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Performance of combined double seasonal univariate time series models for forecasting water demand

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Abstract: In this article, we examine the daily water demand forecasting performance of double seasonal univariate time series models (Holt-Winters, ARIMA and GARCH) based on multi-step ahead forecast mean squared errors. A within-week seasonal cycle and a within-year seasonal cycle are accommodated in the various model specifications to capture both seasonalities. We investigate whether combining forecasts from different methods for different origins and horizons could improve forecast accuracy. The analysis is made with daily data for water consumption in Granada, Spain.

Keywords: ARIMA; Combined forecasts; Double seasonality; Exponential Smoothing; Forecasting; GARCH; Water demand.

1. Introduction

Water demand forecasting is of great economic and environmental importance. Many factors can influence directly or indirectly water consumption. These include rainfall, temperature, demography, land use, pricing and regulation. Weather conditions have been widely used as inputs of multivariate statistical models for hydrological time series modelling and forecasting.

Maidment and Miaou (1986), Filides, Randall and Stubbs (1997), Zhou, McMahan, Walton and Lewis (2000), Jain, Varshney and Joshi (2001) and Bougadis, Adamowski and Diduch (2005) adopted regression and time series models for water demand forecasting by using climate effects as explanatory variables for their models. Wong, Ip, Zhang and Xia (2007) used a non-parametric approach based on the transfer function model to forecast a time series of riverflow. Jain and Kumar (2007) and Coulibary and Baldwin (2005) employed artificial neural networks methods for hydrological time series forecasting. Such methods are useful for assessing water demand under some stability conditions. However, their ability to project demand into the future may be limited as a result of weather conditions variability and changes in consumer behavior and technology.

Water demand is highly dominated by daily, weekly and yearly seasonal cycles. The univariate time series models based on the historical data series can be quite useful for short-term demand forecasting as we accommodate the various periodic and seasonal cycles in the model specifications and forecasts. To avoid their sensibility to changes in weather conditions and other seasonal patterns, we may combine forecasts derived from the most accurate forecasting methods for different forecast origins and horizons. Combining forecasts can reduce errors by averaging of individual forecasts (Clemen, 1989, Armstrong 2001) and is particularly useful when we are uncertain about which forecasting method is better for future prediction. Some relevant works on combined forecasts of univariate time series models are by Makridakis and Winkler (1983), Sanders and Ritzman (1989), Lobo (1992) and Makridakis, Chatfield, Hibon, Lawrence, Mills, Ord and Simons (1993).

In this paper, we examine the water demand forecasting performance of double seasonal univariate time series models based on multi-step ahead forecast mean squared errors. We investigate whether combining forecasts from different
methods and from different origins and horizons could improve forecast accuracy. Our interest in this problem arose from the time series competition organized by Spanish IEEE Computational Intelligence Society at the SICO’2007 Conference.

The remainder of the paper is organized as follows. Section 2 describes the dataset used in the study. Section 3 discusses the methodology used in time series modelling and forecasting. Section 4 presents the empirical results. Section 5 offers some concluding remarks.

2. Data

We analyze the daily water consumption series in Spain from 1 January 2001 to 30 June 2006 (2006 observations). We have drop February 29 in the leap year 2004 in order to maintain 365 days in each year. This series is plotted in Figure 1. The dataset was obtained from the Spanish IEEE Computational Intelligence Society (http://www.congresocedi.es/2007/).

We use the first 1976 observations from 1 January 2001 to 31 May 2006 as training sample for model estimation, and the remaining 30 observations from 1 June 2006 to 30 June 2006 as post-sample for forecast evaluation. The series exhibits periodic behavior with a within-week seasonal cycle of 7 periods and a within-year cycle of 365 periods. The observed increases (decreases) in demand in the summer (winter) days seem to be caused by good (bad) weather. The analysis of weekly seasonality shows a consumption drop in demand on Saturdays and Sundays as a result of the shutdown of industry.

Figure 2 shows the sample autocorrelations (ACF) and the sample partial autocorrelations (PACF) for the training sample. The ACF decays very slowly at regular lags and at multiples of seasonal periods 7 and 365. The PACF has a large spike at lag 1 and cut off to zero after lag 2. This suggests both a weekly seasonal difference \((1 - B^7)\) and a yearly seasonal difference \((1 - B^{365})\) to achieve stationarity. Figures 3 and 4 present the double seasonal differenced series \((1 - B^7)(1 - B^{365})Y_t\) and their estimated ACF and PACF functions.

3. Methodology

3.1. Forecast evaluation

Denote the actual observation for time period \(t\) by \(Y_t\) and the forecasted value for the same period by \(F_t\). The mean squared error (MSE) statistic for the post-sample period \(t = m+1, m+2, \ldots, m+h\) is defined as follows:

\[
MSE = \frac{1}{h} \sum_{t=m+1}^{m+h} (Y_t - F_t)^2. \tag{1}
\]

This statistic is used to evaluate the out-of-sample forecast accuracy using a training sample of observations of size \(m < n\) (where \(n\) is the sample size) to estimate the model, and then computing recursively the one-step ahead forecasts for time periods \(m + 1, m + 2, \ldots\) by increasing the training sample by one. For \(k\)-step ahead forecasts, we begin at the start of the training sample and we compute the forecast errors for time periods \(t = m + k, m + k + 1, \ldots\) using the same recursive procedure.

3.2. Random walk

The naïve version of the random walk model is defined as

\[
F_{t+1} = Y_t. \tag{2}
\]

This purely deterministic method uses the most recent observation as a forecast, and is used as a basis for evaluating of time series models described below.

3.3. Exponential smoothing

Exponential smoothing is a simple but very useful technique of adaptive time series forecasting. Standard seasonal methods of exponential smoothing includes the Holt-Winters’ additive trend, multiplicative trend, damped additive trend and damped multiplicative trend (see Gardner, 2006). We implemented the double seasonal versions of the Holt-Winters’ exponential smoothing (Taylor, 2003) in order to take into account the two seasonal cycle periods in the daily water consumption: a within-week cycle of 7 days and a within-year cycle of 365 days. In an application to half-hourly electricity demand, Taylor (2003) used a within-day seasonal cycle of 48
Figure 1. Daily water demand in Spain for the period 1 January 2001 to 30 June 2006

Figure 2. ACF and PACF of the water demand series
Figure 3. Water demand series after yearly seasonal differencing and weekly seasonal differencing

Figure 4. ACF and PACF of the differenced water demand series
half-hours and a within-week seasonal cycle of 336 half-hours.

The double seasonal additive methods outperformed the double seasonal multiplicative methods. Within the double seasonal additive methods, the additive trend was found to be the best for one-step ahead forecasting.

The forecasts for Taylor’s exponential smoothing for double seasonal additive method with additive trend are determined by the following expressions:

\[
L_t = \alpha(Y_t - S_{t-7} - D_{t-365}) + (1 - \alpha)(L_{t-1} + T_{t-1})
\]

\[T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}\]

\[S_t = \gamma(Y_t - L_t - D_{t-365}) + (1 - \gamma)S_{t-7}\]

\[D_t = \delta(Y_t - L_t - S_{t-7}) + (1 - \delta)D_{t-365}\]

\[F_{t+h} = L_t + T_t \times h + S_{t+h-7} + D_{t+h-365} + \phi^h\]

\[\times [Y_t - (L_{t-1} - T_{t-1} - S_{t-7} - D_{t-365})]\]

where \(L_t\) is the smoothed level of the series; \(T_t\) is the smoothed seasonal index for weekly period \((s_1 = 7)\); \(S_t\) is the smoothed seasonal index for yearly period \((s_2 = 365)\); \(\alpha\) and \(\beta\) are the smoothing parameters for the level and trend; \(\gamma\) and \(\delta\) are the seasonal smoothing parameters; \(\phi\) is an adjustment for first-order autocorrelation; and \(F_{t+h}\) is the forecast for \(h\) periods ahead, with \(h \leq 7\). We initialize the values for the level, trend and seasonal periods as follows:

\[L_{365} = \frac{1}{365} \sum_{t=1}^{365} Y_t\]

\[T_{365} = \frac{1}{365^2} \left( \sum_{t=366}^{730} Y_t - \sum_{t=1}^{365} Y_t \right)\]

\[S_1 = \frac{Y_1}{L_7}, S_2 = \frac{Y_2}{L_7}, ..., S_7 = \frac{Y_7}{L_7}\]

\[D_1 = \frac{Y_1}{L_{365}}, D_2 = \frac{Y_2}{L_{365}}, ..., D_{365} = \frac{Y_{365}}{L_{365}}\]

The smoothing parameters \(\alpha, \beta, \gamma, \delta\) and \(\phi\) are chosen by minimizing the MSE statistic for one-step-ahead in-sample forecasting using a linear optimization algorithm.

### 3.4. ARIMA model

We implemented a double seasonal multiplicative ARIMA model (see Box, Jenkins and Reinsel, 1994) of the form:

\[
\phi_p(B)\Phi_P(B^{s_1})\Pi_{P_2}(B^{s_2})(1 - B)^d \times (1 - B^{s_1})^{D_1} (1 - B^{s_2})^{D_2} (Y_t - c) = \theta_q(B)\Theta_{Q_1}(B^{s_1})\Psi_{Q_2}(B^{s_2}) \varepsilon_t
\]

where \(c\) is a constant term; \(B\) is the lag operator such that \(B^k Y_t = Y_{t-k}\); \(\phi_p(B)\) and \(\theta_q(B)\) are regular autoregressive and moving average polynomials of orders \(p\) and \(q\); \(\Phi_P(B^{s_1})\), \(\Pi_{P_2}(B^{s_2})\), \(\Theta_{Q_1}(B^{s_1})\) and \(\Psi_{Q_2}(B^{s_2})\) are seasonal autoregressive and moving average polynomials of orders \(P_1\), \(P_2\), \(Q_1\) and \(Q_2\); \(s_1\) and \(s_2\) are the seasonal periods; \(d\), \(D_1\) and \(D_2\) are the orders of integration; and \(\varepsilon_t\) is a white noise process with zero mean and constant variance. The roots of the polynomials \(\phi_p(B) = 0\), \(\Phi_P(B^{s_1}) = 0\), \(\Pi_{P_2}(B^{s_2}) = 0\), \(\Theta_{Q_1}(B^{s_1}) = 0\) and \(\Psi_{Q_2}(B^{s_2}) = 0\) should lie outside the unit circle. This model is often denoted as ARIMA\((p,d,q)\times(P_1,D_1,Q_1)s_1\times(P_2,D_2,Q_2)s_2\).

We examine the sample autocorrelations and the partial autocorrelations of the differenced series in order to identify the integer values \(p\), \(q\), \(P_1\), \(Q_1\), \(P_2\) and \(Q_2\). After identifying a tentative ARIMA model, we estimate the parameters by Marquardt nonlinear least squares algorithm (for details, see Davison and MacKinnon, 1993). We check the adequacy of the model by using suitable fitted residuals tests. We use the Schwarz Bayesian Criterion (SBC) for model selection.

### 3.5. GARCH model

In many practical applications to time series modelling and forecasting, the assumption of nonconstant variance may be not reliable. The models with nonconstant variance are referred to as conditional heteroscedasticity or volatility models. To deal with the problem of heteroscedasticity in the errors, Engle (1982) and Bollerslev (1986) proposed the autoregressive conditional heteroskedasticity (ARCH) and the generalized ARCH (or GARCH) to model and forecast the conditional variance (or volatility). The
GARCH\((p,q)\) model assumes the form:
\[
\sigma_i^2 = \omega + \sum_{j=1}^{p} \beta_j \sigma_{i-j}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{i-i},
\]
(9)
where \(p\) is the order of the GARCH terms and \(q\) is the order of the ARCH terms. The necessary conditions for the model (9) to be variance and covariance stationary are: \(\omega > 0; \beta_j \geq 0, j = 1, \ldots, p; \alpha_i \geq 0, j = 1, \ldots, q; \text{and} \sum_j \beta_j + \sum_i \alpha_i < 1\). Last summation quantifies the shock persistence to volatility. A higher persistence indicates that periods of high (slow) volatility in the process will last longer. In most economical and financial applications, the simple GARCH(1,1) model has been found to provide a good representation of a wide variety of volatility processes as discussed in Bollerslev, Chou and Kroner (1992).

In order to capture seasonal and cyclical components in the volatility dynamics, we implemented a seasonal-periodic GARCH model of the form:
\[
\sigma_i^2 = \omega + \beta_1 \sigma_{i-1}^2 + \alpha_1 \varepsilon_{i-1}^2 + \alpha \gamma \varepsilon_{i-7}^2 + \alpha_3 \varepsilon_{i-365}^2 + \sum_{k=1}^{M} \lambda_k \cos \left(\frac{2\pi k S_i}{7}\right) + \varphi_k \sin \left(\frac{2\pi k S_i}{7}\right) + \upsilon_k \cos \left(\frac{2\pi k D_i}{365}\right) + \varphi_k \sin \left(\frac{2\pi k D_i}{365}\right) + \lambda_k' \varepsilon_{i-7}^2 \cos \left(\frac{2\pi k S_i}{7}\right) + \varphi_k' \varepsilon_{i-7}^2 \sin \left(\frac{2\pi k S_i}{7}\right) + \upsilon_k' \varepsilon_{i-365}^2 \cos \left(\frac{2\pi k D_i}{365}\right) + \varphi_k' \varepsilon_{i-365}^2 \sin \left(\frac{2\pi k D_i}{365}\right),
\]
(10)
where \(S_i\) and \(D_i\) are repeating step functions with the days numerated from 1 to 7 within each week, and from 1 to 365 within each year, respectively. A similar approach was used by Campbell and Diebold (2005) to model conditional variance in daily average temperature data, and by Taylor (2006) to forecast electricity consumption. In the empirical study, we set \(M = 3\) for the Fourier series. We estimate the model by the method of maximum likelihood, assuming a generalized error distribution (GED) for the innovations series (see Nelson, 1991).

### 3.6. Combining forecasts
We examine whether combining forecasts from the various univariate methods for different forecast origins and horizons could provide more accurate forecasts than the individual methods being combined. The forecasts can be combined by using simple and optimal weights.

#### 3.6.1. Simple combination
We consider all possible combinations of the forecast methods Holt-Winters (HW), ARIMA (A) and GARCH (G), and we compute the simple (unweighted) average of the forecasts for one to seven days ahead,
\[
F_t^S = \frac{F_t^{(HW)} + F_t^{(A)} + F_t^{(G)}}{3},
\]
(11)
where \(F_t^{(\cdot)}\) is the forecasted value of method \(\cdot\) in time period \(t\). This approach is simple and useful when we have no evidence about which forecasting method is more accurate. We drop the random walk (the worst method) of the combination.

#### 3.6.2. Optimal combination
We consider two approaches for computing optimal weights. Firstly, we compute the optimal combination of the forecasts using weights by the inverse of the MSE of each of the individual methods (see Makridakis and Winkler, 1983), as follows:
\[
F_t^{MSE} = \left[ (MSE - MSE^{(HW)}) F_t^{(HW)} + (MSE - MSE^{(A)}) F_t^{(A)} + (MSE - MSE^{(G)}) F_t^{(G)} \right] / MSE,
\]
(12)
where \(MSE = MSE^{(HW)} + MSE^{(A)} + MSE^{(G)}\) is the sum of the post-sample forecast mean squared errors of the three methods.

Secondly, we compute optimal combination of the post-sample forecasts using weights by the inverse of each of the forecast squared errors (SE)
of each of the individual methods, as follows:

\[ F_t^{SE} = \left[ (SE_t - SE_t^{(HW)})F_t^{(HW)} \right. \]
\[ \quad + (SE_t - SE_t^{(A)})F_t^{(A)} \]
\[ \left. \quad + (SE_t - SE_t^{(G)})F_t^{(G)} \right] \]
\[ \pm 2SE_t, \]  

(13)

where \( SE_t = SE_t^{(HW)} + SE_t^{(A)} + SE_t^{(G)} \) is the sum of the post-sample forecast squared errors of the three methods in time period \( t \).

4. Empirical study

4.1. Estimation results

The implementation of the double seasonal Holt-Winters method to the water demand series \( Y_t \) gives the values: \( \alpha = 0.000, \beta = 0.755, \gamma = 0.303, \delta = 0.294 \) and \( \phi = 0.607 \).

After evaluating different ARIMA formulations, we apply the following multiplicative double seasonal ARIMA model:

\[ (1 - \phi_1B - \phi_2B^2 - \phi_4B^4)(1 - \Phi_1B^7 - \Phi_2B^{14}) \]
\[ \times (1 - B^7)(1 - B^{365})(Y_t - c) \]
\[ = (1 - \theta_3B^9)(1 - \Theta_3B^{21})(1 - \Psi_1B^{365})\varepsilon_t \]

This model can be represented as ARIMA(4,0,9) \( \times (2,1,3)_7 \times (0,1,1)_{365} \), with \( \phi_3 = 0, \theta_1 = \cdots = \theta_8 = 0, \) and \( \Theta_1 = \Theta_2 = 0 \).

The estimated results and diagnostic checks are shown in Table 1. All the parameter estimates are significant at the 5\% significance level. The residual autocorrelation function (ACF) and partial autocorrelation function (PACF) exhibit no patterns up to order 7. The Ljung-Box statistic, \( Q = 18.31 \), based on 20 residual autocorrelations is not significant at the conventional levels. These results suggest that the model is appropriate for modeling the water demand series.

We then fitted a significant parameter ARIMA-GARCH model of the form:

\[ (1 - \phi_1B - \phi_2B^2 - \phi_4B^4)(1 - \Phi_1B^7 - \Phi_2B^{14}) \]
\[ \times (1 - B^7)(1 - B^{365})(Y_t - c) \]
\[ = (1 - \theta_3B^9)(1 - \Theta_3B^{21})(1 - \Psi_1B^{365})\varepsilon_t \]

and

\[ \sigma_t^2 = \omega + \beta_1\varepsilon_{t-1}^2 + \alpha_1\varepsilon_t^2 + \alpha_{365}\varepsilon_{t-365}^2 \]
\[ + \varphi_1^2 + \varphi_2^2 + \varphi_3^2\varepsilon_{t-365}^2 \]
\[ \times \sin\left(\frac{2\pi D_t}{365}\right) + \varphi_3^2\varepsilon_{t-365}^2 \]
\[ \sin\left(\frac{6\pi D_t}{365}\right). \]

The model estimates and diagnostic checks are given in Table 2. The Ljung-Box test statistics show evidence of no serial correlation in the residuals (mean equation) and no serial correlation in the squared residuals (variance equation) up to order 20. Thus, we conclude that this model is also adequate for the data.

4.2. Forecast evaluation results

The performance of the estimated univariate methods were evaluated by computing MSE statistics for multi-step forecasts from 1 to 7 days ahead.

Table 3 and Figure 5 give the forecasts results for the post-sample period from 1 June 2006 to 30 June 2006. An initial interpretation of the results suggests that the ability to forecast water demand did not decrease as the forecast horizon increased, except from 1 to 2 days ahead.

The ARIMA and GARCH models appear to have the same forecast performance especially for short-term forecasts (one to two days ahead). For one to four days ahead forecasts, the stochastic models ARIMA and GARCH performed better than the Holt-Winters method. In contrast, the Holt-Winters outperformed the ARIMA and GARCH models in long horizons. The random walk model ranked last for any of the forecast horizons considered.

The optimal combination of Holt-Winters, ARIMA and GARCH weighted by inverse squared errors is more accurate than the various simple combinations, except for 7-step ahead forecasting in which the Holt-Winters outperformed the optimal combined forecasting. For one day ahead, the average MSE for the individual forecasting methods (HW, ARIMA and GARCH) was 0.36 while it was 0.33 for the optimal combined forecasts – a error reduction of 8.33\%.

For two and three days ahead forecasts, combining reduced the MSE by 12.77\% and 10.64\%, respectively.

Table 4 and Figure 6 give the forecast results
for each of the 7 days of the week in the same period. The results suggest that the Thursdays exhibit irregular demand patterns in the post-sample period used in this study. From the data, we found that the water consumption decreased 10.37% on the first Thursday of the post-sample period (1 June 2006), whereas it increased 4.22% and 18.44% on the following Thursdays (8 June 2006 and 15 June 2006, respectively). Possible reasons for this unusual pattern are weather changes and any restrictions on water demand.

In terms of the day of the week effect on forecasting performance, the optimal combination HW-A-G (SE) appears to be most useful for Monday, Tuesday and Wednesday forecasts – combining reduced the MSE of multi-step ahead averaged forecasts by 12.15%, 45.45% and 14.60%, respectively, when compared with the average of the individual methods. The Holt-Winters appears to be the most appropriate method for Thursday, Friday and Saturday forecasts and the GARCH model appears to be the best method for Sunday forecasts.

5. Conclusions

In this article we compared the forecast accuracy of individual and combined univariate time series models for multi-step ahead daily water demand forecasting. We implemented double seasonal versions of the Holt-Winters, ARIMA and GARCH models in order to accommodate the two seasonal effects (within-week cycle of 7 days and within-year cycle of 365 days) on the variability of the data.

The empirical results suggest that the optimal combined forecasts can be quite useful especially for short-term forecasting. However, the forecasting performance of this approach is not consistent over the seven days of the week. The deterministic method Holt-Winters and the stochastic method GARCH can be used independently to improve forecast accuracy on Thursdays to Saturdays and Sundays, respectively.

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REFERENCES

Figure 5. Comparison of multi-step ahead forecasts for post-sample period

Figure 6. Comparison of multi-step ahead averaged forecasts for each of the seven days of the week


Table 1
Seasonal ARIMA model estimates for water demand series

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lag</th>
<th>Estimate</th>
<th>Standard error</th>
<th>Residual ACF</th>
<th>Residual PACF</th>
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</thead>
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<tr>
<td>c</td>
<td></td>
<td>-0.004</td>
<td>0.007</td>
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<td>0.004</td>
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<td>4</td>
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<td>$\Psi_1$</td>
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</table>

$R^2_{\text{adjusted}} = 0.662; Q(20) = 18.31 (0.11).$

Notes: $Q(20)$ is the Ljung-Box statistic for serial correlation in the residuals up to order 20; $p$-value in parentheses.

Table 2
Seasonal-periodic GARCH model estimates for water demand series

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lag</th>
<th>Estimate</th>
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<table>
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<th>Estimate</th>
<th>Standard error</th>
<th>Sq. residual ACF</th>
<th>Sq. residual PACF</th>
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$R^2_{\text{adjusted}} = 0.657; Q(20)=19.20 (0.08); Q^2(20)=13.61 (0.33).$

Notes: $Q(20)$ ($Q^2(20)$) is the Ljung-Box statistic for serial correlation in the residuals (squared residuals) up to order 20; $p$-value in parentheses.
Table 3
MSE for multi-step-ahead forecasts for post-sample period

<table>
<thead>
<tr>
<th>Forecast horizon</th>
<th>RW</th>
<th>HW</th>
<th>ARIMA</th>
<th>GARCH</th>
<th>Simple combination</th>
<th>Optimal combin.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>HW-A</td>
<td>HW-G</td>
</tr>
<tr>
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<td>0.38</td>
<td>0.35</td>
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<td>0.45</td>
</tr>
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<td>0.42</td>
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<td>0.44</td>
<td>0.44</td>
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<td>0.43</td>
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Table 4
MSE for multi-step ahead forecasts for each day of the week in post-sample period

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<th>Forecast horizon</th>
<th>Day of the week</th>
<th>RW</th>
<th>HW</th>
<th>ARIMA</th>
<th>GARCH</th>
<th>Simple combination</th>
<th>Optimal combin.</th>
</tr>
</thead>
<tbody>
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<td>HW-G</td>
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