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Abstract

Recent research has proposed several ways in which overconfident traders can persist in competition with rational traders. This paper offers an additional reason: overconfident traders do better than purely rational traders at exploiting mispricing caused by liquidity or noise traders. We examine both the static profitability of overconfident versus rational trading strategies, and the dynamic evolution of a population of overconfident, rational and noise traders. Replication of overconfident and rational types is assumed to be increasing in the recent profitability of their strategies. The main result is that the long-run steady-state equilibrium always involves overconfident traders as a substantial positive fraction of the population.

Keywords: Survivorship, Natural Selection, Overconfident Traders, Noise traders

JEL classification: G00, G14

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1 Introduction

Several recent papers have argued that investor overconfidence or shifts in confidence offer a possible explanation for a range of anomalous empirical patterns in securities markets.[1] An important general objection to such approaches is that rational traders ought to make profits at the expense of the irrational ones, so that irrationality should in the long run be eliminated as a significant factor in the market.[2]

This paper offers a new reason for the possible long-run survival of overconfident traders in competition with rational traders. The basic idea is that risk averse, overconfident traders trade more aggressively based on valid information than do rational traders. As a result, overconfident traders are better able to exploit risky profit opportunities created by the trades of liquidity-motivated traders or the mistakes of noise traders.[3] Overconfident investors

[1] Odean (1998) examines investor overconfidence, overreactions, and the high volume of trade in securities markets. Daniel, Hirshleifer and Subrahmanyam (1998, 2000) examine the consistency of overconfidence and shifts in confidence with abnormal post-event returns in event studies, short horizon stock price momentum, long run reversal, short- versus long-horizon correlations between accounting performance and later stock price performance, and the relative ability of size, fundamental/price ratios, and risk measures to predict future returns. Odean (1998) and Daniel, Hirshleifer and Subrahmanyam (1998) also examine conditions under which there will be excess or insufficient volatility of security returns relative to a rational benchmark.

[2] Luo (1998) provides a model of natural selection in which irrational traders lose money and the market evolves toward long run efficiency. Also, Figlewski (1978, 1982) finds that owing to wealth shifts among traders with diverse information, informational efficiency may or may not be achievable depending on the correlation of signals received by the traders and depending on the degree of traders' risk aversion.

[3] Apart from this informational benefit, overconfident investors who underestimate risk can potentially benefit from exploiting the risk premium on a positive net supply risky asset (i.e., investing heavily in the 'market portfolio'). This non-informational effect was noted previously in DeLong, Shleifer, Summers and Waldmann (1990), discussed below. We rule out this effect by assuming here that the risky security is in zero net supply.

trade aggressively both because they underestimate risk and because they overestimate the conditional expected value from their trading strategies. Since the information they exploit is valid, their more aggressive use of it (either long or short on the risky asset) causes them to earn higher expected profits (though lower expected utility). Their expected profits are limited by the fact that if there are too many overconfident traders, or if their confidence is too extreme, their trading pushes price against them excessively. Rational traders then profit by trading in opposition to overconfident traders. If trader types replicate according to the profitability of their strategies, we show that overconfident traders survive in the long run, and can even drive out rational traders completely.

Several authors, beginning with De Long, Shleifer, Summers, and Waldmann (1990, 1991), have offered other distinct arguments as to why imperfectly rational traders, including overconfident ones, may survive in the long run.[4] De Long et al (1991) examine traders who are overconfident in the sense that they underestimate risk. As a result of underestimating risk, these traders hold more of the risky asset (e.g., the market portfolio). Since the risky asset earns higher expected return, these traders can do well relative to rational traders.

Our approach differs from De Long et al (1991) in the following respects.

[4]For example, noise traders themselves create a risk in the price that discourages rational traders from betting against them. Noise traders bear a disproportionate amount of the risk that they themselves create, and therefore may earn a correspondingly higher risk premium. In this sense they can "create their own space." De Long et al (1990) point out that, as a result, noise traders can earn higher expected profits than rational traders. Palomino (1996) shows that small market size can further enhance the survivorship of noise traders.

First, we model overconfidence as overestimation of the precision of private information signals. We therefore derive beliefs endogenously about the payoff of the risky asset. Overconfidence in our sense implies underestimation of risk, consistent with their assumption. It also implies incorrect conditional means. Their assumption of a noise component of trades is implicitly consistent with misassessment of conditional means. However, by deriving beliefs endogenously, our model goes further by constraining the relation between errors in mean assessments and underestimation of risk. In our model the misperceptions of both first and second moments are determined endogenously through Bayes rule.

Second, we model prices endogenously. We would often expect irrational traders who trade in a certain direction (e.g., buying a hot internet start-up) to push the price unfavorably to themselves. This influence on price tends to reduce the long-run expected profits to irrational trading. For example, on Friday November 13, 1998, in the first hours of trading after the initial public offering of TheGlobe.com, the price quickly leaped from the offer price of \$9 to a price of \$97, reflecting enthusiasm on the part of individual investors. By the end of the day the price had fallen by about 1/3, so many investors who bought in the aftermarket took heavy losses. Given the possible adverse effects on price, it is interesting to see whether irrational traders can survive despite having an influence on price.

Third, the results of De Long et al (1991) are driven mainly by a non-informational effect, that overconfident individuals who underestimate risk tilt their portfolios heavily toward the 'market' (high risk/high return) security. In our paper, the high profits of the overconfident arise from the overreaction in their assessments of mean, so that these investors *exploit their information more aggressively* in either a long or short direction. (This effect is reinforced by their underestimation of risk, but would work even if they did not underestimate risk. In contrast, underestimation of risk is essential to the result of De Long et al (1991).) In so doing, they gain profits by exploiting the mispricing created by liquidity/noise traders. In our model overconfidence is profitable even if the risky security is in zero net supply; it is not a matter of investing more heavily in the market portfolio, but of exploiting information more intensely.

Kyle and Wang (1997) provide a distinct reason for the survival of overconfident traders based on imperfect competition in securities markets. An informed trader who knows he is trading against an overconfident informed opponent chokes back on his trades, to the benefit of the overconfident trader.[5] An informed trader knows that the price execution in the direction indicated by his signal will be less favorable by virtue of the fact that the overconfident informed trader will trade aggressively based on the same signal. Being perceived as overconfident is in effect like being a Stackelberg leader, which generates oligopoly profits. Benos (1999) develops this theme to examine cases of imperfectly correlated signals. Fischer and Verrecchia (forthcoming) examine 'heuristic' traders who, owing to overconfidence or base-rate underweighting, overreact to new signals. In their paper as well, overreaction creates a 'first mover' advantage for heuristic traders owing to

[5] Wang (1997) extends the Kyle and Wang (1997) framework to a dynamic setting to show that overconfident traders can survive in the long run.

imperfect competition. In all three papers, holding constant the behavior of other traders, trading more aggressively *reduces* expected profits; the only benefit of overconfidence comes from being *perceived* as such by other informed traders. Furthermore, in all three papers, the effects described are only important if the mass of informed traders, and especially overconfident traders, is high enough to influence prices significantly.

In contrast with the commitment approach of these papers, in our model traders are perfectly competitive. Traders observe the market price and take it as given. Thus, a trader does not limit the size of his position out of fear that an overconfident informed trader will trade intensely in the same direction. The benefit to overconfidence in our model is that overconfident traders are willing to take on more risk, and hence better exploit the mispricing generated by the trades of 'noise' or liquidity traders. Unlike the commitment approach discussed above, in our model this benefit applies even if there is only a very small measure of informed traders. In other words, the profits arise not from the *commitment* to be aggressive (and the desirable effects of such commitment upon the behavior of other traders), but directly from the aggressiveness of the trading strategy.[6]

In principle, an overconfident trader ought to learn based on his past performance that the precision of his signal is not as great as he thought. If such learning were rational, overconfidence would disappear. This line of reasoning clashes with the extensive empirical evidence from psychology, based on

[6] Benos (1999) describes his model (the same applies to the other two papers as well) as follows: "Our result comes from a first mover advantage, not from excessive risk-taking." In our model, the result comes from the combination of misperception of means and aggressive risk-taking, not from a first mover advantage.

both experimental, survey and case research, that most individuals tend to be overconfident.[7] We do not model the process by which individuals learn about their own abilities; see however, the analyses of Daniel, Hirshleifer and Subrahmanyam (1998) and Gervais and Odean (2000). These models are based on evidence from psychology that in updating beliefs about their own abilities, individuals tend to credit themselves for favorable outcomes strongly, and to blame external factors for unfavorable outcomes (Daniel, Hirshleifer and Subrahmanyam (1998) discuss several such studies.) This phenomenon is termed self-attribution bias. This bias in the learning process explains why overconfidence exists persistently. Such an effect tends to maintain the importance of overconfidence in a dynamic steady state even if overconfident traders lose money. Our approach differs in that we do not allow a trader's confidence to grow over time. Nevertheless, overconfident traders can thrive profitably.

Section 2 of the paper describes the basic model with three types of agents (rational, overconfident and liquidity/noise traders) who invest in a risk-free and a risky asset. Section 3 models the long-run survival of overconfidence in an evolutionary process where trader types replicate according to their expected profits. Section 4 describes results, and Section 5 gives concluding remarks.

[7] In a review of psychology and finance, DeBondt and Thaler (1995) comment that "perhaps the most robust finding in the psychology of judgment is that people are overconfident." Odean (1998) provides a detailed summary of the evidence.

2 The Static Model

Consider a one-period competitive market consisting of two types of securities, a risk-free security with a constant payoff equal to one, and a risky security with a payoff equal to θ, where θ is a normally distributed random variable with mean $\overline{\theta}$ and variance σ_θ^2. There are three types of agents: rational traders, overconfident traders, and liquidity/noise traders. Both rational and overconfident traders receive a signal with respect to the payoff of the risky asset, denoted as s, where $s = \theta + \epsilon$, and where ϵ is normally distributed with $E(\epsilon) = 0$ and $Var(\epsilon) = \sigma_\varepsilon^2$ and is independent of θ. The distribution of θ is known to both rational and overconfident traders. The rational traders correctly perceive the distribution of ϵ (i.e., $E_r(\epsilon) = 0$ and $Var_r(\epsilon) = \sigma_\epsilon^2$ where the subscript r indicates a rational trader). Overconfident traders believe that the variance of the residual error (ϵ) is smaller than the true residual error variance (i.e., $Var_c(\epsilon) = \sigma_c^2 < \sigma_\epsilon^2$, where the subscript c indicates an overconfident trader). We assume that $\sigma_c^2 > 0$, i.e., overconfident traders recongnize that their signal is imperfect.

Both rational and overconfident traders choose a portfolio to maximize expected utility of wealth (denoted as w_i for $i = r, c$) at the end of the period, based on their interpretation of the signal, s. Trader i's utility function is assumed to be exponential, $U(w_i) = -e^{-aw_i}$, $a > 0$, where a is the coefficient of absolute risk aversion. Together with normality, this implies a mean-variance expected utility function. The wealth at the end of the time period for each trader is the sum of the initial wealth (denoted as w) and the gain derived from the two types of assets. Since the payoff of the risk-free asset is always one, for trader i ($i = r, c$), $w_i = w + X_i(\theta - p)$, where X_i is trader i's demand for the risky asset and p is its price . Therefore, for $i = r, c$, trader i's demand function X_i is the solution to

$$\begin{aligned} &\max_{X_i} E_i(w_i\,|s) - \frac{a}{2} Var_i(w_i\,|s), \\ s.t. \quad w_i &= w + X_i(\theta - p). \end{aligned}$$

Since θ and s are independent and normally distributed,

$$E_i(\theta\,|s) = \overline{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_i^2}(\theta - \overline{\theta} + \epsilon) \quad \text{and} \quad Var_i(\theta\,|s) = \frac{\sigma_\theta^2 \sigma_i^2}{\sigma_\theta^2 + \sigma_i^2},$$

where $\sigma_i^2 = \sigma_\epsilon^2$ for $i = r$ and $\sigma_i^2 = \sigma_c^2$ for $i = c$. This further implies that trader i's demand function is

$$X_i = \frac{\overline{\theta} + \eta_i(\theta - \overline{\theta} + \epsilon) - p}{a v_i}, \tag{1}$$

where

$$\eta_i = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_i^2}, \text{ and } v_i = \frac{\sigma_\theta^2 \sigma_i^2}{\sigma_\theta^2 + \sigma_i^2}.$$

Note that $\eta_c > \eta_r$ and $v_c < v_r$, hence, $|X_c| > |X_r|$. In other words, overconfident traders' higher conditional mean and lower conditional variance about the risky asset's payoff result in taking a larger long or short position.

The total demand for all liquidity/noise traders is x, where $E(x) = 0$ and $Var(x) = \sigma_x^2$. Within the subset of the population consisting of rational and overconfident traders, let λ denote the proportion of rational traders, and $1 - \lambda$ the proportion of overconfident traders. Assuming the supply of risky assets is zero, the market clearing condition for the risky asset is

$$\lambda X_r + (1 - \lambda) X_c = -x. \tag{2}$$

Substituting equation (1) into equation (2), the equilibrium price of the risky asset can be solved as

$$p = \frac{\widehat{\theta}_r \lambda v_c + (1-\lambda)\widehat{\theta}_c v_r + a v_r v_c x}{\lambda v_c + (1-\lambda) v_r},$$

where $\widehat{\theta}_i = \overline{\theta} + \eta_i(\theta - \overline{\theta} + \epsilon)$ for $i = r, c$. Note that $E(p) = \overline{\theta}$. Furthermore, the expected profit for trader i can be calculated as

$$\pi_i(\lambda) = E(X_i(\theta - p)) \quad \text{for } i = r, c.$$

Specifically, for the rational trader

$$\pi_r(\lambda) = \frac{\sigma_\epsilon^2 \sigma_\theta^2 \left[(\lambda - 1)^2 \left(\sigma_\epsilon^2 - \sigma_c^2\right)^2 + a^2 \sigma_x^2 \left(\sigma_c^2\right)^2 \left(\sigma_\theta^2 + \sigma_\epsilon^2\right) \right]}{a \left[\lambda \sigma_\theta^2 \sigma_c^2 + (1-\lambda)\sigma_\epsilon^2 \sigma_\theta^2 + \sigma_c^2 \sigma_\epsilon^2\right]^2},$$

and for the overconfident trader

$$\pi_c(\lambda) = \frac{\sigma_\epsilon^2 \sigma_\theta^2 \left[\lambda\left(\lambda - 1\right) \left(\sigma_\epsilon^2 - \sigma_c^2\right)^2 + a^2 \sigma_x^2 \sigma_c^2 \sigma_\epsilon^2 \left(\sigma_\theta^2 + \sigma_c^2\right) \right]}{a \left[\lambda \sigma_\theta^2 \sigma_c^2 + (1-\lambda)\sigma_\epsilon^2 \sigma_\theta^2 + \sigma_c^2 \sigma_\epsilon^2\right]^2}.$$

The expected profit for both types of traders is a function of the variance of the risky asset's payoff, the variance of noise trading, the variance of the signal perceived by the rational traders, the variance of the signal perceived by the overconfident traders, the degree of risk aversion and the relative size of overconfident traders to the rational traders.

The difference in the expected profits for both types of traders is shown to be

$$\pi_r(\lambda_t) - \pi_c(\lambda_t) = \frac{\sigma_\epsilon^2 \sigma_\theta^2 \left[\left(1 - \lambda_t\right) \left(\sigma_\epsilon^2 - \sigma_c^2\right)^2 - a^2 \sigma_\theta^2 \sigma_c^2 \sigma_x^2 \left(\sigma_\epsilon^2 - \sigma_c^2\right) \right]}{a \left[\lambda_t \sigma_\theta^2 \sigma_c^2 + (1-\lambda_t)\sigma_\epsilon^2 \sigma_\theta^2 + \sigma_c^2 \sigma_\epsilon^2\right]^2}.$$

3 The Dynamic Model

To examine the long-run survival of overconfident traders, we embed the static model of Section 2 within an evolutionary process in which trader types replicate differentially over time according to the profitability of their strategies.[8] This is reasonable because the existing traders tend to use the strategy which has proven to be profitable and the new traders also tend to imitate the profitable strategy as well. The natural selection process works as follows. If the expected profit for trader i is greater (less) than for trader j, for $i, j \in \{r, c\}$ and $j \neq i$, then in the next time period the proportion of trader i increases (decreases). If the expected profits for both sets of traders are equal, then the proportion of traders remains the same in the following period. Specifically, let the fraction of the population of rational traders follow the following dynamics:

$$\lambda_{t+1} = \lambda_t + f(\pi_r(\lambda_t) - \pi_c(\lambda_t); \lambda_t), \tag{3}$$

where $f(\cdot)$ maps from $(-\infty, +\infty) \times [0, 1] \to [0, 1]$ is a continuous function with the following properties:

$$\begin{aligned}
(i) \quad f(\cdot) &= 0 \text{ if } \pi_r(\lambda_t) - \pi_c(\lambda_t) = 0, \text{ and } \lambda_t \in (0, 1), \\
(ii) \quad f(\cdot) &< 0 \text{ if } \pi_r(\lambda_t) - \pi_c(\lambda_t) < 0 \text{ and } \lambda_t > 0, \\
(iii) \quad f(\cdot) &= 0 \text{ if } \lim_{\lambda_t \to 0^+} (\pi_r(\lambda_t) - \pi_c(\lambda_t)) \leq 0, \\
(iv) \quad f(\cdot) &> 0 \text{ if } \pi_r(\lambda_t) - \pi_c(\lambda_t) > 0 \text{ and } \lambda_t < 1,
\end{aligned}$$

[8] Our assumption that replication is based on profits rather than utility is based on the notion that opportunities to observe others may be limited, and that with a single observation it is easier to estimate profit than risk or utility.

$$(v) \quad f(\cdot) \quad = \quad 0 \text{ if } \lim_{\lambda_t \to 1^-} (\pi_r(\lambda_t) - \pi_c(\lambda_t)) \geq 0.$$

The above class of dynamics is general enough to encompass standard ones such as replicator dynamics and many other types of selection dynamics used in evolutionary game theory (see Taylor and Jonker (1978), Weibull (1995)). It is similar to that used by Luo (1999).[9]

This type of selection dynamics is based on the difference in the average outcomes of the population of rational traders and the population of overconfident traders. With a continuum of individuals, the average outcome of a population is equal to the expected profit of one trader in that population. Thus, this selection dynamic can be derived in a setting where trader types replicate based on the realized profits of individuals. For example, suppose that at the end of each period, each trader randomly samples (or meets) another trader. If the sampled trader is the opposite type to the given trader (i.e., one is overconfident and the other is not), and if the sampled trader made greater realized profit in that time period, then the given trader will switch to be the type of this sampled trader with some positive probability; otherwise the given trader remains the same type. This evolutionary process based on the realized profits at the individual level produces a selection dynamic based on the average outcome of populations.

The dynamic equilibrium is defined to be either an interior fixed point (case (i) above) or a corner solution (case (iii) or case (v) above) of the above dynamics. We denote the dynamic equilibrium as λ.

[9] Examples of individual micro behavior (imitation or adaptation) that produce this type of population-based selection dynamics can be found in standard evolutionary game theory textbooks (see also Taylor and Jonker (1978), Weibull (1995), and Luo (1999)).

4 Results

This section relates the long-run proportion of surviving overconfident traders to the underlying parameters of the model, such as the degree of overconfidence, noise volatility, and the volatility of underlying security payoffs.

Proposition 1 *For all positive parameter values $(a, \sigma_x^2, \sigma_c^2, \sigma_\theta^2, \sigma_\epsilon^2)$, there is a unique dynamic equilibrium. Regardless of the initial fraction of overconfident traders $1-\lambda_t$, the market always converges to this dynamic equilibrium. The equilibrium has the following properties:*

(i) If $a^2\sigma_x^2\sigma_c^2\sigma_\theta^2 < \sigma_\epsilon^2 - \sigma_c^2$, then the dynamic equilibrium is the interior fixed point where $\lambda = 1 - \frac{a^2\sigma_x^2\sigma_c^2\sigma_\theta^2}{\sigma_\epsilon^2 - \sigma_c^2}$.

(ii) If $a^2\sigma_x^2\sigma_c^2\sigma_\theta^2 \geq \sigma_\epsilon^2 - \sigma_c^2$, the dynamic equilibrium is the corner point where $\lambda = 0$ (all overconfident traders).

(iii) The dynamic equilibrium cannot be the corner point where $\lambda = 1$.

(iv) The higher is the volatility of the underlying security payoff (σ_θ^2), the higher is the proportion of overconfident traders in the equilibrium.

(v) The more volatile is liquidity/noise trading (the higher is σ_x^2), the higher is the proportion of overconfident traders in the equilibrium.

(vi) The greater the confidence of the overconfident traders, the lower is the proportion that survive in the equilibrium.

Proof. See the Appendix ■

We now comment on the results in order:

(i) If $a^2\sigma_x^2\sigma_c^2\sigma_\theta^2 < \sigma_\epsilon^2 - \sigma_c^2$, then for all positive parameters, no matter where λ_t starts in the interval $(0, 1)$, it evolves into the interior fixed point where the rational traders and the overconfident traders coexist. Thus, so long as there is some liquidity/noise trading, and overconfidence is not too severe, the overconfident traders will persist in the long run. Intuitively, overconfident traders place greater weight on the signal optimistically, and therefore take a bigger (or more risky) position and better exploit the misvaluation created by liquidity/noise traders than do rational traders. Consequently, overconfident traders survive in the long run. However, if there are too many overconfident traders, the prices would be pushed against them excessively and rational traders would gain at the expense of overconfident traders by trading in the opposite direction to the overconfident traders. Hence, the rational traders survive in the long run as well.

(ii) If $a^2\sigma_x^2\sigma_c^2\sigma_\theta^2 \geq \sigma_\epsilon^2 - \sigma_c^2$, for all positive parameter values, rational traders are driven out of the market and only overconfident traders survive in the long run. For example, if σ_c^2 is close to σ_ϵ^2 (i.e., low overconfidence), then overconfident traders do not trade too aggressively; consequently, the survival of overconfident traders is favored. This inequality also indicates that overconfident traders could be so favored by high risk aversion (high a), high volatility of liquidity/noise trading (high σ_x^2) or high volatility in the underlying asset value (high σ_θ^2) that rational traders are driven out of the market completely.

(iii) As long as liquidity/noise trading is present, $\lambda = 1$ cannot be an equilibrium, since overconfident traders can better exploit noise than rational traders and survive in the market. However, we have shown (proof available on request) that if liquidity/noise trading vanishes ($\sigma_x^2 = 0$) or overconfidence is extreme ($\sigma_c^2 = 0$), then overconfident traders are driven out of the market.[10]

(iv) The higher is the volatility coming from the underlying security payoff, the larger is the proportion of surviving overconfident traders.[11]

As volatility of the underlying security payoff increases, rational traders are not able to infer as clearly that a high price indicates overvaluation. This increases the riskiness for them of a contrarian strategy of selling when price is high and buying when price is low. The perceived risk of this strategy is reduced by observation of the private information signal, but this perceived risk reduction is greater for the overconfident. As a result, higher volatility of underlying security payoff increases the relative expected profitability for the overconfident. Figure 1 plots the increasing relationship between the long-run proportion of surviving overconfident traders and σ_θ^2.

(v) As the volatility of liquidity/noise trading increases, the equilibrium proportion of overconfident traders increases as well. Noise creates misvaluation, which offers greater profit opportunities for overconfident traders than

[10] In this model, liquidity/noise trading is constant through time (i.e., σ_x is constant and positive). We do not apply a selection dynamic to such traders because the inclusion of such traders implicitly reflects the notion that many or even all individuals are subject to shocks in the need for cash for consumption purposes. However, an alternative perspective is that there are traders who trade in a random independent fashion. In some settings such traders can make money (see DeLong et al (1990, 1991)), but in others they are eventually eliminated (see Luo (1998)). The long-run performance of such traders also depends on the degree of the traders' risk aversion and the correlation of the signals received by traders (see Figlewski (1978)). Even if there is economic natural selection against noise trading, the inclusion of a group of noise traders in the model can be viewed as reflecting random errors to which individuals tend to be subject.

[11] This is also consistent with Luo (2000), who looks at the convergence of the futures price to the spot price in a dynamic model of natural selection according to wealth. She finds that when volatility in the underlying spot market increases, the interval around the spot price (where the future price eventually converges to) gets larger. This allows more room for the survival of irrational traders (e.g., overconfident traders).

for fully rational ones. Figure 1 plots the increasing relationship between the long-run proportion of surviving overconfident traders and σ_x^2.

(vi) If overconfident traders are too confident, their trading becomes too aggressive, and the equilibrium proportion of surviving overconfident traders decreases. Figure 2 plots the increasing relationship between the long-run proportion of surviving overconfident traders and the perceived error variance σ_c^2 (an inverse measure of overconfidence).

5 Conclusion

Recent research has proposed several ways in which overconfident traders can persist despite competition from rational traders. This paper offers an additional reason: overconfident traders do better than purely rational traders at exploiting misvaluation caused by liquidity or noise trading. Using a model of a perfectly competitive asset market involving rational traders, overconfident traders and liquidity/noise traders, we examine both the static profitability of overconfident versus rational trading strategies, and the dynamic evolution of the population of traders. Different investor types are assumed to become more prevalent when their strategies are more profitable.

In some cases there is an interior equilibrium with both rational and overconfident traders. If the degree of risk aversion, the volatility of liquidity/noise trading or the volatility of the underlying security payoff becomes sufficiently large, rational traders are driven out of the market and only overconfident traders survive. The higher the noise volatility and the higher the volatility of the underlying security payoff, the larger is the proportion of surviving overconfident traders. The more intense is their confidence, the lower is the proportion of surviving overconfident traders. Finally, our main result is that unless the degree of overconfidence is infinite, the long-run steady-state equilibrium always involves overconfident traders surviving as a positive fraction of the population.

Figure 1
The relationship between the surviving overconfident traders and either volatility or noise

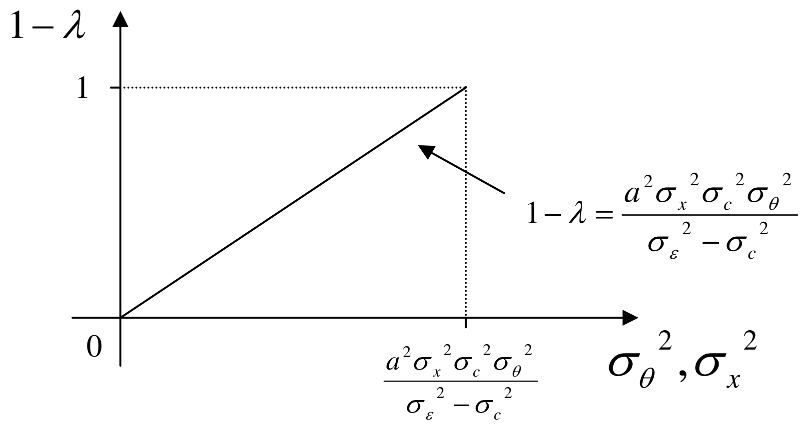
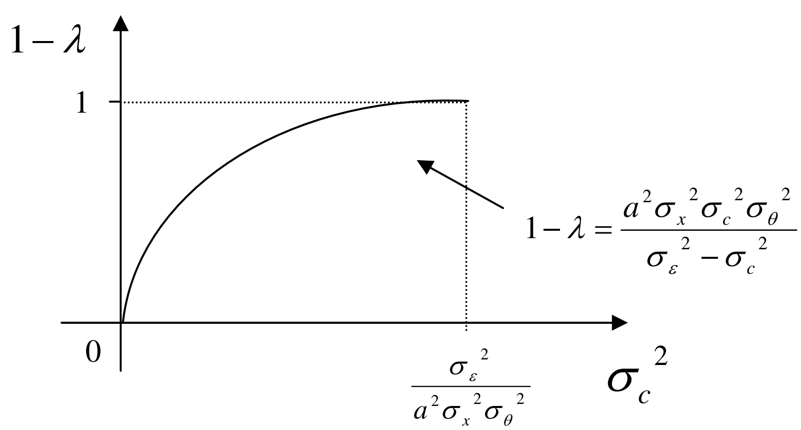


Figure 2
The relationship between the surviving overconfident traders and overconfidence level



Appendix

Proof of Proposition 1:

(i) Solving $\pi_r(\lambda) = \pi_c(\lambda)$ results in an interior fixed point $\lambda = 1 - \frac{a^2\sigma_x^2\sigma_c^2\sigma_\theta^2}{\sigma_\epsilon^2 - \sigma_c^2}$. If $a^2\sigma_x^2\sigma_c^2\sigma_\theta^2 < \sigma_\epsilon^2 - \sigma_c^2$, then for all positive parameter values, $\lambda \in (0,1)$. Furthermore, since for any $\lambda_t < \lambda$, $\pi_r(\lambda_t) > \pi_c(\lambda_t)$ and for any $\lambda_t > \lambda$, $\pi_r(\lambda) < \pi_c(\lambda)$, using the dynamics defined in equation (3), no matter where the λ_t starts in the interval $(0,1)$, the market converges to this interior fixed point.

(ii) For the corner solution corresponding to $\lambda = 0$ to be an equilibrium, using the definition of the dynamics, it requires $\lim_{\lambda_t \to 0^+} (\pi_r(\lambda_t) - \pi_c(\lambda_t)) \leq 0$. This is true under the parameter restrictions that $a^2\sigma_x^2\sigma_c^2\sigma_\theta^2 \geq \sigma_\epsilon^2 - \sigma_c^2$. Furthermore, since for any $\lambda_t > 0$, $\pi_r(\lambda_t) < \pi_c(\lambda_t)$, using the dynamics defined in equation (3), no matter where the λ_t starts in the interval $(0,1)$, the market converges to this equilibrium where $\lambda = 0$.

(iii) For the corner solution $\lambda = 1$ to be an equilibrium, the dynamics require that $\lim_{\lambda_t \to 1^-} (\pi_r(\lambda_t) - \pi_c(\lambda_t)) \geq 0$. For all positive parameters,

$$\lim_{\lambda_t \to 1^-} (\pi_r(\lambda_t) - \pi_c(\lambda_t)) = \frac{-\left(\sigma_\theta^2\right)^2 \sigma_\epsilon^2\sigma_x^2 a^2 \left(\sigma_\epsilon^2 - \sigma_c^2\right)}{\sigma_c^2 \left(\sigma_\theta^2 + \sigma_\epsilon^2\right)^2} < 0.$$

Therefore, the corner solution corresponding to $\lambda = 1$ cannot be an equilibrium.

(iv) At the interior fixed point, $\frac{\partial\lambda}{\partial(\sigma_\theta^2)} = -\frac{a^2\sigma_c^2\sigma_x^2}{\sigma_\epsilon^2 - \sigma_c^2} < 0$. Hence, (iv) follows.

(v) When $a^2\sigma_x^2\sigma_c^2\sigma_\theta^2 < \sigma_\epsilon^2 - \sigma_c^2$ the dynamic equilibrium has the interior fixed point $\lambda = 1 - \frac{a^2\sigma_x^2\sigma_c^2\sigma_\theta^2}{\sigma_\epsilon^2 - \sigma_c^2}$. Since $\frac{\partial\lambda}{\partial(\sigma_x^2)} = -\frac{a^2\sigma_c^2\sigma_\theta^2}{\sigma_\epsilon^2 - \sigma_c^2} < 0$, (v) follows.

(vi) At the interior fixed point, $\frac{\partial\lambda}{\partial(\sigma_c^2)} = -\frac{a^2\sigma_\theta^2\sigma_x^2\sigma_\epsilon^2}{(\sigma_\epsilon^2 - \sigma_c^2)^2} < 0$. Hence, (vi) follows.

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