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27 April 2009

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* This work is partially supported by Research Grants HP2007-0022; SEJ2006-15401-C04-01 (Ministerio de Ciencia y Tecnología and FEDER), SA0870A08 (Junta de Castilla y León), and PGIDIT07PXIB300095PR (Xunta de Galicia).

Abstract. We consider a pure exchange economy with a finite set of types of agents which have incomplete and asymmetric information on the states of nature. Our aim is to describe the equilibrium price formation and how the lack of information may affect the allocation of resources. For it, we adapt to an asymmetric information scenario a variant of the Shapley-Shubik game introduced by Dubey and Geanakoplos (2003).

JEL Classification: C72; D51

Keywords: Competitive equilibrium, differential information, Nash equilibrium.

1 Introduction

We consider a pure exchange economy with a finite number of types of agents and commodities. The economy extends over two periods and agents arrange contracts at the first period that may be contingent on the realized state of nature in the second period. Agents have incomplete information on a finite set of the states of nature and this information may differ across agents (differential information economies). After the realization of the state of nature, a particular agent do not necessarily know (or is unable to prove in a court of law) which state of nature has actually occurred, (because, for example, she receives a signal that may be identical for different states). Therefore they are restricted to sign contracts that are compatible with their private information.

For these economies, Radner (1968) defined and established the existence of a notion of Walrasian expectations equilibrium (or Radner equilibrium), an analogous concept to the Walrasian equilibrium in Arrow-Debreu model with symmetric information.

Recently, there has been a resurgent interest on Walrasian expectations equilibrium and, in parallel to the Arrow-Debreu model, questions concerning the existence and characterization of Radner equilibrium by means of cooperative solutions has been obtained (see Allen and Yannelis, 2001, Einy, Moreno and Shitovitz, 2001, Hervés-Beloso, Moreno-García and Yannelis, 2005a and Hervés-Beloso, Moreno-García and Yannelis, 2005b).

It is important to notice that in sharp contrast with others asymmetric information models, as for example the rational expectations equilibrium model (Radner (1979)), in the framework of Walrasian expectation equilibrium, it is assumed that prices do not reveal any private information ex ante. They rather reflect agents' informational asymmetries since they have been obtained by maximizing utility taking into account the private information of each agent.

Our aim in this paper is to use a market-game approach to study the behavior of these markets and to analyze the mechanism of price formation.

The wide literature on market games uses the principles of game theory to motivate or justify the description of markets in which certain behavioral characteristics, such us price-taking behavior, are assumed. Most of these works show how strategic interactions by rational agents leads to a competitive equilibrium situation. One of the advantages of building a strategic foundation for perfect competition is that we will be forced to describe the process completely and explain how the market equilibrium is reached.

We remark that equilibrium price formation for economies with differential information become particularly interesting due to the role which prices are called to play in this scenario. As prices may differ in states of nature that an agent does not distinguish, prices affect not only the allocation of wealth but also could affect the private information of the agent.

In order to explain the equilibrium price formation, we adapt a variant of the Shapley-Shubik game introduced by Dubey and Geanakoplos (2003). We describe new rules for price formation and the corresponding allocations, which underly the differentiated information structures. First, we define a market game where the formation of price does not support any informative role of prices. This mechanism allows to observe the equilibrium price formation and how the lack of information may affect the allocation of wealth.

One objection to the Radner model is that, since prices prices may be different in states of nature that an agent does not distinguish, a price system may refine the private information of such consumer. With this concern in mind, we describe a new game where the mechanism specifying the price formation leads to a particular price system which is compatible with the common information structure. We refer to this refinement as non-revealing Radner equilibrium. (See Faias and Moreno-García (2008)). A non-revealing equilibrium price system, for which we show an existence result, avoids this criticism and gives consistency to the model.

In fact, for both types of games we show existence of Nash equilibrium and then the corresponding market equilibrium solutions (that is, Radner or Walrasian expectations equilibrium and non-revealing Radner equilibrium, respectively) are obtained as a limit of a sequence of Nash equilibria. We remark that these limit results provide new existence proofs for both, Radner and non-revealing equilibria.

Our assumptions are the same as in Radner (1968), however, in order to obtain the existence of the refinement called non-revealing Radner equilibrium, we require the compatibility of the total endowment allocation with the common information. This additional assumption is automatically fulfilled when the initial endowments do not depend on states of nature.

Regarding related work, Fugarolas et al. (2009) also undertake a non-cooperative approach to differential information economies by extending Schmeidler's (1980) work to the differential information setting. However, as in Schmeidler's result, the existence of Nash equilibrium is obtained as a consequence of the existence of the Walrasian expectations equilibrium and, since prices are included in the strategy sets, no explicit price formation rule is obtained.

The remaining of the paper is organized as follows. In Section 2 we describe the differential information model and the notion of market equilibrium. In Section 3 we state an associated game a la Shapley-Shubik and we prove existence of Nash equilibrium. In Section 4 we prove that the limit of a sequence of Nash equilibria results in a Radner equilibrium. In Section 5, we define a new game where the price resulting from the interaction among consumers are compatible with the common information structure. For this game we also show existence of Nash equilibrium and, finally, a non-revealing equilibrium is obtained as limit of a sequence of Nash equilibia.

2 The model

Let us consider an economy $\mathcal{E}$ with differential information. Let Ω be the set of states of nature that describes the uncertainty. Ω is finite with cardinality k and there is a finite number of goods, L, in each state. There is a a continuum of agents that trade the L commodities at each state of nature $\omega \in \Omega$.

The private information structure of each agent is described by a partition of the set of states Ω. Given a partition $\mathcal{P}$ of Ω, a commodity bundle $x = (x(\omega))_{\omega\in\Omega} \in (\mathbb{R}^{\ell}_{+})^{k}$ is said to be

$\mathcal{P}$-measurable when it is constant on the elements of the partition .[1]

The set of agents is represented by the unit real interval $I = [0,1] = \bigcup_{i=1}^{n} I_i$, where $I_i = \left[\frac{i-1}{n}, \frac{i}{n}\right)$, if $i \neq n$, and $I_n = \left[\frac{n-1}{n}, 1\right]$. We consider the Lebesgue measure μ on the Borel subsets of I. Each agent $t \in I_i$ is characterized by her private information $\mathcal{P}_t = \mathcal{P}_i$, her initial endowments $e_t = e_i \in \mathbb{R}_+^{Lk}$ and preference relation over the consumption space, which is represented by a utility function $U_t = U_i : \mathbb{R}_+^{Lk} \to \mathbb{R}_+$. We will refer to agents belonging to the subinterval I_i as agents of type i.

The economy lasts for two periods $\tau = 0, 1$. Consumption takes place at $\tau = 1$. At $\tau = 0$ there is uncertainty about the states of nature and the agents make contracts (agreements) that are contingent on the realized state of nature that occurs at $\tau = 1$. Thus, the contracts are specified ex-ante.

An agent $t \in I_i$ with information given by the partition $\mathcal{P}_i$ is not able to distinguish those states of nature that are in the same element of $\mathcal{P}_i$. Given a sate $\omega \in \Omega$, let $E_i(\omega)$ denote the event in the partition $\mathcal{P}_i$ which contains the state ω. We say that a consumption bundle $x \in (\mathbb{R}_+^L)^k$ is compatible with the information of agents of type i if, given any state ω, we have $x(\omega) = x(\omega')$ for every $\omega' \in E_i(\omega)$. Let us denote by $\mathcal{X}_i$ the set which consists in the bundles that are compatible with the information structure of agents of type i. That is,

$$\mathcal{X}_i = \left\{ x \in (\mathbb{R}_+^L)^k | x \text{ is } \mathcal{P}_i\text{-measurable} \right\}.$$

We state the following assumptions:

(U) For every i the utility U_i is a continuous, concave and monotone[2] function.

(E) $e_i \gg 0$ and $e_i \in \mathcal{X}_i,\ i = 1, \ldots, n$. That is, every agent is initially endowed with strictly positive amounts of every commodity and e_i is $\mathcal{P}_i$-measurable for every type i of consumers.

An allocation x is a μ-integrable function that associates to each agent t a consumption bundle x_t. We refer to an allocation x as *physically feasible* if $\int_I (x_t - e_t) d\mu(t) \leq 0$, and as *informationally feasible* if $x_t \in \mathcal{X}_i$, for every $t \in I_i$ and every i. A *feasible allocation* is both physically and informationally feasible.

Each agent $t \in I$ behaves as a price-taker and maximizes her utility functions restricted to the allocations in her budget set. Given a price system $p \in \mathbb{R}_+^{Lk}$ that specifies a commodity price $p(\omega) \in \mathbb{R}_+^L$ at each state $\omega \in \Omega$, the budget set of an agent of type i is given by

$$B_i(p) = \{x \in \mathcal{X}_i \mid \sum_{\omega \in \Omega} p(\omega) \cdot (x(\omega) - e_i(\omega)) \leq 0\}.$$

Next we define a competitive equilibrium notion in the sense of Radner where traders must balance the budget ex-ante. For it, we stress that although commodity prices, that agents

[1]That is, $x(\omega) = x(\omega')$, for all $\{\omega, \omega'\} \subseteq S$, for some $S \in \mathcal{P}$.

[2]$x \gg y$ implies $U_i(x) > U_i(y)$

take as given, can be different across the states of nature, the market cannot communicate any information through the price system .[3]

Definition 2.1 *A pair (p, x), where p is a price system and x is a feasible allocation, is a competitive or a Radner equilibrium if the bundle x_t maximizes U_t on $B_t(p)$, for almost all $t \in I$.*

Radner equilibrium is an ex ante concept. Notice that we assume free disposal. It is well known that if we impose the condition of non-free disposal then a Radner equilibrium might not exist with positive prices (see, for example, Glycopantis, Muir and Yannelis (2003)). However, allowing for negative prices one can dispense with the free disposal assumption.

Finally, given our atomless economy $\mathcal{E}$, let us consider an economy $\mathcal{E}_n$ with a finite number n of agents. In the differential information economy $\mathcal{E}_n$ each agent i is characterized by an initial endowments e_i, the utility funtion U_i and a private information structure given by the partition $\mathcal{P}_i$. We have that if (p, x) is a competitive equilibrium for the continuum economy $\mathcal{E}$ then (p, z) is a competitive equilibrium for $\mathcal{E}_n$, where the allocation $z = (z_i, i = 1, \dots n)$ is given by $z_i = \dfrac{1}{\mu(I_i)} \displaystyle\int_{I_i} x_t d\mu(t)$. Reciprocally, if (p, z) is a competitive equilibrium for the economy $\mathcal{E}_n$ with n consumers, then (p, x) is a competitive equilibrium for $\mathcal{E}$ where x is the step function given by $x_t = z_i$ for every consumer $t \in I_i$. Therefore, if we consider an economy with n consumers associated to the n-type continuum economy then the equilibrium solutions for the continuum and the discrete approach are equivalent (see Hervés-Beloso et. al, 2005, for details)

3 An Associated Game a la Shapley-Shubik

Following Shapley-Shubik (1977) approach each commodity in each state of nature is traded at a trade-post, so there is a post for each commodity in each state of nature. Each consumer deliver to the post the endowment of commodity ℓ in each state $\omega \in \Omega$ for sale and fiat money to purchase the consumption goods. Consumers place their entire initial bundle for sale and then each post for commodity ℓ in the state ω receives the corresponding total endowment in the economy, i.e., $e^\ell(\omega) = \displaystyle\int_I e_t^\ell(\omega) d\mu(t)$. As in Dubey and Geanakoplos (2003) and in order to trigger the market we assume that and external agent place 1 unit of fiat money at each post.

[3]As in Maus (2004), we may argue that agents do not infer any new information from prices. Consumers observe prices according to their action possibilities, which are determined by their private information. Consider an agent with information given by $\mathcal{P}$ and let E be an event, that is, E is a subset of state that such a consumer cannot differentiate. Let $\#E$ denote the cardinality of the event E. Then this agent perceives the price system p under her information $\mathcal{P}$ as $(p(E))_{E \in \mathcal{P}_i}$, with $p(E)$ representing the same observed price in each state belonging to the event $E \in \mathcal{P}_i$, given by the average price $\dfrac{1}{\#E} \displaystyle\sum_{w \in E} p(\omega)$.

The trading-posts and the bank, that borrows at zero price the fiat money and put in each trading post one unit of money, are dummy players. They have no choices to make and so, they do not optimize.

In our scenario, consumers deliver to a central post the endowment of every commodity ℓ in each state $\omega \in \Omega$ for sale and, in addition, the individuals choose strategies that precise the amount of fiat money to purchase the corresponding consumption goods. Since a particular consumer may be not able to distinguish all the sates of nature, the central post (playing also the role of an additional dummy agent) deliver in each of the $k \times l$ trading posts the corresponding initial endowments and fiat money of each agent.

The strategic variable of each agent is the amount of fiat money that she wants to spend in each commodity. Precisely, to purchase commodity ℓ at the state ω each agent t deliver to the post fiat money $\theta_t^\ell(\omega)$ that she borrows at zero interest. In order to have compact strategy sets we impose an upper bound on borrowing. Thus, each agent can not borrow more than M units of fiat money. Therefore, the strategy set of each consumer $t \in I$ is given by the set

$$\xi_t(M) = \xi(M) = \{\theta \in \mathbb{R}_+^{Lk} \text{ such that } \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} \theta^\ell(\omega) \leq M\}.$$

We remark that, in this setting, the role of money is just “means of payment”.

Prices are determined by the actions of traders. Actually, given a strategy profile $\Theta = (\theta_t, t \in I)$, the price for each commodity ℓ in each state of nature $\omega \in \Omega$ arises in each post according to the next rule:

$$p^\ell(\omega) = \frac{\theta^\ell(\omega) + 1}{e^\ell(\omega)} > 0,$$

where $\theta^\ell(\omega) = \displaystyle\int_I \theta_t^\ell(\omega) d\mu(t)$. Let $p(\Theta) = (p^\ell(\omega),\ \omega \in \Omega,\ \ell = 1, \ldots, L)$.

Each agent t receives a bundle compatible with her information structure which means that the consumption bundle is constant in the states that belong to the same event. Let us consider an agent $t \in I_i$, and recall that $E_i(\omega)$ denote the event in the partition $\mathcal{P}_i$ which contains the state ω. The amount of commodity ℓ assigned to an individual $t \in I_i$ in the sate ω is given by:

$$x_t^\ell(\omega) = \min\left\{\frac{\theta_t^\ell(\omega')}{p^\ell(\omega')},\ \omega' \in E_i(\omega)\right\}.$$

Let $x_t(\Theta) = (x_t^\ell(\omega),\ \omega \in \Omega,\ \ell = 1, \ldots, L)$ be the bundle allocated to consumer t when Θ is the strategy profile.

The agent $t \in I_i$ also receives money from the sale of her endowment, thus, his net deficit is given by,

$$d_t(\Theta) = \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} \theta_t^\ell(\omega) - \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} p^\ell(\omega) e_t^\ell(\omega).$$

The payoff of each agent $t \in I_i$ for each strategy profile Θ is,

$$\Pi_t(\Theta) = U_i(x_t(\Theta)) - d_{t+}(\Theta),$$

where $d_{t+} = \max\{0, d_t\}$. The use of maximum to define the payoff function means that agents do not ascribe utility to fiat money, but are penalized in the case of default.

Now, let us show that the mechanism that we propose guarantees that for every strategy profile the resulting allocation of commodities is feasible, that is, physically and informationally feasible. Note that the informational feasibility follows trivially since for each agent the mechanism assigns the same bundle in states that belong to the same event of the private partition. The final allocation for agents is physically feasible, in fact, for every commodity $\ell = 1, \dots, L$ and every state $\omega \in \Omega$, the following inequality holds

$$\int_I x_t^\ell(\omega) d\mu(t) \le \int_I \frac{\theta_t^\ell(\omega)}{p^\ell(\omega)} d\mu(t) = \int_I \frac{\theta_t^\ell(\omega)}{\theta^\ell(\omega) + 1} e^\ell(\omega) d\mu(t) \le e^\ell(\omega).$$

Let $\mathcal{G}(M) \equiv \{(\Pi_t, \xi_t(M)) = \xi(M), t \in I\}$ denote the game previously described. Given a strategy profile $\Theta : I \to \xi(M)$ we denote by $\Theta \setminus \alpha_t$ the strategy profile which coincides with Θ except for player t who chooses α_t instead of $\Theta(t)$. A strategy profile Θ is a Nash equilibrium in the game $\mathcal{G}(M)$ if for almost all $t \in I$ we have $\Pi_t(\Theta) \ge \Pi_t(\Theta \setminus \alpha_t)$ whatever $\alpha_t \in \xi(M)$ may be.

Before showing a Nash equilibrium existence result for the game $\mathcal{G}(M)$, we state a Lemma obtaining a property of these equilibria that will be used in the convergence result presented in the next section.

Lemma 3.1 *If the profile $\Theta = (\theta_t, t \in I)$ is a Nash equilibrium for the game $\mathcal{G}(M)$, then for every commodity ℓ and type i we have $\dfrac{\theta_t^\ell(\omega)}{p^\ell(\omega)} = \dfrac{\theta_t^\ell(\bar\omega)}{p^\ell(\bar\omega)}$ for any $\omega \in E_i(\bar\omega)$ for almost all $t \in I_i$.*

Proof. Assume that the statement of the Lemma does not hold. Then there exist a Nash equilibrium $\Theta = (\theta_t, t \in I)$ and a positive measure set J of agents of a type j such that, for every $t \in J \subset I_j$ one has $\dfrac{\theta_t^\ell(\omega)}{p^\ell(\omega)} \neq \dfrac{\theta_t^\ell(\bar\omega)}{p^\ell(\bar\omega)}$ for some commodity ℓ and some states ω and $\bar\omega$ such that $\omega \in E_j(\bar\omega)$[4] For each $t \in J$, and each commodity ℓ let A_t^ℓ be the set of states at which the minimum of $\left\{\dfrac{\theta_t^\ell(\omega)}{p^\ell(\omega)} \text{ with } \omega \in E_j(\bar\omega)\right\}$ is attained. Recall that one player is not able to alter the price by modifying her strategy unilaterally. For each $t \in J$ let us consider a strategy α_t given by

$$\alpha_t^\ell(\omega) = \begin{cases} \theta_t^\ell(\omega) \text{ if } \omega \text{ does not belong to } E_j(\bar\omega) \\ \\ \theta_t^\ell(\omega) - \varepsilon_t \text{ if } \omega \text{ does not belong to } A_t^\ell \\ \\ \theta_t^\ell(\omega) + \delta_t \text{ if } \omega \text{ belongs to } A_t^\ell \end{cases}$$

We can choose $\varepsilon_t > 0$ and $\delta_t > 0$ in such a way that $d_t(\Theta) = d_t(\Theta \setminus \alpha_t)$ and

$$\min\left\{\frac{\alpha_t^\ell(\omega)}{p^\ell(\omega)} \text{ with } \omega \in E_j(\bar\omega)\right\} > \min\left\{\frac{\theta_t^\ell(\omega)}{p^\ell(\omega)} \text{ with } \omega \in E_j(\bar\omega)\right\}.$$

[4]The commodity ℓ and the states ω and $\bar\omega$ may depend on t.

Therefore, every player $t \in J$ has an incentive to deviate from the profile Θ which is a contradiction with the conditions of Nash Equilibrium.

Q.E.D.

A strategy profile Θ is called symmetric if every agent of the same type selects the same strategy, that is, $\Theta(t) = \theta_i$ for every $t \in I_i$. If it is the case, we write $\Theta = (\theta_1, \ldots, \theta_n) \in (\xi(M))^n$.

Theorem 3.1 *For every $M \in \mathbb{R}_+$ the set of symmetric Nash equilibria for the game $\mathcal{G}(M)$ is non-empty.*

Proof. Let $\mathcal{B}_t$ be a correspondence which associates to each symmetric strategy profile the best replies of the player $t \in I$. That is, given the strategy profile $\Theta = (\theta_1, \ldots, \theta_n) \in (\xi(M))^n$

$$\mathcal{B}_t(\Theta) = \arg\max_{\alpha_t \in \xi(M)} \Pi_t(\Theta \setminus \alpha_t)$$

Note that, by symmetry, $\mathcal{B}_t$ is the same for every player $t \in I_i$ and we denote $\mathcal{B}_i$. By definition, $p(\Theta) = p(\Theta \setminus \alpha_t)$ which allows us to obtain that $x_t(\Theta \setminus \alpha_t)$ is concave in α_t and $d_t(\Theta \setminus \alpha_t)$ is linear in α_t. Then, by assumption (U), we have that the payoff function Π_t is concave in the strategy selected by player t. This implies that $\mathcal{B}_t$ takes non-empty-convex values provided that $\xi(M)$ is a convex and compact set.

Moreover, the payoff function Π_t is a continuous function. Then, the maximun theorem allows us to conclude that the correspondence $\mathcal{B}_i$, from $\xi(M))^n$ to $\xi(M)$, is upper semi-continuous for every $i = 1, \ldots, n$.

Finally, let us consider the correspondence $\mathcal{B} = (\mathcal{B}_1, \ldots, \mathcal{B}_n)$. By Kakutani's theorem $\mathcal{B}$ has a fixed point, which actually is a symmetric Nash equilibrium.

Q.E.D.

4 Radner equilibrium as a limit of a sequence of Nash equilibria

In this section, we show that a Radner equilibrium can be obtained as the limit of a sequence of prices and allocations resulting from the sequence of symmetric Nash equilibria of the games $\mathcal{G}(M)$ when M goes to infinity. For it, given a price system $p \in \mathbb{R}_+^{Lk}$, let $\|p\| \equiv \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} p^\ell(\omega)$.

Theorem 4.1 *For each integer M, let $\Theta_M = (\theta_{M,t},\ t \in I)$, be the a symmetric Nash equilibrium for the game $\mathcal{G}(M)$. Let $(p(M), x(M))$ be the corresponding sequence of prices and allocations which is defined by this sequence of Nash equilibria. Then, there exists a subsequence of $(p(M)/\|p(M)\|, x(M))$ which converges to a price system p and an allocation x, such that (p, x) is an equilibrium for the economy $\mathcal{E}$.*

Proof. Since $\Theta_M = (\theta_{M,t},\ t \in I)$ is a symmetric Nash equilibrium for the game $\mathcal{G}(M)$, we have $\theta_{M,t} = \theta_{M,i}$ for every $t \in I_i$ and every type i of players. This equilibrium define the prices $p(M) = (p^\ell_M(\omega), \ell = 1, \ldots, L,\ \omega \in \Omega)$ which leads to the allocation $x(M) = (x_i(M),\ i = 1, \ldots, n)$ and net deficits $(d_i(M), i = 1, \ldots, n)$.

The definition of the game ensures that

$$\int_I x_t(M) d\mu(t) = \sum_{i=1}^n \mu(I_i) x_i(M) = \frac{1}{n} \sum_{i=1}^n x_i(M) \leq e = \sum_{i=1}^n \mu(I_i) e_i = \frac{1}{n} \sum_{i=1}^n e_i.$$

The remaining endowment at the trading post $e - \int_I x_t(M) d\mu(t)$ is delivered to the external agent. Thus, the consumption bundles allocated to consumers are uniformly bounded.

Note that if a player selects the strategy $\theta = 0$ then she spends and consumes nothing. This implies that $U_i(e) - d_{i+}(M) \geq U_i(x_i(M)) - d_{i+}(M) \geq U_i(0)$ and then $d_{i+}(M)$ is bounded from above by $U_i(e) - U_i(0)$.

Now, for each M let us consider the sets of types defined as follows:

$$D(M) = \{i \in \{1, \ldots, n\} \text{ such that } d_i(M) > 0\} \quad \text{and}$$

$$S(M) = \{i \in \{1, \ldots, n\} \text{ such that } d_i(M) < 0\}.$$

That is, $D(M)$ is the subset of types agents who are in deficit and $S(M)$ is the set of agents who are in surplus. It holds trivially the next equality

$$\sum_{i=1}^n d_i(M) = \sum_{i \in D(M)} d_i(M) + \sum_{i \in S(M)} d_i(M).$$

On the other hand, $0 = Lkn + \sum_{i=1}^n d_i(M) = Lkn - \sum_{i \in S(M)} -d_i(M) + \sum_{i \in D(M)} d_i(M)$, which implies that $\sum_{i \in S(M)} -d_i(M) = Lkn + \sum_{i \in D(M)} d_i(M)$ is also uniformly bounded from above. Since $d_{i+}(M)$ is uniformly bounded it it follows that so is $-d_i(M)$. Finally, we can conclude that $d_i(M)$ is uniformly bounded.

Thus if we consider a sequence $(x_i(M), d_i(M), i = 1, \ldots, n)_M$ with M converging to infinity, there exists a converging subsequence with limit $(x_i, d_i, i = 1, \ldots, n)$. We write $x_i(M) \to x_i$ and $d_i(M) \to d_i$, for each type i. Moreover, the sequence $\frac{p(M)}{\|p(M)\|}$ has also a convergent subsequence with limit p. We write , $x_i(M) \to x_i,\ d_i(M) \to d_i$, for each type i, and $\frac{p(M)}{\|p(M)\|} \to p$.

We remark that since $x_i(M)$ belongs to $\mathcal{X}_i$ for every M and every i, the limit allocation x is informationally feasible, that is, x_i is $\mathcal{P}_i$-measurable for every type i of agents. It remains to show that $(p, (x_t)_{t \in I})$, with $x_t = x_i$ for every $t \in I_i$ is a Radner equilibrium.

Since $L > 0$ the set $S(M)$ is nonempty. Moreover, every agent of type i in $S(M)$ must be bidding all the money that they can borrow. Otherwise, such an agent could increase the bidding in every commodity in each state of nature what entails a strictly increase in the consumption quantities of his bundle without incurring any default, and by consequence her payoff will increase which contradict the fact that we are in a symmetric Nash equilibrium. Since any agent of type $i \in S(M)$ is in surplus we have that $p(M)e_i > M$ which implies $\|p(M)\| \to \infty$ when $M \to \infty$.

Recall that the allocation $x(M)$ is defined as

$$x^\ell_{M,i}(\bar{\omega}) = \min\left\{\frac{\theta^\ell_{M,i}(\omega)}{p^\ell_M(\omega)}, \omega \in E_i(\bar{\omega})\right\}.$$

The Lemma 3.1, stated in the previous section, allows us to ensure that minimum above considered is attained at every $\omega \in E_i(\bar{\omega})$, that is

$$\frac{\theta^\ell_{M,i}(\omega)}{p^\ell_M(\omega)} = \frac{\theta^\ell_{M,i}(\bar{\omega})}{p^\ell_M(\bar{\omega})}, \text{ for every } \omega \in E_i(\bar{\omega}),\ i = 1, \dots n.$$

Then, we conclude that actually $x^\ell_{M,i}(\omega) = \dfrac{\theta^\ell_{M,i}(\omega)}{p^\ell_M(\omega)}$, that is $\theta^\ell_{M,i}(\omega) = p^\ell_M(\omega)x^\ell_{M,i}(\omega)$.

Now, we can write

$$\begin{aligned}
\frac{d_i(M)}{\|p(M)\|} &= \frac{\sum_{\omega\in\Omega}\sum_{\ell=1}^{L}\theta^\ell_{M,i}(\omega) - \sum_{\omega\in\Omega}\sum_{\ell=1}^{L}p^\ell_M(\omega)e^\ell_i(\omega)}{\|p(M)\|} \\
&= \frac{\sum_{\omega\in\Omega}\sum_{\ell=1}^{L}p^\ell_M(\omega)x^\ell_{M,i}(\omega) - \sum_{\omega\in\Omega}\sum_{\ell=1}^{L}p^\ell_M(\omega)e^\ell_i(\omega)}{\|p(M)\|} \\
&= \frac{p(M)}{\|p(M)\|}(x_i(M) - e_i)
\end{aligned}$$

Since $\|p(M)\| \to \infty$ and $d_i(M)$ is uniformly bounded for every type i, it follows that $\dfrac{p(M)}{\|p(M)\|}(x_i(M) - e_i) \to 0$, that is, $p(x_i - e_i) = 0$ for all $i \in \{1, \dots, n\}$.

Note that $px_i = pe_i > 0$, provided that $e_i \gg 0$. To finish the proof, let us show that $U_i(y) \leq U_i(x_i)$ for any bundle $y \in B_i(p)$ for every i. For it, let us take any real number $\lambda \in (0, 1)$ and a bundle $y \in B_i(p)$. Then, $\lambda py \leq \lambda pe_i = \lambda px_i < px_i$. This implies that $\dfrac{p_M}{\|p(M)\|}\lambda y < \dfrac{p_M}{\|p(M)\|}x_i(M)$ and thus $p(M)\lambda y < p(M)x_i(M) \leq M$, for all M large enough. For each M, let us consider the strategy given by $\alpha^\ell_M(\omega) = p^\ell_M \lambda y^\ell(\omega)$. Note that $\sum_{\omega\in\Omega}\sum_{\ell=1}^{L}\alpha^\ell_M(\omega) = p(M)\lambda y$ and then $\alpha_M \in \xi(M)$. By selecting $\alpha_t = \alpha_M$, we have

$$d_t(\Theta_M \setminus \alpha_t) = \left[\sum_{\ell=1}^{L}\alpha^\ell_M - p(M)\right]_+ \leq [p(M)x_i(M) - p(M)e_i]_+ = d_t(\Theta_M),$$

for any $t \in I_i$. Therefore, since Θ_M is a Nash equilibrium, $U_i(x_i(M)) \geq U_i(\lambda y)$. Finally, passing to the limit and observing that $\lambda < 1$ was arbitrary, we conclude that $U_i(x_i) \geq U_i(y)$.

Q.E.D.

5 Non-revealing equilibrium

In the previous sections we have considered that contracts are made ex-ante and prices, although can differ for different states, do not reveal information to agents. In this Section, we show an existence result of equilibrium where prices are measurable with respect the common information structure, that is, we show that the set of non-revealing equilibria is non-empty.

Consider again, as before, a differential information economy $\mathcal{E}$ with a continuum of agents but a finite number of types. Each agent $t \in I_i$ is is characterized by the private information $\mathcal{P}_i$, initial endowments $e_i \in \mathbb{R}_+^{Lk}$ and preference relation over the consumption space, which is represented by a utility function $U_i : \mathbb{R}_+^{Lk} \to \mathbb{R}_+$.

Let $\mathcal{P}_{\mathcal{C}}$ denote the associated common information, which is the meet of the partitions $(\mathcal{P}_i, i = 1, \ldots, n)$, and we write $\mathcal{P}_{\mathcal{C}} = \bigwedge_{i=1}^n \mathcal{P}_i$[5]. Then, E is said to be a common information event if $E_i(\omega) \subset E$ for every state $w \in E$ and for every type i of agents. Note that $\{\omega\} \in \mathcal{P}_{\mathcal{C}}$ if and only if $E_i(\omega) = \{\omega\}$ for every i or, equivalently, information does not lead directly to any consumption restriction at the state ω for any agent.

Let us consider a price system p which is $\mathcal{P}_{\mathcal{C}}$-measurable. That is, $p(\omega) = p(\omega')$ for any $\omega' \in E(\omega)$, where $E(\omega)$ is the common information event that contains ω. We remark that if prices are considered to have an informative role, then these prices that are $\mathcal{P}_{\mathcal{C}}$-measurable are precisely the prices that do not reveal any information to agents. In other words, if consumers could refine their information through prices, the only price systems which do not provide any additional information to any consumer are those which are compatible with the common information structure.

Next, we precise a non-informational equilibrium solution defined by prices that are compatible with the common information.

Definition 5.1 *A non-revealing equilibrium for the economy $\mathcal{E}$ with differential information is a price system p and a feasible allocation of commodities x such that*

(i) the commodity price system p is $\mathcal{P}_{\mathcal{C}}$-measurable,

(ii) every agent $t \in I$ maximizes U_t on the budget constraint $B_t(p)$ and

(iii) $\displaystyle\int_I x_t(\omega) d\mu(t) \leq \int_I e_t(\omega) d\mu(t)$ *for every state ω.*

Note, as we have already remarked, in this case equilibrium prices actually do not reveal any information to consumers provided that we just consider non-enlightening prices, i.e., equilibrium prices are required to be compatible with the common information $\mathcal{P}_{\mathcal{C}}$.

[5]The meet is the largest σ-algebra which is contained in each σ-algebra generated by $\mathcal{P}_i$, for every i. That is, $\mathcal{P}_{\mathcal{C}}$ is the finest partition of the set of states that is coarser than each $\mathcal{P}_i$.

5.1 A new market game with a non-enlightening prices rule: existence of Nash equilibium

In this subsection we define a sequence of market games associated to the economy $\mathcal{E}$, with a new price formation rule leading to non-enlightening prices, that is, prices which are compatible with the common information $\mathcal{P}_{\mathcal{C}}$. Thus, these games will allow us to deal with market equilibrium where prices might acquire an informative role and then address the problem of existence of non-revealing equilibrium, where prices are $\mathcal{P}_{\mathcal{C}}$-measurable and therefore consumers cannot infer any additional information from prices.

We use the same notation stated in the Sections 3 and 4. As in the previous game $\mathcal{G}(M)$, let us state an upper bound M on borrowing for fiat money. The new game $\mathcal{G}_{\mathcal{NR}}(M)$ with non-revealing price formation is defined by the same strategy sets as $\mathcal{G}(M)$ but differs form it in the payoff functions since the mechanism defining prices and allocations are modified as follows:

Given a strategy profile $\Theta = (\theta_t, t \in I)$ and a sate ω the price for the commodity ℓ in this state is given by

$$p^\ell(\omega) = \max\left\{\frac{\theta^\ell(\omega')+1}{e^\ell(\omega')},\ \ \omega' \in E(\omega)\right\}^6$$

In this case, the amount of commodity ℓ assigned to an individual $t \in I_i$ in the state ω is given by:

$$x_t^\ell(\omega) = \min\left\{\frac{\theta_t^\ell(\omega')}{p^\ell(\omega')},\ \omega' \in E_i(\omega)\right\} = \frac{1}{p^\ell(\omega)}\min\left\{\theta_t^\ell(\omega'),\ \omega' \in E_i(\omega)\right\},$$

where the last equality is due to the fact that p is $\mathcal{P}_{\mathcal{C}}$-measurable, which implies that p is $\mathcal{P}_i$-measurable for every $i = 1, \ldots, n$.

As in $\mathcal{G}(M)$ agents do not ascribe utility to fiat money, but are penalized in the case of default. To define the penalizations in this new game, let us define for each common information event E the function d_t^E as follows:

$$d_t^E(\Theta) = \#E\sum_{\ell=1}^{L}\max_{\omega\in E}\left\{\theta_t^\ell(\omega)\right\} - \sum_{\omega\in E}\sum_{\ell=1}^{L}p^\ell(\omega)e_t^\ell(\omega),$$

where $\#E$ denotes the cardinal of E. For each agent $t \in I$, the penalizations in the non-revealing game is given by $d_{t+}(\Theta)$, being

$$d_t(\Theta) = \sum_{E\in\mathcal{P}_C} d_t^E(\Theta).$$

Then the payoff of each agent $t \in I_i$ for each strategy profile Θ is $\Pi_t(\Theta) = U_i(x_t(\Theta)) - d_{t+}(\Theta)$, where $x_t(\Theta)$ and $d_{t+}(\Theta)$ are, respectively, the bundle and penalizations previously defined.

[6] We remark that if the initial allocation e is $\mathcal{P}_{\mathcal{C}}$-measurable, then the price rule can be recasted as $p^\ell(\omega) = \dfrac{1}{e^\ell(\omega)}\left(1+\max\left\{\theta^\ell(\omega'),\ \omega' \in E(\omega)\right\}\right)$

Lemma 5.1 *If the profile $\Theta = (\theta_t, t \in I)$ is a Nash equilibrium for the game $\mathcal{G}_{\mathcal{NR}}(M)$, then θ_t is $\mathcal{P}_i$-measurable for almost all agent $t \in I_i$ and every type $i = 1, \ldots, n$.*

Proof. Assume that the statement of the Lemma does not hold. Then there exist a Nash equilibrium $\Theta = (\theta_t, t \in I)$ and a positive measure set J of agents of a type j such that, for every $t \in J \subset I_j$ one has that θ_t is not $\mathcal{P}_j$-measurable. That is, for every $t \in J$ one has $\theta_t(\omega) \neq \theta_t(\bar{\omega})$ for some states ω and $\bar{\omega}$ such that $\omega \in E_j(\bar{\omega})$. Hence for each $t \in J$, we have $\theta_t^\ell(\omega) \neq \theta_t^\ell(\bar{\omega})$ for some commodity ℓ.[7] For each $t \in J$ and each commodity ℓ let H_t^ℓ be the set of states at which the minimum of $\left\{\theta_t^\ell(\omega) \text{ with } \omega \in E_j(\bar{\omega})\right\}$ is attained. For each $t \in J$ let us consider a strategy α_t given by

$$\alpha_t^\ell(\omega) = \begin{cases} \theta_t^\ell(\omega) \text{ if } \omega \text{ does not belong to } E_j(\bar{\omega}) \\ \\ \theta_t^\ell(\omega) - \varepsilon_t \text{ if } \omega \text{ does not belong to } H_t^\ell \\ \\ \theta_t^\ell(\omega) + \delta_t \text{ if } \omega \text{ belongs to } H_t^\ell \end{cases}$$

We can choose $\varepsilon_t > 0$ and $\delta_t > 0$ in such a way that $d_t(\Theta) \geq d_t(\Theta \setminus \alpha_t)$[8] and

$$\min\left\{\alpha_t^\ell(\omega) \text{ with } \omega \in E_j(\bar{\omega})\right\} > \min\left\{\theta_t^\ell(\omega) \text{ with } \omega \in E_j(\bar{\omega})\right\}.$$

On the other hand, we have that $p(\Theta) = p(\Theta \setminus \alpha_t)$ provided that if an agent modifies her strategy unilaterally, the price system does not change. Therefore, every player $t \in J$ has an incentive to deviate from the profile Θ which is a contradiction with the conditions of Nash Equilibrium.

Q.E.D.

Theorem 5.1 *For every $M \in \mathbb{R}_+$ the set of symmetric Nash equilibria for the game $\mathcal{G}_{\mathcal{NR}}(M)$ is non-empty.*

We omit the proof of this Theorem, since it follows as the proof of Theorem 3.1. which shows the existence result of symmetric Nash equilibrium for the game $\mathcal{G}(M)$.

5.2 Non-revealing equilibrium as limit of Nash equilibria

In which follows, we show that a non-revealing equilibrium of the economy $\mathcal{E}$ can be obtained as a limit of a sequence of prices and allocations which results from a sequence of Nash equilibria. For it, we state the following assumption:

[7] As in the proof of Lemma 3.1, the states ω and ω' and the commodity ℓ may depend on t.

[8] Let H_t denote the set of states in $E_j(\bar{\omega})$ which do not belong to H_t^ℓ and # denotes cardinal of the corresponding set. Note that $E_i(\bar{\omega}) \subseteq E(\bar{\omega})$, where $E(\bar{\omega})$ is the common information event which contains $\bar{\omega}$. Then, we can take δ_t in such a way that $\delta_t \leq \dfrac{\varepsilon_t \# H_t}{\# H_t^\ell}$ and $\max\limits_{\omega \in H_t^\ell} \left\{\theta_t^\ell(\omega) + \delta_t\right\} \leq \max\limits_{\omega \in E(\bar{\omega})} \left\{\theta_t^\ell(\omega)\right\}$ since this last maximum is not attained in H_t^ℓ.

(M) The total initial endowment $e = \dfrac{\sum_{i=1}^{n} e_i}{n}$ is $\mathcal{P}_{\mathcal{C}}$-measurable.

Theorem 5.2 *For each integer M, let $\Theta_M = (\theta_{M,t},\ t \in I)$, be the a symmetric Nash equilibrium for the game $\mathcal{G}_{\mathcal{NR}}(M)$. Let $(p(M), x(M))$ be the corresponding sequence of prices and allocations which is defined by this sequence of Nash equilibria. Then, there exists a subsequence of $(p(M)/\|p(M)\|, x(M))$ which converges to a price system p and an allocation x, such that (p, x) is a non-revealing equilibrium for the economy $\mathcal{E}$.*

Proof. To show this result, we can adapt the same proof of Theorem 4.1 taking into account that the following assertions are verified:

(i) Since e is compatible with the common information structure $\mathcal{P}_{\mathcal{C}}$, the definition of the price formation rule in the game $\mathcal{G}_{\mathcal{NR}}(M)$ allows us to obtain the physical feasibility of $x(M)$ provided that the next inequalities hold

$$\begin{aligned}
\sum_{i=1}^{n} x_{M,i}^{\ell}(\omega) &= \sum_{i=1}^{n} \frac{\theta_{M,i}^{\ell}(\omega)}{p_M^{\ell}(\omega)} = \sum_{i=1}^{n} \frac{\theta_{M,i}^{\ell}(\omega)}{\displaystyle\max_{\omega' \in E(\omega)} \frac{1+\theta_M^{\ell}(\omega')}{e^{\ell}(\omega')}} \\
&= \sum_{i=1}^{n} \frac{\theta_{M,i}^{\ell}(\omega)}{1+\displaystyle\max_{\omega' \in E(\omega)} \theta_M^{\ell}(\omega')} e^{\ell}(\omega) \le \sum_{i=1}^{n} \frac{\theta_{M,i}^{\ell}(\omega)}{1+\theta_M^{\ell}(\omega)} e^{\ell}(\omega) \\
&\le n e^{\ell}(\omega),
\end{aligned}$$

where the first equality follows by applying Lemma 5.1. whereas the third equality is implied by the $\mathcal{P}_c$-measurability of e which requires $e(\omega) = e(\omega')$ for any $\omega' \in E(\omega)$.

(ii) Each $x(M)$ is informationally feasible and then the limit allocation is also informationally feasible.

(iii) Applying lemma 5.1, we obtain that $\theta_{M,i}(\omega) = \theta_{M,i}(\omega')$ for any $\omega' \in E_i(\omega)$, for every type i.

(iv) The price formation rule in $\mathcal{G}_{\mathcal{NR}}(M)$ allows us to ensure

$$\begin{aligned}
\sum_{i=1}^{n} \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} p^{\ell}(\omega) e_i^{\ell}(\omega) &= \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} p^{\ell}(\omega) \sum_{i=1}^{n} e_i^{\ell}(\omega) \\
&= \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} n \max_{\omega \in E} \left\{\theta^{\ell}(\omega) + 1\right\} \\
&= \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} \max_{\omega \in E} \left\{\sum_{i=1}^{n} \theta_i^{\ell}(\omega) + n\right\} \\
&\ge \sum_{E \in \mathcal{P}_C} \sum_{\ell=1}^{L} \sum_{i=1}^{n} \#E \max_{\omega \in E} \left\{\theta_i^{\ell}(\omega)\right) - Ln \sum_{E \in \mathcal{P}_C} \#E
\end{aligned}$$

Then, by the definition of the penalizations, we have the following inequality:

$$\sum_{i\in S(M)} -d_i(M) \leq Ln \sum_{E\in\mathcal{P}_C} \#E + \sum_{i\in D(M)} d_i$$

Therefore, we can conclude that the sequence of $d_i(M)$ is uniformly bounded.

(v) The definition of the mechanism of price formation allows us to ensure that each $p(M)$ is $\mathcal{P}_{\mathcal{C}}$-measurable and then the limit of the convergent subsequence is also a non-enlightening price system.

Q.E.D.

References

Allen, B., Yannelis, N.C. (2001): On Differential Information Economies. Economic Theory, 18, 263-273.

Dubey, P., Geanakoplos, J. (2003): From Nash to Walras via Shapley-Shubik. Journal of Mathematical Economics 39, 391-400.

Einy, E., Moreno, D., Shitovitz, B. (2001): Competitive and core allocations of large economies with differential information. Economic Theory, 18, 321-332.

Faias, M,, Moreno-García, E.: Incomplete Financial Markets and Differential Information Economic Theory, in press. DOI: 10.1007/s00199-008-0429-6

Fugarolas, G., Hervés Beloso, C., Moreno-García, E., Torres-Martínez, J.P. (2009): A Market Game Approach to Differential Information Economies. Economic Theory 38, 321-330.

Glycopantis, D., Muir, A., Yannelis, N.C. (2003): On extensive form implementation of contracts in Differential Information Economies, Economic Theory 21, 495-526.

Hervés-Beloso, C., Moreno-García, E., Yannelis, N.C. (2005): An Equivalence Theorem for a Differential Information Economy. Journal of Mathematical Economics 41, 844-856.

Hervés-Beloso, C., Moreno-García, E., Yannelis, N.C. (2005) Existence and Incentive Compatibility of the Walrasian Expectations Equilibrium in Infinite Dimensional Commodity Spaces Economic Theory, 26, 361-381.

Maus, S. (2004): Exchange economies with asymmetric information: Competitive Equilibrium and the core. Economic Theory, 24, 395-418.

Radner, R. (1968): Competitive Equilibrium under Uncertainty. Econometrica 36, 31-58.

Radner, R. (1979): Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices . Econometrica 47(3), 655-678.

Schmeidler, D. (1980): Walrasian Analysis via Strategic Outcome Functions, Econometrica 48, 1585-1593.

Shapley, L.S., Shubik, M. (1977): Trade Using One Commodity as a Means of Payment. Journal of Political Economy 85, 937-968.