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Comparing Cournot and Bertrand Competition in a Unionized Mixed Duopoly*

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Abstract

We investigate a differentiated mixed duopoly in which private and public firms can choose to strategically set prices or quantities by facing a union bargaining process. For the case of a unionized mixed duopoly, only the public firm is able to choose a type of contract irrespective of whether the goods are substitutes or complements in the equilibrium. Thus, we show that social welfare under Bertrand competition is always determined by the public firm's dominant strategy, wherein the Bertrand competition entails higher social welfare than the Cournot competition. Moreover, there are multiple Nash equilibria in the contract stage of the game. Finally, our main results hold irrespective of the nature of goods, with the exception of when a sufficiently large parameter of complements is employed, the ranking of private firm's profit is not reversed, which is contrast to the standard findings.

JEL: C7, D43, H44, J51, L13.

Keywords: Wage Bargaining, Union, Cournot-Bertrand Competition, Mixed Duopoly.

1 Introduction

For a pure duopoly¹, Singh and Vives (1984) were the first to show that Bertrand competition is more efficient than Cournot competition when goods are differentiated (see also Cheng (1985) and Okuguchi (1987)). Their study determined that Cournot equilibrium profits are greater than Bertrand equilibrium profits when goods are substitutes and vice versa when goods are complements. Moreover, they established that when private firms play the downstream duopoly game, the dominant strategy for private firms in a pure duopoly is to choose quantity contracts if goods are substitutes. Dastidar (1997), Qiu (1997), Lambertini (1997), Häckner (2000), and Amir and Jin (2001), among others, have analyzed counterexamples based on the framework of Singh and Vives (1984) by allowing for a wider range of cost and demand asymmetries². These works, which deal with the choice of strategic variables for prices or quantities, suggested important implications for the determination of market outcomes.

However, none of these papers have considered the case in which both private and public firms choose to set prices or quantities in a *unionized* mixed duopoly³. Thus, the present paper will be modeled around a noncooperative game in which the choice of strategic variables is set in a unionized mixed duopoly. There are several studies of mixed oligopolies where a public

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¹In this paper, in order to distinguish the concept of mixed duopoly, we use "*pure Cournot and Bertrand competition*" in a duopoly," when all firms are private firms.

²In particular, Zanchettin (2006) found that Singh and Vives' (1984) result that firms always make larger profits under quantity competition than under price competition fails to hold. Wang (2008) showed that while profit ranking between price and quantity competition can be partially reversed, the traditional result of Singh and Vives (1984) that firms always choose a quantity in a two-stage game continues to hold in the enlarged parameter space.

³Only one exception is, to the best of the author's knowledge, Ghosh and Mitra (2008), who compared Cournot with Bertrand competition in a mixed duopoly where an endogenous type of contract is not provided and there is no trade union in the mixed oligopoly. Later, we will mention several differences between their study and ours.

firm traditionally maximizes the social welfare, while the private firms compete with the public firm for maximizing their own profits⁴. In this paper, we address the issue of whether or not the standard results of the ranking of Cournot- and Bertrand-equilibrium outcomes under a differentiated mixed duopoly hold up where the public firm and the private firm are unionized. More specifically, we illustrate the way in which the choice of strategic variables for setting prices or quantities affects social welfare in a unionized mixed duopoly. The papers that are closest to the present model of unionized mixed duopolies are authored by De Fraja (1993), Haskel and Sanchis (1995), Haskel and Szymanski (1993), and Ishida and Matsushima (2009).

There also have been some attempts, namely, López (2007) and López and Naylor (2004), to introduce union utility into a model of the choice of strategic variables for setting prices or quantities. López and Naylor (2004) compared pure Cournot and Bertrand equilibria in a downstream differentiated duopoly in which wages are paid by each downstream player as per outcome of a strategic bargain with its upstream labor-union. They showed that Singh and Vives' (1984) result holds unless unions are powerful and place considerable weight on the wage argument in their utility function. Since there can exist a dominant strategy for *each private firm* in a pure duopoly when they choose either quantity or price contract depending on unions' bargaining power and weight on the wage argument, López (2007) analyzed the more general case of a profit-maximizing upstream player that sells input to a duopoly in exchange for a negotiated input-price⁵. However, recent studies in the literature have not investigated the issue of how private and public firms play the noncooperative game of choosing strategic variables in the context of wage bargaining in a mixed duopoly. Consequently, our paper differs from previous works on unionized mixed oligopolies that focused on privatization without public and private firms' choices of strategic variables. Furthermore, this paper analyzes the robustness of either Singh and Vives (1984) or the López (2007) and López and Naylor (2004) considering the framework of the unionized mixed duopoly.

The timing of the game of the present paper is as follows. In the first stage, the private firm and the public firm simultaneously commit to choosing a strategic variable for either the price or the quantity (these are the two relevant types of contract) that is to be set in the unionized mixed duopoly. In the second stage, each union independently bargains over its wages, keeping in mind each strategic variable of the private and public firms. In the third stage, each firm chooses its quantity or price simultaneously with the other firms in order to maximize its objective given its knowledge of the strategic variable of the public and private firms and of the wage levels in previous stages.

Given this three-stage game model, we demonstrate that the public firm will always opt for a Bertrand type of contract and the private firm will choose either a Bertrand or a Cournot type of contract, regardless of whether the goods are substitutes or complements in the first stage. This

⁴See De Fraja and Delbono (1990) and Nett (1993) for general reviews of mixed oligopoly models. For recent literature on mixed oligopolies, see Barcena-Ruiz (2007), Matsumura (1998), Matsumura and Kanda (2005), Matsumura and Matsushima (2004, 2006), Lu and Poddar (2006), etc.

⁵On the other hand, Manasakis and Vlassis (2006) extended Singh and Vives' (1984) framework by assuming no ex-ante commitment over the type of contract that each firm will offer consumers. In the context of a unionized symmetric duopoly, they argued that the mode of competition in which equilibrium emerges is the one that entails the most beneficial outcome for both the firm and its labor union, given the choice of the rival firm/union pair. In addition, motivated by the institutional diversity of unionization structures and the growth of foreign direct investment (FDI), the current bargaining process between firms and unions has been developed independently. As identified by Naylor (1998, 1999), Zhao (1995), Skaksen and Sørensen (2001), Haucap and Wey (2004), and Leahy and Montagna (2000), the amount of domestic production is decided through union bargaining when a firm undertakes FDI. In another related paper, the relationship between the amount of production and the union has been explored among domestic private firms (Naylor, 2002). Many studies have considered a unionized international oligopoly; see for instance, Straume (2003), Ishida and Matsushima (2008), Mukherjee and Suetrong (2007), and the references therein.

is because only the public firm is able to choose the type of contract irrespective of whether the goods are substitutes or complements. This effect allows the public firm to employ a strategic commitment with respect to the contract choice (i.e., Bertrand competition), thereby improving social welfare when all firms choose different types of contracts. This result, when the choice of strategic variables is set in a unionized mixed duopoly, differs from the standard findings of Singh and Vives (1984), López (2007), and López and Naylor (2004). However, as Singh and Vives (1984) demonstrated, the outputs as well as social welfare of both public and private firms under the Bertrand competition are higher than those under the Cournot competition. In addition, the main results of this study hold irrespective of the nature of goods, with the exception of when a sufficiently large parameter of complements is employed. In this case, the ranking of a private firm's profit is not reversed, which is contrast to the standard findings of Singh and Vives (1984), López (2007), and López and Naylor (2004). This is because of the dominant strategy in which only the public firm will choose the Bertrand contract, regardless of the nature of the goods; it should be noted that there is no dominant strategy for the private firm.

The main result in the present paper contrasts with Singh and Vives' (1984) conclusion in which a dominant strategy for the private firms in a pure duopoly is to choose the quantity contract if goods are substitutes and vice versa. This occurs because we endogenously treat the type of contract as it exists in a unionized mixed duopoly. That is, since the cost of production is endogenously determined by the introduction of trade unions, it is necessary to incorporate the role of unions to derive these results in the case of a mixed duopoly: Due to the imposition of the budget constraint on the public firm, the public and private firms' profits and unions' utilities cancel each other in the social welfare function. Hence, the public firm's objective function is endogenously determined by a representative consumer's utility.

To the best of the author's knowledge, only one study has attempted to compare Bertrand and Cournot outcomes in a mixed oligopoly when the goods are substitutes: Ghosh and Mitra (2008). More specifically, Ghosh and Mitra (2008) derived their results by comparing Cournot and Bertrand competition in a mixed oligopoly wherein the endogenous type of contract is not determined by the public firm and there is no trade union. Hence, Ghosh and Mitra (2008) demonstrated that the rankings of the profit and social welfare of firms is exogenously determined. The theoretical results of the present study, however, consider the problem in a differentiated mixed duopoly in which both private and public firms can choose to strategically set prices or quantities by facing a union bargaining process. Therefore, this paper differs from the existing literature in at least two important aspects. First, the existing studies on mixed oligopolies consider an exogenous type of contract rather than an endogenous one. Second, while previous works have focused on reverse results with regard to the Cournot-Bertrand profit differential in a pure oligopoly market, this paper not only investigates a case in which both private and public firms choose to set prices or quantities regardless of the nature of goods but also determines how social welfare is affected by the type of contract structure.

The organization of the paper is as follows. In Section 2, we describe the model. Section 3 presents fixed-timing games regarding the type of contract. Section 4 determines firms' endogenous choices of strategic variables. Concluding remarks appear in Section 5.

2 The Basic Model

The basic structure is a differentiated duopoly model, which is a simplified version of Singh and Vives' model (1984). Consider two single-product firms that produce differentiated products that are supplied by a public firm (firm 0) and a private firm (firm 1). We assume that the

representative consumer's utility is a quadratic function given by

$$U = x_i + x_j - \frac{1}{2}(x_i^2 + 2cx_ix_j + x_j^2),$$

where x_i denotes the output of firm i ($i = 0, 1$). The parameter $c \in (0, 1)$ is a measure of the degree of substitutability among goods, while a negative $c \in (-1, 0)$ implies that goods are complements. In the main body of analysis we focus on the imperfect substitutability case of $c \in (0, 1)$. However, we will mention later imperfect complementarity case of $c \in (-1, 0)$ since it is easy to calculate when goods are complements⁶. Thus, the inverse demand is characterized by

$$p_i = 1 - cx_j - x_i; \quad i \neq j, i = j = 0, 1, \quad (1)$$

where p_i is firm i 's market price and x_i denotes the output of firm i ($i = 0, 1$). Hence, we can obtain the direct demands as

$$x_i = \frac{(1 - c) + cp_j - p_i}{1 - c^2} \quad (2)$$

provided the quantities are positive.

To analyze the union's wage bargaining, we also assume that the public and private firms are unionized and that the wages, w_i are determined as a consequence of bargaining between firms and unions. Let \bar{w} denote the reservation wage. Thus, we assume that the union sets the wage while public and private firms unilaterally decide the level of employment. Taking \bar{w} as given, the union's optimal wage-setting strategy, w_i , regarding firm i is defined as:

$$u_i = (w_i - \bar{w})^\theta x_i; \quad i = 0, 1, \quad (3)$$

where θ is the weight that the union attaches to the wage level. As suggested by Haucap and Wey (2004), Leahy and Montagna (2000), and Lommerud *et al.* (2003), for simplicity of exposition, we assume that the union possesses full bargaining power (i.e., $\theta = 1$) for the wage level and $\bar{w} = 0$ to show our results in the simple way⁷.

The firms are homogeneous with respect to productivity. Each firm adopts a constant returns-to-scale technology where one unit of labor is turned into one unit of the final good. The price of labor (i.e., wage) that firm i has to pay is denoted by w_i , $i = 0, 1$.

To specify the public firm 0's objective function SW , and each firm's profit π_i , as

$$SW = U - \sum_{i=0}^1 px_i + \sum_{i=0}^1 (\pi_i + u_i),$$

$$\pi_i = (p_i - w_i)x_i, \quad i = 0, 1,$$

where $U - \sum_{i=0}^1 px_i$ is the consumer surplus, and each firm π_i is the profit of both the private and public firm, and u_i is the union's utility for both the private and public firm. The objective function of the public firm is the sum of consumer surplus, profit of all firms and the union's utility for all the firms.

⁶The detailed computations of complements are available from author upon request; The Appendix C will not be included in the main paper since the inference can be easily verified by reversing signs of c . Furthermore, we exclude independent case since the choice of strategic variables for setting prices or quantities does not change social welfare in a unionized mixed duopoly.

⁷As Naylor (1998, 1999), Haucap and Wey (2004), Leahy and Montagna (2000), and Lommerud *et al.* (2003) suggested, this is because wage claims are decided by the elasticity of labor demand rather than the firm's profit. See also Oswald and Turnbull (1985).

This study considers that each firm can make two types of binding contracts with consumers, as described by Singh and Vives (1984) and López (2007). Thus, a three-stage game is conducted. The timing of the game is as follows. In the first stage, the private and public firms simultaneously commit to choosing strategic variable, i.e., either price or quantity (which determines the type of contract), to set in the unionized mixed duopoly. In the second stage, each union independently bargains over its wage, $w_i, i = 0, 1$, keeping in mind each strategic variable of the private and public firms. In the third stage, each firm chooses its quantity or price simultaneously, in order to maximize its objective knowledge of the strategic variable of the public and private firms and of the wage levels in previous stages. As in Singh and Vives (1984) and López (2007), we adopt the same assumption. i.e., that there are prohibitively high costs associated with changing the type of contract in the first stage.

3 Bargaining in a Mixed Duopoly

3.1 Results: Fixed Contract Motives with Solutions of Substitutes

Before the use of the type of contract for applying in the model and identifying the point of equilibrium, four different cases of contract games are explained. In Bertrand competition, firms set prices, whereas in Cournot competition, firms set quantities. In mixed cases, firm 0 sets the price and firm 1 sets the quantity, and vice versa. Such game is solved by backward induction, i.e., the solution concept used is the subgame perfect Nash equilibrium (SPNE).

3.1.1. [Competition Game in Cournot]: Assume that firm $i(i = 0, 1)$ faces the inverse demand functions given by $p_i = 1 - cx_j - x_i$. In the third stage, the public firm's objective is to maximize the social welfare which is defined as the sum of consumer surplus, each firm's profit, and each union's utility:

$$SW = U - p_0x_0 - p_1x_1 + \pi_1 + u_1 + \pi_0 + u_0 = U.$$

Given w_1 for the private firm (firm 1), the public firm's maximization problem is as follows:

$$\max_{x_0} SW = U \quad \text{s.t.} \quad (p_0 - w_0)x_0 \geq 0.$$

The constraint implies that there is some lower-bound restriction on the public firm's profit, i.e., the public firm faces a budget constraint⁸. Therefore, since we assume that each firm's output is a positive value as in (2), we can rewritten the budget constraint as follows: $(p_0 - w_0)x_0 \geq 0 \Leftrightarrow 1 - cx_1 - x_0 - w_0 \geq 0$.

Denoting the multiplier of the budget constraint λ^{cc} , the Lagrangian equation can be written as

$$L = x_1 + x_0 - \frac{(x_1^2 + x_0^2 + 2cx_0x_1)}{2} + \lambda^{cc}(1 - x_0 - cx_1 - w_0).$$

Given wage level w_0 in the third stage, the first-order conditions are given by

$$\frac{\partial L}{\partial x_0} = 1 - cx_1 - x_0 - \lambda^{cc} = 0 \tag{4}$$

$$\frac{\partial L}{\partial \lambda^{cc}} = 1 - cx_1 - x_0 - w_0 = 0. \tag{5}$$

⁸If both unions aim to simultaneously maximize the wage-level and the public firm's union does not face the budget constraint with a simple Stone-Geary utility function $u_i = (w_i - \bar{w})^\theta x_i$, the public firm's union can limitlessly raise its wage because the optimal output level of the public firm is independent of the wage.

On the other hand, the first-order condition for the private firm is given by

$$\frac{\partial \pi_1}{\partial x_1} = 0 \Leftrightarrow x_1 = \frac{1}{2}(1 - cx_0 - w_1). \quad (6)$$

Solving the first-order conditions (5) and (6), we obtain,

$$x_0 = \frac{2 - c - 2w_0 + cw_1}{2 - c^2}, \quad (7)$$

$$x_1 = \frac{1 - c - w_1 + cw_0}{2 - c^2}, \quad (8)$$

$$\lambda^{cc} = 1 - cx_1 - x_0. \quad (9)$$

To solve for Lagrangian equation, the budget constraint is momentarily binding. We check ex-post that the omitted this constraint is binding.

In the second stage of this case, each wage is set to maximize its firm's union utility: $u_i = x_i w_i$. To do this, the two independent maximization problems should be considered simultaneously. Using (7) and (8), the problem for union i is defined as

$$\begin{aligned} \max_{w_0} u_0 = w_0 x_0 &= \frac{w_0(2 - c - 2w_0 + cw_1)}{2 - c^2}, \\ \max_{w_1} u_1 = w_1 x_1 &= \frac{w_1(1 - c - w_1 + cw_0)}{2 - c^2}. \end{aligned}$$

This implies the following first-order condition

$$w_0 = \frac{2 - c + cw_1}{4}, \quad w_1 = \frac{1 - c + cw_0}{2}. \quad (10)$$

Straightforward computation yields each an equilibrium wage, denoted as w_i^{cc} is obtained by solving (10), and substituting w_i^{cc} into (7) and (8) yields the equilibrium output x_i^{cc} . Thus, we have the following result which is the same results as in Ishida and Matsushima (2009).

Lemma 1 (Ishida and Matsushima, 2009): *Suppose that goods are substitutes and each firm's union is allowed to engage in decentralized bargaining. Then, the equilibrium wage, output and price levels are given by*

$$\begin{aligned} w_0^{cc} &= \frac{4 - c - c^2}{8 - c^2}, & w_1^{cc} &= \frac{4 - 2c - c^2}{8 - c^2}; \\ x_0^{cc} &= \frac{2(4 - c - c^2)}{(2 - c^2)(8 - c^2)}, & x_1^{cc} &= \frac{4 - 2c - c^2}{(2 - c^2)(8 - c^2)}; \\ p_0^{cc} = w_0^{cc} &= \frac{4 - c - c^2}{(8 - c^2)}, & p_1^{cc} &= \frac{12 - 6c - 7c^2 + 2c^3 + c^4}{(2 - c^2)(8 - c^2)}. \end{aligned}$$

This Lemma 1 suggests that the budget constraint is binding. That is, substituting Lemma 1 into (9) then we have

$$\lambda^{cc} = \frac{8 - 2c - 6c^2 + c^3 + c^4}{(2 - c^2)(8 - c^2)} > 0$$

which shows that the public firm sets the output that yields zero profit in equilibrium.

Finally, noting that $SW^{cc} = U^{cc}$ and π_1^{cc} , we can compute the social welfare and private firm's profit, SW^{cc} and π_1^{cc} as follows;

$$SW^{cc} = \frac{304 - 144c - 256c^2 + 92c^3 + 67c^4 - 12c^5 - 6c^6}{2(8 - c^2)^2(2 - c^2)^2}, \quad (11)$$

$$\pi_1^{cc} = \frac{(4 - 2c - c^2)^2}{(8 - c^2)^2(2 - c^2)^2}. \quad (12)$$

3.1.2. [Competition Game in Bertrand]; Consider that firm i faces the following direct demand function

$$x_i = \frac{(1 - c) + cp_j - p_i}{1 - c^2}.$$

In the third stage, by maximization social welfare (respectively, profit) each firm sets its price as a best response to any price chosen by its private firm (respectively, the public firm). The public firm's objective is given as in the previous case as follows:

$$\begin{aligned} \max_{p_0} U &= \frac{(1 - c) + cp_1 - p_0}{1 - c^2} + \frac{(1 - c) + cp_0 - p_1}{1 - c^2} \\ &\quad - \frac{1}{2} \left\{ \frac{[(1 - c) + cp_1 - p_0]^2 + [(1 - c) + cp_0 - p_1]^2 + 2c[(1 - c) + cp_0 - p_1][(1 - c) + cp_1 - p_0]}{(1 - c^2)^2} \right\} \\ \text{s.t.} \quad &\frac{(p_0 - w_0)[(1 - c) + cp_1 - p_0]}{1 - c^2} \geq 0. \end{aligned}$$

Similar to the budget constraint of 3.1.1. [Competition Game in Cournot], we can rewritten the budget constraint as follows: $p_0 - w_0 \geq 0$. Denoting the multiplier of the budget constraint λ^{bb} and repeating the same process as in Competition Game in Cournot yields the first-order conditions of the Lagrangian equation with respect to λ^{bb} and p_0 :

$$\frac{\partial L}{\partial p_0} = 0 \Leftrightarrow \lambda^{bb} = \frac{-(cp_1 - p_0)}{1 - c^2}, \quad (13)$$

$$\frac{\partial L}{\partial \lambda^{bb}} = p_0 - w_0 = 0. \quad (14)$$

On the other hand, the first-order condition for the private firm is given by

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Leftrightarrow p_1 = \frac{1 - c + cp_0 + w_1}{2}. \quad (15)$$

By using x_i and solving the these two equations (14) and (15) problems yields

$$x_0 = \frac{(1 - c)(2 + c) + cw_1 - (2 - c^2)w_0}{2(1 - c^2)}, \quad (16)$$

$$x_1 = \frac{1 - c - w_1 + cw_0}{2(1 - c^2)}. \quad (17)$$

In the second stage of this case, each wage is set to maximize the its own firm's union: $u_i = x_i w_i$. In the analysis that follows, we again focus on the union's maximization problem.

Using (16) and (17), the problems for union i are defined as

$$\max_{w_0} u_0 = w_0 x_0 = \frac{w_0[(1-c)(2+c) + cw_1 - (2-c^2)w_0]}{2(1-c^2)}, \quad (18)$$

$$\max_{w_i} u_1 = w_1 x_1 = \frac{w_1(1-c-w_1+cw_0)}{2(1-c^2)}. \quad (19)$$

The best reply functions for the public firm 0 and the private firm 1 are $w_0 = \frac{(1-c)(2+c)+cw_1}{2(2-c^2)}$ and $w_1 = \frac{1-c+cw_0}{2}$, respectively. Thus, straightforward computation yields each an equilibrium wage, denoted as w_i^{bb} is obtained by maximizing (18) and (19), and substituting w_i^{bb} into (16) and (17) yields the equilibrium output x_i^{bb} and price p_i^{bb} . Thus, we have the following result.

Lemma 2: *Suppose that goods are substitutes and each firm's union is allowed to engage in decentralized bargaining. Then, the equilibrium wage, output and price levels are given by*

$$\begin{aligned} w_0^{bb} &= \frac{4-c-3c^2}{8-5c^2}, & w_1^{bb} &= \frac{4-2c-3c^2+c^3}{8-5c^2}; \\ x_0^{bb} &= \frac{8-2c-10c^2+c^3+3c^4}{2(1-c^2)(8-5c^2)}, & x_1^{bb} &= \frac{4-2c-3c^2+c^3}{2(1-c^2)(8-5c^2)}; \\ p_0^{bb} &= \frac{4-c-3c^2}{8-5c^2}, & p_1^{bb} &= \frac{12-6c-9c^2+3c^3}{2(8-5c^2)}. \end{aligned}$$

Substituting Lemma 2 into (13) then we have

$$\lambda^{bb} = \frac{8-14c+9c^3-3c^4}{2(8-5c^2)(1-c^2)} > 0 \quad \text{if } c < 0.90,$$

which shows that the budget constraint is binding⁹. Since the Lagrange multiplier is marginal social welfare, when $c \geq 0.90$, the Lagrange multiplier is negative, which means that relaxing the constraint makes social welfare even lower.

Finally, noting that $SW^{bb} = U^{bb}$ and π_1^{bb} , we can compute the social welfare and private firm's profit as SW^{bb} and π_1^{bb} respectively;

$$SW^{bb} = \frac{304 - 144c - 816c^2 + 316c^3 + 787c^4 - 222c^5 - 320c^6 + 50c^7 + 45c^8}{8[(1-c^2)(8-5c^2)]^2}, \quad (20)$$

$$\pi_1^{bb} = \frac{(4-2c-3c^2+c^3)^2}{4(1-c^2)(8-5c^2)^2}. \quad (21)$$

3.1.3. [Firm 0 sets price, firm 1 sets quantity]: Let firm 0 optimally choose its price as a best response to any quantity chosen private firm 1, and let private firm 1 optimally choose its quantity as a best response to any price chosen public firm 0. The demand function that each firm i faces are given by

$$x_0 = 1 - cx_1 - p_0 \quad \text{and} \quad p_1 = 1 - c + cp_0 - (1-c^2)x_1. \quad (22)$$

⁹It can be easily checked by which the public firm's profit is zero; $\pi_0^{bb} = [\frac{4-c-3c^2-(4-c-3c^2)}{8-5c^2}]x_0^{bb} = 0 \times x_0^{bb} = 0$.

In stage three, by maximization social welfare (respectively, profit) each firm sets its price as a best response to any price chosen by its private firm (respectively, the public firm). The public firm's objective is given as in the previous case as follows:

$$\begin{aligned} \max_{p_0} U &= 1 - cx_1 - p_0 + x_1 - \frac{1}{2}[(1 - cx_1 - p_0)^2 + x_1^2 + 2cx_1(1 - cx_1 - p_0)] & (23) \\ \text{s.t. } & (p_0 - w_0)(1 - cx_1 - p_0) \geq 0 \Leftrightarrow p_0 - w_0 \geq 0. \end{aligned}$$

Denoting the multiplier of the budget constraint λ^{bc} and repeating the same process as in previous cases yields the first-order conditions of the Lagrangian equation with respect to p_0 and λ^{bc} :

$$\frac{\partial L}{\partial p_0} = -p_0 + \lambda^{bc} = 0, \quad (24)$$

$$\frac{\partial L}{\partial \lambda^{bc}} = p_0 - w_0 = 0. \quad (25)$$

On the other hand, the private firm's objective function as follows:

$$\max_{x_1} \pi_1 = [1 - c + cp_0 - (1 - c^2)x_1 - w_1]x_1.$$

Thus, the first-order condition for the private firm is given by

$$\frac{\partial \pi_1}{\partial x_1} = 0 \Leftrightarrow x_1 = \frac{1 - c + cp_0 - w_1}{2(1 - c^2)}. \quad (26)$$

Substituting the pair (x_1, p_0) into the pair (x_0, p_1) yields

$$x_0 = \frac{(1 - c)(2 + c) + cw_1 - (2 - c^2)w_0}{2(1 - c^2)}, \quad (27)$$

$$p_1 = \frac{1 - c + cw_0 + w_1}{2}. \quad (28)$$

Then, optimal wages are set to maximize union's firm including the public union's utility: $u = x_i u_i$. The best reply functions for the private firm 1 and the public firm 0 are as follows.

$$w_1 = \frac{1 - c + cw_0}{2} \quad \text{and} \quad w_0 = \frac{(1 - c)(2 + c) + cw_1}{2(2 - c^2)}.$$

Straightforward computation yields the same results as in Lemma 2 since best reply functions are the same, i.e., (16) equals to (27) and (17) equals to (26). Thus, $w_i^{bc} = w_i^{bb}$, $x_i^{bc} = x_i^{bb}$ and $p_i^{bc} = p_i^{bb}$. Therefore, we obtain the same social welfare and the private firm's profit, i.e., $SW^{bc} = SW^{bb}$ and $\pi_1^{bc} = \pi_1^{bb}$. Substituting equilibrium values into (24) yields

$$\lambda^{bc} = p_0^{bc} = \frac{4 - c - 3c^2}{8 - 5c^2} > 0. \quad (29)$$

Thus, we have the following result.

Proposition 1: *Suppose that goods are substitutes and each firm's union is allowed to engage in decentralized bargaining. Then, regardless of the value of c , both the equilibrium private firm's profit and the social welfare in the case of [Firm 0 sets price, firm 1 sets quantity] are equal to those in the case of [Competition Game in Bertrand].*

Proposition 1 suggests that regardless of the private firm's strategic choice between price and quantity, the public firm can commit to choosing the competition game in Bertrand even though the optimal output is not satisfied when $c \in [0.90, 1)$. Thus, the type of contract of the private firm is determined by the public firm's choice of the type of contract.

Due to the imposition of the budget constraint on the public firm, the wage level of the public firm increases with its price and vice versa. Suppose that the wage level of the public firm is increased (i.e., the price level of the public firm is increased). This effect makes the public firm have an incentive to reduce its own output because of the best response function of the private firm (i.e., Eqs. (16) and (27)). Notice that it is necessary that the private firm has to increase its own price level –this arises because of Eq. (28). Thus, given the private firm's wage level, the private firm has an incentive to decrease its output level when the price of the private firm is increased. Thus, one of two cases arises: Competition in Bertrand or [Firm 0 sets price, firm 1 sets quantity]. Hence, Proposition 1 indicates that an important role is played by the budget constraint that is imposed on the public firm.

3.1.4. [Firm 1 sets price, firm 0 sets quantity]: Let firm 1 optimally choose its price as a best response to any quantity chosen private firm 0, and let private firm 0 optimally choose its quantity as a best response to any price chosen public firm 1. The demand function that each firm i faces is given by

$$x_1 = 1 - cx_0 - p_1 \quad \text{and} \quad p_0 = 1 - c + cp_1 - (1 - c^2)x_0. \quad (30)$$

Thus, the public firm's objective is given as in the previous case as follows:

$$\begin{aligned} \max_{x_0} U &= 1 - cx_0 - p_1 + x_0 - \frac{[(1 - cx_0 - p_1)^2 + x_0^2 + 2cx_0(1 - cx_0 - p_1)]}{2} \\ \text{s.t.} \quad &1 - c + cp_1 - (1 - c^2)x_0 - w_0 \geq 0. \end{aligned}$$

Denoting the multiplier of the budget constraint λ^{cb} , the Lagrangian equation can be written as

$$L = 1 - cx_0 - p_1 + x_0 - \frac{[(1 - cx_0 - p_1)^2 + x_0^2 + 2cx_0(1 - cx_0 - p_1)]}{2} + \lambda^{cb}[1 - c + cp_1 - (1 - c^2)x_0 - w_0].$$

Taking w_i as given, the first-order conditions are given by

$$\frac{\partial L}{\partial x_0} = 1 - c - x_0 + c^2x_0 - \lambda^{cb}(1 - c^2) = 0, \quad (31)$$

$$\frac{\partial L}{\partial \lambda^{cb}} = 1 - c + cp_1 - (1 - c^2)x_0 - w_0 = 0. \quad (32)$$

In the second stage of this case, from the profit and social welfare optimization for each firm, we obtain the pair (p_1, x_0) as

$$p_1 = \frac{1 + w_1 - cx_0}{2} \quad \text{and} \quad x_0 = \frac{1 - c + cp_1 - w_0}{1 - c^2}. \quad (33)$$

Substituting (p_1, x_0) into (32) yields

$$p_1 = \frac{1 - c + w_1 - c^2w_1 + cw_0}{2 - c^2} \quad \text{and} \quad x_0 = \frac{2 - c - 2w_0 + cw_1}{2 - c^2}. \quad (34)$$

In addition, substituting (34) into x_1 yields

$$x_1 = \frac{1 - c + cw_0 - w_1}{2 - c^2}. \quad (35)$$

Then, optimal wages are set to maximize union's firm including the public union's utility: $u = x_i u_i$. This implies the following equilibrium wages:

$$w_0^{cb} = \frac{4 - c - c^2}{8 - c^2}, \quad w_1^{cb} = \frac{4 - 2c - c^2}{8 - c^2}.$$

Thus, each price is given by

$$p_0^{cb} = \frac{8 - 2c - 6c^2 + c^3 + c^4}{(2 - c^2)(8 - c^2)}, \quad p_1^{cb} = \frac{12 - 6c - 7c^2 + 2c^3 + c^4}{(2 - c^2)(8 - c^2)}.$$

Similar to the case of 3.1.3. [Firm 0 sets price, firm 1 sets quantity], straightforward computation yields the same results as in Lemma 1 since best reply functions in Lemma 1 equal to (34) and (35). Thus, $w_i^{cc} = w_i^{cb}$ and $x_i^{cc} = x_i^{cb}$. Therefore, we obtain the same social welfare and the private firm's profit, i.e., $SW^{cc} = SW^{cb}$ and $\pi_1^{cc} = \pi_1^{cb}$. Substituting equilibrium values into (31) then we have

$$\lambda^{cb} = \frac{8 - 6c - 6c^2 + 2c^3 + c^4}{(1 + c)(2 - c^2)(8 - c^2)} > 0 \quad \text{if } c < 0.89.$$

which shows that the budget constraint is binding. Since the Lagrange multiplier, λ^{cb} is marginal social welfare, when $c \geq 0.89$, the Lagrange multiplier is negative, which means that relaxing the constraint makes social welfare even lower. Thus, we have the following result.

Proposition 2: *Suppose that goods are substitutes and each firm's union is allowed to engage in decentralized bargaining. Then, regardless of the value of c , both the equilibrium private firm's profit and the social welfare in the case of [Firm 1 sets price, firm 0 sets quantity] are equal to those in the case of [Competition Game in Cournot].*

Similar to Proposition 1, Proposition 2 suggests that regardless of the private firm's strategic choice, the public firm can commit to choosing the competition game in Cournot even though the optimal output is not satisfied when $c \in [0.89, 1)$.

Due to the imposition of the budget constraint on the public firm, the wage level of the public firm increases with its price and vice versa. Suppose that the wage level of the public firm is increased. This effect makes the public firm have an incentive to reduce its own output level because of the best response function of the private firm (i.e., Eqs. (5) and (34)). This means that it is necessary that the private firm has to increase its own price level—which is because of Eq. (34). Thus, given the private firm's wage level, the private firm has an incentive to increase its output level when the price of the private firm is increased. One of two cases arises: Competition in Cournot or [Firm 1 sets price, firm 0 sets quantity].

3.2 Results: Fixed Contract Motives with Solutions of Complements

By repeating the same process of preceding subsections, the explanation under the case of complements is largely analogous to the one underlying the case of substitutes for four fixed types of contracts. The setting when the goods are complements can now be briefly depicted¹⁰.

¹⁰The detailed computations of complements are available from author upon request. The Appendix C will not be included in the main paper since the inference can be easily verified by reversing signs of c .

The equilibrium social welfare levels and private firm's profit are characterized by

$$SW^{cc} = SW^{cb} = \frac{304 + 144c - 256c^2 - 92c^3 + 67c^4 + 12c^5 - 6c^6}{2(8 - c^2)^2(2 - c^2)^2}; \quad (36)$$

$$SW^{bb} = SW^{bc} = \frac{304 + 144c - 816c^2 - 316c^3 + 787c^4 + 222c^5 - 320c^6 - 50c^7 + 45c^8}{8[(1 - c^2)(8 - 5c^2)]^2}; \quad (37)$$

$$\pi_1^{cc} = \pi_1^{cb} = \frac{(4 + 2c - c^2)^2}{(8 - c^2)^2(2 - c^2)^2}, \quad \pi_1^{bb} = \pi_1^{bc} = \frac{(4 + 2c - 3c^2 - c^3)^2}{4(1 - c^2)(8 - 5c^2)^2}, \quad (38)$$

where each budget constraint is binding when all firms choose different types of contracts. Compared to the case of substitutes, the Lagrange multipliers are always positive values in the case of complements.

Before determining endogenous choices of strategic variables, we compare the public firm's price and total output under Bertrand competition with the private firm's price and total output under Cournot competition, respectively. Main findings comparing the each price and total output between Cournot and Bertrand can be summarized in the following proposition (all calculations are in the Appendix A).

Proposition 3: *Suppose that goods are either substitutes or complements and each firm's union is allowed to engage in decentralized bargaining. Then, irrespective of the nature of goods,*

$$X^C < X^B, \quad p_0^{cc} = p_0^{cb} < p_1^{cc} = p_1^{cb}, \quad p_0^{cc} = p_0^{cb} > p_0^{bb} = p_0^{bc}, \text{ and } p_1^{cc} = p_1^{cb} > p_1^{bb} = p_1^{bc},$$

where $X^C = x_1^{cc} + x_0^{cc} = x_1^{cb} + x_0^{cb}$ and $X^B = x_1^{bb} + x_0^{bb} = x_1^{bc} + x_0^{bc}$.

Proposition 3 suggests that regardless of the nature of goods, the private firm's price is always higher than the public firm because of the imposition of a budget constraint, and that both firms' prices are greater under Cournot than under Bertrand competition. Moreover, it shows that the public and private firms' total output under Bertrand competition are higher than under Cournot competition. Proposition 3 is analogous to that in the standard setting the equilibrium prices are higher under Cournot than under Bertrand competition, while introducing the public firm into a unionized mixed duopoly indicates that the private firm's price is always higher than the public firm's price.

4 Endogenous Choice of Contract with Substitutes and Complements in a Mixed Duopoly

Once the equilibria for four fixed types of contract and social-welfare levels are derived as per the preceding section, the type of contract can be determined endogenously by taking each social-welfare level and each private firm's profit as given. Therefore, we will consider the cases of substitutes and complements at the same time.

For employing the three-stage game, let "C" and "B" represent, respectively, Cournot and Bertrand competition with regard to each firm's choice. In this section, the SPNE will be found in the first stage for any given pair of competition types. Thus, the payoff matrix for the contract game can be represented by the following table (Table 1).

Table 1: Contract Game

		Firm 0			
		C		B	
Firm 1	C	π_1^{cc}, SW^{cc}	π_1^{bc}, SW^{bc}	π_1^{bc}, SW^{bc}	π_1^{bc}, SW^{bc}
	B	π_1^{cb}, SW^{cb}	π_1^{bb}, SW^{bb}	π_1^{bb}, SW^{bb}	π_1^{bb}, SW^{bb}

Since the comparison of each SW^{lm} , $lm = cc, cb, bb, bc$ in Table 1 becomes complicated, we need to use numerical examples to illustrate the impact of either degree of substitutability or complementary. Using this computation, the numerical analysis of Table 2 shows that the social welfare is always greater under Bertrand competition than under Cournot competition regardless of which the goods are substitutes or complements¹¹.

Table 2: Numerical Examples in Contract Game

Value of c	Case of Substitutes		Case of Complements	
	$SW^{cc} = SW^{cb}$	$SW^{bb} = SW^{bc}$	$SW^{cc} = SW^{cb}$	$SW^{bb} = SW^{bc}$
0.01	0.590961548	0.5909708	0.596586892	0.596596394
0.05	0.580272253	0.580492137	0.60844028	0.608691546
0.15	0.556489786	0.558260535	0.642039279	0.644686716
\vdots	\vdots	\vdots	\vdots	
0.51	0.496803576	0.51300658	0.836753368	0.907093362
0.52	0.495591709	0.512384336	0.844717028	0.920167739
\vdots	\vdots	\vdots	\vdots	
0.89	0.463538455	0.508345527	1.387048603	3.074774601
0.90	0.463017987	0.508359617	1.413992966	3.353424932
\vdots	\vdots	\vdots	\vdots	
0.98	0.459719889	0.503867899	1.680392056	15.58484322
0.99	0.459434736	0.502165707	1.721717336	30.86412902

Hence, choosing Bertrand competition is best the public firm can do, regardless of the private firm's choice of contract and of whether the goods are substitutes or complements. We can therefore show that $SW^{cc} = SW^{cb} < SW^{bb} = SW^{bc}$ under either substitutes or complements. As in Table 1, for the public firm, choosing Cournot is strictly dominated by choosing Bertrand. So the public firm never chooses a Cournot type of contract. Clearly, there can be sustained multiple SPNEs in this stage since $\pi_1^{bb} = \pi_1^{bc}$ and $\pi_1^{cc} = \pi_1^{cb}$: (B, B) and (C, B). Multiple SPNEs of the three-stage game in a unionized mixed duopoly are found and stated in the following proposition.

Proposition 4: *Suppose that goods are either substitutes or complements and each firm's union is allowed to engage in decentralized bargaining. In that case, the public firm always chooses a Bertrand type of contract and the private firm chooses either a Bertrand or a Cournot type of contract regardless of whether the goods are substitutes or complements in the first stage.*

By the restriction of one's attention to the subgame perfect Nash equilibrium of the three-stage game, one significant result can be derived from Proposition 4: only the public firm chooses a type of contract that does not depends on the value of c in the equilibrium. The sustaining of

¹¹For another straightforward calculations, all calculations are in the Appendix B.

multiple SPNEs from $\pi_1^{bb} = \pi_1^{bc}$ and $\pi_1^{cc} = \pi_1^{cb}$ plays an important role in the derivation of the result.

Note that the case where the goods are substitutes; Even in a situation in which the public firm chooses a Cournot (respectively, Bertrand) type of contract and the private firm chooses a Bertrand (respectively, Bertrand) type of contract, wherein $c \in [0.89, 1)$ (respectively, $c \in [0.90, 1)$), the Bertrand competition entails higher social welfare than the Cournot competition. Moreover, this kind of a choice of strategic commitment by the public firm results in the improvement of social welfare when all the firms choose different types of contracts. This is because there exists a dominant strategy for the public firm to choose a Bertrand type of contract, while there is no a dominant strategy for a private firm. Hence, Proposition 4 holds irrespective of the nature of goods the ranking of social welfare is not reversed. As a result, under a unionized mixed duopoly, social welfare under Bertrand competition will always be determined by the public firm's dominant strategy, wherein the Bertrand competition entails higher social welfare than the Cournot competition.

In our framework of a mixed duopoly, there exists a dominant strategy only for the public firm that chooses the Bertrand competition, regardless of the nature of goods; there is no dominant strategy for a private firm. However, in the setting of Singh and Vives (1984), a dominant strategy exists for both firms that choose the Cournot (or Bertrand) competition if the goods are substitutes (or complements). Specifically, López (2007) demonstrated that given the degree of substitutability, *each* firm can have a dominant strategy depending on the parameter of the bargaining power and the weight that the union attaches to the wage level. Nevertheless, although we have assumed that the union possesses a full bargaining power ($\theta = 1$) for the wage level to present our results in a simple manner, it can be interpreted that unions place considerable weight on the wage argument in their utility function, as depicted by López (2007). Thus, our findings partially assess the robustness of López's (2007) argument with respect to the introduction of a unionized mixed duopoly. However, López's analysis (2007) suggests that there can be a sustained unique Bertrand equilibrium¹²; on the contrary, in this paper, the private firm's profit is determined by the public firm's choice of type of contract. Moreover, there are multiple sustained SPNEs; this differs from the result of a unique equilibrium arrived at by Singh and Vives (1984). On the other hand, López (2007) demonstrated that when there is no dominant strategy for the private firm, a Nash equilibria pair of Cournot and Bertrand strategies can be sustained.

The results obtained in this paper are the same as those obtained by Singh and Vives' (1984) and López and Naylor's (2004) conclusion in the sense that the Bertrand competition entails higher social welfare than Cournot competition. However, the social welfare of a Cournot competition can be determined by each private firm's dominant strategy. Moreover, López and Naylor (2004) demonstrated that when the goods are substitutes, profits obtained under a Bertrand equilibrium are greater than those obtained in a Cournot equilibrium, if the unions are sufficiently powerful and place considerable weight on the wage argument in their utility function; this is a reverse of Singh and Vives' (1984) result. Their results hold without both making choice of strategic variable and framework of mixed duopoly. By comparing the profits obtained under

¹²The general asymmetric Nash-bargaining over wages between each union and its corresponding firm solves: $w_i = \arg \max u_i^\beta \pi_i^{1-\beta}$, where $\beta \in (0, 1)$ is the union's Nash bargaining parameter. Note that we use full bargaining power ($\beta = \theta = 1$) for the wage level and that $\bar{w} = 0$. Thus, if we introduce the general case ($\beta = \theta \in (0, 1)$) into the privatization framework, the analysis will be very complicated when we compare the mixed duopoly with privatization (i.e., a pure duopoly). Since all private firms have zero profits when $\beta = \theta = 1$ as in the models described by López and Naylor (2004, p. 685) and López (2007, pp. 492-493), it is left for future research to develop a comparison between privatization and a unionized mixed duopoly using the general asymmetric Nash-bargaining over wages. As a start, under a unionized mixed duopoly, this paper analyzes an endogenous type of contract in simple cases of union utility.

the Cournot and Bertrand equilibrium in a unionized mixed duopoly, the following proposition can be stated.

Proposition 5: *Suppose that goods are either substitutes or complements and each firm's union is allowed to engage in decentralized bargaining. Then,*

$$\begin{aligned} \pi_1^{cc} = \pi_1^{cb} > \pi_1^{bb} = \pi_1^{bc} & \text{ if the goods are substitutes,} \\ \pi_1^{cc} = \pi_1^{cb} > \pi_1^{bb} = \pi_1^{bc} & \text{ if the goods are complements and } c < \hat{c} \equiv 0.92. \end{aligned}$$

Compared to the analysis in López (2007), we show that except for the case of $c \geq \hat{c} \equiv 0.92$, Cournot equilibrium of private firm's profit are greater than Bertrand equilibrium of private firm's profit as Singh and Vives'(1984) result.

The Proposition 3, 4 and 5 are in contrast to the propositions reported by Ghosh and Mitra (2008); they compared Cournot and Bertrand competition in a mixed oligopoly without trade unions and without an endogenous type of contract. Moreover, they primarily demonstrated that despite the ambiguity in price ordering between Bertrand and Cournot competition for private firms, a comparison of quantities and profits yields unambiguous results. In other words, the public firm's output is higher under a Cournot competition, whereas the private firm's output is lower under the same circumstances. In addition, the profits of both the firms are lower under Cournot competition than under Bertrand competition. It should be noted that Ghosh and Mitra (2008) focused on the case of substitutes. The primary results of this study show that the output and social welfare of public and private firms under Bertrand competition are higher than under Cournot competition; this is because only the public firm is able to choose a type of contract irrespective of the nature of the goods. Contrary to Ghosh and Mitra (2008), we indicated that regardless of the nature of goods, the private firm's price is always higher than the public firm's price and that the prices of both the firms are greater under the Cournot competition than under the Bertrand competition. This is because of the imposition of a budget constraint. Consequently, this paper differs from previous works (including the analysis of Ghosh and Mitra (2008)) on unionized mixed oligopolies that focused on privatization without the choice of strategic variables by both public and private firms. Thus, it was found that there are multiple Nash equilibria in the contract stage of the game when there is a unionized mixed duopoly and that the primary results obtained in this study differ from those obtained by Ghosh and Mitra (2008), based on the exogenous type of contract.

In our setting, an endogenous type of contract is determined by the public firm, regardless of the private firm's strategic choice between price and quantity. Moreover, since the cost of production is endogenously determined by introducing trade unions, it is necessary to include the role of unions in order to derive Propositions 4 and 5 in the case of a mixed duopoly. Since the public firm has an inherent tendency to expand output, its union becomes more aggressive in wage bargaining than the private firm's union does. However, due to imposition of a budget constraint on the public firm, the profits of both public and private firms and the utilities of the unions cancel each other on the social welfare function. Hence, the public firm's objective function is endogenously determined by the representative consumer's utility. Consequently, regardless of which firm's profit and which union's utility is larger, social welfare can always be sustained under Bertrand competition equilibrium.

5 Concluding Remarks

We investigated a differentiated mixed duopoly in which private and public firms can choose to strategically set prices or quantities by facing a union bargaining process. A choice of strategic

variables was proposed endogenously in the first stage. For the case of a unionized mixed duopoly, only the public firm is able to choose a type of contract irrespective of whether the goods are substitutes or complements in the equilibrium. This effect leads the public firm to use strategic commitment of Bertrand competition that can improve social welfare when all firms choose different types of contracts. Moreover, there are multiple Nash equilibria in the contract stage of the game. This result contrasts with the findings of Singh and Vives (1984), López (2007), and López and Naylor (2004), in whose researches it is a dominant strategy for each private firm in a pure duopoly to choose either a quantity or a price contract. Hence, our main results hold irrespectively the nature of goods, except with sufficiently large parameter of complements, the ranking of private firm's profit is not reversed. This occurs because we relinquish Singh and Vives' assumption and deal with the type of contract as it exists in a unionized mixed duopoly. Thus, our paper also suggests that private firm's profit is determined by public firm's choice of contract.

We conclude by discussing the limitations of our paper. We have used the simplifying assumption that private and public firms are symmetric due to identical production costs and a decentralized unionization structure. By making these assumptions, we do not take into account any cost difference that may arise from the mixed bargaining that occurs between private and public firms. It is worth noting that the centralized (or, conversely, decentralized) wage-setting process assumes that the centralized (or, conversely, decentralized) union sets a single (or, conversely, different) wage for all firms when public firms are less efficient than private firms. Thus, this paper does not investigate whether or not the degree of the centralization of union bargaining matters for private firms when they choose different bargaining regimes and use different strategic variables. Moreover, we have not extended the model to consider a situation where the public firm competes with both domestic and foreign private firms. An extension of our model in these directions is for future research.

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Appendix A: List of Calculations of Proposition 3 and 5

List of calculations of complements

$$\begin{aligned}
 x_0^{cc} = x_0^{cb} &= \frac{2(4+c-c^2)}{(2-c^2)(8-c^2)}, & x_1^{cc} = x_1^{cb} &= \frac{4+2c-c^2}{(2-c^2)(8-c^2)}; \\
 p_0^{cc} = p_0^{cb} &= \frac{4+c-c^2}{(8-c^2)}, & p_1^{cc} = p_1^{cb} &= \frac{12+6c-7c^2-2c^3+c^4}{(2-c^2)(8-c^2)}; \\
 x_0^{bb} = x_0^{bc} &= \frac{8+2c-10c^2-c^3+3c^4}{2(1-c^2)(8-5c^2)}, & x_1^{bb} = x_1^{bc} &= \frac{4+2c-3c^2-c^3}{2(1-c^2)(8-5c^2)}; \\
 p_0^{bb} = p_0^{bc} &= \frac{4+c-3c^2}{8-5c^2}, & p_1^{bb} = p_1^{bc} &= \frac{12+6c-9c^2-3c^3}{2(8-5c^2)}.
 \end{aligned}$$

Comparison of each price: substitutes (respectively, complements)

$$\begin{aligned}
 p_0^{cb} = p_0^{cc} > p_0^{bb} = p_0^{bc} &\Leftrightarrow 4c^2 + 3c^4 - c^6 > 0 \text{ (respectively, } p_0^{cb} = p_0^{cc} > p_0^{bb} = p_0^{bc} \Leftrightarrow 4c^2 - 3c^3 - c^4 > 0), \\
 p_0^{cb} = p_0^{cc} < p_1^{cc} = p_1^{cb} &\Leftrightarrow 0 < 4 - 4c - c^2 + c^3 \text{ (respectively, } p_0^{cb} = p_0^{cc} < p_1^{cc} = p_1^{cb} \Leftrightarrow 0 < 4 + 4c - c^2 - c^3), \\
 p_1^{cb} = p_1^{cc} > p_1^{bb} = p_1^{bc} &\Leftrightarrow 42c^2 - 16c^3 - 26c^4 + 16c^5 - c^6 - 3c^7 > 0 \\
 \text{(respectively, } p_1^{cb} = p_1^{cc} > p_1^{bb} = p_1^{bc} &\Leftrightarrow 42c^2 + 16c^3 - 26c^4 - 16c^5 + c^6 - 3c^7 > 0).
 \end{aligned}$$

Comparison of total output: substitutes (respectively, complements)

$$\begin{aligned}
 x_0^{cb} + x_1^{cb} = x_0^{cc} + x_1^{cc} = X^C &< X^B = x_0^{bb} + x_1^{bb} = x_0^{bc} + x_1^{bc} \\
 \Leftrightarrow -32c^2 + 132c^3 - 152c^4 - 6c^5 - 43c^6 - 2c^7 - 3c^8 &< 0 \\
 \text{(respectively, } -32c^2 + 88c^3 - 72c^4 - 54c^5 + 43c^6 + 2c^7 - 3c^8 &< 0).
 \end{aligned}$$

Comparison of each profit: substitutes (respectively, complements)

$$\begin{aligned}
 \pi_1^{cb} = \pi_1^{cc} > \pi_1^{bb} = \pi_1^{bc} &\Leftrightarrow \\
 2048c^2 - 2048c^3 - 2560c^4 + 3072c^5 + 576c^6 - 944c^7 - 192c^8 + 456c^9 - 112c^{10} + 120c^{11} + 15c^{12} + 6c^{13} - c^{14} &> 0 \\
 \text{(respectively, } \pi_1^{cb} = \pi_1^{cc} > \pi_1^{bb} = \pi_1^{bc} &\Leftrightarrow \\
 2048c^2 + 2048c^3 - 2560c^4 - 3072c^5 + 576c^6 + 944c^7 - 192c^8 - 456c^9 - 112c^{10} - 120c^{11} + 15c^{12} - 6c^{13} - c^{14} &> 0 \\
 \text{if } c < \hat{c} \equiv 0.92; \text{ otherwise, } \pi_1^{cb} = \pi_1^{cc} < \pi_1^{bb} = \pi_1^{bc} &).
 \end{aligned}$$

Appendix B: Comparison of Social Welfare

Comparison of complements between $SW^{cc} = SW^{cb}$ and $SW^{bb} = SW^{bc}$

$$SW^{cc}(= SW^{cb}) - SW^{bb}(= SW^{bc}) = -324576c^2 - 32768c^3 + 60416c^4 + 92160c^5 - 2890508c^6 - 96128c^7 + 1632c^8 + 44848c^9 + 13560c^{10} - 8168c^{11} - 6614c^{12} - 44c^{13} + 1240c^{14} + 100c^{15} - 90c^{16} < 0.$$

Comparison of substitutes between $SW^{cc} = SW^{cb}$ and $SW^{bb} = SW^{bc}$

$$SW^{cc}(= SW^{cb}) - SW^{bb}(= SW^{bc}) = -324576c^2 + 32768c^3 + 60416c^4 - 92160c^5 - 2890508c^6 + 96128c^7 + 1632c^8 - 44848c^9 + 13560c^{10} + 8168c^{11} - 6614c^{12} + 44c^{13} + 1240c^{14} - 100c^{15} - 90c^{16} < 0.$$

Since comparing each SW^{lm} ; $lm = cc, cb, bb, bc$ becomes complicated, so we need to use numerical examples to illustrate the impact of the degree of c . Using this computation, the numerical analysis of Table A-1 shows that regardless of the nature of goods, Bertrand competition entails higher social welfare than the Cournot competition.

Table A-1: Numerical Examples in Contract Game

value of c	Case of Complements $SW^{cc}(= SW^{cb}) - SW^{bb}(= SW^{bc})$	Case of Substitutes $SW^{cc}(= SW^{cb}) - SW^{bb}(= SW^{bc})$
0.01	-32.48975752	-32.42423995
0.02	-130.0827676	-129.5590692
0.03	-292.9540688	-291.1890716
⋮	⋮	⋮
0.51	-133105.4002	-129245.6055
0.52	-142442.4177	-138495.983
⋮	⋮	⋮
0.97	-2652325.594	-2652026.908
0.98	-2808479.518	-2808289.03
0.99	-2972575.467	-2972484.67

It is available from author upon request

Appendix C: Case of Complements

For the reviewers and editor, this appendix will not be included in the main paper. However, this is only available for the reviewers and editor: the case of complements. In this case where we have been abbreviated, we present on separate page.

6 The Basic Model: Case of Complements

Consider two single-product firms producing differentiated products that are supplied by a public firm 0 and a private firm 1. We assume that the representative consumer's utility is a quadratic function given by

$$U = x_i + x_j - \frac{1}{2}(x_i^2 + 2cx_ix_j + x_j^2),$$

where x_i denotes the output of firm $i = 0, 1$. The parameter $c \in (-1, 0)$ is a measure of the degree of complementarity among goods. Thus, the inverse demand is characterized by

$$p_i = 1 + cx_j - x_i; \quad i \neq j, i = j = 0, 1, \quad (39)$$

where p_i is firm i 's market price and x_i denotes the output of firm $i = 0, 1$. Hence, we can obtain the direct demands as

$$x_i = \frac{(1+c) - cp_j - p_i}{1-c^2} \quad (40)$$

provided that quantities are positive.

To analyze the union's wage bargaining, we also use same assumption as in the main body of our paper. Taking \bar{w} as given, the union's optimal wage setting strategy w_i regarding firm i is defined as

$$u_i = (w_i - \bar{w})^\theta x_i; \quad i = 0, 1, \quad (41)$$

where θ is the weight that the union attaches to the wage level.

Therefore, the public firm 0's objective function SW , and each firm's profit π_i are given by

$$SW = U - \sum_{i=0}^1 px_i + \sum_{i=0}^1 (\pi_i + u_i),$$

$$\pi_i = (p_i - w_i)x_i, \quad i = 0, 1.$$

7 Bargaining in a Mixed Duopoly: Case of Complements

7.1 Results: Fixed Contract Motives with Solutions (Case of Complements)

A 3.1.1. [Competition Game in Cournot]: Assume that each firm i faces the inverse demand functions given by $p_i = 1 + cx_j - x_i$. In the third stage, the public firm's objective is to maximize welfare which is defined as the sum of consumer surplus, each firm's profit, and each union's utility:

$$SW = U - p_0x_0 - p_1x_1 + \pi_1 + u_1 + \pi_0 + u_0 = U.$$

Given w_1 for the private firm, the public firm's maximization problem is as follows:

$$\max_{x_0} SW = U \quad \text{s.t.} \quad (p_0 - w_0)x_0 \geq 0.$$

The constraint implies that there is some lower-bound restriction on the public firm's profit, i.e., the public firm faces a budget constraint. Therefore, since we assume that each firm's output is a positive value as in (40), we can rewrite the budget constraint as follows: $(p_0 - w_0)x_0 \geq 0 \Leftrightarrow 1 + cx_1 - x_0 - w_0 \geq 0$.

Denoting the multiplier of the budget constraint λ^{cc} , the Lagrangian equation can be written as

$$L = x_1 + x_0 - \frac{(x_1^2 + x_0^2 - 2cx_0x_1)}{2} + \lambda^{cc}(1 - x_0 + cx_1 - w_0).$$

Given wage level w_0 in the third stage, the first-order conditions are given by

$$\frac{\partial L}{\partial x_0} = 1 + cx_1 - x_0 - \lambda^{cc} = 0 \quad (42)$$

$$\frac{\partial L}{\partial \lambda^{cc}} = 1 + cx_1 - x_0 - w_0 = 0. \quad (43)$$

On the other hand, the first-order condition for the private firm is given by

$$\frac{\partial \pi_1}{\partial x_1} = 0 \Leftrightarrow x_1 = \frac{1}{2}(1 + cx_0 - w_1). \quad (44)$$

Solving the first-order conditions (43) and (44), we obtain,

$$x_0 = \frac{2 + c - 2w_0 - cw_1}{2 - c^2}, \quad (45)$$

$$x_1 = \frac{1 + c - w_1 - cw_0}{2 - c^2}, \quad (46)$$

$$\lambda^{cc} = 1 + cx_1 - x_0. \quad (47)$$

To solve for Lagrangian equation, the budget constraint is momentarily binding. We check ex-post that the omitted this constraint is binding.

In the second stage of this case, using (45) and (46), the problem for union i is defined as

$$\begin{aligned} \max_{w_0} u_0 = w_0x_0 &= \frac{w_0(2 + c - 2w_0 - cw_1)}{2 - c^2}, \\ \max_{w_1} u_1 = w_1x_1 &= \frac{w_1(1 + c - w_1 - cw_0)}{2 - c^2}. \end{aligned}$$

This implies the following first-order condition

$$w_0 = \frac{2 + c - cw_1}{4}, \quad w_1 = \frac{1 + c - cw_0}{2}. \quad (48)$$

Straightforward computation yields each an equilibrium wage, denoted as w_i^{cc} is obtained by solving (48), and substituting w_i^{cc} into (45) and (46) yields the equilibrium output x_i^{cc} . Thus, we have the following result.

Lemma A-1: *Suppose that goods are complements and each firm's union is allowed to engage in decentralized bargaining. Then, the equilibrium wage, output and price levels are given by*

$$\begin{aligned} w_0^{cc} &= \frac{4+c-c^2}{8-c^2}, & w_1^{cc} &= \frac{4+2c-c^2}{8-c^2}; \\ x_0^{cc} &= \frac{2(4+c-c^2)}{(2-c^2)(8-c^2)}, & x_1^{cc} &= \frac{4+2c-c^2}{(2-c^2)(8-c^2)}; \\ p_0^{cc} &= w_0^{cc} = \frac{4+c-c^2}{(8-c^2)}, & p_1^{cc} &= \frac{12+6c-7c^2-2c^3+c^4}{(2-c^2)(8-c^2)}. \end{aligned}$$

This Lemma A-1 suggests that the budget constraint is binding. That is, substituting Lemma A-1 into (47) then we have

$$\lambda^{cc} = \frac{8+2c-6c^2-c^3+c^4}{(2-c^2)(8-c^2)} > 0,$$

which shows that the public firm sets the output that yields zero profit in equilibrium.

Finally, noting that $SW^{cc} = U^{cc}$ and π_1^{cc} , we can compute the social welfare and private firm's profit, SW^{cc} and π_1^{cc} as follows;

$$SW^{cc} = \frac{304 + 144c - 256c^2 - 92c^3 + 67c^4 + 12c^5 - 6c^6}{2(8-c^2)^2(2-c^2)^2}, \quad (49)$$

$$\pi_1^{cc} = \frac{(4+2c-c^2)^2}{(8-c^2)^2(2-c^2)^2}. \quad (50)$$

A 3.1.2. [Competition Game in Bertrand]; Consider that firm i faces the following direct demand function

$$x_i = \frac{(1+c) - cp_j - p_i}{1-c^2}.$$

In the third stage, by maximization social welfare (respectively, profit) each firm sets its price as a best response to any price chosen by its private firm (respectively, the public firm). The public firm's objective is given as in the previous case as follows:

$$\begin{aligned} \max_{p_0} U &= \frac{(1+c) - cp_1 - p_0}{1-c^2} + \frac{(1+c) - cp_0 - p_1}{1-c^2} \\ &\quad - \frac{1}{2} \left\{ \frac{[(1+c) - cp_1 - p_0]^2 + [(1+c) - cp_0 - p_1]^2 - 2c[(1+c) - cp_0 - p_1][(1+c) - cp_1 - p_0]}{(1-c^2)^2} \right\} \end{aligned}$$

$$\text{s.t. } p_0 - w_0 \geq 0.$$

Denoting the multiplier of the budget constraint λ^{bb} and repeating the same process as in Competition Game in Cournot yields the first-order conditions of the Lagrangian equation with respect to λ^{bb} and p_0 .¹³

$$\frac{\partial L}{\partial p_0} = 0 \Leftrightarrow \lambda^{bb} = \frac{cp_1 + p_0}{1-c^2}, \quad (51)$$

$$\frac{\partial L}{\partial \lambda^{bb}} = p_0 - w_0 = 0. \quad (52)$$

¹³The calculations is as follows.

$[(1+c) - cp_1 - p_0]^2 = (1+c)^2 - 2cp_1 - 2p_0 - 2c^2p_1 - 2cp_0 + c^2p_1^2 + 2cp_0p_1 + p_0^2$

On the other hand, the first-order condition for the private firm is given by

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Leftrightarrow p_1 = \frac{1 + c - cp_0 + w_1}{2}. \quad (53)$$

By using x_i and solving the these two equations (52) and (53) problems yields

$$x_0 = \frac{(1 + c)(2 - c) - cw_1 + (2 - c^2)w_0}{2(1 - c^2)}, \quad (54)$$

$$x_1 = \frac{1 + c - w_1 - cw_0}{2(1 - c^2)}. \quad (55)$$

In the second stage of this case, using (54) and (55), the problems for union i are defined as

$$\max_{w_0} u_0 = w_0 x_0 = \frac{w_0[(1 + c)(2 - c) - cw_1 + (2 - c^2)w_0]}{2(1 - c^2)}, \quad (56)$$

$$\max_{w_i} u_1 = w_1 x_1 = \frac{w_1(1 + c - w_1 - cw_0)}{2(1 - c^2)}. \quad (57)$$

The best reply functions for the public firm 0 and the private firm 1 are $w_0 = \frac{(1+c)(2-c)-cw_1}{2(2-c^2)}$ and $w_1 = \frac{1+c-cw_0}{2}$, respectively. Thus, straightforward computation yields each an equilibrium wage, denoted as w_i^{bb} is obtained by maximizing (56) and (57), and substituting w_i^{bb} into (54) and (55) yields the equilibrium output x_i^{bb} and price p_i^{bb} . Thus, we have the following result.

Lemma A-2: *Suppose that goods are complements and each firm's union is allowed to engage in decentralized bargaining. Then, the equilibrium wage, output and price levels are given by*

$$\begin{aligned} w_0^{bb} &= \frac{4 + c - 3c^2}{8 - 5c^2}, & w_1^{bb} &= \frac{4 + 2c - 3c^2 - c^3}{8 - 5c^2}; \\ x_0^{bb} &= \frac{8 + 2c - 10c^2 - c^3 + 3c^4}{2(1 - c^2)(8 - 5c^2)}, & x_1^{bb} &= \frac{4 + 2c - 3c^2 - c^3}{2(1 - c^2)(8 - 5c^2)}; \\ p_0^{bb} &= \frac{4 + c - 3c^2}{8 - 5c^2}, & p_1^{bb} &= \frac{12 + 6c - 9c^2 - 3c^3}{2(8 - 5c^2)}. \end{aligned}$$

$[(1 + c) - cp_0 - p_1]^2 = (1 + c)^2 - 2cp_0 - 2p_1 - 2c^2p_0 - 2cp_1 + c^2p_0^2 + 2cp_0p_1 + p_1^2$ and $[(1 + c) - cp_1 - p_0][(1 + c) - cp_0 - p_1] = (1 + c)^2 - (1 + c)cp_0 - (1 + c)p_1 - (1 + c)cp_1 + c^2p_1p_0 + cp_1^2 - (1 + c)p_0 + cp_0^2 + p_1p_0$. Thus,

$$\frac{\partial SW^{bb}}{\partial p_0} = \frac{-2(1 - c^2)(1 + c) - \{-2 - 2c - 2c - 2c^2 + 2cp_1 + 2p_0 + 2c^2p_0 + 2cp_1 - 2c[-(1 + c)c - (1 + c) + c^2p_1 + 2cp_0 + p_1]\}}{2(1 - c^2)^2}$$

$$+ \lambda^{bb} = 0$$

$$\Leftrightarrow \frac{\partial SW^{bb}}{\partial p_0} = \frac{-1 - c + c^2 + c^3 - \{-1 - c + c^2 + c^3 + cp_1 + p_0 + c^2p_0 + cp_1 - c[c^2p_1 + 2cp_0 + p_1]\}}{(1 - c^2)^2} + \lambda^{bb} = 0$$

$$\Leftrightarrow \frac{\partial SW^{bb}}{\partial p_0} = \frac{-\{cp_1 + p_0 + c^2p_0 + cp_1 - c^3p_1 - 2c^2p_0 - cp_1\}}{(1 - c)^2} + \lambda^{bb} = 0$$

$$\Leftrightarrow \frac{\partial SW^{bb}}{\partial p_0} = \frac{-\{cp_1 - c^3p_1 + p_0 - c^2p_0\}}{(1 - c^2)^2} + \lambda^{bb} = 0 \Leftrightarrow \frac{\partial SW^{bb}}{\partial p_0} = \frac{(1 - c^2)\{cp_1 + p_0\}}{(1 - c^2)^2} = \lambda^{bb}$$

$$\Leftrightarrow \frac{\partial SW^{bb}}{\partial p_0} = \frac{cp_1 + p_0}{1 - c^2} = \lambda^{bb} \text{ when the goods are complements.}$$

Thus, $\frac{\partial SW^{bb}}{\partial p_0} = \frac{-(cp_1 - p_0)}{1 - c^2} = \lambda^{bb}$ when the goods are substitutes.

Substituting Lemma A-2 into (51) then we have

$$\lambda^{bb} = \frac{8 + 14c - 9c^3 - 3c^4}{2(8 - 5c^2)(1 - c^2)} > 0,$$

which shows that the budget constraint is binding.

Finally, noting that $SW^{bb} = U^{bb}$ and π_1^{bb} , we can compute the social welfare and private firm's profit as SW^{bb} and π_1^{bb} respectively;

$$SW^{bb} = \frac{304 + 144c - 816c^2 - 316c^3 + 787c^4 + 222c^5 - 320c^6 - 50c^7 + 45c^8}{8[(1 - c^2)(8 - 5c^2)]^2}, \quad (58)$$

$$\pi_1^{bb} = \frac{(4 + 2c - 3c^2 - c^3)^2}{4(1 - c^2)(8 - 5c^2)^2}. \quad (59)$$

A 3.1.3. [Firm 0 sets price, firm 1 sets quantity]: Let firm 0 optimally choose its price as a best response to any quantity chosen private firm 1, and let private firm 1 optimally choose its quantity as a best response to any price chosen public firm 0. The demand function that each firm i faces are given by

$$x_0 = 1 + cx_1 - p_0 \quad \text{and} \quad p_1 = 1 + c - cp_0 - (1 - c^2)x_1. \quad (60)$$

In stage three, by maximization social welfare (respectively, profit) each firm sets its price as a best response to any price chosen by its private firm (respectively, the public firm). The public firm's objective is given as in the previous case as follows:

$$\begin{aligned} \max_{p_0} U &= 1 + cx_1 - p_0 + x_1 - \frac{1}{2}[(1 + cx_1 - p_0)^2 + x_1^2 - 2cx_1(1 + cx_1 - p_0)] \quad (61) \\ \text{s.t.} \quad &p_0 - w_0 \geq 0. \end{aligned}$$

Denoting the multiplier of the budget constraint λ^{bc} and repeating the same process as in previous cases yields the first-order conditions of the Lagrangian equation with respect to p_0 and λ^{bc} :

$$\frac{\partial L}{\partial p_0} = -p_0 + \lambda^{bc} = 0, \quad (62)$$

$$\frac{\partial L}{\partial \lambda^{bc}} = p_0 - w_0 = 0. \quad (63)$$

On the other hand, the private firm's objective function as follows:

$$\max_{x_1} \pi_1 = [1 + c - cp_0 - (1 - c^2)x_1 - w_1]x_1.$$

Thus, the first-order condition for the private firm is given by

$$\frac{\partial \pi_1}{\partial x_1} = 0 \Leftrightarrow x_1 = \frac{1 + c - cp_0 - w_1}{2(1 - c^2)}.$$

Thus, we obtain the pair (x_1, p_0) written as

$$x_1 = \frac{1 + c - cp_0 - w_1}{2(1 - c^2)}, \quad (64)$$

$$p_0 = w_0. \quad (65)$$

Substituting the pair (x_1, p_0) into the pair (x_0, p_1) yields

$$x_0 = \frac{(1+c)(2-c) - cw_1 - (2-c^2)w_0}{2(1-c^2)}, \quad (66)$$

$$p_1 = \frac{1+c - cw_0 - w_1}{2}. \quad (67)$$

Then, optimal wages are set to maximize union's firm including the public union's utility: $u = x_i u_i$. The best reply functions for the private firm 1 and the public firm 0 are as follows.

$$w_1 = \frac{1+c - cw_0}{2} \quad \text{and} \quad w_0 = \frac{(1+c)(2-c) - cw_1}{2(2-c^2)}.$$

Straightforward computation yields the same results as in Lemma A-2 since best reply functions are the same, i.e., (54) equals to (66) and (53) equals to (67). Thus, $w_i^{bc} = w_i^{bb}$, $x_i^{bc} = x_i^{bb}$ and $p_i^{bc} = p_i^{bb}$. Therefore, we obtain the same social welfare and the private firm's profit, i.e., $SW^{bc} = SW^{bb}$ and $\pi_1^{bc} = \pi_1^{bb}$. Substituting equilibrium values into (62) yields

$$\lambda^{bc} = p_0^{bc} = \frac{4+c-3c^2}{8-5c^2} > 0. \quad (68)$$

Thus, we have the following result.

Proposition A-1: *Suppose that goods are complements and each firm's union is allowed to bargain collectively. Then, both the equilibrium private firm's profit and social welfare in the case of [Firm 0 sets price, firm 1 sets quantity] are equal to those in the case of [Competition Game in Bertrand].*

This Proposition A-1 suggests that regardless of the choice of the private firm's strategic variable between the price and quantity, the public firm can commit to choose the competition game in Bertrand. Thus, the type of contract of the private firm is determined by public firm's choosing the type of contract.

A 3.1.4. [Firm 1 sets price, firm 0 sets quantity]: Let firm 1 optimally choose its price as a best response to any quantity chosen private firm 0, and let private firm 0 optimally choose its quantity as a best response to any price chosen public firm 1. The demand function that each firm i faces is given by

$$x_1 = 1 + cx_0 - p_1 \quad \text{and} \quad p_0 = 1 + c - cp_1 - (1-c^2)x_0. \quad (69)$$

Thus, the public firm's objective is given as in the previous case as follows:

$$\begin{aligned} \max_{x_0} U &= 1 + cx_0 - p_1 + x_0 - \frac{[(1 + cx_0 - p_1)^2 + x_0^2 - 2cx_0(1 + cx_0 - p_1)]}{2} \\ \text{s.t.} \quad &1 + c - cp_1 - (1 - c^2)x_0 - w_0 \geq 0. \end{aligned}$$

Denoting the multiplier of the budget constraint λ^{cb} , the Lagrangian equation can be written as

$$L = 1 + cx_0 - p_1 + x_0 - \frac{[(1 + cx_0 - p_1)^2 + x_0^2 - 2cx_0(1 + cx_0 - p_1)]}{2} + \lambda^{cb}[1 + c - cp_1 - (1 - c^2)x_0 - w_0].$$

Taking w_i as given, the first-order conditions are given by

$$\frac{\partial L}{\partial x_0} = 1 + c - x_0 + c^2 x_0 - \lambda^{cb}(1 - c^2) = 0, \quad (70)$$

$$\frac{\partial L}{\partial \lambda^{cb}} = 1 + c - cp_1 - (1 - c^2)x_0 - w_0 = 0. \quad (71)$$

In the second stage of this case, from profit and social welfare optimization for each firm, we obtain the pair (p_1, x_0) as

$$p_1 = \frac{1 + w_1 + cx_0}{2} \quad \text{and} \quad x_0 = \frac{1 + c - cp_1 - w_0}{1 - c^2}. \quad (72)$$

Substituting (p_1, x_0) into (71) yields

$$p_1 = \frac{1 + c + w_1 - c^2w_1 - cw_0}{2 - c^2} \quad \text{and} \quad x_0 = \frac{2 + c - 2w_0 - cw_1}{2 - c^2}. \quad (73)$$

In addition, substituting p_1 into x_1 yields

$$x_1 = \frac{1 + c - cw_0 - w_1}{2 - c^2}. \quad (74)$$

Then, optimal wages are set to maximize union's firm including the public union's utility: $u = x_i u_i$. This implies the following equilibrium wages:

$$w_0^{cb} = \frac{4 + c - c^2}{8 - c^2}, \quad w_1^{cb} = \frac{4 + 2c - c^2}{8 - c^2}.$$

Thus,

$$p_0^{cb} = \frac{4 + c - c^2}{(8 - c^2)}, \quad p_1^{cb} = \frac{12 + 6c - 7c^2 - 2c^3 + c^4}{(2 - c^2)(8 - c^2)}.$$

Similar to the case of 3.1.3. [Firm 0 sets price, firm 1 sets quantity], straightforward computation yields the same results as in Lemma A-1 since best reply functions in Lemma A-1 equal to (43) and (44). Thus, $w_i^{cc} = w_i^{cb}$ and $x_i^{cc} = x_i^{cb}$. Therefore, we obtain the same social welfare and the private firm's profit, i.e., $SW^{cc} = SW^{cb}$ and $\pi_1^{cc} = \pi_1^{cb}$. Substituting equilibrium values into (70) then we have

$$\lambda^{cb} = \frac{1 - x_0(1 - c)}{(1 - c)} = \frac{8 + 6c - 6c^2 - 2c^3 + c^4}{(1 - c)(2 - c^2)(8 - c^2)} > 0,$$

which shows that budget constraint is binding. Thus, we have the following result.

Proposition A-2: *Suppose that goods are complements and each firm's union is allowed to bargain collectively. Then, both the equilibrium private firm's profit and social welfare in the case of [Firm 1 sets price, firm 0 sets quantity] are equal to those in the case of [Competition Game in Cournot].*