Performance of combined double seasonal univariate time series models for forecasting water consumption

Caiado, Jorge

January 2009

Performance of combined double seasonal univariate time series models for forecasting water demand

Jorge Caiado[a]

[a]Center for Applied Mathematics and Economics (CEMAPRE), Instituto Superior de Economia e Gestão, Technical University of Lisbon, Rua do Quelhas 6, 1200-781 Lisboa, Portugal. Tel.: +351 21 392 2715. E-mail: jcaiado@iseg.utl.pt

Abstract: In this article, we examine the daily water demand forecasting performance of double seasonal univariate time series models (Holt-Winters, ARIMA and GARCH) based on multi-step ahead forecast mean squared errors. A within-week seasonal cycle and a within-year seasonal cycle are accommodated in the various model specifications to capture both seasonalities. We investigate whether combining forecasts from different methods for different origins and horizons could improve forecast accuracy. The analysis is made with daily data for water consumption in Granada, Spain.

Keywords: ARIMA; Combined forecasts; Double seasonality; Exponential Smoothing; Forecasting; GARCH; Water demand.

1. Introduction

Water demand forecasting is of great economic and environmental importance. Many factors can influence directly or indirectly water consumption. These include rainfall, temperature, demography, land use, pricing and regulation. Weather conditions have been widely used as inputs of multivariate statistical models for hydrological time series modelling and forecasting.

Maidment and Miaou (1986), Fildes, Randall and Stubbs (1997), Zhou, McMahou, Walton and Lewis (2000), Jain, Varshney and Joshi (2001) and Bougadis, Adamowski and Diduch (2005) adopted regression and time series models for water demand forecasting by using climate effects as explanatory variables for their models. Wong, Ip, Zhang and Xia (2007) used a non-parametric approach based on the transfer function model to forecast a time series of riverflow. Jain and Kumar (2007) and Coulibary and Baldwin (2005) employed artificial neural networks methods for hydrological time series forecasting. Such methods are useful for assessing water demand under some stability conditions. However, their ability to project demand into the future may be limited as a result of weather conditions variability and changes in consumer behavior and technology.

Water demand is highly dominated by daily, weekly and yearly seasonal cycles. The univariate time series models based on the historical data series can be quite useful for short-term demand forecasting as we accommodate the various periodic and seasonal cycles in the model specifications and forecasts. To avoid their sensibility to changes in weather conditions and other seasonal patterns, we may combine forecasts derived from the most accurate forecasting methods for different forecast origins and horizons. Combining forecasts can reduce errors by averaging of individual forecasts (Clemen, 1989, Armstrong 2001) and is particularly useful when we are uncertain about which forecasting method is better for future prediction. Some relevant works on combined forecasts of univariate time series models are by Makridakis and Winkler (1983), Sanders and Ritzman (1989), Lobo (1992) and Makridakis, Chatfield, Hibon, Lawrence, Mills, Ord and Simons (1993).

In this paper, we examine the water demand forecasting performance of double seasonal univariate time series models based on multi-step ahead forecast mean squared errors. We investigate whether combining forecasts from different methods and from different origins and horizons could improve forecast accuracy. Our interest in this problem arose from the time series competition organized by Spanish IEEE Computational Intelligence Society at the SICO'2007 Conference.

The remainder of the paper is organized as follows. Section 2 describes the dataset used in the study. Section 3 discusses the methodology used in time series modelling and forecasting. Section 4 presents the empirical results. Section 5 offers some concluding remarks.

2. Data

We analyze the daily water consumption series in Spain from 1 January 2001 to 30 June 2006 (2006 observations). We have drop February 29 in the leap year 2004 in order to maintain 365 days in each year. This series is plotted in Figure 1. The dataset was obtained from the Spanish IEEE Computational Intelligence Society (http://www.congresocedi.es/2007/).

We use the first 1976 observations from 1 January 2001 to 31 May 2006 as training sample for model estimation, and the remaining 30 observations from 1 June 2006 to 30 June 2006 as post-sample for forecast evaluation. The series exhibits periodic behavior with a within-week seasonal cycle of 7 periods and a within-year cycle of 365 periods. The observed increases (decreases) in demand in the summer (winter) days seem to be caused by good (bad) weather. The analysis of weekly seasonality shows a consumption drop in demand on Saturdays and Sundays as a result of the shutdown of industry.

Figure 2 shows the sample autocorrelations (ACF) and the sample partial autocorrelations (PACF) for the training sample. The ACF decays very slowly at regular lags and at multiples of seasonal periods 7 and 365. The PACF has a large spike at lag 1 and cut off to zero after lag 2. This suggests both a weekly seasonal difference $(1 - B^7)$ and a yearly seasonal difference $(1 - B^{365})$ to achieve stationarity. Figures 3 and 4 present the double seasonal differenced series $(1 - B^7)(1 - B^{365})Y_t$ and their estimated ACF and PACF functions.

3. Methodology

3.1. Forecast evaluation

Denote the actual observation for time period t by Y_t and the forecasted value for the same period by F_t. The mean squared error (MSE) statistic for the post-sample period $t = m+1, m+2, ..., m+h$ is defined as follows:

$$MSE = \frac{1}{h} \sum_{t=m+1}^{m+h} (Y_t - F_t)^2. \quad (1)$$

This statistic is used to evaluate the out-of-sample forecast accuracy using a training sample of observations of size $m < n$ (where n is the sample size) to estimate the model, and then computing recursively the one-step ahead forecasts for time periods $m + 1$, $m + 2$, ... by increasing the training sample by one. For k-step ahead forecasts, we begin at the start of the training sample and we compute the forecast errors for time periods $t = m + k$, $m + k + 1$, ... using the same recursive procedure.

3.2. Random walk

The naïve version of the random walk model is defined as

$$F_{t+1} = Y_t. \quad (2)$$

This purely deterministic method uses the most recent observation as a forecast, and is used as a basis for evaluating of time series models described below.

3.3. Exponential smoothing

Exponential smoothing is a simple but very useful technique of adaptive time series forecasting. Standard seasonal methods of exponential smoothing includes the Holt-Winters' additive trend, multiplicative trend, damped additive trend and damped multiplicative trend (see Gardner, 2006). We implemented the double seasonal versions of the Holt-Winters' exponential smoothing (Taylor, 2003) in order to take into account the two seasonal cycle periods in the daily water consumption: a within-week cycle of 7 days and a within-year cycle of 365 days. In an application to half-hourly electricity demand, Taylor (2003) used a within-day seasonal cycle of 48

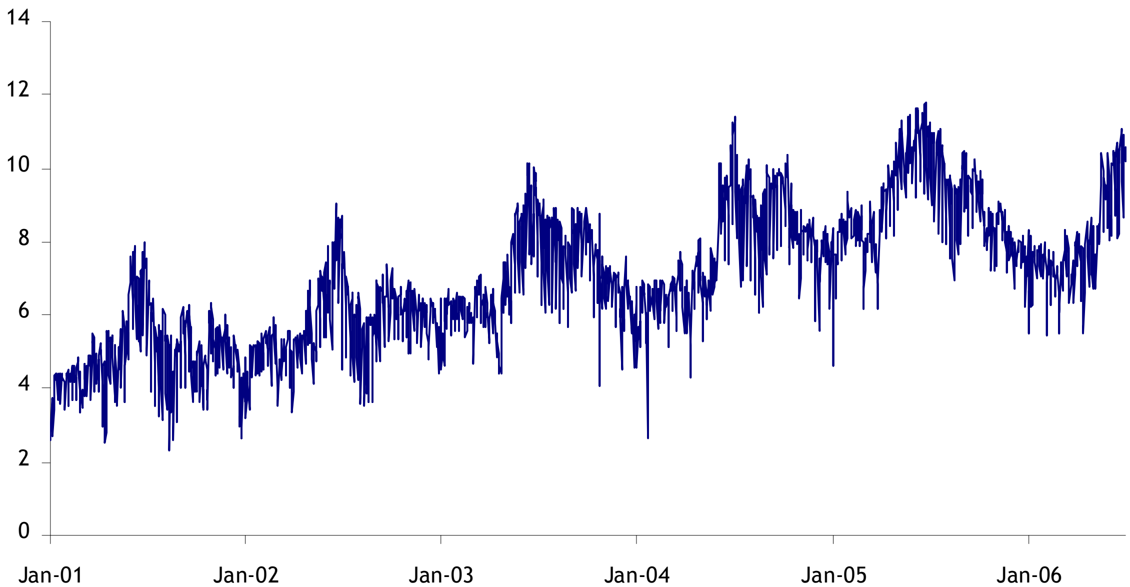

Figure 1. Daily water demand in Spain for the period 1 January 2001 to 30 June 2006

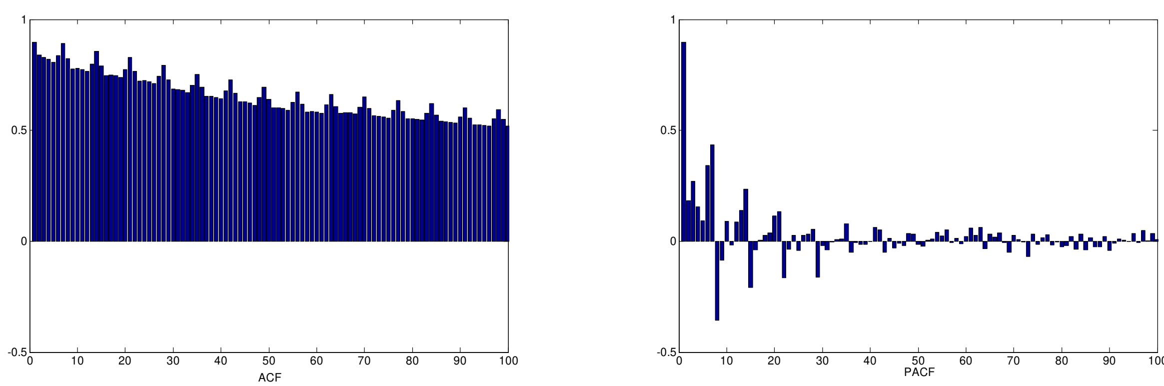

Figure 2. ACF and PACF of the water demand series

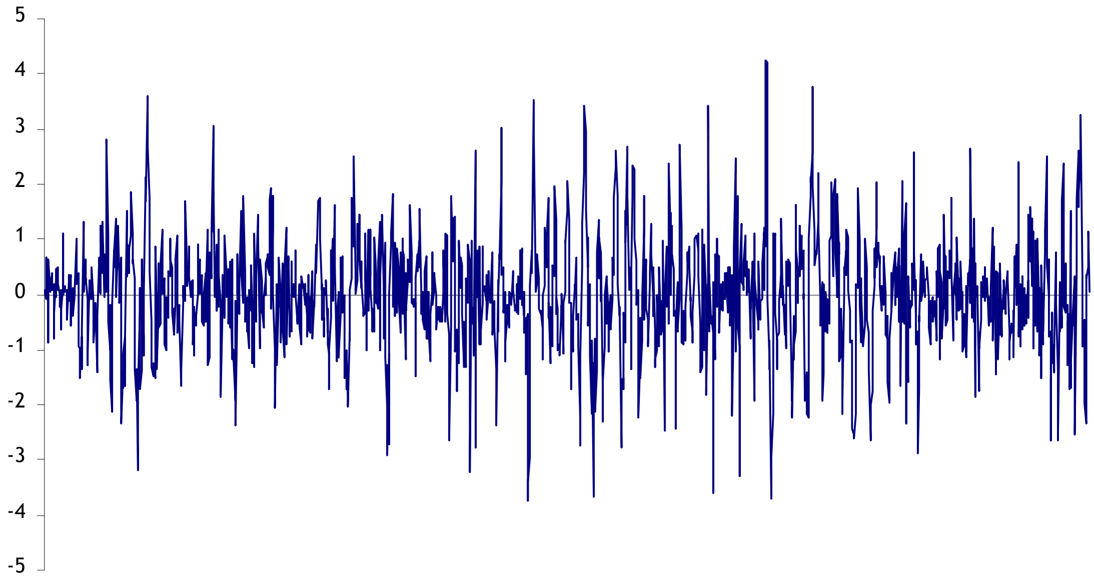

Figure 3. Water demand series after yearly seasonal differencing and weekly seasonal differencing

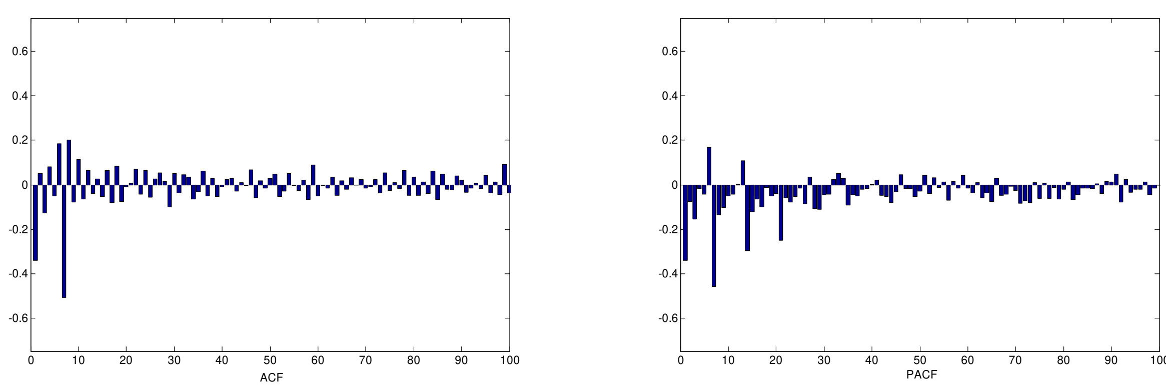

Figure 4. ACF and PACF of the differenced water demand series

half-hours and a within-week seasonal cycle of 336 half-hours.

The double seasonal additive methods outperformed the double seasonal multiplicative methods. Within the double seasonal additive methods, the additive trend was found to be the best for one-step ahead forecasting.

The forecasts for Taylor's exponential smoothing for double seasonal additive method with additive trend are determined by the following expressions:

$$L_t = \alpha(Y_t - S_{t-7} - D_{t-365}) + (1-\alpha)(L_{t-1} + T_{t-1}) \quad (3)$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \quad (4)$$

$$S_t = \gamma(Y_t - L_t - D_{t-365}) + (1-\gamma)S_{t-7} \quad (5)$$

$$D_t = \delta(Y_t - L_t - S_{t-7}) + (1-\delta)D_{t-365} \quad (6)$$

$$F_{t+h} = L_t + T_t \times h + S_{t+h-7} + D_{t+h-365} + \phi^h \times [Y_t - (L_{t-1} - T_{t-1} - S_{t-7} - D_{t-365})] \quad (7)$$

where L_t is the smoothed level of the series; T_t is the smoothed additive trend; S_t is the smoothed seasonal index for weekly period ($s_1 = 7$); D_t is the smoothed seasonal index for yearly period ($s_2 = 365$); α and β are the smoothing parameters for the level and trend; γ and δ are the seasonal smoothing parameters; ϕ is an adjustment for first-order autocorrelation; and F_{t+h} is the forecast for h periods ahead, with $h \leq 7$. We initialize the values for the level, trend and seasonal periods as follows:

$$L_{365} = \frac{1}{365}\sum_{t=1}^{365} Y_t$$

$$T_{365} = \frac{1}{365^2}\left(\sum_{t=366}^{730} Y_t - \sum_{t=1}^{365} Y_t\right)$$

$$S_1 = \frac{Y_1}{L_7}, S_2 = \frac{Y_2}{L_7}, ..., S_7 = \frac{Y_7}{L_7}$$

$$D_1 = \frac{Y_1}{L_{365}}, D_2 = \frac{Y_2}{L_{365}}, ..., D_{365} = \frac{Y_{365}}{L_{365}}$$

The smoothing parameters α, β, γ, δ and ϕ are chosen by minimizing the MSE statistic for one-step-ahead in-sample forecasting using a linear optimization algorithm.

3.4. ARIMA model

We implemented a double seasonal multiplicative ARIMA model (see Box, Jenkins and Reinsel, 1994) of the form:

$$\phi_p(B)\Phi_{P_1}(B^{s_1})\Pi_{P_2}(B^{s_2})(1-B)^d \times(1-B^{s_1})^{D_1}(1-B^{s_2})^{D_2}(Y_t - c) = \theta_q(B)\Theta_{Q_1}(B^{s_1})\Psi_{Q_2}(B^{s_2})\varepsilon_t \quad (8)$$

where c is a constant term; B is the lag operator such that $B^k Y_t = Y_{t-k}$; $\phi_p(B)$ and $\theta_q(B)$ are regular autoregressive and moving average polynomials of orders p and q; $\Phi_{P_1}(B^{s_1})$, $\Pi_{P_2}(B^{s_2})$, $\Theta_{Q_1}(B^{s_1})$ and $\Psi_{Q_2}(B^{s_2})$ are seasonal autoregressive and moving average polynomials of orders P_1, P_2, Q_1 and Q_2; s_1 and s_2 are the seasonal periods; d, D_1 and D_2 are the orders of integration; and ε_t is a white noise process with zero mean and constant variance. The roots of the polynomials $\phi_p(B) = 0$, $\Phi_{P_1}(B^{s_1}) = 0$, $\Pi_{P_2}(B^{s_2}) = 0$, $\theta_q(B) = 0$, $\Theta_{Q_1}(B^{s_1}) = 0$ and $\Psi_{Q_2}(B^{s_2}) = 0$ should lie outside the unit circle. This model is often denoted as ARIMA$(p,d,q)\times(P_1,D_1,Q_1)_{s_1}\times(P_2,D_2,Q_2)_{s_2}$.

We examine the sample autocorrelations and the partial autocorrelations of the differenced series in order to identify the integer values p, q, P_1, Q_1, P_2 and Q_2. After identifying a tentative ARIMA model, we estimate the parameters by Marquardt nonlinear least squares algorithm (for details, see Davison and MacKinnon, 1993). We check the adequacy of the model by using suitable fitted residuals tests. We use the Schwarz Bayesian Criterion (SBC) for model selection.

3.5. GARCH model

In many practical applications to time series modelling and forecasting, the assumption of nonconstant variance may be not reliable. The models with nonconstant variance are referred to as conditional heteroscedasticity or volatility models. To deal with the problem of heteroscedasticity in the errors, Engle (1982) and Bollerslev (1986) proposed the autoregressive conditional heteroskedasticity (ARCH) and the generalized ARCH (or GARCH) to model and forecast the conditional variance (or volatility). The

GARCH(p,q) model assumes the form:

$$\sigma_t^2 = \omega + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \tag{9}$$

where p is the order of the GARCH terms and q is the order of the ARCH terms. The necessary conditions for the model (9) to be variance and covariance stationary are: $\omega > 0$; $\beta_j \geq 0$, $j = 1, ..., p$; $\alpha_i \geq 0$, $j = 1, ..., q$; and $\sum_{j=1}^{p} \beta_j + \sum_{i=1}^{q} \alpha_i < 1$. Last summation quantifies the shock persistence to volatility. A higher persistence indicates that periods of high (slow) volatility in the process will last longer. In most economical and financial applications, the simple GARCH(1,1) model has been found to provide a good representation of a wide variety of volatility processes as discussed in Bollerslev, Chou and Kroner (1992).

In order to capture seasonal and cyclical components in the volatility dynamics, we implemented a seasonal-periodic GARCH model of the form:

$$\begin{aligned} \sigma_t^2 \quad = \quad & \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_7 \varepsilon_{t-7}^2 + \alpha_{365} \varepsilon_{t-365}^2 \\ & + \sum_{k=1}^{M} \lambda_k \cos\left(\frac{2\pi k S_t}{7}\right) + \varphi_k \sin\left(\frac{2\pi k S_t}{7}\right) \\ & + u_k \cos\left(\frac{2\pi k D_t}{365}\right) + v_k \sin\left(\frac{2\pi k D_t}{365}\right) \\ & + \lambda_k' \varepsilon_{t-7}^2 \cos\left(\frac{2\pi k S_t}{7}\right) \\ & + \varphi_k' \varepsilon_{t-7}^2 \sin\left(\frac{2\pi k S_t}{7}\right) \\ & + u_k' \varepsilon_{t-365}^2 \cos\left(\frac{2\pi k D_t}{365}\right) \\ & + v_k' \varepsilon_{t-365}^2 \sin\left(\frac{2\pi k D_t}{365}\right), \end{aligned} \tag{10}$$

where S_t and D_t are repeating step functions with the days numerated from 1 to 7 within each week, and from 1 to 365 within each year, respectively. A similar approach was used by Campbell and Diebold (2005) to model conditional variance in daily average temperature data, and by Taylor (2006) to forecast electricity consumption. In the empirical study, we set $M = 3$ for the Fourier series. We estimate the model by the method of maximum likelihood, assuming a generalized error distribution (GED) for the innovations series (see Nelson, 1991).

3.6. Combining forecasts

We examine whether combining forecasts from the various univariate methods for different forecast origins and horizons could provide more accurate forecasts than the individual methods being combined. The forecasts can be combined by using simple and optimal weights.

3.6.1. Simple combination

We consider all possible combinations of the forecast methods Holt-Winters (HW), ARIMA (A) and GARCH (G), and we compute the simple (unweighted) average of the forecasts for one to seven days ahead,

$$F_t^S = \frac{F_t^{(HW)} + F_t^{(A)} + F_t^{(G)}}{3}, \tag{11}$$

where $F_t^{(\cdot)}$ is the forecasted value of method $(\cdot)$ in time period t. This approach is simple and useful when we have no evidence about which forecasting method is more accurate. We drop the random walk (the worst method) of the combination.

3.6.2. Optimal combination

We consider two approaches for computing optimal weights. Firstly, we compute the optimal combination of the forecasts using weights by the inverse of the MSE of each of the individual methods (see Makridakis and Winkler, 1983), as follows:

$$\begin{aligned} F_t^{MSE} \quad = \quad & \Big[(MSE - MSE^{(HW)}) F_t^{(HW)} \\ & + (MSE - MSE^{(A)}) F_t^{(A)} \\ & + (MSE - MSE^{(G)}) F_t^{(G)} \Big] \\ & \div 2MSE, \end{aligned} \tag{12}$$

where $MSE = MSE^{(HW)} + MSE^{(A)} + MSE^{(G)}$ is the sum of the post-sample forecast mean squared errors of the three methods.

Secondly, we compute optimal combination of the post-sample forecasts using weights by the inverse of each of the forecast squared errors (SE) of each of the individual methods, as follows:

$$\begin{aligned} F_t^{SE} &= \Big[(SE_t - SE_t^{(HW)})F_t^{(HW)} \\ &\quad +(SE_t - SE_t^{(A)})F_t^{(A)} \\ &\quad +(SE_t - SE_t^{(G)})F_t^{(G)}\Big] \\ &\quad \div 2SE_t, \end{aligned} \tag{13}$$

where $SE_t = SE_t^{(HW)} + SE_t^{(A)} + SE_t^{(G)}$ is the sum of the post-sample forecast squared errors of the three methods in time period t.

4. Empirical study

4.1. Estimation results

The implementation of the double seasonal Holt-Winters method to the water demand series Y_t gives the values: $\alpha = 0.000$, $\beta = 0.755$, $\gamma = 0.303$, $\delta = 0.294$ and $\phi = 0.607$.

After evaluating different ARIMA formulations, we apply the following multiplicative double seasonal ARIMA model:

$$\begin{aligned} &(1-\phi_1 B - \phi_2 B^2 - \phi_4 B^4)(1-\Phi_1 B^7 - \Phi_2 B^{14}) \\ &\times(1-B^7)(1-B^{365})(Y_t - c) \\ &= (1-\theta_9 B^9)(1-\Theta_3 B^{21})(1-\Psi_1 B^{365})\varepsilon_t \end{aligned}$$

This model can be represented as ARIMA$(4,0,9) \times (2,1,3)_7 \times (0,1,1)_{365}$, with $\phi_3 = 0$, $\theta_1 = \cdots = \theta_8 = 0$, and $\Theta_1 = \Theta_2 = 0$. The estimated results and diagnostic checks are shown in Table 1. All the parameter estimates are significant at the 5% significance level. The residual autocorrelation function (ACF) and partial autocorrelation function (PACF) exhibit no patterns up to order 7. The Ljung-Box statistic, $Q = 18.31$, based on 20 residual autocorrelations is not significant at the conventional levels. These results suggest that the model is appropriate for modeling the water demand series.

We then fitted a significant parameter ARIMA-GARCH model of the form:

$$\begin{aligned} &(1-\phi_1 B - \phi_2 B^2 - \phi_4 B^4)(1-\Phi_1 B^7 - \Phi_2 B^{14}) \\ &\times(1-B^7)(1-B^{365})(Y_t - c) \\ &= (1-\theta_9 B^9)(1-\Theta_3 B^{21})(1-\Psi_1 B^{365})\varepsilon_t \end{aligned}$$

and

$$\begin{aligned} \sigma_t^2 &= \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_{365}\varepsilon_{t-365}^2 \\ &\quad +\varphi_1 \sin\left(\frac{2\pi D_t}{365}\right) + \varphi_3' \varepsilon_{t-365}^2 \sin\left(\frac{6\pi D_t}{365}\right). \end{aligned}$$

The model estimates and diagnostic checks are given in Table 2. The Ljung-Box test statistics show evidence of no serial correlation in the residuals (mean equation) and no serial correlation in the squared residuals (variance equation) up to order 20. Thus, we conclude that this model is also adequate for the data.

4.2. Forecast evaluation results

The performance of the estimated univariate methods were evaluated by computing MSE statistics for multi-step forecasts from 1 to 7 days ahead.

Table 3 and Figure 5 give the forecasts results for the post-sample period from 1 June 2006 to 30 June 2006. An initial interpretation of the results suggests that the ability to forecast water demand did not decrease as the forecast horizon increased, except from 1 to 2 days ahead.

The ARIMA and GARCH models appear to have the same forecast performance especially for short-term forecasts (one to two days ahead). For one to four days ahead forecasts, the stochastic models ARIMA and GARCH performed better than the Holt-Winters method. In contrast, the Holt-Winters outperformed the ARIMA and GARCH models in long horizons. The random walk model ranked last for any of the forecast horizons considered.

The optimal combination of Holt-Winters, ARIMA and GARCH weighted by inverse squared errors is more accurate than the various simple combinations, except for 7-step ahead forecasting in which the Holt-Winters outperformed the optimal combined forecasting. For one day ahead, the average MSE for the individual forecasting methods (HW, ARIMA and GARCH) was 0.36 while it was 0.33 for the optimal combined forecasts – a error reduction of 8.33%. For two and three days ahead forecasts, combining reduced the MSE by 12.77% and 10.64%, respectively.

Table 4 and Figure 6 give the forecast results for each of the 7 days of the week in the same period. The results suggests that the Thursdays exhibit irregular demand patterns in the post-sample period used in this study. From the data, we found that the water consumption decreased 10.37% on the first Thursday of the post-sample period (1 June 2006), whereas it increased 4.22% and 18.44% on the following Thursdays (8 June 2006 and 15 June 2006, respectively). Possible reasons for this unusual pattern are weather changes and any restrictions on water demand.

In terms of the day of the week effect on forecasting performance, the optimal combination HW-A-G (SE) appears to be most useful for Monday, Tuesday and Wednesday forecasts – combining reduced the MSE of multi-step ahead averaged forecasts by 12.15%, 45.45% and 14.60%, respectively, when compared with the average of the individual methods. The Holt-Winters appears to be the most appropriate method for Thursday, Friday and Saturday forecasts and the GARCH model appears to be the best method for Sunday forecasts.

5. Conclusions

In this article we compared the forecast accuracy of individual and combined univariate time series models for multi-step ahead daily water demand forecasting. We implemented double seasonal versions of the Holt-Winters, ARIMA and GARCH models in order to accommodate the two seasonal effects (within-week cycle of 7 days and within-year cycle of 365 days) on the variability of the data.

The empirical results suggest that the optimal combined forecasts can be quite useful especially for short-term forecasting. However, the forecasting performance of this approach is not consistent over the seven days of the week. The deterministic method Holt-Winters and the stochastic method GARCH can be used independently to improve forecast accuracy on Thursdays to Saturdays and Sundays, respectively.

Acknowledgment: The author gratefully acknowledges the helpful comments of the participants in the Spanish IEEE Computational Intelligence Society at the SICO'2007 Conference. This research was supported by a grant from the Fundação para a Ciência e a Tecnologia (FEDER/POCI 2010).

REFERENCES

1. Armstrong, J. (2001). "Combining forecasts", in *Principles of Forecasting: A Handbook for Researchers and Practitioners*, J. S. Armstrong (ed.), Kluwer Academic Publishers.
2. Bollerslev, T. (1986). "Generalized autoregressive conditional heteroskedasticity", *Journal of Econometrics*, 31, 307-327.
3. Bollerslev, T., Chou, R. and Kroner, K. (1992). "ARCH modeling in Finance", *Journal of Econometrics*, 52, 5-59.
4. Bougadis, J., Adamowski, K. and Diduch, R. (2005). "Short-term municipal water deamand forecasting", *Hydrological Processes*, 19, 137-148.
5. Box, G., Jenkins, G. and Reinsel, G. (1994). *Time Series Analysis: Forecasting and Control*, 3rd ed., Prentice-Hall, New Jersey.
6. Campbell, S. and Diebold, F. (2005). "Weather forecasting for weather derivatives", *Journal of the American Statistical Association*, 100, 6-16.
7. Clemen, R. (1989). "Combining forecasts: a review and annoted bibliography", *International Journal of Forecasting*, 5, 559-584.
8. Coulibaly, P. and Baldwin, C. (2005): "Nonstationary hydrological time series forecasting using nonlinear dynamic methods", *Journal of Hydrology*, 307, 164-174.
9. Davison, R. and MacKinnon, J. (1993). *Estimation and Inference in Econometrics*, Oxford University Press, Oxford.
10. Engle, R. (1982). "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation", *Econometrica*, 50, 987-1008.
11. Fildes, R., Randall, A. and Stubbs, P. (1997). "One-day ahead demand forecasting in the utility industries: Two case studies", *Journal of the Operational Research Society*, 48, 15-24.
12. Gardner Jr., E. (2006). "Exponential smooth-

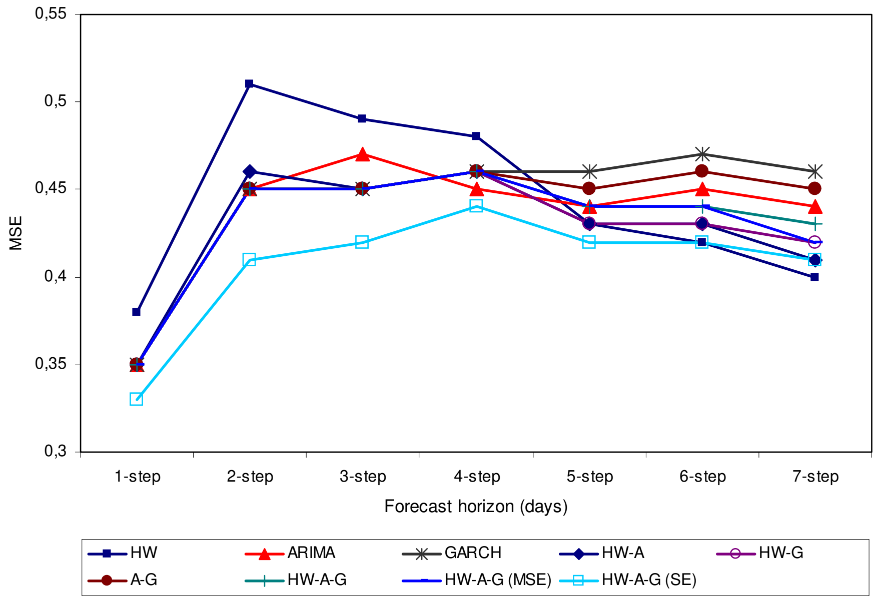

Figure 5. Comparison of multi-step ahead forecasts for post-sample period

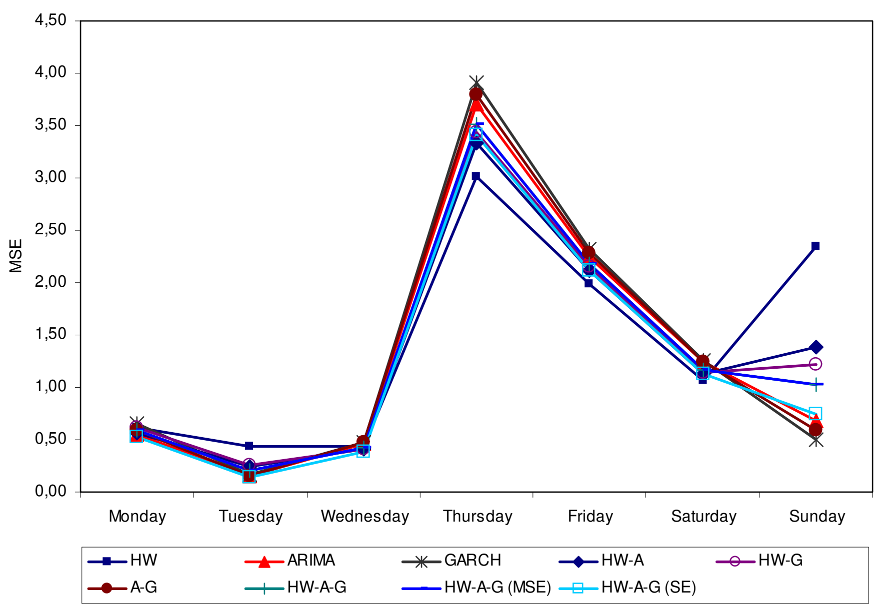

Figure 6. Comparison of multi-step ahead averaged forecasts for each of the seven days of the week

ing: The state of the art - Part II", *International Journal of Forecasting*, 22, 637-666.

13. Jain, A., Varshney, A. and Joshi, U. (2001). "Short-term water demand forecast modeling at IIT Kanpur using artificial neural networks", *Water Resources Management*, 15, 299-231.
14. Jain, A. and Kumar, A.M. (2007). "Hybrid neural network models for hydrologic time series forecasting", *Applied Soft Computing*, 7, 585-592.
15. Lobo, G. (1992). "Analysis and comparison of financial analysts, time series, and combining forecasts of annual earnings", *Journal of Business Research*, 24, 269-280.
16. Maidment, D. and Miaou, S. (1986). "Daily water use in nine cities", *Water Resources Research*, 22, 845-851.
17. Madridakis, S. and Winkler, R. (1983). "Average of forecasts: Some empirical results", *Management Science*, 29, 987-996.
18. Madridakis, S., Chatfield, C., Hibon, M., Lawrence, M., Mills, T., Ord. K. and Simons, L. (1993). "The M2-competition: A real-time judgmentally based forecasting study", *International Journal of Forecasting*, 9, 5-22.
19. Nelson, D. (1991). "Conditional heteroskedasticity in asset returns: a new approach", *Econometrica*, 59, 347-370.
20. Sanders, N. and Ritzman, L. (1989). "Some empirical findings on short-term forecasting: Technique complexity and combinations", *Decision Sciences*, 20, 635-640.
21. Taylor, J. (2003). "Short-term electricity demand forecasting using double seasonal exponential smoothing", *Journal of the Operational Research Society*, 54, 799-805.
22. Taylor, J. (2006). "Density forecasting for the efficient balancing of the generation and consumption of electricity", *International Journal of Forecasting*, 22, 707-724.
23. Wong, H., Ip, W., Zhang, R. and Xia, J. (2007). "Non-parametric time series models for hydrological forecasting", *Journal of Hidrology*, 332, 337-347.
24. Zhou, S., McMahon, T., Walton, A. and Lewis, J. (2000). "Forecasting daily urban water demand: a case study of Melbourne", *Journal of Hidrology*, 236, 153-164.

Table 1
Seasonal ARIMA model estimates for water demand series

Model: ARIMA(4,0,9)×(2,1,3)$_7$×(0,1,1)$_{365}$				Residual ACF		Residual PACF	
Parameter	Lag	Estimate	Standard error	Lag	Estimate	Lag	Estimate
c		-0.004	0.007	1	0.004	1	0.004
ϕ_1	1	0.592	0.025	2	0.009	2	0.009
ϕ_2	2	0.134	0.027	3	-0.020	3	-0.020
ϕ_4	4	0.061	0.023	4	0.001	4	0.001
θ_9	9	-0.053	0.024	5	-0.026	5	-0.025
Φ_1	7	-0.757	0.023	6	0.015	6	0.015
Φ_2	14	-0.561	0.029	7	-0.010	7	-0.010
Θ_3	21	-0.366	0.032				
Ψ_1	365	-0.644	0.023				
R^2 adjusted = 0.662; $Q(20) = 18.31$ (0.11).							

Notes: $Q(20)$ is the Ljung-Box statistic for serial correlation in the residuals up to order 20; p-value in parentheses.

Table 2
Seasonal-periodic GARCH model estimates for water demand series

Model: ARIMA(4,0,9)×(2,1,3)$_7$×(0,1,1)$_{365}$—GARCH(1,1)×(0,1)$_{365}$							
	Mean equation			Residual ACF		Residual PACF	
Parameter	Lag	Estimate	Standard error	Lag	Estimate	Lag	Estimate
c		-0.011	0.008	1	-0.007	1	0.007
ϕ_1	1	0.502	0.029	2	0.023	2	0.023
ϕ_2	2	0.137	0.030	3	-0.028	3	-0.028
ϕ_4	4	0.075	0.024	4	-0.026	4	-0.026
θ_9	9	-0.064	0.023	5	-0.042	5	-0.040
Φ_1	7	-0.747	0.023	6	0.026	6	0.027
Φ_2	14	-0.534	0.028	7	-0.006	7	-0.006
Θ_3	21	-0.346	0.031				
Ψ_1	365	-0.640	0.025				
	Variance equation			Sq. residual ACF		Sq. residual PACF	
Parameter	Lag	Estimate	Standard error	Lag	Estimate	Lag	Estimate
ω		0.107	0.028	1	0.012	1	0.012
α_1	1	0.103	0.037	2	-0.030	2	-0.031
β_1	1	0.483	0.108	3	0.028	3	0.029
α_{365}	365	0.109	0.032	4	0.018	4	0.016
φ_1		0.026	0.011	5	0.008	5	0.009
φ'_3	365	0.062	0.035	6	-0.023	6	-0.023
GED		1.361	0.055	7	0.015	7	0.015
R^2 adjusted = 0.657; $Q(20)$=19.20 (0.08); $Q^2(20)$=13.61 (0.33).							

Notes: $Q(20)$ ($Q^2(20)$) is the Ljung-Box statistic for serial correlation in the residuals (squared residuals) up to order 20; p-value in parentheses.

Table 3
MSE for multi-step-ahead forecasts for post-sample period

Forecast horizon	RW	HW	ARIMA	GARCH	Simple combination HW-A	HW-G	A-G	HW-A-G	Optimal combin. MSE	SE
1-step	0.96	0.38	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.33
2-step	1.55	0.51	0.45	0.45	0.46	0.45	0.45	0.45	0.45	0.41
3-step	1.82	0.49	0.47	0.45	0.45	0.45	0.45	0.45	0.45	0.42
4-step	2.09	0.48	0.45	0.46	0.46	0.46	0.46	0.46	0.46	0.44
5-step	2.23	0.43	0.44	0.46	0.43	0.43	0.45	0.44	0.44	0.42
6-step	1.91	0.42	0.45	0.47	0.43	0.43	0.46	0.44	0.44	0.42
7-step	1.33	0.40	0.44	0.46	0.41	0.42	0.45	0.43	0.42	0.41
Average	1.70	0.44	0.44	0.44	0.43	0.43	0.44	0.43	0.43	0.41

Table 4
MSE for multi-step ahead forecasts for each day of the week in post-sample period

Forecast horizon	Day of the week	RW	HW	ARIMA	GARCH	Simple combination HW-A	HW-G	A-G	HW-A-G	Optimal combin. MSE	SE
1-step	Monday	16.18	2.33	1.18	1.25	1.71	1.75	1.21	1.55	1.54	1.34
	Tuesday	0.28	0.53	0.20	0.19	0.34	0.34	0.19	0.29	0.28	0.21
	Wednesday	0.18	0.14	0.25	0.26	0.19	0.20	0.26	0.21	0.22	0.20
	Thursday	3.15	4.19	5.26	5.40	4.71	4.78	5.33	4.93	4.94	4.84
	Friday	0.47	0.37	0.54	0.54	0.45	0.45	0.54	0.48	0.48	0.35
	Saturday	3.00	0.23	0.64	0.58	0.39	0.37	0.61	0.45	0.46	0.40
	Sunday	1.20	1.26	0.40	0.33	0.70	0.61	0.36	0.53	0.53	0.41
4-step	Monday	3.86	0.42	0.43	0.54	0.42	0.48	0.48	0.46	0.46	0.44
	Tuesday	2.66	0.15	0.16	0.17	0.15	0.15	0.16	0.16	0.16	0.12
	Wednesday	8.39	0.48	0.69	0.77	0.58	0.62	0.73	0.64	0.64	0.59
	Thursday	11.27	3.63	3.79	4.14	3.71	3.88	3.96	3.85	3.85	3.73
	Friday	1.83	1.78	1.88	1.94	1.83	1.86	1.91	1.87	1.87	1.84
	Saturday	4.14	1.29	1.21	1.26	1.25	1.28	1.24	1.25	1.25	1.25
	Sunday	10.23	3.23	1.10	0.81	2.03	1.82	0.95	1.56	1.55	1.18
7-step	Monday	0.30	0.19	0.24	0.38	0.21	0.28	0.30	0.26	0.26	0.25
	Tuesday	0.15	0.07	0.06	0.08	0.06	0.06	0.06	0.06	0.06	0.04
	Wednesday	1.09	0.27	0.39	0.29	0.33	0.28	0.34	0.31	0.31	0.29
	Thursday	13.60	2.54	3.33	3.42	2.92	2.96	3.38	3.08	3.07	2.99
	Friday	7.91	2.14	2.25	2.38	2.19	2.26	2.32	2.26	2.25	2.17
	Saturday	4.19	1.43	1.48	1.59	1.46	1.51	1.54	1.50	1.50	1.49
	Sunday	0.70	1.14	0.29	0.22	0.63	0.51	0.26	0.42	0.44	0.31
Average	Monday	4.79	0.61	0.55	0.65	0.57	0.62	0.59	0.59	0.59	0.53
	Tuesday	4.13	0.44	0.16	0.17	0.25	0.26	0.16	0.21	0.21	0.14
	Wednesday	4.89	0.43	0.46	0.48	0.41	0.41	0.47	0.42	0.42	0.39
	Thursday	7.52	3.01	3.71	3.91	3.33	3.43	3.80	3.51	3.51	3.41
	Friday	5.21	1.99	2.25	2.32	2.12	2.15	2.28	2.18	2.18	2.11
	Saturday	4.02	1.06	1.24	1.26	1.13	1.14	1.25	1.17	1.17	1.13
	Sunday	6.10	2.35	0.68	0.50	1.38	1.22	0.59	1.02	1.02	0.75