On the Macroeconomic and Welfare Effects of Illegal Immigration

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Abstract

This paper investigates the macroeconomic and welfare effects of illegal immigration on the native born within a dynamic general equilibrium framework with labor market frictions. A key feature of the model is that job competition is allowed for between domestic workers and illegal immigrants. We calibrate the model to match some key statistics of the postwar U.S. economy. The model predicts that in the long run illegal immigration is a boon, but the employment opportunities of domestic workers are strongly negatively affected. The model also predicts that the level of domestic consumption has a U-shaped relationship with the share of illegal immigrants.

Keywords: Economic Growth, Immigration, Welfare, Search, Unemployment

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1 Introduction

Illegal immigration is a contentious issue facing most developed economies. In the United States, for instance, scholars have heatedly debated the pros and cons of illegal immigration for years. The main economic argument in support of immigration is that it helps increase the supply of labor, reduces the cost of production and hence is good for the economy. Primary opposing arguments include supposed high rates of use of welfare programs, immigrant poverty and job competition. Much of the discussion is motivated by concerns for the welfare effects of illegal immigration on the native born. However, most research applying partial-equilibrium analysis has only addressed slices of this problem through analyzing the effects of immigration on labor-market outcomes. There is only a small set of theoretical studies that address this issue of illegal immigration in a general equilibrium context. These studies have noticeable limitations. Among them, Ethier (1986) and Bond and Chen (1987) are carried out within a static context and they pay particular attention to problems and prescriptions for border control. Following the Ramsey tradition, subsequent research supplements the literature by investigating this issue within a one-sector dynamic general equilibrium framework. These studies include Hazari and Sgro (2003), Moy and Yip (2006), and Palivos (2009).

One common limitation among all existing studies is that they assume full employment in domestic labor market. These models thus ignore the effect of illegal immigration on the employment opportunities of domestic workers. In fact, one common argument in general against immigration is that immigrants harm the employment opportunities of native workers. Studies failing to address this issue cannot capture the whole picture of the effects of illegal immigration. The primary objective of this paper is to develop a dynamic general equilibrium model that can be used to evaluate the displacement effects of illegal immigration on native workers. To the best of our knowledge, so far no such theoretical model has been developed.

To achieve this objective, this study builds upon the contributions of Shi and Wen (1997) and models illegal immigration in a standard dynamic general equilibrium model with labor market frictions. One key feature of our model that differentiates it from the previous literature is that we allow both domestic workers and illegal immigrants to search for jobs at the same time, which in turn leads to job competition between them and consequently *increases* the unemployment rate of native workers.

In the model economy, each individual has three alternative, mutually exclusive uses of one indivisible unit of time: searching for a job, working for a firm, or enjoying leisure. Firms hire both domestic and illegal immigrant workers. The labor market is subject to search-matching frictions. Once unemployed domestic workers and job vacancies are matched, the terms of employment contracts are determined through bilateral bargaining. We assume that firms are able to distinguish illegal immigrants from domestic workers and face a punishment for hiring the former if being caught. The wage rate for illegal immigrants is thus equated to the wage rate of domestic workers minus the expected value of the punishment. We characterize the search equilibrium and prove the existence and uniqueness of stationary equilibrium.

The model generates important theoretical predictions due to the incorporation of illegal immigrants. Within this dynamic general equilibrium framework, we are able to show analytically that the long-run level of the unemployment rate for domestic citizens is increasing in the share of illegal immigrants in the total population for the case in which natives and illegal immigrant workers are perfect substitutes in the production process. We also uncover that the entry of illegal immigrants makes domestic workers face tighter labor markets in the long run.

To develop the quantitative implications of our model, we numerically solve and calibrate the model to match some key statistics of the postwar U.S. economy. Palivos (2009) finds that illegal immigration necessarily lowers the long-run level of per capita consumption and welfare of domestic citizens. In sharp contrast, the quantitative predictions of this study indicate that the long-run level of consumption of domestic citizens has a U-shaped relationship with the share of illegal immigrants. In other words, an increase in the number of illegal immigrants first reduces and then raises the long-run level of per capita domestic consumption. This finding is due to the presence of four effects at work. (1) A positive exploitation effect. When there is an increase in the number of illegal immigrants, a greater number of unemployed illegal immigrants are searching for jobs. In contrast, the change in the number of domestic workers searching for jobs is small. This leads to a tighter labor market which in turn leads to more fierce competition for jobs. To successfully secure a job, both domestic and foreign labor would have to lower their wages. This raises the firm's profits which are then distributed to domestic households as dividends. This effect adds to domestic consumption. (2) A negative capital-using-up effect. This is due to the fact that in the domestic economy some capital has to be used to produce output for the consumption of illegal migrants. This effect reduces current output which could have been used for domestic consumption and investment. (3) A negative wage depressing effect. As mentioned above, when more illegal immigrants enter into the economy, the competition for jobs becomes more severe. Thus, the wages for domestic labor are pushed down. (4) A negative displacement effect. As unemployed domestic labor and migrants compete for jobs, the chance for unemployed domestic workers to find a job is reduced. This effect reduces domestic consumption. Empirical studies on this topic often focus on (3) and (4) (for instance, see Hotchkiss and Myriam 2008). These four effects work together to determine the relationship between the long-run level of consumption of domestic citizens and the share of illegal immigrants. Under the baseline parameterization, the negative effects dominate when the population fraction of illegal immigrants is small. Thus, an increase in illegal immigration would lead to a drop in consumption. However, as the share of illegal immigrants continues to increase, the long-run level of domestic consumption would rise as the positive effect eventually dominates. This gives rise to the U-shaped relationship between the long-run level of consumption of domestic citizens and the share of illegal immigrants.

In order to shed some light on the welfare effects of illegal immigration, we compute the consumption-equivalent level of utility of domestic households and find that illegal immigration has a positive welfare effect. In particular, we compare two scenarios: (1) the economy stays at the steady state with no illegal immigrants forever; and (2) at $t = 0$, the host country admits a certain fraction of illegal immigrants and the economy gradually converges to the new steady state. The welfare measure of illegal immigration is calculated for a wide variety of combinations of labor supply elasticity and population share of illegal immigrants. For instance, we find that the domestic households would require a .746-percent increase in their consumption under scenario (1) in every period when the labor supply elasticity is 0.4 and when there is an increase in the population share of illegal immigrants from zero to 5 percent. The model also generates a prediction on employment opportunities of domestic workers. It predicts that employment opportunities of domestic workers are strongly negatively affected in the long run. Specifically, a greater number of domestic workers will leave the labor force when there is an increase in illegal immigration. In contrast, the labor force participation rate for illegal immigrants experiences a slight decrease. This result turns out to be qualitatively supported by the existing empirical evidence (for instance, see Borjas et al., 2007).

The remainder of the paper is structured as follows. Section 2 presents the search-theoretic model of unemployment and analyzes the search equilibrium. Section 3 studies the welfare effect of illegal immigration on domestic citizens and discusses the quantitative implications of the model. Finally, Section 4 offers some concluding remarks.

2 The Model

Consider an economy that is inhabited by two types of households, i.e., domestic (D) and illegal immigrant (M) households.[1] The number of each type of households is normalized to one. Each household consists of many infinitely lived agents. We use $L(t)$ and $M(t)$ to denote the size of each domestic and immigrant household at any time $t \geq 0$, respectively. We call $N(t) = L(t) + M(t)$ the total population. Both $L(t)$ and $M(t)$ are assumed to be growing at the same constant rate $g > 0$.[2] The share of illegal immigrants in the total population is constant over time $m = \frac{M(t)}{N(t)}$. Each household member at each point in time is endowed with one indivisible unit of time that has three alternative, mutually exclusive uses: searching for a job, working for a firm, or enjoying leisure. Throughout we use a superscript $i \in \{D, M\}$ to indicate these two types of households. The variable $s_1^i(t)$ is the fraction of the household's time in work, and $s_2^i(t)$ is the fraction of the household's time in search. The variable $s_1^i(t)$ is also referred to as the search effort. Accordingly, at the aggregate level, a representative domestic household spends $L_1(t) = s_1^D(t)L(t)$ of its total amount of time in search, and $L_2(t) = s_2^D(t)L(t)$ in work. Similarly, define $M_1(t) = s_1^M(t)M(t)$ and $M_2(t) = s_2^M(t)M(t)$ as the respective aggregate amount of time in search and in work for a representative illegal immigrant household. The domestic labor participation rate and unemployment rate can be termed as $s_1^D(t) + s_2^D(t)$ and $\frac{s_1^D(t)}{s_1^D(t)+s_2^D(t)}$, respectively.

Aggregate output $Y(t)$ is produced according to the Cobb-Douglas production technology that takes as inputs aggregate capital $K(t)$ and aggregate labor $L_2(t) + M_2(t)$,

$$Y(t) = [K(t)]^{\epsilon}[L_2(t) + M_2(t)]^{1-\epsilon},\ \epsilon \in (0, 1),$$

[1] Although both legal and illegal immigrants act as a factor substitute for native labor of similar skill, in this model, we only consider illegal immigrants because they work as a *cheaper* production substitute for domestic workers of the same level of labor productivity.

[2] Imposing this assumption is to ensure balanced growth properties of the model.

where ϵ is the capital share of national income. Domestic labor $L_2(t)$ and illegal migrants $M_2(t)$ are assumed to be perfect substitutes in production.[3]

2.1 Domestic Household's Utility Maximization Problem

In each period, each household member derives utility from consumption and disutility from working and searching for jobs. The momentary utility function of a typical agent is given by

$$u[c(t), s_1^D(t) + s_2^D(t)] = \log c(t) - \xi \frac{[s_1^D(t) + s_2^D(t)]^{1+\phi}}{1+\phi}, \;\; \xi > 0, \;\text{ and }\; \phi > 0, \tag{1}$$

where ϕ denotes the inverse of labor supply elasticity, and ξ is a preference parameter.

The household's total discounted utility is described by

$$U = \int_0^\infty e^{-(\rho-g)t} u[c(t), s_1^D(t) + s_2^D(t)]dt. \tag{2}$$

The variables $C(t)$ and $c(t) = C(t)/L(t)$ are aggregate and individual consumption of the domestic household, respectively.[4] The parameter $\rho > 0$ is the discount rate, and $\rho - g$ the effective discount rate, which is assumed to be greater than zero.

A worker receives a wage rate $w(t)$ when he enters an employment relationship. Let $r(t)$ denote the rate of return to capital net of depreciation at time t, and $\Pi(t)$ be the amount of dividends that a household receives by owning the firm. Thus, the momentary budget constraint faced by a representative domestic household is

$$\dot{K}(t) + C(t) = w(t)L_2(t) + r(t)K(t) + \Pi(t).$$

Dividing it by the size of population $N(t)$ gives the budget constraint in per capita terms as

$$\dot{k}(t) + k(t)g + c(t)\alpha = w(t)s_2^D(t)\alpha + r(t)k(t) + \pi(t), \tag{3}$$

where $\dot{\varrho}(t) \equiv \frac{d\varrho(t)}{dt}$ is the time derivative of the variable $\varrho(t)$, $\alpha = L(t)/N(t)$ is the ratio of domestic

[3] The same assumption is also adopted in Hazari and Sgro (2003), Moy and Yip (2006), and Palivos (2009).

[4] Following Merz (1995), we assume that there are a large number of agents in each household. They pool their income and care only about the household's utility. By doing so they provide each other with complete insurance against variations in labor income due to unemployment.

to total population, $\pi(t) = \Pi(t)/N(t)$ is dividend per capita and $k(t) = K(t)/N(t)$ is capital per capita.[5]

The number of employed domestic workers evolves according to

$$\dot{L}_2(t) = \gamma(t)L_1(t) - \theta L_2(t), \tag{4}$$

where $\gamma(t)$ is the rate at which unemployed workers find jobs and $\theta > 0$ is the job destruction rate. In equilibrium, $\gamma(t)$ is determined by the aggregate numbers of job vacancies and unemployed workers. In the utility maximization problem, however, $\gamma(t)$ is taken as given by a representative household. Upon dividing by $N(t)$, an individual's employment evolves according to the law of motion:

$$\dot{s}_2^D(t) = \gamma(t)s_1^D(t) - \theta s_2^D(t) - gs_2^D(t). \tag{5}$$

The representative domestic household's optimization problem is to choose a set of time paths $\{c(t),\, s_1^D(t),\, k(t),\, s_2^D(t)\}$ so as to maximize (2) subject to (3), (5) and two initial conditions: $k(0) > 0$, $1 > s_2^D(0) > 0$. Let $\lambda(t)$ and $\psi(t)$ be the costate variables. They denote the shadow prices of household's employment and capital, respectively. The first-order conditions of the representative household's optimization problem with respect to $\{c(t),\, s_1^D(t),\, k(t),\, s_2^D(t)\}$ and the associated transversality conditions (TVC) are given by

$$\begin{aligned}
u_c^{'}(t) &= \psi(t)\alpha, && (6)\\
u_{s_1^D}^{'}(t) &= -\lambda(t)\gamma(t), && (7)\\
\dot{\lambda}(t) &= (\rho+\theta)\lambda(t) - [\psi(t)w(t)\alpha + u_{s_1^D}^{'}(t)], && (8)\\
\frac{\dot{\psi}(t)}{\psi(t)} &= \rho - r(t), && (9)\\
\lim_{t\to\infty} e^{-(\rho-g)t}\lambda(t)s_2^D(t) &= 0, && (10)\\
\lim_{t\to\infty} e^{-(\rho-g)t}\psi(t)k(t) &= 0. && (11)
\end{aligned}$$

Equation (7) states the rule for the household to decide how much effort it should put into search. It requires the marginal cost of search to be equal to the marginal benefit of search.

[5] As defined earlier, $m = \frac{M(t)}{N(t)}$. Thus, $m = 1 - \alpha$ holds true for each time period.

Given the separable utility function form in (1), combining (6) and (9) and rearranging terms yield a simple expression for the Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho, \tag{12}$$

where the elasticity of intertemporal substitution of consumption is 1. This condition describes the evolution of individual's consumption. In other words, it states that if r exceeds ρ, then individual consumption will expand over time. By using (6), (7), and (8), we obtain

$$\dot{\lambda}(t) = (\rho+\theta)\lambda(t) + [w(t)\frac{u_c^{'}(t)}{u_{s_1^D}^{'}(t)} + 1]\gamma(t)\lambda(t). \tag{13}$$

An important implication of (13) is that in order to compensate for the search cost the wage rate has to be set above the marginal rate of substitution between leisure and consumption.[6]

2.2 Immigrant Household's Utility Maximization Problem

Similar to the domestic households, in each period each immigrant household member derives utility from consumption and disutility from working and searching for jobs. The momentary utility function of a typical immigrant agent is given by

$$u[c^M(t), s_1^M(t) + s_2^M(t)] = \log c^M(t) - \xi\frac{[s_1^M(t) + s_2^M(t)]^{1+\phi}}{1+\phi}, \;\xi > 0, \text{ and } \phi > 0. \tag{14}$$

The migrant household's total discounted utility is characterized by

$$U = \int_0^\infty e^{-(\rho-g)t} u[c^M(t), s_1^M(t) + s_2^M(t)]dt. \tag{15}$$

The variables $C^M(t)$ and $c^M(t) = C^M(t)/M(t)$ are aggregate and individual consumption of immigrant household, respectively. Under the conventional assumptions in the literature, illegal migrants are paid at the wage rate $w_M(t)$ which is distinct from that paid to domestic labor, $w(t)$.[7] This is due to the fact that in most developed countries, firms have to pay a fine once they are caught

[6] It can be shown that if $w = -\frac{u_{s_1^D}^{'}(t)}{u_c^{'}(t)}$ as in a typical neoclassical model, the shadow price of employment $\lambda(t)$ can grow without bound.

[7] See also Hazari and Sgro (2003), Moy and Yip (2006), and Palivos (2009).

hiring illegal migrants. In a competitive market, the wage rate $w_M(t)$ is equated to the marginal product of the illegal immigrants minus the expected value of the punishment. In this study, as described later, the wage rate for domestic workers is endogenously determined through a Nash bargaining process. Given the assumption that firms are able to distinguish illegal immigrants from domestic workers, the wage rate for illegal immigrants is therefore equated to the wage rate of domestic workers minus the expected value of the punishment. Moreover, it's assumed that illegal migrants do not accumulate capital in the host country. This can be justified by the fact that in most developed countries illegal immigrants find no way to legally establish credit and own assets.[8] The budget constraint that a representative migrant household faces is therefore

$$C^M(t) = w_M(t)M_2(t). \tag{16}$$

Dividing it by $N(t)$ gives the budget constraint in per capita terms as

$$c^M(t) = w_M(t)s_2^M(t). \tag{17}$$

Analogous to (4), the number of employed illegal immigrants evolves according to

$$\dot{M}_2(t) = \gamma(t)M_1(t) - \theta M_2(t). \tag{18}$$

Upon dividing by $N(t)$, an individual's employment evolves as follows:

$$\dot{s}_2^M(t) = \gamma(t)s_1^M(t) - (\theta + g)s_2^M(t). \tag{19}$$

Let $\tilde{\lambda}(t)$ be the costate variable of illegal immigrant household's employment. The maximization conditions for the representative immigrant household with respect to $\left\{s_1^M(t),\, s_2^M(t)\right\}$ and the

[8] This assumption is also made in Hazari and Sgro (2003), Moy and Yip (2006), and Palivos (2009). In Hazari and Sgro (2003) and Moy and Yip (2006), it's assumed that immigrants do not save and hence their consumption is equal to their income. Palivos (2009) assumes that immigrants do save but they channel all their savings abroad. The capital accumulation process in the host country is not affected by the illegal immigrants' consumption-saving decisions in either way. Therefore, it doesn't matter how illegal immigrant households split their income.

associated TVC are

$$
\begin{aligned}
u_{s_1^M}(t) &= -\tilde{\lambda}(t)\gamma(t), && (20)\\
\dot{\tilde{\lambda}}(t) &= (\rho+\theta)\tilde{\lambda}(t) - [u_{c^M}(t)w_M(t) + U_{s_2^M}(t)], && (21)\\
\lim_{t\to\infty} e^{-(\rho-g)t}\tilde{\lambda}(t)s_2^M(t) &= 0. && (22)
\end{aligned}
$$

In particular, (20) governs illegal immigrant household's optimal decision on the search effort.

2.3 Production

In this economy, there are a large number of identical firms. Firms hire both domestic and foreign labor from the labor market to produce output. In order to hire labor, the firm has to post job vacancies $V(t)$. Each vacancy costs $d > 0$ units of output. The probability that a firm finds an unemployed worker is $\mu(t)$. Similar to $\gamma(t)$, $\mu(t)$ is determined by the aggregate numbers of job vacancies and unemployed workers in equilibrium. However, in the profit maximization problem, $\mu(t)$ is taken as given by a representative firm. The law of motion of a firm's employment is given by:

$$\dot{L}_2(t) + \dot{M}_2(t) = \mu(t)V(t) - \theta[L_2(t) + M_2(t)]. \quad (23)$$

Taking the factor prices as given, a representative firm chooses a set of time paths $\{K(t),\ L_2(t),\ M_2(t),\ V(t)\}$ so as to maximize its present value of the future profit streams. Formally, this is given by

$$Max\ \Gamma = \int_0^\infty e^{-\int_0^t r(\tau)d\tau}\Pi(t)dt,$$

subject to (23), and

$$\Pi(t) = F[K(t), L_2(t) + M_2(t)] - [r(t)+\delta]K(t) - w(t)L_2(t) - w_M(t)M_2(t) - pM_2(t) - dV(t). \quad (24)$$

The parameter δ is the rate of capital depreciation, and $p \in (0,1)$ the probability that a firm which employs illegal migrants gets detected.[9] The fine for employing illegal migrants is normalized to one per illegal immigrant worker. Let $\chi(t)$ and $\Omega(t)$ be the costate variables of firm's employment

[9] This probability can surely be affected by a country's enforcement budget. In the present model, it's assumed to be constant.

of domestic and foreign labor, respectively. Interior solutions of the above maximization problem are characterized by the first-order conditions

$$F_k'(t) = r(t) + \delta, \tag{25}$$

$$\chi(t) = \frac{d}{\mu(t)}, \tag{26}$$

$$\Omega(t) = \chi(t), \tag{27}$$

$$\dot{\chi}(t) = [r(t) + \theta]\chi(t) + w(t) - F_{L_2}(t), \tag{28}$$

$$\dot{\Omega}(t) = [r(t) + \theta]\Omega(t) + w_M(t) + p - F_{M_2}(t). \tag{29}$$

Equation (25) is the usual condition which states that the rental rate on capital is equated to the marginal product of capital. Equation (26) governs the firm's optimal vacancy decisions. The marginal cost of vacancy d equals the marginal benefit of vacancy $\chi(t)\mu(t)$. Equation (28) demonstrates that if there is no vacancy maintaining cost for the firm i.e., $d = 0$, we would obtain the standard neoclassical productivity condition for labor $w(t) = F_{L_2}(t)$. In that case, firms would post an infinite number of vacancies and there will not any search frictions in the labor market. With positive d, however, the wage rate for domestic workers $w(t)$ is less than the marginal product of labor $F_{L_2}(t)$ in this model.

The relationship between the wage rates paid to domestic worker $w(t)$ and illegal immigrant $w_M(t)$ is given by

$$w_M(t) = w(t) - p. \tag{30}$$

The wage rate $w(t)$ for domestic workers is determined through a Nash bargaining process which will become clear later on. As p is positive, it follows that wage rate $w_M(t)$ paid to illegal migrants is strictly lower than that paid to domestic labor. Notice that the above condition also indicates that the punishment of hiring illegal immigrants is completely borne by the illegal immigrants themselves. Firms therefore do not suffer directly from employing illegal immigrant workers.

2.4 Matching and Wage Determination

The labor market is subject to search-matching frictions. Vacant jobs and unemployed workers are brought together in a pair-wise fashion by a stochastic search-matching process. The search part follows from the fact that both domestic workers and immigrants invest some time and effort in searching for jobs. Meanwhile, firms seek workers to fill vacant job positions. The matching part of the process is derived from a matching function which pairs the unemployed workers with vacancies. For analytical convenience, we employ a Cobb-Douglas matching function with constant returns-to-scale.[10] The number of successful job matches Φ is determined by the following matching function:

$$\Phi[V(t), L_1(t) + M_1(t)] = \gamma_0[V(t)]^\eta[L_1(t) + M_1(t)]^{1-\eta}, \qquad \eta \in (0,1)$$

where $V(t)$ is the number of vacancies, $L_1(t)+M_1(t)$ is the number of unemployed workers searching for jobs, η is the elasticity of vacancy in job matches, and $\gamma_0 > 0$ is assumed to be constant over time.

Given the Cobb-Douglas matching function, the vacancy-matching rate $\mu(t)$ and the job-finding rate $\gamma(t)$ are obtained as follows:

$$\mu(t) = \frac{\Phi(t)}{V(t)} = \gamma_0[x(t)]^{\eta-1}, \tag{31}$$

$$\gamma(t) = \frac{\Phi(t)}{L_1(t) + M_1(t)} = \gamma_0[x(t)]^\eta, \tag{32}$$

$$\Rightarrow \quad \mu(t) = \frac{\gamma(t)}{x(t)},$$

where the ratio between the vacancies and the unemployed workers, $x(t) = \frac{V(t)}{L_1(t)+M_1(t)}$, is conventionally labeled as the tightness of the labor market. Intuitively, it captures the pressure that unemployed workers and firms face in the labor market. Specifically, workers and employers face a tighter labor market when $x(t)$ is smaller. The above expressions make clear the dependence of the rates $\mu(t)$ and $\gamma(t)$ on the tightness of the labor market $x(t)$. In particular, $\mu(t)$ falls with $x(t)$ and $\gamma(t)$ rises with $x(t)$.

With the use of this matching function, the equilibrium outcomes are not Pareto optimal. This is due to the presence of search externalities inherent in the model. The intuition is as follows. On the one hand, with more unemployed workers participating in search, firms will be beneficial since vacancies are more likely to be filled. However, unemployed workers will suffer as their chance to

[10] The Cobb-Douglas matching function is also empirically verified. See, for instance, Blanchard and Diamond (1989).

match themselves with a job is reduced. On the other hand, with more open vacancies, unemployed workers win while firms searching for workers lose. Thus, the decentralized outcome is not efficient because workers and firms do not take into consideration the costs that they impose on others. The activities that generate a negative externality are carried out to a greater extent than are socially desirable.

Unemployed domestic workers and vacant jobs meet in pairs. A successful job match generates a surplus for both unemployed domestic workers and employers. How is this surplus shared between them? It is a matter of bargaining. The standard search and matching model assumes that by choosing a proper wage rate this surplus is maximized according to the Nash solution to a bargaining problem. In particular, if a firm hires a domestic worker, then the surplus to the firm from employing him is $f^{'}_{s_2^D}(t) - w(t)$.[11] On the other hand, if a domestic worker chooses to work for a firm, then the gain to him from accepting the job is $w(t) - [-\frac{u^{'}_{s_2^D}(t)}{u^{'}_c(t)}]$.[12] Hence, there exists a possibility of a mutually advantageous deal. Let $\beta \in (0,1)$ and $1-\beta$ represent the relative bargaining powers of domestic labor and firms, respectively. A domestic worker and a firm jointly determine the employment contract under the assumption that each firm-worker pair takes the behavior of other such pairings as given. The optimal wage contract under Nash bargaining is derived by solving

$$\underset{w(t)}{Max}\ \{(1-\beta)\log[f^{'}_{s_2^D}(t) - w(t)] + \beta\log[w(t) - (-\frac{u^{'}_{s_2^D}(t)}{u^{'}_c(t)})]\}.$$

The solution of this is given by

$$w(t) = \beta f^{'}_{s_2^D}(t) + (1-\beta)[-\frac{u^{'}_{s_2^D}(t)}{u^{'}_c(t)}]. \tag{33}$$

The optimal wage is a weighted average of the worker's marginal product of labor and reservation wage, which is the marginal rate of substitution between consumption and leisure. If domestic workers have relative stronger bargaining strength, i.e., β is closer to one, then the optimal wage is closer to the marginal product of labor. In this model, illegal immigrants have no bargaining

[11] As denoted above, $k(t) = K(t)/N(t)$, the Cobb-Douglas production function in per capita terms can be written as $f[k, \alpha s_2^D + (1-\alpha)s_2^M] = k^{\epsilon}[\alpha s_2^D + (1-\alpha)s_2^M]^{1-\epsilon}$.

[12] The derivative of u with respect to $s_2^D(t)$ is negative i.e., $u^{'}_{s_2^D(t)} < 0$. The expression $[-\frac{u^{'}_{s_2^D(t)}}{u^{'}_{c(t)}}]$ represents the domestic workers' endogenized reservation wage.

power, i.e., they are not allowed to bargain over the wage with the firms. Rather, as mentioned above, their wage rate $w_M(t)$ is determined by (30).

2.5 Market Equilibrium

In this subsection, we provide all the necessary ingredients of this model as follows:

Definition *A search equilibrium consists of a set of time paths* $\{c(t), c^M(t), k(t), v(t), s_1^D(t), s_2^D(t), s_1^M(t), s_2^M(t) \mid t \geq 0\}$, *prices* $\{r(t), w(t), w_M(t) \mid t \geq 0\}$, *profit* $\{\pi(t) \mid t \geq 0\}$, *and matching rates* $\{\gamma(t), \mu(t) \mid t \geq 0\}$ *such that*

1. *Given* $\{r(t), w(t), \pi(t), \gamma(t) \mid t \geq 0\}$, $\{c(t), k(t), s_1^D(t), s_2^D(t) \mid t \geq 0\}$ *solves the domestic household's problem.*

2. *Given* $\{w_M(t), \gamma(t) \mid t \geq 0\}$, $\{c^M(t), s_1^M(t), s_2^M(t) \mid t \geq 0\}$ *solves the immigrant household's problem.*

3. *Given* $\{r(t), w(t), w_M(t), \mu(t) \mid t \geq 0\}$, $\{k(t), v(t), s_2^D(t), s_2^M(t) \mid t \geq 0\}$ *solves the firm's problem.*

4. *The wage rate* $w(t)$ *is determined by (33).*

5. *The matching rates are given by (31) and (32).*

6. *All markets clear.*

   (a) *The goods market clears at every* $t \geq 0$, i.e.,

   $$C(t) + \dot{K}(t) + \delta K(t) = F[K(t), L_2(t) + M_2(t)] - C^M(t) - pM_2(t) - dV(t) \text{ for all } t \geq 0.$$

   (b) *In the labor market, in equilibrium, the flows of workers into employment must equal the flows of vacancies matched with unemployed agents, i.e.,* $\gamma(t)[L_1(t) + M_1(t)] = \mu(t)V(t)$. *(4), (18), and (23) indicate that the total supply for labor equals the demand for labor at every* $t \geq 0$.

2.6 Characterization of Equilibrium

After simple manipulations and substitutions, the equilibrium defined above is summarized by the following seven differential equations which together determine the dynamic properties of $\Psi \equiv [c(t), k(t), x(t), s_1^D(t), s_2^D(t), s_1^M(t), s_2^M(t)]^T$.

$$\frac{\dot{c}(t)}{c(t)} = f_k'(t) - \delta - \rho, \tag{34}$$

$$\dot{k}(t) = f(t) - k(t)g - k(t)\delta - c(t)\alpha - [\beta f_{s_2^D}'(t) + (1-\beta)\frac{u_{s_2^D}'(t)}{u_c'(t)}]s_2^M(t)(1-\alpha) - dv(t), \tag{35}$$

$$\dot{x}(t) = \frac{x(t)}{1-\eta}(\rho+\theta) - \frac{\gamma(t)(1-\beta)}{d(1-\eta)}[f_{s_2^D}(t) - \frac{u_{s_2^D}'(t)}{u_c'(t)}], \tag{36}$$

$$\dot{s}_1^D(t) = (\rho+\theta)\frac{u_{s_1^D}'(t)}{u_{s_1^D}''(t)s_1^D(t)} + \frac{u_c'(t)w(t)\gamma(t)}{u_{s_1^D}''(t)s_1^D(t)} + \frac{u_{s_2^D}'(t)\gamma(t)}{u_{s_1^D}''(t)s_1^D(t)}, \tag{37}$$

$$\dot{s}_2^D(t) = \gamma(t)s_1^D(t) - (\theta+g)s_2^D(t), \tag{38}$$

$$\dot{s}_1^M(t) = (\rho+\theta)\frac{u_{s_1^M}'(t)}{u_{s_1^M}''(t)s_1^M(t)} + \frac{u_{c^M}'(t)w_M(t)\gamma(t)}{u_{s_1^M}''(t)s_1^M(t)} + \frac{u_{s_2^M}'(t)\gamma(t)}{u_{s_1^M}''(t)s_1^M(t)}, \tag{39}$$

$$\dot{s}_2^M(t) = \gamma(t)s_1^M(t) - (\theta+g)s_2^M(t). \tag{40}$$

The three initial conditions of this system are $k(0)$, $s_2^D(0)$, and $s_2^M(0)$.[13]

Proposition 1. *A unique steady state* Ψ^* *exists.*

All proofs can be found in the Appendix. Notice that this unique steady state is in per capita terms. All aggregate variables, such as $K(t), C(t)$, are still growing at rate $g > 0$.

3 Quantitative Analysis

In this section, we develop the quantitative implications of our model. Thus, we first numerically solve and calibrate the model to match some key statistics of the postwar U.S. economy. Then we discuss those quantitative predictions in order. Specifically, we answer the following three questions:

1. Will the long-run level of consumption of domestic citizens decrease in the population share of illegal immigrants?

[13]The mathematical derivations of these differential equations are available from the author upon request.

2. How domestic workers' employment opportunities are affected by illegal immigration flows in the long run?

3. What is the welfare effect of illegal immigration on the host country?

3.1 Parameterization

This subsection presents the procedure used to parameterize the model. The specific numerical values to the parameters of the model are assigned so that the model can match as closely as possible some key statistics for the U.S. economy for the postwar period. In particular, the model aims to match U.S. facts on the labor participation rate, the unemployment rate, the average capital-output ratio, and the real interest rate.

There are eleven parameters which need to be assigned in this model: the preference parameters ρ, ϕ, and ξ, the production parameters ϵ and δ, the search-matching parameters γ_0, η, θ, the rate of population growth g, the bargaining power of domestic labor β and the unit cost of vacancy d. As a time period is normalized to be one quarter, each parameter is interpreted quarterly.

The preference parameter ξ is set equal to 3.7939 to match the steady-state labor force participation rate of 0.68. This is consistent with the U.S. labor force participation rate for the population aged 16 years old and over in the postwar period.[14] We choose the depreciation rate on capital $\delta = 0.0108$ so that the quarterly capital-output ratio in the steady state is equal to 12, which roughly matches the average capital-output ratio in postwar U.S. data (Cooley et al., 1995). The unit cost of vacancy d is set at 2.064 to achieve the steady-state unemployment rate of 0.06, which matches the U.S. quarterly average unemployment rate in the postwar period.[15] We use the discount rate $\rho = 0.01$ so that the steady-state annual interest rate is roughly 4% (Siegel 2002).[16]

The value of the capital's share of national income ϵ is set to 0.25, which falls in the range in Gollin (2002).[17] We set the value of $\phi = 2.5$ to obtain a labor supply elasticity of $\varkappa = 0.4$ (Killingsworth 1983). We also allow $\varkappa$ to take different values 0.2, 0.7 and 1. The parameter γ_0 is commonly normalized to one. As indicated in Blanchard and Diamond (1989), the parameter for

[14] Source: U.S Bureau of Labor Statistics
<http://data.bls.gov/PDQ/servlet/SurveyOutputServlet?data_tool=latest_numbers&series_id=LNS11300000>

[15] U.S. Bureau of Labor Statistics has documented the annual average unemployment rate from 1948 to the present.

[16] Siegel (2002) suggests the average of the real return to stock and long-term bonds over the period 1946-2001 is 0.042.

[17] Gollin (2002) indicates that the labor shares for most countries fall in the range of 0.65-0.80.

the elasticity of vacancy in job matches is 0.6, hence $\eta = 0.6$. We use the exogenous job destruction rate $\theta = 0.05$, which resembles the quarterly employment-unemployment transition probability (Shi and Wen 1999). The value of bargaining power of labor β is set to 0.5, a value commonly used in the literature. We use the rate of population growth $g = 0.0027$ as the annual population growth rate in the postwar US is roughly 1%. The baseline parameterization is summarized in Table 1.

Table 1: Baseline Parameter Values

Preferences	$\rho = 0.01, \phi = 2.5, \xi = 3.7939.$
Production	$\epsilon = 0.25, \delta = 0.0108.$
Matching	$\gamma_0 = 1, \eta = 0.6, \theta = 0.05.$
Others	$g = 0.0027, \beta = 0.5, d = 2.064.$

3.2 Local Dynamics

We next examine local dynamics by linearizing the system of differential equations in the neighborhood of the steady state. As stated in Proposition 1, the nonlinear dynamic system has a unique steady state at $\Psi^* \equiv (c^*, k^*, x^*, s_1^{D*}, s_2^{D*}, s_1^{M*}, s_2^{M*})^T$. Let J be the 7×7 Jacobian matrix evaluated at the steady state Ψ^*.

The dynamic properties of the linearized system is determined by the eigenvalues of the Jacobian matrix J. The predetermined variables are $k(t)$, $s_2^D(t)$, and $s_2^M(t)$. Saddle-path stability requires that the number of stable eigenvalues be exactly the same as the number of predetermined variables. Therefore, the matrix J needs to have three stable eigenvalues and four unstable eigenvalues in order to ensure the existence of a unique transition path.

In the quantitative exercise, we allow the population share of illegal immigrants m to vary between 0 and 0.5.[18] In all of these experiments we obtain three stable eigenvalues and four unstable eigenvalues. By allowing $\varkappa$ to take values in $\{0.2, 0.4, 0.7, 1\}$, we find that the above result is robust with respect to changes in the labor supply elasticity. The values of the stable eigenvalues are reported in Table 2. Thus, the unique steady state is saddle-path stable under the baseline parameterization.

[18] In this study, we only consider the case in which the number of illegal immigrants is less than that of domestic citizens. Therefore, we allow m to vary from 0 through 0.5.

3.3 Macroeconomic Effects

In this subsection, we develop the quantitative implications of the model. In particular, we focus on the steady-state effects of illegal immigration. In order to demonstrate the economic impact of illegal immigration on domestic residents, we now perform some comparative static experiments. In the first comparative static experiment we are concerned with the effects on the long-run level of domestic consumption when there is an increase in the share of illegal immigrants in the population. Specifically, by allowing the fraction of immigration m to take values from 0 through 0.5, we compute a series of steady states to capture the response of domestic consumption to an influx of illegal immigrants.

The quantitative prediction of the present model is that the long-run level of consumption of domestic citizens has a U-shaped relationship with the share of illegal immigrants (see Figure 2). In other words, an increase in the number of illegal immigrants first reduces and then raises the long-run consumption of the domestic citizens.[19]

The intuitions of these results are as follows. The presence of illegal migration has four effects. The first one is the exploitation effect. As shown in Figure 2, when there is an increase in the number of illegal immigrants, a greater number of unemployed illegal immigrants are searching for jobs. In contrast, the change in the number of domestic workers searching for jobs is small. This leads to a tighter labor market which in turn leads to more fierce competition for jobs. To successfully secure a job, both domestic and foreign labor would have to lower their wages. The firms therefore make more profits. In turn, domestic citizens receive more dividends which can be used for consumption and investment. This effect adds to domestic consumption. Second, the capital-using-up effect. This is due to the fact that the illegal immigrants do not save in the domestic economy. Some capital has to be used to produce output for the consumption of illegal migrants. This effect reduces current output which could have been used for domestic consumption and investment. Third, the displacement effect. As unemployed domestic labor and migrants compete for jobs, the chance for unemployed domestic workers to find a job is reduced. This effect lowers their consumption. Fourth, the wage depressing effect of illegal immigrants.[20] As more

[19] As the number of illegal immigrants increases, the variable α decreases and vice versa.

[20] This wage depressing effect of illegal immigrant workers has been documented in Hotchkiss and Myriam (2008). Borjas (2003) also concludes that a 10-percent increase in labor supply could reduce wages by 3-4 percent.

illegal immigrants enter into the economy, the competition for jobs becomes more severe. Thus, the wages of domestic labor are pushed down. The net impact of illegal immigration on domestic consumption hinges upon the magnitude of these four effects. If the exploitation effect dominates, domestic consumption will rise. Otherwise, it will fall.

More precisely, the steady-state equilibrium value of the domestic consumption c^* is determined by

$$c^* = w^* s_2^{D*} + (\rho - g)\frac{k^*}{\alpha} + \frac{\pi^*}{\alpha}. \tag{41}$$

To understand the intuition of this comparative static finding, we differentiate (41) with respect to α and obtain

$$\begin{aligned}\frac{dc^*}{d\alpha} = {} & \underbrace{[w^* \overset{\text{positive}}{\frac{ds_2^{D*}}{d\alpha}}}_{\text{the displacement effect}} + \underbrace{\overset{\text{positive}}{s_2^{D*}\frac{dw^*}{d\alpha}}]}_{\text{the wage depressing effect}} + \underbrace{[\underbrace{\frac{1}{\alpha}\frac{d\pi^*}{d\alpha}}_{\text{negative}} + \underbrace{(-\frac{1}{\alpha}\frac{\pi^*}{\alpha})}_{\text{negative}}]}_{\text{the exploitation effect}} \\ & + \underbrace{[\underbrace{\frac{1}{\alpha}(\rho - g)\frac{dk^*}{d\alpha}}_{\text{positive}} + \underbrace{(-\frac{1}{\alpha}(\rho - g)\frac{k^*}{\alpha})}_{\text{negative}}]}_{\text{the capital-using-up effect}} \end{aligned} \tag{42}$$

Equation (42) implies that there are three results generated by an increase in the population share of illegal immigrants (a decrease in α) in the U.S. The first result is that domestic labor income falls. This is due to the displacement and negative wage depressing effects, which are captured by the two terms in the first square bracket of (42), respectively. The second square bracket reflects the positive exploitation effect. The reason is that as more illegal migrants are in the U.S., domestic households receive more dividends which can be used for consumption and investment. The last square bracket shows how the domestic capital income is affected by the inflows of illegal migrants. When the share of illegal immigrants goes up, the capital per worker k^* declines while the capital per domestic citizen $\frac{k^*}{\alpha}$ rises, which generates additional income for domestic households.[21]

Figure 2 summarizes the responses of the key variables in this model ($c^*, x^*, k^*, w^*, s_1^{D*} + s_2^{D*}, s_1^{M*} + s_2^{M*}, \frac{s_1^{D*}}{s_1^{D*}+s_2^{D*}}, \frac{s_1^{M*}}{s_1^{M*}+s_2^{M*}}$) to a gradual increase in illegal immigration.[22] When it goes up, the model predicts that workers and employers face a tighter labor market, i.e., x^* goes down,

[21] In the steady state, $r = \rho$. Therefore, the capital income is solely determined by the quantity of capital.

[22] For different values of $\varkappa$ (e.g. $\varkappa = 0.2, 0.7$, and 1), see corresponding Figure 1, 3,and 4 in the appendix.

the capital per worker k^* reduces, the unemployment rate for domestic labor $\frac{s_1^{D*}}{s_1^{D*}+s_2^{D*}}$ rises, the wage rate w^* for domestic labor drops, the fraction of domestic residents in search s_1^{D*} rises first and then declines, and the labor force participation rate of domestic residents $s_1^{D*} + s_2^{D*}$ falls. Notice that comparing with domestic residents, we observe that there is only a small reduction in the labor force participation rate for illegal immigrants $s_1^{M*} + s_2^{M*}$. The reason for this is that with more illegal immigrants in search, domestic workers find the opportunity cost of searching for jobs becomes higher so that it's optimal to withdraw from supplying labor and to enjoy leisure instead.[23] This result turns out to be consistent with the existing empirical evidence. Borjas et al. (2007) report that a 10-percent immigrant-induced increase in the supply of a particular skill group is associated with a reduction in the black employment rate of 3.5 percentage points, and a 1.6 percentage point reduction in the employment rate of white men.

Our results are in sharp contrast with those obtained in previous studies. Analyzing the issue of illegal immigration under the full employment assumption, Hazari and Sgro (2003) conclude that illegal immigration necessarily lowers the long-run per capita domestic consumption. Palivos (2009) obtains an unambiguous positive effect of illegal immigration. It raises the consumption and welfare of domestic workers. Palivos (2009) also considers a case in which a binding minimum wage only applies to unskilled workers. His finding is that illegal immigration decreases domestic consumption.

3.4 Welfare Effects

In order to answer the question of how illegal immigration affects domestic welfare, we compute and compare, using a consumption-equivalent measure as in Lucas (1987), the level of utility of domestic households under two scenarios.

Let $\left\{c(t;m),\ s_1^D(t;m),\ s_2^D(t;m)\right\}$ denote the equilibrium time paths when the population share of illegal immigrants is m. The lifetime utility of the representative domestic household is given by

$$U(m) = \int_0^\infty \{\log c(t;m) - \xi \frac{[s_1^D(t;m) + s_2^D(t;m)]^{1+\phi}}{1+\phi}\} e^{-(\rho-g)t} dt.$$

[23] The gap between domestic worker's wage and reservation wage is narrowed with more immigrants flowing into the country.

The consumption-equivalent measure $\kappa(m)$ is defined by

$$\begin{aligned}\int_0^\infty \{\log[1+\kappa(m)]c^{'}(t;0) - \xi\frac{[s_1^{D'}(t;0)+s_2^{D'}(t;0)]^{1+\phi}}{1+\phi}\}e^{-(\rho-g)t}dt &= U(m) \\ \frac{1}{\rho-g}\log[1+\kappa(m)] + U(0) &= U(m)\end{aligned}$$

If $\kappa(m) > 0$, then $U(0) < U(m)$ which means that the domestic households are better off in the presence of illegal immigrants. In particular, the domestic households would require a $\kappa(m)$-percent increase in $c^{'}(t;0)$ in every period so as to make themselves indifferent between $m = 0$ and $m > 0$. Hence, illegal immigrants create a welfare gain to the host country's economy. On the contrary, if $\kappa(m) < 0$, then $U(0) > U(m)$. The domestic household are now willing to surrender $\kappa(m)$-percent of $c^{'}(t;0)$ in every period so as to expel the illegal immigrants. This means that illegal immigrants lead to a welfare loss to the host country.

Suppose the economy starts at the steady state with $m = 0$. The two scenarios that we consider are as follows:

1. The economy stays at the steady state with $m = 0$ forever.

2. At $t = 0$, the host country admits $m > 0$ fraction of illegal immigrants and the economy gradually converges to the new steady state. Hence, $U(m)$ is computed based upon the transition path. To account for the transition path, the procedure described in Cooley and Ohanian (1997) is carried out.

Given a specific numerical value of m, we can simply calculate the corresponding $\kappa(m)$.[24] Table 3 shows the welfare measure of illegal immigration. Three results can be drawn from Table 3. First, it is instructive to note that illegal immigration induces important net gains among domestic citizens for any values of $(m, \varkappa)$. For instance, when $\varkappa = 0.4$ and when there is an increase in m from zero to 5 percent in the US, the domestic households would require a .746-percent increase in $c^{'}(t;0)$ in every period. Second, these gains increase in the share of illegal immigrants in the population for each fixed $\varkappa$ that we consider. Third, $\kappa(m)$ clearly depends upon the magnitude

[24] We further restrict our attention to the case in which m can alter only from 0 through 20%. This is due to the fact that in the traditional host countries, nearly 24.6% of the population in Australia, 22.5% in New Zealand, 18.9% in Canada, and 12.3% in the United States is foreign-born (United Nations 2004). Among the foreign-born, only a fraction of them are illegal immigrants.

of the labor supply elasticity. In particular, for each fixed $m > 0$, $\kappa(m)$ decreases with the labor supply elasticity $\varkappa$.

Table 3: Welfare measure of illegal immigration[a]

	$\varkappa = 0.2$	$\varkappa = 0.4$	$\varkappa = 0.7$	$\varkappa = 1$
m	$\kappa(m)$	$\kappa(m)$	$\kappa(m)$	$\kappa(m)$
5%	0.756%	0.746%	0.73%	0.727%
10%	2.02%	1.978%	1.833%	1.78%
20%	6.684%	6.34%	5.98%	5.75%[b]

[a] Values of other parameters remain the same as in Table 1.

In order to shed some light on the above computational results, we differentiate the domestic household's utility with respect to m and obtain the following expression (43). As we require the economy to move from zero to an arbitrary amount of illegal immigration, we evaluate (43) at $m = 0$. Equation (43) reveals that the effect of illegal immigration on domestic welfare depends on two factors: (1) the change in the level of per capita consumption of domestic citizens, and (2) the change in the domestic labor participation rate.

$$\frac{du[c(t), s_1^D(t) + s_2^D(t)]}{dm} = u_c(t)\frac{dc(t)}{dm}|_{m=0} + u_{s_1^D+s_2^D}(t)\frac{d[s_1^D(t) + s_2^D(t)]}{dm}|_{m=0}. \tag{43}$$

These two factors jointly determine the welfare effect of illegal immigration. In general, it's not possible to obtain definite results analytically. We thus resort to numerical exercises. We focus on one particular example and examine the transition paths of consumption and leisure. The example that we consider here is when $\varkappa = 0.4$ and when there is an increase in m from zero to 5 percent. Under the baseline parameterization, illegal immigration lowers domestic consumption level throughout the entire transition. It first reduces and then raises the labor force participation rate during the transition (see Figure 5). By (43), we know that the overall welfare effect is ambiguous as these two changes tend to move domestic welfare in opposite directions. Nevertheless, according to our simulation, the positive welfare effect dominates. Thus, illegal immigration induces a welfare gain to the host country's economy. This welfare gain comes from an increase in leisure.

4 Concluding Remarks

This paper contributes to the existing literature on welfare effect of illegal immigration on domestic workers by introducing illegal immigration into a standard dynamic general equilibrium framework with labor market frictions. We therefore construct and calibrate a search-theoretic model. In the model economy, illegal immigrants enter domestic production as perfect substitutes for domestic workers. They are allowed to spend their one indivisible unit of time in searching for a job, working for a firm, or enjoying leisure in each period. Firms hire both domestic and illegal immigrant workers. Once unemployed domestic workers and vacant jobs are paired with each other, they jointly determine the wage rate through bilateral Nash bargaining. As we assume that firms are able to distinguish illegal immigrants from domestic workers and face a punishment for hiring the former if being caught. The wage rate for illegal immigrants is thus equated to the wage rate of domestic workers minus the expected value of the punishment.

We characterize the search equilibrium and prove the existence and uniqueness of stationary equilibrium. In contrast to the previous studies, our analysis reveals three striking results. First, although illegal immigration is indeed a boon to the United States, it significantly harms the employment opportunities of domestic workers. Namely, it increases the unemployment rate for domestic workers. Furthermore, it forces them to face a tighter labor market and even to leave the labor force. Second, we quantitatively prove that the long-run level of consumption of domestic citizens has a U-shaped relationship with the share of illegal immigrants. Third, illegal immigration's negative impact on native wages has been found in this framework. This result turns out to be qualitatively consistent with the empirical evidence.

To close the paper, we like to point out one line of future research. In this study, we assume that domestic workers and illegal immigrants are perfect substitutes. However, empirical evidence documents that even with the same level of education, they are not perfect substitutes.[25] Therefore, the analysis will become more interesting if illegal immigrants can be modeled as a separate factor

[25] For a related dicussion, see, among others, Borjas (2003) and Card and Lemieux (2001).

of production. Moreover, in real life, the debate over illegal immigration has also concerned with its distributional effects. Assuming that domestic and foreign labor differ in terms of their production skills, the distributional impact of illegal immigration on domestic workers can be analyzed in a search-theoretic framework. Nevertheless, this extension would not be trivial. We have to consider a two-sector version of the search model. This could significantly increase the dimension of the dynamic system.

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Appendix

Proof of Proposition 1.

In a steady-state equilibrium, consumption, c^*, capital, k^*, market tightness, x^*, fraction of domestic worker in search, s_1^{D*}, fraction of domestic workers in work, s_2^{D*}, fraction of illegal immigrants in search, s_1^{M*}, and fraction of illegal immigrant in work, s_2^{M*}, are constant over time.

Given the functional forms on utility and production functions, the steady state $(\Psi^* \equiv (c^*, k^*, x^*, s_1^{D*}, s_2^{D*}, s_1^{M*}, s_2^{M*})^T)$ can be described by the following seven equations with seven unknowns:

$$
\begin{aligned}
\frac{k^*}{\alpha s_2^{D*} + (1-\alpha) s_2^{M*}} &= \left(\frac{\epsilon}{\rho+\delta}\right)^{\frac{1}{1-\epsilon}}, & (44)\\
c^*\alpha &= \left(\frac{\epsilon}{\rho+\delta}\right)^{\frac{\epsilon}{1-\epsilon}}[\alpha s_2^{D*} + (1-\alpha)s_2^{M*}] - \beta(1-\epsilon)\alpha\left(\frac{\epsilon}{\rho+\delta}\right)^{\frac{\epsilon}{1-\epsilon}} s_2^{M*}(1-\alpha) & (45)\\
&\quad -(1-\beta)\xi c^*(s_1^{D*} + s_2^{D*})^{\phi} s_2^{M*}(1-\alpha) - dx^*[\alpha s_1^{D*} + (1-\alpha)s_1^{M*}] - k^* g - k^*\delta, &\\
x^*(\rho+\theta) &= \frac{\gamma(x^*)(1-\beta)}{d}\left[(1-\epsilon)\alpha\left(\frac{\epsilon}{\rho+\delta}\right)^{\frac{\epsilon}{1-\epsilon}} - \xi c^*(s_1^{D*} + s_2^{D*})^{\phi}\right], & (46)\\
\rho+\theta+\gamma(x^*) &= \frac{\beta(1-\epsilon)\left(\frac{\epsilon}{\rho+\delta}\right)^{\frac{\epsilon}{1-\epsilon}}\alpha\gamma(x^*)}{\xi c^*(s_1^{D*} + s_2^{D*})^{\phi}} + (1-\beta)\gamma(x^*), & (47)\\
\gamma(x^*)s_1^{D*} &= (\theta+g)s_2^{D*}, & (48)\\
\rho+\theta+\gamma(x^*) &= \frac{\gamma(x^*)}{\xi s_2^{M*}(s_1^{M*} + s_2^{M*})^{\phi}}, & (49)\\
\gamma(x^*)s_1^{M*} &= (\theta+g)s_2^{M*}. & (50)
\end{aligned}
$$

Determination of a unique steady state

The equations (44)-(50) together determine a unique steady state with the following steps.

Step 1, First find the expression for the marginal rate of substitution $\left(\frac{u'_{s_2^{D*}}}{u'_{c^*}}\right)$ by using (46). Substituting it into (47), we can solve for a unique x^*.

Step 2, Combining (49) together with (50) pins down a unique s_1^{M*}.

Step 3, s_2^{M*} is obtained by substituting s_1^{M*} into (50).

Step 4, Substituting (44), (48), and the expression for marginal rate of substitution $\left(\frac{u'_{s_2^{D*}}}{u'_{c^*}}\right)$ into (45) yields a unique s_2^{D*}.

Step 5, The solution for s_1^{D*} can be attained by substituting s_2^{D*} into (47).

Step 6, k^* is obtained by substituting s_2^{D*} and s_2^{M*} into (44).

Step 7, Finally, substituting s_1^{D*}, s_2^{D*} and x^* into the expression for the marginal rate of substitution $(\frac{u'_{s_2^{D*}}}{u'_{c^*}})$ yields c^*.

Finding a unique solution for x^*

$\dot{x} = 0$ implies

$$x^*(\rho+\theta) = \frac{\gamma(x^*)(1-\beta)}{d}[(1-\epsilon)\alpha(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}} - \xi c^*(s_1^{D*} + s_2^{D*})^{\phi}]. \tag{51}$$

Equation (51) can reexpressed as

$$\xi c^*(s_1^{D*} + s_2^{D*})^{\phi} = (1-\epsilon)\alpha(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}} - \frac{dx^*(\rho+\theta)}{\gamma(x^*)(1-\beta)}. \tag{52}$$

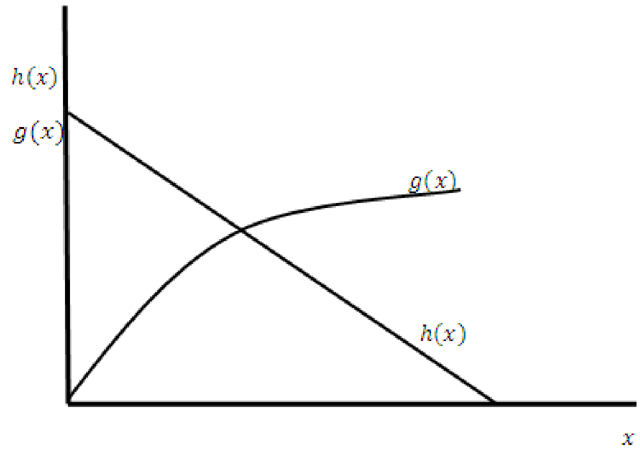
$\dot{s}_1^D = 0$ implies

$$\rho+\theta+\gamma(x^*) = \frac{\beta(1-\epsilon)(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}\alpha\gamma(x^*)}{\xi c(s_1^{D*} + s_2^{D*})^{\phi}} + (1-\beta)\gamma(x^*). \tag{53}$$

Substituting (52) into (53) yields

$$\begin{aligned} \rho+\theta+\gamma(x^*) \quad &= \quad \frac{\beta(1-\epsilon)(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}\alpha\gamma(x^*)}{(1-\epsilon)\alpha(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}} - \frac{dx^*(\rho+\theta)}{\gamma(x^*)(1-\beta)}} + (1-\beta)\gamma(x^*) \\ &\Rightarrow \quad -\beta x^*\gamma_0 + \frac{(1-\beta)(1-\epsilon)(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}\alpha}{d}\gamma_0 = (\rho+\theta)x^{*1-\eta}. \end{aligned} \tag{54}$$

Let $g(x^*) = (\rho+\theta)x^{*1-\eta}$ and $h(x^*) = -\beta x^*\gamma_0 + \frac{(1-\beta)(1-\epsilon)(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}\alpha}{d}\gamma_0$. Draw both $g(x^*)$ and $h(x^*)$ in the following diagram.



The curve for $g(x^*)$ is strictly increasing, but the curve for $h(x^*)$ is strictly decreasing with respect to x^*. In addition, $h(0) > g(0)$. If a solution to (54) exists then it is unique. In the above graph, it's easy to observe that there exists a unique intersection point between the above two curves. Thus, (54) yields a unique solution for x^*.

In order to establish the effect of an increase in illegal immigration on the labor market tightness, $\frac{\partial x^*}{\partial \alpha}$, one can differentiate (54) with respect of α.

$$
\begin{aligned}
-\beta x^* \gamma_0 + \frac{(1-\beta)(1-\epsilon)(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}\alpha}{d}\gamma_0 &= (\rho+\theta)x^{*1-\eta} \\
-\beta\gamma_0\frac{\partial x^*}{\partial \alpha} + \frac{(1-\beta)(1-\epsilon)(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}}{d}\gamma_0 &= (\rho+\theta)(1-\eta)x^{*-\eta}\frac{\partial x^*}{\partial \alpha} \\
\frac{\partial x^*}{\partial \alpha} &= \frac{\gamma_0(1-\beta)(1-\epsilon)(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}}{d[(\rho+\theta)(1-\eta)x^{-\eta}+\beta\gamma_0]} > 0. \qquad (55) \\
M &\uparrow \Rightarrow \alpha \downarrow \Rightarrow x^* \downarrow
\end{aligned}
$$

From (55), we show that there is a positive relationship between x^* and α. Therefore, more illegal immigrants induce a tighter labor market.

Finding the domestic unemployment rate

The unemployment rate for domestic labor is defined by $\frac{s_1^{D*}}{s_1^{D*}+s_2^{D*}}$. With the aid of $\gamma(x^*)s_1^{D*} = \theta s_2^{D*} + g s_2^{D*}$, the domestic unemployment rate can be reexpressed as

$$UR^{D*} = \frac{s_1^{D*}}{s_1^{D*}+s_2^{D*}} = \frac{1}{1+\frac{\gamma(x^*)}{\theta+g}}. \qquad (56)$$

Differentiating (56) with respect to α gives

$$\begin{aligned} \frac{\partial UR^{D*}}{\partial \alpha} &< 0 \\ M \quad \uparrow &\Rightarrow \alpha \downarrow \Rightarrow x^* \downarrow \Rightarrow \gamma(x^*) \downarrow \Rightarrow UR^{D*} \uparrow \end{aligned} \tag{57}$$

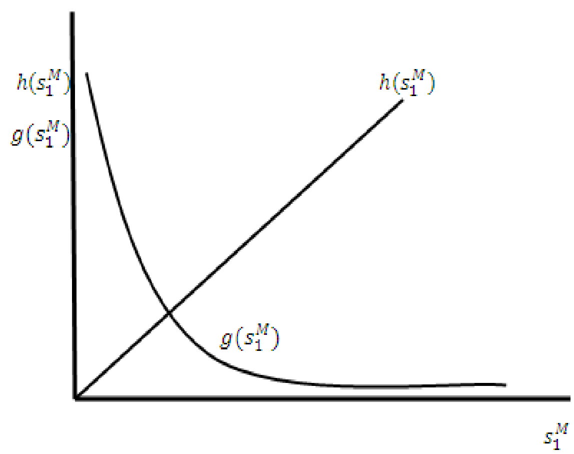
It shows that the long-run level of the unemployment rate for domestic citizens rises with more illegal immigrants entering into the economy.

Finding a unique solution for s_1^{M*}

Combining $\dot{s}_1^M = 0$ together with $\dot{s}_2^M = 0$ yields

$$\left.\begin{aligned} \dot{s}_1^M &= 0 \\ \dot{s}_2^M &= 0 \end{aligned}\right\} \Rightarrow \frac{\rho + \theta + \gamma(x^*)}{\theta + g} s_1^{M*} = \frac{1}{\xi}(1 + \frac{\gamma(x^*)}{\theta + g})^{-\phi} (s_1^{M*})^{-\phi}. \tag{58}$$

Let $h(s_1^{M*}) = \frac{\rho+\theta+\gamma(x^*)}{\theta+g} s_1^{M*}$ and $g(s_1^{M*}) = \frac{1}{\xi}(1 + \frac{\gamma(x^*)}{\theta+g})^{-\phi}(s_1^{M*})^{-\phi}$. Draw both $h(s_1^{M*})$ and $g(s_1^{M*})$ in the diagram below.



The curve for $h(s_1^{M*})$ is strictly increasing, but the curve for $g(s_1^{M*})$ is strictly decreasing with respect to s_1^{M*}. In addition, $g(0) > h(0)$. If a solution to (58) exists, then it is unique. It's easy to observe that there exists a unique intersection point between the above two curves which jointly determine a unique solution for s_1^{M*}.

Finding a unique solution for s_2^{M*}

Using the expression of $s_2^{M*} = \frac{\gamma(x^*) s_1^{M*}}{\theta+g}$, we can find a unique s_2^{M*}.

Finding the effect of an increase in illegal immigration on the fraction of migrants in search

Differentiating (58) with respect of α yields

$$\begin{aligned}
\frac{\rho+\theta+\gamma(x^*)}{\theta+g}s_1^{M*} &= \frac{1}{\xi}(1+\frac{\gamma(x^*)}{\theta+g})^{-\phi}(s_1^{M*})^{-\phi} \\
\Rightarrow \quad \frac{\partial s_1^{M*}}{\partial\alpha} &= -\frac{s_1^{M*}+\xi^{-1}\phi(1+\frac{\gamma(x^*)}{\theta+g})^{-\phi-1}(s_1^{M*})^{-\phi}}{\rho+\theta+\gamma(x^*)+\xi^{-1}(\theta+g)\phi(1+\frac{\gamma(x^*)}{\theta+g})^{-\phi}(s_1^{M*})^{-\phi-1}}\frac{\partial\gamma(x^*)}{\partial\alpha} \\
\Rightarrow \quad \frac{\partial s_1^{M*}}{\partial\alpha} &< 0. \qquad (59) \\
M \quad \uparrow \quad &\Rightarrow \alpha\downarrow\Rightarrow s_1^{M*}\uparrow
\end{aligned}$$

From (59), we observe a negative relationship between α and s_1^{M*}. Thus, when free entry is allowed, more illegal immigrants search for jobs in the long run.

Finding the effect of an increase in illegal immigration on the fraction of migrants in work

Equation (50) shows $\gamma(x^*)s_1^{M*}=\theta s_2^{M*}+gs_2^{M*}$.

Differentiating (50) with respect to α gives

$$\begin{aligned}
s_1^{M*}\frac{\partial\gamma(x^*)}{\partial\alpha}+\gamma(x^*)\frac{\partial s_1^{M*}}{\partial\alpha} &= (\theta+g)\frac{\partial s_2^{M*}}{\partial\alpha} \\
\Rightarrow \quad \frac{\partial s_2^{M*}}{\partial\alpha} &= \frac{s_1^{M*}}{\theta+g}\frac{\partial\gamma(x^*)}{\partial\alpha}+\frac{\gamma(x^*)}{\theta+g}\frac{\partial s_1^{M*}}{\partial\alpha} \\
\Rightarrow \quad \frac{\partial s_2^{M*}}{\partial\alpha} &= [\frac{s_1^{M*}}{\theta+g}+\frac{\gamma(x^*)}{\theta+g} \\
&\quad (-\frac{s_1^{M*}+\xi^{-1}\phi(1+\frac{\gamma(x^*)}{\theta+g})^{-\phi-1}(s_1^{M*})^{-\phi}}{\rho+\theta+\gamma+\xi^{-1}(\theta+g)\phi(1+\frac{\gamma(x^*)}{\theta+g})^{-\phi}(s_1^{M*})^{-\phi-1}})]\frac{\partial\gamma(x^*)}{\partial\alpha} \\
\Rightarrow \quad \frac{\partial s_2^{M*}}{\partial\alpha} &> 0. \\
M \quad \uparrow \quad &\Rightarrow \alpha\downarrow\Rightarrow s_2^{M*}\downarrow
\end{aligned}$$

The adverse consequence of the entry of illegal immigrants is that the fraction of illegal immigrants in work becomes lower.

Finding a unique solution for s_2^{D*}

Utilizing both $\frac{\dot{c}}{c}=0$ and $\dot{s}_1^D=0$, we can derive

$$\begin{aligned}
&\underbrace{\alpha[(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}-(\frac{\epsilon}{\rho+\delta})^{\frac{1}{1-\epsilon}}(g+\delta)-dx^*\frac{\theta+g}{\gamma(x^*)}]}_{A}s_2^{D*}-(1-\epsilon)(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}\alpha s_2^{M*}(1-\alpha)\\
&+(1-\alpha)s_2^{M*}[(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}-(\frac{\epsilon}{\rho+\delta})^{\frac{1}{1-\epsilon}}(g+\delta)]+\frac{dx^*(\rho+\theta)}{\gamma(x^*)}s_2^{M*}(1-\alpha)-dx^*(1-\alpha)s_1^{M*}\\
=\;&\alpha[(1-\epsilon)(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}\alpha-\frac{dx^*(\rho+\theta)}{\gamma(x^*)(1-\beta)}](1+\frac{\theta+g}{\gamma(x^*)})^{-\phi}\xi^{-1}(s_2^{D*})^{-\phi}. \qquad (60)
\end{aligned}$$

Let $A=\alpha[(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}-(\frac{\epsilon}{\rho+\delta})^{\frac{1}{1-\epsilon}}(g+\delta)-dx^*\frac{\theta+g}{\gamma(x^*)}]$

A direct calculation shows $(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}(1-\frac{g+\delta}{\rho+\delta}\epsilon)>(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}(1-\epsilon)$ as $\rho>g$. Using the expression for $\xi c^*(s_1^{D*}+s_2^{D*})^\phi$ given by (52), we have

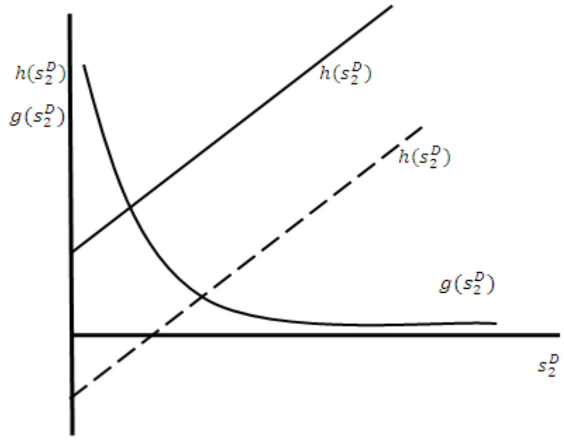
$$\begin{aligned}
\xi c^*(s_1^{D*}+s_2^{D*})^\phi &= (1-\epsilon)\alpha(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}-\frac{dx^*(\rho+\theta)}{\gamma(x^*)(1-\beta)}\\
&\Rightarrow (1-\epsilon)(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}>\frac{dx^*(\rho+\theta)}{\gamma(x^*)(1-\beta)\alpha}>0.
\end{aligned}$$

Therefore,

$$\begin{aligned}
(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}}(1-\frac{g+\delta}{\rho+\delta}\epsilon) &> \frac{dx^*(\rho+\theta)}{\gamma(x^*)(1-\beta)\alpha}>\frac{dx^*(\theta+g)}{\gamma(x^*)}\\
&\Rightarrow A>0.
\end{aligned}$$

as $0<\alpha<1$ and $0<\beta<1$.

Let LHS and RHS of (60) be $h(s_2^{D*})$ and $g(s_2^{D*})$, respectively.



There is a unique intersection point between the curves for $h(s_2^{D*})$ and $g(s_2^{D*})$, which leads to a unique solution for s_2^{D*}.

Finally, the unique s_2^{D*} implies the unique s_1^{D*}, k^*, and c^* as $s_1^{D*} = \frac{(\theta+g)}{\gamma(x^*)} s_2^{D*}$, $k^* = (\frac{\epsilon}{\rho+\delta})^{\frac{1}{1-\epsilon}}[\alpha s_2^{D*} + (1-\alpha)s_2^{M*}]$, and $c^* = \frac{1}{\xi(s_1^{D*}+s_2^{D*})^{\phi}}[(1-\epsilon)\alpha(\frac{\epsilon}{\rho+\delta})^{\frac{\epsilon}{1-\epsilon}} - \frac{dx^*(\rho+\theta)}{\gamma(x^*)(1-\beta)}]$.

The above procedures demonstrate that a steady state of this dynamic system can be uniquely determined. ■

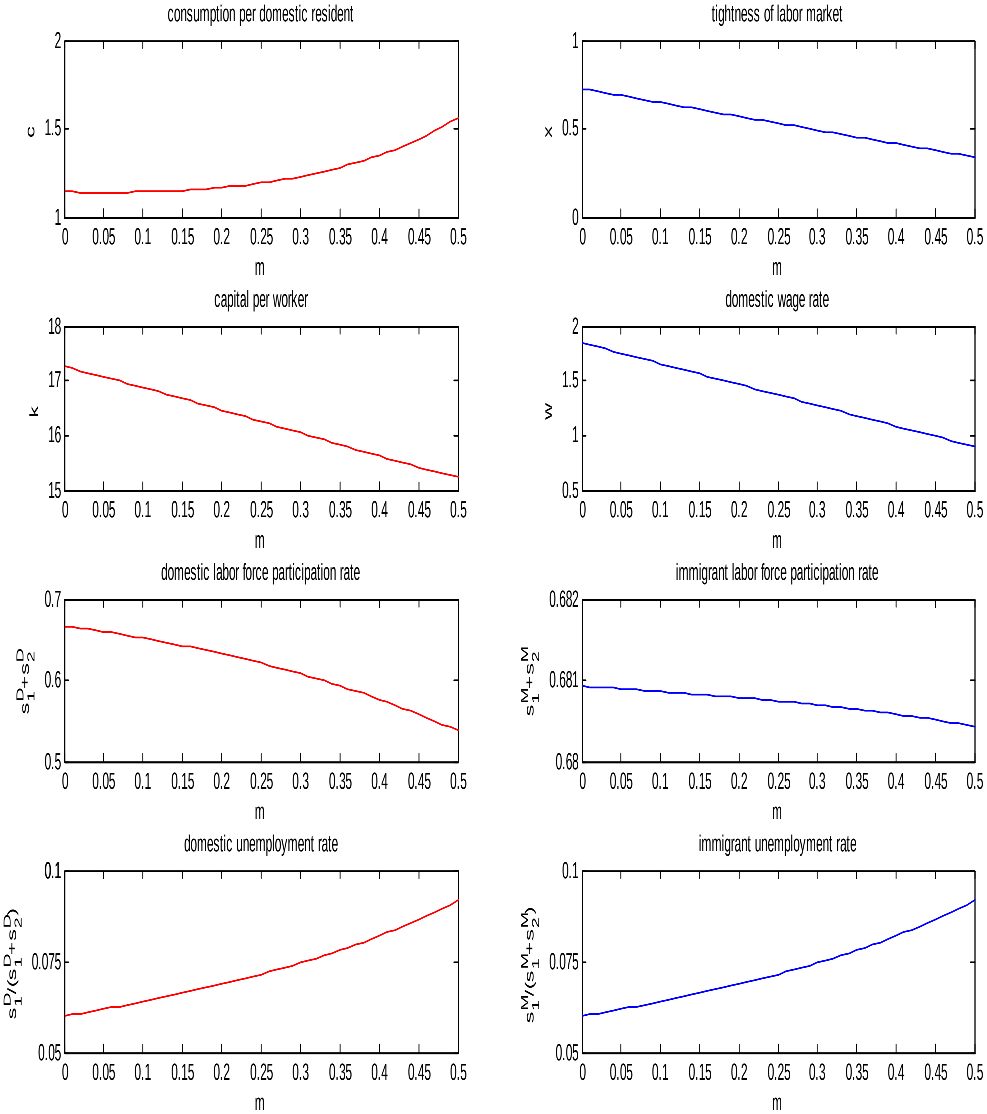


Figure 1: Long-run effects of illegal immigration when $\varkappa = 0.2$.

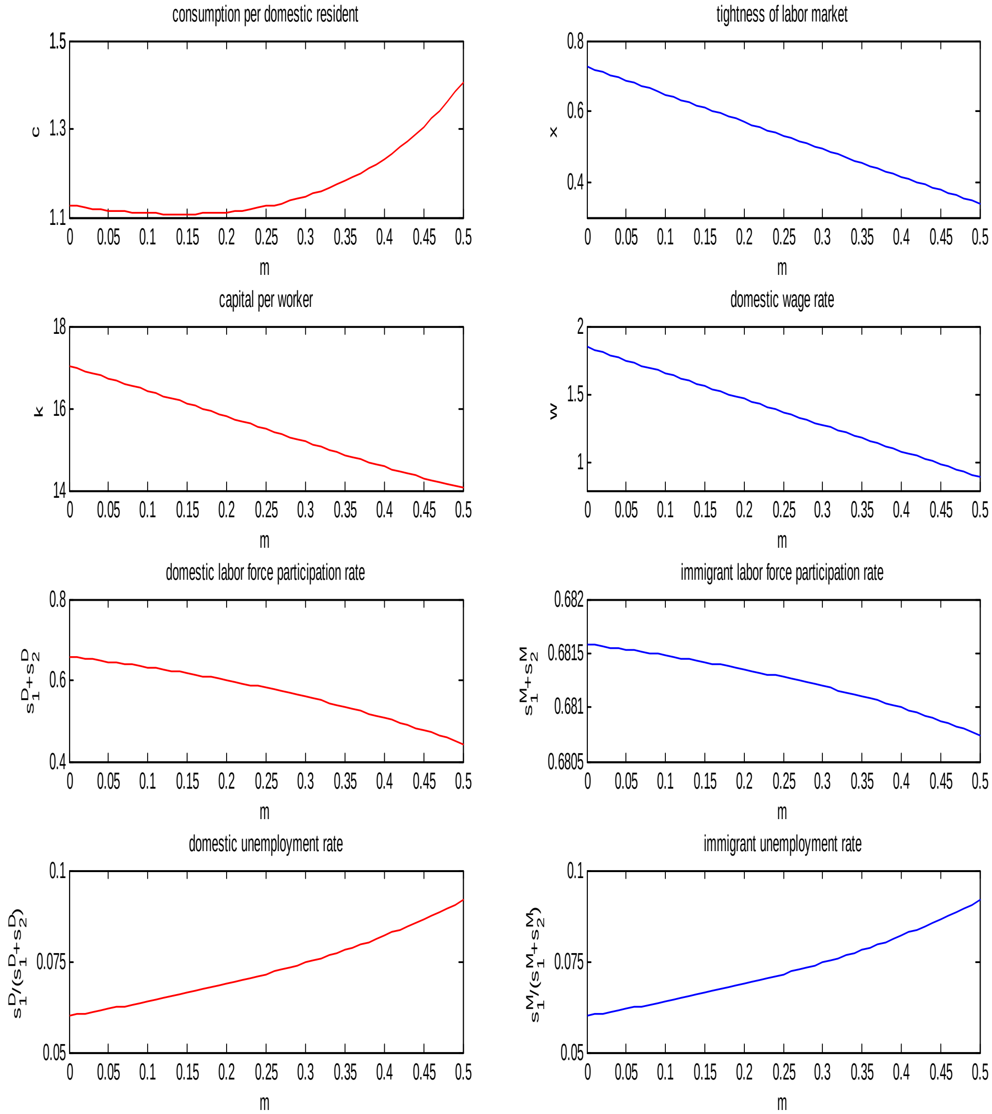


Figure 2: Long-run effects of illegal immigration when $\varkappa = 0.4$.

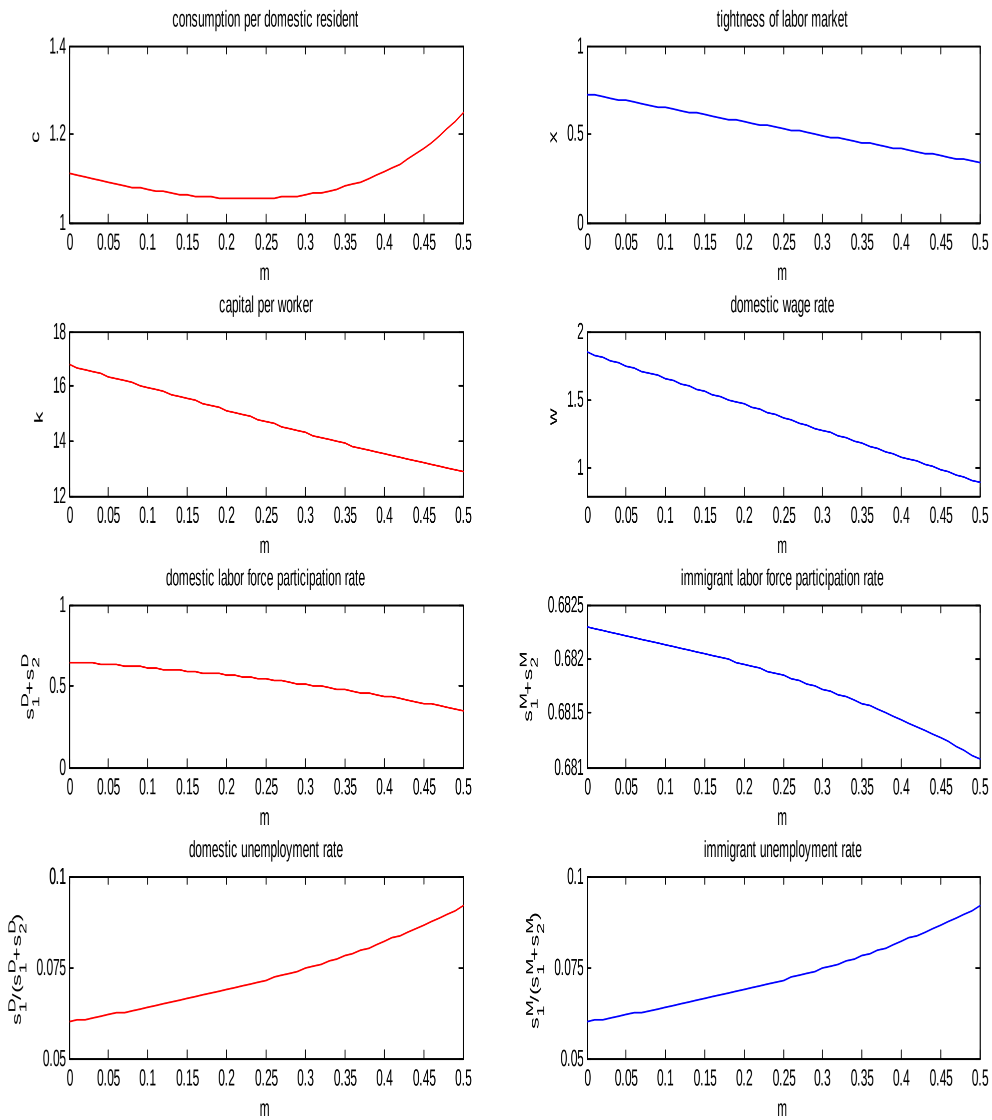


Figure 3: Long-run effects of illegal immigration when $\varkappa = 0.7$.

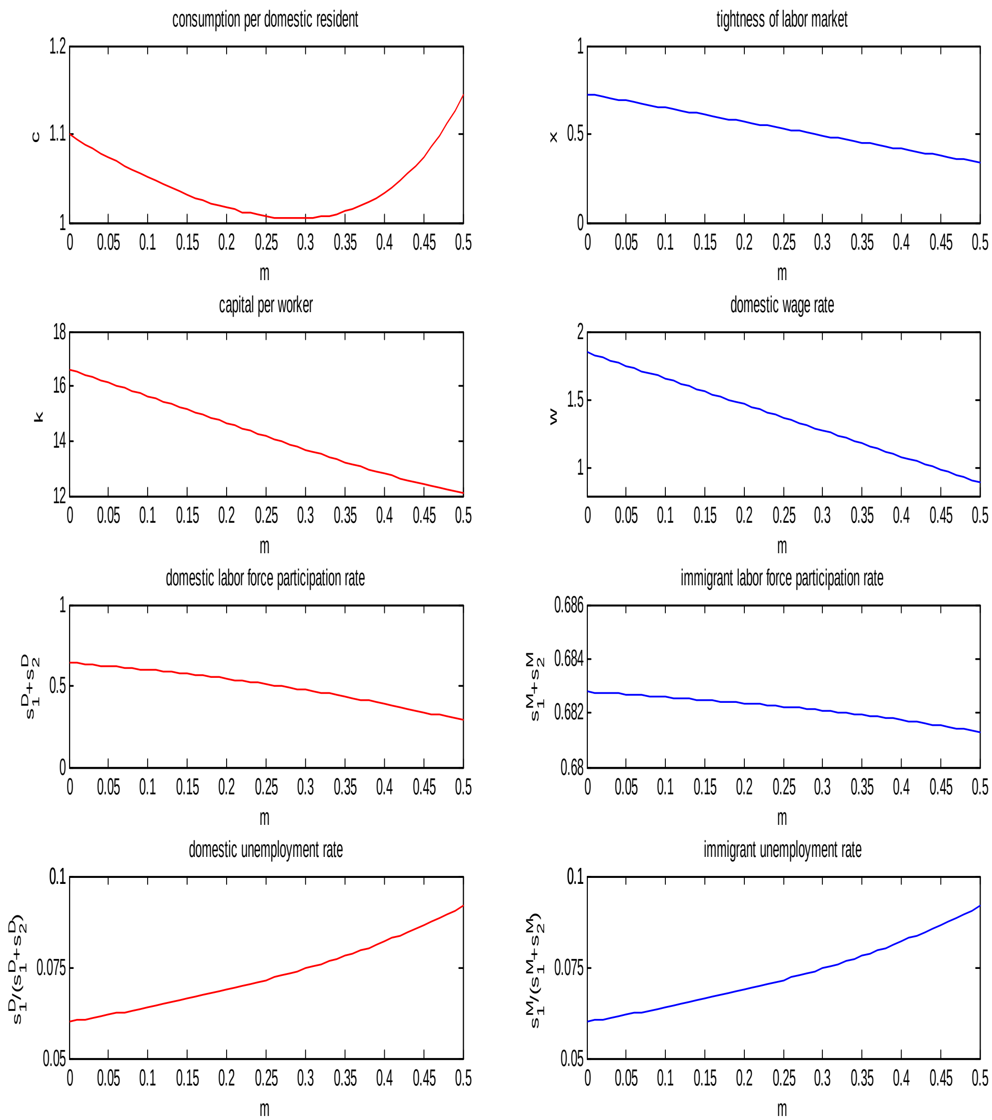


Figure 4: Long-run effects of illegal immigration when $\varkappa = 1$

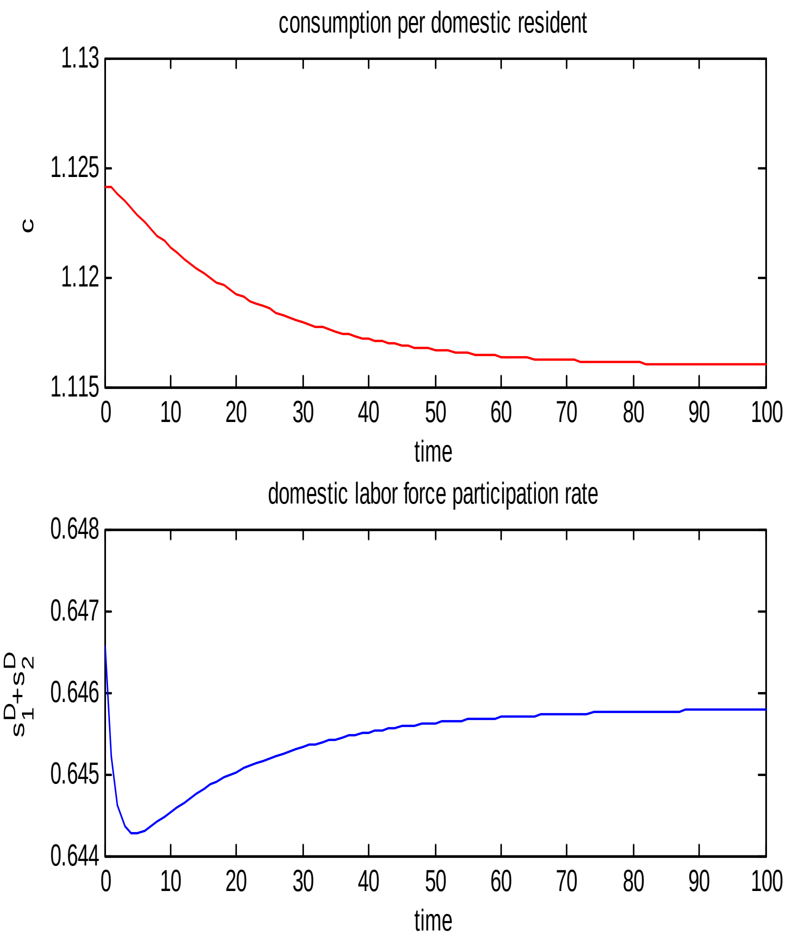


Figure 5: Transition dynamics after a 5% increase in illegal immigration.

Table 2: Stable Eigenvalues

	$\varkappa = 0.2$			$\varkappa = 0.4$			$\varkappa = 0.7$			$\varkappa = 1$		
m	Root 1	Root 2	Root 3	Root 1	Root 2	Root 3	Root 1	Root 2	Root 3	Root 1	Root 2	Root 3
0	−0.9755	−0.8002	−0.0449	−1.0476	−0.7344	−0.0500	−1.1240	−0.6660	−0.0574	−1.1770	−0.6162	−0.0647
0.10	−0.9192	−0.7510	−0.0432	−0.9912	−0.6919	−0.0478	−1.0688	−0.6322	−0.0545	−1.1230	−0.5891	−0.0611
0.20	−0.8597	−0.6988	−0.0417	−0.9314	−0.6463	−0.0457	−1.0096	−0.5957	−0.0515	−1.0652	−0.5601	−0.0572
0.30	−0.7966	−0.6431	−0.0403	−0.8673	−0.5971	−0.0437	−0.9459	−0.5562	−0.0484	−1.0025	−0.5287	−0.0529
0.40	−0.7290	−0.5832	−0.0392	−0.7981	−0.5437	−0.0418	−0.8765	−0.5128	−0.0452	−0.9339	−0.4943	−0.0482
0.50	−0.6560	−0.5185	−0.0385	−0.7227	−0.4854	−0.0401	−0.8000	−0.4647	−0.0420	−0.8578	−0.4559	−0.0434