since we can always find a q such that $z > q > \delta z$.

with buyer A, where $x \leq n$, then the game between the firm and buyer A has a unique perfect equilibrium where the price is 1.

We consider the subgame where the firm has already reneged for $(x-1)$ times with buyer A. Suppose that the supremum of the A-agent's payoff, in any subgame perfect equilibrium of the game, is z, and that $z > 0$. We again consider two cases depending on whether the supremum is being reached or not.

Case 1. Suppose that the supremum is actually being reached. Thus there exists some subgame where there was no further reneging by the firm, the firm offers a price of $1 - z$ to buyer A, and the offer is accepted. Now consider the deviant offer $1 - z'$ such that

$$\begin{aligned} z' &> \delta z, & (3) \\ 1 - z' &> 1 - z \geq \delta. & (4) \end{aligned}$$

Now if buyer A accepts the offer, will the firm accept the acceptance? In case of acceptance, the payoff of the firm is $(1 - z') + \frac{NP}{1-\delta}$. Whereas if it rejects the offer its payoff is, at most, $\delta + \frac{NP}{1-\delta}$. Since $1 - z' \geq \delta$, it will accept the acceptance.[6] Therefore, the buyer A accepts the offer and the firm gains.

Case 2. If the supremum is not being reached we can again make an argument similar to that above.

Thus $z = 0$, and the outcome must involve a price of 1 being agreed upon without any delay. ■

[6]Suppose that $1 - z < \delta$, then the firm should renege on the offer. Since, by the induction hypothesis, the firm obtains 1 in the next period while bargaining with buyer A, and the outcomes of the games with the B-agents are not affected, it gains.

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