The influence of different forms of government spending on distribution and growth

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The influence of different forms of government spending on distribution and growth

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Abstract: This paper deals with the influence of different types of government expenditure on growth. It widens that proposed by the literature which follows the lines set by Barro (1990) because it adds the changes working through the demand side, generated by the variations in the distribution of the net income of the economy, to those working through the supply side, generated by the variations in factor productivity. The analysis considers a government sector with a balanced budget and an autonomous and nonlinear investment function, interpreted along a Kaleckian and a Classical-Harrodian line. It shows under which conditions different types of government expenditure are beneficial or detrimental for economic growth, comparing some results with those reached by Barro (1990) and points out the emergence of phenomena like multiple equilibria, hysteresis and low growth traps.

Key words: Distribution, Growth, Government expenditure, post-Keynesian theory, Nonlinearity

JEL Classifications: E12, E25, E62, O41

1. Introduction

This paper deals with the role of different types of government expenditure in post-Keynesian analysis. This subject has been largely overlooked by this tradition of thought, in spite of the attention paid to it by its founders. As Pressman (1994) notices, Keynes underlined that there are economic and political reasons for preferring certain kinds of expenditures to others. Kaldor (1958, pp. 136-137; 1966; 1967; and 1971) pointed out that the composition of government expenditure has important effects on long-run growth. For him, a large government consumption can transform the economy into one with low investment, with some undesirable consequences on its international competitiveness and long-run growth due to the fact that the capital goods sector tends to enjoy higher rates of variation in productivity than the consumption goods sector.

The views of Keynes and Kaldor, which provide interesting insights into the complexity of the growth processes, were presented in a descriptive way, nor have they been subsequently formalised by other authors. A formal treatment of the role of the government sector in the post-Keynesian theory of growth refers to one kind of expenditure without analysing the effects on the coefficients of production (see You and Dutt, 1996; Lavoie, 2000; Commendatore, Panico and Pinto, 2005).

A more detailed account of the influence of government expenditure on growth can be found in the literature that follows the lines set by Barro (1990). This author assumes that government expenditure enters the production function and is complementary with private inputs. It has two opposite effects on the rate of growth, one positive, working through the increase in the productivity of private capital, and one negative, working through the reduction of saving due to the variation in tax revenues. The economic mechanisms captured by this analysis only refer to the effects on the rate of growth emerging in the production or supply side of the economy. Those produced by the variations in income distribution and effective demand are absent.

In what follows an attempt is made to develop an analysis that also takes into account the influence on the rate of growth of the variations in income distribution and effective demand generated by changes in government expenditure. Like in Barro (1990), here too government expenditure enters the production function, even though not directly as an input, but indirectly by affecting the coefficients of production. Moreover, the increase in productivity does not necessarily lead to an increase in the rate of growth of the economy. In our analysis, however, this result is due to the assumption that government expenditure affects factor productivity and to other effects, generated by the variations in income distribution and effective demand, which are absent in Barro's analysis and can work in different ways. When government expenditure does not affect the coefficients of production (we define this kind of expenditure "unproductive")[1], its variations cause a transfer of income from the private to the government sector. When it affects the coefficients of production (we define this kind of expenditure "productive") its variations can also cause a re-distribution of income between profit-earners (or capitalists) and wage-earners (or workers), depending on how the increase in productivity is appropriated by the two groups. In both cases, a change in effective demand occurs because the propensity to consume of the profit-earners (or of the capitalist class) is smaller than the propensity to spend of the government sector and the propensity to consume of the wage-earners (or of the working class). This change in turn causes a variation in the rate of growth of the economy.

The analysis presented below develops a model that introduces a government sector, which works with a balanced budget, and an autonomous investment function, which is nonlinear. This second assumption allows the model to reproduce a variety of complex phenomena. Some of them, related to the analysis of what characterizes the equilibrium solutions, are examined in this paper. They clarify different aspects of how government intervention affects the rate of growth of the economy, underlining the possible occurrence of multiple equilibrium solutions, low growth traps and hysteresis. Some others, related to the analysis of the dynamic processes, are examined in a different essay (Commendatore, Panico and Pinto, 2009). They clarify the different ways in which government intervention can affect the stability of the economy, underlining the possible occurrence of regular and irregular growth cycles.

The model can be interpreted along Kaleckian and Classical-Harrodian lines. The first interpretation considers the state of long-term expectations of investors as exogenously given, driven for instance by entrepreneurs' animal spirits. The second considers that investors' expectations are related to the "warranted rate of growth", in the sense that the expected level of demand and output of the economy is the one corresponding to that rate.

[1] Devarajan, Swaroop and Zou (1996, pp. 316-317) points out that in some path-breaking empirical works of the 1980s and the early 1990s the distinction between "productive" and "unproductive" government expenditure was related to that between "capital" and "current" government expenditure. In the subsequent years the difference between productive and unproductive changed and became related to the ability of each type of expenditure to influence positively the rate of growth of the economy. Within this approach every kind of expenditure could be productive or unproductive depending on its share in total expenditure. In what follows we take a different line and define "productive" and "unproductive" expenditures according to their ability to affect productivity.

Both interpretations underline, in opposition to the literature following the lines set by Barro (1990), the role played by income distribution and the fact that the rate of growth crucially depends on the influence of government expenditure on after-tax profits. In the Kaleckian interpretation the rate of growth moves in the contrary direction to after-tax profits. In the Classical-Harrodian interpretation the opposite tendency occurs. The different result depends on the investment function.

In the Kaleckian interpretation proposed below the degree of capital utilization is what matters, so that, on account of the low propensity to consume of the profit-earners, effective demand rises when after-tax profits decrease, leading entrepreneurs to invest more. This expansionary influence on effective demand is able to produce structural breaks, movements away from " low growth traps" and hysteresis effects.

In the Classical-Harrodian interpretation a prominent role is played by the "warranted rate of growth", which is linked to saving in such a way as to establish a direct relationship between the rate of growth and after-tax profits. In this interpretation there are no structural breaks and hysteresis effects, but it is possible to identify a size of the government sector that maximizes the rate of growth. There is here some similarity with the results reached by Barro (1990), in the sense that the relationship between the rate of growth and the size of the government sector has the shape of a bell. In the Classical-Harrodian analysis however the shape of this relationship depends on the variations in the distribution of income within the economy, rather than on the assumption that the marginal productivity is decreasing.

Finally, both the Kaleckian and the Classical-Harrodian analyses make it possible to extend to an economy with the government sector some growth regimes identified by the post-Keynesian literature.

The paper is so organised. Section 2 presents the basic model. Section 3 deals with its Kaleckian interpretation showing how "productive" and "unproductive" government expenditures can affect the rate of growth within this framework. Section 4 deals with the Classical-Harrodian interpretation of the model and its results. Section 5 presents some conclusions.

2. The model

We consider a single-good closed economy with two inputs of production: labour (*L*) with a perfectly elastic supply and fixed capital (*K*) that does not depreciate. Technical progress is excluded and the production function is of a Leontief type. $1/a$ and $1/b$ represent the capital and labour coefficients, respectively. Moreover in each period the capital stock is not fully utilised and $u = Y/Y_P$, where u is the degree of capacity utilization, Y is the current output and Y_P is the potential output. Under these assumptions the following conditions hold:

$$Y = bL \le aK = Y_P$$

Income is distributed between wages and profits: $Y = wL + rK$, where w is the wage rate and r is the rate of profit. By normalising with respect to output we get $\pi + \omega = 1$, where π is the share of profits and ω is the share of wages in national income. Moreover, the following relationship between the rate of profit and capital utilisation holds:

$$r = \frac{\pi Y}{K} = \pi a u$$

The wage rate is a function of labour productivity and depends on the bargaining power of the unions:

$$w = w(b) = \overline{w} b^{\lambda} \quad with \quad \lambda \ge 0 \quad and \quad \overline{w} > 0 \tag{1}$$

where λ is the wage-productivity elasticity and measures the ability of the unions to capture labour productivity improvements. When $\lambda < 1$ workers only capture a portion of the increase in productivity; when instead $\lambda \ge 1$ the increase in the wage rate is equal or higher than the rate of variation of productivity.

The model assumes that workers do not save and the investment function is not linear:

$$s = s_{\pi} r(1-\tau) = s_{\pi}\pi(1-\tau)au \tag{2}$$

$$g = \alpha + \phi(u) \tag{3}$$

where s is the ratio between saving and capital, s_π is the propensity to save out of profits, τ is the tax rate and g is the rate of variation of capital, which depends on an autonomous term (α) and on a nonlinear component ($\phi(u)$) which enjoys the following properties:[2]

$$\phi(\tilde{u}) = 0, \quad \phi' > 0, \quad and \quad \phi'' \geq (<)\, 0 \quad for \quad u \leq (>)\, \tilde{u}$$

where $0 < \tilde{u} \leq 1$ is the "normal" degree of capacity utilization, interpreted as the optimal degree of capacity utilization given the existing technology.

Equation (3) assumes that investment is an "S-shaped" function of the degree of capacity utilisation (see Figures 1 and 4 below): when capacity utilisation is low the propensity to invest is weak, it improves when capacity utilisation rises, and slows down again when capital utilisation is high.[3]

The autonomous component of the investment function can be interpreted along two different lines. Following a Kaleckian interpretation, $\alpha = \overline{\alpha}$ reflects entrepreneurs' animal spirits and is taken as given like the state of long-term expectations in Keynesian models (see Rowthorn, 1981; Dutt, 1984; Amadeo, 1986; and Lavoie 1992). Along a Classical-Harrodian interpretation, $\alpha = \tilde{g}$, where $\tilde{g} = s_\pi \tilde{r}(1-\tau)$ is the warranted rate of growth and $\tilde{r} \equiv \pi a \tilde{u}$ is the rate of profit corresponding to normal capacity utilisation (for a similar interpretation, see Commendatore, D'Acunto, Panico and Pinto, 2003; and Shaikh, 2007).

The model further assumes that the government sector operates with a balanced budget:[4]

$$\tau = \gamma \tag{4}$$

where taxation and government expenditure γ are measured in terms of net income.

[2] For simplicity, we do not incorporate the rate of profit into the investment function. The introduction of this variable in the investment function does not qualitatively modify the results reached below.
[3] A nonlinear investment function, justified by the presence of increasing installation costs, of the increasing risk of entering new projects and by the difficulty of getting additional funding with the same net wealth, can be found in Kalecki (1937), Kaldor (1940) and Chang and Smith (1973). Recently, other works have used it (Bischi, Dieci, Rodano and Saltari, 2001; Gong, 2001; Bruno, 2005). In Commendatore (2006), a different nonlinearity, concerning firms' mark-up, is introduced in a Kaleckian model and in a model with Classical-Harrodian features.
[4] The effects of government deficits and debt on growth, assuming a linear investment function and "unproductive" government expenditure, are explored by You and Dutt (1990) and Lavoie (2000) in Kaleckian models and by Commendatore, Panico and Pinto (2005) in the post-Keynesian theory of growth and personal distribution.

Like Barro (1990, p. S107), we assume that the government enhances input productivity by purchasing goods and services that are freely provided to the private sector.[5] Unlike Barro, however, in our analysis the influence of government expenditure is described as a positive externality that changes the input coefficients. Moreover, we only focus on the case in which it influences b, the labour productivity:[6]

$$b = b(\gamma) \quad with \quad b(0) > 0, \quad b' \geq 0 \quad and \quad b'' \leq 0 \tag{5}$$

When government expenditure affects labour productivity, the wage share may vary too, depending on the bargaining power of the unions, i.e. on the value of λ. This assumption makes it possible, using equations (1), (4) and (5), to describe pre-tax profits as a function of government expenditure:

$$\pi = \pi(\gamma) = 1 - \bar{w}\left[b(\gamma)\right]^{\lambda-1} \tag{6}$$

where

$$\pi'(\gamma) = (1-\lambda)[1-\pi(\gamma)]b'(\gamma) \gtreqless 0 \quad \text{for} \quad \lambda \lesseqgtr 1 \tag{7}$$

If the wage rate increases less than labour productivity, pre-tax profits increase. The opposite occurs when the wage rate increases more than labour productivity.

The influence of government expenditure on after-tax profits, $\hat{\pi} = \pi(1-\gamma)$, can be described as follows:

$$\hat{\pi}' = \pi'(1-\gamma) - \pi \tag{8}$$

The sign of this derivative depends on b', the effect of government expenditure on labour productivity, and on λ, the elasticity of wages to changes in productivity. We identify three cases in which, for all values of γ, the sign is negative:

1. Government expenditure does not affect labour productivity, i.e. $b' = 0$, from which it follows $\pi' = 0$;

[5] Taking into account the existence of capital owned by the government sector would increase the complexity of the analysis. We leave this study to further research.
[6] The analysis of the influence of government expenditure on the productivity of capital is not qualitatively different from that developed here.

2. Government expenditure affects labour productivity, $b' > 0$, and wages rise at least at the same rate as productivity, $\lambda \geq 1$, which implies $\pi' \leq 0$, i.e. that pre-tax profits do not increase;
3. Government expenditure affects labour productivity, $b' > 0$, and wages rise less than productivity, $\lambda < 1$, but the increase in pre-tax profits, $\pi' > 0$, is not sufficient to counteract the rise in taxation. In this case, after-tax profits decrease, $\tilde{\pi}' < 0$.

The sign of this derivative is instead positive in one case: when $b' > 0$, $\lambda < 1$ and the increase in pre-tax profits more than compensate the rise in taxation. It follows the increase of after-tax profits, $\tilde{\pi}' > 0$. In this case, since $\pi'' < 0$, it is possible to identify a level of government expenditure, $0 < \gamma^m < 1$, which maximises after-tax profits and changes the sign of the derivative from positive to negative. Thus, the sign is positive only for $0 \leq \gamma < \gamma^m < 1$. When instead $\gamma > \gamma^m$, we have a fourth case in which after-tax profits decrease when government expenditure rises because the increase of pre-tax profits becomes again smaller than the rise in taxation.[7]

The equilibrium solutions for the degree of capacity utilisation, u^*, and for the rate of growth, g^*, are obtained from equations (2), (3) and (4) by taking γ as an independent variable and imposing the condition $g = s$.

The dynamic behaviour of the system is generated by the variation in the degree of capital utilisation in the face of a discrepancy between demand and supply, i.e. between investment and saving:

$$u_{+1} - u = \theta(g - s) \tag{9}$$

where $\theta > 0$ is the speed at which capacity utilisation adjusts to the discrepancy between saving and investment.

[7] Notice that when $b'>0$ and $\lambda<1$, two cases can occur: (i) after-tax profits decrease for any $0 \leq \gamma \leq 1$; or (ii) after tax-profits increase for $0 \leq \gamma < \gamma^m$ and decrease for $\gamma^m < \gamma \leq 1$. This is shown as follows: consider first that since $b'>0$, $b'' \leq 0$ and $\lambda<1$, it follows that $\pi'>0$ and $\pi''=-(1-\lambda)[\pi' b' - (1-\pi) b'']<0$. Moreover, by differentiating equation (8), we obtain $\tilde{\pi}''=\pi''(1-\gamma)-2\pi'$, which is negative for any $\gamma \leq 1$. If $\tilde{\pi}'(0)<0$, since $\tilde{\pi}''<0$, it follows that $\tilde{\pi}'<0$ for any $0 \leq \gamma \leq 1$. On the contrary, if $\tilde{\pi}'(0)>0$, since $\tilde{\pi}'(1)<0$ and $\tilde{\pi}''<0$ for $\gamma \leq 1$, there exists a unique value of γ, $0<\gamma^m<1$, such that $\tilde{\pi}' \gtreqless 0$ for $\gamma \lesseqgtr \gamma^m$.

In what follows we assume that $\theta < \theta^F \equiv 2\left[s_\pi \hat{\pi} a - \phi'(u^*)\right]^{-1}$. In this case, the equilibrium solutions are stable if the slope of the saving function is steeper than that of the investment function, i.e.

$$s_\pi \hat{\pi} a > \phi'(u^*) \tag{10}$$

Commendatore, Panico and Pinto (2009) abandons this assumption to analyze the dynamic processes generated by the model and the occurrence of regular and irregular cycles.

3. The Kaleckian interpretation

In the Kaleckian interpretation the autonomous component of the investment function represents entrepreneurs' animal spirits. With $\alpha = \overline{\alpha}$, the equilibrium solutions are:[8]

$$u^* = \frac{\overline{\alpha} + \phi(u^*)}{s_\pi \hat{\pi} a} \tag{11}$$

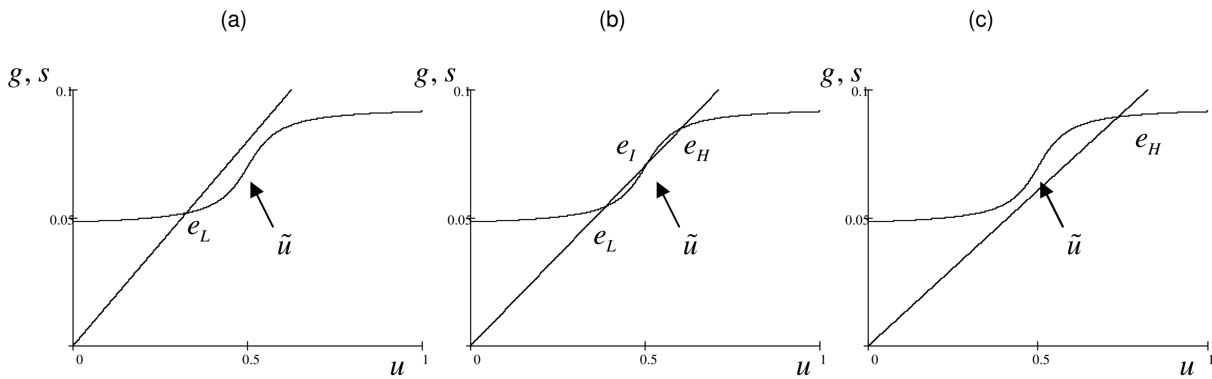
$$g^* = s_\pi \hat{\pi} a u^* \tag{12}$$

There can be up to three solutions depending on the value of γ. They are indentified in Figure 1 by $e_L \equiv \left(u_L^*, g_L^*\right)$, $e_I \equiv \left(u_I^*, g_I^*\right)$ and $e_H \equiv \left(u_H^*, g_H^*\right)$, the points of intersection between the saving and the investment functions.[9] The intermediate equilibrium solution e_I is unstable because it violates condition (10). For e_L and e_H, instead, the stability condition (10) holds. When three solutions exist, the economy converges either to the low or to the high equilibrium solution depending on the initial conditions: if $0 < u < u_I$ the economy converges towards e_L, if $u_I < u \leq 1$ towards e_H. The convergence towards e_L makes it possible to talk of a "low growth equilibrium trap".

[8] We exclude the possibility that demand exceeds the maximum level of capacity utilisation.

[9] In Figure 1 the saving and investment functions are plotted by taking α as given and assuming that γ takes the following values: (a) γ= 0, (b) γ= 0.12 and (c) γ= 0.24. The values of the other parameters are the following: $\tilde{u} = 0.5$, α= 0.07, a = 0.5, $\overline{w} = 0.6$ and $s_\pi = 0.8$. As an explicit form for the nonlinear component of the investment function we choose $\phi(u) = \beta_1 \arctan(\beta_2(u - \tilde{u}))$, where $\beta_1 = 0.015$ and $\beta_2 = 15$; and as an explicit form for the average labour productivity function we choose $b(\gamma) = b_0 + b_1 \arctan(b_2\gamma)$, where $b_0 = 1$, $b_1 = 0$ and $b_2 = 7.5$. For these values of b_0, b_1 and b_2 we have that $b' = 0$. For reasons of space we do not provide here a graphical representation where the values of the parameters allow $b' > 0$. This graphical representation would be similar to Figures 1(a)-(c).

Figure 1



The influence of government expenditure on the equilibrium solutions is described by the following derivatives :

$$\frac{du^*}{d\gamma} = -\frac{s_\pi a \hat{\pi}' u^*}{s_\pi \hat{\pi} a - \phi'(u^*)} \tag{13}$$

$$\frac{dg^*}{d\gamma} = -\frac{s_\pi a \hat{\pi}' u^*}{s_\pi \hat{\pi} a - \phi'(u^*)} \phi'(u^*) \tag{14}$$

For the stable equilibrium solutions e_L and e_H, the sign of the derivatives depends on the effect of government expenditure on after-tax profits. If $\hat{\pi}' < 0$ the sign of the two derivatives is positive: government expenditure has an expansionary effect on capital utilisation and growth. If $\hat{\pi}' > 0$ the opposite occurs.

As said in the previous section, $\hat{\pi}' < 0$ when the conditions described in the following Table hold:

Table 1

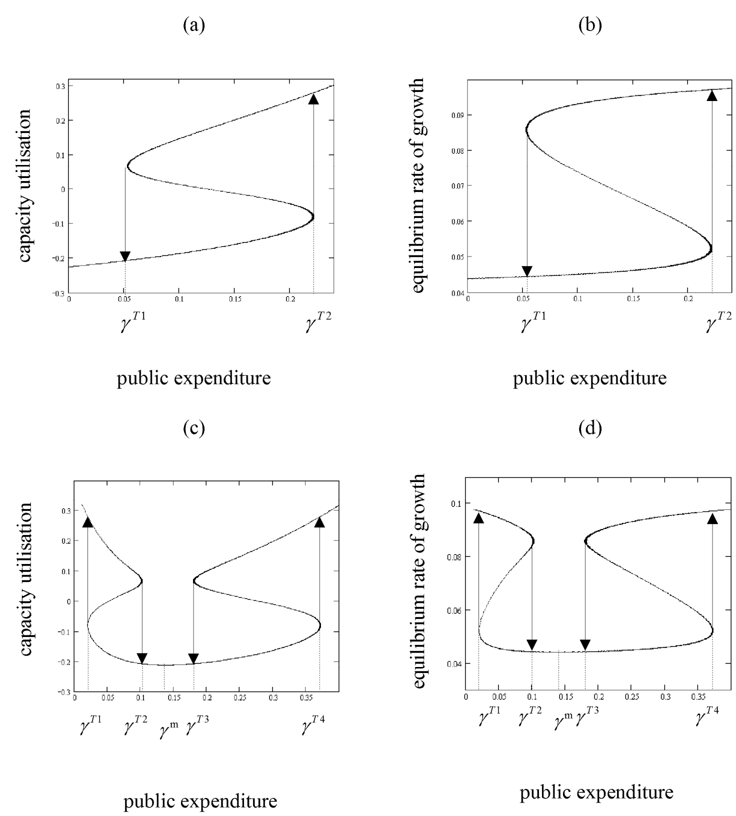
Case 1	$b' = 0$		$0 \le \gamma < 1$
Case 2	$b' > 0$	$\lambda \ge 1$ i.e. $\pi' \le 0$	$0 \le \gamma < 1$
Case 3	$b' > 0$	$\lambda < 1$ i.e. $\pi' > 0$	$0 \le \gamma < 1$
Case 4	$b' > 0$	$\lambda < 1$ i.e. $\pi' > 0$	$0 < \gamma^m < \gamma < 1$

The expansionary influence on capital utilisation and growth occurring in these cases extends the Kaleckian "paradox of costs" to the analysis of an economy in which the government sector plays an active role (see Rowthorn, 1981). The "paradox of costs" holds for the two stable equilibrium solutions, but not for the unstable intermediate solution e_I.

The results generated by the Kaleckian interpretation of our model can be further appreciated by considering three other graphical representations, in which the equilibrium values u^* and g^* are related to the values of γ.

Figures 2(a) and 2(b) describe the equilibrium values of u^* and g^* at different levels of γ when after-tax profits always decrease, $\hat{\pi}' < 0$.[10] In both figures, for $0 \leq \gamma < \gamma^{T1}$, there is one equilibrium solution, which corresponds to point e_L of Figure 1(a).

Figure 2



[10] Figures 2(a) and 2(b) assume $0 \leq \gamma \leq 0.24$ and, for the other parameters, the same values used in Figure 1. Notice too that in Figure 2 and in the following Figures 3, 4, 6 and 7, the vertical axis represents the difference between the equilibrium and the normal degree of capacity utilisation.

Both the degree of capacity utilisation and the rate of growth increase with government expenditure. For $\gamma^{T1} < \gamma < \gamma^{T2}$, owing to a "fold bifurcation", two new equilibrium solutions emerge (corresponding e_I and e_H of Figure 1(b)). The degree of capital utilisation and the rate of growth increase with γ when the equilibrium solutions e_L and e_H prevail. On the contrary they decrease when the equilibrium solution e_I prevails. Finally, for $\gamma > \gamma^{T2}$, owing to the occurrence of another "fold bifurcation", there is again one equilibrium solution, which corresponds to point e_H of Figure 1(c). Once again the degree of capacity utilisation and the rate of growth increase with government expenditure.

Figures 2(a) and 2(b) also show that for $0 \le \gamma < \gamma^{T2}$, if the low equilibrium solution prevails, an increase in government expenditure raises capital utilisation and growth. When γ overtakes γ^{T2} a structural break occurs, meaning that the economy shifts to the high equilibrium solution, where, owing to the shape of the equilibrium curves, it remains even if $\gamma^{T1} \le \gamma < \gamma^{T2}$. This result shows the existence of hysteresis effects.

Figures 2(c) and 2(d) describe the equilibrium values of u^* and g^* at different levels of γ when after-tax profits increase for $0 \le \gamma < \gamma^m < 1$ and decrease for $\gamma > \gamma^m$.[11] In the case $\tilde{\pi}' > 0$, for the stable equilibrium solutions, e_L and e_H, the degree of capital utilisation and the rate of growth decrease when government expenditure increases. On the contrary, if $\tilde{\pi}' < 0$, variations of government expenditure have a positive effect on the degree of capital utilisation and the rate of growth.

These results again confirm the validity of the "paradox of costs" for the stable equilibrium solutions. Moreover they confirm that in Kaleckian analysis growth is always driven by demand. It is driven by government expenditure when changes in this variable have a negative effect on after-tax profits. This is shown in Figures 2(a) and 2(b) for all values of public expenditure and in Figures 2(c) and 2(d) for $\gamma > \gamma^m$. When their changes have instead a positive effect on after-tax profits,

[11] Figures 2(c), 2(d), like Figures 3 and 4, are plotted for $0 \le \gamma \le 0.4$. The other parameters take the following values: $\tilde{u} = 0.5$, $\alpha = 0.07$, $a = 0.5$, $\overline{w} = 0.72$, $s_\pi = 0.8$, $\beta_1 = 0.02$, $\beta_2 = 15$, $b_0 = 1$, $b_1 = 1.15$ and $b_2 = 7.5$. Moreover, in Figures 2(c) and 2(d) $\lambda = 0.6$ and in Figure 4 $\lambda = 0.628$, whereas in Figure 3 the wage-productivity elasticity is varied within the interval $0.6 \le \lambda \le 0.85$.

growth is driven by the rise in the wage share generated by the reduction in the dimension of government intervention. This is shown in Figures 2(c) and 2(d) for $0 \leq \gamma < \gamma^m < 1$.

Figures 2(c) and 2(d) also show the existence of four bifurcation points, two of them, γ^{T1} and γ^{T2}, located at the left, the other two, γ^{T3} and γ^{T4}, located at the right of γ^m. A structural break occurs at each bifurcation point: the economy shifts either from high to low growth or from low to high growth, with associated hysteresis effects.

While in Figure 2 we assume that the wage-productivity elasticity is given, in Figure 3 we let λ vary to verify how different assumptions on workers' ability to appropriate changes in productivity influence the relationship between government expenditure, on the one side, and capacity utilisation and growth, on the other. An increase in λ implies a movement towards the origin of the axes and a consequent increase in the degree of capital utilisation and in the rate of growth. This too is a typical Kaleckian result: it confirms that an improvement in the wage share has a positive effect on growth.

Figure 3

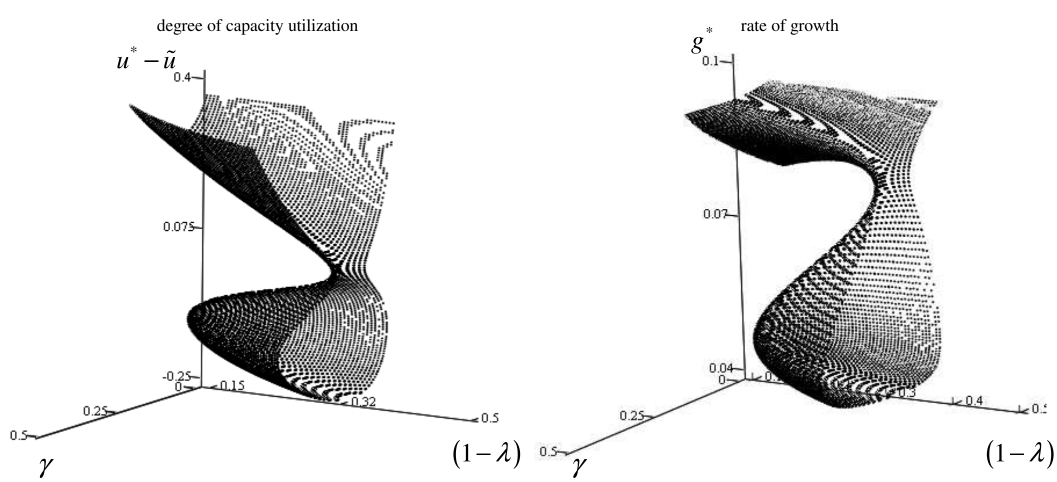
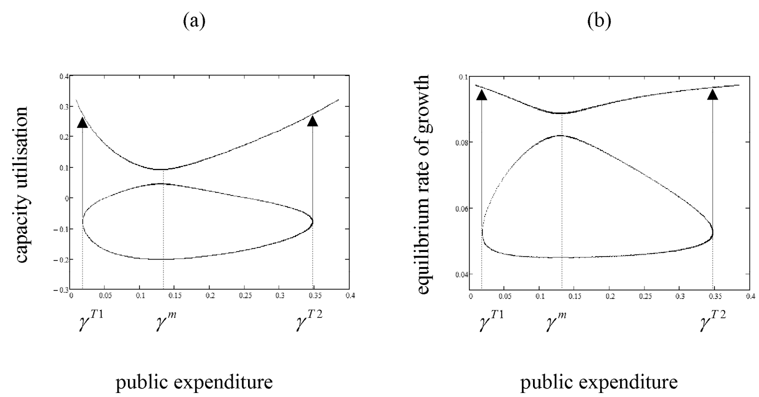


Figure 3 also clarifies that for values of λ close to 1, we get that variations in government expenditure only generate movements from low to high equilibrium solutions, provided that the former exist. On the contrary, for values of λ close to 0, unlike what happens in Figure 2(c) and 2(d), we only have two structural breaks and hysteresis effects generating a "high growth trap".

This result can be more clearly appreciated by looking at Figure 4, which is another section of Figure 3 for a value of λ higher than that assumed in Figure 2.

Figure 4



4. The Classical-Harrodian interpretation

In the Classical-Harrodian interpretation the expected rate of growth of demand is the warranted rate of growth, $\alpha = \tilde{g} = s_\pi \hat{\pi} a \tilde{u}$ and the solutions for u and g are

$$u^* = \tilde{u} + \frac{\phi(u^*)}{s_\pi \hat{\pi} a} \tag{15}$$

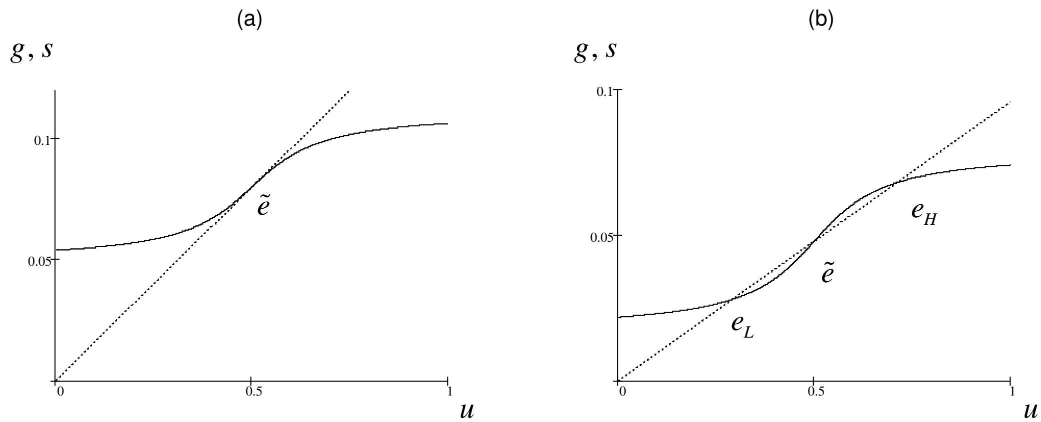
$$g^* = \tilde{g} + \phi(u^*) \tag{16}$$

There can be either one or three equilibrium solutions, denoted in Figure 5 by $\tilde{e} \equiv (\tilde{u}, \tilde{g})$, $e_L \equiv (u_L^*, g_L^*)$ and $e_H \equiv (u_H^*, g_H^*)$. The first solution corresponds to the normal capacity utilisation and to the warranted rate of growth. The other two solutions correspond to a level of capacity utilisation different from normal.[12] The number of solutions depends on the value of γ. In Figure 5(a), the equilibrium solution $\tilde{e}$ satisfies condition (10) and is globally stable. In Figure 5(b) there

[12] In Figure 5, γ takes the following values: (a) $\gamma = 0$ and (b) $\gamma = 0.4$; the value of the other parameters are $\tilde{u} = 0.5$, $a = 0.5$, $s_\pi = 0.8$, $\bar{w} = 0.6$, $\beta_1 = 0.02$, $\beta_2 = 7.5$, $b_0 = 1$, $b_1 = 0$ and $b_2 = 7.5$. Owing to $b_0 = 1$ and $b_1 = 0$, we have that $b' = 0$.

are three solutions. The one corresponding to normal capacity utilisation ($\tilde{e}$) is unstable; the other two (e_L and e_H) are symmetrical with respect to $\tilde{e}$ and are locally stable because condition (10) holds. The economy converges either to e_L or e_H depending on the initial value of *u*: if it is smaller than $\tilde{u}$, the adjustment process leads to the low equilibrium solution; otherwise, it leads to the high equilibrium solution.

Figure 5



The effects of government expenditure on the equilibrium degree of capacity utilization and rate of growth are described by the following derivatives:

$$\frac{du^*}{d\gamma} = -\frac{s_\pi a \hat{\pi}'}{s_\pi \hat{\pi} a - \phi'(u^*)}(u^* - \tilde{u}) \tag{17}$$

$$\frac{dg^*}{d\gamma} = s_\pi \hat{\pi}' a \tilde{u}\left(1 - \frac{u^* - \tilde{u}}{s_\pi \hat{\pi} a - \phi'(u^*)}\frac{\phi'(u^*)}{\tilde{u}}\right) \tag{18}$$

They show that the influence of government intervention on the economy is more complex than in the Kaleckian interpretation. The signs of the derivatives do not depend only on the influence on after-tax profits and the equilibrium degree of capital utilisation and rate of growth may change in opposite directions.

The sign of derivative (17) is positive for $u > \tilde{u}$ and negative for $u < \tilde{u}$ if $\hat{\pi}' < 0$: government expenditure has an expansionary effect on degree of capital utilisation when this is already above its normal value. The opposite occurs if $\hat{\pi}' > 0$, a case described by Figure 6(c) when $\gamma < \gamma^m$. The results of the Kaleckian interpretation presented above are thus confirmed when the economy works at a degree of capital utilization greater than normal.

The sign of derivative (18) is positive when $\hat{\pi}' > 0$ and $s_\pi \hat{\pi} a\tilde{u} > \phi'(u^*)u^*$. The expansionary influence of government expenditure depends on the relative weight of after-tax profits and the degree of capital utilisation on investment decisions. As a matter of fact, we can write the previous condition as follows, by noticing that $\partial g^* / \partial \hat{\pi} = s_\pi a\tilde{u}$ and $\partial g^* / \partial u^* = \phi'(u^*)$:

$$\eta_{\hat{\pi}} \equiv \frac{\partial g^*}{\partial \hat{\pi}} \frac{\hat{\pi}}{g^*} > \eta_{u^*} \equiv \frac{\partial g^*}{\partial u^*} \frac{u^*}{g^*} \tag{19}$$

where $\eta_{\hat{\pi}}$ is the elasticity of the rate of growth with respect to after-tax profits and η_u is the elasticity of the rate of growth with respect to the degree of capital utilisation. Condition (19) is always satisfied when the equilibrium solutions $\tilde{e}$ and e_L^* prevail. When the equilibrium solution e_H^* prevails, condition (19) does not necessarily hold.

Instead, if $\hat{\pi}' < 0$ the sign of derivative (18) is negative when $\eta_{\hat{\pi}} > \eta_{u^*}$. This condition again is satisfied for $\tilde{e}$ and e_L^*, whereas it does not necessarily hold for e_H^*. When the sign of derivative is positive the results of the Classical-Harrodian interpretation resemble those of the Kaleckian one, because the degree capital utilisation plays a more prominent role that after-tax profits in determining investment decisions.

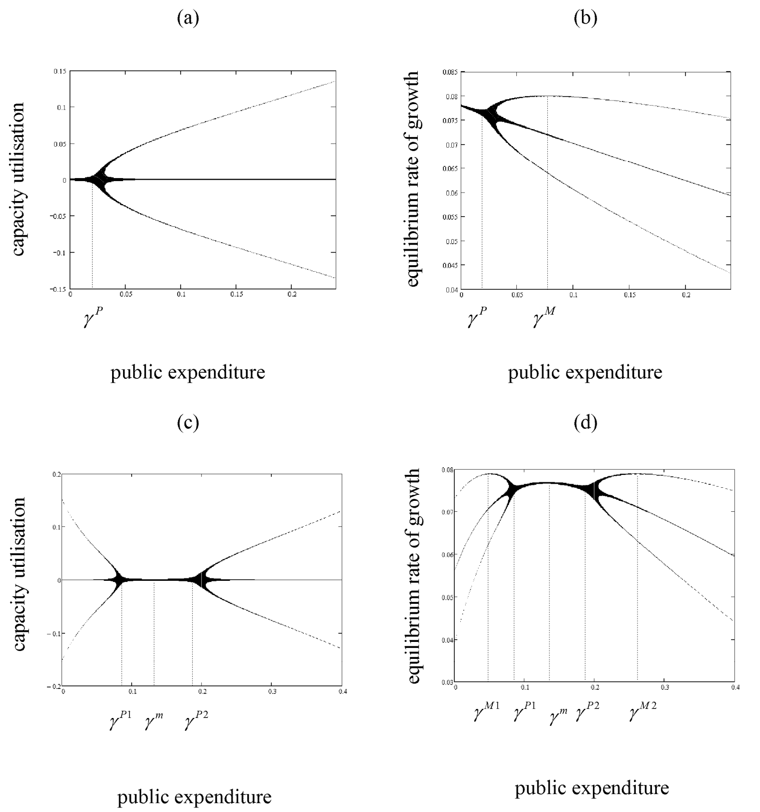
The analysis of Figures 6 and 7 further clarify the relationship between the size of the government sector and the equilibrium values of capital utilisation and the rate of growth.[13]

Figures 6(a) and 6(b) deal with the case of always decreasing after-tax profits, $\hat{\pi}' < 0$. For $0 \le \gamma < \gamma^P$ there is one equilibrium solution, as in Figure 5(a). This solution corresponds to the

[13] Figures 6(a) and 6(b) are plotted for $0 \le \gamma \le 0.24$ and, for the other parameters, the same values used in Figure 5. Figures 6(c), 6(d) and 7 are plotted for $0 \le \gamma \le 0.4$. The other parameters take the following values: $\tilde{u} = 0.5$, $a = 0.5$, $\bar{w} = 0.72$, $s_\pi = 0.8$, $\beta_1 = 0.02$, $\beta_2 = 7.5$, $b_0 = 1$, $b_1 = 1.15$ and $b_2 = 7.5$. Moreover, $\lambda = 0.6$ in Figures 6(c) and 6(d) whereas in Figure 7 the wage-productivity elasticity is varied within the interval $0.6 \le \lambda \le 0.85$.

warranted rate of growth $\tilde{e}$, which is asymptotically stable. When γ overtakes γ^P two new equilibrium solutions emerge owing to a pitchfork bifurcation. They correspond to e_L and e_H of Figure 5(b), which are symmetrical with respect to $\tilde{e}$ and locally stable, unlike $\tilde{e}$ that is now unstable.

Figure 6



Figures 6(a) and 6(b) also show that the relationship between the size of the government sector and u^* and g^* have different trends. In Figure 6(a) for $\gamma > \gamma^P$ the low equilibrium solution of the degree of capital utilisation is monotonically decreasing, the intermediate solution is constant and the high equilibrium solution is monotonically increasing. On the contrary, in Figure 6(b) for $\gamma > \gamma^P$ the low and the intermediate equilibrium solutions of the rate of growth are monotonically decreasing, while the high equilibrium solution is increasing for $\gamma < \gamma^M$ and decreasing for $\gamma > \gamma^M$. The value of government expenditure γ^M, which maximises the rate of growth, makes $\eta_{\tilde{\pi}} = \eta_{u^*}$.

The trends of the low and the intermediate equilibrium solutions in Figure 6(b) point out that the equilibrium rate of growth decreases when government expenditure increases and after-tax profits decrease. One can thus claim that there is a profit-led growth when the size of the government sector decreases. Moreover, for the low equilibrium solutions we have that both g^* and u^* move in the same direction as profits. Badhuri and Marglin (1990) define this regime "exhilarationist".

Profit-led growth also holds for the high equilibrium solution when $\gamma > \gamma^M$ and $\eta_{\bar{\pi}} > \eta_{u^*}$. In this case g^* and u^* move in the opposite direction, as occurs in the "conflictual stagnationist" regime, defined by Badhuri and Marglin (1990).

The occurrence of a profit-led regime for the stable equilibrium solutions differentiates the results of the Classical-Harrodian interpretation from those of the Kaleckian interpretation.

On the contrary, for $\gamma^P < \gamma < \gamma^M$, the equilibrium rate of growth increases when government expenditure increases and after-tax profits decrease. Growth is thus led by the government sector, with the further support of the wage share when government expenditure affects labour productivity and wages rise at least at the same rate as productivity. The opposite trends of the rate of growth and of after-tax profits make it possible to say that in this particular case, as in the Kaleckian analysis, the "paradox of costs" holds. Yet in all other cases the increase in the size of the government sector is not beneficial to growth.

Figures 6(a) and 6(b) also shows that, unlike what happens in the Kaleckian analysis, in the Classical-Harrodian interpretation there are no structural breaks and then no escape from "low growth traps" and no hysteresis effects.

Figures 6(c) and 6(d) deal with the case of after tax profits increasing when $0 \leq \gamma < \gamma^m < 1$ and decreasing when $\gamma > \gamma^m$. For $0 \leq \gamma < \gamma^{P1}$ there are three equilibrium solutions, as in Figure 5(b). When γ overtakes γ^{P1}, a pitchfork bifurcation occurs and there is one asymptotically stable equilibrium solution.

Figures 6(c) and 6(d) also show that the relationships between the size of the government sector and u^* and g^* have different trends. In Figure 6(c) the relationship between the size of the government sector and capital utilisation is increasing when the low equilibrium solution prevails, constant for the warranted equilibrium solution, and decreasing for the high one. Notice that for $\gamma > \gamma^m$ the increase in γ generates a reduction of after-tax profits ($\hat{\pi}' < 0$) as occurs in Figure 6(a), and the relationship between γ and u^* has the same trend as that described by that Figure.

Figure 6(d) shows that the relationship between the size of the government sector and the equilibrium rate of growth has a more complex behaviour than that between the size of the government sector and the degree of capital utilisation. The low equilibrium solutions show an increasing relationship for $\gamma < \gamma^m$ and a decreasing one for $\gamma > \gamma^m$. The trend of this relationship thus depends on the effect of government expenditure on after-tax profits, and is similar to that of the relationship between the size of government sector and the equilibrium degree of capital utilisation.

The trends of the warranted and the low equilibrium solutions are similar: they both depend on the influence of government expenditure on after-tax profits. The relationship has the shape of a bell and resembles what can be found in Barro (1990), without necessarily depending on the assumption that marginal productivity is decreasing. Growth is maximised when after-tax profits reach their maximum level, unlike what happens in Kaleckian analysis where growth reaches its minimum level when after-tax profits are maximised.

The high equilibrium solutions show that for $\gamma < \gamma^{M1}$ there is an increasing relationship of government expenditure with the equilibrium rate of growth and with after-tax profits. For $\gamma^{M1} < \gamma < \gamma^{P1}$, instead, the relationship of government expenditure with the equilibrium rate of growth is decreasing, while that with after-tax profits is increasing. In γ^{P1} a pitchfork bifurcation occurs and a unique equilibrium solution emerges. When $\gamma > \gamma^m$ the relationship between γ and g^* is similar to that described in Figure 6(b) as the increase in the size of the government sector generates a reduction in after-tax profits ($\hat{\pi}' < 0$).

The trends of the low and the warranted equilibrium solutions in Figure 6(d) point out that there is profit-led growth. For the low equilibrium solutions we have again an "exhilarationist" regime.

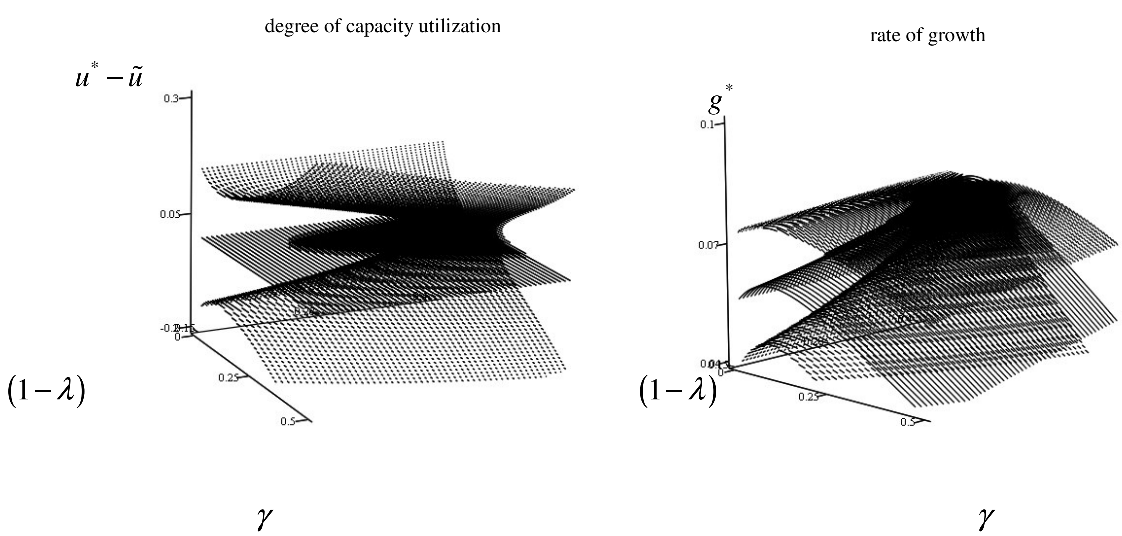
Profit-led growth also holds for the high equilibrium solutions when $\gamma < \gamma^{M1}$ and when $\gamma > \gamma^{M2}$. In both cases the elasticity of the rate of growth with respect to after-tax profits is greater than the elasticity with respect to the equilibrium degree of capital utilisation ($\eta_{\hat{\pi}} > \eta_{u^*}$). This result confirms once again the prominent role of the re-distributive effects in this analysis and indicates that for these values of γ a "conflictual stagnationist" regime prevails.

A different growth regime prevails for the other values of γ in the high equilibrium solutions. For $\gamma^{M1} < \gamma < \gamma^{P1}$ the equilibrium rate of growth decreases when government expenditure rises. When instead $\gamma^{P2} < \gamma < \gamma^{M2}$ a rise in the government expenditure has a positive effect on the equilibrium rate of growth. In both cases $\eta_{\tilde{\pi}} < \eta_{u^*}$ and the equilibrium rate of growth moves in the opposite direction of after-tax profits and in the same direction as the equilibrium degree of capital utilisation. Growth is wage-led for $\gamma^{M1} < \gamma < \gamma^{P1}$. It is driven instead by government expenditure, with the further support of the wage share, for $\gamma^{P2} < \gamma < \gamma^{M2}$. The "paradox of costs" holds in both cases, so that we have here the same result reached in the Kaleckian analysis presented above.

In Figures 6(c) and 6(d), like in Figures 6(a) and 6(b), there are no structural breaks and no hysteresis effects. None the less, the possibility that in some cases variations in γ generate a movement from an equilibrium growth path to another cannot be excluded.

Finally, to verify how different assumptions on workers' ability to appropriate changes in productivity influence the effects of government expenditure on capacity utilisation and growth, in Figure 7 we let the wage-productivity elasticity (λ) vary.

Figure 7



If it increases, we move towards the origin of the axis, where there are three equilibrium solutions for all values of γ and shifts from one equilibrium growth path to another, through changes in the size of the government sector, must be excluded.

To sum up, the results of the Classical-Harrodian interpretation tend to diverge from those of the Kaleckian one on account of the different role attributed to after-tax profits and to the degree of capital utilization in the investment function. The former interpretation emphasizes the role in investment decisions of the rate of profit associated with the "warranted rate of growth", whereas the latter considers the role of the rate of profit associated with a degree of capital utilization different from normal. Thus, in the Classical-Harrodian analysis the rate of capital accumulation tends to move in the contrary direction to that of the Kaleckian analysis, except in some particular cases, pointed out above, in which the degree of capital utilization can play a more prominent role than after-tax profits in affecting investment decisions.

The following Table summarises the results achieved in this Section, comparing them with those reached in the previous Section.

Table 2

	Kaleckian case		Classical/Harrodian case	
	u^*	g^*	u^*	g^*
$\hat{\pi}' < 0$	low: positive intermediate: negative high: positive	low: positive intermediate: negative high: positive	low: negative normal: no effect high: positive	low: negative normal: negative high: first positive then negative
$\hat{\pi}' > 0$	low: negative intermediate: positive high: negative	low: negative intermediate: positive high: negative	low: positive normal: no effect high: negative	low: positive normal: positive high: first positive then negative

The results also point out the existence of differences between the two interpretations as to the occurrence of structural breaks and hysteresis effects and as to the growth regimes that can emerge. With respect to the Kaleckian one, in the Classical-Harrodian analysis there are no structural breaks and no hysteresis effects. Moreover, a profit-led regime appears in addition to those emerging in the Kaleckian analysis.

Finally, the Classical-Harrodian analysis allows the identification of a size of the government sector that maximizes the rate of growth, as occurs in Barro (1990). This however depends on the influence of changes in government expenditure on income distribution and effective demand, which is overlooked by the literature following the lines set by Barro (1990).

5. Conclusions

The analysis presented above of the effects of the different types of government expenditure on growth widens that proposed by the literature that follows the lines set by Barro (1990). It adds the changes working through the demand side, generated by the variations in the distribution of the net income of the economy, to those working through the supply side, generated by the variations in factor productivity.

The analysis has considered a government sector with a balanced budget and an autonomous and nonlinear investment function, interpreted along a Kaleckian and a Classical-Harrodian line. It shows that for both interpretations the influence of government expenditure on the rate of growth depends on that on after-tax profits. Moreover, it points out that, due to their specification of the investment function, the two interpretations can generate different results, the most important of which are summarized below, recalling that for convenience of exposition we have defined "unproductive" the expenditure that does not affect labor productivity and "productive" the expenditure that has a positive effect on it:

- In the Kaleckian analysis, for both "productive" and "unproductive" government expenditure, the rate of growth moves in the contrary direction to after-tax profits, whereas in the Classical-Harrodian analysis the opposite tendency occurs.
- In the Kaleckian analysis the influence of "unproductive" expenditure is always beneficial to growth, whereas in the Classical-Harrodian analysis it tends to be detrimental.
- The influence of "productive" expenditure is more complex and tends to be different in the two analyses. None the less, when in the Classical-Harrodian analysis the degree of capital utilization achieves a more prominent role than after-tax profits in investment decisions, the influence of "productive" expenditure is the same as in the Kaleckian one.
- The Kaleckian analysis shows, for both kinds of expenditure, the occurrence of phemomena (structural breaks, movements away from the "low growth traps" and hysteresis effects), which do not emerge in the Classical-Harrodian one.

- Finally, when expenditure is "productive" it is possible to identify in the Classical-Harrodian analysis a size of the government sector which maximizes the rate of growth. This result resembles that reached by Barro (1990), but depends on the variations in the distribution of income within the economy, rather than on the assumption that marginal productivity is decreasing. Moreover, in the Classical-Harrodian analysis the maximum rate of growth is associated with the maximum level of after-tax profits, whereas in the Kaleckian analysis when after-tax profits reach their maximum level, the rate of growth is minimized.

The different results underline the complexity of the problem considered and the importance of investigating all forces at work.

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