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Parallel Market Premia and Misalignment of Official Exchange Rates*

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Due to restrictive foreign exchange policy and active parallel markets for foreign exchange in some developing countries, it is often believed that the real official exchange rate is undervalued (overvaluing the domestic currency). Since an overvalued domestic currency depresses official current account below its equilibrium level, while an undervalued domestic currency may be inflationary, it is important to be able to detect real official rate misalignment as early as possible. This is difficult in economies where parallel markets for foreign currencies are well established, and where it is therefore difficult to observe underlying capital flows. We show that the direction of real official rate misalignment is identifiable without knowing the size of capital flow in the parallel market. We indicate that a change in the parallel market premium is an appropriate indicator of change in real official rate misalignment.

I. Introduction

The issue of exchange rate misalignment -- the sustained departure of the actual real official exchange rate from its equilibrium value -- is capturing policy-makers' attention in many developing countries. Overvaluation of the domestic currency (where the exchange rate is the price of foreign currency) depresses the official current account balance and makes the government more dependent on foreign aid, while overvaluation of foreign currency contributes to domestic inflation. Setting the real official exchange rate at the right level therefore matters a lot in many developing countries. However, the extent of official exchange rate misalignment is difficult to identify in economies with well-established informal or black or parallel foreign exchange markets, since it is difficult to observe the underlying flows of international payments in such economies.[1]

One approach to the issue is to use the premium of the parallel market rate over the official rate as an indicator of misalignment of the official rate. Such an approach is supported empirically by the work Kamin (1993) and Edwards (1989).[2]

In this paper we analyse whether one can reliably expect the parallel market premium to be correlated with real official exchange rate misalignment. To do so we follow the approach of Lizondo (1987), and of Kharas and Pinto (1989) in modelling both the official and the parallel

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1. We use the terms of informal and black and parallel interchangeably.
2. Analysing devaluation events in a number of developing countries, both Kamin and Edwards found that the parallel market premium often rises very rapidly right before an increase in the official exchange rate and then falls off right afterwards. This finding was interpreted by some as a positive correlation between the parallel market premium and real official rate misalignment.

markets for foreign exchange. Our approach is different in two major ways: first, we endogenize the under invoicing of export revenue, making it dependent on the parallel market premium. Second, we include workers' remittances from abroad as an additional source of foreign currency, also divided endogenously between official and parallel markets.

Section II specifies our model of the official and parallel foreign exchange markets. Section III derives stability and steady state properties of the complete model. Section IV extracts implications for behaviour of the parallel market premium, and in particular whether it will reliably indicate real official rate misalignment. Section V concludes. An appendix contains some proofs and derivations of results in sections III and IV.

II. The Macroeconomic Model

Our model of foreign exchange markets is derived from a macroeconomic model made up of the optimal rules of the various decision-making agents in the economy. Of these agents, private sector producers choose output and input levels for both home goods and export goods. Inputs include both labour and imported inputs. Firms also set how much of any export revenue to divert to the foreign exchange market by under-invoicing export sales. Nationals working abroad choose how much of their foreign earnings to channel through the official foreign exchange market, and how much to divert to the parallel market instead. Households choose how much of their total financial wealth to hold in domestic currency, and how much to hold in foreign currency. Finally, the government determines its fiscal stance and financing, and the rules for pricing and rationing in the official foreign exchange market. We will deal with each in turn, and then assemble the various decision rules into equilibrium conditions for the two parts of the foreign exchange market.

1. Domestic Producers' Decisions

Domestic firms can produce for home consumption or for export. Exports are exogenously determined, as they depend mainly on the amount of rainfall and international terms of trade. Home goods (Y) have a Cobb-Douglas production function with imported producer goods I_p and domestic L_y as the only inputs:

$$Y \leq I_p^{(1-\alpha)} L_y^{\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

The domestic currency purchase price of imported producer goods is the foreign currency price P_m multiplied by the exchange rate relevant for marginal purchases. Marginal purchases of imported producer goods will always be made at the parallel market exchange rate (that is, price of foreign currency)b, since the government imposes fixed import quotas for purchases at the official exchange rate, and the quotas are never sufficient to satisfy all demand.

Firms have a choice about converting the foreign currency revenue from their exports P_xX (where P_x is the foreign price of exports) into domestic currency: they can do so in the official market at the official exchange rate e or in the parallel market at the higher rate b. They can divert a portion ϕ of their foreign currency revenue from the official market to the parallel market, despite foreign exchange controls, by under-invoicing. The size of ϕ chosen will reflect both the expected gain from conversion at the higher parallel rate b, and the probability of being detected and penalized (losing the entire transaction ($\phi b P_x X$) through confiscation). We assume that the probability of detection rises with the proportion of under-invoicing, and for simplicity at exactly half the rate, i.e., the probability is $\phi/2$, and the expected confiscation amount is therefore $[(\phi/2)\phi b P_x X]$. For convenience in what follows, we capture the expected gain from converting in the parallel market by the ratio $\pi = b/e$, commonly referred to as the parallel market premium.

Firms' decision rules for all the choices above are found by maximizing their profit function

$$\text{Max } \{P_y Y + [\phi b(1-\phi/2) + (1-\phi)e]P_x X - bP_m I_p - W(L_x + L_y)\} \tag{2}$$

with respect to L_y, I_p and ϕ, subject to all the constraints above and to the usual non-negativity restrictions. The first order conditions for I_p, L_y and ϕ are:

$$P_y(1-\alpha)I_p^{-\alpha}L_y^{\alpha} = P_m b, \tag{3}$$

$$P_y \alpha Y/L_y = W, \tag{4}$$

$$\phi = (1 - 1/\pi). \tag{5}$$

With constant average labour productivity of home goods Y/L_y, it can be shown using equation (3) and (4) that the price of home goods is a constant multiple of the domestic currency price of imports:

$$P_y = \sigma P_m b, \tag{6}$$

where $\sigma = [(1/\alpha^{\alpha})(1-\alpha)^{\alpha-1}(\alpha Y/L_y)^{\alpha}]^{(1/(1-\alpha))}$.

Equation(3) can be rearranged to solve for imports of producer goods. Substituting for P_y from equation (6) as well, the optimal level of imported producer goods is a constant multiple of output Y:

$$I_p = \sigma(1-\alpha)Y. \tag{7}$$

2. Decisions of Workers Abroad

The total inflow of workers remittances K is assumed fixed in total, but workers choose the proportion θ to be exchanged at the parallel market rate rather than at the official exchange rate. The choice reflects a trade-off extra gain from the parallel market premium, and extra expected risk of loss from being detected and punished. The probability of being detected diverting workers' remittances to the parallel market is assumed to rise with the proportion diverted, and for simplicity at exactly half the rate, or $\theta/2$. The punishment for being caught is confiscation of the entire amount diverted, $b\theta K$ so the expected confiscation loss is $(\theta/2)b\theta K$.

The optimal proportion θ can be found by maximizing workers' net revenue (in domestic currency) with respect to θ:[3]

$$\text{Max}\,[\theta b(1-\theta/2)+(1-\theta)e]\,K. \tag{8}$$

The first order condition can be solved for the relationship between θ and the parallel market ratio or premium π:

$$\theta = 1-(1/\pi). \tag{9}$$

Equation (9) provides the same value for workers' optimal diversion rate θ as equation (5) above provided for firms' optimal diversion rate ϕ. This is not surprising, as both groups face the same form of tradeoff.

3. Household Portfolio Allocation

Households choose between domestic and foreign assets, a portfolio allocation decision. Following Dornbusch (1983), and to reflect under- development of the financial system, households' nominal financial asset portfolio H is assumed to consist only of domestic money holdings M, and foreign money holdings F. Foreign residents do not hold domestic money, so the entire stock of M is held by domestic households. Since households buy foreign currency F only in the parallel market, and therefore value it at parallel market exchange rate b, the domestic currency value of households' nominal wealth H can be expressed as:

$$H = M + bF. \tag{10}$$

Let λ be the fraction of financial wealth H that households want to hold in foreign currency. Both foreign currency F and domestic currency M earn zero interest, but F will provide a return whenever the parallel market rate b changes. The fraction λ will therefore rise with the expected rate of increase of the parallel market rate. We use the perfect foresight assumption (following Lizondo (1987) and Dornbusch (1983)), so λ also rises with actual increases in the parallel market rate.

In equilibrium, desired holdings of foreign money λH must equal the actual stock bF of foreign money being held, so we can solve for $H = bF/\lambda$. Replacing H in equation (10) and rearranging to solve for M, we get:

$$M = [(1-\lambda)/\lambda]\,bF. \tag{11}$$

If is useful to divide both sides of equation (11) by e to convert to (official) foreign currency values, giving:

3. The first order condition is $K(b(1-\theta/2)-\theta b(1/2)-e)=0$, assuming the net revenue function has an optimum within the permissible range $0<\theta<1$.

$$m = [(1 - \lambda)/\lambda]\pi F, \tag{12}$$

where $m = M/e$. The fraction λ is a function of the rate of increase in parallel market rate, but this can be broken down into appreciation of the official exchange rate $\hat{e}$ and of the parallel market premium $\hat{\pi}$. Letting $\Lambda(\hat{\pi} + \hat{e})$ stand for the relationship of $(1-\lambda)/\lambda$, we can rewrite equation (12) as

$$m = \Lambda(\hat{\pi} + \hat{e})\pi F, \qquad \Lambda' < 0. \tag{13}$$

Equation (13) is the portfolio-balance or the asset market equilibrium condition. It indicates that the higher the expected rate of increase of the parallel market rate (that is, depreciation of domestic currency in the parallel market) is, the lower is the ratio of domestic money holdings to foreign currency holdings.

4. Government Decisions

The government determines much of the context for decisions of other agents in the economy, and also acts as a separate agent. For instance, the government decrees and administers both an import quota system and a set of foreign exchange controls which regulate entry into the official foreign exchange market. In this market the government buys foreign currencies from households at the official exchange rate e, and allocates it either to pay for government imports (G) or to sell to households for officially sanctioned imports (I_{off}). The government can buy from only two sources: private sector export revenue X (we set P_x=1), and workers' remittances K form abroad; of these, only the portions $(1-\phi)$ of X and $(1-\theta)$ of K flow to the official market[4]. The rest is diverted to the parallel market. Foreign aid is constant and exogenous and for simplicity is fixed at zero.

We assume that government spending G is entirely on imports, including payment of interest on foreign debt, and that no new foreign debt is being incurred. Further, we assume that any of G that is not financed by taxes must be financed by borrowing from the central bank. Taxes are assumed to be lump-sum. Since G is all spent on imports, we assume that both G and tax revenue T are fixed in foreign currency units. That is, both G/e=g and T/e=t are constants.

The change in the stock of domestic money $\dot{M}$, is equal to the change in central bank domestic credit, $\dot{D}$, plus change in (domestic currency value of) foreign reserves, $e\dot{R}$, held by the government. The change in domestic credit reflects government borrowing from the central bank to finance its deficit, $G - T$ or (in foreign currency) $g - t$. That is,

$$\dot{M} = \dot{D} + e\dot{R}, \tag{14}$$

4. While the government does collect some revenue from exports and form workers abroad through occasional confiscations of foreign exchange diverted to the parallel market these amounts are assumed to leak back into the private sector through bribes and corruption rather than to reduce the government deficit.

but dividing both sides by e and substituting $G - T$ for $\dot{D}$, we get

$$\dot{M}/e = g - t + \dot{R}. \tag{15}$$

Now $\dot{M}/e = \dot{m} + m\hat{e}$, where the hat denotes a proportional growth rate, so we can write this expression in terms of the rate of growth of the money supply in foreign currency units:

$$\dot{m} = (g - t) - m\hat{e} + \dot{R}. \tag{16}$$

The value of $\dot{R}$ is determined in the official foreign exchange market, to which we now turn.

5. Foreign Exchange Markets

a. *The Official Market:*

The official foreign exchange market is operated by the government as described above. The change in official foreign exchange reserves $\dot{R}$ will be determined as a residual to match the current account balance.[5] This is the fractions of export revenue and workers' remittances channelled through the official exchange market [$(1-\phi)eX$ and $(1-\theta)eK$, respectively], less government imports (G) and private sector purchases of quota-sanctioned imports (I_{off}), all valued in foreign currency. In order not to leave the import quota undetermined, we assume that the government sets private sector import quotas at a fixed proportion δ of the total foreign currency inflow to the official market. Algebraically, the official current account balance can be expressed as:

$$\dot{R} = (1-\delta)[(1-\phi)X + (1-\theta)K] - g. \tag{17}$$

We can substitute the optimal levels of ϕ and θ (both equal to $1 - 1/\pi$) from the decision rules of firms and workers that have been derived in equations (5) and (9) above, to give the final form of the official current account balance:

$$\dot{R} = (1-\delta)(X+K)/\pi - g. \tag{18}$$

b. *The Parallel Market:*

In the parallel foreign exchange market, the supply of foreign currency consists of the portion ϕ of export revenue X that is diverted from the official market by under-invoicing, and the portion θ of workers' remittances K which workers divert to the parallel market. Demand consists of the current account flow of imports not permitted under the import quota policy, and increases in households' stock of foreign currency.

The current account balance in the parallel market must match the private sector's accumulation

5. There are no capital account transactions in the official market, so the current account balance and the overall balance are identical.

of foreign currency $\dot{F}$, since that is all that is left and the parallel market rate adjusts to balance the parallel market. This could be arrived at by netting inflows and outflows to and from the parallel market, which have been explained above. Alternatively, it can be found as a residual by subtracting total imports (official and unofficial, private sector and government)plus accumulation of official reserves, from the total inflow of foreign currency to the economy. The latter approach is simpler:

$$\dot{F} = X + K - I - g - \dot{R}, \tag{19}$$

where I is total private sector imports measured in foreign currency units and consists of imports of producer goods I_p and of consumer goods I_e. Breaking I into its components and substituting for $\dot{R}$ from equation (18):

$$\dot{F} = (1 - (1 - \delta)/\pi)(X + K) - (I_p + I_e), \tag{20}$$

We can substitute for I_p form equation (7) to get

$$\dot{F} = (1 - (1 - \delta)/\pi)(X + K) - (\sigma(1 - \alpha)Y + I_e). \tag{21}$$

Equation (21) completes the model, which consist of the differential equation system (13), (16), (18) and (21) in $\dot{m}, \dot{F}, \dot{R}$ and $\hat{\pi}$. To simplify later analysis, we reduce the differential equation system to three equations by substituting for $\dot{R}$ into equation (16) giving

$$\dot{m} = \frac{(1 - \delta)}{\pi}(X + K) - m\hat{e} - t. \tag{22}$$

The third differential equation is equation (13), repeated here:

$$m = \Lambda(\hat{e} + \hat{\pi})\pi F, \qquad \Lambda' < 0. \tag{13}$$

The rate of change of the official exchange rate e could be totally exogenous. More generally, we could postulate that the government will adjust the official rate towards the parallel rate, closing the gap at some constant rate f. That is,[6]

$$\hat{e} = f(\pi - 1), \qquad f \geq 0. \tag{23}$$

If we substitute for $\hat{e}$ from equation (23) into equations (22) and (13), we get instead

$$\dot{m} = ((1 - \delta)/\pi)(X + K) - mf(\pi - 1) - t, \tag{24}$$

6. A fixed exchange rate system is the special case where f=0.

$$m = \Lambda[f(\pi - 1) + \hat{\pi}]\pi F, \qquad \Lambda' < 0. \tag{25}$$

In the next section we establish the steady state and stability of the system represented by either equations (21), (22) and (13), or equation (21), (24) and (25).

III. Steady State and Stability

The steady-state values of π, F, and m vary considerably in the two versions with which we ended the last section. If the official exchange rate adjusts at all towards the parallel rate, then in the steady state it must have arrived, and the parallel premium π is exactly 1. Substituting that into equation (24) when $\dot{m} = 0$ gives

$$(1 - \delta)(X + K) = t, \tag{26}$$

i.e., the government finances all of its imports from taxes. When $\hat{\pi} = 0$ and $\pi = 1$, equation (25) gives

$$m = \Lambda[0] F, \tag{27}$$

which constrains only the relative sizes of private sector holding of domestic and foreign currency.

If we assume instead that the rate of official exchange rate increase is fixed and non-zero, even in the steady state, then the parallel market premium need not be 1 and specific steady state values of, F, π and m can be found from equations (13), (21) and (22). Setting $\dot{F} = 0$ in equation (21) and solving for π:

$$\pi = \frac{(1 - \delta)(X + K)}{(X + K) - (\sigma(1 - \alpha)Y + I_e)}. \tag{28}$$

From equation (19) it is clear that when $\dot{F} = 0$ and $\dot{R} = 0$ the denominator in equation (28) can be reduced to, g, so that:

$$\bar{\pi} = \frac{(1 - \delta)}{g}(X + K). \tag{29}$$

Setting $\dot{m} = 0$ in equation (22) and substituting for π from equation (29) we get

$$\bar{m} = \frac{[g - t]}{\hat{e}}. \tag{30}$$

Setting $\hat{\pi} = 0$ in equation (13) and substituting (29) and (30) into (13) and solving for F:

$$\overline{F} = \frac{(g - t)g}{\hat{e}\Lambda(\hat{e})(1 - \delta)(X + K)}. \tag{31}$$

Equation (29) shows that the parallel market premium rises if the government allocates a smaller share δ of total official foreign currency receipts to private sector import quotas. That is, tightening of import quotas on private sector imports, as practised by many developing countries facing a shortage of foreign currency, will by itself lead to a higher premium level in the steady state as seekers of foreign currency move from the official to the parallel market. Since $\Lambda'(\hat{e}) < 0$, from equations (30) and (31) it is clear that any increase in the official exchange rate will lower the ratio of domestic currency to foreign currency held in private sector portfolio.

Establishing the stability of the model is more difficult, and the mathematics is left to the appendix. Analysis of the characteristic polynomial of the linear approximation of this model shows that the dynamic model has one positive and two negative roots. That means the steady state discussed above is a saddle-point solution, therefore the economy can (re-) converge to the steady state from a distance away. With that result in hand, we will move to consider the main question of this paper.

IV. Comparative Dynamics and Real Official Rate Misalignment

The main question of this paper is whether movements of the real official exchange rate towards and away from its long run equilibrium level will coincide with reductions and increases, respectively, in the parallel market premium. If the answer is yes, then changes in parallel market premiums indicate changes in misalignment of real official exchange rates. To address this question we use a crawling-peg exchange rate system.

With the official exchange rate adjusting towards the parallel rate as in equation (23), the basic dynamic equations of the model are:

$$\dot{F} = (1-(1-\delta)/\pi)(X+K)-(\sigma(1-\alpha)Y+I_e), \tag{21}$$

$$\dot{m} = ((1-\delta)/\pi)(X+K)-mf(\pi-1)-t, \tag{24}$$

$$m = \Lambda[f(\pi-1)+\hat{\pi}]\pi F, \quad \Lambda' < 0. \tag{25}$$

Assuming initially the stock of domestic money supply is held constant, equations (21) and (25), provides a stability locus for combinations of π and F. The slope of the $\hat{\pi} = 0$ locus in (π, F) space is

$$d\pi/dF = -\Lambda\pi/(\pi F\Lambda' f+m/\pi). \tag{32}$$

This slope is negative as in Figure 1 if the rate of adjustment f of the official exchange rate is sufficiently small. This locus would be shifted to the right by an increase in m (as higher level of F would be required to maintain the same steady state level of π).

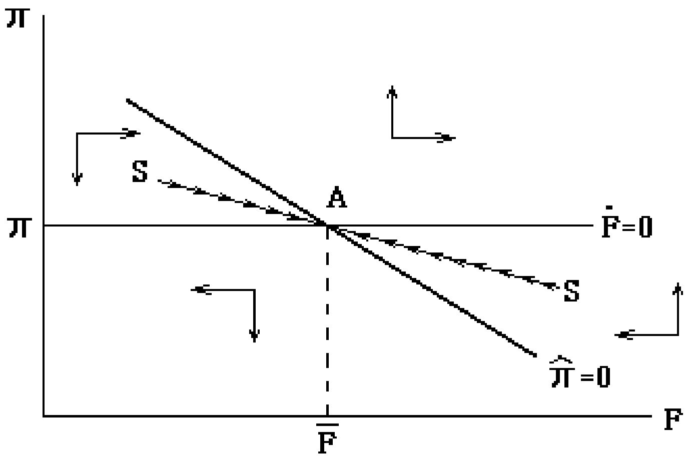

Figure 1 Stability Loci and Phase Paths

Movements of F and π from positions off the stability loci are best explained intuitively. Points above the $\dot{F}=0$ curve imply a premium so high as to make for a surplus in the parallel market and thus a growing stock of foreign currency, while points below the $\dot{F}=0$ curve imply a declining stock. Points to the right of the $\hat{\pi}=0$ locus indicate excess demand for foreign currency in the parallel market and thus an increasing parallel premium ($\hat{\pi}>0$), while points to the left imply a falling parallel market premium ($\hat{\pi}<0$).

Now consider a rightward shift of the $\hat{\pi}=0$ locus, such as would be caused by greater domestic money supply m. The economy will eventually move from the old steady state at A to a new one at the right-hand side of point A, but by what path? In particular, will the change in parallel premium reflect the extent of deviation of the real official rate from its long run steady state.

In order to determine the adjustment path of the parallel market premium b, we use the dominant and non-dominant eigenvector approach, suggested by Calvo (1987b) and others, explained as follows. For each of the two negative roots corresponding to the saddle-path solution (analysed in the appendix), there is a corresponding eigenvector.
If the system were to start on either of the eigenvectors it would travel along that vector toward the steady state, since both are solutions to the dynamic model. It follows that the path to a new steady state cannot cross either of the eigenvector rays.

Furthermore, on its way to the new steady state the adjustment path will always converge asymptotically to the dominant eigenvector (the one with smallest absolute root), unless the economy happens to start right on the other, non-dominant eigenvector. We can therefore determine the adjustment path of the parallel market premium by determining the slope of the dominant eigenvector. The mathematics behind this result is contained in the proposition in part (ii) of the Appendix.

The adjustment path of the parallel market premium is shown in Figure 2. It shows an economy initially at point A, which was originally but is no longer a steady state position. The economy has been shocked by an unexpected increase in domestic money supply. As a result, private agents anticipate increases in the parallel exchange rate, and therefore in the return on holdings of foreign currency; this raises desired foreign currency holdings F. Extra accumulation of F would put upward pressure on the premium, so the $\hat{\pi} = 0$ locus shifts to the right. The new steady state is at point C.

The impact effect is to raise the parallel market premium (a free or 'jump' variable) above its long-run equilibrium level to a point such as B. Private foreign currency holding F is a predetermined variable that can change gradually over time by a accumulation. Two adjustment processes move the economy from B towards C after the impact effect. The first is the gradual accumulation of foreign currency holdings by the private sector. The second is the gradual crawl of the official rate to maintain a steady premium level. As the official rate e rises, it approaches closer to the parallel rate b, and the parallel premium π falls towards its long-run equilibrium level.

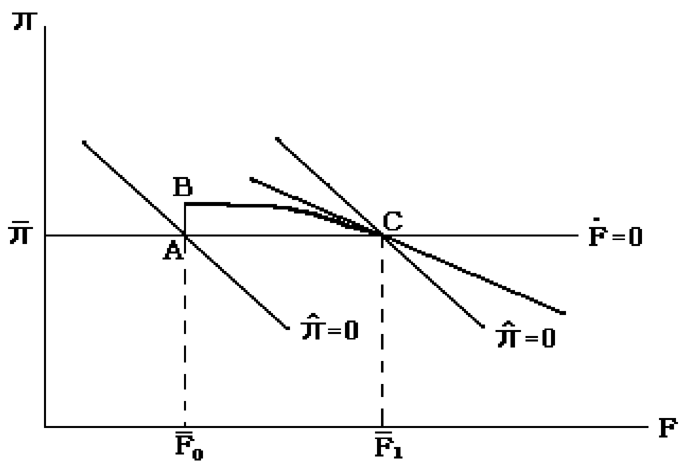

Figure 2 The Adjustment Path of the Parallel Market Premium

All that remains is to discuss real official rate misalignment explicitly. The real official exchange rate is defined as the relative price of traded goods to home goods. P_y can be used to approximate the price of home goods, and P_m the price of traded goods, so the real official exchange rate is

$$\varepsilon = e\frac{P_m}{P_y} \quad , \tag{33}$$

Substituting $P_y = \sigma b P_m$ from equation (6),

$$\varepsilon = (1/\sigma\pi), \qquad \text{where} \quad \sigma > 0. \tag{34}$$

Define real official rate misalignment as the absolute value of the deviation of the real rate from its long-run steady-state level[7], that is $(\varepsilon - \bar{\varepsilon})$. The adjustment path of the real official rate corresponding to the unexpected rise in domestic money supply can be found by using Figure 2 and equation (34). So initially, the real official rate falls below its long-run steady state level (due to the rise in the premium level). Over time, the gradual increase of the nominal official rate e lowers the parallel premium π, so the real official rate returns towards its steady-state level. That is, the real official rate misalignment decreases as the parallel market premium falls. It follows that change in real official rate misalignment follows a similar pattern to the change in parallel market premium.

V. Conclusion

If changes in parallel market rate premiums are positively associated with changes in real official rate misalignment, then the latter can be identified without knowing the size of capital flows in the parallel market. We show that when domestic inflation is mainly affected by the parallel rate rather than the official market rate, change in the parallel market premium does indicate change in real official rate misalignment. Our result does not mean that eliminating the parallel market premium will automatically eliminate real official rate misalignment, though holding the parallel market rate premium constant will help keep real official rate misalignment constant as well.

7. In this definition of real official rate misalignment the long-run steady state real exchange rate is assumed constant. For more detailed discussion justifying this assumption see Williamson (1985), pp. 13-26.

Appendix: Mathematical Results

$$m = \Lambda(\hat{\pi} + \hat{e})\pi F, \qquad \Lambda' < 0, \tag{13}$$

$$F = \frac{\pi - 1 + \delta}{\pi}(X + K) - [\sigma(1 - \alpha)Y + I_c], \tag{21}$$

$$m = \frac{1 - \delta}{\pi}(X + K) - t - m\hat{e}. \tag{22}$$

We linearize these three equations around the steady-state values of π and F and m, to give the following matrix equation:

$$\begin{pmatrix} \hat{\pi} \\ \dot{F} \\ \dot{m} \end{pmatrix} = \begin{pmatrix} \hat{\pi}_\pi & \hat{\pi}_F & \hat{\pi}_m \\ \dot{F}_\pi & \dot{F}_F & \dot{F}_m \\ \dot{m}_\pi & \dot{m}_F & \dot{m}_m \end{pmatrix} \begin{pmatrix} \pi - \bar{\pi} \\ F - \bar{F} \\ m - \bar{m} \end{pmatrix}.$$

The values of the partial derivatives in the Jacobian matrix can be determined as follows:

$$\frac{d\hat{\pi}}{dF} = -\frac{\Lambda}{\Lambda' F} > 0, \qquad \frac{d\hat{\pi}}{d\pi} = -\left[\frac{\Lambda}{\Lambda' \pi}\right] > 0, \qquad \frac{d\hat{\pi}}{dm} = \frac{1}{\Lambda' \pi F} < 0,$$

$$\frac{d\dot{F}}{d\pi} = \frac{(1 - \delta)(X + K)}{\pi^2} > 0, \qquad \frac{d\dot{F}}{dF} = 0, \qquad \frac{d\dot{F}}{dm} = 0,$$

$$\frac{d\dot{m}}{d\pi} = -(1 - \delta)\frac{X + K}{\pi} < 0, \qquad \frac{d\dot{m}}{dF} = 0, \qquad \frac{d\dot{m}}{dm} = -\hat{e} < 0.$$

A necessary condition for a saddle-point solution is that determinant of the Jacobian matrix be positive, which is satisfied in this care:

$$|J| = \dot{m}_m[\,-\hat{\pi}_F\,\dot{F}_\pi\,] > 0.$$

When the determinant of the Jacobian matrix is positive, either there are two roots that are negative and the third is positive or all three roots are positive. In the latter case, the steady state is unstable. To rule out instability, we analyse the characteristic polynomial P(r) of the Jacobian matrix as:

$$P(r) = \begin{vmatrix} \hat{\pi}_\pi - r & \hat{\pi}_F & \hat{\pi}_m \\ F_\pi & -r & 0 \\ \dot{m}_\pi & 0 & \dot{m}_m - r \end{vmatrix},$$

where r is a characteristic root of the equation system.
The polynomial's properties include:

$$P(r) = \gamma_0 + \gamma_1 r + \gamma_2 r^2 + \gamma_3 r^3,$$

$$\gamma_0 = r_1 r_2 r_3 > 0 \quad (\det |J|),$$

$$\gamma_1 = r_1 r_2 + r_1 r_3 + r_2 r_3,$$

$$\gamma_2 = r_1 + r_2 + r_3 \quad \text{(trace)}.$$

If the sum of cross-products $\gamma_1 < 0$, then at least one of the three roots is negative, and the case of instability is ruled out. Using the equation for γ_1 and the determinant equation, we can show that:

$$\gamma_1 = [\, -F_\pi \hat{\pi}_F + \hat{\pi}_\pi \dot{m}_m - \dot{m}_\pi \hat{\pi}_m] < 0,$$

which rules out the possibility of an unstable solution, and leaves only the saddle-point solution.

Dominant Eigenvectors

The solution for the eigenvectors corresponding to the two negative roots is specified by writing the general solution of the system of the differential equations (13), (21) and (22):

$$\begin{pmatrix} \pi - \bar{\pi} \\ F - \bar{F} \\ m - \bar{m} \end{pmatrix} = \begin{pmatrix} h_{11} & h_{21} & h_{31} \\ h_{12} & h_{22} & h_{32} \\ h_{13} & h_{23} & h_{33} \end{pmatrix} \begin{pmatrix} w_1 \exp(r_1 t) \\ w_2 \exp(r_2 t) \\ w_3 \exp(r_3 t) \end{pmatrix}, \tag{35}$$

where w_i, $i = 1, 2, 3$ are arbitrary scalars, and $h_i = (h_{i1}, h_{i2}, h_{i3})$ is the eigenvector associated with the root r_i. As shown before, the saddle-point solution has two negative roots, denoted by r_1 and r_2. Since $r_3>0$, then a necessary condition for convergence of the dynamic system requires $w_3 = 0$.
The eigenvector are found by solving the characteristic equation:

$$|J - r_i I| h_i = 0,$$

where I is identity matrix, J is the Jacobian matrix from part (i) of this appendix. Substituting for J gives:

$$\begin{pmatrix} \left[\frac{\Lambda}{\Lambda'\pi} - r_i\right] & -\frac{\Lambda}{\Lambda' F} & \frac{1}{\Lambda'\pi F} \\ \frac{(1-\delta)(X+K)}{\pi^2} & -r_i & 0 \\ -\left[\frac{(1-\delta)(X+K)}{\pi^2}\right] & 0 & -[\hat{e}+r_i] \end{pmatrix} \begin{pmatrix} h_{i1} \\ h_{i2} \\ h_{i3} \end{pmatrix} = 0,$$

whose second equation can be rearranged to find the eigenvector slopes h_{i1}/h_{i2}:

$$\frac{h_{i1}}{h_{i2}} = \frac{r_i \pi^2}{(1-\delta)(X+K)} < 0 \quad \text{for } i = 1, 2.$$

Proposition:

If $|r_1| > |r_2|$, then for any initial values of π and F such that

$$\frac{(\pi_0 - \bar{\pi})}{(F_0 - \bar{F})} \neq \frac{h_{11}}{h_{12}}$$

then

$$\lim_{t\to\infty} \frac{\pi_t - \bar{\pi}}{F_t - \bar{F}} = \frac{h_{21}}{h_{22}} < 0.$$

That is, unless the initial values of π and F lie exactly on the non-dominant eigenvector 1, the slope of the adjustment path $(\pi_t - \bar{\pi})/(F_t - \bar{F})$ asymptotically converges to the slope of the dominant eigenvector (h_{21}/h_{22}), which has been shown to be negative.
The proof of the proposition is shown as follows: From the equations of (35) we get

$$\frac{\pi_t - \bar{\pi}}{F_t - \bar{F}} = \frac{h_{11}W_1\exp(r_1 t) + h_{21}W_2\exp(r_2 t)}{h_{12}W_1\exp(r_1 t) + h_{22}W_2\exp(r_2 t)}.$$

Dividing the numerator and denominator of the equation above by $W_2\exp(r_2 t)$:

$$\frac{\pi_t - \bar{\pi}}{F_t - \bar{F}} = \frac{h_{11}\frac{W_1}{W_2}\exp(r_1 - r_2)t + h_{21}}{h_{12}\frac{W_1}{W_2}\exp(r_1 - r_2)t + h_{22}}.$$

Since $(r_1 - r_2) < 0$, then

$$\lim_{t \to \infty} \frac{\pi_t - \bar{\pi}}{F_t - \bar{F}} = \frac{h_{21}}{h_{22}} < 0.$$

Inspection of the equation for the eigenvector slopes will show that the dominant eigenvector must be the flatter of the two, since the only difference between the two slopes comes from the different roots, and the root of the dominant eigenvector is smaller in absolute value.

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