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Abstract

Since Leeper's (1991, *Journal of Monetary Economics* 27, 129-147) seminal paper, an extensive literature has argued that if fiscal policy is *passive*, that is, guarantees public debt stabilization irrespectively of the inflation path, monetary policy can independently be committed to inflation targeting. This can be pursued by following the Taylor principle, i.e., responding to upward perturbations in inflation with a more than one-for-one increase in the nominal interest rate. This paper considers an optimizing framework in which the government can only finance public expenditures by levying distortionary taxes. It is shown that households' participation constraints and Laffer-type effects may render passive fiscal policies unfeasible. For any given target inflation rate, there exists a threshold level of public debt beyond which monetary policy independence is no longer possible. In such circumstances, the dynamics of public debt can be controlled only by means of higher inflation tax revenues: inflation dynamics in line with the fiscal theory of the price level *must* take place in order for macroeconomic stability to be guaranteed. Otherwise, to preserve inflation control around the steady state by following the Taylor principle, monetary policy *must* target a higher inflation rate.

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1 Introduction

The interaction between fiscal and monetary rules is one of the most controversial issues for policy design. Since Leeper's (1991) seminal contribution, modern theory has argued that if fiscal policy is *passive*, that is, guarantees public debt stabilization irrespectively of the inflation path, monetary policy can independently be committed to inflation targeting, for example, by managing the nominal interest rate on the basis of a Taylor-type rule (Taylor, 1993). Notably, a Taylor-type rule prescribes to implement an active monetary policy, responding to increases in inflation with a more than one-for-one increase in the nominal interest rate (the so-called Taylor principle). Conversely, if fiscal policy is *active*, that is, does not guarantee public debt stabilization for each dynamic path of inflation, monetary policy should be passive, responding to increases in inflation with a less than one-for-one increase in the nominal interest rate in order to rule out explosive dynamics for public debt. These results are known as Leeper's active/passive *dichotomy*, and have been proved to hold in economies with either flexible or sticky prices (Woodford, 2003). The type of fiscal feedback rules commonly used in the literature to model government's policy involves the adoption of lump-sum taxes.

The main contribution of this paper is to show that in the realistic case in which lump-sum taxes are unavailable, it might be unfeasible to implement passive fiscal policies. This result comes from two relevant implications of distortionary taxes when agents optimize: (*i*) the emergence of households' participation constraints; (*ii*) the occurrence of Laffer-type effects generated by both tax and interest-rate feedback rules.

The central implication of our analysis is that, for any given target inflation rate, there exists a threshold level of public debt beyond which monetary policy independence is no longer possible. Under these circumstances, the dynamics of public debt can be controlled only by means of higher inflation tax revenues. Specifically, two possible scenarios arise: if the central bank is intended to preserve inflation control *around* the steady state by adopting the Taylor principle, it *must* fix a sufficiently higher target inflation rate; otherwise, inflation dynamics of the type studied by the "fiscal theory of the price level" (eg., Leeper 1991; Sims 1994; Woodford 1994, 1995, 2003; Cochrane, 1998, 2005; Leeper and Yun, 2006) *must* occur in order for macroeconomic stability to be ensured.

To illustrate our analytical results and provide transparent economic rationales, we organize the paper as follows. In Section 2, we describe a continuous-time general equilibrium optimizing framework with lump-sum taxation and discuss the central features of Leeper's dichotomy. A continuous-time setup proves to be more convenient for the arguments developed in the present paper. A discrete-time setup would not alter the essence of our analysis, but would complicate economic intuitions, due to issues pertaining to timing conventions.

In Sections 3-6, we remove the recourse by the government to lump-sum taxes as an operating instrument to implement passive fiscal policies. In Section 3, we demonstrate that a passive fiscal policy cannot be based on a tax system that has only debt as tax base. In this case, a passive fiscal policy violates the households' participation constraint in the bonds market. Such a constraint requires that the "net" interest rate is positive, a feature that can be satisfied only by an active fiscal policy. In Section 4, we consider the implications of using income taxes to stabilize public debt, in the simplified case of an endowment economy. Using income, as opposed to debt, as tax base can allow to preserve monetary policy independence. The resulting participation constraint becomes binding only for quite high levels of public debt. In Section 5, we relax the assumption of endowment economy. We demonstrate that there exists a lower threshold level of public debt beyond which a passive fiscal policy is no longer feasible and monetary policy independence disappears, due to the presence of Laffer effects on tax revenues. The consequences for monetary policy design are analyzed in Section 6. Our conclusions are summarized in Section 7.

2 The basic framework

In this Section, we set up a simple continuous-time optimizing framework with lump-sum taxation. In this context, we reconsider Leeper's dichotomy. In the subsequent Sections, we shall employ this model as a benchmark to study the consequences of distortionary taxation.

2.1 Agents

Consider an endowment economy with a private sector and a public sector. The private sector consists of a continuum of identical infinitely lived households.[1] The representative household has preferences given by the following lifetime utility function:

$$U = \int_0^{\infty} e^{-\rho t} \left[u(c, m) + f(g) \right] dt, \tag{1}$$

where c is real private consumption, m are real money balances, and g is real government consumption expenditure. The instantaneous utility function $u(c, m) + f(g)$ is increasing in the three arguments and concave: $\partial u/\partial c = u_c > 0$, $\partial u/\partial m = u_m > 0$, $df/\partial g = f' > 0$, $\partial^2 u/\partial c^2 = u_{cc} < 0$, $\partial^2 u/\partial m^2 = u_{mm} < 0$, $d^2 f/dg^2 = f'' < 0$. Consumption and real balances are Edgeworth complements, so that $u_{cm} > 0$.

The household's instant budget constraint in real terms is given by

$$c + \dot{b} + \dot{m} = (i - \pi) b + y - \tau + \tau_h - \pi m, \tag{2}$$

where b is the stock of interest-bearing bonds, i is the nominal interest rate paid on bonds, π is the inflation rate, y is a constant endowment of perishable goods, τ are lump-sum taxes, and τ_h are government transfers.[2] The right-hand-side of (2) represents disposable income; the left-hand-side shows the uses of disposable income: consumption and saving; the latter takes the form of increases in the stock of real bonds and real balances. The household is prevented from engaging in Ponzi's games.

The public sector's budget constraint in real terms is given by

$$g + \tau_h + (i - \pi) b - \tau - \pi m = \dot{b} + \dot{m}. \tag{3}$$

Now, the left-hand-side of (3) represents government deficit net of inflation tax revenues; the right-hand-side shows how the public sector can finance its deficit: by issuing interest-bearing bonds and printing money.

[1] Several issues on monetary and fiscal policy design in non-Ricardian economies in which new generations are born over time are studied by Benassy (2007).

[2] The budget constraint in real terms (2) is derived dividing by the price level the budget constraint in nominal terms,

$$C + \dot{B} + \dot{M} = iB + Y - T + T_h,$$

where upper-case letters represent the corresponding nominal variables. Standard algebra leads to (2).

2.2 Agents' choices

The private sector chooses paths for private consumption, real balances, and bonds so as to maximize (1) subject to the budget constraint (2) and the transversality conditions, given the constant stream of the endowment y, and the initial conditions $m(0) = m_0$ and $b(0) = b_0$. Optimization yields

$$\begin{cases} (1) & u_c(c,m) = \lambda, \\ (2) & u_m(c,m) = \lambda i, \\ (3) & \dot{\lambda} = \lambda(\rho + \pi - i). \end{cases} \tag{4}$$

Consistently with Leeper (1991), the choices of the public sector are described by two rules, one pertaining to monetary policy, the other to fiscal policy.

The monetary authority fixes the nominal interest rate i in order to control the inflation rate π around the target inflation rate π^*. To facilitate the analysis, and without loss of generality, we assume $\pi^* > 0$. We summarize such a feedback rule as

$$i = \phi(\pi), \tag{5}$$

where $\phi(\pi)$ is continuous, non-decreasing, and strictly positive. Monetary policy is defined as *active* when the monetary authority reacts more than proportionally to changes in inflation, $di/d\pi = \phi' > 1$, according to the so-called Taylor principle. Monetary policy is defined as *passive* when the opposite occurs, $\phi' < 1$.

Let now consider fiscal policy. Public consumption g and transfers τ_h are assumed to be exogenous and constant. Taxes are described by the feedback rule

$$\tau = \bar{\alpha} + \alpha b, \tag{6}$$

where $\bar{\alpha}$ is a constant parameter and $\alpha \geq 0$ captures the degree of reactiveness of taxes to public debt. Fiscal policy is defined as *passive* when rule (6) guarantees stability of public debt around the steady state for each dynamic path of inflation. That is, a passive fiscal policy must respect the condition

$$\left.\frac{\partial \dot{b}}{\partial b}\right|_{(\pi^*,\, b^*)} < 0. \tag{7}$$

Conversely, fiscal policy is defined as *active* when the fiscal rule (6) is such that

$$\left.\frac{\partial \dot{b}}{\partial b}\right|_{(\pi^*,\, b^*)} > 0. \tag{8}$$

2.3 Equilibrium

Combining the two constraints (2) and (3), and imposing equilibrium in the bonds and money markets, one obtains the goods' market equilibrium condition:

$$y = c + g. \tag{9}$$

Since y and g are both exogenous and constant, it follows that $\dot{c} = 0$. Thus, from (4.1) and (4.2), we can derive the relationships between m and i, i.e., the money demand function, and between λ and i:[3]

$$\begin{cases} (1) \quad m = m(i) \quad \text{with} \quad m' < 0, \\ (2) \quad \lambda = \lambda(i) \quad \text{with} \quad \lambda' < 0. \end{cases} \tag{10}$$

We can now derive the equation describing inflation dynamics. Time differentiating (10.2), using the costate equation (4.3) and the monetary policy rule (5), we obtain

$$\dot{\pi} = H(\pi)\left[\phi(\pi) - \pi - \rho\right], \tag{11}$$

where $H(\pi) = -\lambda/\lambda'\phi' > 0$.

To derive the equilibrium equation describing public debt dynamics, we start from money demand (10.1); using the monetary policy rule (5), differentiating with respect to time, using the inflation dynamics equation (11), substituting into the budget constraint (3), and taking into account the fiscal policy rule (6), we obtain

$$\dot{b} = \left[\phi(\pi) - \pi - \alpha\right] b + g + \tau_h - \bar{\alpha} + K(\pi)\left[\phi(\pi) - \pi - \rho\right] - \pi m\left[\phi(\pi)\right], \tag{12}$$

where $K(\pi) = \lambda m'/\lambda' > 0$.

The dynamics of the economy is described by the system of differential equations (11) and (12) in the variables (π, b). Since the monetary authority controls the nominal interest rate, money supply is endogenous, and adjusts to demand. Money demand turns out to depend on the inflation rate according to the function $m\left[\phi(\pi)\right]$. The inflation rate π results to be "chosen"

[3] For analytical details, see Appendix A.

indirectly by the private sector, thus being a jump variable. The level of public debt b is instead the state variable in the system. We can then define a perfect-foresight equilibrium as a pair of functions $\{\pi(t), b(t)\}$ that satisfy (11)-(12), given the initial condition $b(0) = b_0$ and the transversality conditions.

The system is in steady state when $\dot{b} = 0$ and $\dot{\pi} = 0$. From (11), the steady-state value of inflation π^* is implicitly defined by

$$\phi(\pi^*) = \rho + \pi^*. \tag{13}$$

Using (13) into (12) yields the steady-state value of debt b^*:

$$b^* = \frac{\bar{\alpha} - g - \tau_h + \pi^* m(\rho + \pi^*)}{\rho - \alpha}. \tag{14}$$

As in Leeper (1991), the parameter $\bar{\alpha}$ is chosen to make b^* positive, and can be interpreted as a "scale" parameter.

The system (11)-(12) and its steady-state solution (13)-(14) enable us to specify when fiscal policy is passive and when it is active. We must compute the partial derivative of $\dot{b}$ with respect to b, evaluated at the steady state (π^*, b^*). If the value of this derivative is negative, fiscal policy is passive, and viceversa. We have

$$\left.\frac{\partial \dot{b}}{\partial b}\right|_{(\pi^*,\, b^*)} = \rho - \alpha, \tag{15}$$

Therefore, fiscal policy is passive if $\alpha > \rho$. Note the economic meaning of this condition: the *implicit* marginal tax rate on bonds must be greater than the return on bonds.

2.4 Dynamics

To study the dynamics of the system (11)-(12), let linearize it around the steady state (π^*, b^*):

$$\begin{pmatrix} \dot{\pi} \\ \dot{b} \end{pmatrix} = J \begin{pmatrix} \pi - \pi^* \\ b - b^* \end{pmatrix}. \tag{16}$$

The Jacobian J is

$$J = \begin{bmatrix} H^*(\phi' - 1) & 0 \\ A_{21} & \rho - \alpha \end{bmatrix}, \tag{17}$$

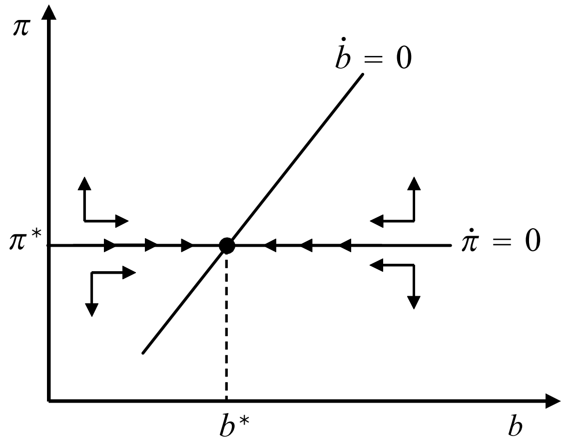

Figure 1: Passive fiscal policy and active monetary policy

where $A_{21} = (b^* + K^*)(\phi' - 1) - m^*\left(1 - \eta^*_{m/\pi}\right)$, with $\eta^*_{m/\pi} = |(\pi^*/m^*)\, m'\phi'|$ denoting the elasticity of money demand with respect to inflation, evaluated at (π^*, b^*).

Since b is a state variable and π a jump variable, we have a saddle path if the following condition holds:

$$\det J = H^*(\phi' - 1)(\rho - \alpha) < 0. \tag{18}$$

Condition (18) is satisfied *either* if « $\alpha > \rho$ and $\phi' > 1$ » (passive fiscal policy and active monetary policy) *or* if « $\alpha < \rho$ and $\phi' < 1$ » (active fiscal policy and passive monetary policy). These are the two cases that specify Leeper's dichotomy. To see it at work, suppose to start from a value $b_0 \neq b^*$.

When $\alpha > \rho$, the solution of the system (16) is given by

$$\begin{cases} b = b^* + (b_0 - b^*)\, e^{-(\alpha-\rho)t}, \\ \pi = \pi^*. \end{cases} \tag{19}$$

Since, by assumption, fiscal policy is passive, the monetary authority is perfectly able to control inflation according to the Taylor principle ($\phi' > 1$). The phase diagram of the system (16) is presented in Figure 1. The slope of the locus $\dot{b} = 0$, given by $(\rho - \alpha)/A_{21}$, depends on the sign of A_{21} which can be positive or negative. This is because inflation has two opposite effects on the level of public debt: on the one hand, it increases interest payments by the government, since, by assumption, $\phi' > 1$; on the other hand, it increases inflation tax. In Figure 1, we have drawn the locus $\dot{b} = 0$ with positive slope, as it is more likely to occur when inflation is relatively low. Nevertheless, this slope has no relevance for the system dynamics, since in this case the saddle path coincides with the locus $\dot{\pi} = 0$. Finally, equation (19) implies that the velocity through which debt converges to the steady state is an increasing function of α.

When $\alpha < \rho$, we must have $\phi' < 1$ for saddle-path stability to occur. The solution of the system (16) becomes

$$\begin{cases} b = b^* + (b_0 - b^*)\, e^{H^*(\phi'-1)t}, \\ \pi = \pi^* + S(\phi', \alpha)(b - b^*). \end{cases} \tag{20}$$

where $S(\phi', \alpha) = -\mathrm{Tr}J/A_{21} > 0$ measures the slope of the saddle path, which is greater than the slope of the locus $\dot{b} = 0$. Since now fiscal policy is active ($\alpha < \rho$), the monetary authority cannot follow the Taylor principle. Assuming $b_0 > b^*$, the jump in inflation above the target π^* allows the real public debt to decrease gradually and converge to the steady state. This is because

$$\left.\frac{\partial \dot{b}}{\partial \pi}\right|_{(\pi^*,\, b^*)} = A_{21} < 0. \tag{21}$$

The intuition is as follows. Inflation decreases the real interest rate $\phi(\pi) - \pi$, increases inflation tax, and hence increases the monetary financing of deficit.[4] The associated phase diagram is illustrated in Figure 2. The jump in inflation needed to ensure stability of real public debt is in accordance with the so-called "fiscal theory of the price level".[5] In synthesis, when fiscal policy is active, inflation dynamics depends on fiscal variables.

To summarize, Leeper's dichotomy establishes that monetary policy is able to control inflation consistently with the target level π^*, provided that fiscal policy takes the burden of controlling public debt. In the opposite case, it is monetary policy that must take the burden of bringing public debt back to the level b^*. Monetary policy can obtain this result only by allowing inflation to jump above the level π^* when $b(t) > b^*$.

[4] The term A_{21} can be decomposed in two parts. The first part is negative only when $\phi' < 1$. The second part is negative if the economy is on the upward-sloping side of the Laffer curve for seignorage, as it is efficient.

[5] See Woodford (2003, pp. 311-319) for a discussion.

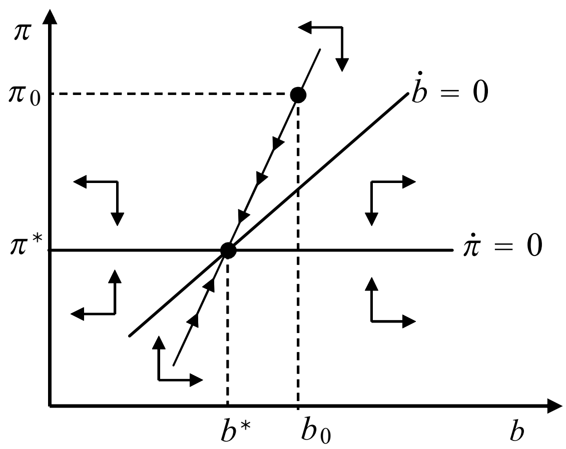

Figure 2: Active fiscal policy and passive monetary policy

Thus, Leeper's dichotomy states that a necessary condition for monetary policy *independence* in the presence of public debt is that fiscal policy is *passive*. In the next Sections, we explore the constraints that the fiscal authority can face in implementing a passive policy, as soon as we relax the simplified case of lump-sum taxation.

3 Bonds taxation and the participation constraint

We have seen that fiscal policy is passive when its primary objective is public debt stabilization. To obtain this result, the *implicit* marginal tax rate α must be greater than ρ, the steady-state real return on public bonds. This condition raises a feasibility problem for a passive fiscal policy, which emerges when we remove the assumption of lump-sum taxation, thus enabling optimizing households to take into account the interaction between their choices and the level of taxation.[6] Intuitively, the household will never demand an asset with a negative real return.[7]

[6] As long as taxation is lump sum and equilibrium is competitive, so that the single household is atomistic, the exogenous parameter α does not appear in the solution of the private agents' maximizing problem. This is because the representative household is not able to internalize taxation into its optimal choice.

[7] Money is the only exception. There can be a positive demand for money also in the presence of a negative return due to inflation, since money has a positive marginal utility.

Let us analyze the argument. Suppose that fiscal policy obtains its revenues by setting the nominal public debt as tax base,[8] with a tax rate equal to α. The households' participation constraint in the bonds market imposes

$$\alpha < i. \tag{22}$$

We shall now show that, by rewriting the model with such a participation constraint, fiscal policy cannot be passive.

The representative household's instant budget constraint in real terms is now

$$c + \dot{b} + \dot{m} = (i - \pi - \alpha)\, b + y - \bar{\alpha} + \tau_h - \pi m. \tag{23}$$

Performing optimization yields

$$\begin{cases} (1) & u_c(c,m) = \lambda, \\ (2) & u_m(c,m) = \lambda\,(i-\alpha)\,, \\ (3) & \dot{\lambda} = \lambda\,(\rho + \pi + \alpha - i)\,. \end{cases} \tag{24}$$

The government's budget constraint is now

$$g + \tau_h + (i - \pi - \alpha)\, b - \bar{\alpha} - \pi m = \dot{b} + \dot{m}. \tag{25}$$

In equilibrium, optimality conditions (24.1) and (24.2) can be written in implicit form as follows:

$$\begin{cases} (1) & m = m\,(i-\alpha) \quad \text{with} \quad m' < 0, \\ (2) & \lambda = \lambda\,(i-\alpha) \quad \text{with} \quad \lambda' < 0. \end{cases} \tag{26}$$

The closed-form differential-equation system in the variables (π, b) is then given by

$$\dot{\pi} = H\,(\pi)\,[\phi\,(\pi) - \pi - \alpha - \rho]\,, \tag{27}$$

$$\dot{b} = [\phi\,(\pi) - \pi - \alpha]\, b + g + \tau_h + K\,(\pi)\,[\phi\,(\pi) - \pi - \alpha - \rho] - \pi m\,[\phi\,(\pi)]\,. \tag{28}$$

The steady-state solutions are

$$\phi\,(\pi^*) = \alpha + \rho + \pi^*, \tag{29}$$

[8] It can be shown that the same argument applies by assuming taxation on real bonds, or on nominal (or real) interest payments. For an analysis of macroeconomic stability under Taylor rules in a New Keynesian framework with nominal interest taxation, see Edge and Rudd (2007).

$$b^* = \frac{\bar{\alpha} - g - \tau_h + \pi^* m \left(\alpha + \rho + \pi^*\right)}{\rho}. \tag{30}$$

It immediately follows that

$$\left.\frac{\partial \dot{b}}{\partial b}\right|_{(\pi^*,\ b^*)} = \rho. \tag{31}$$

This proves that fiscal policy cannot be passive, for households internalize bonds taxation into their optimal decisions.

The implications for monetary policy are straightforward. Now, the Jacobian is

$$J = \begin{bmatrix} H^* \left(\phi' - 1\right) & 0 \\ A_{21} & \rho \end{bmatrix}. \tag{32}$$

The emergence of a saddle path requires $\phi' < 1$, that is, a passive monetary policy. The monetary authority is no longer able to control inflation.

To conclude, a passive fiscal policy cannot rely on debt taxation only. There are two alternatives to ensure macroeconomic stability. The first is to combine debt taxation with an inflationary path brought about by a passive monetary policy, along the lines depicted by the fiscal theory of the price level. But in this case, the monetary authority cannot be independent, i.e., cannot adopt a Taylor-type rule with $\phi' > 1$, in order to set inflation equal to the target level π^*. The second alternative is to raise revenues from another tax base.

4 Income taxation in an endowment economy

Let us focus on the implications of using income taxes as instrument of a passive fiscal policy.[9] For now, let us maintain the simplified hypothesis of endowment economy. So the analysis of this Section is directly comparable with Leeper's (1991), and serves to introduce the issues addressed in the next Section.

Let $\tau_y < 1$ be the tax rate on income. The household's budget constraint is given by

$$c + \dot{b} + \dot{m} = (i - \pi)\, b + (1 - \tau_y)\, y + \tau_h - \pi m. \tag{33}$$

[9] Linnemann (2006) studies the dynamic effects of alternative fiscal rules in a New Keynesian model with income taxation.

Since y is exogenous, the optimality conditions are exactly the same as in Section 2.

The government's budget constraint is given by

$$g+\tau_h+(i-\pi)\,b-\tau_y y-\pi m=\dot{b}+\dot{m}. \tag{34}$$

Fiscal policy is now described in terms of a feedback rule in which income taxation reacts to public debt:

$$\tau_y y=\bar{\alpha}+\alpha b. \tag{35}$$

The differential-equation system is the same as in Section 2. Hence, using income, as opposed of debt, as tax base allows to reestablish Leeper's dichotomy, so that a passive fiscal policy allows monetary policy independence. However, this result is subject to the following remark.

The steady-state marginal tax rate, τ_y^*, depends on the target inflation rate π^*, independently set by the monetary authority, and on the steady-state level of public debt b^*:

$$\tau_y^*=\rho\frac{b^*}{y}+\frac{g+\tau_h}{y}-\frac{\pi^* m\,(\rho+\pi^*)}{y}. \tag{36}$$

From (36), the fiscal rule may violate the participation constraint, which imposes $\tau_y<1$. Because $\partial\tau_y^*/\partial b^*>0$, it emerges a limit on the level of steady-state public debt. Let b_y^{M} be the threshold value of public debt beyond which the participation constraint is violated. From (36), it follows that

$$b_y^{\mathrm{M}}=\frac{y-g-\tau_h+\pi^* m\,(\rho+\pi^*)}{\rho}. \tag{37}$$

If $b_0>b_y^{\mathrm{M}}$, it is not feasible to implement a passive fiscal policy, for $\tau_y\,(0)\,y=\bar{\alpha}+\alpha b_0>1$, which violates the constraint $\tau_y<1$. A central bank intended to follow the Taylor principle has to accept a higher steady-state inflation rate in order to raise the monetary financing, thereby ensuring $b_0\leq b_y^{\mathrm{M}}$.

The foregoing remark, it can be argued, is "purely" theoretical. The condition $b_0>b_y^{\mathrm{M}}$ could be, in fact, empirically implausible. However, recall that thus far we have assumed an endowment economy. Households' optimal decisions for consumption and saving do not affect the level of y, thereby not influencing fiscal revenues. Such an independence between households' optimal decisions and fiscal revenues no longer holds in a production economy. We shall examine the consequences in what follows.

5 Laffer effects and monetary policy independence

Consider an economy populated by a continuum of identical household-firms. The production technology of the representative household-firm is given by

$$y = l, \tag{38}$$

where l represents labor supply. The household's lifetime utility function takes the following form:

$$U = \int_0^\infty e^{-\rho t} \left[u(c, m) + f(g) - v(l) \right] dt, \tag{39}$$

where $u(c, m)$ is linearly homogeneous, so that $u_{cc}u_{mm} - u_{cm}^2 = 0$, $v'(l) > 0$ and $v''(l) > 0$.

Using (38), the household-firm's flow budget constraint is given by (33), and the optimality conditions associated with the maximization problem become

$$\left\{ \begin{array}{ll} (1) & u_c(c, m) = \lambda, \\ (2) & u_m(c, m) = \lambda i, \\ (3) & v'(y) = \lambda (1 - \tau_y), \\ (4) & \dot{\lambda} = \lambda (\rho + \pi - i). \end{array} \right. \tag{40}$$

The government's budget constraint is given by (34). Fiscal policy is described by rule (35).

In equilibrium, conditions (40.1)-(40.3) can be expressed in implicit form as[10]

$$\left\{ \begin{array}{llll} (1) & y = y(i, \tau_y) & \text{with} & y_i < 0,\ y_{\tau_y} < 0, \\ (2) & m = m(i, \tau_y) & \text{with} & m_i < 0,\ m_{\tau_y} < 0, \\ (3) & \lambda = \lambda(i) & \text{with} & \lambda' < 0. \end{array} \right. \tag{41}$$

[10]For analytical details, see Appendix B.

Using (41.1), the fiscal policy rule takes the following form:

$$\tau_y y\left(i, \tau_y\right) = \bar{\alpha} + \alpha b. \tag{42}$$

Differentiating with respect to time yields

$$\dot{\tau}_y = \frac{\alpha}{y\left(1 - \eta_{y/\tau_y}\right)} \dot{b} - \frac{\tau_y y_i}{y\left(1 - \eta_{y/\tau_y}\right)} \dot{i}, \tag{43}$$

where $\eta_{y/\tau_y} = \left|\left(\tau_y/y\right) y_{\tau_y}\right|$ denotes the elasticity of output with respect to the marginal rate. We assume $\eta_{y/\tau_y} < 1$, i.e., that the economy is on the upward-sloping side of the Laffer curve, for it results to be efficient. Therefore, we can write

$$\tau_y = \tau\left(b, i\right) \quad \text{with} \quad \tau_b > 0, \tau_i > 0. \tag{44}$$

Dynamics can then be expressed in terms of the following differential-equation system:

$$\dot{\pi} = H\left(\pi\right)\left[\phi\left(\pi\right) - \pi - \rho\right], \tag{45}$$

$$\dot{b} = \frac{\left[\phi(\pi) - \pi - \alpha\right] b + g + \tau_h - \bar{\alpha} + K(\pi, b)\left[\phi(\pi) - \pi - \rho\right] - \pi m\left\{\phi(\pi), \tau\left[b, \phi(\pi)\right]\right\}}{1 + m_{\tau_y}\tau_b}, \tag{46}$$

where $K\left(\pi, b\right) = \lambda\left(m_i + m_{\tau_y}\tau_i\right)/\lambda' > 0$.

The steady-state solutions are given by

$$\phi\left(\pi^*\right) = \rho + \pi^*, \tag{47}$$

$$b^* = \frac{\bar{\alpha} - g - \tau_h + \pi^* m\left[\rho + \pi^*, \tau\left(b^*, \rho + \pi^*\right)\right]}{\rho - \alpha}. \tag{48}$$

It follows that

$$\begin{aligned} \left.\frac{\partial \dot{b}}{\partial b}\right|_{(\pi^*,\, b^*)} &= \rho - \alpha - \pi^* m_{\tau_y}\tau_b \\ &= \rho - \alpha\left[1 + \frac{\pi^* m_{\tau_y}}{y^*\left(1 - \eta^*_{y/\tau_y}\right)}\right], \end{aligned} \tag{49}$$

where we have used the fact that, from (44) evaluated at the steady state, $\tau_b = \alpha/y^*\left(1 - \eta^*_{y/\tau_y}\right)$.

To facilitate our discussion on dynamic stability, and make the present analysis easily comparable with the results that apply in the benchmark model of Section 2, let us restrict attention to the case in which

$$y^* \left(1 - \eta^*_{y/\tau_y}\right) > \left|\pi^* m_{\tau_y}\right|. \tag{50}$$

This condition simply says that the increase in fiscal revenues generated by an increase in the tax rate is greater than the decrease in inflation tax brought about by the associated fall in money demand. Therefore, *total* revenues, i.e., fiscal revenues plus inflation tax, are assumed to raise following an increase in the tax rate. If condition (50) holds, then a passive fiscal policy requires

$$\alpha > \frac{\rho}{1 + \pi^* m_{\tau_y} / y^* \left(1 - \eta^*_{y/\tau_y}\right)}. \tag{51}$$

Since $\pi^* m_{\tau_y} / y^* \left(1 - \eta^*_{y/\tau_y}\right) < 0$, the feedback parameter α must be greater than in the endowment-economy case. The reason is clear. An increase in public debt causes the tax rate to raise via the fiscal policy feedback rule. The increase in the tax rate brings about a decrease in output and hence in money demand. This crowds out inflation tax, thereby requiring a more aggressive reaction by the fiscal authority. The foregoing mechanism implies that the higher the elasticity of output with respect to the tax rate, the higher parameter α ensuring a passive fiscal policy, as it is apparent from (51).

The Jacobian is given by

$$J = \begin{bmatrix} H^* (\phi' - 1) & 0 \\ B_{21} & \rho - \alpha \left[1 + \frac{\pi^* m_{\tau_y}}{y^* \left(1 - \eta^*_{y/\tau_y}\right)}\right] \end{bmatrix}, \tag{52}$$

where

$$B_{21} = \frac{(b^* + K^*)(\phi' - 1) - m^* \left(1 - \eta^*_{m/\pi}\right) - \pi^* m_{\tau_y} \tau_i \phi'}{1 + m_{\tau_y} \tau_b}$$

does not affect the two eigenvalues of the matrix and hence the conditions for saddle-path stability. The latter occurs if the following condition applies:

$$\det J = H^* (\phi' - 1) - \rho - \alpha \left[1 + \frac{\pi^* m_{\tau_y}}{y^* \left(1 - \eta^*_{y/\tau_y} \right)} \right] < 0. \tag{53}$$

Condition (53) is verified either if

$$\alpha > \frac{\rho}{1 + \pi^* m_{\tau_y} / y^* \left(1 - \eta^*_{y/\tau_y} \right)} \qquad \text{and} \qquad \phi' > 1$$

or if

$$\alpha < \frac{\rho}{1 + \pi^* m_{\tau_y} / y^* \left(1 - \eta^*_{y/\tau_y} \right)} \qquad \text{and} \qquad \phi' < 1.$$

If fiscal policy is passive, i.e., $\alpha > \rho / \left[1 + \pi^* m_{\tau_y} / y^* \left(1 - \eta^*_{y/\tau_y} \right) \right]$, monetary policy independence is ensured.

However, for a given target inflation rate independently set by the monetary authority, the occurrence of Laffer-type effects poses a limit on the level of steady-state public debt. Let indicate it by b_l^{M}. We shall demonstrate that beyond such a limit, a passive fiscal policy becomes unfeasible.

To prove this result, first notice that in the steady state it must be that

$$\tau_y^* y \left(\rho + \pi^*, \tau_y^* \right) + \pi^* m \left(\rho + \pi^*, \tau_y^* \right) = \rho b^* + g + \tau_h. \tag{54}$$

It follows that

$$b_l^{\mathrm{M}} = \frac{\max\limits_{\tau_y^*} \left[\tau_y^* y \left(\rho + \pi^*, \tau_y^* \right) + \pi^* m \left(\rho + \pi^*, \tau_y^* \right) \right] - g - \tau_h}{\rho}. \tag{55}$$

Maximization of total revenues with respect to the tax rate occurs when

$$y^* \left(1 - \eta^*_{y/\tau_y} \right) = -\pi^* m_{\tau_y}. \tag{56}$$

Since $m_{\tau_y} < 0$, total revenues are maximized on the left-hand-side of the Laffer curve. This is precisely because, for a given target inflation rate, higher tax rates generate a negative spillover on inflation tax.

Now, substituting (56) into (49) yields

$$\left.\frac{\partial \dot{b}}{\partial b}\right|_{\left(\pi^*,\, b_l^{\mathrm{M}}\right)} = \rho. \tag{57}$$

This demonstrates our finding: if $b_0 > b_l^{\mathrm{M}}$, fiscal policy cannot be passive, for total revenues cannot be sufficient to reduce public debt over time.

Remarkably, the presence of Laffer effects on tax revenues causes the threshold level of public debt to be lower with respect to the endowment-economy case. That is, we have

$$b_l^{\mathrm{M}} < b_y^{\mathrm{M}}. \tag{58}$$

A central policy implication emerges. If $b_0 > b_l^{\mathrm{M}}$, the dynamics of public debt can be controlled *only* by means of inflation tax revenues. Monetary policy independence is no longer possible.

6 Maximum debt, inflation targeting, and the fiscal theory of the price level

From (55), b_l^{M} is a function of the target inflation rate π^*, $b_l^{\mathrm{M}} = b_l^{\mathrm{M}}(\pi^*)$. To study this function, we can apply the envelop theorem. We have

$$\begin{aligned} \frac{db_l^{\mathrm{M}}}{d\pi^*} &= \frac{\tau^{\mathrm{M}} y_i + m^* + \pi^* m_i}{\rho} \\ &= \frac{m^*}{\rho}\left(1 - \eta^*_{m/\pi} - \frac{\tau^{\mathrm{M}} y^*}{\pi^* m^*}\eta^*_{y/\pi}\right), \end{aligned} \tag{59}$$

where

$$\tau^{\mathrm{M}} = \arg\max_{\tau_y^*}\left[\tau_y^* y\left(\rho + \pi^*, \tau_y^*\right) + \pi^* m\left(\rho + \pi^*, \tau_y^*\right)\right].$$

From (59), $db_l^{\mathrm{M}}/d\pi^* > 0$ as long as $\eta^*_{m/\pi} - \left(\tau^{\mathrm{M}} y^*/\pi^* m^*\right)\eta^*_{y/\pi} < 1$. We let π^{M} be the value of the inflation rate such that $\eta^*_{m/\pi} - \left(\tau^{\mathrm{M}} y^*/\pi^* m^*\right)\eta^*_{y/\pi} = 1$, that is, $db_l^{\mathrm{M}}/d\pi^* = 0$.

Function $b_l^{\mathrm{M}}(\pi^*)$ is illustrated in Figure 3, and has the following interpretation. For $\pi^* = 0$, we have $\eta^*_{m/\pi} = \eta^*_{y/\pi} = 0$, so that $db_l^{\mathrm{M}}/d\pi^* = m^*/\rho > 0$. As long as π^* raises, both elasticities $\eta^*_{m/\pi}$ and $\eta^*_{y/\pi}$ increase. This is because

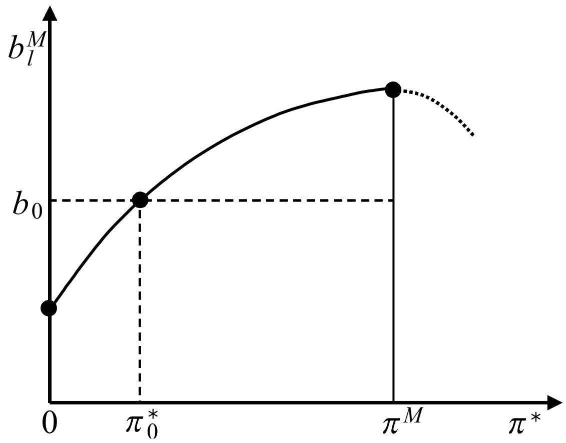

Figure 3: Maximum debt and inflation target

the increase in inflation causes the nominal interest rate to raise, leading to a fall in both money demand and output. As a result, total revenues, that is, fiscal revenues plus inflation tax, increase as long as $\pi^* < \pi^{\mathrm{M}}$, reach a maximum at $\pi^* = \pi^{\mathrm{M}}$, and decrease as along as $\pi^* > \pi^{\mathrm{M}}$. Two implications for the design of monetary policy rules arise.

First, if the monetary authority is intended to adopt the Taylor principle in order to maintain inflation control around the steady state, and at the same time avoid explosive paths in public debt, it *must* set an inflation target such that $\pi_0^* \leq \pi^* \leq \pi^{\mathrm{M}}$, thereby ensuring $b_0 \leq b_l^{\mathrm{M}}$.

Second, if the monetary authority sets a target inflation rate such that $\pi^* < \pi_0^*$, then we have $b_0 > b_l^{\mathrm{M}}$, and macroeconomic stability is guaranteed *only* by inflation dynamics along the lines of the fiscal theory of the price level. In fact, the Jacobian evaluated at $\left(\pi^*, b_l^{\mathrm{M}}\right)$ is given by

$$J_{\left(\pi^*,\, b_l^{\mathrm{M}}\right)} = \begin{bmatrix} H^*\left(\phi' - 1\right) & 0 \\ B_{21} & \rho \end{bmatrix}. \tag{60}$$

Saddle-path stability requires that monetary policy is passive, $\phi' < 1$. Violating the Taylor principle allows the inflation rate to jump up in order to rule out explosive dynamics in public debt. Nevertheless, in this second case the monetary authority clearly looses inflation control around the steady state.[11]

[11] Our results also suggest that when the level of public debt is quite high, an independent monetary policy may not be the best one because coordination issues between monetary

7 Conclusions

Distortionary taxation has negative consequences for monetary policy independence. It is well known, at least since the original contribution by Leeper (1991), that inflation control by an independent monetary authority requires a passive fiscal policy, ensuring stability of public debt for each time path of inflation. In this paper, we have demonstrated that when only distortionary revenue sources are available for the government, passive fiscal policies may not be feasible. This result comes about because of both households' participation constraints and Laffer-type effects. We have shown that there exists a threshold level of public debt beyond which monetary policy independence vanishes. We have examined the implications for policy design. The central bank can maintain inflation control around the steady state, by applying the Taylor principle, *provided* it targets a higher inflation rate. Otherwise, inflation *must* endogenously jump up in line with the fiscal theory of the price level.

and fiscal authorities may become relevant. Of course, when the two policies are coordinated, they must be designed according to welfare considerations. Welfare-maximizing monetary and fiscal policy rules in a model with sticky prices and distortionary taxation are studied by Schmitt-Grohé and Uribe (2007).

Appendix A

Consider the two optimality conditions (4.1) and (4.2). Differentiating with respect to time, recalling that $\dot{c} = 0$, we can write the results in matrix notation:

$$\begin{pmatrix} u_{cm} & -1 \\ u_{mm} & -i \end{pmatrix} \begin{pmatrix} \dot{m} \\ \dot{\lambda} \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ \dot{i} \end{pmatrix}. \tag{A.1}$$

Let $\Delta = u_{mm} - u_{cm} i < 0$. Then we have

$$\dot{m} = \frac{\lambda \begin{vmatrix} 0 & -1 \\ \dot{i} & -i \end{vmatrix}}{\Delta} = \frac{\lambda}{\Delta} \dot{i}, \tag{A.2}$$

$$\dot{\lambda} = \frac{\lambda \begin{vmatrix} u_{cm} & 0 \\ u_{mm} & \dot{i} \end{vmatrix}}{\Delta} = \frac{\lambda u_{cm}}{\Delta} \dot{i}. \tag{A.3}$$

We can thus write (10).

Appendix B

Consider the three optimality conditions (4.1)-(4.2). Differentiating with respect to time and imposing the goods' market equilibrium condition, we can express the results as

$$\begin{pmatrix} u_{cc} & u_{cm} & -1 \\ u_{cm} & u_{mm} & -i \\ v'' & 0 & -(1-\tau_y) \end{pmatrix} \begin{pmatrix} \dot{y} \\ \dot{m} \\ \dot{\lambda} \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ \dot{i} \\ -\dot{\tau}_y \end{pmatrix}. \tag{B.1}$$

Let $\Psi = v'' (u_{mm} - u_{cm} i) < 0$. Hence, we have

$$\begin{aligned} \dot{y} &= \frac{\lambda \begin{vmatrix} 0 & u_{cm} & -1 \\ \dot{i} & u_{mm} & -i \\ -\dot{\tau}_y & 0 & -(1-\tau_y) \end{vmatrix}}{\Psi} \\ &= \frac{\lambda (1-\tau_y) u_{cm}}{\Psi} \dot{i} - \frac{\lambda}{v''} \dot{\tau}_y, \end{aligned} \tag{B.2}$$

$$\dot{m} = \frac{\lambda \begin{vmatrix} u_{cc} & 0 & -1 \\ u_{cm} & \dot{i} & -i \\ v'' & -\dot{\tau}_y & -(1-\tau_y) \end{vmatrix}}{\Psi} = \frac{\lambda \left[v'' - (1-\tau_y)\, u_{cc}\right]}{\Psi}\dot{i} + \frac{\lambda \left(u_{cm} - u_{cc} i\right)}{\Psi}\dot{\tau}_y, \tag{B.3}$$

$$\dot{\lambda} = \frac{\lambda \begin{vmatrix} u_{cc} & u_{cm} & 0 \\ u_{cm} & u_{mm} & \dot{i} \\ v'' & 0 & -\dot{\tau}_y \end{vmatrix}}{\Psi} = \frac{\lambda v'' u_{cm}}{\Psi}\dot{i}. \tag{B.4}$$

We can thus write (41).

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