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Exogenous coalition formation in the e-marketplace based on geographical proximity

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Abstract

This paper considers a model for exogenous coalition formation in e-marketplaces. Using the informational advantage e-retailer creates coalitions of customers based on geographical proximity. Most of the literature regards this process as endogenous: a coalition leader bundles eventual purchases together in order to obtain a better bargaining position. In contrast - and in response to what is being observed in business practice - we analyse a situation in which an existing e-retailer exogenously forms customers’ coalitions. Results of this study are highly encouraging. Namely, we demonstrate that even under highly imperfect warehouse management schemes leading to contagion effects, suggested combined delivery service may offer significant efficiency gains as well as opportunities for Pareto-improvement.

Keywords: Coalition formation, e-commerce, multi-agent systems, consumer satisfaction, demand planning, warehouse management.

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1 Introduction

Over the last decade, electronic marketplaces have witnessed considerable growth and development in terms of both volume and value. However, in spite of the very diverse contributions from the scientific world on the obvious opportunities of virtual marketplaces, those used in practice are still usually very simple (cfr. Tsvetovat, Sycara, Chen and Ying (2000)). Although current efficiency and low costs of communication offer numerous possibilities for economic agents to meet and cooperate, companies and services enabling such collaboration remain still on a relatively sophisticated and, thus, rare level. Few examples include Ag Guild from Chicago,\(^1\) US Iowa-based E-Markets\(^2\) or Accompany.com and Mercata\.dcom \(^3\) as well as Aerogistics\.com \(^4\). A related theoretical literature has focused mainly on the opportunities for volume discount, essentially proving that in terms of business practice there is room for an additional intermediary. Internet has turned this intermediary into a virtual marketplace, where perhaps contributors (depending on mechanism design) are able to appropriate all the benefits.

In this paper, we consider the coalition formation problem in e-marketplaces from a different perspective. We study whether the orders of different customers from the same geographical location (e.g. ZIP code) could be pooled together and generate savings on shipment cost under highly demanding assumptions of imperfect warehouse management system as well as contagion effects due to delivery defaults. Based on an informational advantage seller has over the buyers, it may exogenously form coalitions of orders (not buyers), creating room for shipment cost reductions. We demonstrate that combining orders can be beneficial to the e-retailers even when potential savings are redistributed to customers and such a strategy leads to a general Pareto improvement. The proposed mechanism is not only able to increase profits in the long-run but contributes to other meta-objectives (e.g. price comparative advantage, enlargement of the potential customer base and increased loyalty of current clients).

Our setting differs considerably from the ones analysed in the literature. The key idea behind our approach is to allow the seller use the informational advantage it has over the dispersed clients, to partially or fully control the coalition formation process. This contrasts with, say, the case of Ag Guild, where orders are created spontaneously. Moreover, the benefits from coalition formation in

\(^{1}\)Ag Guild is a corporation founded by 35 farmers, with an average of 150 acres land each. Every producer pledges ten percent of his or her output to the management of the guild which specializes in organizing production and marketing of corn and soybean crops. The guild buys production inputs and sells final products. Companies which buy and process agriculture find it easier to negotiate with one large seller than with 35 smaller farmers. Similarly, the guild, as a buyer of production inputs, has the power to negotiate better prices by aggregating demand. Farmers can pool orders via the Internet. The first farmer specifies the type of product he wants to buy, e.g., seed, and how long the order may be open. Then other farmers can add their names to the list. Cfr. Robinson (2000)

\(^{2}\)The leader in agricultural-based e-commerce, serving more than 14,000 agrifood companies, grain elevators, and producers. Cfr. Robinson (2000)

\(^{3}\)They allow potential purchasers to form buying coalitions, and offer volume discounts on the size of the group.

\(^{4}\)It allows manufacturers of aerospace components to form consortia to bid for larger contracts.
our setting will come solely from shipment cost reductions (combined shipment discount) - while we do not neglect potential volume discounts, they are not modeled in the paper.

The motivation for such choice is that literature has convincingly demonstrated that volume discount is a mechanism fostering coalition formation. Nonetheless, taking a longer term perspective, a forum where buyers can pool their orders and who profit from volume discounts is essentially an intermediary and will develop into a retailer over time. With repeated activities and the emergence of some operation costs (as well as taxes!) it becomes - in effect - a regular shop. In other words, the examples literature quotes demonstrate that there is room for one more intermediary in the purchasing process by using informational advantages to obtain a volume discount. Conversely, this paper attempts to demonstrate that even without volume discount combined shipment guarantees Pareto improvements even under quite demanding assumptions.

The remainder of this paper is structured as follows. Section 2 presents a literature review focusing on contributions to coalition formation in e-marketplaces. In Section 3, we present the design of the model, including both the buyer decision and exogenous coalition formation mechanisms. Based on this framework Section 4 presents the simulation setting and parametrisation assumptions, while Section 5 presents simulation results and Section 6 offers a discussion of model sensitivity to parametrisation. Finally, in Section 8, we conclude with some insights into future research directions.

2 Literature review and motivation

Coalition formation has been an subject of extensive game theoretic research for years (e.g. Moulin (1988), Osborne and Rubinstein (1999) and Bloch (1996)). It also became of interest in the emerging Multi-Agent System (MAS) literature with the works of Shehory and Kraus (1996) and Yamamoto and Sycara (2001). However, only a limited number of papers have been published on coalition formation in e-marketplaces.6

Yamamoto and Sycara (2001) propose a buyer coalition formation scheme, GroupBuyAuction, which enables a large number of buyers who want to buy a certain good or a type of a good to form coalitions. In this setting, each buyer specifies a set of (substitutable) goods, one of which he would be willing to purchase, together with their reservation prices. Based on this information the leader of the auction group divides all buyers into coalitions in such a way that each coalition purchases a desired quantity of a particular good profiting from any volume discounts; the resulting surplus is distributed in a stable way between participating buyers. In reality it seems unlikely that such a mechanism could grow in popularity, mainly due to the costs incurred by the leader of the group and the issue of trust.

See (Moulin 1995) for review of the coalition formation literature.

6He and loerger (2000) provides an excellent but general survey and analysis of the state-of-the-art agent-mediated e-commerce.
Li, Chawala, Rajan and Sycara (2003) extend this work, discussing the desired mechanism properties of coalition formation in an e-marketplace from the perspective of cooperative and non-cooperative game theory. These desirable properties include stability (being in the core) and incentive compatibility with good efficiency. Li and Sycara (2002) discuss algorithms of coalition formation in combinatorial auctions analyzing a setting where each buyer places a bid on a combination of items with a reservation cost, and sellers offer price discounts based on the volume of each item. Finally, Tsvetovat et al. (2000) also considers the creation of spontaneous coalitions of similar-minded customers coming together to procure goods at a volume discount (“buying clubs” as in the Ag Guild example). This study focuses on the economic incentives for the creation of such groups and present a flexible test-bed system that could be used to implement and test coalition formation and multilateral negotiation protocols.

We take a different perspective on coalition formation in e-marketplaces. Namely, we study how an e-retailer could increase profits and/or achieve other objectives by pooling together orders of customers from the same geographical location. Thus, we acknowledge an informational advantage for the retailer concerning the geographical distribution of the purchases. This advantage provides the basis for inducing coalition formation exogenously among the customers. The role of the e-retailer in our paper in some cases may resemble to some extent the role of the leader in (Yamamoto and Sycara 2001), because e-retailer divides clients into coalitions. However, in our model, depending on the strategy chosen, clients can (but do not have to) know that their orders were pooled together.

The distinction between volume discount and combined shipment discount we introduce is actually quite significant. The first concept concerns economies of scale irrespectively of shipment, while the latter refers to the lowering of transportation costs. Note that a coalition of producers in the Ag Guild case takes advantage of economies of scale both in volume (the volume discount) and shipment (the combined shipment discount). Why should one focus on perceived as potentially smaller combined shipment discount instead of perceived as potentially bigger volume discount?

The practice of retailers and e-retailers seems to suggest that actually the former does not provide sources for long-term comparative advantage in the e-economy. In fact, both retailers and e-retailers seek to emphasise arguable product differentiation by competing on the full range of marketing variables within their power to determine. These include, in addition to price structures and levels, variables such as: the products assortment; branding and promotion, store outlet or website design, warranties, payment securities, delivery options, loyalty campaigns and other relationship marketing techniques, after-sales services, product upgrades, etc. Brynjolfsson and Smith (2000) suggest that previous experience (i.e. reliability) is especially important as they find that customers are willing to pay premium prices for books from online retailers that they

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7Accompany offered only volume discounts. We should stress that in this paper by volume discount we mean volume discount offered to an organized group of clients and not to a single client. We acknowledge that volume discount can be offered to a single client with a sufficiently big order but we will disregard such a strategy of a retailer in this paper.
have dealt with previously. In economic terms, these efforts aim to enable retailers to gain some monopolistic power and so increase their profit margins. To this end, the study of Pan, Ratchford and Shankar (2002) on the example of 105 online retailers found that price dispersion is still considerable and persistent. Consequently, e-markets seem to be quite similar to other types of commerce, by avoiding profit-destroying competition, especially competitive pricing. In contrast, shipment discounts does not affect the profit margin of e-retailer, as in most of the cases shipment service is provided externally to the retailer and charged separately to the customer.

With intensifying competition, retailers’ mark-ups are naturally driven down, which implies that volume discounts cannot constitute a profit stimulating strategy in the long term perspective. While differentiation strategies constitute one way to overcome this shortcoming of Bertrand-Nash equilibrium, they are costly and require a considerable upfront (and sunk!) investment. This model suggests another way, i.e. benefiting from shipment cost reductions. They are typically born by buyers (retailer adds this cost to a price of the product, while the amount of shipment costs depends on delivery time chosen by the buyer) and external to retailers’ warehouse management technology - thus, external to his cost structure.

In this paper we take the position that, since an e-retailer is already there, he himself can take the role of coalition creator. The reason he would consider taking such a role is the same reason he would consider manipulation of any marketing variable within his power to influence: doing so may be attractive to potential customers and thus provide a competitive advantage over other retailers who do not employ this strategy. Because exploiting volume discounts already constitutes the basic activity of the e-retailer we consider the possibility of generating additional profits from shipment discount to the client from the same geographical location. Our strategy can be summarized as follows:

1. Basic intuition suggests that combining orders to obtain shipment discounts should be always profitable as it leads to improving the outlooks of e-retailers and possibly customers (a Pareto-improvement). The question is about the scale of this improvement and whether it is sufficient to provide incentives to e-retailers;

2. Answer to this question depends on (i) frequency of same-location orders, (ii) potential gains from combined shipment and (iii) costs of implementing such a solution. In the paper we abstract from addressing the last point, as it depends mainly on highly company-specific conditions. Nonetheless, points (i) and (ii) are addressed from both theoretical perspective and in as far as the sensitivity of the simulation results is concerned;

3. Our approach is based on probability and coalition formation theories. We propose an algorithm to combine orders from the same geographical location;

4. To confront the model with realistic assumptions, we consider two warehouse management systems: a perfect one that allows no delivery delays and a realistic one, which minimises retailer risks at the expense of delivery delay. In the case of combined shipment, delivery
delays imply so-called "contagion effect" (some orders await over time limit another one with which they are combined). We consider this rather a realistic way of modelling retailers behaviour;

5. We show that when these real-world imperfections are introduced to the model, combining orders still provides room for Pareto improvement. This counter-intuitive finding is a consequence of the fact that although failures, such as a delay, may indeed affect adversely consumer satisfaction and thus future loyalty, gains from combined shipment are indeed significant; and

6. We also allow the retailer to redistribute the gains from combined shipment in the form of price reduction. The protocol for sharing them in facto provides additional reinforcing mechanism. Downward sloping demand function implies that price reductions increase sales, boosting profits.8

3 Model assumptions

For a potential (private) consumer, purchasing goods in an e-marketplace has both advantages and disadvantages compared to shopping in a high-street retail outlet. The advantages usually comprise of lower prices, variety, availability (24 hours, 7 days a week), while among the disadvantages, one typically lists both a lack of professional advice and the perceived risk of fraud in commercial Internet transactions. Moreover, it may impossible to actually assess all the characteristics of the item on the Internet (for instance, how silent is the laptop that we buy on the net). In this paper we focus only on prices and delivery times. The interplay of these factors plays a crucial role in a decision to buy something on-line, because many buyers are not likely to seek e-opportunities involving considerable waiting if the difference between on-line and traditional shopping is not sufficiently high.

Consider a case where an e-retailer tries to increase profits (short-term or/and long-term) by offering to clients a combined delivery service (CDS). Designing a coalition formation mechanism and redistribution of such a discount between customers or/and itself is be the sole responsibility of the e-retailer. Following Bennett (1985), we distinguish between exogenous and endogenous coalition formations.9

In our setting there are two conditions for a coalition to be formed, i.e. for the goods to be dispatched together: (i) they must be placed by customers residing in the same geographic location;

8To be exact, in economic terms the necessary condition for the profits to grow is that the price elasticity of demand exceeds unity. However, in our setting demand function is not explicit, so this condition is binding.

9If the e-retailer pools orders of consumers without their participation in choosing coalitions then such a coalition is exogenous. When consumers take part in a decision process to group orders without incorporating the e-retailer in the coalition formation process, such a coalition is endogenous. Note that the model of Yamamoto and Sycara (2001) concerns exogenous coalition formation as it is the leader of a group who divides its participants into coalitions.
and (ii) their delivery times have to be the same or lie within a sufficiently-short interval.\textsuperscript{10} This raises a basic question on the profitability of such a combined delivery system, i.e., how many orders are actually placed from the same location and with a similar delivery time? Clearly, the answer depends on many assumptions, e.g., a definition of every location, population density, seasonal variables, etc. However, one can easily demonstrate that profits from combined delivery service can actually be considerable.\textsuperscript{11}

3.1 The potential for exogenous coalition formation

The main vehicle for discussing the likelihood of same-location purchase occurrence is the so-called birthday paradox. Consider a certain number of people - what is the probability that any two people of this group have birthday on the same day? Actually, this number is strikingly high and already for 23 people exceeds 50%. For the purpose of this paper birthday paradox demonstrates in the following problem: with \( r \) localisations populated by \( n \) people\textsuperscript{12} what is the probability of two orders arriving on the same day from a location? Consequently, the likelihood of two orders coming on the same day from the same location is naturally given by \( 1 - \frac{r!}{r^n(r-n)!} \). Importantly, we do not specify anywhere in the model any rule for the process of orders allocation.

Basing on the birthday paradox one can say that the occurrence of same-location purchases is likely. To address how profitable this may be, one needs to consider the shipment costs properties. In the remainder of this paper, we build a model of an e-retailer introducing a combined delivery service (CDS).\textsuperscript{13} To formalize the concept of such a discount let \( c_1, c_2, ..., c_n \) be the costs of shipment of \( n \) goods to the same area separately. Since in principle goods can differ both in size and in weight, delivery costs may differ as well. Assume that the courier service provider offers a discount for combined delivery of these \( n \) goods at a price \( f(c_1 + c_2 + ... + c_n) \) which is a function of a sum of separate shipment costs meeting the following conditions:

\[
0 < f(c_1 + c_2 + ... + c_n) < c_1 + c_2 + ... + c_n, \\
f'(c_1 + c_2, ..., c_n) > 0 \quad \text{and} \quad f''(c_1 + c_2, ..., c_n) > 0.
\]

In words, the only assumption necessary for the combined delivery shipment to produce costs reductions is that the cost function is increasing and concave. In the real world case, most delivery companies set pricing strategies along intervals (usually, weight based). Prices are fixed within these intervals and - if anything - depend on delivery zones. Although this is not a continuous func-

\textsuperscript{10} In principle, in the real world it may frequently happen that customers buy for locations different from their own, e.g. gifts. However, this would not introduce any change to our model, because the pivotal characteristic of an order in this model is the address to which the it is to be dispatched. For the purpose of clarity we keep this simplified setting.

\textsuperscript{11} We pursue with some numerical examples in the analytical section.

\textsuperscript{12} All notations are summarised in a Table 1 at the end of this section.

\textsuperscript{13} Naturally, we need to assume that a courier service provider is willing to offer a discount for the combined dispatches to the same location, e.g., to the same ZIP-code.
tion (derivatives do not exist) it guarantees quasi-concavity over the vast majority of cases. Thus, it seems that the specification suggested above resembles to a large extent real world solutions.\footnote{To prevent excessive complexity of the model we do not maintain the interval structure, because it would require allocating ”weight” property to the purchases. This would constitute rather arbitrary additional parameterisation in the model and was thus avoided.}

### 3.2 Consumer satisfaction

In order to incorporate a long term perspective in the retailer’s behavior this paper introduces a novel approach taking into account objectives beyond short term profits. Namely, our model explicitly includes consumer satisfaction from on-line shopping, which deteriorates with delays on declared delivery. This allows to capture the impact of punishment following from ”disappointing” the client by delaying his delivery, which is important because once the combined delivery service is introduced, e-retailer are subjected to evident incentives of delaying some orders due to possible shipment costs reductions to the detriment of consumer satisfaction. Such strategy is punished in our model, since any subsequent purchase of each customer is evaluated \emph{vis-à-vis} based on his past experience.

As argued in the literature (see, for instance, Zeithaml, Parasuraman and Malhotra (2002) and Santos (2003)) consumer satisfaction driven by service quality is especially important in e-commerce. This is because the online comparison (but not assessment) of technical features of products is essentially costless, feasible, and easier than comparisons of products through traditional channels. Price may be important in initially attracting customers, but if a company does not provide good service, customers, trivially, do not come back (Reibstein 2002).\footnote{There are many models of consumer satisfaction in the literature, e.g., (Fornell, Johnson, Anderson, Cha and Bryant 1996), (Bruhn and Grund 2000), (Martensen, Gronholdt and Kristensen 2000) or (Hackl, Scharitzer and Zuba 2000), most of which deal with Consumer Satisfaction Index (CSI) for high street commerce, which is focused on physical settings. In contrast, (Hsu 2007) constructs the equivalent e-CSI, which takes into account specific issues of the e-marchandise, including, for instance, the fact that each online transaction involves a number of third parties, such as credit card clearance firms and delivery companies. Thus, there is always a possibility of failure being virtually independent from e-retailer.}

For the purpose of this paper we develop a relatively simple but comprehensive measure of consumer satisfaction. Let $stf_n(t)$ denote satisfaction of the consumer $n$ at time $t$ from total up-to-date services of the e-retailer and assume that at the beginning of simulations $stf_n(0) = 1$. Every time an e-purchase is made and delivered, customer updates $stf_n(t)$ taking into account the (i) utility that was expected from e-purchase, (ii) the utility actually experienced and (iii) utility which would have been experienced, had the good been purchased from the traditional retailer. In particular, if the good ordered is delivered within the promised time range, while the price/delivery ratio was competitive, consumer’s satisfaction increases and \emph{vice versa} for the opposite. For example, assume that the good $m$ was delivered at time $t$ to consumer $n$. Then, after each good ordered with the e-retailer is delivered, the satisfaction of the consumer is updated according to the following rule:
∀_{t(arrival)} stf_{n}(t+1) = stf_{n}(t) \frac{E_{(t)}(U_{e})}{E_{(t)}(U_{e}) - \varepsilon(U_{e}, E_{(t)}(U_{e}), U_{s}) (E_{(t)}(U_{e}) - U_{s})} \quad (2)

where $E_{(t)}(U_{e})$ denotes expected utility from price and delivery time when purchasing the good at the e-retailer at the moment of order, $U_{e}$ is the utility the consumer actually experienced (after the good arrived) and $U_{s}$ is a potential utility from the price of the good when bought in the high street shop.\(^{16}\) and $\varepsilon(U_{e}, E_{(t)}(U_{e}), U_{s})$ is a parameter value of which depends on the eventual difference between the values of expected and actual utility as well as potential high street utility in the following way:

1. If the actually experienced utility is lower than expected (i.e. $U_{e} \leq E_{(t)}(U_{e})$) then $stf_{n}(t)$ is positively updated and $\eta$ grows with the difference between actual and expected utility;

2. If actual utility is higher than expected but it is still lower than utility from the purchase in the high shop, i.e. $E_{(t)}(U_{e}) < U_{e} < U_{s}$, then update is moderately negative; or

3. If due to any e-retailer failure actual utility is even higher than this from the high street shop then satisfaction deteriorates considerably faster.

The rationale behind formula (3) is as follows. Every successful purchase, i.e. when the e-retailer kept up with the promised delivery time, increases trust and contentment of a consumer and makes it more likely that the next purchase will also be made on-line. Thus, satisfaction $stf_{n}(t)$ grows and this growth is proportional to the difference between expected and actual utility. In contrast, if delivery is delayed satisfaction decreases. The potential satisfaction from the same purchase made at high street shop is the natural benchmark point to which consumers relate deterioration of their satisfaction. Intuitively, if the actual utility is still below the expected one, consumer feels disappointed but the purchase can be still considered a better alternative when compared to the traditional retailer (available alternative). However, if actual utility is worse than high-street purchase then on-line shopping turns out to be the worst alternative. In such a case, satisfaction from services of the e-retailer deteriorates fastest.

As depicted by Figure (1), this in-built satisfaction mechanism allows consumers to be more (or less) eager to buy from the e-retailer when compared to the high street shop based on past experience. On the other hands, consumers are not directly Bayesian in the sense that in making their purchase decisions they always believe the declared delivery time. Thus, they do not update

\(^{16}\)Note that this formulation of satisfaction function allows for convenient re-scaling of the preferences. With assigned values of $\varepsilon$ 10% of increase in satisfaction will not lead to increase in propensity to buy from e-retailer of the same size. Since changes in the utility are rather minuscule this mechanism seems seems necessary.
their beliefs indirectly (about expected default of the e-retailer) but rather directly (about expected
utility of subsequent purchase from the e-retailer).\textsuperscript{17} In other words, consumers in this model are
forward looking but do not have projections about probabilities of future outcomes - they focus
on projections of future satisfaction levels instead.

### 3.3 Model structure

The diagram in Figure (2) presents the general structure of the model. The e-retailer buys $\bar{m}$
goods and sells them to $\bar{n}$ consumers grouped in $\bar{r}$ ZIP-codes.\textsuperscript{18} Consumers have the choice of
buying goods from high street shop or e-retailer based on preferences for delivery time and price.
In order to make purchase decisions consumers need to have sufficient income, which is distributed
among them in every period with normal distribution.

[ Figure (2) about here ]

Purchases may be made from e-retailers and high-street shops, while the goods purchased are
not distinguishable with respect to suppliers (essentially, from the point of view of the consumers,
item is identical in e-shop and in real shop with only prices and delivery times potentially differing).
Consumers are equipped with preferences, from which it is derived what they want to buy and
from whom (depending on price/time preference). Income distribution is the most indiscretionary
way of allowing for purchases to happen over time (and not in one point in time). Arriving income
values were calibrated in a way enabling repeated purchases (the same consumer will have an
opportunity to make purchase decisions more than once).

Once the decisions about e-purchase are made, e-retailer has the choice of pulling orders made
from the same location. If he does and his warehouse management system is imperfect (delays
occur), contagion effect may materialise. This is the crucial element of this simulation: delays
in some orders may transmit to delays in others if they are pulled into one delivery package.
Consequently, consumer dissatisfaction may spread beyond the natural scope of one disappointed
client. If this effect proves smaller than the shipment cost reductions, combined delivery service
introduces an efficiency gain. However, this may not necessarily be a strict Pareto improvement,
if some clients observe \textit{ex post} lower satisfaction scores than in benchmark situation of no orders
pulling.

### 3.4 Consumer choice

To enable modelling of the consumer decision making process, utility functions (choice criteria)
were specified. Utility accounts for price, waiting time and the interaction of the two. More

\textsuperscript{17}See Brynjolfsson and Smith (2005) for a review of multi-category choice behaviour and the use of Bayesian
methods.

\textsuperscript{18}Note that it is irrelevant how many suppliers the e-retailer has, as possible combined deliveries from the sup-
pliers providing more than one good are beyond the e-retailer’s control (they follow from the suppliers warehouse
management system and thus cannot influence decisions by the e-retailer).
explicitly, both the price to pay and the waiting time are economic “bads”, as they provide
dissatisfaction to the consumer, resulting in (dis)utility. Furthermore, decreasing marginal utility
must hold to ensure that disutility increases, but at a decreasing pace from both price and waiting
time growths. Any convex utility function allowing for an interaction is thus acceptable (cfr. Holahan (1988)).

Since both the price-to-pay and the delivery period provide negative utility, it is more conve-
nient to work with disutility curves. Let us consider a typical consumer $i$ with a disutility curve
given by:

$$U_i = \alpha_i p^2 + \beta_i d^2 + \gamma_i p \cdot d,$$

(3)

where $p$ denotes a price of a good purchased, $d$ stands for the waiting time, while $\alpha$, $\beta$ and $\gamma$
denote consumer specific preferences. Evidently, $d$ takes the values from 0 in the case of traditional
retailer to a considerable number of days in the case of e-marketplaces. Such a quadratic form
of a disutility function is commonly used in economics: the smaller are the values of either
$p$ and/or $d$, the better-off is the consumer. Moreover, the cross term is needed to warrant imperfect
substitution between price and delivery time.

Since neither price nor time of delivery can be negative, indifference maps need to be located
in the top-right quadrant of the p-d plane. Nonetheless, utility levels analyzed in this paper
are negative. Thus, the further away one gets from the origin, the lower the satisfaction level.\(^{19}\)
Consequently, origin constitutes the preferred location for each consumer, while higher disutility
levels are justified by fixed utility levels derived from consuming particular goods, which remain
beyond the scope of this paper.

Coefficients $\alpha$ and $\beta$ in equation (3) capture the elasticity of each consumer to changes in price
and in delivery times. The interaction term of $p \cdot d$ captures a possible interplay for every consumer
in his trade-off profile: some of the buyers might be willing to wait somewhat longer if a price of
the good could be diminished as a result. On the other hand, to some of the buyers it might seem
justified to pay more in order to receive purchases sooner. Consequently, $\gamma$ can differ substantially
from consumer to consumer both in terms of size and in terms of signs. This is depicted on Figure
(3) with left panel representing utility levels for negative $\gamma$ values, while right one corresponding
to a positive interplay between price and waiting time.\(^ {20}\)

[ Figure (3) about here ]

Obviously, the price $p$ contains both the actual wholesale price for the e-retailer and the delivery
costs. Let us define the wholesale price as a cost $\hat{p}$ and shipment expense as $\hat{s}(t)$, while the latter
must be dependent on the period of delivery (the longer the waiting period, the lower the shipment

\(^{19}\)Importantly, utility functions cannot be concave - their convexity follows necessarily from inverting the decreasing
marginal utility principle. Namely, once the time of delivery grows from 100 days to 99, utility must change less
than in the case of twofold growth from just 24 hours shipment period.

\(^{20}\)Note that curves do not show the values of the utilities; these could only be observed in the third dimension.
The graph represents the shape of the utility curve maps.
cost). In addition, the delivery time $d$ consists of handling time $w$ and shipment time $z$. Thus, $p = \hat{p} + s(t)$ and $d = w + z$ which transforms (3) to:

$$U_i = \alpha_i(\hat{p}^2 + s^2(t)) + \beta_i(w^2 + z^2) + \gamma_i(s(t) \cdot t + s(t) \cdot w) + 2\beta w \cdot z + 2\alpha \hat{p} \cdot s(t) + \gamma(\hat{p} \cdot w + \hat{p} \cdot z). \tag{4}$$

In equation (4), the first term measures the utility of direct and indirect purchase costs, while the second reflects the negative utility derived from waiting. The third term corresponds to the combined effect of waiting and the costs of shipment and accounts for the substitution effect from the interplay of cost and waiting. Since, in principle, $\gamma$ can be both negative and positive, the model allows the interplay to have both possible impacts on the disutility of the buyers.

The term $2\beta w \cdot z$ captures the fact that high street shops always enjoy a comparative advantage over e-retailers with respective to delivery times: the negative sign of $\beta$ requires that if any waiting whatsoever occurs either due to processing by the e-retailer or due to shipment, the consumer will always prefer purchasing the good from a high-street shop at the same price. Straightforward assumptions concerning the differences between high street shops and the e-marketplace impose that for each reservation price (i.e., the utility derived from possessing a good) there are several alternative combinations of the price to be paid and the time of delivery that can provide uniform utility level to buyers. In addition, in the case of high street shops, $d = 0$, and so the mark-up between wholesale costs and prices listed may be higher for each reservation price, making it possible to cover the higher costs of operations. Moreover, consumers with the same reservation price can differ in terms of weight associated with price and waiting, thus, imposing necessary differentiation of utility function parameters.

Finally, the last two terms in the utility function require explanation. In this model - as in real world - costs of shipment are assumed to be set independently of the value of goods ordered (they only depend on the quantity and are fixed with respect to the value of goods - different parametrizations only consider the relation between transportation costs and a mark-up). Consequently, delivery times are similarly independent of the product value. Thus, by assumption, these two vectors (product value $\hat{p}$ and shipment cost $s$) need to be orthogonal ($\hat{p} \perp s$), which makes their product equal to zero by definition. Finally, similar reasoning can be applied to the last term $\gamma(\hat{p} \cdot w + \hat{p} \cdot z)$. Namely, since costs of shipment are to be independent of the value of purchase, then so does the shipment time (hence $\hat{p} \perp w$). By the same token, nothing would justify considering the waiting time of the retailer dependent on the product value (hence $\hat{p} \perp z$).

Final remark considers the coalition leadership. One could consider a structure in which the higher consumers’s relative gains from coalition (the lower the value of ones purchases), the higher the propensity to take effort to induce coalition emergence. In other words, following Gamson (1961) and later contributions by Yamamoto and Sycara (2001), Li et al. (2003) and Tsvetovat

\[21\text{Please note, that this specification allows the e-retailer to incorporate in the utility of buyers the "cheating" of the retailer: in principle, the retailer could extend the waiting time in order to hide the fact that he forms coalitions beyond declared preferences of customers. If he decides to do that, the consumer takes this into account as well, since this is the total delivery time that matters for his utility.} \]
et al. (2000) one could in principle allow consumer to encourage purchases from the same location, resulting in endogenous coalition formation mechanisms. However, in this paper we chose to focus on potential benefits to the e-retailer from the evident informational advantage in his possession. Obtaining information on potential purchases from a consumer’s location is obviously costly, while - from the business point of view - e-retailers already enjoy a hold of this advantage.

3.5 Retailer, warehouse management, demand planning and shipment

Only a few retail companies can afford large and varied stocks. Many of them have liquidity problems which inhibits keeping stocks. Retailers operate on relatively small margins, thus they rarely invest in goods that cannot be cashed relatively soon. Moreover, the variety of goods in the offer is increasing while their life cycles shorten on an accelerating pace. All these elements make keeping stock excessively expensive. Consequently, one of the main problems of the e-retailer is to coordinate deliveries from suppliers with customers’ orders: warehouse management and demand planning.22

For simplicity, our solution to these problems is intuitive and aims to meet only a few basic requirements: (i) the e-retailer does not keep goods in stock all the time - i.e., sometimes waiting time is non-zero; (ii) under some specific circumstances delays are possible. Perfect systems - with no delays and no defaults - can be designed only in theory, as all these warehouse management and demand planning algorithms follow from stochastic expectations based on past experience and cannot foresee the future perfectly. Therefore, in reality, for most e-retailers, both of the above requirements hold - not all the goods are always in stock and defaults occur.

Introduction of a combined delivery service (CDS) introduces additional complexity into the warehouse management and demand systems. From the e-retailer point of view one order is going to be composed of (possibly) much bigger variety of goods of different types. Accordingly, we assume e-retailer purchases quantity \( q_p(m,t) \) of every type of goods \( m \) at a wholesale price \( p_p(m,t) \). Moreover, we assume that there exists a certain minimal order quantity \( q_p^{\text{min}}(m,t) \) above which the transport from the producer becomes profitable enough for the e-retailer. There is also a certain delivery time of good \( m \) from the producers to the dispatching unit of the e-retailer - in our specification, the waiting time.

We assume the following ordering policy of the e-retailer. Let \( \text{initial stock}_m(t_1) \) be the initial stock of good \( m \) kept by the retailer at a certain time \( t_1 \) (either at the beginning of the simulation or after last order from the producer entered the stock) and assume that \( \text{initial stock}_m \ll q_p^{\text{min}}(m) \). Assume further that at some time \( t_2 > t_1 \) the e-retailer runs out of stock of goods \( m \). Referring to this experience, e-retailer can extrapolate the pace of future purchases of this good and estimate

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22One of the strategies adopted by e-retailers to circumvent inventory problems is drop shipping (see Khouja (2001)). In such a case, a retailer simply forwards customers’ orders to the manufacturer who fills the orders directly to the customers. Such a strategy would be obviously very difficult to apply in our setting as only goods produced by the same manufacturer could be pooled.
when the next delivery from the producer of good $m$ should take place, i.e. when future purchases will exceed $q_p^{\text{min}}(m) − \text{intended initial stock}_m$ (where $\text{intended initial stock}_m$ states for the number of items of good $m$ on stock after clearing all orders). Once the good from the offer is no longer in stock, a time estimate of the next delivery time is declared on the web in the form of waiting time announcement. To construct this estimate of the next delivery time, the e-retailer has to refer to the average time in which one item of good $m$ is purchased $E(pt_m)$, which can be easily computed as:

$$\frac{t_2 − t_1}{\text{number of purchases of good } m \text{ between } t_1 \text{ and } t_2}.$$  \hfill (5)

where $t_2 > t_1$. Then, waiting time $w(m, t)$ for a delivery of goods from producer $m$ announced by the e-retailer at time $t$ can be computed as:  

$$w(m, t) = E(pt_m)q_p^{\text{min}}(m)$$  \hfill (6)

Clearly, in the above warehouse management system is in a sense "commitment" because in this setting, it is virtually impossible to default on promised delivery time. Whenever a good is no longer in stock, the new arrival time is openly communicated to the consumers. Obviously, it will result in positive stocks from time to time if the e-retailer is not able to sell as many goods as it has been expected before the next transport is brought from the supplier. Importantly, although consumers are never deceived, this system imposes considerable cost on the e-retailer due to excessive stocks.

To circumvent this problem, alternative warehouse management system is also introduced, in which goods are brought from the producer if and only if the amount of items ordered is greater than or equal to $q_p^{\text{min}}(m)$. Consequently, positive stocks never occur. This system is in a sense "doomed to default", as at a certain point in time the e-retailer will make a delivery promise on which he will subsequently default in order to avoid positive stocks. Therefore some of the customers will face extended delivery times which adversely affects satisfaction. The "punishment" to the e-retailer will come in the form of increased purchases from the high street shop and thus lower customer base over the long term.

The "doomed to default" system allows us to introduce the crucial element in this model - the notion of failure to the e-retailer operations and thus consumer dissatisfaction with potential contagion in case of combined delivery service. Although, in this scenario, the e-retailer bears no unnecessary warehousing costs, with orders combined for dispatching, delays on some of them transmits automatically to defaults on others. In contrast, "commitment" mechanism justifies no

\footnote{Note that the above producers’ delivery scheme allows the e-retailer to pursue a whole range of stock policies.}

\footnote{Warehouse management systems that carry no stock have been already proposed in the literature. See, for instance, (Barnes-Schuster and Bassok 1997) and (Mitra and Chatterjee 2004). Our approach, is of course, simpler, however resorts to the similar idea.}

\footnote{Note that even top of the line e-retailers do not satisfy all the clients due to various reasons. See, for instance, opinion tags provided for every e-seller at Amazon.}
updating on the side of consumers, as goods are always delivered when promised, but it imposes
the burden associated with positive stocks at certain points in time. Figure (4) shows the example
of stock evolution in both systems. Negative values of stock mean that the good was ordered by
a consumer but is not on stock and is to be brought in the next transport from a producer.

3.6 Combined Delivery Service

Spontaneous endogenous coalition formation emerges from an efficiency gain as perceived by the
buyers. Such coalitions can only be formed if gains exceed the aggregated informational cost (at
least in the \textit{ex ante} perception). Coalitions triggered by an e-retailer emerge when the latter
observes an informational advantage at no additional cost of obtaining it - by providing combined
service delivery, shipment costs may be reduced to the customers' advantage. This provides an
argument finance-wise, to and convince the buyers - they either move along their indifference
curves to another price/delivery combination or to a lower disutility level due to (1) lower waiting
time at the same price, (2) lower price with the same waiting time, or (3) a combination of
the two. Obviously, spontaneous endogenous coalitions of buyers will only be formed if costs
are exceeded by the efficiency gain. In the case of combined delivery service coordinated by the
retailer, information costs converge to zero. Naturally, coalitions like this should only be stable
over the long run if they constitute a Pareto improvement, \textit{i.e.} none of the parties is worse off and
at least one of the stakeholders improves his situation. \footnote{Depending on the distribution of the efficiency gains, the satisfaction of consumers may be affected differently, but will never be lower. The mechanism of incorporating consumer satisfaction into the retailer optimisation problem allows to measure the "punishment" for defaulting on the offered transaction criteria. In the sensitivity analysis we demonstrate the effect on consumer satisfaction distribution in our model to adress implicitly the Pareto improvement problem.} It is easy to demonstrate, that such
a result is always achieved under the "commitment" scenario when all the goods are delivered as
planned. In contrast, under "doom-to-default" scenario, Pareto-improvement cannot be \textit{à priori}
guaranteed as some orders will be delayed, while some others will be affected by contagion.

We consider two types of combined delivery services. In CDS I, the e-seller pools together
similar orders but neither offers shipping discounts to clients nor informs them about the combined
delivery. This enables the e-seller to increase profits immediately, however, we may expect that if
delivery defaults are taken into account, this strategy will affect adversely consumer satisfaction.
Conversely, in CDS II we allow e-retailers to combine orders with differentiated order dates. In
real world, each e-retailer announces ”handling” time, which is the period between order approval
and order shipment. In principle, this period could serve to pull orders arriving within a short
interval of time, extending the potential number of coalitions. Finally, in both types of CDS in
the simulation we allow the e-retailer to return the gain from shipment cost reduction back to
the customers in an egalitarian way (no additional redistribution mechanism is introduced). This
reimbursement implies higher consumer satisfaction on one way and more purchases throughout the simulation horizon on the other.

[ Table (1) about here. ]

4 Simulation settings

In this section we present the most important aspects of the simulation setup including the consumer decision making process, combined delivery service procedures and parametrisation. E-retailer sells a vector \( m \equiv [1, 2, ..., m, ..., M] \) of different types of goods at prices \( \hat{p}_e(t) \equiv [\hat{p}_e(1, t), \hat{p}_e(2, t), ..., \hat{p}_e(m, t)]^T \) and shipment costs \( \hat{s}_e(t) \equiv [\hat{s}_e(1, t), \hat{s}_e(2, t), ..., \hat{s}_e(m, t)]^T \) to consumers from set \( \mathcal{N} \equiv \{1, 2, ..., n, ..., N\} \) populating (possibly in a semi-random way) the set of areas \( \mathcal{R} \equiv \{1, 2, ..., r, ..., R\} \) (ZIP-areas). Total prices of the goods for the consumer are denote by \( p_e(t) \equiv [p_e(1, t), p_e(2, t), ..., p_e(m, t)]^T \) i.e. the prices that include shipment costs, or \( p_e(t) = \hat{p}_e(t) + \hat{s}_e(t) \).

The basic public offer of e-retailer \( \Phi(t) \) at time \( t \) is composed of vectors of prices, corresponding shipment costs, waiting time and a shipment time, i.e. \( \Phi(t) = [\hat{p}_e(t), \hat{s}_e(t), w(t), z] \) where \( w(t) \equiv [w(1, t), w(2, t), ..., w(m, t)]^T \) is a vector of waiting times and \( z \equiv [z(1), z(2), ..., z(m)]^T \) vector of shipment times.\(^{27}\) Of course, if good \( m \) is on stock then \( w(m, t) = 0 \). Waiting time and shipment time combined are called delivery time and are denoted by \( d(t) \equiv w(t) + z \).

It is assumed that the utility of a consumer \( n \) from buying good \( m \) at prices \( p_e(m, t) \) or \( p_e(m, t) \) and delivery time \( d_s(m, t) \) or \( d_e(m, t) \) either from high street retailer \( s \) or e-retailer \( e \), respectively, is defined according to 4. Each consumer \( n \) lives only in one ZIP-area \( r \) (which will be denoted as \( n_r \)), obtains an income \( j_n \in [\underline{j}, \bar{j}] \), \( \underline{j} \leq \bar{j} \) per period \( (j_n \) is randomly predetermined at the beginning of simulations) and buys only one type of good \( m \in M \). Once enough money is collected to afford good \( m \), the consumers orders it either from the e-retailer or chooses a high street shop which in the model stands for all the (“rest of the world”) competition. An order of consumer \( n \) purchasing good \( m \) from the e-retailer at a certain time \( \hat{t} \) will be denoted by \( \theta(n_m) = [m, p_m, \hat{t} + d_n(m, \hat{t})] \) where \( \hat{t} + d_n(m, \hat{t}) \) is a delivery date promised by the e-shop at time \( \hat{t} \), i.e.: \( \hat{t} + d_n(m, \hat{t}) \equiv \hat{t} + w(m, \hat{t}) + z(m) \).

Simulations focus on the core variables of interest i.e. the profit of e-retailer and the consumer satisfaction. In order to demonstrate the stability of results next section shows simulations performed for differentiated parameters of the utility function, a spectrum of population density and different ratios of shipment cost to retailer markup.

4.1 Consumer decision making

Let \( J_n(t) \) be the total wealth of consumer \( n \) at time \( t \). A purchase of a good is made simply once a total wealth of a consumer surpasses the price of the good. We assume the following decision

\(^{27}\) For simplicity we assume that shipment time is constant in this model.
rule of consumer $n$:

$$U_s(n, m, t) \geq \frac{U_e(n, m, t)}{stf_n(t)}$$

i.e. consumer $n$ compares both utilities adjusting them with the satisfaction parameter and buys good $m$ from e-retailer if both $J_n(t) \geq p_e(m, t)$ and (7) hold. If any of these two conditions do not hold and, at the same time, $J_n(t) \geq p_s(m, t)$ then the good is bought from the high street shop. Positive experience of a consumer with the e-retailer (i.e. $stf_n(t) > 1$) decreases disutility from the e-purchase.

The important feature of the above consumer satisfaction measure and decision making process is that it may create a (long term) customer loyalty which has vital consequences on long-term e-commerce profitability.\footnote{According to Reichheld and Schefter (2000), mainly due to enormous multi-dimensional competition, acquiring customers on the internet is expensive, and a creation of a base of loyal customers, which come back over the years, is the first-order condition for long-term success. However, Ribbink, Liljander and Streukens (2004) point out that only few companies seem to succeed in creating e-loyalty, and, as far as now, little is known about the mechanisms involved in generating it.} In our approach, a decision to purchase a good at e-retailer is an interplay between relationships between e/hs current price, e-delivery time (embodied in the utility) and client’s past e-loyalty stock-piled in $stf_n(t)$. By lowering prices (and/or delivery times), the e-retailer has always a possibility to rebuild consumer satisfaction which deteriorated due to various failures in the past, however, such a behaviour is very costly in the long term. The issue of loyalty deterioration is especially important in our model, as goods bundling means that any failure in on-time delivery is profiliated to a group of customers and non only one of them. A delay can be interpreted as any failure from the e-retailer side, and, in other words, depicts the overall e-service quality related to goods delivery.

4.2 Combined Delivery Service

The crucial problem in implementing the combined delivery system concerns the algorithm of pulling together orders from the same location (e.g. ZIP-code area). Let $\Theta_r(t)$ be the set of current orders (on-going orders) in ZIP-code $r$ at time $t$. Then, this set can be partitioned in disjoint groups of orders pulled together. Of course, many of such coalitions can be so-called trivial coalitions - i.e. consist of only one order, which means that no pooling was possible. More formally, a coalition is any non-empty subset of $\Theta_r(t)$, and will be denoted by $C_k(t)$. The cardinality of a coalition $C_k(t)$ is the number of orders (players) in this coalition and will be denoted by $|C_k(t)|$.\footnote{Please note that in our setting one customer can have only one current order at a time.}

Definition 1 A dynamic coalition structure $\pi(\Theta_r(t)) := \{C_1(t), C_2(t), ..., C_m(t)\}$ is a partition of the orders/customers set $\Theta_r(t)$ into coalitions at time $t$; hence, for every time $t$ coalitions

$$\pi(\Theta_r(t)) := \{C_1(t), C_2(t), ..., C_m(t)\}$$
within coalition structure it satisfy: $C_k(t) \neq \emptyset$ for $k = 1, 2, \ldots, m$; $\bigcup_{k=1}^{m} C_k(t) = \Theta_r(t)$ and $C_k(t) \cap C_l(t) = \emptyset$ if $k \neq l$.

Let $C_k(t) \in \Theta_r(t)$ and let $S_{C_k}(t)$ be the sum of shipment costs of all the customers in coalition $C_k(t)$ at time $t$ under the condition that all the packages are sent separately (i.e. in a standard way, without CDS). More formally, $S_{C_k}(t) = \sum_{m} s_m(m_n)$. Let the negotiated cost of courier service provided assure convexity, for example the following shorthand formula for a discount function (2):

$$CDS_{C_k}(t) = \frac{1}{2} \sqrt{S_{C_k}(t)} \left(\sqrt{S_{C_k}(t)} + 1\right),$$

(8)

Hence, the surplus (or saving) from $CDS$ can be easily computed as:

$$S_{C_k}(t) - CDS_{C_k}(t) = \frac{1}{2} \sqrt{S_{C_k}(t)} \left(\sqrt{S_{C_k}(t)} - 1\right).$$

(9)

[Figure (5) about here.]

Figure (5) is an example of the costs of shipment with and without $CDS$ system for $C_k(t)$ which consist from 1 to 5 players, i.e. $|C_k(t)| = 1, 2, \ldots, 5$. The difference between $S_{C_k}(t) - CDS_{C_k}(t)$ depicted as a shadowed area on Figure (5) represents the efficiency gain due to $CDS$. Of course, the surplus exists only for integer numbers on the horizontal axis. Importantly, from the point of view of the e-retailer, every coalition $C_k(t) \in \Theta_r(t)$ is seen as orders pulled together. Thus, denote it by:

$$\theta(C_k(t)) = \left[ \sum_{C_k} m, \sum_{C_k} p_m, t + d(C_k(\hat{t})) \right],$$

(10)

where $\hat{t}$ is the time when coalition $C_k(t)$ was created and $d(C_k(\hat{t}))$ is its delivery time.

### 4.2.1 Mechanisms for exogenous coalition formation

Following rules were imposed on combined shipment mechanism.

**CDS I** Let customer $n_m$ from ZIP-code $r$ make an order $\theta(n_m) = [m, p_m, t + d_n(m, t)]$ at time $t$ by the e-retailer and let $\Theta_r(t)$ be the set of all current orders from ZIP-code $r$ partitioned by the e-retailer into a dynamic coalition structure $\pi(\Theta_r(t))$. In CDS I, if $\theta(n_m)$ is not the only order from this ZIP-code or $\Theta_r(t) \neq \{\theta(n_m)\}$, then the e-retailer pools order $\theta(n_m)$ with such a (possibly trivial) coalition $C_k(t) \in \Theta_r(t)$ for which the delivery time of customer $n_m$ and coalition $C_k(t)$ is the same. Note that this choice is a bijection, since there cannot exist two different coalition of orders from $\Theta_r(t)$ with the same delivery time. In CDS I no party risks anything.

It is clear that CDS I is an effective mechanism since it leads to a Pareto improvement. It is also an exogenous coalition formation mechanism that could be called natural, since it only makes the use of the fact that the information about orders is centralized in the retailer’s IT systems.
However, its use is limited only to orders with the same delivery date. If the e-retailer wants to increase the room for pooling coalitions he could artificially create this room, by differentiating the dispatching time from the availability time, for example by introducing handling time.

**CDS II** Assume for simplicity that the e-retailer decides to add additional time to delivery time \( d(m,t) \) of every good called "order handling" and denoted by \( h_m \geq 1 \). Consequently, \( d(t) \equiv w(t) + z + h(t) \) and from the point of view of the e-retailer the delivery time of good \( m \) to a client \( n \) is no longer a point in time (a day) but a time span \( (d_n(m) - h_m, d_n(m)) \). In other words, we assume that handling time \( h_m \) does not serve as expected maximum handling time and can be shortened on a case by case basis, thus allowing the e-retailer to gain space for pulling together orders.\(^{31}\) Importantly, an e-retailer will not indulge the temptation to forgo some of the clients (and thus some of the revenues) by prolonging total delivery time in order to gain more space for exogenous coalition formations (and thus cost reduction). The construction of the utility function as well as the mechanism of consumer satisfaction both insure a "punishment" in the form of revenue loss if handling time is excessively prolonged. Thus, benefits follow from the cost reductions on coalitions with forced delays, but the prolonged waiting times and delays turn customers away from the e-retailer to the high street shop. Therefore, there is always a profit maximizing optimum, especially over the longer run.

The algorithm for CDS II runs as follows:

1. If \( \theta(n_m) \) is not the only order from this ZIP-code, or \( \Theta_r(t) \neq \{\theta(n_m)\} \), then the e-retailer pools order \( \theta(n_m) \) with such a (possibly trivial) coalition \( C_k(t) \in \Theta_r(t) \) for which there exist a common element between delivery time spans of customer \( n_m \) and coalition \( C_k(t) \), or
   \[ \langle d_m(n) - h_m, d_m(n) \rangle \cap \langle d_m(C_k(t)) - h_m, d_m(C_k(t)) \rangle \neq \emptyset; \]

2. If there are more than one coalitions satisfying condition (1), denote the set of them by \( C'(t) \). Then, the order \( \theta(n_m) \) is pulled together with the coalition of the highest cardinality, or
   \[ \max_{C_k(t) \in C'(t)} |C_k(t)|; \]

3. If there are more than one coalitions satisfying condition (2), denote the set of them by \( C''(t) \in C'(t) \). Then, this coalition is chosen for which order \( \theta(n_m) \) has has the longest common element of delivery time spans, or
   \[ \max_{C_k(t) \in C''(t)} \{ \langle d_m(n) - h_m, d_m(n) \rangle \cap \langle d_m(C_k(t)) - h_m, d_m(C_k(t)) \rangle \}; \]

\(^{30}\)This approach conforms with the real-world observation that e-marketplaces typically send the message "in stock, dispatched in \( h_m \) days" rather than "in stock in \( h_m \) days, dispatched immediately after".

\(^{31}\)This coalition formation mechanism is quite common for courier companies, where the promised handling time reflects the internal target handling time. It allows to flexibly adjust the order of orders handling as well as possible overtime of the workers, subject to the promised maximum and temporary work load.
4. If there are more than one coalitions satisfying condition (3), denote the set of them by \( C''(t) \). Then, this coalition is chosen for which the final delivery time is the longest, i.e.

\[
\max_{C_k(t) \in C''(t)} \{d_m(C_k(t))\}
\]

It is easy to show that the above algorithm turns the problem of the coalition choice for \( \theta(n_m) \) into bijection. The intuition behind it is straightforward. To maximize a surplus from the CDS the e-retailer pulls the new order with this (possibly trivial) coalition with a common part of a time span (condition 1) which has the highest cardinality, i.e. has the highest number of players (condition 2). Such a strategy directly follows from (9) which is an increasing function of a coalition’s cardinality. Furthermore, if there is more than one coalition satisfying condition (2) then the e-retailer opts for the longest common element of delivery time spans in order to (logistically) ease the process and minimize the probability of any default (condition 3). For the same reason, he also favours longest delivery time (condition 4).

Figure (6) presents the example where:

\[
\Theta_r(t-1) = \Theta_r(2) = \{\theta \left( C_1^{t-1} = (1_{m_1}, 2_{m_2}) \right), \theta \left( C_2^{t-1} = (3_{m_3}, 4_{m_4}) \right)\},
\]

with \( \theta(C_1) = \{\lfloor m_1 + m_2, p_1 + p_2, 2 + 2 \rfloor\}, \theta(C_2) = \lfloor m_3 + m_4, p_3 + p_4, 2 + 2 \rfloor\} \) and the new order \( \theta(5_{m_5}) = \lfloor m_5, p_5, 3 + 3 \rfloor \) is placed at \( t = 3 \). In other words, by time \( t = 3 \) there have been 5 current orders placed, 4 of which have been already divided by the e-retailer into two coalitions \( C_1 \) and \( C_2 \) and there is a decision being made about what to do with a new order \( \theta(5_{m_5}) \). As \( \theta(5_{m_5}) \) has a common element with both coalitions, i.e. both \( C_1 \) and \( C_2 \) satisfy condition 1, and they both also satisfy conditions 2 and 3 as having the same cardinality as well as common element of the delivery time span, the e-retailer chooses to pool \( \theta(5_{m_5}) \) with \( C_2 \) as final delivery date of this coalition is longer. Hence, \( C_2 = (3_{m_3}, 4_{m_4}, 5_{m_5}) \).

4.2.2 CDS I and CDS II with transfers

Under both CDS scenarios described above one should expect cost reduction compared to the case where all orders are shipped separately. Therefore, surplus is created. Depending on the type of mechanism designed e-retailer can further use this surplus to (i) boost his profits; (ii) lower the prices ex ante to all customers with the amount proportional to the expected surplus, thus positively influencing the competitive edge vis-a-vis the high street shop; (iii) decrease the prices ex ante for the coalition members (return the appropriate funds to their accounts).\(^\text{(32)}\) If the e-retailer decides to distribute the surplus, he can still decide on the distribution mechanism, withholding part of the gain.

\(^\text{(32)}\)For the sake of simplicity we assume that e-retailer has no investment needs that could be turned into smoother warehouse management system, better CRM, etc.
4.3 Parametrization

A natural benchmark corresponds to the situation in which no CDS is provided. We assume the following values for basic parameters: number of goods $\bar{m} = 30$; number of consumers $\bar{n} = 2000$; number of ZIP-areas $\bar{r} = 300$; e-retailer prices $p_c(t)$ are randomly chosen from the set $\{21, 22, ..., 27\}$, i.e. $21 \leq p_c(t) \leq 27$; shipment cost is the same for every good, $\hat{s}_c(t) = 2.4$; while shipment time $z$ is set at 11 for both benchmark scenario and CDS I. For the case of CDS II we assume that total waiting time is still 11 days, but it consist of 3 days handling time and 8 days actual shipment time.\(^\text{33}\) For every good $q_p^{\text{min}}(m) = 20$, initial stock $m$ is chosen randomly, and intended initial stock $m$ is 0 for simplicity. The price comparative advantage of the e-retailer to the high-street shop $p_E/p_{HS}$ is set to 0.8475. Although this choice may seem arbitrary, the main motivation was to assure that both markets (electronic and traditional) exist and none of them dominates. For the chosen parameters, the comparative advantage of 0.8475 serves as a guarantee that none of the channels is effectively threatened by the other. The mutual relations between the sizes of both markets and the choice of comparative advantage is depicted by Figure (7), where point $A$ denotes chosen specification.

[Figure (7) about here.]

As far as consumers’ parameters are concerned, we may assume without a loss of generality that $\alpha = 1$ (i.e. price plays a role of the *numeraire* unit for other variables). In other words, it is not the price per se that is important, but the size of the relation between prices and other variables. Preference parameters of the utility curves are allocated to the consumers $\beta_n \in (0.9, 1.1)$ and $\gamma_n \in (-0.1, 0.1)$. Furthermore, income $j_n$ is randomly allocated from a set $\{1, 2, ..., 50\}$ where at every point in time income of consumer $n$ grows or not with this number with even odds.

When analyzing this parametrization, there are few main points that require justification. There are two important groups of parameters. The first one concerns the rate of income arrival (resulting in a number of total purchases made by clients), whereas the second one decides on the share of e-purchases. To the first group consists of average number of consumers per ZIP-code (determined by $\bar{n}$ and $\bar{r}$), price levels (defined by vector $p_c(t)$) and income $j_n$. Values of these parameters determine how often average consumer has enough income to purchase a good either from the e-retailer or the high-street shop and this, in turn, is one of the factors determining the probability with which the e-retailer will have an occasion to pool orders together in every ZIP-code. For the above parametrization there are on average 6,66 consumers per ZIP-code and each of them makes a (e- or traditional) purchase once in every three weeks (21 days). The sensitivity of this group of parameters is checked (see Subsection 6) by varying the average number of consumers in ZIP-codes. Similar effect would be obtained by changing either the average price level or income.\(^\text{33}\)

\(^{33}\) Sensitivity of the results to the chosen handling/shipment time division is presented in Section 6 along with other parameters choice.
To the group of parameters that determine the number of e-purchases in relation to traditional shopping belong the relationship of the price comparative advantage of the e-retailer to the high-street shop $p_E/p_{HS}$ to the web shipment cost to price level ratio $\hat{s}_e(t)/p_E$, and finally the average waiting time for the goods ordered via internet. In each of these cases (utility function parameters, handling time, shipment cost and competitive advantage) the choice for the parameters was a consequence of the pursuit to assure comparable number of e-purchases and traditional purchases.

As we argue, it is not the value of shipment cost per order that matters here, nor is it the price vector of the goods, but the relation of between this ratio and the comparative advantage of the e-retailer\textsuperscript{34}. And, once again, Figure (7) supports our specification. Nonetheless, section 6 presents results stability with respect to this ratio as well.

Secondly, the values of $\beta$ and $\gamma$ were chosen in such a way, that consumers chose from both types of the retailers, sometimes switching between traditional intermediary and the e-seller. Since, to our best knowledge, studies over the exact value of $\gamma$ are not available, we decided to center it at zero. Consequently, both types of substitution effects discussed in Subsection 3.4 occur. Of course, the intervals over which $\beta$ and $\gamma$ are distributed were chosen arbitrarily, but one needs to bear in mind that these parameters play the role of semi-elasticities. The higher their values, the less responsive the consumer to changes in $p_E/p_{HS}$ ratio (i.e. the price competitive advantage is less important). Therefore, the chosen sets of moderate values assure that most of the clients will not be by definition inclined to buy only from the e-retailer or only in a high street shop\textsuperscript{35}.

5 Results

Table 2 compares main results of no CDS with CDS I and CDS II - columns (1), (2) and (3) under the commitment scenario. In addition, based on the initial simulations, relative size of reduction was estimated, allowing to incorporate ex ante reduction into pricing strategies of the e-retailer (CDS I as well as CDS II with transfers). Namely, the results of a general price cut for all customers were estimated and these ”cuts” were distributed among the clients in the form of price discount (average percentage to every initial list price).

Table (2) about here.

In the benchmark scenario a total of 2000 customers made 18 143 HS-purchases and 16 432 E-purchases. In other words, the setting is parametrized in a way allowing approximately half of

\textsuperscript{34}It was assumed in the model specification that the shipment cost amounts to 2.4, while the average price of the goods purchased equals 24. However, not these values per se but their relation constitutes the key driving factor.

\textsuperscript{35}Some consumers with extreme values of $\beta$ and $\gamma$ buy goods either only from high-street shop or only from the e-retailer. However, the whole spectrum of consumers in between sometimes chooses one and sometimes the other marketplace. Such a parametrization ensures us that changes in delivery times and prices have a visible effect on sales (if most consumers were extremely dedicated to one of the shops then changes in any of these values would not affect sales).
the purchases to be conducted via the internet. In addition, results seem fairly robust to the choice of \( \alpha, \beta \) and \( \gamma \) parameters as more than 60% of consumers buy goods from both the e-retailer and high street shops. Those are clients who the e-retailer should primarily fight for. Since in this model we do not control for costs other than supplies, calculating e-retailer’s profit seems only marginally valuable and therefore is not reported.

Mechanism CDS I shows no change in sales and number of transactions as only natural coalitions are implemented by the e-retailer (orders made from the same location and on exactly the same date). Therefore, the only difference to be observed concerns the costs of shipment, which are significantly reduced. Mechanism CDS II fosters slightly both sales and costs of supplies because more coalitions may be formed. This facilitates purchases by the clients (the growth of 27 orders in total) due to possibly shorter waiting times for some clients (the frequency of purchases increases). More importantly, shipment costs are significantly lowered and both the number and the average size of the coalition grows considerably.

The results of columns (1), (2) and (3) clearly demonstrate that exogenous formation of coalitions creates efficiency gains, lowering significantly shipment costs. There is also an observable increase in consumer satisfaction comparing benchmark and CDS I to CDS II (1.0213 and 1.0254), due to benefits some clients incur from shorter than 11 days delivery times in some cases. Correspondingly, the reduction due to combined shipment ranges from 0.47% of input purchases for CDS I (in the simulated example: £1 909) to 1,72% of input purchases for CDS II (in the simulated example: £5 103).

Naturally, the shipment cost reduction can be redistributed from the e-retailer to the consumers. Result for this scenario are presented in columns (4) for CDS I and (5) for CDS II. Consumers automatically buy more from the e-retailer, but total purchases increase due to income effect induced by price reductions. Namely, with the average price decreasing by 0.47% for CDS I and 1.72% for CDS II, relative income of the consumers subsequently grows which fosters the growth of purchasing power and thus purchases. Obviously, this is a fading out pattern. Nonetheless, more coalitions are formed with fairly comparable coalition size, resulting in significant average shipment cost reductions.

As suggested earlier, the ex ante reduction can alternatively be distributed only among the coalition members (e.g. returned to their accounts). Evidently, the calculated reductions of £1 909 in the case of CDS I and £5 103 for CDS II might result in higher eventual price reductions if their coverage is reduced to coalition members only. Similarly, income effect will be stronger among these consumers. With all the reservations described earlier, such a distribution strategy would boost the consumer satisfaction to 1.0233 and 1.0254 for CDS I and CDS II respectively.

\[36\] In addition, we are not modeling the profit maximizing behavior of the e-retailer, hence we are unable to ascertain if price reduction is rational. More precisely, e-retailers in our model simply provide a variety of goods at certain pre-defined prices and do not have any explicit pricing strategy. Therefore, we cannot undermine or confirm the validity of the amount of the price reductions. Standard profit maximising behaviour could make these reductions even larger if the demand elasticity was above unity for respective prices.

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Simulation results show that the size of average coalition as well as the number of non-trivial coalitions remain fairly stable (average coalition size reach to 2.09 and 2.35, while the number of pooled deliveries to 1456 and 4651).

5.1 Combined delivery systems under ”doomed to default” scenario

As suggested earlier, the warehouse management system in which clients are only offered goods that are already in stock (or, equivalently, are informed of the waiting times including the delivery time from the suppliers) can never result in deception of the customers. More explicitly, recalling Figure (1), under the ”commitment” scenario, customers can never arrive right of the expected disutility. Correspondingly, only with CDS II can they appear left of this point. However, under ”doomed to default” scenario with CDS II some clients might be forced to wait longer for two reasons already: (i) their good is not in stock and will only arrive once the sufficient number of orders is collected by the e-retailer; (ii) their good is in stock but their shipment is combined with another order that is not in stock for reasons described in (i). The latter option is equivalent to contagion effect, where the warehouse management system shortcomings spread through the exogenous coalition formation mechanism. Thus, it seems particularly interesting to inquire this scenario.

Results of simulation under ”doomed to default” scenario with the same parameters as previously. Similarly to Table (2), no combined delivery service are compared to CDS I and CDS II - columns (1), (2) and (3) respectively. Similarly as above, general *ex ante* price reduction are reported in columns (4) and (5) for CDS I and CDS II, respectively.

As expected, these results demonstrate a decrease in consumers’ satisfaction due to delivery delays. Interestingly, the size of the contagion effect must be negligible compared to the number of clients who benefit from earlier arrivals, since consumers’ satisfaction is highest under CDS II scheme. Nonetheless, the number of e-purchases is lower (due to lower overall coefficient of consumers’ satisfaction from interactions with the e-retailer), while the e-retailer’s share in the overall sales falls short of the outcomes under ”commitment” scenario.

The most important conclusion, however, concerns the possible size of the contagion effect. If orders are pooled while no delays can occur (as under ”commitment” scenario), CDS can only introduce benefits. However, if delays are in principle possible (as under ”doomed to fail” scenario), one delayed delivery can be transmitted to other clients, thus deteriorating their satisfaction as much as the satisfaction of the consumer who ordered this particular good. Of course, the more orders in coalitions, the wider the range of the contagion effect. On the other hand, the larger the coalitions, the higher the shipment cost reduction and thus possible compensation to the consumers can also be higher.

[Table (3) about here.]
The size of the contagion effect due to the CDS combined with the possible price reductions for consumers can be inferred from the satisfaction of consumers in Table (3). The satisfaction falls comparing to the benchmark when CDS I is introduced exactly due of the contagion effect. Conversely, satisfaction increases after the introduction of CDS II because some orders are delivered before time (it is more beneficial for the e-retailer to shorten the handling time - at an additional expense - for some orders than to deliver them later, when other ordered goods arrive in stock), thus overweighing the impact of delivery defaults.

6 Analysis of sensitivity to parametrisation

As it was pointed out before there are three important assumptions concerning the benchmark parametrization which might have significantly influenced the results, either biasing them or excessively increasing their size. These are: (i) population density (the number of customers at each location), (ii) handling time length (the relation of handling time to shipment time)\textsuperscript{37} and (iii) the comparative advantage ratio (the ratio between shipment cost and price of products in relation to the price edge enjoyed by e-retailers). The sensitivity analysis with respect to these three assumptions shall now be presented.

6.1 The effect of population density

The results concerning the ratio of number of consumers to the number of locations are presented in the panel on Figure (8). As can be inferred, satisfaction is fairly stable for all the densities. Average size of non-trivial coalitions obviously grows with the increase of population density, but room for shipment cost reductions appears as soon as there are at least two people per one location. The positive values for the density of one result from stochastically probable but rather unfeasible in reality occurrence, that one person places two separate orders within a time sufficiently short to permit combined delivery. Similar holds for the average cost of shipment, as depicted by Figure (9) and for average size of combined order, as depicted by Figure (10).

\[\text{Figures (8), (9) and (10) about here.}\]

From the analysis of these graphs, one could not support the hypothesis that results presented in Table (2) and (3) are driven by the choice of 2000 consumers populating 300 areas. Interestingly, results are fairly stable over warehouse management system chosen (WMS 1 and WMS 2), distinguishable only for CDS I and CDS II cases.

\textsuperscript{37}For the sake of argument in the case of CDS II it was assumed that handling does not prolong waiting times, which still cannot exceed 11 days. However, the longer the handling time, the larger the "space" for coalition formation and thus possible range of cost reductions.
6.2 The effect of handling time

If one considers the choice of handling time duration despite the initial choice of 3/8 days proportion, here too results seem to be robust as well. Of course, differences are stark between "commitment" and "doomed to default" scenarios, but remain essentially stable within each of these cases.

[Figures (11), (12) and (13) about here.]

Naturally, the higher the time, the lower the average consumer satisfaction - Figure (11), the size of average combined order - Figure (12), and consequently the shipment savings - Figure (13). This is mainly because the purchases over the internet will be less frequent. On the other hand, Figure (12) suggests a fading out pattern, which implies that even for longer waiting times, there is still room for e-markets.

6.3 The effect of comparative edge

Finally, it seems that the simulation results are independent of the assumed comparative advantage ratio with respect to the shipment/price relation. More specifically, consumer satisfaction remains essentially unaffected, as depicted by Figure (14), while average combined order size and shipment cost reductions grow mildly the more competitive e-retailer becomes - Figure (15) and Figure (16), respectively. Both of these patterns follow intuitively from the model specification. We observe no stark changes or unpredicted drops/hikes with the change of this ratio.

[Figures (14), (15) and (16) about here.]

This finding is actually a very strong confirmation of the chosen parametrisation. Namely, an argument raised against the simulation results might have been put, that customers only buy on-line, because we have artificially exaggerated the comparative pricing edge by the e-retailers. This "inflated" market base - in turn - created room for irrationally high gains from combined delivery service and the resulting shipment cost reductions. In fact, the main driving force behind willingness to buy on-line - consumer satisfaction - is extremely stable over the alternative specifications. Results of combined order size and shipment cost are naturally susceptible to CDS I and CDS II inclusion, but they are stable over the warehouse management systems. This suggests that potential benefits for e-retailers to abuse consumers' trust in the declared delivery times are not outweighed by the disappointment "punishment".

6.4 The effects on welfare

Last, but not least, one needs to analyse if combined delivery service is indeed welfare improving. In order to guarantee long-term profitability of this undertaking, e-retailers would need to be assured that over the longer perspective consumers observe gains in terms of satisfaction. Average
satisfaction - as reported above - is only a synthetic measure. Consequently, it could actually occur that e-retailers lose some clients, while some of them observe explosively high utility levels. To inquire how is general welfare affected by combined delivery service, we have analysed the distribution of consumer satisfactions under all scenarios. This is depicted by Figure (17).

(Figure (17) about here.)

Four panels represent distributions (kernel density estimates) for varied scenarios in this model. Naturally, the case of no combined delivery service overlaps perfectly with CDS1 under "commitment" warehouse management system (top left panel). Consistently, scenarios allowing CDS are shifted to the right with better (higher satisfaction) outcomes more frequent. This implies directly, that consumers - as a whole - are better off upon the introduction of combined delivery service. Consequently, at least weak Pareto improvement can be proven - e-retailers increase their profits while customers as a group are at the very least in a comparable situation.

6.5 Business case

As demonstrated by the sensitivity analysis, our results do not seem to be driven in any way by the choice of the crucial parameters. This property is especially important from the business perspective. Namely, our conclusions of considerable economic gains due to the shipment cost reductions (especially in the scenario allowing transfers back to the customers) are robust to the potential specificities of particular markets. Putting some real-world numbers allows to obtain a business-wise conclusion from the model.

Namely, consider that over \( w \) weeks (5\( w \) working days) there are 3 orders made from a certain area with separate shipment costs \( c_1, c_2 \) and \( c_3 \), which enable some form of combined delivery (sufficiently close period of time between orders). For such a setting, there are five possible configurations of the orders. Assuming that the purchases are independent events, probabilities of every possible combination (each separately, all together and three pairs) one can easily compute their probabilities. Computing the theoretical expected value of savings from combining shipments yields:

\[
EV(CDS) = \frac{10w - 1}{25w^2} (c_1 + c_2 + c_3) - \frac{5w - 1}{25w^2} (f(c_1 + c_2) - f(c_2 + c_3) + f(c_1 + c_3)) \quad (12)
\]

\[
- \frac{1}{25w^2}f(c_1 + c_2 + c_3),
\]

where the exact value of expected savings depends on the cost function \( f(.) \) as well as the ratio of respective costs \( c_1 : c_2 : c_3 \). For a square root function of the sum of shipment cost, \( i.e. \)

\[38\]In principle, to prove strict Pareto improvement, instead of consumer satisfaction distribution, one should focus on the distribution of the difference between satisfaction under no CDS scenario and all eight others. However, in our setting income arrivals, prices and purchases are governed by a stochastic scenario, which implies that each time a simulation is run, "a consumer" is not the same as in the previous analysis. Therefore, calculating the difference is virtually impossible.
\[ f(c_1 + c_2 + ... + c_n) < \sqrt{c_1 + c_2 + ... + c_n} \] and with \( c_1 = 1L; c_2 = 2L \) and \( c_3 = 3L \) one obtains an expected shipment cost reduction of 11.24%. If one considers that e-retailers operate on profit margins of approximately 3-6%, where shipment costs provide on average up to 10% of price paid by the consumers, such a reduction in expenses may serve to actually double the eventual profit margin \textit{ex post} or to halve the mark-up forced on consumers \textit{ex ante}.

The applicability of suggested combined delivery schemes is actually quite probable. Naturally, none of the delivery service providers would be interested in implementing (on e-retailers behalf) combined delivery service, because exactly his profits (resulting from economies of scale) are ripped off. However, e-retailers may use publicly available "access points" (like Boots in the UK, post offices, drug stores networks, even traditional retailers operating under the same brand name, e.g. Barnes&Nobles or Borders). In many countries, purchases from e-retailers are delivered to the closest post-offices (eg. Germany, Poland), while in some others one already observes attempts to lower shipment costs by making it possible to the customers to pick up ordered items from relatively frequently visited sites. With the decrease in product prices, this is likely to become an increasingly important area of comparative advantage with reference to traditional retailers and e-competitors. We believe one is likely to observe more of such initiatives emerging.

### 7 Other extensions and future work

In the previous sections we have analysed two interesting but relatively simple combined delivery service schemes. However, one can think of a whole spectrum of algorithms to pool orders, comprising even treachery by the retailer in waiting time information disclosure. One such examples would be CDS with surplus and information sharing, where buyers are informed about the CDS system even before they make a decision to purchase a good and they are offered the surplus from CDS as a form of compensation in the case of prolonging the delivery period. In the short-term this CDS strategy is not directly profitable to the e-seller, but in the long-term can bring more profits from increased demand which follows from two main sources: (i) higher propensity to buy in the e-marketplace and (ii) lowering of the effective prices.

While making a decision to make a purchase the client is offered to participate in CDS III by choosing one of the options from the set \( \Omega \):

\[
\Omega = \left\{ (0, 0, 1), (\hat{\mu}_1, \tilde{d}_1, \tilde{r}_1), (\hat{\mu}_2, \tilde{d}_2, \tilde{r}_2), \ldots, (\hat{\mu}_k, \tilde{d}_k, \tilde{r}_k) \right\}
\]  

where \( \hat{\mu}_i \) are discounts offered against delays \( \tilde{d}_i \) and \( \tilde{r}_i \) indicate the probability that the offered discount is actually achieved for \( i = 1, 2, \ldots, k \), \textit{i.e.} the odds that an order will be pooled together. If \( \tilde{r}_i \) than all the risk is born by the e-retailer and are not visible to the client. Conversely, a mechanism where the risk is announced by the e-retailer and born by the customer is also possible, as well as all the intermediate solutions.

Whatever risk sharing rule is introduced, orders are subsequently pulled according to CDS II,
subject to the consent of customers. Note, that contrary to the case of CDS I and CDS II, in CDS III, even under the “commitment” scenario, delays are possible in principle, but they are not going to deteriorate consumers’ satisfaction as delays are now consulted with the customer. In all forms of CDS III it is crucial to appropriately evaluate Ω elements, i.e., correctly predict the probability of success, which obviously can follow only from the registers of actual sales. Evidently, the negotiations mechanism requires certain effort at the side of consumers. Nonetheless, this form of combined delivery service mechanism could be particularly valuable in B2B e-marketplaces, where customers do exercise the communication effort on an everyday basis while risk sharing and information disclosure are common characteristics of relations even without CDS.

The exogenous coalitions formed by the e-retailer can also be supplemented with the endogenous coalitions emerging between the customers. There are, however, difficulties in designing an efficient and user friendly protocol of shipment endogenous coalition formation. Note that whereas shipment savings can be considerable for the e-retailer, they are certainly less important from the individual point of view.

The results of this paper are undoubtedly influenced by the form of the utility function imposed. Notably, the parametrization of α, β and γ implicitly defines how many of the customers are interested in the e-purchases at all, how many of them are willing to switch subject to the particular offer, etc. Furthermore, the quadratic function influences the size of the discount savings. It can be argued that the linear quadratic form of the (dis)utility function punishes small delays of goods with a very long delivery time too greatly. Hence, it could be interesting to consider more carefully the specific characteristic of time in the utility function. Observing the sensitivity of results to parametrization of the utility function as well as to the functional form of the curve seems crucial to confirm the generality of this paper findings.

Finally, as was already mentioned in section 3.4, in this setting customers are not Bayesian in the sense that they have explicit expectations formation mechanism. In the case of a delivery delay, their opinion about the e-retailer is influenced, thus influencing the customer’s future choice between an e-marketplace retailer and a high street shop. However, they are not in any sense evaluating the information about delivery date provided by the e-retailer. In this sense, they unambiguously trust e-retailer, while the mechanism of consumer satisfaction is more of a retaliation scheme than forward looking device. Extending the framework to comprise the expectations component seems a valid direction of future research.

8 Conclusion

Spontaneous coalition formation can be performed by buyers on their own. People exhibiting common characteristics (e.g. inhabiting one ZIP area) could meet and agree on their needs, subsequently posting a combined order on the e-marketplace. Obviously, they would not notify the e-retailer about the coalition they have formed and they alone would enjoy the benefits of any
volume discount as well as any shipment discount, if a combined delivery service was available. This scenario, however, involves a coordination effort, thus imposing a necessary cost, a cost that is not subadditive in a sense that all coalition members need to bear it irrespectively of the number of buyers already forming a coalition. Moreover, unlike in some of the coalition formation approaches, this cost cannot be overcome by the means of silent participation strategy. One could argue, that these costs are still sufficiently high to prevent the emergence of endogenous coalitions in our everyday life, thereby explaining why one rarely finds them in reality.

Although a convincing argument, this paper provides an alternative explanation. So far in the literature it was assumed that this cost is overweight by the coalition members’ “fee” in favour of the most active agent bundling the orders together. Consequently, a more active agent is nothing but a shop - one more intermediary facilitating the flow of goods. This paper argues that an exogenous coalition formation mechanism can be implemented with the retailer taking the role of the coordinating agent, bundling orders from similar locations in order to benefit from a shipment discount operating still on his attributive volume discount.

Collecting purchases from different locations, an e-retailer is in possession of a unique advantage vis-à-vis potential coalition members, namely he knows already what purchase orders have already been placed. This allows him in principle to offer a combined delivery service to buyers that came later with their purchase orders, thus overcoming the informational cost. Importantly, coordination issue can pose an obstacle to coalition formation also in another aspect. Notably, some of the potential coalition members may have delivery times considerably shorter than some others, thus threatening the stability of a coalition. Therefore, optimal stock levels are affected adversely by the introduction of CDS; for a retailer with lower stock levels coalitions can be formed less frequently than for those with higher availability of items ceteris paribus.

This paper demonstrates that a combined delivery service can constitute an exogenous coalition formation mechanism, while the rentability of this solution depends on the preferences of the consumers as well as - crucially - on the relation of shipment costs to the price of goods purchased. The main findings of this paper are that shipment costs can be reduced within the range of as much as 10-20% (under the assumed parametrisation). Even the application of simple combined delivery shipment (CDS) algorithms can thus significantly boost the rentability in the e-marketplace as well as induce customers to resort to this form of shopping. Consequently, value can be created economy-wide because resources are released from inefficient uses, with Pareto improvements.

Our results suggest - according to a shorthand intuition - that in a perfect world without delivery defaults introducing a combined delivery service brings nothing but a Pareto improvement. However, the results are somewhat stronger, demonstrating that in an imperfect world with delivery defaults, introducing CDS II can actually help to overcome these problems on an

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39If a buyer does not express an interest in a good and does not specify preferred delivery dates as well as shipment costs, no coalition can be formed. Hence, there can be no free-riding in terms of coordinating effort and, unless each buyer takes an active role, a comprehensive coalition cannot be successfully formed. This is not to say, however, that the coordination cost should be homogenous for all buyers.
aggregate scale. Thus, these findings are not susceptible to possible weaknesses of the warehouse management systems. With combined deliveries, any delay may be spread to other customers, thus decreasing their satisfaction from e-purchasing. Simulations show that despite this contagion effect, CDS is still mutually beneficial. However, in this case CDS does not immediately lead to a Pareto improvement, because some clients are worse off due to the contagion effect. Nonetheless, introducing an incentive to the e-retailer to incorporate a longer term perspective in his optimization problem, going beyond short term profit maximization, guarantees that on the aggregate scale consumers benefit from a combined delivery service and so does the e-retailer.
References


Figure 1: Consumer satisfaction depending on the actual arrival date.
Table 1: Notation of parameters and functions

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha, \beta, \gamma$</td>
<td>Parameters of the utility function</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Retailer profits</td>
</tr>
<tr>
<td>$p = \tilde{p} + s(t)$</td>
<td>Total price (product price $\tilde{p}$ and shipment cost $s(t)$)</td>
</tr>
<tr>
<td>$d = w + t$</td>
<td>Total delivery time (waiting time $w$ and transport time $t$)</td>
</tr>
<tr>
<td>$m \equiv [1, 2, \ldots, m, \ldots, \bar{m}]^T$</td>
<td>Types of products available</td>
</tr>
<tr>
<td>$\hat{p}<em>{e/hs}(t) \equiv [\hat{p}</em>{e/hs}(1, t), \ldots, \hat{p}_{e/hs}(\bar{m}, t)]^T$</td>
<td>Vector of product prices ($e$ for e-retailer, $hs$ for high street shop)</td>
</tr>
<tr>
<td>$R \equiv {1, 2, \ldots, \bar{r}}$</td>
<td>Vector of locations</td>
</tr>
<tr>
<td>$N \equiv {1, 2, \ldots, n, \ldots, \bar{n}}$</td>
<td>Vector of consumers</td>
</tr>
<tr>
<td>$j_n \in (\mathbb{1}, \mathbb{j})$, $1 \leq j \leq \mathbb{j}$</td>
<td>Per period of income of consumer assigned randomly</td>
</tr>
<tr>
<td>$\theta(n_m) = [m, p_m, \hat{t} + d_n(m, \hat{t})]$</td>
<td>Order of consumer $n$ purchasing good $m$ from e-retailer at time $\hat{t}$</td>
</tr>
<tr>
<td>$stf_n(t)$</td>
<td>Consumer Satisfaction</td>
</tr>
<tr>
<td>$WMS1$</td>
<td>Warehouse Management System under &quot;commitment&quot; scenario</td>
</tr>
<tr>
<td>$WMS2$</td>
<td>Warehouse Management System under &quot;doomed to default&quot; scenario</td>
</tr>
<tr>
<td>$CDS I$</td>
<td>Combined Delivery Service (same day orders)</td>
</tr>
<tr>
<td>$CDS II$</td>
<td>Combined Delivery Service (intervals for orders combining)</td>
</tr>
<tr>
<td>$CDS (t)$</td>
<td>Combined Delivery Service with transfers to customers</td>
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Table 2: Simulation results under "commitment" scenario

<table>
<thead>
<tr>
<th>Simulated variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<tr>
<td></td>
<td>No CDS</td>
<td>CDS I</td>
<td>CDS II</td>
<td>CDS I (t)</td>
<td>CDS II (t)</td>
</tr>
<tr>
<td>High street purchases</td>
<td>18 143</td>
<td>18 143</td>
<td>17 974</td>
<td>16 894</td>
<td>13 282</td>
</tr>
<tr>
<td>% of high street purchases</td>
<td>52.7%</td>
<td>52.7%</td>
<td>51.9%</td>
<td>48.3%</td>
<td>36.8%</td>
</tr>
<tr>
<td>E-purchases</td>
<td>16 432</td>
<td>16 432</td>
<td>16 628</td>
<td>18 053</td>
<td>22 791</td>
</tr>
<tr>
<td>% of E-purchases</td>
<td>47.3%</td>
<td>47.3%</td>
<td>48.1%</td>
<td>51.7%</td>
<td>63.2%</td>
</tr>
<tr>
<td>Total sales</td>
<td>£407 320</td>
<td>£407 320</td>
<td>£412 116</td>
<td>£444 074</td>
<td>£584 723</td>
</tr>
<tr>
<td>Costs of sales</td>
<td>£348 817</td>
<td>£348 817</td>
<td>£353 060</td>
<td>£381 652</td>
<td>£481 026</td>
</tr>
<tr>
<td>No. of coalitions (a)</td>
<td>0</td>
<td>1383</td>
<td>4498</td>
<td>1661</td>
<td>6195</td>
</tr>
<tr>
<td>No. of orders in coalitions (a)</td>
<td>0</td>
<td>2875</td>
<td>10413</td>
<td>3472</td>
<td>16 369</td>
</tr>
<tr>
<td>Share of orders in coalitions</td>
<td>0</td>
<td>17.50%</td>
<td>62.62%</td>
<td>19.53%</td>
<td>71.82%</td>
</tr>
<tr>
<td>Average size of a coalition</td>
<td>0</td>
<td>2.08</td>
<td>2.35</td>
<td>2.09</td>
<td>2.45</td>
</tr>
<tr>
<td>Costs of shipment (b)</td>
<td>£39 151</td>
<td>£37 242</td>
<td>£32 139</td>
<td>£40 644</td>
<td>£42 270</td>
</tr>
<tr>
<td>Average costs of shipment (b)</td>
<td>£2.40</td>
<td>£2.26</td>
<td>£1.93</td>
<td>£2.25</td>
<td>£1.85</td>
</tr>
<tr>
<td>Average satisfaction</td>
<td>1.0213</td>
<td>1.0213</td>
<td>1.0254</td>
<td>1.0188</td>
<td>1.0212</td>
</tr>
</tbody>
</table>

Notes:
Simulated along the specified parametrisation. For columns (4) and (5) the notation of (t) corresponds to a transfer scenario calculated basing on the assumption that the expected savings are \textit{ex ante} reduced from the prices proportionally to revenues. (a) Only non-trivial coalitions are reported. (b) Total cost of shipment, including single shipments (i.e. trivial coalitions).
Table 3: Simulation results under "doomed to default" scenario

<table>
<thead>
<tr>
<th>Simulated variables</th>
<th>(1) Benchmark</th>
<th>(2) CDS I</th>
<th>(3) CDS II</th>
<th>(4) CDS I (t)</th>
<th>(5) CDS II (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High street purchases</td>
<td>19 658</td>
<td>19 730</td>
<td>19 027</td>
<td>18 255</td>
<td>15 359</td>
</tr>
<tr>
<td>% of high street purchases</td>
<td>57.30%</td>
<td>57.54%</td>
<td>55.26%</td>
<td>52.56%</td>
<td>43.16%</td>
</tr>
<tr>
<td>E-purchases</td>
<td>14 651</td>
<td>14 651</td>
<td>15 402</td>
<td>16 475</td>
<td>20 231</td>
</tr>
<tr>
<td>% of E-purchases</td>
<td>42.70%</td>
<td>42.46%</td>
<td>44.74%</td>
<td>47.44%</td>
<td>56.84%</td>
</tr>
<tr>
<td>Total sales</td>
<td>364 206£</td>
<td>362 228£</td>
<td>382 329£</td>
<td>406 916£</td>
<td>491 838£</td>
</tr>
<tr>
<td>Costs of sales</td>
<td>310 678£</td>
<td>309 130£</td>
<td>326 991£</td>
<td>349 130£</td>
<td>426 295£</td>
</tr>
<tr>
<td>No. of coalitions (a)</td>
<td>0</td>
<td>1096</td>
<td>3980</td>
<td>1412</td>
<td>5816</td>
</tr>
<tr>
<td>No. of orders in coalitions (a)</td>
<td>0</td>
<td>2 253</td>
<td>9 090</td>
<td>2 928</td>
<td>13 870</td>
</tr>
<tr>
<td>Share of orders in coalitions</td>
<td>0</td>
<td>15.47%</td>
<td>59.02%</td>
<td>17.78%</td>
<td>68.65%</td>
</tr>
<tr>
<td>Average size of a coalition</td>
<td>0</td>
<td>2.06</td>
<td>2.28</td>
<td>2.07</td>
<td>2.38</td>
</tr>
<tr>
<td>Costs of shipment (b)</td>
<td>34 809£</td>
<td>33 119£</td>
<td>30 201£</td>
<td>37 231£</td>
<td>38 005£</td>
</tr>
<tr>
<td>Average costs of shipment (b)</td>
<td>2.40£</td>
<td>2.30£</td>
<td>1.99£</td>
<td>2.28£</td>
<td>1.92£</td>
</tr>
<tr>
<td>Average satisfaction</td>
<td>0.9957</td>
<td>0.9912</td>
<td>1.0116</td>
<td>0.9972</td>
<td>1.0169</td>
</tr>
</tbody>
</table>

Notes:
Simulated along the specified parametrisation. For columns (4) and (5) the notation of (t) corresponds to a transfer scenario calculated basing on the assumption that the expected savings are ex ante reduced from the prices proportionally to revenues. (a) Only non-trivial coalitions are reported. (b) Total cost of shipment, including single shipments (i.e. trivial coalitions).
Figure 2: Model structure
Figure 3: Example of utility maps for $\alpha = 10$, $\beta = 10$. Values of $\gamma$ specified to -3 in the left panel and 3 in the right panel.
Figure 4: Stocks depending on the chosen warehouse management system. Left panel demonstrates the "commitment" scenario while right panel depicts the "doomed to default" one.
Figure 5: The surplus created by CDS in a multiplayer setting - a simulation.
Figure 6: The order merging mechanism
Figure 7: The percent of e-purchases depending on the values of utility function parameters.
Figure 8: Sensitivity analysis - population density and consumer satisfaction.

Figure 9: Sensitivity analysis - population density and shipment cost reductions.

Figure 10: Sensitivity analysis - population density and average size of combined order.
Figure 11: Sensitivity analysis - shipment time and consumer satisfaction.

Figure 12: Sensitivity analysis - shipment time and shipment cost reductions.

Figure 13: Sensitivity analysis - shipment time and average size of combined order.
Figure 14: Sensitivity analysis - comparative advantage and consumer satisfaction.

Figure 15: Sensitivity analysis - comparative advantage and shipment cost reductions.

Figure 16: Sensitivity analysis - comparative advantage and average size of combined order.
Figure 17: Consumer satisfaction distributions under varied scenarios (kernel density estimates).