A New Classification of Goods in Sraffa-Price-Systems: Basic Capital Goods, Non-Basic Capital Goods and Consumption Goods

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Abstract

Sraffa (1960) introduced a classification of goods which has been used without modification for a long time. According to him goods have to be classified as basic and non-basic commodities. Among non-basic commodities, he extracts goods which are used directly and indirectly in the production of pure consumption goods. Pure consumption goods are neither used directly and nor indirectly in the production of a commodity. However, Sraffa regards pure consumption goods as non-basic commodities. Thus, one always needs to be careful, i.e. speaking of non-basic commodities one needs to concretize what kind of non-basic commodity it is - a pure consumption good or not. I suggest the introduction of a new classification which does not principally change Sraffa's assumptions in his models, but concretizes definitions of the kinds of goods. I regard Sraffa's basic commodities as "basic capital goods", Sraffa's non-basic commodities, which are used as inputs for production of Sraffa's non-basic commodities as "non-basic capital goods", and Sraffa's non-basic commodities which are not used as inputs for production of any commodity, i.e. Sraffa's pure consumption goods simply as "consumption goods". I understand "capital goods" as being any good which is used as input in the input coefficient matrix of any Sraffa-Price-System within the framework of Sraffa's interpretation at least one time.

Key words: One-Product-Sraffa-Price-System, Multiple-Product-Sraffa-Price-System, basic capital goods, non-basic capital goods, consumption goods, subsystem of basic capital goods, indecomposable matrices

JEL classifications: B12, D33, D46, E11

*Any comments, ideas and suggestions are welcome

1. Preliminaries

We have already briefly determined the various kinds of commodities as basic capital goods, non-basic capital goods and consumption goods. But we need a more detailed definition and analysis of these kinds of goods.

Sraffa (1960) divides all commodities in his models into basic commodities and non-basic commodities. As basic commodities, he considers commodities which are used directly and indirectly in the production of all other commodities, otherwise, the commodities are considered as non-basic. I am introducing a modified approach in defining the kinds of commodities.

All other remaining Sraffa's non-basic commodities which are final goods used only for consumption and not used in production of any commodity we call consumption goods.

For example, Schefold (1989) considers non-basic commodities as Sraffa did: "...Non-basics may have the character of pure consumption goods (e.g. ice-cream) or they may enter the production of other non-basics (e.g. eggs)...". In our models, eggs are treated as capital goods since they are used as inputs. So, in Sraffa's interpretation non-basics are both directly and indirectly, however in our interpretation consumption goods are only directly consumed, while non-basic capital goods are both - directly and indirectly consumed. That means that a Sraffian input matrix in our understanding contains the proportions of both basic and non-basic capital goods as technical input coefficients, but no (or zero) proportions of consumption goods as technical input coefficients.

The classification of goods into Sraffa's basic and non-basic goods in joint product price systems is not similar to that of single product price systems. This classification does not seem to be possible in Sraffa's tradition, and Schefold (1971, 1978c, 1989) made a suggestion of a criterion which allows the classification of joint product price system as basic and non-basic one, i.e. focusing on property of Sraffian input matrix to be indecomposable (basic system) and decomposable (non-basic system). However, he finds out that indecomposability is only a necessary condition, but not sufficient. Schefold provides a definition which is general, i.e. applicable for single and joint production price systems. He defines a basic commodity as one, which is produced under the following conditions:

a) technical independence of the set of processes from the rest of the economy,

b) economic independence of one set of commodities from the others, in that, influences on prices of the independent commodities affect the prices of the others, but not vice versa.

We have to mention also that Manara (1968) earlier generalized the concept of basic subsystems for decomposable joint production price systems, i.e. each decomposable joint production price system contains a subsystem of basic capital goods in definite conditions. However, we will not treat the Manara's aproach because he considered two kinds of goods - basics and non-basics.

2. Basic Capital Goods, Non-Basic Capital Goods and Consumption Goods in One-Product-Sraffa-Price-System

Consider the n-sectoral One-Product-Sraffa-Price-System

$$\tilde{\mathbf{A}} = \left[\hat{\mathbf{A}}, \vec{l}, \mathbf{I}\right] \geqslant 0,\ \hat{\mathbf{A}} = (1+\pi)\mathbf{A}, \left[(1+\pi)\mathbf{A}, \vec{l}\,\right]\vec{p}_{\tilde{\mathbf{A}}} = \vec{p} \ \Leftrightarrow\ (1+\pi)\mathbf{A}\vec{p} + \vec{l}w = \vec{p},$$
$$\pi \geqslant 0,\ \vec{p}_{\tilde{\mathbf{A}}} = (p_1, p_2, \ldots, p_n, w) \geqslant \vec{0},$$

where $\mathbf{A}$ is decomposable. That is, we treat the common subcase of the normalized Sraffian price system in which capital and consumption goods are produced. Taking into account an output matrix $\mathbf{I}$, we can rearrange the matrix $\mathbf{A}$ by the permutation of its rows and columns in order to obtain the matrix $\acute{\mathbf{A}}$ with submatrices, which are crucial to identify basic capital goods, non-basic capital goods and consumption goods:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix},\ \mathbf{I} = \begin{bmatrix} \mathbf{I}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{22} \end{bmatrix} \Rightarrow \acute{\mathbf{A}} = \begin{bmatrix} \acute{\mathbf{A}}_{11} & \mathbf{0} \\ \acute{\mathbf{A}}_{21} & \acute{\mathbf{A}}_{22} \end{bmatrix},\ \mathbf{I} = \begin{bmatrix} \mathbf{I}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{22} \end{bmatrix},$$

where $\acute{\mathbf{A}}_{11},\ \acute{\mathbf{A}}_{22}$ are square submatrices and $\mathbf{I}_{11},\ \mathbf{I}_{22}$ are unit matrices. $\acute{\mathbf{A}}_{11}$ and $\mathbf{I}_{11}$ are of order r , $\acute{\mathbf{A}}_{22}$ and $\mathbf{I}_{22}$ are of order $n - r$. Then the first r goods are basic capital goods, the last $n - r$ goods are non-basic capital goods and consumption goods.

All economists to my knowledge, who are interested in issues concerning Sraffa's basic and non-basic commodities, stop their analysis as they just decomposed $\mathbf{A}$ and obtained an indecomposable matrix $\acute{\mathbf{A}}_{11}$. But what about the matrix $\acute{\mathbf{A}}_{22}$ which is still decomposable?

In fact, $\acute{\mathbf{A}}_{22}$ could be decomposed, that is, we could obtain two square indecomposable matrices $\ddddot{\mathbf{A}}_{22}$ and $\ddddot{\mathbf{A}}_{33}$, which are crucial for identifying the kinds of goods we suggested:

$$\acute{\mathbf{A}} = \begin{bmatrix} \acute{\mathbf{A}}_{11} & \mathbf{0} \\ \acute{\mathbf{A}}_{21} & \acute{\mathbf{A}}_{22} \end{bmatrix},\ \mathbf{I} = \begin{bmatrix} \mathbf{I}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{22} \end{bmatrix} \Rightarrow$$
$$\ddddot{\mathbf{A}} = \begin{bmatrix} \ddddot{\mathbf{A}}_{11} & \mathbf{0} & \mathbf{0} \\ \ddddot{\mathbf{A}}_{21} & \ddddot{\mathbf{A}}_{22} & \mathbf{0} \\ \ddddot{\mathbf{A}}_{31} & \ddddot{\mathbf{A}}_{31} & \ddddot{\mathbf{A}}_{33} \end{bmatrix},\ \mathbf{I} = \begin{bmatrix} \mathbf{I}_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{33} \end{bmatrix},$$

where $\ddddot{\mathbf{A}}_{11},\ \ddddot{\mathbf{A}}_{22},\ \ddddot{\mathbf{A}}_{33}$ are square submatrices and $\mathbf{I}_{11},\ \mathbf{I}_{22},\ \mathbf{I}_{33}$ are unit matrices. $\ddddot{\mathbf{A}}_{11}$ and $\mathbf{I}_{11}$ are of order r, $\ddddot{\mathbf{A}}_{22}$ and $\mathbf{I}_{22}$ are of order $s < n$, $\ddddot{\mathbf{A}}_{33}$ and $\mathbf{I}_{33}$ are of order $n-r-s$ Then the first r goods are basic capital goods, the next s goods are non-basic capital goods and $n-r-s$ are consumption goods.

Let us rewrite the Sraffian price system $(1+\pi)\mathbf{A}\vec{p}+\vec{l}w=\vec{p}$ as a Sraffian price system with three economic subdivisions producing basic capital goods, non-basic capital goods and consumption goods:

$$
\begin{gathered}
(1+\pi)\mathbf{A}\vec{p}+\vec{l}w=\vec{p} \Leftrightarrow \\
(1+\pi)\ddddot{\mathbf{A}}_{11}\vec{p}_1+\vec{l}_1 w=\vec{p}_1 \\
(1+\pi)(\ddddot{\mathbf{A}}_{21}\vec{p}_1+\ddddot{\mathbf{A}}_{22}\vec{p}_2)+\vec{l}_2 w=\vec{p}_2 \\
(1+\pi)(\ddddot{\mathbf{A}}_{31}\vec{p}_1+\ddddot{\mathbf{A}}_{32}\vec{p}_2+\ddddot{\mathbf{A}}_{33}\vec{p}_3)+\vec{l}_3 w=\vec{p}_3,
\end{gathered}
$$

where $\vec{p}_1$ is a column price vector of basic capital goods, $\vec{l}_1$ a column labor input vector of industries producing basic capital goods, $\vec{p}_2$ is a column price vector of non-basic capital goods, $\vec{l}_2$ a labor column input vector of industries producing non-basic capital goods, $\vec{p}_3$ is a column price vector of consumption goods, $\vec{l}_3$ is a column labor input vector of industries producing consumption goods. It is clear that $\ddddot{\mathbf{A}}_{33}$ contains only zero entries.

From the equations stated above, we can see that the price vector of basic capital goods $\vec{p}_1$ is defined independently of prices of non-basic capital goods $\vec{p}_2$ and prices of consumption goods $\vec{p}_3$. In turn, the price vector of non-basic capital goods $\vec{p}_2$ is defined independently of the prices of consumption goods $\vec{p}_3$, but dependently of the prices of non-basic capital goods $\vec{p}_1$. The price vector of consumption goods $\vec{p}_3$ depends on the prices of basic and non-basic capital goods.

Hence, one needs to have basic capital goods or subsystem of basic capital goods in order to make the Sraffian price system consistent. This is why one assumes that Sraffian price system must contain at least one basic capital good to be considered for economic analysis. Recall that Ricardo ([1821] 1951) treated this extreme case of an economy consisting of only one good, i.e. corn, which is a capital basic good in our interpretation.

3. Basic Capital Goods, Non-Basic Capital Goods and Consumption Goods in Multiple-Product-Sraffa-Price-System

We will denote many similar but not exactly the same notions by the same symbols but we will keep in mind that the meaning of them is slightly different.

Consider the n-sectoral Multiple-Product-Sraffa-Price-System

$$
\begin{gathered}
\tilde{\mathbf{A}}=\left[\hat{\mathbf{A}},\vec{l},\mathbf{B}\right]\geqslant 0,\ \hat{\mathbf{A}}=(1+\pi)\mathbf{A},\left[(1+\pi)\mathbf{A},\vec{l}\right]\vec{p}_{\tilde{\mathbf{A}}}=\ \mathbf{B}\vec{p}\ \Leftrightarrow\ (1+\pi)\mathbf{A}\vec{p}+\vec{l}w=\mathbf{B}\vec{p}, \\
\pi\geqslant 0,\ \vec{p}_{\tilde{\mathbf{A}}}=(p_1,p_2,\ldots,p_n,w)\geqslant\vec{0},
\end{gathered}
$$

where $\mathbf{A}$ is decomposable. Suppose, we treat the common subcase of the normalized Sraffian price system in which capital and consumption goods are produced. Taking into account an output matrix $\mathbf{B}$, we can rearrange the matrix $\mathbf{A}$ by the permutation of its rows and columns in order to obtain the matrices $\acute{\mathbf{A}}$ and $\acute{\mathbf{B}}$ with corresponding submatrices, which are of great importance to identify basic capital goods, non-basic capital goods and consumption goods:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \Rightarrow \acute{\mathbf{A}} = \begin{bmatrix} \acute{\mathbf{A}}_{11} & \mathbf{0} \\ \acute{\mathbf{A}}_{21} & \acute{\mathbf{A}}_{22} \end{bmatrix}, \ \acute{\mathbf{B}} = \begin{bmatrix} \acute{\mathbf{B}}_{11} & \mathbf{0} \\ \acute{\mathbf{B}}_{21} & \acute{\mathbf{B}}_{22} \end{bmatrix}$$

where $\acute{\mathbf{A}}_{11},\ \acute{\mathbf{A}}_{22}$ are square input submatrices and $\acute{\mathbf{B}}_{11},\ \acute{\mathbf{B}}_{22}$ are square output submatrices. $\acute{\mathbf{A}}_{11}$ and $\acute{\mathbf{B}}_{11}$ are of order r , $\acute{\mathbf{A}}_{22}$ and $\acute{\mathbf{B}}_{22}$ are of order $n-r.$ Then the first r goods are basic capital goods, the last $n-r$ goods are non-basic capital goods and consumption goods.

However, the matrices $\acute{\mathbf{A}}_{22}$ and $\acute{\mathbf{B}}_{22}$ are still decomposable:

$$\acute{\mathbf{A}} = \begin{bmatrix} \acute{\mathbf{A}}_{11} & \mathbf{0} \\ \acute{\mathbf{A}}_{21} & \acute{\mathbf{A}}_{22} \end{bmatrix}, \ \acute{\mathbf{B}} = \begin{bmatrix} \acute{\mathbf{B}}_{11} & \mathbf{0} \\ \acute{\mathbf{B}}_{21} & \acute{\mathbf{B}}_{22} \end{bmatrix} \Rightarrow$$

$$\ddddot{\mathbf{A}} = \begin{bmatrix} \ddddot{\mathbf{A}}_{11} & \mathbf{0} & \mathbf{0} \\ \ddddot{\mathbf{A}}_{21} & \ddddot{\mathbf{A}}_{22} & \mathbf{0} \\ \ddddot{\mathbf{A}}_{31} & \ddddot{\mathbf{A}}_{31} & \ddddot{\mathbf{A}}_{33} \end{bmatrix}, \ \ddddot{\mathbf{B}} = \begin{bmatrix} \ddddot{\mathbf{B}}_{11} & \mathbf{0} & \mathbf{0} \\ \ddddot{\mathbf{B}}_{21} & \ddddot{\mathbf{B}}_{22} & \mathbf{0} \\ \ddddot{\mathbf{B}}_{31} & \ddddot{\mathbf{B}}_{31} & \ddddot{\mathbf{B}}_{33} \end{bmatrix},$$

where $\ddddot{\mathbf{A}}_{11},\ \ddddot{\mathbf{A}}_{22},\ \ddddot{\mathbf{A}}_{33}$ are square input submatrices and $\ddddot{\mathbf{B}}_{11},\ \ddddot{\mathbf{B}}_{22},\ \ddddot{\mathbf{B}}_{33}$ are square output submatrices. $\ddddot{\mathbf{A}}_{11}$ and $\ddddot{\mathbf{B}}_{11}$ are of order $r,$ $\ddddot{\mathbf{A}}_{22}$ and $\ddddot{\mathbf{B}}_{22}$ are of order $s < n,$ $\ddddot{\mathbf{A}}_{33}$ and $\ddddot{\mathbf{B}}_{33}$ are of order $n-r-s$ Then the first r goods are basic capital goods, the next s goods are non-basic capital goods and $n-r-s$ are consumption goods. Note that $\ddddot{\mathbf{A}}_{33} = \mathbf{0},\ \ddddot{\mathbf{B}}_{33} > \mathbf{0}.$

Let us rewrite the Sraffian joint production price system $(1+\pi)\mathbf{A}\vec{p} + \vec{l}w = \mathbf{B}\vec{p}$ as a Sraffian joint production price system with three economic subdivisions producing basic capital goods, non-basic capital goods and consumption goods:

$$\begin{gathered} (1+\pi)\mathbf{A}\vec{p} + \vec{l}w = \mathbf{B}\vec{p} \Leftrightarrow \\ (1+\pi)\ddddot{\mathbf{A}}_{11}\vec{p}_1 + \vec{l}_1 w = \ddddot{\mathbf{B}}_{11}\vec{p}_1 \\ (1+\pi)(\ddddot{\mathbf{A}}_{21}\vec{p}_1 + \ddddot{\mathbf{A}}_{22}\vec{p}_2) + \vec{l}_2 w = \ddddot{\mathbf{B}}_{21}\vec{p}_1 + \ddddot{\mathbf{B}}_{22}\vec{p}_2 \\ (1+\pi)(\ddddot{\mathbf{A}}_{31}\vec{p}_1 + \ddddot{\mathbf{A}}_{32}\vec{p}_2 + \ddddot{\mathbf{A}}_{33}\vec{p}_3) + \vec{l}_3 w = \ddddot{\mathbf{B}}_{31}\vec{p}_1 + \ddddot{\mathbf{B}}_{32}\vec{p}_2 + \ddddot{\mathbf{B}}_{33}\vec{p}_3, \end{gathered}$$

where $\vec{p}_1$ is a column price vector of basic capital goods, $\vec{l}_1$ a column labor input vector of industries producing basic capital goods, $\vec{p}_2$ is a column price vector of non-basic capital goods, $\vec{l}_2$ a labor column input vector of industries producing both the basic and non-basic capital goods, $\vec{p}_3$ is a column price vector of consumption goods, $\vec{l}_3$ is a column labor input vector of industries producing the basic, non-basic and consumption goods. It is clear that $\ddddot{\mathbf{A}}_{33}$ contains only zero entries.

From the equations stated above, we can see that the price vector of basic capital goods $\vec{p}_1$ is defined independently of prices of non-basic capital goods $\vec{p}_2$ and prices of consumption goods $\vec{p}_3$. In turn, the price vector of non-basic capital goods $\vec{p}_2$ is defined independently of the prices of consumption goods $\vec{p}_3$, but dependently of the prices of non-basic capital goods $\vec{p}_1$. The price vector of consumption goods $\vec{p}_3$ depends on the prices of basic and non-basic capital goods.

Note that in the case of joint production basic capital goods does not need to enter into production of each nonbasic and consumption good, but they enter directly and indirectly into production of themselves.

As we stated above, Sraffa defines basic commodity in his own interpretation as a commodity, which enters directly or indirectly into production of all other commodities, i.e. basic commodity is used in each production process. But this definition is true only for single production price systems. In particular, the subsystem of basic commodities is indecomposable and extracted from decomposable single production price system. Recall the definition of decomposable square matrix, which is of crucial importance for understanding the basic subsystem in case of One-Product-Sraffa-Price-System.

Thus, the two matrices $\mathbf{A}$ and $\mathbf{B}$ are decomposable (reducible, non-basic) if and only if the corresponding permutation matrices $\mathbf{P}$ and $\mathbf{Q}$ exist, so that:

$$\ddddot{\mathbf{A}} = \mathbf{PAQ} = \begin{bmatrix} \ddddot{\mathbf{A}}_{11} & \mathbf{0} & \mathbf{0} \\ \ddddot{\mathbf{A}}_{21} & \ddddot{\mathbf{A}}_{22} & \mathbf{0} \\ \ddddot{\mathbf{A}}_{31} & \ddddot{\mathbf{A}}_{32} & \ddddot{\mathbf{A}}_{33} \end{bmatrix}, \ \ddddot{\mathbf{B}} = \mathbf{PBQ} = \begin{bmatrix} \ddddot{\mathbf{B}}_{11} & \mathbf{0} & \mathbf{0} \\ \ddddot{\mathbf{B}}_{21} & \ddddot{\mathbf{B}}_{22} & \mathbf{0} \\ \ddddot{\mathbf{B}}_{31} & \ddddot{\mathbf{B}}_{32} & \ddddot{\mathbf{B}}_{33} \end{bmatrix},$$

where $\ddddot{\mathbf{A}}_{11}$ and $\ddddot{\mathbf{B}}_{11}$ are square matrices of the same order, and $\ddddot{\mathbf{A}}_{33} = \mathbf{0}$.

Hence, the joint production price system with all necessary economic requirements can be expressed as follows:

$(1+\pi)\mathbf{A}\vec{p} + \vec{l}w = \mathbf{B}\vec{p},\ \mathbf{A} \geqslant \mathbf{0}, \mathbf{B} \geqslant \mathbf{0}, \sum_{j=1}^{n} b_{ij} \geqslant \sum_{j=1}^{n} a_{ij},\ i,j = 1,\ldots,n$ such that:

$$a)\ \left[\ddddot{\mathbf{B}}_{11} - (1+\pi)\ddddot{\mathbf{A}}_{11}\right]\vec{p}_1 = \vec{l}_1 w,$$
$$\vec{p}_1 = \left[\ddddot{\mathbf{B}}_{11} - (1+\pi)\ddddot{\mathbf{A}}_{11}\right]^{-1}\vec{l}_1 w > 0,\ \ddddot{\mathbf{A}}_{11} > \mathbf{0}, \ddddot{\mathbf{B}}_{11} > \mathbf{0}, \vec{l}_1 > \vec{0}$$

$$b)\ \left[\ddddot{\mathbf{B}}_{21} - (1+\pi)\ddddot{\mathbf{A}}_{21}\right]\vec{p}_1 + \left[\ddddot{\mathbf{B}}_{22} - (1+\pi)\ddddot{\mathbf{A}}_{22}\right]\vec{p}_2 = \vec{l}_2 w,$$
$$\ddddot{\mathbf{A}}_{21} \geqslant \mathbf{0}, \ddddot{\mathbf{B}}_{21} \geqslant \mathbf{0}, \ddddot{\mathbf{A}}_{22} \geqslant 0, \ddddot{\mathbf{B}}_{22} \geqslant 0,\ \vec{l}_2 \geqslant \vec{0}$$

$$c)\ \left[\ddddot{\mathbf{B}}_{31} - (1+\pi)\ddddot{\mathbf{A}}_{31}\right]\vec{p}_1 + \left[\ddddot{\mathbf{B}}_{32} - (1+\pi)\ddddot{\mathbf{A}}_{32}\right]\vec{p}_2 + \left[\ddddot{\mathbf{B}}_{33} - (1+\pi)\ddddot{\mathbf{A}}_{33}\right]\vec{p}_3 = \vec{l}_3 w,$$
$$\ddddot{\mathbf{A}}_{31} \geqslant \mathbf{0}, \ddddot{\mathbf{B}}_{31} \geqslant \mathbf{0}, \ddddot{\mathbf{A}}_{32} \geqslant 0, \ddddot{\mathbf{B}}_{32} \geqslant 0, \ddddot{\mathbf{B}}_{33} \geqslant 0, \ddddot{\mathbf{A}}_{33} = 0, \vec{l}_3 \geqslant \vec{0}$$

The first subsystem (a) is basic and indecomposable, and from the necessity that labor inputs for the production of basic capital goods are to be strictly positive follows that the prices of basic capital goods are strictly positive, i.e. if no production of basic capital goods exists, then no economy exists at all! However, the requirements for the existence of production of non basic capital goods and consumption goods are much weeker, i.e. their prices are to be nonnegative.

Recall that we call $\ddddot{\mathbf{A}}_{11}$ indecomposable in the case of One-Product-Sraffa-Price-System because we can determine prices of r capital goods in that Sraffian price system independent of other $n-r$ technical coefficients and prices, if r is a dimension and rank of a matrix $\ddddot{\mathbf{A}}_{11}$.

However, this approach is not sufficient as soon as we begin to consider Multiple-Product-Sraffa-Price-System. Hence, definitions of basic and non-basic capital goods given for One-Product-Sraffa-Price-Systems are no longer valid for the Multiple-Product-Sraffa-Price-Systems. For example, when

$$\ddddot{\mathbf{A}} = \begin{bmatrix} 0.2 & 0.5 & 0 & 0 \\ 0.3 & 0.3 & 0 & 0 \\ 0.1 & 0.1 & 0.4 & 0.6 \\ 0.3 & 0 & 0.3 & 0.2 \end{bmatrix}, \ddddot{\mathbf{B}} = \begin{bmatrix} 0.3 & 0.4 & 0 & 0 \\ 0.4 & 0.3 & 0 & 0 \\ 0.1 & 0.2 & 0.5 & 0.6 \\ 0.3 & 0.3 & 0.3 & 0.3 \end{bmatrix}$$

are input and output matrices in joint production price system with no consumption goods, i.e. subsystem (c) is absent. The condition $\sum_{j=1}^{n} b_{ij} \geqslant \sum_{j=1}^{n} a_{ij}$ for this matrices is fulfilled, but for submatrices $\ddddot{\mathbf{A}}_{11}$ and $\ddddot{\mathbf{B}}_{11}$ it fails. As we see in the case of Multiple-Product-Sraffa-Price-System, each commodity may enter the production of each other, if the corresponding elements of $\ddddot{\mathbf{A}}_{21}$ and $\ddddot{\mathbf{B}}_{21}$ are positive. In the case of One-Product-Sraffa-Price-System, each commodity may enter the production of each other only in a subsystem of basic capital goods within Sraffian input matrix.

Example. Consider the general example of the joint production price system with the production of six goods based on the following data:

$$\mathbf{A} = \begin{bmatrix} 1.6 & 0.34 & 1.08 & 2.58 & 2.72 & 1.96 \\ 2.8 & 0.67 & 1.84 & 4.81 & 5.5 & 3.7 \\ 1.6 & 0.43 & 1.46 & 3.33 & 3.08 & 2.24 \\ 2 & 0.45 & 1.3 & 3.61 & 4.32 & 2.86 \\ 3.4 & 0.47 & 1.64 & 4.41 & 5.6 & 4 \\ 1.8 & 0.14 & 0.58 & 1.68 & 2.52 & 1.86 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 11.2 & 6.1 & 11 & 14.5 & 13.6 & 10.5 \\ 19.3 & 10.5 & 18.9 & 26.5 & 24.8 & 19.2 \\ 8.9 & 4.5 & 8.2 & 12.4 & 11.5 & 9.1 \\ 13.5 & 6.4 & 11.1 & 17.4 & 16.8 & 14 \\ 20.5 & 9.9 & 17.5 & 24 & 24.6 & 19.3 \\ 11.6 & 5.6 & 9.8 & 12.3 & 13 & 10.2 \end{bmatrix}.$$

From these matrices we can not define whether three kinds of the goods mentioned above are represented here. Or the question can be posed: do exist the permutation matrices $\mathbf{P}$ and $\mathbf{Q}$ such that $\ddot{\ddot{\mathbf{A}}} = \mathbf{PAQ}$, $\ddot{\ddot{\mathbf{B}}} = \mathbf{PBQ}$. Yes, such matrices exist:

$$\mathbf{P} = \begin{bmatrix} \frac{-5}{2} & 1 & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{3}{2} \\ -7 & 3 & 4 & -1 & -4 & 7 \\ \frac{-3}{2} & 1 & 1 & \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} \\ \frac{-5}{2} & 1 & 2 & \frac{-1}{2} & \frac{-3}{2} & \frac{5}{2} \\ 6 & -3 & -3 & 2 & 2 & -4 \\ 3 & -1 & -2 & 0.0 & 2 & -3 \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \frac{13}{15} & \frac{-2}{5} & \frac{-1}{5} & \frac{1}{3} & \frac{-1}{5} & \frac{-2}{15} \\ \frac{-14}{15} & \frac{1}{5} & \frac{-7}{5} & \frac{1}{3} & \frac{3}{5} & \frac{1}{15} \\ -\frac{1}{3} & 0.0 & 1 & \frac{-2}{3} & 0.0 & \frac{2}{3} \\ \frac{8}{15} & \frac{-2}{5} & \frac{-1}{5} & \frac{2}{3} & \frac{-1}{5} & \frac{-7}{15} \\ \frac{8}{15} & \frac{-2}{5} & \frac{-1}{5} & \frac{2}{3} & \frac{-1}{5} & \frac{-7}{15} \\ \frac{-14}{15} & \frac{1}{5} & \frac{3}{15} & -\frac{2}{3} & \frac{3}{5} & \frac{1}{15} \end{bmatrix}$$

so that

$$\ddot{\ddot{\mathbf{A}}} \approx \begin{bmatrix} 0.2 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.3 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.1 & 0.1 & 0.1 & 0.2 & 0.0 & 0.0 \\ 0.2 & 0.0 & 0.2 & 0.1 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.1 & 0.02 & 0.0 & 0.0 \\ 0.1 & 0.01 & 0.0 & 0.01 & 0.0 & 0.0 \end{bmatrix}, \quad \ddot{\ddot{\mathbf{B}}} \approx \begin{bmatrix} 0.3 & 0.4 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.4 & 0.3 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.1 & 0.2 & 0.2 & 0.5 & 0.0 & 0.0 \\ 0.3 & 0.1 & 0.1 & 0.2 & 0.0 & 0.0 \\ 0.2 & 0.0 & 0.2 & 0.2 & 1.1 & 0.5 \\ 0.0 & 0.3 & 0.2 & 0.1 & 0.5 & 1.2 \end{bmatrix}$$

Now we can easily identify that the first two goods are basic capital goods and the production of them builds an indecomposable basic joint production subsystem, the next two goods are non-basic capital goods and the remaining last two goods are consumption goods. Choosing the numeraire $p_1 = 1$ and taking $w = 1$, $\vec{l}_1 = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}^T$, $\vec{l}_2 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T$, $\vec{l}_3 = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}^T$ we get the economically meaningful solution to the system, i.e. $\vec{p} \approx \begin{bmatrix} 1 & 0.82 & 0.39 & 0.68 & 0.14 & 0.75 \end{bmatrix}^T$ and $\pi \approx 0.18$.

Thus, the single and joint production price systems are different: in the first case, the inverse matrix $(\mathbf{I}-\mathbf{A})^{-1}$ is automatically nonnegative as soon as the Sraffian input-output matrix $\mathbf{A}$ becomes productive. But this property no longer holds in joint production.

Definition 1. Each good which is used as a positive input in $\ddddot{\mathbf{A}}_{11}$, i.e. in basic processes of indecomposable basic joint production price subsystem $\left[\ddddot{\mathbf{A}}_{11}, \ddddot{\mathbf{B}}_{11}\right]$, and used as a nonnegative input in $\ddddot{\mathbf{A}}_{21}, \ddddot{\mathbf{A}}_{31}$, i.e. in non-basic processes of decomposable joint production price system $[\mathbf{A}, \mathbf{B}]$ is a basic capital good. Its price is independent of prices of other goods which are not basic.

Note that a basic capital good does not need to be used in all processes in order to be regarded as a basic capital good. But in single product price systems, basic capital good must strongly be used in each process of the production price system.

Definition 2. Each good which is used only as a nonnegative input in $\ddddot{\mathbf{A}}_{22}, \ddddot{\mathbf{A}}_{32}$, i.e. in non-basic processes of decomposable joint production price system $[\mathbf{A}, \mathbf{B}]$, but not used as input in basic processes of indecomposable basic joint production price subsystem $\left[\ddddot{\mathbf{A}}_{11}, \ddddot{\mathbf{B}}_{11}\right]$, is a non-basic capital good. Its price is independent of prices of consumption goods but depends on prices of basic capital goods.

Definition 3. Each good which is not used at all as input in basic and non-basic processes of decomposable joint production price system $[\mathbf{A}, \mathbf{B}]$ is a consumption good. Its price depends on prices of both basic and non-basic capital goods.

Thus, we have shown that our classification of goods is also applicable for joint product price systems with some modifications concerning the definitions of the corresponding kinds of goods.

4. Concluding remarks

An analysis of definitions shows that in order to generalize the definitions one needs to choose a criterion. It is clear, that Sraffa's criterion of entering good directly and indirectly into production of other goods cannot be acceptable for this purpose, since this criterion is only sufficient for single product price systems, but not sufficient for joint product price systems. A criterion of indecomposability of production price system as we showed above is also not sufficient for joint product price systems, although it is sufficient for single product price systems. The only criterion which seems to be sufficient for both kinds of systems remains the identification of groups of goods according to criterion whether their prices depend on prices of the goods from other groups. If decompositions and combined processes allow the formation of an appropriate production price system $[\mathbf{A}, \mathbf{B}]$ for joint product price system and $[\mathbf{A}, \mathbf{I}]$ for single product price system, so that subsystems of equations allow the extraction of the initial set of goods (basic capital goods) with implication that prices of these goods absolutely do not depend on prices of the other goods, and the second set of the goods (non-basic capital goods) with admission that their prices depend on the prices of the set of goods extracted firstly, but with implication that their prices do not depend on prices of the third remaining set of goods (consumption goods), then this approach for identifying kinds of goods is unique and applicable for both single and joint production price systems.

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