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5 November 2008

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First draft: November 5, 2008
This version: May 13, 2009

Abstract

This study examines the steady-state growth effect of inflation in an endogenous growth model in which Calvo-type nominal rigidity with endogenous contract duration and monetary friction via wage-payment-in-advance constraint are assumed. On the balanced-growth path in this model, the marginal growth effect of inflation is weakly negative or even positive at low inflation rates because the effect on average markup offsets the negative marginal growth effect through the monetary friction, but the growth effect of inflation is negative and convex at higher inflation rates because the frequency of price adjustment approaches that of the flexible-price economy and the growth effect through the nominal rigidity is dominated by the growth effect through the monetary friction. With a plausible calibration of the structural parameters, this model generates a relationship between inflation and growth that is consistent with empirical evidence, particularly in industrial countries.

Keywords: Inflation and growth; Sticky prices; Endogenous contract duration

JEL classification: E31; O42

*The author appreciates helpful comments and suggestions from Tomoyuki Nakajima, Katsunori Yamada, and seminar participants at Osaka University and Kyushu University. Any remaining errors are the sole responsibility of the author. This study is financially supported by the research fellowships of the Japan Society for the Promotion of Science for young scientists.

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1 Introduction

Recent empirical studies have found that the relationship between inflation and growth is non-linear.[1] The stylized facts are as follows. First, there is a threshold inflation rate above which the marginal effect of inflation on growth is negative and below which the one is nonsignificant or even positive. Second, above the threshold inflation rate, the relationship between inflation and growth is convex in the sence that the negative marginal effect is weaker when inflation is high.

On the other hand, most theoretical studies fail to generate this non-linear relationship. For example, in flexible-price monetary endogenous growth models with cash-in-advance constraint, the marginal growth effect of inflation is always negative, as surveyed in Gillman and Kejak (2005). In monetary endogenous growth models with prototypical Calvo-type nominal rigidity, as in Funk and Kromen (2006) and Kuwahara and Sudo (2007), there is a threshold inflation rate but above it the relationship is concave.

In this paper we show that, with a plausible calibration of the structural parameters, a monetary endogenous growth model with a Calvo-type staggered price setting with endogenous contract duration, as in Levin and Yun (2007), can generate the non-linear relationship consistent across a wide range of inflation with the empirical evidence for industrial countries. In this model, there is a threshold inflation rate below which the marginal effect of inflation on growth is weakly negative or even positive, because at low inflation rates steady-state inflation affects average markup through the nominal rigidity, which offsets the negative marginal growth effect through the monetary friction. However, when inflation is high, nominal rigidity becomes weaker and the situation approaches that of flexible-price economy; hence the marginal effect becomes negative and the inflation-growth relationship is convex. In our numerical result, the threshold inflation rate is about 0.1%, which is consistent with the empirical evidence in Khan and Senhadji (2001) that this rate is below 1% for five-year averaged data in industrial countries.

The rest of the paper is organized as follows. Section 2 describes the economy. Section 3 shows the mechanisms and the numerical results of the growth effect in the flexible-price economy, in the prototypical Calvo-type sticky-price economy, and in the Calvo economy with endogenous contract duration. Section 4 is the conclusion.

2 The Model

The model considered in this study is a simple two-capital endogenous growth model in which monetary friction via wage-payment-in-advance constraint of firms is introduced. Time is discrete. There are three types of agents in this economy: the representative household, monopolistically competitive firms, and the monetary authority. For simplicity, fiscal policy is ignored.

[1]See Section 1 in Hung (2008).

The representative household maximizes the following discounted sum of utility[2]:

$$\sum_{t=0}^{\infty} \beta^t \{\log C_t + \psi \log[(1-n_t)H_t]\}, \tag{1}$$

where C denotes aggregate consumption, $n \in (0,1)$ denotes hours worked, H denotes human capital stock, and $\psi > 0$ and $\beta \in (0,1)$ are exogenous parameters. The intertemporal budget constraint is as follows:

$$\begin{aligned}\frac{B_t}{P_t} + C_t + K_{t+1} - (1-\delta_k)K_t + H_{t+1} - (1-\delta_H)H_t \\ = \frac{i_{t-1}B_{t-1}}{P_t} + w_t n_t H_t + r_t^K K_t + \Phi_t, \end{aligned}\tag{2}$$

where B denotes the quantity of a nominal financial asset that earns the gross nominal interest rate i, K denotes physical capital stock, π denotes gross rate of inflation, w denotes real wage rate, r^K denotes real gross rate of return on physical capital, Φ denotes real dividend income from firms they own, δ_K is an exogenous parameter representing the depreciation rate of physical capital, and δ_H is an exogenous parameter representing the depreciation rate of human capital.

Each individual firm j ($\in [0,1]$) monopolistically supplies the variety j, using a Cobb-Douglas production technology:

$$Y_t(j) = AK_t(j)^{\alpha} Z_t(j)^{1-\alpha}, \tag{3}$$

where A and α denote exogenous parameters representing aggregate productivity and capital share respectively, and where $K(j)$ and $Z(j)$ denote the demand for physical capital and for effective labor respectively, each of which must satisfy the resource constraints, $\int_0^1 K_t(j)di = K_t$ and $\int_0^1 Z_t(j)di = n_t H_t$. It is assumed that workers must be paid their wage bill by cash in advance of production. Hence firm j borrows its nominal wage payment, $P_t w_t Z_t(j)$, from a financial intermediary at the beginning of period t. Repayment occurs at the end of period t at the gross nominal interest rate i_t. Consequently, the total real production cost of firm j is $r_t^K K_t(j) + i_t w_t Z_t(j)$. From the first-order conditions of cost minimization with respect to $K_t(j)$ and $Z_t(j)$, it holds that $r_t^K = \frac{\alpha A\left(\frac{K_t}{n_t H_t}\right)^{\alpha-1}}{\mu_t}$ and $w_t = \frac{(1-\alpha)A\left(\frac{K_t}{n_t H_t}\right)^{\alpha}}{i_t \mu_t}$, where μ denotes average markup, which is defined as the reciprocal of the real marginal cost (the Lagrange multiplier with respect to (3)).

[2]To keep the model tractable, we assume log utility and quality time of leisure. Our final result is robust even if the instantaneous utility function is assumed to be in the CRRA form, $\frac{C_t^{1-\sigma}[(1-n_t)H_t]^{\psi(1-\sigma)}}{1-\sigma}$ or to depend on raw time of leisure, $\frac{C_t^{1-\sigma}(1-n_t)^{\psi(1-\sigma)}}{1-\sigma}$, though its mechanism becomes more complicated.

The aggregate demand index Y is assembled using the Dixit-Stiglitz aggregator, $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ denotes the parameter representing the elasticity of substitution. Hence firm j faces a downward-sloping demand function:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t, \tag{4}$$

where $P(j)$ denotes the price of variety j and the aggregate price level P is defined as $P_t = \left(\int_0^1 P_t(j)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$. Each firm maximizes its profit by optimally setting its price subject to (4). The details will be described later.

At the beginning of period t, financial intermediaries have nominal money balances $P_{t-1}M_{t-1}$ and receive a monetary transfer $P_tM_t - P_{t-1}M_{t-1}$ from the monetary authority, where M denotes real money balances, and lend all their money to firms for their wage payments $\int_0^1 P_t w_t Z_t(j) di$. Hence the loan market clearing condition is $M_t = w_t n_t H_t$.

The aggregate demand consists of aggregate consumption, aggregate physical capital investment, aggregate human capital investment, and aggregate menu cost[3]: hence,

$$Y_t = C_t + K_{t+1} - (1-\delta_K)K_t + H_{t+1} - (1-\delta_H)H_t + (1-\xi)\Omega_t. \tag{5}$$

The monetary authority sets the inflation rate $\{\pi_t\}$.[4]

3 Growth Effect of Inflation

Given μ and i, the steady-state growth rate of output γ is determined by:

$$\gamma = \beta r, \qquad \text{(Euler equation)} \tag{6}$$

$$r = \frac{\alpha A \left(\frac{K}{nH}\right)^{\alpha-1}}{\mu} + 1 - \delta_K, \qquad \text{(No-arbitrage condition)} \tag{7}$$

$$r = \frac{(1-\alpha) A \left(\frac{K}{nH}\right)^{\alpha}}{i\mu} + 1 - \delta_H. \qquad \text{(No-arbitrage condition)} \tag{8}$$

Equation (6) implies that households' saving behavior determines the growth rate of output, depending only on the real rate of interest, r. Equations (7) and (8) imply that arbitrage between physical capital, human capital, and financial assets determines the physical-capital-to-effective-labor ratio and real rate of interest for a given μ and i[5]; hence

[3]The final term of RHS in (5) denotes aggregate menu cost. The datails are described later.

[4]This assumption is equivalent to assuming that the monetary authority sets nominal interest rate $\{i_t\}$ or money growth rate $\{\eta_t\} \equiv \{\frac{P_tM_t}{P_{t-1}M_{t-1}}\}$. The stability of the equilibrium depends on the monetary policy rule, but we ignore the details of the rule because in this paper we focus on the steady state.

[5]Note that real rate of interest depends only on $\frac{K}{nH}$ because we assume quality time for utility from leisure. If it is assumed that utility from leisure depends only on raw time, $1-n_t$, then the determination of the real interest rate becomes more complicated. However, our main results is numerically robust.

inflation has a growth effect if inflation affects the real rate of interest through a change in the nominal rate of interest and/or average markup. In the following subsections, we consider a) the flexible-price economy in order to see the growth effect through changes in the nominal rate of interest, and b) the sticky-price economy in order to see the effect through changes in average markup.

3.1 Flexible-price Economy

Let us consider the flexible-price economy, in which inflation affects the real rate of interest only through the change of the nominal interest rate because steady-state average markup is constant, $\mu = \frac{\theta}{\theta-1}$. The reason why the nominal rate of interest affects the real rate of interest is because there exists a monetary friction according to the wage-payment-in-advance assumption. We refer to this growth effect of inflation as a *nominal interest rate effect*. The relationship between inflation rate and real and nominal rate of interest is described by the Fisher equation:

$$i = r\pi. \qquad \text{(Fisher equation)} \tag{9}$$

Substituting (9) into (8), it holds that:

$$r = \frac{1}{2}\left(1 - \delta_H + \sqrt{(1-\delta_H)^2 + \frac{4}{\pi\mu}(1-\alpha)A\left(\frac{K}{nH}\right)^{\alpha}}\right). \tag{10}$$

Given π, equations (7) and (10) determine real rate of interest. Figure 1 illustrates the determination of the real interest rate. When π rises, (10) shifts downward and r falls. Therefore, the marginal nominal interest rate effect of inflation on growth rate is necessarily negative as in standard monetary endogenous growth models.[6]

3.2 Sticky-price Economy with Exogenous Contract Duration

In this subsection we consider the sticky-price economy with exogenous contract duration (the prototypical Calvo model), in which each firm can reset its price with the probability $1-\xi$ and in which ξ is constant. In this economy, inflation has an effect on real interest rate and growth, since the existence of nominal rigidity causes an inflationary effect on average markup, in addition to its nominal interest rate effect. We refer to the effect on average markup as the *markup effect*. As illustrated in Figure 2, a rise of μ makes r fall

[6] This monetary friction works similarly to the cash-in-advance constraint in standard monetary endogenous growth models. In our model, the cash-in-advance constraint of households does not affect growth because cash-in-advance constraint affects only hours worked, n. If we assume that utility from leisure depends only on raw time, the cash-in-advance constraint has the same growth effect as in standard monetary endogenous growth models.

because (7) and (10) shift downward. Therefore, a rise (fall) of average markup gives rise to a fall (rise) of growth rate of output.

In this economy, for a given π, the economy-wide average markup μ is determined by the optimal pricing behavior of firms and the price level equation as follows:[7]

$$\tilde{\mu} = \frac{\theta}{\theta - 1} \frac{1 - \beta\xi\pi^{\theta-1}}{1 - \beta\xi\pi^{\theta}}, \qquad \text{(Optimal pricing behavior)} \qquad (11)$$

$$\mu^{1-\theta} = \xi \left(\frac{\mu}{\pi}\right)^{1-\theta} + (1-\xi)\tilde{\mu}^{1-\theta}, \qquad \text{(Price level equation)} \qquad (12)$$

where $\tilde{\mu} \equiv \frac{\tilde{P}}{P}\mu$ denotes the optimal markup set by firms that can reset their prices. From these equations, average markup is described as:

$$\mu = \frac{\theta}{\theta - 1} \frac{1 - \beta\xi\pi^{\theta-1}}{1 - \beta\xi\pi^{\theta}} \left(\frac{1 - \xi\pi^{\theta-1}}{1 - \xi}\right)^{\frac{1}{\theta-1}}, \qquad (13)$$

hence average markup depends only on inflation rate.[8] As shown in Panel C of Figure 3, the relationship between inflation and average markup is U-shaped.[9] The intuition of the U-shaped average markup is as follows. Equation (12) implies that the economy-wide average markup μ depends on both the average markup of firms that *cannot* reset their nominal prices μ/π and the markup of firms that *can* reset their nominal prices $\tilde{\mu}$. On the one hand, μ/π is decreasing in π for a given μ. This is because the average relative price of firms that cannot reset their nominal prices falls at that inflation rate, but real marginal cost is constant on a balanced-growth path. On the other hand, from (11), we see that $\tilde{\mu}$ is increasing in π. The reason is that when the inflation rate is high, firms that can reset their nominal prices set their markup higher because they are concerned with the possibility that their markup would keep declining in the future when they cannot reset their prices. Because of these two opposing effects, inflation has a U-shaped impact on economy-wide average markup.

Since a rise of average markup brings a fall of real interest rate and growth, the marginal markup effect on growth is positive at low inflation rates and negative at high inflation rates. Figure 3 shows the numerical result of this relationship for various values of ξ.[10] We can see that the U-shaped average markup becomes flatter as ξ decreases

[7]The derivation of (11) and (12) is in Appendix A.

[8]Our assumption of log utility simplifies the analysis. If instantaneous utility has a more general form, $\frac{C_t^{1-\sigma}[(1-n_t)H_t]^{\psi(1-\sigma)}}{1-\sigma}$, then average markup depends not only on inflation but also on the growth rate of output; hence the mechanism becomes more complicated. However, even in this case, our results are robust.

[9]This U-shaped relationship follows if β is sufficiently near 1. The analytical proof is in Appendix B.

[10]The values of the structural parameters are in Table 1 and the calibration strategy is in Appendix C. The Matlab programs for our numerical analysis are on the author's website (http://sites.google.com/site/hirokiarato/).

and that the markup effect disappears when $\xi = 0$. This is because the decrease of ξ means that nominal rigidity becomes weaker and the situation approaches that of the flexible-price economy. When ξ is sufficiently high, there is a threshold inflation rate below which the marginal growth effect is positive because the markup effect offsets the nominal interest rate effect. However, this relationship is concave in the whole range of inflation, which is inconsistent with empirical evidence at high inflation rates.[11]

3.3 Sticky-price Economy with Endogenous Contract Duration

Finally, we consider the Calvo model with endogenous contract duration as in Levin and Yun (2007). In this model, each firm is allowed to choose not only its price but also its average contract duration (or the probability of changing its price). For simplicity, we assume that the economy is on a balanced-growth path.[12] In each period, firm j can reset the nominal price of its variety with probability $1 - \xi(j)$. Moreover, firms must pay a fixed menu cost $\Omega_t \equiv \omega Y_t$ when they can change their prices. Given these assumptions, each firm maximizes the expected present-value of its current and future profits subject to the demand function (4), choosing its price and the probability of changing its price. Following Levin and Yun (2007), we restrict our analysis to a symmetric Nash equilibrium in which all firms choose the same probability of changing prices; hence $\xi(j) = \xi$ for all j. In this economy, ξ and μ are determined according to (11), (12), and the optimal condition with respect to the contract duration of firms, which is described as:

$$\frac{\tilde{\mu}^{1-\theta}(\pi^{\theta-1} - 1)}{(1 - \beta\xi\pi^{\theta-1})^2} = \frac{\tilde{\mu}^{-\theta}(\pi^{\theta} - 1)}{(1 - \beta\xi\pi^{\theta})^2} - \omega\mu^{1-\theta}, \qquad \text{(Optimal contract duration)} \qquad (14)$$

when the internal solution exists.[13] By allowing firms to choose the frequency of changing prices, firms change their price more frequently as inflation deviates from zero, as shown in Panel D of Figure 4. The reason for this relationship is the existence of fixed menu cost. If inflation is near zero, the loss of profit by unchanging their prices is small because the difference between price-unchanging firms' markup and the optimal one is small. Hence, concerned by fixed menu cost, firms choose a low frequency of price changing. As inflation deviates from zero, the loss of profit by unchanging their prices becomes larger; hence firms choose a higher frequency even if they must pay the menu cost more frequently. If inflation is extremely high, all firms change their prices in every period; hence the situation is the same as that of the flexible-price economy.

[11]Moreover, this model can analyze the growth effect only at moderate inflation. This model has an equilibrium only if $\xi\pi^{\theta-1} < 1$ and $\xi\pi^{\theta} < 1$ because the average markup μ and the degree of relative price dispersion s must be positive and finite (see eq.(12) and eq.(38) in Appendix C).

[12]For the firm's behavior in a stochastic economy, see the working paper version of Levin and Yun (2007).

[13]The derivation of (14) is in Appendix A.

Varying the frequency of changing prices makes the markup effect more complex. In addition to the U-shaped markup effect in the previous subsection, there is the effect that this U-shaped relationship becomes flatter as inflation deviates from zero. Figures 4 and 5 indicate the numerical result, which is consistent with the empirical evidence. First, there is a threshold inflation rate of about 0.1% in a year at which the marginal growth effect changes from positive to negative. Readers may question why at severe deflation (below minus 0.1% in year) the marginal effect is negative. We think that the reason for the positive or nonsignificant marginal effect of growth in empirical studies is the infrequency in the number of observations of severe deflation. Since most of the observations below the threshold inflation rate are distributed around zero inflation, the regression analysis would show an upward-sloping or nonsignificant relationship between inflation and growth. Second, above the threshold inflation rate, the relationship between inflation and growth is decreasing and convex because the markup effect is weaker when inflation is high and the situation approaches the flexible-price economy in which only the nominal interest rate effect affects growth. As a result, this economy can generate the plausible inflation-growth relationship in a wider range of inflation than the sticky-price economy with exogenous contract duration. Third, our model can be calibrated more accurately than the endogenous growth models with the imperfect information in credit market, in which Bose (2002) and Hung (2008) show the existence of a threshold inflation rate. With our calibration of the structural parameters, the threshold inflation rate is about 0.1%. In the empirical study in Khan and Senhadji (2001), the threshold inflation rate is below 1% in industrial countries and 11% in developed countries for five-year averaged data. With this empirical evidence, we conclude that our model can generate the plausible threshold inflation rate in industrial countries.

4 Concluding Remarks

In this paper we show that the monetary endogenous growth model with Calvo-type nominal rigidity with endogenous contract duration can generate the plausible relationship between inflation and growth, especially in industrial countries. However, there are some open questions in our analysis. First, our model suggests the existence of a lower alternative threshold inflation rate below which the marginal growth effect becomes negative. This hypothesis is potentially testable. If we had more observations of deflation, we could test the existence of the alternative threshold inflation rate by dividing the low-inflation observations into two subsamples. Second, our model can not replicate the plausible threshold inflation rate in developing countries, which is shown to be 11% for five-year averaged data in Khan and Senhadji (2001). This result suggests that the analysis for developing countries needs some alternative assumptions of, for example, imperfect information in credit market as in Bose (2002) and in Hung (2008). However, the measurement of the degree of imperfect information is difficult. In order to analyze the growth effect of inflation in developing countries quantitatively, we must obtain more empirical evidence about market structure and imperfect information.

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Appendix

A. Derivation of (11), (12), and (14)

In the prototypical Calvo model, firms that can reset their price at time t choose their steady-state relative price $\tilde{p} \equiv \tilde{P}_t/P_t$, which is constant, to maximize the discounted sum of their expected profit until they reset their prices,

$$\begin{aligned} \Phi_t(\tilde{p}, \xi) &= \sum_{s=0}^{\infty} \left(\frac{\xi}{r}\right)^s \left[\left(\frac{\tilde{p}}{\pi^s}\right)^{1-\theta} Y_{t+s} - \left(\frac{\tilde{p}}{\pi^s}\right)^{-\theta} \frac{Y_{t+s}}{\mu}\right] \\ &= Y_t \sum_{s=0}^{\infty} \left(\frac{\xi\gamma}{r}\right)^s \left[\left(\frac{\tilde{p}}{\pi^s}\right)^{1-\theta} - \left(\frac{\tilde{p}}{\pi^s}\right)^{-\theta} \frac{1}{\mu}\right] \end{aligned} \tag{15}$$

on a balanced-growth path. The first-order condition is described as:

$$\sum_{s=0}^{\infty} \left(\frac{\xi\gamma}{r}\right)^s \left[\pi^{s\theta} - \frac{\theta-1}{\theta} (\mu\tilde{p})\, \pi^{s(\theta-1)}\right] = 0, \tag{16}$$

Using the Fisher equation $\gamma = \beta r$ and the definition of optimal markup $\tilde{\mu} = \mu\tilde{p}$,

$$\sum_{s=0}^{\infty}(\beta\xi)^s\left(\pi^{s\theta} - \frac{\theta-1}{\theta}\tilde{\mu}\pi^{s(\theta-1)}\right) = 0. \tag{17}$$

After some calculation, we obtain (11).

See the aggregate price level equation,

$$P_t = \left(\int_0^1 P_t(j)^{1-\theta}di\right)^{\frac{1}{1-\theta}}. \tag{18}$$

When the probability that each firm can reset their prices is $1-\xi$, (18) can be rewritten as:

$$P_t^{1-\theta} = \xi P_{t-1}^{1-\theta} + (1-\xi)\tilde{P}_t^{1-\theta}. \tag{19}$$

Dividing $(\frac{P_t}{\mu})^{1-\theta}$ into this equation, we can see that (12) holds on a balanced growth path.

Next consider the Calvo model with endogenous contract duration. Assume that firm j can change its price at time t. If it sets its relative price to be $\tilde{p}(j)$, then the discounted sum of expected profit until it resets its price is $\Phi_t(\tilde{p}(j),\xi(j)) - \omega Y_t$. Therefore, the total discounted sum of its profit $V_t(\tilde{p}(j),\xi(j))$ is:

$$\begin{aligned} V_t(\tilde{p}(j),\xi(j)) = (\Phi_t(\tilde{p}(j),\xi(j)) - \omega Y_t) &+ \frac{1-\xi(j)}{r}(\Phi_{t+1}(\tilde{p}(j),\xi(j)) - \omega Y_{t+1}) \\ &+ \frac{1-\xi(j)}{r^2}(\Phi_{t+2}(\tilde{p}(j),\xi(j)) - \omega Y_{t+2}) + \cdots \end{aligned} \tag{20}$$

Hence, $V_t(\tilde{p}(j),\xi(j))$ can be rewritten as:

$$\begin{aligned} V_t(\tilde{p}(j),\xi(j)) &= Y_t\phi(\tilde{p}(j),\xi(j))\left[1 + (1-\xi(j))\frac{\gamma}{r} + (1-\xi(j))\left(\frac{\gamma}{r}\right)^2 + \cdots\right] \\ &= Y_t\phi(\tilde{p}(j),\xi(j))\left[1 + (1-\xi(j))\beta + (1-\xi(j))\beta^2 + \cdots\right] \\ &= Y_t\phi(\tilde{p}(j),\xi(j))\frac{1-\beta\xi(j)}{1-\beta}, \end{aligned} \tag{21}$$

where,

$$\begin{aligned} \phi(\tilde{p}(j),\xi(j)) &\equiv \frac{\Phi_t(\tilde{p}(j),\xi(j)) - \omega Y_t}{Y_t} \\ &= \left\{\sum_{s=0}^{\infty}\left(\frac{\xi(j)\gamma}{r}\right)^s\left[\left(\frac{\tilde{p}(j)}{\pi^s}\right)^{1-\theta} - \left(\frac{\tilde{p}(j)}{\pi^s}\right)^{-\theta}\frac{1}{\mu}\right]\right\} - \omega, \end{aligned} \tag{22}$$

which is constant on balanced-growth path.

The profit maximization problem of firm j has two steps. First, given $\xi(j)$, firm j chooses its optimal relative price $\tilde{p}^*(\xi(j))$. Hence,

$$\begin{aligned}\tilde{p}^*(\xi(j)) &= \arg\max_{\tilde{p}(j)} V_t(\tilde{p}(j), \xi(j)) \\ &= \arg\max_{\tilde{p}(j)} \phi(\tilde{p}(j), \xi(j)).\end{aligned} \tag{23}$$

We can solve this problem as in the prototypical Calvo model and obtain:

$$\tilde{\mu}(\xi(j)) = \frac{\theta}{\theta-1}\frac{1-\beta\xi(j)\pi^{\theta-1}}{1-\beta\xi(j)\pi^{\theta}}. \tag{24}$$

Second, given ξ, firm j chooses its optimal frequency of changing price $1-\xi^*(j)$. Hence,

$$\xi^*(j) = \arg\max_{\xi(j)} \eta_t(\xi(j)), \tag{25}$$

where,

$$\eta_t(\xi(j)) \equiv V_t(\tilde{p}^*(\xi(j)), \xi(j)). \tag{26}$$

By the envelope theorem, the first-order condition is

$$\frac{d\eta_t}{d\xi(j)} = \frac{\partial V_t}{\partial \xi(j)} = 0. \tag{27}$$

By (21), the first-order condition can be written as:

$$\frac{\partial}{\partial \xi(j)}\left[\phi(\tilde{p}^*, \xi(j))\frac{1-\beta\xi(j)}{1-\beta}\right] = 0. \tag{28}$$

Some calculations arrange it as:

$$\frac{\tilde{\mu}^{1-\theta}(\pi^{\theta-1}-1)}{(1-\beta\xi^*(j)\pi^{\theta-1})^2} = \frac{\tilde{\mu}^{-\theta}(\pi^{\theta}-1)}{(1-\beta\xi^*(j)\pi^{\theta})^2} - \omega\mu^{1-\theta}. \tag{29}$$

In a symmetric Nash equilibrium, $\xi^*(j) = \xi$, for all j. Therefore, we obtain (11) and (14) from (24) and (29), respectively.

B. Proof of U-shaped average markup in exogenous contract duration model

Here we prove that if β is sufficiently near 1, there is an inflation rate $\pi^* \in (1, \min\{\frac{1}{\beta}, \xi^{-\frac{1}{\theta}}\})$ such that $\frac{\partial\mu}{\partial\pi} \lesseqqgtr 0$ if $\pi \lesseqqgtr \pi^*$.

From (13), it follows that

$$\frac{\partial\mu}{\partial\pi} = \frac{\theta}{\theta-1}(1-\xi)^{\frac{1}{\theta-1}}\frac{(1-\xi\pi^{\theta-1})^{\frac{1}{\theta-1}-1}}{(1-\beta\xi\pi^{\theta})^2}\xi\pi^{\theta-2}f(\pi), \tag{30}$$

where

$$f(\pi) \equiv \theta\beta(1-\xi\pi^{\theta-1})(\pi-1) - (1-\beta\xi\pi^{\theta})(1-\beta). \tag{31}$$

The sign of $\frac{\partial\mu}{\partial\pi}$ is identical to the $f(\pi)$'s one because there are restrictions that $1-\xi\pi^{\theta} > 0$ and $1-\xi\pi^{\theta-1} > 0$. Differentiating $f(\pi)$, we obtain

$$f'(\pi) = \theta\beta[1-\beta\xi\pi^{\theta-1} - (\theta-1)(\pi-1)\xi\pi^{\theta-2}]; \tag{32}$$

hence, $f'(\pi) > 0$ if $\pi \leq 1$ and $f'(\pi)$ is decreasing if $\pi \geq 1$. Here we see that

$$f(1) = -(1-\beta\xi)(1-\beta) < 0, \tag{33}$$

$$f(\frac{1}{\beta}) = (\theta-1)(1-\beta)(1-\xi\beta^{1-\theta}) > 0. \tag{34}$$

and if β is sufficiently near 1,

$$\lim_{\pi\to\xi^{-\theta}-0} f(\pi) = -\beta^2 + [2+\theta(\xi^{-\frac{1}{\theta}} + \xi^{\frac{1}{\theta}} - 2)]\beta - 1 > 0. \tag{35}$$

Therefore, there is a unique $\pi^* \in (1, \min\{\frac{1}{\beta}, \xi^{-\frac{1}{\theta}}\})$ such that $f(\pi) \lesseqqgtr 0$ if $\pi \lesseqqgtr \pi^*$.

C. Calibration

In order to calibrate our model, in addition to the equilibrium conditions that have already been derived, that is, the Euler equation (6), the no-arbitrage conditions (7) and (8), the Fisher equation (9), the optimal pricing equation (11), the price level equation (12), and the optimal contract duration equation (14), other conditions are needed. First, the optimal labor supply equation,

$$\frac{\psi}{1-n}\frac{C}{H} = \frac{(1-\alpha)A(\frac{K}{nH})^{\alpha}}{i\mu}. \tag{36}$$

Second, the aggregate good market clearing condition,

$$\frac{A(\frac{K}{H})^{\alpha}n^{1-\alpha}}{s} = \frac{\frac{C}{H} + (\gamma-1+\delta_K)\frac{K}{H} + (\gamma-1+\delta_H)}{1-(1-\xi)\omega}, \tag{37}$$

where $s \equiv \int_0^1 (\frac{P_t(j)}{P_t})^{-\theta} dj \geq 1$ denotes the degree of relative price dispersion, which is described as:

$$s = (1-\xi)^{\frac{1}{1-\theta}} \frac{(1-\xi\pi^{\theta-1})^{\frac{\theta}{\theta-1}}}{1-\xi\pi^{\theta}}. \tag{38}$$

The time unit is assumed to be one quarter. α, δ_H, δ_K, ω, and θ are set to the values used in growth and business cycle literature. A, β, and ψ are set such that the annual real interest rate is 3% and that the Frisch elasticity of labor supply is unity (hence $n = 0.5$) at the steady state with $\pi = 1.042^{1/4}$ and $\gamma = 1.0045$. The values of the structural parameters are shown in Table 1.

Table 1: Structural parameters

A	α	β	δ_H	δ_K	ω	ψ	θ
0.0445	0.36	$1.0045/1.03^{1/4}$	0.005	0.025	0.029	807.4	4.33

Figure 1: Nominal interest rate effect (when π increases)

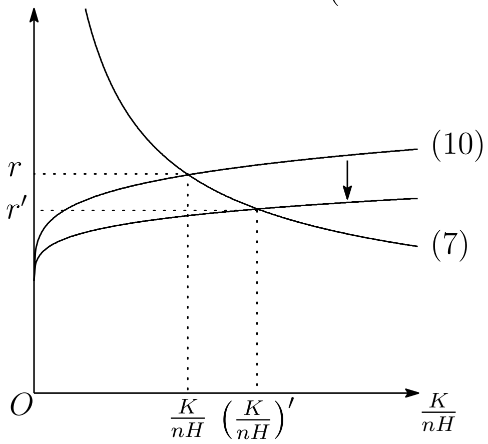


Figure 2: Markup effect (when μ increases)

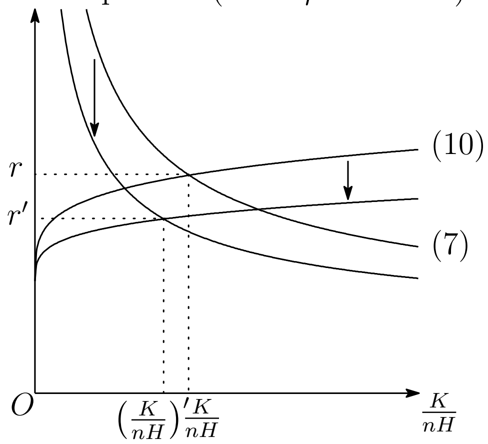
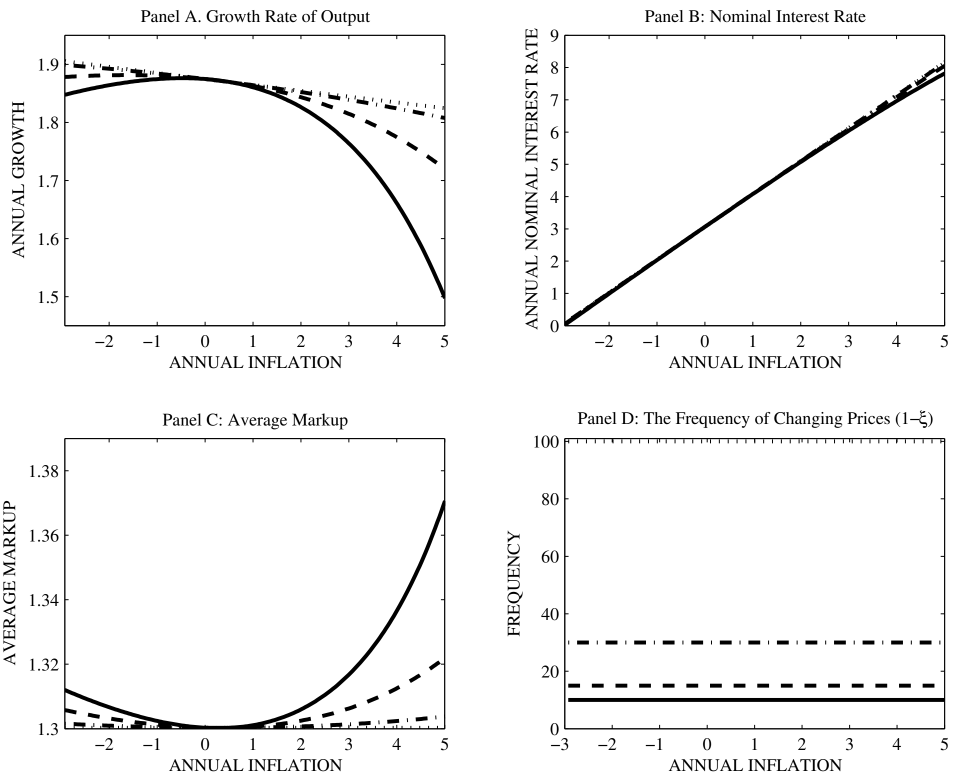


Figure 3: Effects of inflation in the exogenous contract duration model



Note: Solid line when $\xi = 0.9$, broken line when $\xi = 0.85$, dash–dotted line when $\xi = 0.7$, dotted line when $\xi = 0$ (flexible-price economy).

Figure 4: Effects of inflation in the endogenous contract duration model

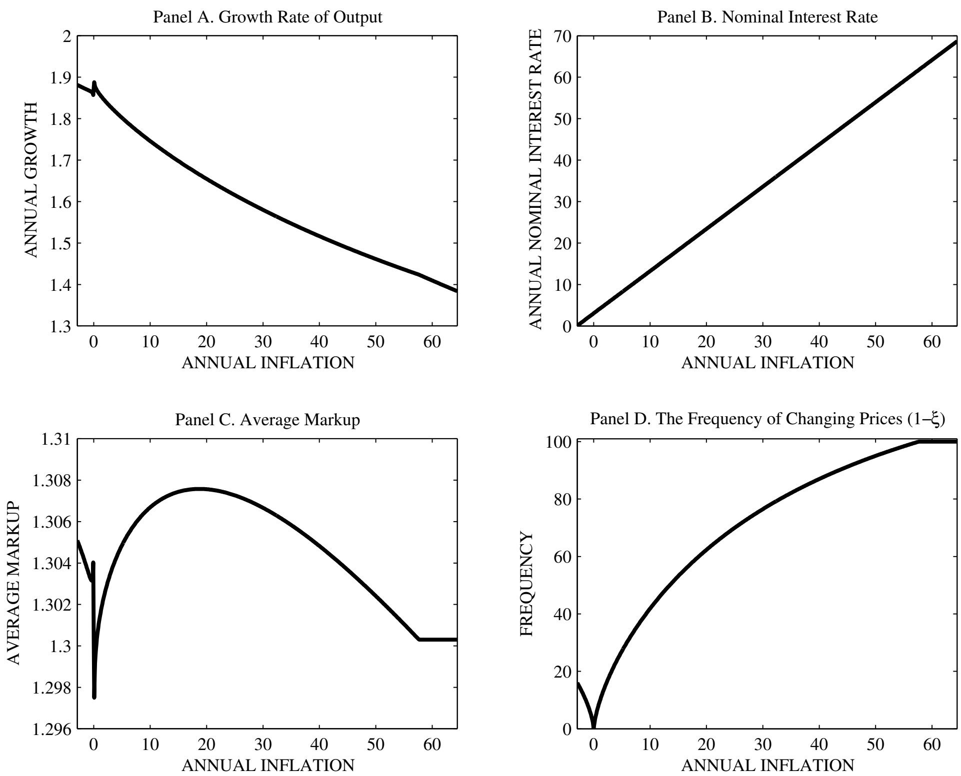


Figure 5: Effects of inflation in the endogenous contract duration model (around zero inflation)