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Heller, Yuval

School of Mathematical Sciences, Tel-Aviv University.

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Yuval Heller

School of Mathematical Sciences, Tel-Aviv University, Tel-Aviv 69978, Israel.
Phone: 972-3-640-5386. Fax: 972-3-640-9357.
Email: helleryu@post.tau.ac.il

Abstract

In many situations a decision maker has incomplete psychological preferences, and the weak axiom of revealed preference (WARP) is often violated. In this paper we relax WARP, and replace it with convex axiom of revealed non-inferiority (CARNI). An alternative x is revealed inferior to y if x is never chosen when y is in the convex hull of the choice set. CARNI requires that an alternative is chosen if it is not inferior to all other alternatives in the convex hull of the choice set. We apply CARNI in two models and axiomatize non-binary choice correspondences. In the first model we impose the standard axioms of expected utility model, except that WARP is replaced by CARNI. We prove that it has a multiple-utility representation: There is a unique convex set of vN-M utilities, such that an alternative is chosen if and only if it is best with respect to one of the utilities in this set. In the second model we impose the axioms of the subjective expected utility, relax WARP in a similar way, and get multiple-prior representation: There is a unique convex set of priors over the state of nature, such that an alternative is chosen if and only if it is best with respect to one of these priors. Both representations are closely-related to psychological insights of justifiable choice: The decision maker has several ways to evaluate acts, each with a different justification. Observable payoff-irrelevant information during the choice triggers her to use a specific "anchoring" justification for the evaluation of the alternatives.

Key words: uncertainty, multiple priors, multiple utilities, incomplete preferences, anchoring, framing, non-binary choice. JEL classification: D81

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1 Introduction

Most existing models of rational choice under risk and uncertainty assume that the decision maker (DM) has complete psychological preferences over all alternatives. [2] That is, every two alternatives are comparable: either they are equivalent or one of them is inferior to the other. As argued by Aumann ([3]), Bewely ([6]), Mandler ([28]), and Dubra, Maccheroni and Ok ([9]), among others, rationality does not imply completeness. The psychological preferences may be incomplete, thereby allowing for the occasional "indecisiveness" of the DM. For example, this is usually the case when the DM has different objective functions (multi-criteria decision making).

Mounting evidence from the psychological literature indicates that when the psychological preferences are incomplete, her choice relies on justifications (rationales). Specifically, the DM has several ways to evaluate alternatives, each with a different justification, and additional payoff-irrelevant information that is observable or available during the choice determines which justification is used. The chosen alternative is the best with respect to this justification. Some examples for justifiable choice are: (1) Availability heuristics (Tversky and Kahneman, [41]) - DMs base their predictions on how easily examples can be brought to mind. (2) Framing effect (Tversky and Kahneman, [42]) - The way the choice problem is presented influences the way the DM evaluates alternatives. (3) Anchoring ([41]) - DMs overly rely on specific information or a specific value, and adjust their evaluations accordingly. (4) Reason-based Choice (Shafir, Simonson and Tversky, [39]) - DMs often seek and construct reasons in order to justify their choices to themselves and to others.

Justifications influence choice over incomparable alternatives in two main aspects: (1) *Taste-justifications* influence the tastes of the DM over the different consequences. (2) *Belief-justifications* influence the belief of the DM over the unknown state of nature.

One example of taste-justifications is the regret considerations that were analyzed in Zeelenberg et al. ([45]). The participants in their experiments had to choose between a safe lottery and a risky lottery. A matching procedure ensured that these gambles were roughly of equal attractiveness when there was a feedback only for the chosen lottery. The results show that having a feedback on the risky lottery (also when it is not chosen) causes people to choose it more often due to regret considerations. A similar phenomena occurs in real-life in the Dutch postal code lottery (Zeelenberg and Pieters, [46]), where one's postcode is the ticket number, and hence even if not participating one may still find out that one would have won had one played.

[2] We use the term psychological preferences to describe the DM's judgment of her welfare. See Mandler ([28]) for a discussion of the difference between psychological preferences and revealed preferences that are observed by choice behavior.

Wright and Bower ([44]) demonstrate belief-justifications that depend on the DM's mood. In their experiments happy or sad moods were induced by having the subject focus on particularly happy or sad personal experiences. They show that the induced mood influence the evaluation of ambiguous events. Relative to control subjects, happy people are "optimistic" i.e., they report higher probabilities for positive ambiguous events and lower probabilities for negative events. Conversely, sad people are "pessimistic".

We model the choices of the DM by a choice correspondence C, which selects in each closed and non-empty set of alternatives A, a non-empty subset of choosable alternatives - $C(A)$. That is, when the DM faces a choice from the alternatives in A, she may choose any of the alternatives in $C(A)$. The weak axiom of revealed preference (WARP) is often violated when the psychological presences are incomplete. For example, if x and y are two incomparable alternatives, and x' is a bit better than x, then it is plausible that only x' and y (but not x) are chosen from the three alternatives, which violates WARP.[3]

Eliaz and Ok ([10]) present a weakening of WARP that is more appropriate to incomplete psychological preferences. An alternative x is revealed inferior to y (according to [10]'s definition), if x is not chosen in any set that includes y. Their *weak axiom of revealed non-inferiority* (*WARNI*) requires that an alternative is chosen if it is not revealed inferior to any chosen alternative.

WARNI implies that the choice correspondence is binary.[4] However, the use of justifications and incomplete preferences often induces non-binary choice correspondence, as in the following example of taste-justifications. Let $\{x, y, z\}$ be 3 restaurants, where x always offers hamburger, y always offers chicken, and z randomly offers either hamburger, chicken or fish. Assume that Alice is indecisive about hamburger and chicken (her choice between them depends on taste-justifications), and that she prefers both of them a bit more than fish. It is plausible that when only restaurants x and z are available Alice sometimes chooses z (when she has a taste-justification for chicken), and similarly that Alice sometimes chooses z when only y and z are available, but she never chooses z when all three alternatives are available (she chooses either x or y depending on her taste-justification).

In this paper we present a convex variation of WARNI that captures this kind of justifiable choice with incomplete preferences. We say that an alternative

[3] Recall that WARP requires that if x and y are two alternatives in the intersection of two sets, x is chosen in the first set and y is chosen in the second set, then both alternatives are chosen in both sets. In the example above, both x and y are in the intersection of $\{x, y\}$ and $\{x, y, x'\}$, x is chosen in the first set, y is chosen in the second set, but x is not chosen in the second set.

[4] A choice correspondence C is binary if there exists a binary relation $\succeq$ such that C maximizes $\succeq$. This is equivalent to requiring that an alternative is chosen in a set if and only if it is chosen in any couple in the set.

x is revealed inferior to y, if x is never chosen when y is in the convex hull of the choice set. The *convex axiom of revealed non-inferiority* (CARNI) requires that an alternative is chosen if it is not revealed inferior to any alternative in the convex hull of the chosen acts. Observe that CARNI is implied by WARP together with the independence axiom.

CARNI implies that the choices of the DM are based on pair-wise comparisons to alternatives in the convex hull of the choice set. There are three reasons for this kind of comparisons, which are demonstrated by the rejection of z in the choice set $\{x, y, z\}$ in the example above:

(1) We assume that each justification triggers a complete linear ordering $\succeq$ (an ordering that satisfies the independence axiom, and is consistent with the incomplete psychological preferences) over the alternatives, and that the chosen alternative is the maximal according to this ordering. The fact that fish are inferior to the other main dishes (or more generally that z is inferior to a mixture of x and y) implies that z cannot maximize $\succeq$ (the linearity of the ordering implies that $y \succ z$ or $x \succ z$). That is, there is no justification that can support the choice of z.
(2) The fact that a random choice between x and y is strictly preferred over z triggers Alice to consider the pair $\{x, y\}$ as strictly better then z, and to limit her choice to either x or y, even if her final choice between x and y would be deterministic.
(3) If Alice may face the same choice problem again in the future, then repeated choices of z are strictly worse than some combination of repeated choices of x and y.[5] This again triggers Alice to limit her choice to the pair $\{x, y\}$.

In this paper we apply CARNI in two axiomatic models of justifiable choice: the first model describes taste-justifications, and the second one describes belief-justifications. In both models we impose standard axioms, and replace WARP with CARNI.

The first model is in a von Newman-Morgenstern framework ([43]), where each alternative is a lottery over a finite set of consequences. The expected utility model assumes that the choice correspondence satisfies four axioms: non-triviality, continuity, independence and WARP. We adopt the first three axioms, and replace WARP with CARNI. Theorem 1 shows that this axiomatization is equivalent to the following representation: There exists a unique (up to linear transformations) convex and closed set U of affine (vN-M) utility functions, such that for every set A and every lottery x, x is chosen in A if and only if it is the best lottery with respect to one of the utilities in U:

$$x \in C(A) \Leftrightarrow \exists u \in U \ u(x) \geq u(y) \ \ \forall y \in A$$

[5] Assuming that Alice's total utility from a sequence of choices is the a discounted sum of the utilities in each choice, with a discount factor closed enough to 1.

The second model is in an Anscombe-Aumann framework ([2]), where each act (alternative) is a function that assigns a lottery in each state of nature. The subjective expected utility model assumes that the choices satisfy five axioms: non-triviality, monotonicity, continuity, independence and WARP. We adopt the first four axioms, and replace WARP with two weaker axioms: CARNI, and weak axiom of revealed *unambiguous* preferences. The latter axiom requires that the weak axiom is satisfied over unambiguous (constant) acts. That is, the DM has complete psychological preferences over these acts. Theorem 2 shows that this axiomatization is equivalent to the following representation: There exists a unique (up to linear transformations) affine (vN-M) utility u, and a unique convex set P of priors (probability distributions over the state of nature), such that for every set A and every act f, f is chosen in A if and only if it is the best act with respect to one of the priors in the set:

$$f \in C(A) \Longleftrightarrow \exists p \in P \ E_p\left(u\left(f\right)\right) \geq E_p\left(u\left(g\right)\right) \ \forall g \in A$$

The interpretation of both representations is in the spirit of the psychological insights mentioned earlier: The DM has several ways to evaluate alternatives, each with a different justification. Each justification triggers the DM to base her evaluation on a specific anchoring utility (prior), that is determined by the observable payoff-irrelevant information during her choice.

Both representations extend existing models of preferences: the first representation extends multiple-utility preferences (Dubra et al., [9]), and the second representation extends Knightian preferences (Bewely, [6]; Lehrer and Teper, [23]). Our models coincide with the existing models for choice over binary sets. However, when there is a larger (non-convex) choice set, the binary choice correspondences that are induced by the existing preferences imply the choice of "unjustified" acts, which are not best with respect to any of the utilities or priors. The contribution of our paper is the addition of CARNI to the original list of axioms,[6] which gives a non-binary choice correspondence that is more appropriate to the psychological insights about justifiable choice.

One can use CARNI to extend other axiomatizations of binary preferences into axiomatizations of non-binary justifiable choice correspondences. Ok, Ortoleva and Riella ([31]) present an axiomatization for a preference that is represented by either multiple priors or multiple utilities, and a few axiomatizations of multiple state-dependent utilities. Seidenfeld, Scharvish and Kadane ([36]) present an axiomatization for a preference that is represented by a set of pairs of state-dependent utilities/priors.[7] It is possible to add CARNI to each of these axiomatic models and get the appropriate justifiable choice rep-

[6] Where each existing axiom is adapted in the obvious way to relate to the choice correspondence instead of the preference relation.

[7] The utilities of the extreme pairs in [36]'s representation are almost state-independent.

resentation.

The paper is organized as follows. Section 2 presents the models and the results. Different aspects of our model, and its relations to the existing literature are discussed in Section 3. Section 4 includes the proofs.

2 Models and Results

2.1 Taste-Justifications

2.1.1 Preliminaries

Let X be a finite set of outcomes (certain prizes).[8] Let $Y = \triangle(X)$ be the set of lotteries over X. The mixture (convex combination) of two lotteries is defined as follows: $(\alpha y + (1-\alpha) z)(x) = \alpha y(x) + (1-\alpha) z(x)$ (where $\alpha \in [0,1]$, $y, z \in Y$ and $x \in X$). Similarly, given $A \subseteq Y$, let $\alpha y + (1-\alpha) A$ denote the set where each $z \in A$ is replaced by $\alpha y + (1-\alpha)z$: $(\alpha y + (1-\alpha) A) = \{\alpha y + (1-\alpha)z | z \in A\}$.

The primitive of the model is a choice correspondence C over Y. The domain of C is all the non-empty closed sets in Y.[9] For each such set $A \subseteq Y$, $C(A)$ is a non-empty subset of A. The interpretation of C is the following: when a DM faces a choice from the acts in A, she chooses one of the acts in $C(A)$, and each act in $C(A)$ might be chosen. That is, the DM considers all the acts in $C(A)$, and only them, as choosable acts. The choice of a specific act in $C(A)$ is not explicitly modeled.[10] When $y \in C(A)$ we say the y is choosable in A, or that the DM (sometimes) chooses y in A, and similarly when $y \notin C(A)$ we say the y is not choosable in A, or that the DM does not choose y in A. Given $A \subseteq Y$, $conv(A)$ denotes the convex hull of A (the smallest convex set that contains A).

2.1.2 Axioms

The following axioms (assumptions) are imposed on the choice correspondence:

A1 *Non-triviality.* There is a lottery $y \in A \subseteq Y$ such that $y \notin C(A)$.

[8] We define X to be finite for simplicity of presentation. Both of our models can be easily extended to compact metric space by adapting the proofs a la Dubra et al. ([9]) and Gilboa et al. ([16]).
[9] We define C only on closed sets because in non-closed sets the Pareto frontier might be an empty set. Our results remain the same if C is defined only on finite (non-empty) sets.
[10] In the model's interpretation the choice of a specific act in $C(A)$ depends on the payoff-irrelevant information that is observable during the choice.

A2 *Continuity.* For any lottery $y \in A$, the set $\{z \in Y | z \in C(\{y, z\})\}$ is closed, and the set $\{z \in Y | \{z\} = C(\{y, z\})\}$ is open.
A3 *Independence.* Let $y \in A \subseteq Y$, $z \in Y$ and $\alpha \in (0, 1)$. $y \in C(A) \Leftrightarrow \alpha z + (1 - \alpha) y \in C(\alpha z + (1 - \alpha) A)$.
A4 *Convex Axiom of Revealed Non-Inferiority (CARNI).* Let $y \in A \subseteq Y$. If for every $z \in conv(C(A))$ there exists a set $B \subseteq Y$ with $y \in C(B)$ and $z \in conv(B)$, then $y \in C(A)$.

Axioms A1-A3 are standard. Axiom A1 requires that C is not trivial (there is a choice set with at least one unchoosable act). Axiom A2 (continuity) is standard. It is equivalent to the requirement that for any lottery $y \in A$, the sets $\{z|z \succeq y\}$ and $\{z|z \preceq y\}$ are closed, where $\succeq$ is the revealed preference relation: $f \succeq g \Leftrightarrow f \in C(f, g)$.

Assume that Alice is going to choose lottery y in A, when she finds out that there is some probability that event E occurs, and in that case she will have to take lottery z. Axiom A3 (independence) requires Alice to choose the mixture of y and z in the new choice problem (the mixture of A and z). That is, to choose lottery y if E does not occur. Observe that violating independence is time-inconsistent.

Most models that generalize expected utility (such as, Machina, [27]) and subjective expected utility (such as, [14,15,26,35]), choose to weaken the independence axiom (and keep WARP). Some support for the independence axiom is found in Raffia's ([32]) results: most people that violate the independence axiom in Ellsberg's paradox, change their choices when presented with an analysis that shows that their original choices counter the independence axiom. Luce and von Winterfeldt ([25]) discuss the experimental violations of the independence axiom in the literature, and show that they are mostly caused by the violation of the assumption of reduction of compound lotteries to normal form, which is implicitly assumed in the existing models (mentioned above) that weaken the independence axiom.

The weak axiom of revealed preferences (WARP) requires that if both lotteries y, z are in $A \cap B$, $x \in C(A)$ and $y \in C(B)$, then $x \in C(B)$. Axioms A1-A3 and WARP imply expected utility representation ([43]): There exists a unique affine (vN-M) utility function u, such that the chosen lotteries are the best according to u. That is, for every closed set $A \subseteq L$ and every lottery $y \in A$:

$$y \in C(A) \Leftrightarrow u(y) \geq u(z) \;\; \forall z \in A$$

In our axiomatization we replace WARP with the weaker CARNI. With an eye to this relaxation we formulate WARP slightly differently:

WARP *(Weak Axiom of Revealed Preference)* - Let $y \in A \subseteq Y$. If there exists $z \in C(A)$ and $B \subseteq Y$ such that $y \in C(B)$ and $z \in B$, then $y \in C(A)$.

WARP is appropriate when the psychological preferences of the DM are complete. In such cases, $y \in C(B)$ and $z \in B$ imply that y is revealed to be weakly-superior to z (i.e., y is as good as z). Thus if z is chosen in A so does y. When the psychological preferences are incomplete, there is a difference between something being superior and it being non-inferior for a DM. Observe that given the independence axiom, WARP is also equivalent to the following convex formulation:

WARP *(equivalent convex formulation)* - Let $y \in A \subseteq Y$. If there exists $z \in conv(C(A))$ and $B \subseteq Y$ such that $y \in C(B)$ and $z \in conv(B)$, then $y \in C(A)$.

Eliaz and Ok ([10]) propose the following axiom:

WARNI (Weak Axiom of Revealed non-inferiority) - Let $y \in A \subseteq Y$. If for every $z \in C(A)$ there exists a set $B \subseteq Y$ with $y \in C(B)$ and $z \in B$, then $y \in C(A)$.

When the psychological preferences are incomplete, $y \in C(B)$ and $z \in B$ only imply that y is revealed non-inferior to z, but it is not necessary that y is weakly-superior to z. WARNI requires that if y is revealed non-inferior to all the chosen alternatives in A, then it must be chosen from A as well. As discussed in the introduction, in some choice situations, it seems more appropriate to require the convex variation of WARNI, where a chosen act has to be non-inferior to all acts in the convex hull of A. This requirement is captured by CARNI, which requires that if y is revealed non-inferior to all the alternatives in $conv(C(A))$, then it must be chosen in A as well. CARNI is especially appealing when the DM follows the independence axiom, and the justification completes her psychological preferences into a linear ordering.

2.1.3 Representation Theorem

Replacing WARP with CARNI yields the following representation.

Theorem 1 *Let C be a choice correspondence over Y. The following are equivalent:*

(1) C satisfies axioms A1-A4 (non-triviality, continuity, independence and CARNI).

(2) There exists a convex and closed set U of affine (vN-M) utility functions, such that for every closed set $A \subseteq L$ and every lottery $y \in A$:

$$y \in C(A) \Leftrightarrow \exists u \in U,\ u(y) \geq u(z)\ \ \forall z \in A$$

That is, a lottery is chosen if and only if it is the best with respect to one of the utilities in U. Moreover:

(a) U is unique up to linear transformations . That is, if both U and V are convex and closed sets that represent the same choice correspondence then $\forall u \in U,\ \exists v \in V$ such that $u = a \cdot v + b$ where $a > 0$ and $b \in R$.

(b) There are two outcomes $\underline{x}, \overline{x} \in X$ such that $\forall u \in U,\ u(\underline{x}) < u(\overline{x})$.

2.2 *Belief-justifications*

2.2.1 *Preliminaries*

In this model we follow the framework of Anscombe-Aumann ([2], as reformulated in Fishburn, [12]). Similar to the first model, X is a finite set of outcomes and $Y = \triangle(X)$ is the set of lotteries. Let S be a finite set of states of nature, and, abusing notation, let $S = |S|$. Let $L = Y^S$ be the set of all functions from states of nature to lotteries. Such functions are referred to as acts. Endow this set with the product topology, where the topology on Y is the relative topology inherited from $[0,1]^X$. Abusing notation, for an act $f \in L$ and a state $s \in S$, we denote by $f(s)$ the constant (unambiguous) act that assigns the lottery $f(s)$ to every state of nature. Similarly for a set $A \subseteq L$ and a state $s \in S$, let $A(s)$ denote the act-wise set of constant acts: $A(s) = \{f(s)\,|f \in A\}$.

Mixtures (convex combinations) of acts are performed point-wise. In particular if $f, g \in L$ and $\alpha \in [0,1]$, then $(\alpha f + (1-\alpha)g)(s) = \alpha f(s) + (1-\alpha)g(s)$ for every $s \in S$. Similarly, Let $(\alpha f + (1-\alpha)A)$ denote the set where each $g \in A$ is replaced by $\alpha f + (1-\alpha)g$: $(\alpha f + (1-\alpha)A) = \{\alpha f + (1-\alpha)g | g \in A\}$. As in the former model, the primitive is a choice correspondence C over L. The domain of C is all the non-empty closed sets in L. For each such set $A \subseteq L$, $C(A)$ is a non-empty subset of A.

2.2.2 *Axioms*

The following six axioms are imposed on the choice correspondence:

B0 *Monotonicity.* Let $f \in A \subseteq L$. $\forall s \in S$, $\{f(s)\} = C(A(s)) \Rightarrow \{f\} = C(A)$.
B1 *Non-triviality.* There is an act $f \in A \subseteq L$ such that $f \notin C(A)$.
B2 *Continuity.* For any act $f \in L$, the set $\{g \in L | g \in C(f,g)\}$ is closed, and the set $\{g \in L | \{g\} = C(\{f,g\})\}$ is open.
B3 *Independence.* Let $f \in A \subseteq L$, $h \in L$ and $\alpha \in (0,1)$. $f \in C(A) \iff \alpha h + (1-\alpha) f \in C(\alpha h + (1-\alpha) A)$.
B4 *Convex Axiom of Revealed Non-Inferiority (CARNI).* Let $f \in A \subseteq L$. If for every $g \in conv(C(A))$ there exists a set $B \subseteq L$ with $f \in C(B)$ and $g \in Conv(B)$, then $f \in C(A)$.
B5 *Weak Axiom of Revealed Unambiguous Preferences* (*WARUP*). Let $A, B \subseteq Y$ and $y, z \in A \cap B$. $y \in C(A)$ and $z \in C(B)$ implies that $y \in C(B)$.

Axioms B0-B3 are all standard. We say that an act f dominates the set A if for every state of nature $s \in S$ $\{f(s)\} = C(A(s))$. That is, for every state of nature s, if the DM knows s, she would uniquely choose act f. Axiom B0 (monotonicity) requires that if f dominates the set A then it is uniquely chosen in A. Axioms B1-B3 are analog to axioms A1-A3, which have been discussed in the first model.

Axioms B0-B3 and WARP imply the subjective expected utility representation ([34], [2]): There exists a unique affine (vN-M) utility function u, and a unique probability distribution (prior) p over the states of nature, such that for every closed set $A \subseteq L$ and every act $f \in A$:

$$f \in C(A) \Longleftrightarrow E_p(u(f)) \geq E_p(u(g)) \;\; \forall g \in A$$

That is f is the best act according to the prior p (and the utility u).

In our axiomatization we replace WARP with two weaker axioms: CARNI (B4) and WARUP (B5). CARNI has been discussed in the first model. WARUP requires WARP to be satisfied only on unambiguous (constant) acts. That is, if y, z are two acts in the intersection of two sets, where both sets only include constant acts, then if y is chosen in the first set and z is chosen in the second set, then y must be chosen also in the second set.

2.2.3 Representation Theorem

Replacing WARP with CARNI and WARUP yields the following Theorem :

Theorem 2 *Let C be a choice correspondence over L. The following are equivalent:*

(1) C satisfies axioms B0-B5.
(2) There exists a unique non-degenerate affine (vN-M) utility function u, and a unique set $P \subseteq \triangle(S)$ of probability distributions (priors) over S, such that for every closed set $A \subseteq L$ and every act $f \in A$:

$$f \in C(A) \Leftrightarrow \exists p \in P, \text{ s.t. } E_p(u(f)) \geq E_p(u(g)) \;\; \forall g \in A$$

That is, an act is chosen if and only it is best according to one of the priors in P (and the utility u).

3 Discussion

3.1 Primitive of the Model and Binariness

In most existing literature, the primitive of the model of rational choice under uncertainty is a preference order ([2,6,15,16,21,26,35]). This implicitly assumes that the choice correspondence of a rational DM is induced from a binary preference relation. That is, $f \in C(A) \Leftrightarrow \forall g \in A \; f \succeq g$, where $\succeq$ is the revealed preference: $f \succeq g \Leftrightarrow f \in C(\{f,g\})$. In this paper we demonstrate why this assumption may be too strong in situations when the psychological preferences of the DM are incomplete, and why the primitive of the model should be a non-binary choice correspondence, in which the choices of the DM over the couples in a set A do not determine her choice in A.

Examples for other models with non-binary choice in the literature include the social choice models of Batra and Pattanaik ([4]) and Deb ([8]) and Nehring [30]'s model for preference relation between an act and a set of acts. The choice correspondence in our models has a *global* binariness property that is not shared by the existing models mentioned above: the choices of the DM over all the couples in the global set L (or at-least over all the couples in $conv(A)$) determine her choices in A.

3.2 Properties of CARNI

Luce and Raiffa ([24, 13.3]) present a list of 9 reasonable axioms for a rational choice correspondence under uncertainty. Satisfying all of them is equivalent to the subjective expected utility model ([34]). Our second model satisfies all of these axioms except the convexity of the chosen acts: if both acts f and g are chosen in A, and $\alpha f + (1-\alpha) g$ is an element of A, then $\alpha f + (1-\alpha) g$ is chosen in A. The following example demonstrates why this violation is plausible. Let $|S| = 2$, and $f, g, h \in L$ three acts with the following vN-M utilities: $u(f) = (1,0)$, $u(g) = (0,1)$ and $u(h) = (0.6, 0.6)$. Assume that the DM considers all priors to be possible. Let $A = \{f, g, h, 0.5f + 0.5g\}$. It is plausible that both $f, g \in C(A)$ as the DM believes that the probability of either state of nature may be high, and there are justifications to choose both acts. However, it is not rational to choose $0.5f + 0.5g$ because it has utility $(0.5, 0.5)$, which is strictly dominated by h.

In a dynamic environment in which the DM faces at each stage a new choice problem, violating the weak axiom (by following CARNI) may make the DM vulnerable to *money-pumps*. This can be avoided if the choice from the choosable alternatives at each stage are based on a *status-quo justification*: The DM is triggered to evaluate alternatives according to utilities (or priors) that are consistent with his past choices. This kind of behavior has strong empirical support in the psychological literature. A closely related formal model (for belief justification) is found in Bewley ([6]).

In some choice situations the DM can always base her choice on a random device. This can be modeled by defining the choice correspondence C only over convex (non-empty and closed) sets. That is, when the DM supposedly faces a choice in A, her ability to use a random device enlarges her set of alternatives to $conv(A)$. In such a setup, CARNI is equivalent to WARNI, and all of our results remain the same.

CARNI can be decomposed into four independent axioms. That is, a choice correspondence satisfies CARNI if and only if it satisfy the following 4 axioms:

E1 Contraction (Sen's property α) - $A \subseteq B$, $f \in C(B) \Rightarrow f \in C(A)$.
E2 Irrelevant acts invariant (Aizerman's property) - $A \subseteq B$, $C(B) \subseteq A \Rightarrow C(A) \subseteq C(B)$.
E3 Convex Expansion - $\cup A_n$ is convex, $f \in \cap A_n$, and $\forall n$ $f \in C(A_n) \Rightarrow f \in C(\cup A_n)$.
E4 Invariance to mixtures - $f \in C(A) \Rightarrow f \in C(conv(A))$.

Axiom E1 requires that if an alternative is chosen in some set, it must also be chosen also in a smaller subset. It implies that the two possible definitions of revealed preferences coincide: $f \succeq g \Leftrightarrow f \in C(\{f,g\}) \Leftrightarrow \exists A$, s.t. $g \in A$ and $f \in C(A)$. Axiom E2 requires that choice is invariant to the addition of irrelevant (=unchosen) alternatives in the following sense: if f is chosen in some set, and the choice set is extended such that all the new alternatives are not chosen, then f must be chosen in the larger set. Sen ([38]) showed that Axioms E1 + E2 imply that the revealed preference is quasi-transitive (that is: $f \succ g$, $g \succ h \Rightarrow f \succ h$). Aizerman and Malishevski ([1]) showed that when the grand set is finite, axioms E1+E2 are equivalent to the following representation: there is a set of orderings, such that an alternative is chosen if and only if it maximizes one of the orderings.

The standard expansion axiom requires that if an alternative is chosen in some sets, then it must also be chosen in the union of these sets:

E3' Standard expansion - $f \in \cap A_n$, $\forall n$ $f \in C(A_n) \Rightarrow f \in C(\cup A_n)$

Axiom E3 limits this requirement only for convex sets.Aizerman and Malishevski showed (when the grand set is finite) that a choice correspondence satisfies axioms E1+E2+E3' if and only if it is rationalized by a quasi-transitive relation. Eliaz and Ok had presented an axiom, WARNI, which is equivalent to these three axioms.

Axiom E4 requires that choice is invariant to the addition of mixtures. That is, if an alternative is chosen in some set, then it must be chosen when a mixture of existing alternatives is added to the choice set. Seidenfeld, Schervish and Kadane ([37]) have recently presented an axiomatic model for choice under uncertainty, where they require axioms E1, E2 and E4 (but not E3), in addition to some standard axioms (non-triviality, continuity, independence, monotonicity and domination), and get a representation where the set of justifications (pairs of state dependent utilities/priors in [37]) is non-convex. The addition of axiom E3 implies the convexity of the set of justifications.

3.4 Attitude to Uncertainty

Consider the following example: $|S| = 2$, $X = \{\underline{x}, \overline{x}\}$, $\overline{x} = C(\underline{x}, \overline{x})$, $f = (0.5\underline{x} + 0.5\overline{x}, 0.5\underline{x} + 0.5\overline{x})$ and $g = (\underline{x}, \overline{x})$. Act f gives unambiguous probability 0.5 to get the better outcome $\overline{x}$, while f gives $\overline{x}$ with the ambiguous probability that state 2 occurs. Assume that P, the set of possible priors, includes $(0.5, 0.5)$.

Gilboa and Schmeidler's model ([15]) predicts that people would strictly prefer f over g, i.e., people are uncertainty averse, as experimentally observed in Ellsberg's paradox. Our model of belief-justifications predicts that both acts are choosable, and that the attitude to uncertainty depends on the relevant justification. An experimental support for this prediction is found in Heath and Tversky ([17]), where it is shown that people may be uncertainty averse or uncertainty-seekers, and that it depends on payoff-irrelevant observable information. Specifically, people prefer ambiguous events over equiprobable chance events when they consider themselves knowledgeable in the area that is the source of the uncertainty, and they prefer chance events when they consider themselves ignorant or uninformed.

3.5 Relation with Equilibrium Notions in the Learning Literature

Conjectural equilibrium (Battigalli, 1987) is a solution concept that extends Nash equilibrium in the learning literature.[11] In such an equilibrium each player receives some partial information regarding the action profile played by the other players (such as information how similar players have played in similar games in the past). This provides a player with a set of possible strategy profiles that the others might play (the profiles that are consistent with his partial information). In equilibrium each player plays an action that is the best response to one of the possible strategy profiles (and the chosen action profile is consistent with the information of the players).

The behavior of the players in such equilibria is usually modeled by subjective expected utility. However, it can also be modeled by belief-justifications: The state of the world captures uncertainty about both nature and the actions of the other players. Each player has a set of possible priors, and she chooses an action that is optimal given one of the priors in this set.[12] This alternative modeling may sometimes have a more appealing interpretation. For example,

[11] Closely related solution concepts are Fudenberg and Levine's [13] self-confirming equilibrium and Kalai and Lehrer's [19] subjective equilibrium.

[12] In our model we assume that the set of priors is convex. This assumptions holds, in most applications of conjectural equilibrium, where each player knows the distribution of some part of the action profile, or more generally, each player knows the expected value of some random variables that depend on the action profile.

consider the case where all players share symmetric information about the action profile. Subjective expected utility model requires that each player would have a different prior, and that players are not aware about the priors of the other players. Contrary to that, belief-justification model allows the players to share a common set of possible priors (and to use private payoff-irrelevant information to choose a specific anchoring prior).

3.6 Related Literature

In our models, choices are derived from multiple justifications (rationales) with the following properties: (1) Each justification is represented by an ordering. (2) The chosen acts are best with respect to one of the justifications. (3) Each justification may be used in all choice problems. Some related models for choice with multiple justifications are:

- Kalai, Rubinstein and Spiegler ([20]) - The DM has several justifications, and each of them is used in a disjoint subset of choice problems.
- Manzini and Mariotti ([29]) - The DM has several justifications that are used sequentially in a fixed order. Each justification is represented by an incomplete preference relation.
- Rubinstein and Salant ([33]) - The DM has a set of justifications, and she uses one of the justifications according to the framing of the choice problem (such as the order in which the acts are presented).
- Cherpanov, Feddersen and Sandroni ([7]) - The DM has several justifications, but only one preference relation. The chosen act is the most preferred among all the justifiable acts.

Unlike these models, we work with a more structured framework and this allows us to impose more structure on the justifications: the set of justifications is convex and closed, and each justification is a linear ordering.

4 Proofs

4.1 Taste-Justifications

In this subsection we prove Theorem 1. We have to show that axioms A1-A4 (non-triviality, continuity, independence and CARNI) are sufficient for the representation of taste-justifications. The other direction is immediate. Let $\succ$ denote the *revealed strict preference relation* that is induced from C: $y \succ z \Leftrightarrow \{y\} = C(\{y,z\})$ $(y \neq z)$. The following lemma shows the global (convex) binariness property of C.

Lemma 3 *Let C be a choice correspondence that satisfies CARNI. For each*

$y \in A \subseteq Y$, $y \in C(A) \Leftrightarrow \neg\exists z \in conv(A)\, s.t.\, z \succ y$

PROOF. "$\Rightarrow$ part": Let $y \in C(A)$ and $z \in conv(A)$. Assume to the contrary that $z \succ y$ s$\Rightarrow$ $\{z\} = C(\{y,z\})$. This implies (by CARNI) that for every $B \subseteq Y$ with $z \in conv(B)$, $y \notin C(B)$. Specifically $y \notin C(A)$ (a contradiction). "$\Leftarrow$ part": Let $y \in A \backslash C(A)$. This implies (by CARNI) that there is $z \in conv(C(A)) \subseteq conv(A)$ such that for every $B \subseteq Y$ with $z \in conv(B)$, $y \notin C(B)$. Specifically, $y \notin C(\{y,z\}) \Rightarrow z \succ y$.

The following lemma shows that $\succ$ satisfies transitivity, non-triviality, continuity and independence.

Lemma 4 *Let C be a choice correspondence that satisfies axioms A1-A4, and let $\succ$ be the revealed strict preference. Then $\succ$ satisfies the following properties:*

C1 *Non-triviality* - There are $y, z \in Y$ such that $y \succ z$.
C2 *Continuity* - For each $y \in Y$ the sets $\{z | z \succ y\}$ and $\{z | z \prec y\}$ are open.
C3 *Independence - For any $y, z, w \in Y$ and any $\alpha \in (0,1)$: $y \succ z$ implies $\alpha w + (1-\alpha) y \succ \alpha w + (1-\alpha) z$*
C4 *Transitivity - For any $y, z, w \in Y$: $y \succ z$ and $z \succ w$ implies that $y \succ w$.*

PROOF. Axioms C1-C3 are immediately implied from the analog properties of C (A1-A3) and from lemma 3. C4 (transitivity) is proven as follows. Let $y \succ z$ *and* $z \succ w$. By lemma 3 $w, z \notin C(\{y,z,w\})$. This implies $\{y\} = C(\{y,z,w\})$. Assume to the contrary that $w \in C(\{w,y\})$. CARNI implies that $w \in C(\{y,z,w\})$ and we get a contradiction.

The following proposition shows that $\succ$ has a unique multiple utility representation. The proposition is a direct adaptation of the results of Dubra et al. for weak preference relation ($\succeq$). The proof, which is very similar to [9, Sect. 4], is omitted.

Proposition 5 *Let $\succ$ be a strict binary relation over Y. The following are equivalent:*

(1) $\succ$ satisfies axioms C1-C4 (transitivity, non-triviality, continuity and independence).
(2) There exists a convex and closed set U of affine (vN-M) utility functions, such that for every two lotteries $y, z \in Y$:

$$y \succ z \Longleftrightarrow \forall u \in U,\ u(y) > u(z)$$

Moreover:
(a) U is unique up to linear transformations. That is if both U and V are convex and closed sets that represent the same choice correspondence then $\forall u \in U,\ \exists v \in V$ such that $u = a \cdot v + b$ where $a > 0$ and $b \in R$.

(b) There are two outcomes $\underline{x}, \overline{x} \in X$ such that $\forall u \in U$, $u(\underline{x}) < u(\overline{x})$

We use Prop. 5 to finish the proof of Theorem 1, by showing that axioms A1-A4 are sufficient for the taste-justification representation. Let C be a choice correspondence that satisfies these axioms, and let $\succ$ be the revealed strict preference. Let U be the unique (up to linear transformations) convex and closed set of utilities of Prop. 5. We have to show for each $y \in A \subseteq Y$, $y \in C(A) \Leftrightarrow \exists u \in U$, s.t. $u(y) \geq u(z)\ \ \forall z \in A$. This is done as follows:

$$
\begin{aligned}
y \in C(A) &\Longleftrightarrow \neg\exists z \in conv(A)\, s.t.\, z \succ y && (1)\\
&\Longleftrightarrow \forall z \in conv(A)\ \exists u \in U \,\text{such that}\, u(y) \geq u(z) && (2)\\
&\Longleftrightarrow \min_{z \in conv(A)} \max_{u \in U} (u(y) - u(z)) \geq 0 \\
&\Longleftrightarrow \max_{u \in U} \min_{z \in conv(A)} (u(y) - u(z)) \geq 0 && (3)\\
&\Longleftrightarrow \exists u \in U \,\text{such that}\, \forall z \in conv(A),\ u(y) \geq u(z) && (4)\\
&\Longleftrightarrow \exists u \in U \,\text{such that}\, \forall z \in A,\ u(y) \geq u(z)
\end{aligned}
$$

Where (6) is implied by lemma 3, (7) is due to Prop. 5, (8) is implied by a by Sion's Minimax Theorem ([40]), and (4) is implied by the linearity of u.

4.2 Belief-Justifications

In this subsection we prove Theorem 2. We have to show that axioms B0-B5 are sufficient for the representation of belief-justifications. The other direction is immediate. Let $\succ$ denote the revealed strict preference that is induced from C. Observe that lemma 3 is valid in this framework as well, and thus CARNI implies that $\forall f \in A \subseteq L$:

$$f \in C(A) \Leftrightarrow \neg\exists g \in conv(A)\, s.t.\, g \succ f \tag{5}$$

The following proposition shows that $\succ$ satisfies monotonicity, non-triviality, continuity, independence, transitivity and complete-transitivity over unambiguous acts .

Lemma 6 *Let C be a choice correspondence that satisfies axioms B0-B5, and let $\succ$ be the revealed strict preferences. Then $\succ$ satisfies the following properties:*

D0 *Monotonicity.* Let $f, g \in L$. $\forall s \in S\ f(s) \succ g(s)$ implies $f \succ g$.
D1 *Non-triviality.* There are acts $f, g \in L$ s.t. $f \succ g$.
D2 *Continuity.* For any $f \in L$, the sets $\{g | g \succ f\}$ and $\{g | g \prec f\}$ are open.
D3 *Independence. Let $f, g \in L$. $f \succ g$* if and only if $\alpha h + (1-\alpha) f \succ \alpha h + (1-\alpha) g$ for every $h \in L$ and $\alpha \in [0,1]$.
D4 *Transitivity. $\forall f, g, h \in L$ $f \succ g$ and $g \succ h$ implies $f \succ h$.*

D5 *Complete-transitivity over unambiguous acts.* Let $y, z, w \in Y$. $\neg z \succ y$ and $\neg w \succ z$ implies that $\neg w \succ y$.

PROOF. Properties D0-D3 are immediately implied by the analog properties B0-B3. D4 (transitivity) is implied by CARNI (as in lemma 4). D5 is implied by B5 (WARUP) as follows: assume to the contrary that $\neg z \succ y$, $\neg w \succ z$ and $w \succ y$. This implies that $y \in C(\{y,z\})$ and $y \notin C(\{y,z,w\})$. If $\{w\} = C(\{y,z,w\})$ then we get a contradiction to CARNI ($z \in C(\{z,w\}) \Rightarrow z \in C(\{y,z,w\})$). Thus $z \in C(\{y,z,w\})$, and this implies by WARUP that $y \in C(\{y,z,w\})$ and we get a contradiction.

The following proposition shows that $\succ$ has a unique multiple-prior representation. The proposition is a direct adaptation of the result of Ok et al. for weak preference a la Bewely ([6]). The proof, which is very similar to [31, Sect. 6], is omitted.

Proposition 7 *Let $\succ$ be a strict binary relation over L. The following are equivalent:*

(1) $\succ$ satisfies axioms D0-D5.
(2) There exists a unique (up to linear transformations) non-degenerate vN-M utility u, and a unique convex and closed set P of priors over the state of nature, such that for every two acts $f, g \in L$:

$$f \succ g \Leftrightarrow \forall p \in P,\ E_p(u(f)) > E_p(u(g))$$

We use Prop. 7 to finish the proof of Theorem 2, by showing that axioms B0-B5 are sufficient for the belief-justification representation. Let C be a choice correspondence that satisfies these axioms, and let $\succ$ be the revealed strict preference. Let u be the unique (up to linear transformations) utility, and let P be the unique convex and closed set of priors of Prop. 7. We have to show for each $f \in A \subseteq L$, $f \in C(A) \Leftrightarrow \exists p \in P$, s.t. $E_p(u(f)) \geq E_p(u(g))\ \ \forall g \in A$. This is done as follows:

$$\begin{aligned}
f \in C(A) &\Longleftrightarrow \forall g \in conv(A)\ \neg g \succ f & (6)\\
&\Longleftrightarrow \forall g \in conv(A)\ \exists p \in P \,\text{such that}\, p \cdot u(f) \geq p \cdot u(g) & (7)\\
&\Longleftrightarrow \min_{g \in conv(A)} \max_{p \in P} (p \cdot u(f) - p \cdot u(g)) \geq 0 &\\
&\Longleftrightarrow \max_{p \in P} \min_{g \in conv(A)} (p \cdot u(f) - p \cdot u(g)) \geq 0 & (8)\\
&\Longleftrightarrow \exists p \in P \,\text{such that}\, \forall g \in conv(A),\ p \cdot u(f) \geq p \cdot u(g) & (9)\\
&\Longleftrightarrow \exists p \in P \,\text{such that}\, \forall g \in A,\ p \cdot u(f) \geq p \cdot u(g) &
\end{aligned}$$

Where (6) is implied by (5), (7) is due to Prop. 7, (8) is implied by a by Sion's Minimax Theorem ([40]), and (9) is implied by the linearity of u.

References

[1] Aizerman MA, Malishevski AV, 1981. General theory of best variants choice. IEEE Trans. Automat Control: 1030-1041.

[2] F. J. Anscombe, and R. J. Aumann, 1963. A definition of subjective probability, *Annals of Mathematical Statistics* 34, 199-295.

[3] Aumann R. J., Utility theory without the completeness axiom. *Econometrica* 30 (3), 445-462.

[4] Batra R.N., Pattanaik P.K., 1972. On some suggestions for having non-binary social choice functions. *Theory and Decision* 3, 1-11.

[5] Battigalli P., 1987. Comportanmento rezionale ed equilibrio nei glochi e nelle situazioni sociali, unpublished dissertation, Milano, Bocconi University.

[6] Bewley T.F., 2002. Knightian decision theory - part 1. *Decisions in economics and finance* 25, 79-110.

[7] Cherepanov V., Feddersen T., Sandroni A., 2008. Rationalization. mimeo.

[8] Deb R, 1983. Binariness and rational choice. *Mathematical Social Sciences* 5(1), 97-105.

[9] Dubra J., Maccheroni F., Ok E., 2003. Expected utility theory without the completeness axiom. *Journal of economic Theory* (115), 118-133.

[10] Eliaz, K., Ok, E.A., 2005. Indifference or indecisiveness? Choice-theoretic foundations of incomplete preferences. *Games Econ Behav* 56, 61-86.

[11] Ellsberg D., 1961. Risk, ambiguity and the Savage axioms. *Quarterly Journal of Economics* 75, 643-669.

[12] Fishburn, P. C. 1970, *Utility Theory for Decision Making.* Wiley, New-York.

[13] Fudenberg D., Levine D.K., 1993. Self-confirming equilibrium, *Econometrica* 61, 523-545.

[14] Ghirardato P., Maccheroni F., Marinacci M., 2004. Differentiating ambiguity and ambiguity attitude, *Journal of Economic Theory* 118, 133-173.

[15] Gilboa I., Schmeidler D., 1989. Maxmin expected utility with a non-unique prior, *Journal of Mathematical Economics* 18, 141-153.

[16] Gilboa I., Maccheroni F., Marinacci M., Schmeidler D., 2008. Objective and Subjective rationality in a multiple prior model, manuscript.

[17] Heath C, Tversky A, Preference and belief: ambiguity and competence in choice under uncertainty, Journal of Risk and Uncertainty, 1991.

[18] Inada K, 1970. Majority rule and rationality, *Journal of Economic Theory* 2, 27-40.

[19] Kalai E., Lehrer E., 1993. Subjective equilibrium in repeated games, Econometrica 61 (1993), 1231-1240.

[20] Kalai G., Rubinstein A., Spiegler R., 2002. Rationalizing choice functions by multiple rationales. Econometrica 70 (6), 2481-2488.

[21] Kilbanoff P., Marinacci M., Mukerji S., 2005. A smooth model of decision making under ambiguity, *Econometrica* 73, 1849-1892.

[22] Kim T., Richter M., 1985. Nontransitive nontotal consumer theory, *Journal of Economic Theory* 38, 324-363.

[23] Lehrer E., Teper R., 2009. Justifiable preferences. manuscript. http://www.math.tau.ac.il/~teperroe/Papers/just.pdf

[24] Luce R.D., Raiffa H., 1957. *Games and Decisions*. Wiley, New York.

[25] Luce R D, von Winterfeldt D, 1994. What common grounds exists for descriptive, perspective, and normative utility theories. Management Science, 40 (2), 263-279.

[26] Maccheroni F., Marinacci M., Rustchini A., 2006. Ambiguity aversion, robustness, and the variational representation of preferences, *Econometrica* 74, 1447-1498.

[27] Machina MJ, 1982. "Expected Utility" Analysis without the Independence Axiom. *Econometrica* 50(2), 277-323.

[28] Mandler M., 2005. Incomplete preferences and rational intransitivity of choice. *Games and Economic Behavior*, 255-277.

[29] Manzini P., Mariotti M., 2007. Sequantually rationalizable choice. *American Economic Review* 97 (5), 1824-1839.

[30] Nehring K, 1997. Rational choice and revealed preference without binariness. *Social Choice and Welfare* 14, 403-425.

[31] Ok EA, Ortoleva P, Riella G, 2008. Incomplete prefernces under uncertainty: indecisiveness in beliefs vs. tastes. mimeo.

[32] Raiffa H, 1961. Risk ambiguity, and the Savage axioms: comment. The Quarterly Journal of Economics. 75 (4), 690-694.

[33] Rubinstein A., Salant Y., 2008. Choice with frames. *Review of Economic Studies* 75, 1287-1296.

[34] Savage L.J., 1954. *The Foundations of Statistics*, Wiley, New-York.

[35] Schmeidler D., 1989. Subjective probability and expected utility without additivity, Econometrica 57, 571-587.

[36] Seidenfeld T, Schervish M, Kadane JB, 1995. A representation of partially ordered preferences. *The Annals of Statistics* 23 (6), 2168-2217.

[37] Seidenfeld T, Schervish M, Kadane JB, 2009. Coherent choice functions under uncertainty, to appear in *Synthese*.

[38] Sen A., 1971. Choice functions and revealed prefernces. *The Review of Economic Studies* 38 (3), 307-317.

[39] Shafir E, Simonson I, Tversky A, 1993. Reason-based choice. *Cognition* 49, 11-36.

[40] Sion M., 1958. On general Minimax theorems, Pac. J. Math. 8, 171-176

[41] Tversky, A, Kahneman, D, 1974. Judgments under uncertainty: Heuristics and biases. Science, 185, 1124-1131

[42] Tversky A, Kahneman D, 1981. The framing of decisions and the psychology of choice, Science, 211, 453-458.

[43] von Neumann J, Morgenstern O, 1944. Theory of Games and Economic Behavior, Princeton University Press, Princeton, 1944.

[44] Wright WF, Bower GH, 1992. Mood effects on subjective probability assessment. *Organizational Behavior and Human Decision Processes,* 52 (2), 276-291.

[45] Zeelenberg M, Beattie J, Pligt J, Vries NK, 1996. Consequences of regret aversion: effects of expected feedback on risky decision making. *Organzational behavior and human decision processes* 65(2), 148–158,

[46] Zeelenberg M, Pieters R, 2004. Consequences of regret aversion in real life: the case of the Dutch postcode lottery. *Organizational Behavior and Human Decision Processes* 93, 155–168.